Understanding Quantum Technologies

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Olivier Ezratty

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Olivier Ezratty advises and trains various public and industry organizations in the development of their innovation strategies in the quantum technologies realm. He brings them a rare 360° understanding of the scientific, technology, market and ecosystems dimensions of this burgeoning and complex domain.

He covered many other topics since 2005, like digital television, Internet of things and artificial intelligence. As such, he carried out various strategic advisory missions of conferences or training in different verticals and domains such as **media and telecoms** (Orange, Bouygues Telecom, TDF, Astra), **finance and insurance** (BPCE, Société Générale, Swiss Life, Crédit Agricole, Crédit Mutuel-CIC, Generali), **industry and services** (Schneider, Camfil, Vinci, NTN-STR, Econocom, ADP, Air France, Airbus) and the **public sector** (CEA, Météo France, Bpifrance, Business France).

He became a quantum technologies specialist in 2018 with many complementary activities:

- **Author** of the reference book **Understanding Quantum Technologies** (September 2021, 2022 and 2023) following three previous editions in French in 2018, 2019 and 2020. The 2021, 2022 and 2023 editions are also available in paperback version on Amazon.
- **Trainer and teacher** on quantum technologies for **Capgemini Institut** and for **CEA INSTN**. In September 2021, he took in charge an elective curriculum on quantum technologies for **EPITA**, an IT engineering school in France.
- **Speaker** in a large number of quantum technology events since 2018 such as the Q2B Paris organized by QC Ware, France Quantum and other events, on top of presentations at Société Générale, BNP, Crédit Agricole, Michelin, Adéo, L'Oréal, FIECC, IHEDN, Business France, CentraleSupelec, Avolta Partners, IHEDN, etc.
- **Producer** of two series of podcasts on quantum technologies along with Fanny Bouton (in French): a monthly « Quantum » on tech news (since September 2019) and Decode Quantum, with entrepreneurs and researchers since March 2020, with a total of over 100 episodes as of July 2023.
- **Cofounder** of the **Quantum Energy Initiative** with Alexia Auffèves (CNRS MajuLab Singapore), Robert Whitney (CNRS LPMMC) and Janine Splettstoesser (Chalmers University, Sweden).
- **Expert** for **Bpifrance** to evaluate quantum collaborative projects and startups.
- **Ambassador** for France 2030 since February 2022, the French government innovation strategy plan.

He also lectures in various universities such as CentraleSupelec, Ecole des Mines de Paris, Telecom Paristech, Les Gobelins, HEC, Neoma Rouen and SciencePo, on artificial intelligence, entrepreneurship and product management (until 2020) and on quantum technologies (since 2018), in French and English as needed. He is also the author of many open source ebooks in French on entrepreneurship (2006-2019), the CES of Las Vegas yearly report (2006-2020) and on artificial intelligence (2016-2021).

Olivier Ezratty started in 1985 at **Sogitec,** a subsidiary of the Dassault group, where he was successively Software Engineer, then Head of the Research Department in the Communication Division. He initialized developments under Windows 1.0 in the field of editorial computing as well as on SGML, the ancestor of HTML and XML. Joining **Microsoft France** in 1990, he gained a strong experience in many areas of the marketing mix: products, channels, markets and communication. He launched in France the first version of Visual Basic in 1991 and Windows NT in 1993. In 1998, he became Marketing and Communication Director of Microsoft France and in 2001, of the Developer Division, which he created in France to launch the .NET platform and promote it to developers, higher education and research, as well as to startups. Olivier Ezratty is a software engineer from **Centrale Paris** (1985), which became CentraleSupelec in 2015.

Credits

Cover illustration: personal creation associating a Bloch sphere describing a qubit and the symbol of peace (my creation, first published in 2018) above a long list of over 400 scientists and entrepreneurs who are mentioned in the ebook.

This document contains nearly 1,000 illustrations. I have managed to give credits to their creators as much as possible. Most sources are credited in footnotes or in the text. Only scientists' portraits are not credited since it is quite hard to track it. I have added my own credit in most of the illustrations I have created. In some cases, I have redrawn some third-party illustrations to create clean vector versions or used existing third-party illustrations and added my own text comments. The originals are still credited in that case.

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Content

This is the **third volume** of the book "Understanding Quantum Computing", that is also downloadable from https://www.oezratty.net/wordpress/2023/understanding-quantum-technologies-2023/. The downloadable PDFs are available in a single volume A4 and Letter version, containing the three volumes in sequential order. You can also download compressed PDFs for the three volumes in A4 and Letter formats. Their size fits into the constraints of ebook readers like the Kindle from Amazon.

This book printed version separates volume 1, volume 2 and volume 3.

The **first volume** contains an history of quantum physics, some quantum physics 101 and everything about quantum computing basics and engineering.

This **second volume** contains the parts dedicated to quantum computing hardware, enabling technologies, unconventional computing solutions which are potential alternate routes between classical and quantum computing, quantum telecommunications, quantum cryptography, post-quantum cryptography and quantum sensing.

The **third volume** contains the parts dedicated to algorithms, software tools, case studies, an inventory of quantum investments per country, various societal topics, corporate adoption methodologies and quantum fake sciences, glossary, an index and bibliography.

The three-volume index and glossary are consolidated at the end of the third volume.

The book is split into three volumes to make its printing easier, some online printing services including Amazon being limited to a maximum of 600 pages. Here, we have three volumes of respectively 322, 530 and 532 pages, covers included.

You can order the printed version of this book in three volumes on all **Amazon** sites with searching for the book title, edition 2023.

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Quantum algorithms

It is now time to put aside quantum hardware that was the main topic of the previous parts of this book and to turn to quantum algorithms and software!

Gate-based quantum computers use quantum algorithms, some of which being theoretically much more efficient than their equivalents designed for classical computers. There are not that many algorithms and their relative performance compared to classical algorithms is not always obvious to prove. It is even sometimes contested. The assertion "*quantum computers are faster than classical computers*" is therefore debatable and must be discussed and analyzed on a case-by-case basis.

Richard Feynman described the idea of creating quantum simulators in 1982[3151]. His idea was to create devices using the effects of quantum mechanics to simulate them, which would be almost impossible with traditional computers. This corresponds today to so-called quantum simulators, a specific breed of analog quantum computers. But we're dealing here mostly with gate-based quantum computing, based on **Yuri Manin** and **Paul Benioff** ideas from 1979-1981 and later refined by **David Deutsch** between 1985 and 1992.

Mathematicians have been working since the mid- and late 1980s on creating algorithms for quantum computers and simulators, long before any quantum hardware was available.

The first quantum algorithms were published in the early 1990s, while the first two-qubit quantum systems appeared around 2000/2002. Researchers have been regularly creating new algorithms for the past 25 years, regardless of the relatively slow progress with hardware (Figure 727). Launched in 2011, the Quantum Algorithm Zoo identifies 65 algorithms in the scientific literature and 435 related scientific references as of October 2022 when it was last updated, organized in 4 algorithms groups (algebraic and number theory, oracle-based problems, approximation and simulations, optimization - numerics and machine learning). The list is maintained by Stephen Jordan, a Microsoft Quantum researcher. This is still a modest number compared to the thousands of non-quantum algorithms[3152]. Even though most classical computing developers don't know and use many algorithms in practice!

Quantum algorithms creation is thus a parallel research path with hardware progress. This is not the first time in history. The emblematic **Ada Lovelace** did formalize the first algorithms and lines of code to run on **Charles Babbage**'s machine, which only saw the light of day in 2002 in London, 153 years after its conception (video) (see the sample program in Figure 728). In 1842/1843, she had annotated a translation of her own of a paper by the Italian **Luigi Federico Menabrea** describing Babbage's machine. It took 102 years for the first electronic computers to see the light of day at the end of World War II, the ENIAC! A beautiful game... of patience!

It is also reminiscent of **Leonardo da Vinci**'s helicopter designs dating from 1487-1490. A first human-powered helicopter created by the University of Toronto flew in 2013, AeroVelo (video) followed by another fairly close specimen from the University of Maryland flying in 2018 (video)! So, more than five centuries apart! And even taking into account the flight of the first motorized helicopter in 1907, the time lag is still over four centuries. This same University of Maryland is one of the most advanced in the world in quantum computers based on trapped ions!

[3151] See Simulating Physics with Computers, Richard Feynman, 1982 (22 pages).

[3152] For an extensive coverage of the key gate-based quantum algorithms, see Lecture Notes on Quantum Algorithms by Andrew M. Childs, April 2021 (181 pages) and Quantum Computing Lecture Notes by Ronald de Wolf, 2021 (184 pages).

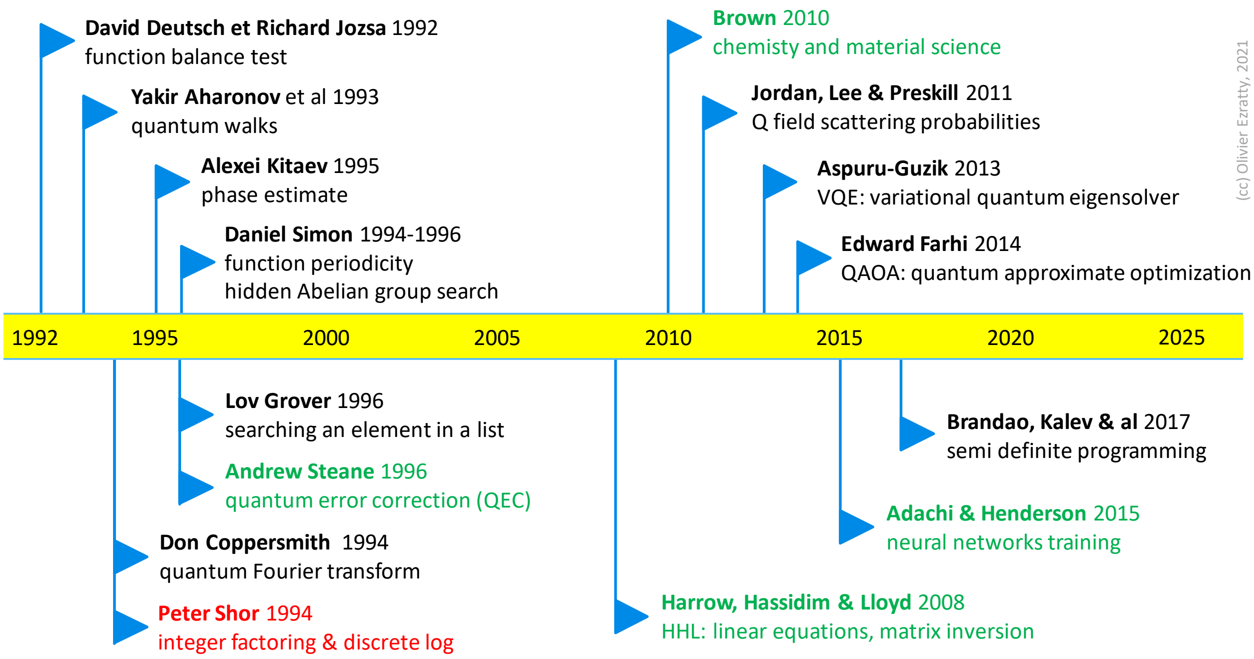


Figure 727: a quantum computing algorithms creation timeline. It is a three-decade story. (cc) Olivier Ezratty, 2021.

After the war, history repeated itself in part for much of the work in the vast field of artificial intelligence, where researchers were also working on algorithms, especially neural network-based algorithms, before any computers could execute them properly on a useful scale such as for objects recognition in images. The first computers running perceptrons, the ancestors of today's artificial neural networks, were rudimentary. The rise of deep learning since 2012 is partly linked to the power of machines and GPUs able to train such neural networks. Hardware has once again joined algorithms that were ahead of their time.

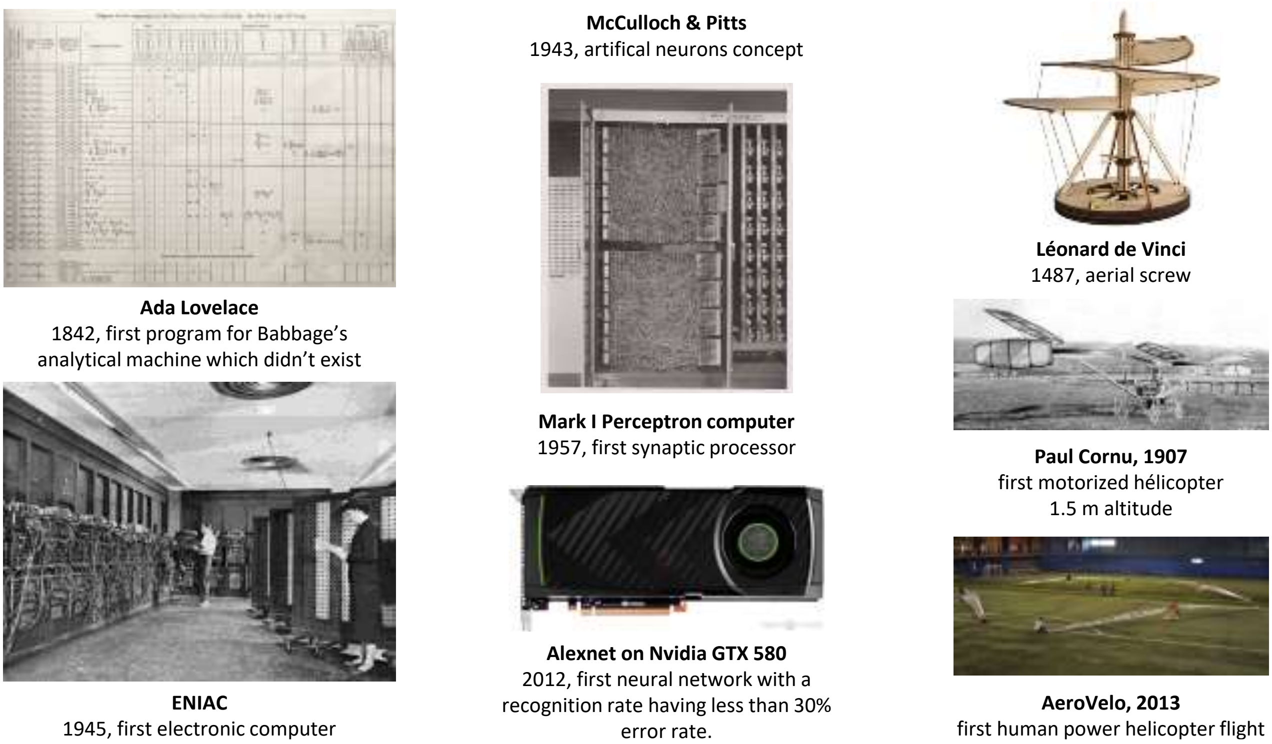


Figure 728: a perspective on the time gap between algorithms creation and their underlying hardware. One century between Ada Lovelace's Bernoulli equations programming and the advent of the first electronic computer. And 6 decades to implement neural networks practically. Same for helicopters in another domain! (cc) Olivier Ezratty with various image sources. 2020.

Even today, many of the quantum algorithms that are invented are not yet executable on a large problem scale on current quantum computers or on classical computing quantum emulators. There are not enough quality qubits available to be of any use and, more importantly, to be more powerful in any dimension than classical computers. Supercomputers can emulate about 50 qubits in state vector mode, but no operational quantum computer can reach this number with error corrected qubits.

In another analogy with the History of Computer Science, we are still programming quantum computers with rather low layers of machine language, a bit like machine language or macro-assembler used 30 to 50 years ago, or more recently, for those who program low-level embedded systems or peripheral drivers. Today's quantum algorithms are mid- to low-level logical chunks of quantum code. Their assembly is not even yet done in practice.

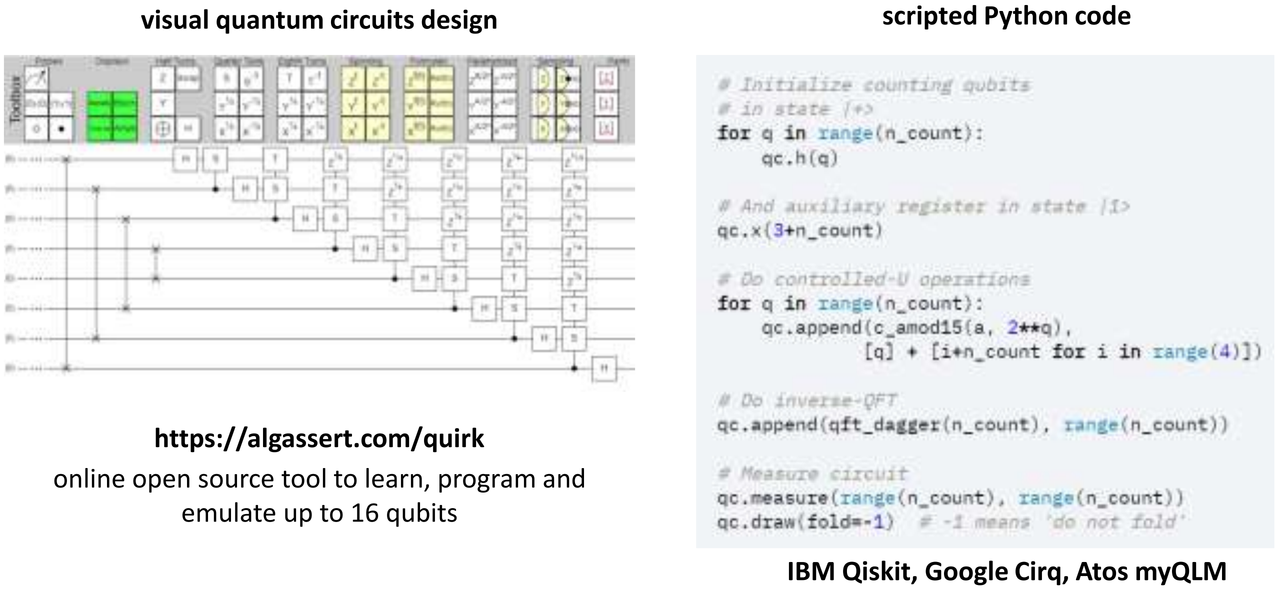


Figure 729: gate-based programming can be done graphically with tools like Quirk, mostly for learning purpose and also, to visualize interactively qubits values (Bloch sphere, vector state, density matrix) in emulation mode. Scripted code with Python is used for professional programming. (cc) Olivier Ezratty, 2022.

The creation of quantum algorithms requires a capacity for abstraction that is beyond that of classical algorithms and programs, even taking into accounts classical object-based or events-based programming. We'll have to groom a new generation of mathematicians and developers capable of reasoning with the mathematical formalism of quantum programming as quantum computers mature. They will have to be able to conceptualize algorithms that are not easy to mentalize, even when using graphical tools like Quirk and the likes (Figure 729, *left*). Most of the time, though, quantum algorithms won't be simple language translation from classical programming language. They are also bound to solve problems that classical computers and classical programming languages can't solve efficiently.

There is currently an interesting difference between classical computing and gate-based quantum software development. While classical computing can use various levels of abstraction for both coding and data structures, quantum computing currently uses only one coding abstraction level, gate-based circuits, as shown in Figure 730. From the highest coding level using frameworks down to gate-based code generated by compilers and transpilers, qubits and qubit gates are always used, while in classical computing, nearly nobody cares about bits[3153]. Quantum software tools can get rid of qubit formalism only with high level libraries and tools that are fed with user data. One day, the abstraction level of quantum programming may rise to a point where it is no longer necessary to understand the low-level intricacies of quantum gates, Hilbert spaces, Hamiltonians and quantum interferences. But this is just a conjecture!

[3153] See Neither Software Developers nor Users Need to Know about Bits by Jack Krupansky, March 2023.

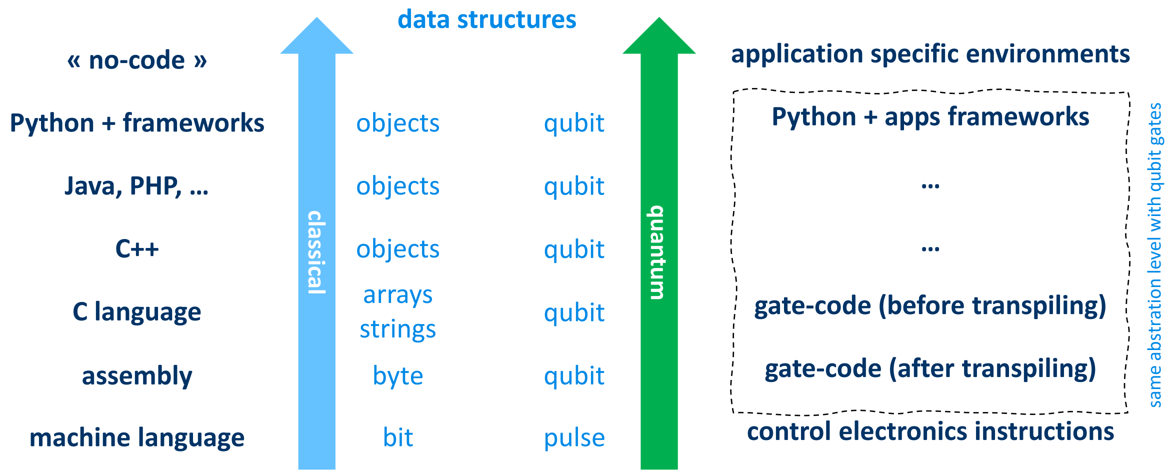


Figure 730: quantum languages and data abstraction levels. (cc) Olivier Ezratty, 2023.

One could wonder how to write the first "**Hello world**" program using a quantum computer. One example can be found in Microsoft's C# code samples[3154] but the "Hello world" part is entirely classical! The real "Hello World!" template of quantum programming is to create your first Bell state with coupling a H and CNOT gate and generate a two qubit entangle state $(|00\rangle + |11\rangle)/\sqrt{2}$.

Today's classical quantum algorithms use quantum gates. But there are other quantum computing paradigms:

- **Quantum annealing problem solving** such as those for D-Wave machines which are based on the initialization of relations between average quality qubits and on the search for a minimum energy based in particular on the tunnel effect. The basic approach consists in two steps: reformulating an initial problem into a QUBO of Ising problem, and then solve it by a dedicated quantum approach[3155]. We will describe it when discussing about D-Wave.
- **Analog quantum simulators** are used to simulate quantum phenomena, for example to predict the organization of atoms in molecules. These include cold atom quantum simulators. An algorithm here is about preparing the state of the qubits in the system and their link weights. It is a process similar to D-Wave quantum annealing, with variations on the degrees of liberty handled in the system and qubits coherence.
- **Continuous variable quantum computers** that use quantum objects whose physical quantity can be measured as a continuous, not binary, quantity. This creates yet another programming model. They are mainly based on photons[3156].
- **Topological quantum computers**, which do not yet exist. This is the research path of Microsoft and some research laboratories, especially in China. We cover this on page 448. It should still be programmable with gate-based classical code.
- **Variational algorithms** combining traditional algorithms and quantum algorithms running on any of the above system[3157]. These are algorithms mostly designed for gate based NISQ QPUs and a dedicated part of this section is dedicated to them.

[3154] See The Q# quantum programming language user guide.

[3155] See Ising formulations of many NP problems by Andrew Lucas, 2014 (27 pages) which shows how optimization problems can be formalized into Ising problems. QUBO optimization problems and Ising models are formally equivalent.

[3156] See for example Perspective: Toward large-scale fault-tolerant universal photonic quantum computing by S. Takeda et al, April 2019 (13 pages) and Continuous-variable quantum neural networks by Nathan Killoran et al, June 2018 (21 pages) which deals with the use of continuous variable qubits to create neural networks.

[3157] See Hybrid Quantum Computation by Arun, 2011 (155 pages).

We will mention **Quantum inspired algorithms** which are algorithms running on classical computers that are inspired by quantum algorithms, formalism or mathematics for solving complex problems. Their creation started long before the first experimental quantum computers were created.

In practice, the noisy intermediate scale quantum computers (NISQ) that are emerging now and will dominate the landscape for at least a good decade cannot run "deep" algorithms. Namely, because quantum gates and readout error rates is too high and limits the number of quantum gates that can be chained, even when using quantum error mitigation techniques. This is the case for **VQE** (Variational Quantum Eigensolver), **QAOA** (Quantum Approximate Optimization Algorithm), and various **Quantum Machine Learning** algorithms (Support Vector Machine, Principal Component Analysis and Quantum Variational Autoencoder). We will study some of them in a dedicated part related to NISQ algorithms, starting page 919. We will also look at how quantum software handle data input starting page 867 and output and how it is being debugged (Figure 731).

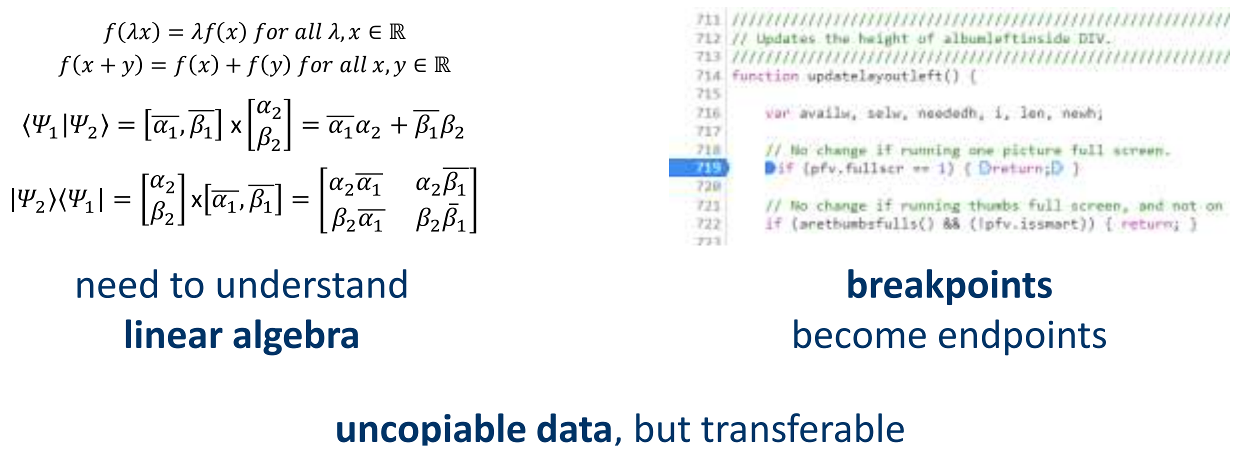


Figure 731: the key differences with quantum programming. A need to understand linear algebra and do some maths, different debugging techniques and coping with the impossibility to copy data and playing with the probabilistic nature of quantum measurement. (cc) Olivier Ezratty, 2022-2023.

We explain in Figure 732 and Figure 733 how to differentiate gate-based and analog quantum computing models, first with the three main models and then with making analogies between analog quantum computing and neural networks training.

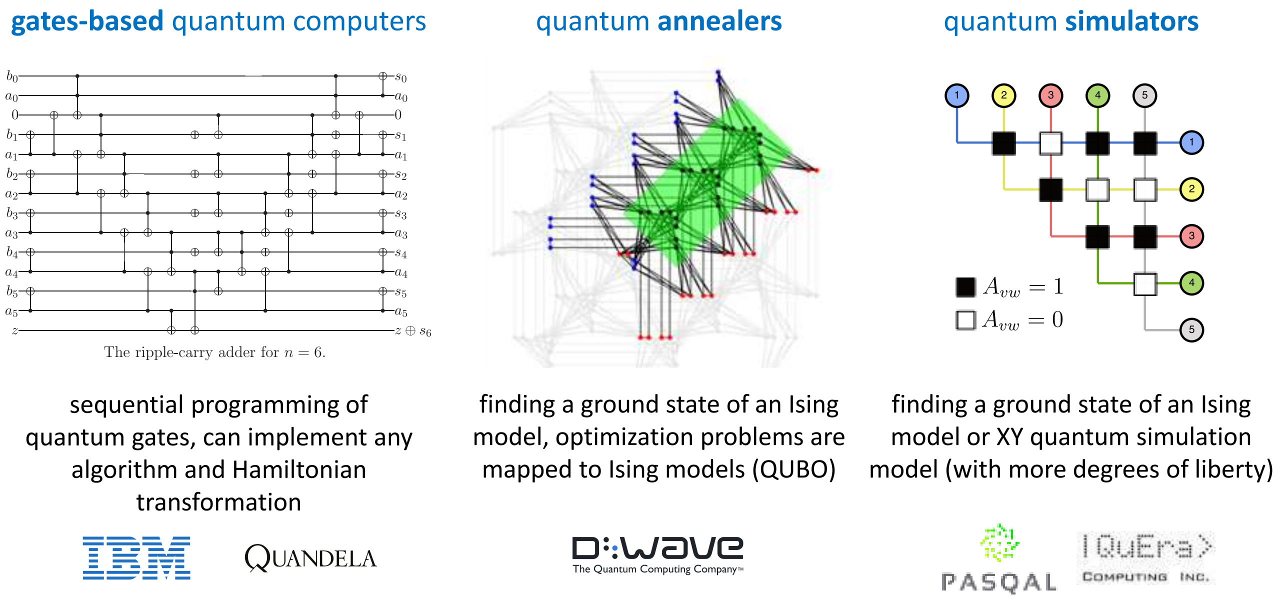
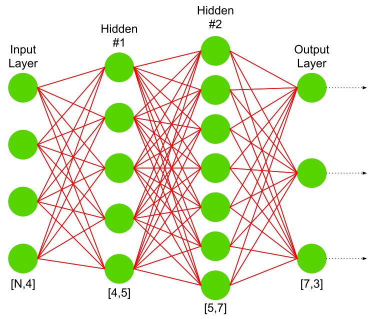
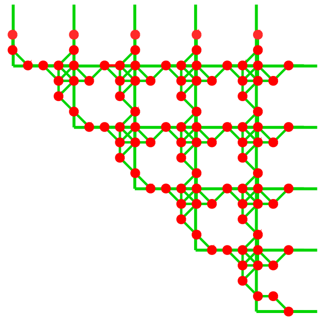


Figure 732: how to differentiate the three main quantum computing paradigms (gate-based, annealing, and simulations). The main difference between the first and two others deal with sequential programming in gate-based model while the two others use a classical preparation of sorts of networks of qubits for which we are searching for an energy minimum. (cc) Olivier Ezratty, 2023.





	deep learning	analog quantum computing
variables	neural network hyperparameters neuron activation functions training data	link weights between qubits and qubit weight qubit connectivity / graph training data that conditions the above values in classical computing
unknowns	neurons link weights	find minimum system energy / Hamiltonian ground state qubit value (-1 or +1 / 0 or 1)
training	minimize cost function with gradient descent and backpropagation	solving BQM or Ising models, classical-quantum hybrid algorithm

Figure 733: here is a way to understand the similarities and differences between deep learning neural networks and analog computing (annealing and simulations). The main difference sits with the variables (green) and unknowns (red). They are reversed! With analog quantum computing, the variables are the links between qubits while these are the unknowns in neural networks. The unknown in analog quantum computing are the qubit weight (0 or 1, or -1 and -1 depending on the model, Ising or QUBO) while with neural networks, the neuron characteristics like their activation function are their variables. (cc) Olivier Ezratty, 2023.

Algorithms classes

Before getting deep into quantum algorithms, let's take a detour by covering their practical usefulness known to date and for which category of quantum hardware they are designed. Then, how they are organized and what is the basic algorithms toolbox available to developers.

Classes and use cases

Here's a simple classification of high-level algorithms by use cases[3158].

Oracle function-based algorithms can fasten the search of a needle in a haystack and find a solution of some complex problem. Some are useful, some are not. The most famous oracle-based algorithms are Deutsch-Jozsa, Simon, Bernstein-Vazirani and Grover.

Quantum physics simulations is a broad field with applications in inorganic and organic chemical processes optimization and new material designs. This is based on simulating at the lowest level the interactions between atoms in molecules and crystal structures or magnetism, which themselves depend on the laws of quantum mechanics. This may someday help invent new solutions such as more efficient batteries that can be charged more quickly and with greater energy density, craft chemical processes for carbon capture or nitrogen fixation or create superconducting materials operating at room temperature.

Biological molecule simulation requires a much larger number of qubits, and therefore are positioned in the longer term. Quantum simulation may eventually help run simulations of biological molecules. This will start with the simulation of peptides, then polypeptides, and finally proteins folding and interactions. Biological molecules have the particularity of being overly complex, with structures that can reach tens of thousands of atoms. The top of the line would be the ability to simulate the assembly and then the operation of a ribosome, which is more than 100,000 atoms. It is the most magical molecular structure in living organisms, the one that assembles amino acids to build

[3158] There are many such classifications around. I've used the most common one.

proteins from the messenger RNA code resulting from the transposition of gene DNA. This would be followed by the simulation of the functioning of a whole cell. But we are here bordering on science fiction.

Algorithms to solve complex optimization problems, particularly combinatorial problems like finding an optimal route for deliveries or automated drive, aka the traveling salesman problem. Such algorithms can also optimize the design of integrated circuits where one generally seeks to minimize the links between functional blocks, transistors and to minimize energy consumption, forms of sub-constrained optimization adapted to quantum processing. This category of algorithms can find solutions to combinatorial optimization problems like discrete log, traveling salesperson problem (TSP), QUBO problems, and continuous optimization problems used in many other fields (linear programming, gradient descent, convex optimizations, and semidefinite programming.

Quantum machine learning algorithms which under some circumstances could be more efficient than machine learning algorithms running on classical hardware, including GPGPUs and TPUs. This can impact both training machine learning and deep learning models and running inferences.

Integer factoring problems relate to cryptography and breaking public encryption keys like RSA keys with Shor's integer factoring algorithm. These may be implemented over a very long-term, when highly scalable gate-based quantum computers are available.

Hybrid algorithms, as already mentioned, use a mix of quantum and classical algorithms with some variations presented in Figure 734. As a matter of fact, all quantum algorithms are hybrid since they are driven by some classical computer that prepares the data and the quantum code. What is referred to as being a hybrid algorithm is often a variational algorithm – mostly in the NISQ realm - that has a classical and quantum loop that ends at some point when a cost function converges to an optimum. But oracle-based algorithms when the oracle is retrieving classical data and even Shor's integer factoring algorithm also have a significant classical part and are thus also hybrid classical/quantum algorithms.

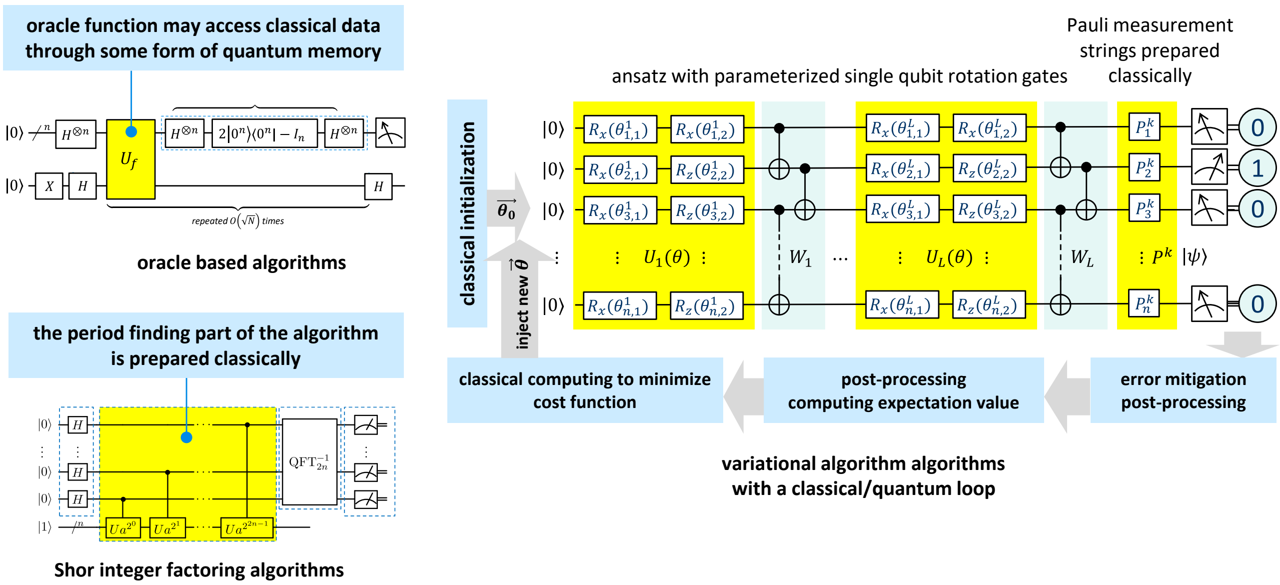


Figure 734: there are many sorts of hybrid algorithms. The dominant one is the variational algorithm with an ansatz that is prepared classically to compute a function cost that is then used classically in a loop to optimize the ansatz. But oracle based algorithms need to access classical data, and even Shor's integer factoring algorithm has a part prepared classically and dynamically, for the period finding function. (cc) Olivier Ezratty, 2023.

Otherwise, many quantum algorithms require some classical data preparation and loading that can be an heavyweight task handled by classical computers before tasking a QPU with a simpler task using prepared data. This is particularly true with quantum machine learning algorithms, whether they rely or not on some variational process. There are also vertical and horizontal hybridization techniques like with parallelizing processing on several specialized QPUs[3159].

Quantum inspired algorithms are classical algorithms like genetic algorithms that are inspired by quantum properties, particularly interferences which are key characteristics of quantum algorithms.

Algorithms and quantum computing paradigms

As shown in Figure 735, there's a relation between these broad classes of algorithms and the class of quantum computers they can run on. Reusing the quantum algorithms paradigm classification used earlier in this book, this gives an idea of what works where and when, given these computer classes span from available systems (quantum annealing), to NISQ systems in the short to mid-term, to very long-term availability (large scale quantum computing). It is still an open question to find relevant and useful algorithms running on NISQ systems, if not reaching a quantum advantage with these[3160].

You can also create analogies to figure out what is quantum computing from a mind model standpoint, as shown in Figure 736. The first one is simple: it is all about mathematics and linear algebra, manipulating large matrices and vectors. A quantum gate is a (unitary) matrix multiplied by a (state) vector. You then decompose a large problem into a series of such unitaries. The second model is related to analog electronics. With quantum computing, you happen to reuse similar concepts, manipulating signals phase, amplitude, decomposition (QFT) or recomposition (inverse QFT) and interferences. At last, quantum annealing, and quantum analog simulations have some similarities with neural networks, with some differences, already shown in Figure 733.

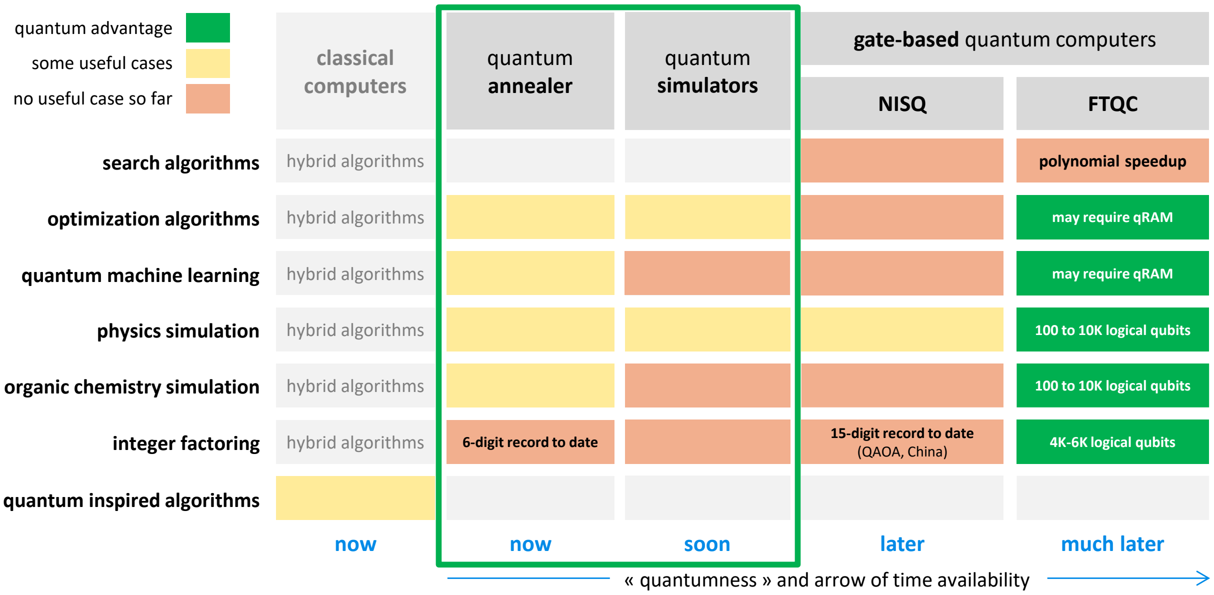


Figure 735: classes of quantum algorithms, the quantum computing paradigm (gate-based, simulation annealing) they can run on and a time scale for their practical availability. Surprisingly, integer factoring algorithms are also available on quantum annealers and simulators, but it may not scale as well as future FTQC systems. (cc) Olivier Ezratty, 2021-2023.

[3159] See Classification of Hybrid Quantum-Classical Computing by Frank Phillipson et al, October 2022 (8 pages).

[3160] See the review paper Noisy intermediate-scale quantum (NISQ) algorithms by Kishor Bharti, Alán Aspuru-Guzik et al, Review of Modern Physics, October 2021 (91 pages) and Simultaneous quantum circuits execution on current and near-future NISQ systems by Yasuhiro Ohkura et al, Dec2021 (10 pages) which addresses some limitations of NISQ systems with running smaller QPUs in parallel.

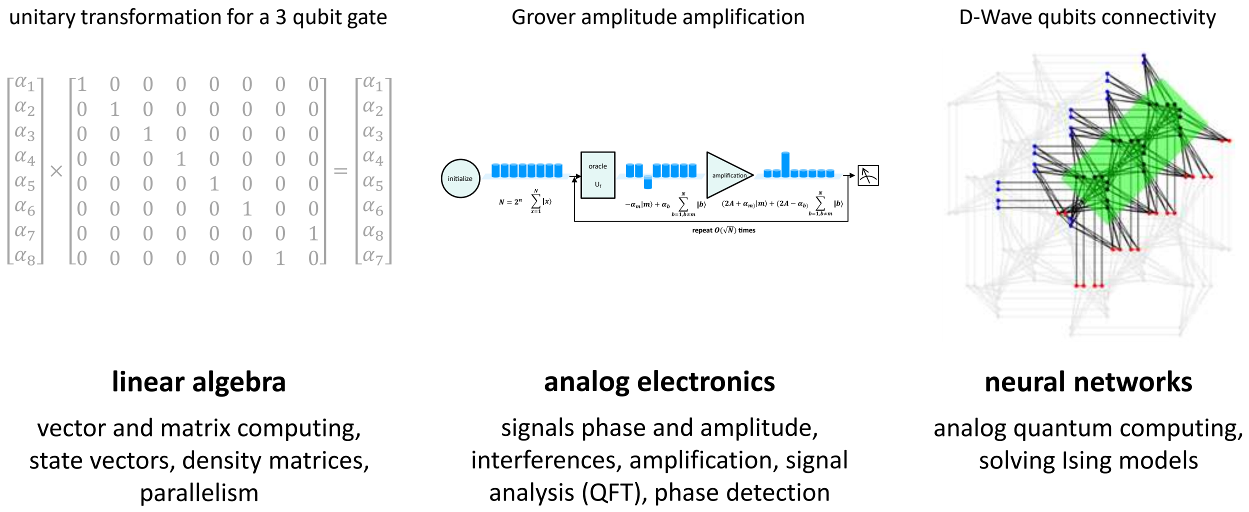


Figure 736: three analogies to understand what quantum computing is. The two first relate to gate-based quantum computing and the last one to quantum annealing and quantum simulation. (cc) Olivier Ezratty, 2023-2024.

Algorithms process and compilation

As we have seen in a previous section describing the structure of gate-based quantum computing, page 205, a quantum algorithm is built with three key parts: data initialization or preparation, computing and qubits readouts. This is always done on a data structure called a quantum register, made of N qubits. Data initialization, preparation and computing are implemented with quantum gates.

Results. The algorithm result comes from the classical measurement of some qubits giving out a mix of 0s and 1s bits. In general, it is necessary to run several times the algorithm entirely and compute an average of the generated results. How many times must it be done? It depends on the nature of the algorithm and the speed at which we'll move from a probabilistic output (one run) to a deterministic result (average of several runs).

Time constraints. The algorithm run must be compatible with the quantum computer characteristics. The main ones are qubits numbers, gates and readout fidelities and coherence time. These parameters will condition the usable depth of computing, *aka*, the number of series of gates that can be executed. This verification is generally performed by quantum code compilers. It will also have to consider the error correction codes that will be implemented in the hardware, either autonomously or through the control of the code compiler that will drive all logical qubits programming.

Gates conversion. Compilers play another key role: they translate the qubit gates used by the programmer into the set of physical qubit gates implemented at the hardware layer. Many quantum gates used by developers will be converted by the compiler into a set of universal quantum gates supported by the quantum computer. This will multiply the number of executed physical quantum gates compared to what shows up in the initial algorithm.

Geometry. They also take into account the physical geometry of qubits, i.e. how are they connected together. A simple two-qubit gate might require chaining a lot of SWAP gates because the two related qubits are far from each other in the quantum register physical layout.

Efficiency. An important consideration in creating quantum algorithms is to ensure that they are more efficient than their optimized counterparts for traditional computers or supercomputers. There are theories to verify this to evaluate the exponential, polynomial, logarithmic or linear rise in computing time as a function of the size of the problem to be solved, or a combination of all four. But nothing can replace experience!

Everything is linear. Quantum algorithms are practical applications of linear algebra, the branch of mathematics that handles vector spaces and matrix-based linear transformations. They are applied in large dimensional spaces, the vectors that define the states of qubit registers. Mathematically speaking, a qubit is a 2-dimensional vector space using complex number and N-qubit register manipulates a vector in a 2^N dimensional space of complex numbers. Their manipulation is based on matrix-based calculations that allow the qubit state to be modified without reading the content of the qubits. One way to look at a gate-based quantum algorithm would be the following: it is about finding the shortest path on a hypersphere of dimension 2^N from an initialized register to the problem solution[3161].

Conditional programming. Since quantum algorithms usually prohibits reading intermediate results, conditional programming is not obvious. Like running a given calculation depending on the value of some intermediate. However, multi qubits quantum gates (CNOT & co) are tools allowing conditional programming, but in another fashion than with classical computing. Conditional branching can be implemented in some situations and is used with hybrid algorithms and with ancilla qubits for intermediate values measurements. It also plays a role in the MBQC programming paradigm.

Algorithms toolbox

All these algorithms are based on a small set of classical low-level algorithms that we'll describe in detail in the Basic algorithm's toolbox section starting page 867:

- **Quantum Fourier Transforms** (QFT), which helps find periods in a signal. It is used in the famous Shor integer factoring, in many other algorithms (HHL, QML), discrete log search, solving the hidden subgroup problem (HSP) and even for simple reversible arithmetic[3162].
- **Quantum Phase Estimation** are relying on a QFT to find the eigenvalues or eigenvalues' phase of a unitary matrix or quantum subcircuit. It is used in HHL and many other quantum linear algebra algorithms.
- **Quantum Amplitude amplification** is used to amplify and select the desired state of a quantum superposition. It is used in the Grover algorithm and with combinatorial searches like the traveling salesperson problem search (TSP).
- **Quantum Phase Kickback** is an interference trick used in most oracle-based search algorithms and quantum walks, and then in quantum machine learning.
- **Linear equations solving** and the famous HHL algorithm and various partial derivative equation solving algorithms.
- **Hamiltonian Simulations** are used to find a point of equilibrium of a complex system such as in quantum physics simulation, neural networks training, the search for optimal paths in networks or process optimization. It can be implemented in all quantum paradigms: annealing, simulation and gate-based computing (with a FTQC QPE or a NISQ VQE).
- **BQM** or binary quadratic model is the mechanism used to drive quantum annealers. This is the underlying model used with D-Wave machines. Many physics simulation (Ising models), combinatorial and optimization problems (QUBO problems) can be translated or converted into BQM, as explained in Figure 738.
- **Quantum teleportation** is also an algorithm basic, used mostly in cryptography and telecommunication. It will also play a key role in distributed quantum computing and also in some non-telecom related algorithms. It is used in the hard to understand "gate teleportation" trick.

[3161] It even seems to have a name: minimizing Wasserstein complexity as seen in Wasserstein Complexity of Quantum Circuits by Lu Li, Seth Lloyd et al, August 2022 (14 pages).

[3162] See A New Approach to Multiplication Opens the Door to Better Quantum Computers by Kevin Harnett, 2019.

We'll add here several other key basic algorithms components:

- **Data preparation**: how is data loaded in an algorithm? This is particularly important for quantum machine learning and optimization algorithms.
- **Uncompute trick**: consists in reversing some parts of an algorithm after it is run. It allows to get rid of garbage states and cleaning up ancilla qubits.
- **Oracle**: are binary functions implemented as unitaries that can be used for parallelizing their operation on all computational state basis (all combinations of 0s and 1s in part of a qubits register). They are used in Grover, Deutsche-Josza, Berstein-Vazirani, Simon and QAE algorithms.

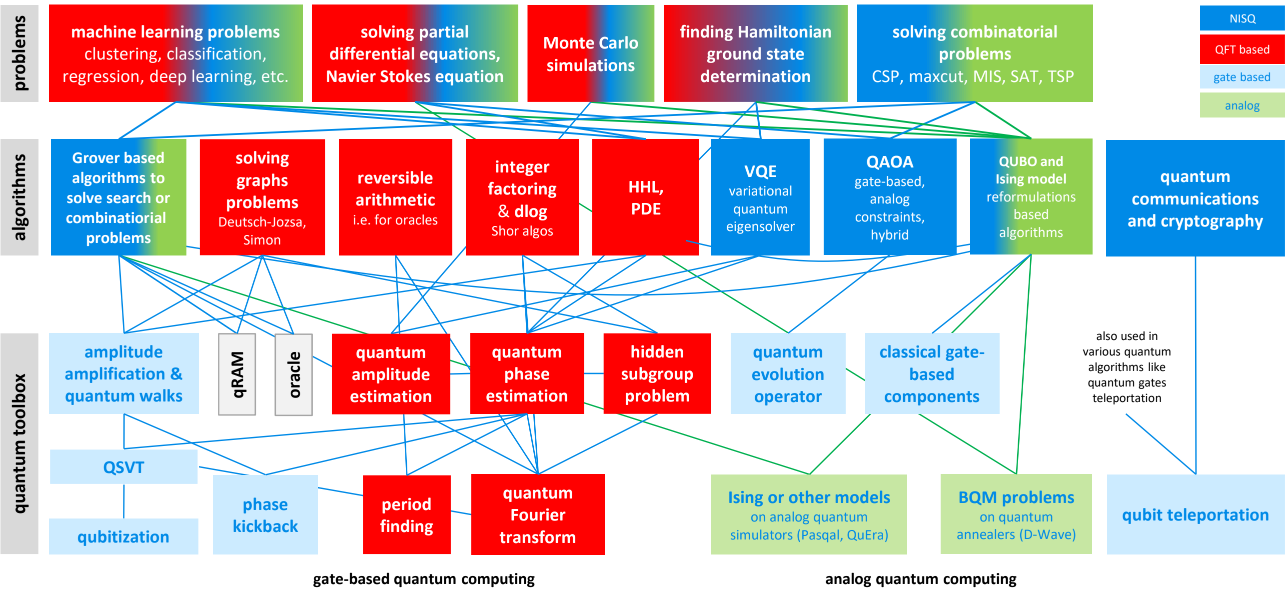


Figure 737: a quantum approaches inventory with problems (top), key algorithms (middle) and basic algorithm components (bottom). The color code is red for FTQC problems/algorithms relying on the QFT (quantum Fourier transform), blue for problems/algorithms corresponding to other gate-based algorithms that may work in NISQ QPUs and green for problems/algorithms related to analog quantum computing paradigms like quantum annealing (ala D-Wave) and quantum simulations (ala Pasqal). Many high-level problems can be solved using an FTQC QFT based version and some analog equivalents, usually based on a QUBO or Ising problem formulation. (cc) Olivier Ezratty, 2023-2024, inspired by a schema found on Quantum Computing Algorithms by Andreas Baertschi, 2019 (45 slides).

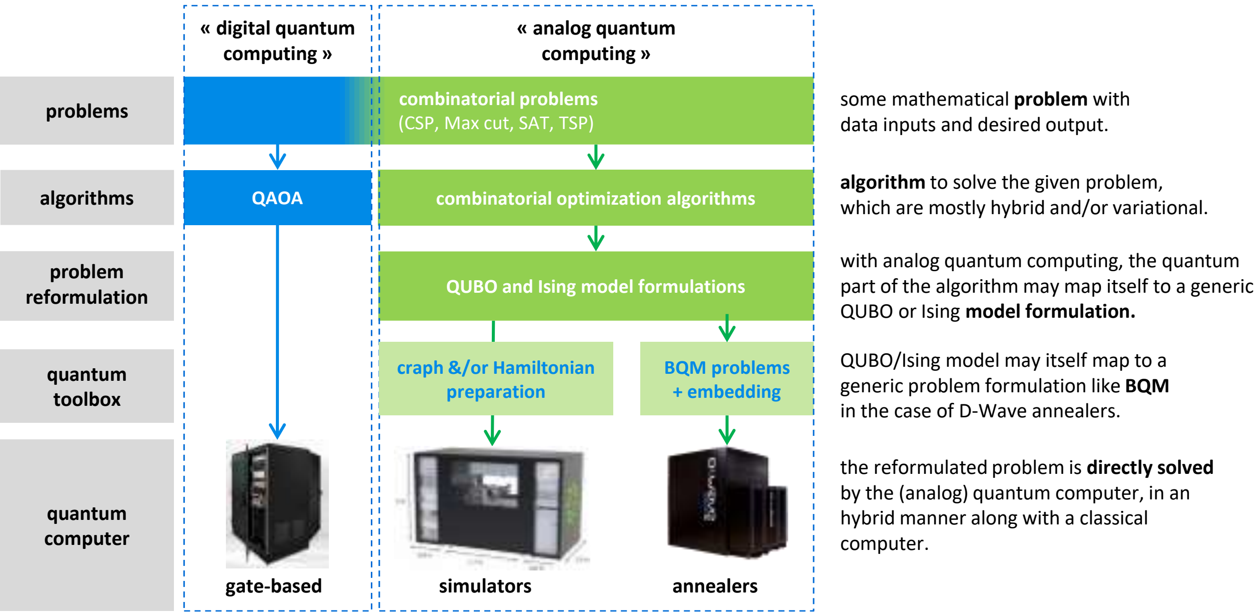


Figure 738: some clarification between the notion of problem, algorithms, and models, particularly in the analog quantum computing domain. In that domain, the quantum part of a hybrid algorithm is mainly a problem reformulation into another generic problem that is directly solved by the quantum computer in a single step. (cc) Olivier Ezratty, 2023-2024.

Classifying quantum algorithms is a tedious task due to the many dependencies they have on each other. For example, a QFT is used in HSP and phase estimate algorithms which themselves are used in integer factoring and linear equations solving.

I have found many different if not inconsistent algorithms classifications in the available literature (Wikipedia, John Preskill, Algorithm Zoo, etc). Some for example consider oracle-based algorithms as a separate algorithm class when other split these algorithms in various classes depending on the sub-algorithms they are using.

The relationship between these low-level algorithms and higher-level ones is showcased in Figure 737. It shows qRAM for quantum RAM, which is not an algorithm per se, but a hardware tool that is indispensable to run the related algorithms, particularly Grover algorithm and a lot of quantum machine learning algorithms.

The chart in Figure 739 shows a more detailed connection between the QFT and the many algorithms that rely on it (those algorithms relying on a QFT are in red rectangles in Figure 737).

Quantum algorithms are classifiable and explainable at a high level, but their detailed understanding is not easy. You must develop some conceptual capacity in a rather analog world[3163].

Algorithm	Description	Reference
Algorithms Based on QFT		
Shor's; $O\left(n^2 (\log N)^3\right)$	Integer factorization (given integer N find its prime numbers); discrete logarithms, hidden subgroup problem, and order finding	Peter W. Shor, "Algorithms for Quantum Computation Discrete Log and Factoring," AT&T Bell Labs, shor@research.att.com
Simon's; *exponential*	Exponential quantum-classical separation. Searches for patterns in functions	Simon, D.R. (1995), *"On the power of quantum computation"*, Foundations of Computer Science, 1996 Proceedings., 35th Annual Symposium on: 116–123, retrieved 2011-06-06
Deutsch's, Deutsch's – Jozsa, an extension Deutsch's algorithm	Depicts quantum parallelism and superposition. "Black Box" inside. Can evaluate the input function, but cannot see if the function is balanced or constant	David Deutsch (1985). "Quantum Theory, the Church-Turing Principle and the Universal Quantum Computer". Proceedings of the Royal Society of London A. 400: 97 David Deutsch and Richard Jozsa (1992). "Rapid solutions of problems by quantum computation". Proceedings of the Royal Society of London A. 439: 553
Bernstein/Vazirani; *polynomial*	Superpolynomial quantum-classical separation	Ethan Bernstein and Umesh Vazirani. *Quantum complexity theory.* In Proc. 25th STOC, pages 11–20, 1993
Kitaev	Abelian hidden subgroup problem	A. Yu. Kitaev. *Quantum measurements and the Abelian stabilizer problem*, arXiv:quant-ph/9511026, 1995
van Dam/Hallgren	Quadratic character problems	Wim van Dam, Sean Hallgren, *Efficient Quantum Algorithms for Shifted Quadratic Character Problems.* CoRR quant-ph/0011067 (2000)
Watrous	Algorithms for solvable groups	John Watrous, Quantum algorithms for solvable groups, **arXiv:quant-ph/0011023**, (2001)
Hallgren	Pell's equation	Sean Hallgren. *Polynomial-time quantum algorithms for pell's equation and the principal ideal problem,* Proceedings of the thirty-fourth annual ACM symposium on the theory of computing, pages 653–658. ACM Press, 2002.
Algorithms Based on Amplitude Amplification		
Grover's; $O\left(\sqrt{N}\right)$	Search algorithm from an unordered list (database) for a marked element, and statistical analysis	Lov Grover, *A fast quantum mechanical algorithm for database search,* In Proceedings of 28th ACM Symposium on Theory of Computing, pages 212–219, 1996
Traveling Salesman Problem; $O\left(\sqrt{N}\right)$	Special case of Grover's algorithm	https://en.wikipedia.org/wiki/Travelling_salesman_problem

Figure 739: Source: Quantum computing (QC) Overview by Sunil Dixit from Northrop Grumman, September 2018 (94 slides).

[3163] Here are a few sources of information to explore the topic: Quantum Computing Applications by Ashley Montanaro from the University of Bristol, 2013 (69 slides), an interesting course on the algorithmic part, An Introduction to Quantum Computing by Phillip Kaye, Raymond Laflamme and Michele Mosca, Oxford, 2017 (284 pages), Lecture Notes on Quantum Algorithms by Andrew M. Childs, University of Maryland, 2017 (174 pages), Quantum Computation and Quantum Information by Nielsen and Chuang, 2010 (10th edition, 704 pages) and A Course in Quantum Computing for the Community College by Michael Locef, 2016 (742 pages) which sets out in great detail the mathematical foundations of linear algebra with complex numbers, Euler formulas, vector and Hilbert spaces, matrix calculus, tensors, eigenvectors and eigenvalues, and quantum algorithms. It takes several weeks to be browsed and understood. It is a course for the second and third year of the Foothill Community College in Los Altos Hills, California (so Bac+1/+2 in French equivalent). In addition, here are some videos on this subject: Quantum Algorithms by Andrew Childs in 2011 (2h31), Language, Compiler, and Optimization Issues in Quantum Computing by Margaret Martonosi, 2015 (39 minutes and slides) and What Will We Do With Quantum Computing? by Aram Harrow, MIT, 2018 (32 minutes).

One key thing that a developer must learn is how to translate customer needs into existing quantum algorithms. How to assemble various quantum algorithms, frequently combined with classical algorithms, is another key skill.

Algorithms figures of merit

It is rarely talked about, but what are the key figures or merits for quantum algorithms? In a generic way, they should showcase either a provable speedup compared with classical algorithms running on classical computers or some other quantitative advantage (precision, error rates for quantum machine learning, energy consumption advantage). They should generate a relatively small-size data output since N qubits generate only N useful bits of information. Finally, there should be some correctness guarantee on the results and of course, it should solve some useful problems. In the case of NISQ algorithms, they should add two requirements: a shallow depth circuit (not many quantum gate cycles) and a resilience to qubit noise[3164].

Basic algorithms toolbox

We'll describe here the overall structure of basic low-level gates-based quantum algorithms. We separate three stages: **data preparation**, **unitary transformations** (caveat: data preparation also relies on unitaries) and **measurement**. We have already covered **error correction** in a previous chapter of this book, starting page 240.

Data preparation

The data preparation stage is also named data loading. Its complexity covers a large range from the simple process of uniform superpositions associated to oracle-based algorithms like Deutsch-Jozsa, Simon or Grover to the most complicated, linked to quantum machine learning algorithms requiring full computational basis state vector amplitude encoding (Figure 740).

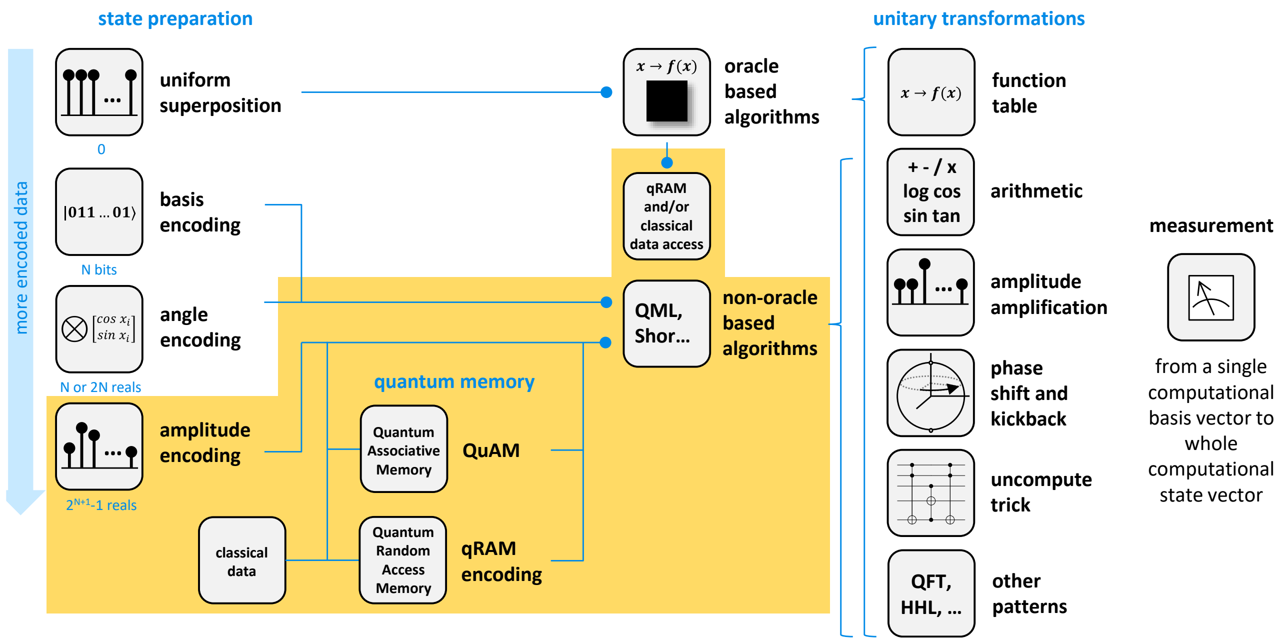


Figure 740: how is data fed into a quantum algorithm depending on whether it uses or not an oracle. (cc) Olivier Ezratty, 2021-2023.

[3164] This inventory is inspired from the second page chart in Towards Quantum Advantage on Noisy Quantum Computers by Ismail Yunus Akhalwaya et al, September 2022 (32 pages) which presents a NISQ QML-based algorithm matching all these figures of merit. But … requiring 96 qubits with 99.99% 2-qubit gate and measurement fidelities which are yet to come, even with trapped ions.

Data loading is only implemented with non-oracle-based algorithms. It may be a long process for large sets of inputs and a significant number of qubits. It thus may require using some form of quantum memory, a sort of qubits buffer used only for data preparation.

It can use addressable qubits like with qRAM where a program can ask to "*put this information in qubits at index i*". This memory can be a qubit register with a longer coherent lifespan than computing qubits or a classical data structure used along a quantum circuit to load a specific addressable quantum state. The data is not necessarily stored in some qubits. When the data is loaded in quantum memory, this one must be transferred to the computing qubits. This is a data transfer and not a data copy process due to the no-cloning theorem. All in all, quantum memory is just some sort of intermediate memory used before computing.

Let's first look at the various techniques used for data encoding[3165] (Figure 741).

Uniform superpositions correspond to the simplest qubits register initialization with a register state where all the computational basis states have the exact same amplitude. It is used by an oracle-based algorithm where the "real data" sits in the oracle function f(x) that outputs such and such values depending on the entry (usually, a 1 for a single entry and 0 for all the others). The oracle can evaluate this function simultaneously for all superposed computational basis values in the prepared superposed register. This superposition is done by applying Hadamard gates on all computing qubits where some data must be prepared.

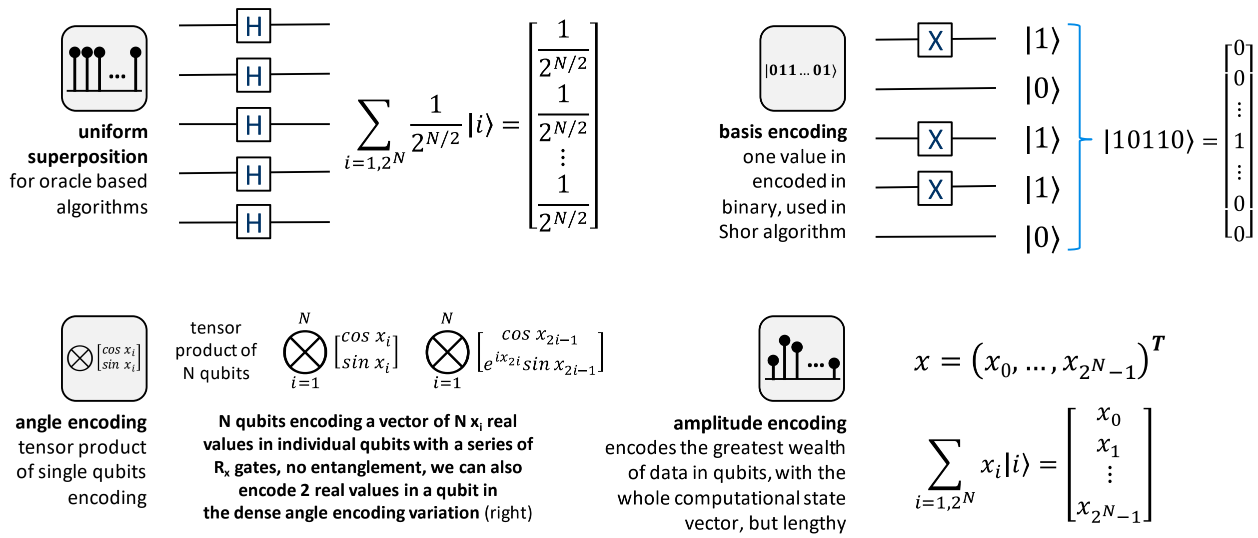


Figure 741: details on the four ways to encode data in a qubit register, the most resource and time consuming being amplitude encoding. (cc) Olivier Ezratty, 2021.

Basis encoding consists in directly transferring N classical bits in N qubits, using a set of X gates (Y gates would also make it), to change individual qubits from $|0\rangle$ to $|1\rangle$. It creates a simple encoding of a computational basis single state, combining 0s and 1s matching classical bits. The 2^N dimensions computational basis state vector thus contains only zeros and a single one related to this combination. Before this encoding, we select the method to encode the problem data which can be for example a floating-point number or an integer in a given number of classical bits before converting them on a computational basis state. Such a basis encoding is used in Shor's algorithm to provide the integer that must be factorized.

[3165] Here are the various sources I used to reconstruct this map: Loading Classical Data into a Quantum Computer by John Cortese and Timothy Braje, 2018 (38 pages), Circuit-centric quantum classifiers by Maria Schuld, Krysta Svore et al, 2018 (17 pages), Robust data encodings for quantum classifiers by Ryan LaRose and Brian Coyle, 2018 (24 pages), Towards a Pattern Language for Quantum Algorithms by Frank Leymann, 2019 (12 pages), Quantum linear systems algorithms: a primer by Danial Dervovic et al, 2018 (55 pages) and The Bitter Truth About Quantum Algorithms in the NISQ Era by Frank Leymann and Johanna Barzen, 2020 (42 pages).

Angle encoding is about encoding a vector of real values of dimension N into N qubits. It is also named product encoding. Each qubit is individually encoded with single qubit gates R_x (which themselves are usually decomposed in simple Pauli and T gates) to encode one of the (Bloch sphere) qubit angle. Since the register is the tensor product of each qubit, with no entanglement, we don't have any exponential gain in the encoding. The dense angle encoding variation uses two angles in the encoding for each qubit and can make use, additionally of R_y and R_z gates for the sake of adding some phase in each qubit state. We end up here with a maximum of 2N real numbers encoded in the N qubits register. And the qubit register is separable, its quantum state being separable into the quantum states of each of its qubits. It reminds us that without entanglement, you can't benefit from the exponential storage (or, better, data handling) capacity of quantum computing. In that case, data encoding requires a depth of $\log_2(N)$ gates.

Amplitude encoding is about creating an arbitrary superposed state associating computational basis states with given real number amplitudes. It is also called an arbitrary state preparation, quantum embedding or wavefunction encoding. It creates a computational state vector with real numbers in several rows. To encode a vector of L real values, you need $N=\text{ceil}(\log_2(L+1))$ qubits. Meaning you round up the $\log_2$ of the vector size and don't use the left-over values in the register vector. Why +1? Because of the normalization constraint, the sum of amplitude being equal to 1. So, with 3 qubits, you have 7 available values, not $8=2^3$. Since the size of your encoded vector may be smaller than the 2^N states of your register, you'll pad the encoded vector with 0s.

To create an arbitrary amplitude set for N qubits, you need at least $\frac{1}{N}2^N$ gates operations combining single and two qubit gates since an arbitrary amplitude encoding will create an entangled state contrarily to a simple product encoding. The usual encoding algorithms use 2^N gates. As a result, unless the encoded data is sparse (with a lot of zeros), data preparation grows exponentially with the number of qubits, erasing any computing advantage we could get afterwards.

It explains why quantum computing is not ideal for any big data computation task, and, at this point, for data intensive machine learning tasks[3166]! There are however some optimized solutions which can for example use some variational methods[3167] [3168], subspace encoding also using variational methods[3169], tensor based methods[3170] or be based on a genetic algorithm[3171].

Encoding precision. Angle and amplitude encoding theoretically deals with real numbers. But what is their precision, particularly on NISQ computers? It is at least bound by the cumulative error rates coming from the encoding qubit gates. It can easily reach a couple %, meaning the encoding precision is limited to a couple digits.

Non linearities. Quantum computing is based on using linear unitaries. This creates limitations on the kinds of computing that can be processed quantumly. But we can handle nonlinearities indirectly. One way lies with the way real numbers are turned into the raw data to be encoded with angle or amplitude encoding. Another way is to use angle encoding with repetition, creating powers of

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[3166] There are some ways to optimize amplitude encoding. See Quantum Resources Required to Block-Encode a Matrix of Classical Data by B. David Clader, William J. Zeng et al, Goldman Sachs, Caltech, AWS and Imperial College London, July 2022 (31 pages) also described in Goldman Sachs and AWS examine efficient ways to load data into quantum computers by Grant Salton et al.

[3167] See Approximate complex amplitude encoding algorithm and its application to classification problem in financial operations by Naoki Mitsuda et al, November 2022 (12 pages). For data loading.

[3168] See Variational preparation of entangled states on quantum computers by Vu Tuan Hai et al, 1University of Information Technology (Vietnam), Tohoku University (Japan), June 2023 (8 pages).

[3169] See Trainability and Expressivity of Hamming-Weight Preserving Quantum Circuits for Machine Learning by Léo Monbroussou, Jonas Landman, Alex B. Grilo, Romain Kukla and Elham Kashefi, September 2023 (23 pages).

[3170] See Data is often loadable in short depth: Quantum circuits from tensor networks for finance, images, fluids, and proteins by Raghav Jumade and Nicolas PD Sawaya, Intel, September 2023 (10 pages).

[3171] See GASP -- A Genetic Algorithm for State Preparation by Floyd M. Creevey et al, February 2023 (8 pages).

encoded values in the computational state vector of the input vector[3172]. Finally, we can apply non linearities on the classical data before it is quantumly encoded. It can also be implemented as a classical Boolean nonlinear circuit embedded in a quantum reversible circuit.

How many registers? In a classical microprocessor, the computing unit is handling data in multiple registers and the arithmetic logical unit can pull data from registers, make calculations and update registers with its results. In a quantum computer, there is usually only one register of N qubits. It can however be logically and dynamically partitioned by the algorithm. There are usually computing qubits and ancilla qubits. Computing qubits contain the input data related to the problem to be solved and that's on this data that most algorithm patterns will be executed, particularly with an oracle. The remaining qubits in the register will be used as accessory qubits and may also contain some part of the algorithm result. And this goes only with logical qubits. We've seen before that quantum error correction is using a lot of ancilla qubits.

Black boxes and oracles

A black box based algorithm is a classical operation encoded with qubit gates that is applied simultaneously to various computational basis states. It is used in the famous Deutsch-Jozsa, Simon and Grover search algorithms. A black box contains reversible quantum equivalents of Boolean and arithmetic functions. It works on n entry qubits x in superposed states and merges its result with m ancilla qubits y, that are usually initialized at 0. It leverages quantum parallelism with input initialized with Hadamard gates (Figure 742). If m=1, the black box outputs a yes or no (1 or 0) and is branded as an "oracle"[3173].

There are many ways to implement an oracle. It can be entirely encoded with qubit gates or access some classical memory or functions, presumably through some qRAM addressing scheme. Presumably since the technology does not exist yet. Even the cost of implementing an entirely quantum oracle is unknown. For instance, just some complicated arithmetic functions can be highly costly in quantum gates since it may require implementing several QFTs (quantum Fourier transforms).

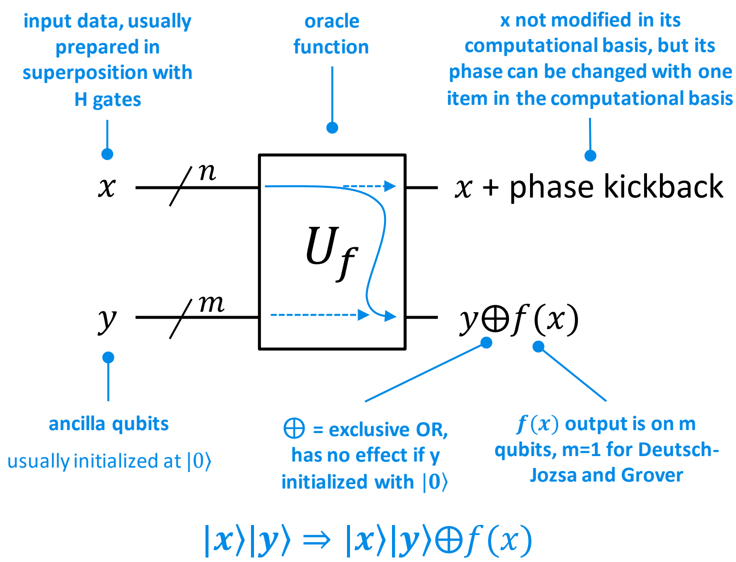


Figure 742: how an oracle function is used in an algorithm, in complement of a phase kickback. (cc) Olivier Ezratty, 2021.

[3172] See Quantum Fourier Networks for Solving Parametric PDEs by Zoë Holmes, Jonas Landman, Natansh Mathur and Iordanis Kerenidis, December 2021-February 2023 (17 pages).

[3173] See Inverse Problems, Constraint Satisfaction, Reversible Logic, Invertible Logic and Grover Quantum Oracles for Practical Problems by Marek Perkowski, May 2020 (62 slides).

Oracle-based algorithms speedup nearly never mention the potential computing overhead coming from the oracle itself. In an ideal world, the oracle implementation complexity should scale linearly and at worst polynomially with the number of handled qubits. This overhead can be highly detrimental to any potential algorithm theoretical speedup. This is particularly concerning for Grover's algorithm that we'll look after later. This algorithm's speedup is only polynomial, before considering the oracle's cost, and of course, quantum error correction (although its cost is "only" polylogarithmic). In the end, Grover's algorithm may not bring any acceleration at all or, at least, provide some acceleration only with prohibitive large problems and computing times. In other words, touting some oracle-based algorithm speedup is like saying that a car drives fast thanks to its aerodynamism, without mentioning anything about its engine specifications and power.

In oracle parlance, an **oracle expansion** consists in considering the oracle quantum circuit to estimate the total algorithm resources like gate cycle, number of Clifford and non-Clifford group gates and the likes[3174].

Output encoding

The literature covering quantum algorithms rarely explains the format of the results they are generating. There are as many variations as in data encoding. This section echoes the one that was dedicated on the various sorts of qubits measurement, page 209. The simplest outcome of a quantum algorithm should be a computational basis vector with a series of $|0\rangle$ and $|1\rangle$, generating a classical bit string like with Grove's search algorithm. In this case, a single run and measurement provides full characterization of this outcome, modulo the error rate of the system. This is the case for only a few classical quantum algorithms as described in Figure 743. However, many quantum algorithms like HHL (linear equations) generate data encoded in amplitude and with some probabilistic outcome. In many cases, exploiting directly an amplitude encoded vector state does not make much sense since you lose any exponential advantage coming from the algorithm.

algorithm	Input	output
Deutsche-Jozsa	oracle function can be entirely quantum (with not much real use case) or access some classical data using a qRAM	function is balanced if all output qubits are at ground state $|0\rangle$
Bernstein-Vazirani		(integer) secret string in basis encoding
Grover		searched item index as integer in basis encoding
Simon		parameters for a linear equation used to find a period, with average of basis encoding
Shor factoring	parametrized period finding function (quantum part)	integer after some classical post-processing
Shor dlog	two integers in basis encoding	integer after some classical post-processing
QFT	series of complex amplitudes with amplitude encoding (any quantum input state)	Fourier coefficients in amplitude encoding, enabling the recovery of the main frequency
HHL	one vector and one matrix amplitude encoding	characteristics of inverted matrix x entry vector (= one vector) in amplitude encoding
VQE	cost function parameters encoded as an Hamiltonian with unitaries (quantum gates)	probabilistic distribution of Pauli strings components of Hamiltonian ground state
QML classification	object vector to classify encoded in amplitude	prediction result as an integer index in basis encoding

Figure 743: various algorithms and the format of their input and output data. Blue marks amplitude encoding of output examples. (cc) Olivier Ezratty, 2021-2024.

[3174] See The bitter truth about gate-based quantum algorithms in the NISQ era by Frank Leymann and Johanna Barzen, Quantum Science and Technology, 2020 (29 pages).

Decoding a full vector state indeed requires running the algorithm several orders of magnitudes of an exponential of the number of qubits. Such an algorithm may be an intermediate one feeding another algorithm, like when HHL is used in the context of some machine learning algorithm. If we keep using quantum data from end to end, then it makes sense to use and create algorithms that output amplitude encoded data[3175]. Shor's integer factoring algorithm is also tricky. Its quantum output after the inverse QFT is a register with some superposition. The circuit is executed several times to obtain some output distribution that is then handled with a classical continued fraction algorithm to retrieve the period used to obtain the integer prime divider.

In some cases, like with VQE algorithms, some generalized measurement of the state vector must be done[3176]. Other algorithms like in computational chemistry output will take the form of expectation values that are real numbers computed with averaging the results of many qubit readout values obtained with many circuits runs. Its overhead requires significant optimizations[3177].

Quantum phase kickback

The role of an oracle is to change the phase of the found item in the computational basis state vector x. Instead of sending the phase to the ancilla qubit y, it is applied to the found result in the source x qubits thanks to the phase kickback mechanism. It is implemented for example in the Grover algorithm that we'll see later.

The Grover operator then amplifies the amplitude of the found item and attenuates the amplitude of the other items in the computational basis, leveraging this phase information injected in the x computational basis vector state. For this to work, the control qubits must be in a superposed state, created by Hadamard gates initialization, the target qubit $|\psi\rangle$ must be an eigenvector of the operator U applied to the target qubit $|\psi\rangle$ using the control qubits[3178].

This simple two qubit configuration explains what's happening (Figure 744). A control-phase gate ends up modifying the phase of the control qubit instead of the phase of the target qubit. It works in the example case since after the X gate being applied to the target qubit, the qubit state becomes an eigenvector of the control-S operation that is executed afterwards[3179].

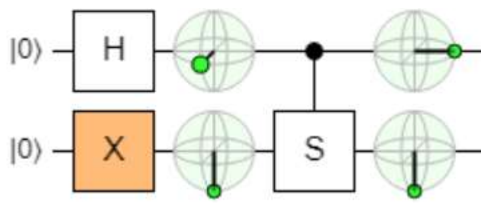


Figure 744: a two-qubit phase kickback.

It is not changed by phase rotation. Since a control-phase changes the global phase of both qubits, the phase modification can only happen on the control qubit. Despite the entanglement created by the control-S gate, the qubits remain separable.

In a general case, when the target qubit $|\psi\rangle$ is an eigenvector of the unitary U (here, a S gate), the target qubit doesn't change after the control-U gate. Literally: $U|\psi\rangle = e^{i\phi}|\psi\rangle$. The control qubit changes and one of its computational state vector amplitudes gets multiplied by the eigenvalue of the eigenvector of U.

[3175] This is what is proposed in Quantum advantage for differential equation analysis by Bobak T. Kiani, Dirk Englund, Seth Lloyd et al, April 2022 (21 pages). The authors use the output of a quantum differential equation solver as the input for a quantum machine learning algorithm.

[3176] See Learning to Measure: Adaptive Informationally Complete Generalized Measurements for Quantum Algorithms by Guillermo García-Pérez et al, PRX, November 2021 (18 pages) that proposes a POVM -based technique to undertake such measurement.

[3177] See Nearly Optimal Quantum Algorithm for Estimating Multiple Expectation Values by William J. Huggins, Ryan Babbush et al, Google AI, PNNL, Stanford and University of Toronto, November 2021-October 2022 (18 pages).

[3178] As explained in Phase Kickback by Eduard Smetanin, November 2019 (4 pages).

[3179] This is explained in A clever quantum trick by Emilio Peláez, January 2021. See also Quantum Phase Kickback - What I told you was true… from a certain point of view by Frank Zickert, March 2021.

Arithmetic

Arithmetic functions can be implemented in a quantum algorithm[3180]. It is frequently used in oracle functions. Many quantum algorithms exist to implement various arithmetic functions: adders[3181 3182], multipliers[3183 3184], dividers, floating point calculus[3185], and even transcendental functions (exponential, logarithm, and trigonometric functions)[3186].

Quantum reversible adders and multipliers can be derived from their classical counterparts like with ripple-carry adders which are built using carry adders (as pictured in Figure 745) and sum blocks (as shown in Figure 746). These blocks use a lot of CNOT, Toffoli and X quantum gates.

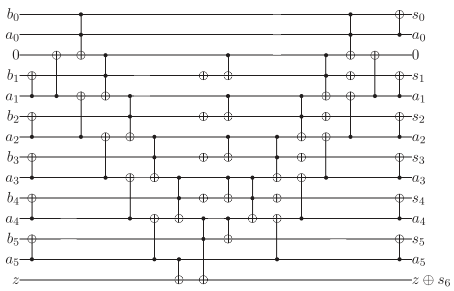
The X gate is equivalent to a classical NOT gate. Combining X and CNOT enables the creation of equivalents to classical XOR and AND two-bit logic operations.

A simple adder of two integers A and B encoded each in N qubits will require at least 3N qubits, the A+B result being encoded in the B qubits and N qubits being used ancilla qubits that are reset to |0⟩ at the end of the module. An alternative consists in using a QFT and inverse QFT which reduce the need for ancilla qubits.

arithmetic operations can be useful in many algorithms

quantum reversible adders/multipliers can be derived from their classical counterparts (ripple-carry adders) or use a QFT and IQFT (inverse QFT) to reduce the need for ancilla qubits

Adder type	Toffoli/T depth	Toffoli/T gates	Qubits required
Majority ripple [12]	$2n$	$2n$	$2n+1$
Prefix-ripple [Section A.1]	n	$3n$	$2n+1$
Carry look-ahead [13]	$4log_2(n)$	$10n$	$4n - log_2(n) - 1$
Fourier transform basis [33]	$3log_2(1/\epsilon)$	$3nlog_2(1/\epsilon)$	n



The ripple-carry adder for $n = 6$.

6-bit ripple-carry addition adder

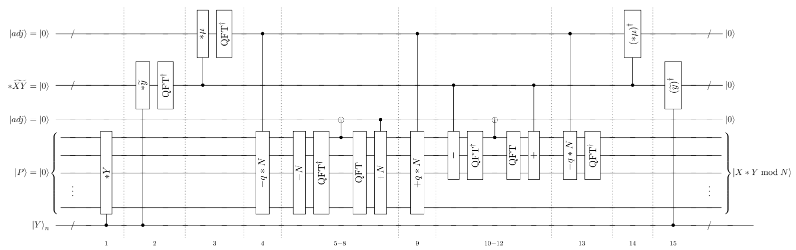


Figure 12: Barrett multiplication circuit using Fourier arithmetic. The numbers in the figure correspond to the steps of Algorithm 2.

Barrett multiplication circuit using QFT

Figure 745: various arithmetic computing can be implemented with quantum algorithms, mostly using a QFT. Sources: A new quantum ripple-carry addition circuit by Steven A. Cuccaro, Thomas G. Draper, Samuel A. Kutin and David Petrie Moulton, 2008 (9 pages) and High performance quantum modular multipliers, Rich Rinesy and Isaac Chuang, 2017 (48 pages).

Most of the adder examples show qubits initialized in their basis states with no superposition and no entanglement. But it is possible to create adders and other math modules that compute superposed states amplitudes[3187].

[3180] See Everything You Always Wanted to Know About Quantum Circuits by Edgard Munoz-Coreas and Himanshu Thapliyal, August 2022 (18 pages) which provides a good overview of arithmetic algorithms among other topics

[3181] See A new quantum ripple-carry addition circuit by Steven A. Cuccaro, et al, 2008 (9 pages).

[3182] See A Higher radix architecture for quantum carry-lookahead adder by SiyiWang, Anubhab Baksi and Anupam Chattopadhyay, Nature Scientific Reports, September 2023 (15 pages).

[3183] See High performance quantum modular multiplier by Rich Rines and Isaac Chuang, 2018 (48 pages).

[3184] See Arithmetic on Quantum Computers: Multiplication by Sashwat Anagolum, December 2018.

[3185] See Efficient Floating Point Arithmetic for Quantum Computers by Raphael Seidel et al, Fraunhofer Institute for Open Communication Systems, December 2021 (15 pages).

[3186] See Quantum circuits for floating-point arithmetic by Thomas Haener, Mathias Soeken, Martin Roetteler and Krysta Svore, 2018 (13 pages) which were patented.

[3187] See Quantum Adder for Superposition States by Xiaowei Lu et al, International Journal of Theoretical Physics, 2018 (10 pages, no free access).

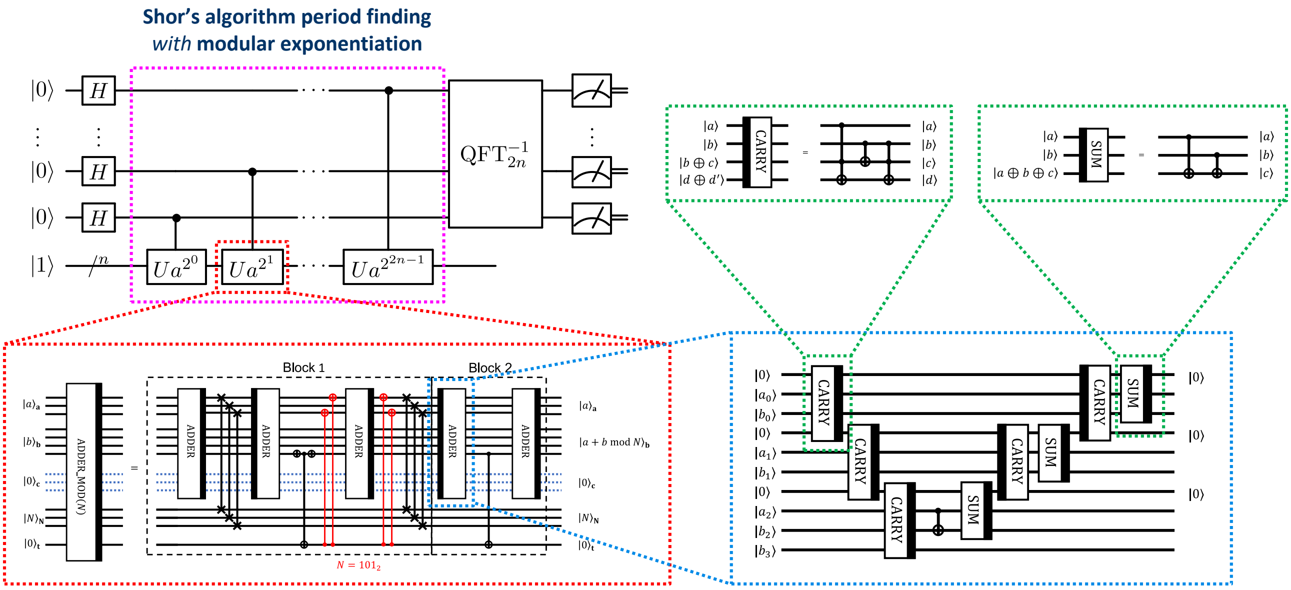


Figure 746: Shor's algorithm period finding decomposition. It is using many ADDER_MOD arithmetic modules which themselves use ADDER modules comprising CARRY and SUM modules. Modules schematics from Efficient realization of quantum primitives for Shor's algorithm using PennyLane library by A. V. Antipov ,E. O. Kiktenko and A. K. Fedorov, July 2022 (20 pages). Added in 2023.

Shor's famous integer factoring algorithm is using a lot of quantum arithmetic's in its period finding component aka modular exponentiation circuit, before its inverse QFT, as shown in Figure 746. It is so compute-intensive that a team in Japan proposed to create special units to run the related multipliers, using fluxonium qubits[3188]. Without however indicating how Shor's algorithm would be implemented in its entirety.

Amplitude Amplification

Amplitude amplification is a gate combination that is frequently connected to the phase kickback mechanism. It consists in amplifying one particular amplitude of the computational state vector of the control qubits that are submitted to an oracle function, at the expense of all the other amplitudes. It is used for example in the Grover operator from the Grover search algorithm as we'll see later. It can replace classical Monte Carlo simulations[3189].

In 2021, a researcher from the Fermi Lab at the DoE found a way to create an amplitude amplification working on a non-Boolean oracle[3190].

Qubitization was introduced by Low and Chuang in 2016 to facilitate the quantum computing of a time evolutive Hamiltonian e^{iHt} to a given error ϵ for particular types of Hamiltonian resulting from the projection of an unitary oracle onto a state created by another oracle [3191]. It enables the creation of optimized amplitude amplification techniques. It led to the creation of the singular value transformation algorithm (QSVT) that brings an exponential speedup with applying polynomial

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[3188] See 4-bit Factorization Circuit Composed of Multiplier Units with Superconducting Flux Qubits toward Quantum Annealing by Daisuke Saida et al, AIST, August 2023 (17 pages).

[3189] See A Survey of Quantum Alternatives to Randomized Algorithms: Monte Carlo Integration and Beyond by Philip Intallura et al, HSBC, Quantum Ventura Inc, March 2023 (22 pages).

[3190] See Non-Boolean Quantum Amplitude Amplification and Quantum Mean Estimation by Prasanth Shyamsundar, February 2021 (36 pages).

[3191] See Hamiltonian Simulation by Qubitization by Guang Hao Low and Isaac L. Chuang, 2016-2019 (23 pages).

transformations to the singular values of a block of a unitary[3192] [3193]. This is related to the notion of SVD (singular value decomposition) of matrices to find the values in their diagonal matrix[3194]. It can be used among other places in various quantum machine learning techniques and with quantum least squares fitting algorithms.

Quantum Signal Processing is a technique also created in 2016 to compute and analyze a Hamiltonian that depends on a value θ that can be viewed as an angle in some signal. The QSP is a simulation algorithm with potential exponential speedup. It is based on a linearization of the operator of a quantum walk using eigenvalue transformation using a constant number of queries[3195] [3196] [3197]. It can help for example to determine the phase of some signal[3198]. It was experimented in a 5 qubit system in 2023[3199].

A Generalized Quantum Signal Processing (GQSP) extension was introduced in 2023 to generalize QSP algorithm with fewer restrictions on the entry matrix[3200]. Other quantum digital processing techniques can avoid using a QFT and provide better speedups for lowpass and high-pass signal filtering[3201].

Quantum Fourier Transform

Classical Fourier transforms are used to decompose a signal into its compound frequencies. In signal theory, this allows us to identify the basic components of some sound by breaking it down into frequencies. In astrophysics, the atomic composition of stars is determined by a decomposition of the light spectrum, but this is done by an optical prism and not by Fourier transform. The same is true for Scio-type near-infrared sensors that determine the composition of food. A prism and the principle of diffraction therefore allow an optical Fourier transform to be performed.

The quantum Fourier transform was invented by **Don Coppersmith** (USA) in 1994, just before Peter Shor used it when designing his integer factoring and dlog algorithms. Shor's factoring algorithm is actually using a reverse quantum Fourier transform, reconstructing a divider of the input integer after using the period finding part of its algorithm.

A QFT is a quantum equivalent of a DFT or a FFT (Fast Fourier Transform). Its inverse operation, an inverse QFT is a QFT executed backwards, with its gates serialized in reverse order.

QFT is everywhere in the algorithm zoo as shown in red in Figure 737, page 865! Many known quantum algorithms are using it, including QPE (quantum phase estimation), HHL (linear equations), Shor's factoring algorithm and most QML algorithms.

[3192] See Quantum singular value transformation and beyond: exponential improvements for quantum matrix arithmetics by András Gilyén, Yuan Su, Guang Hao Low and Nathan Wiebe, 2018 (67 pages).

[3193] See A CS guide to the quantum singular value transformation by Ewin Tang and Kevin Tian, February 2023 (26 pages).

[3194] See Everything about matrix factorizations by Tivadar Danka, 2022 which explains well what is SVD and visually.

[3195] See Optimal Hamiltonian Simulation by Quantum Signal Processing by Guang Hao Low and Isaac L. Chuang, 2016 (6 pages).

[3196] See Methodology of Resonant Equiangular Composite Quantum Gates by Guang Hao Low, Theodore J. Yoder and Isaac L. Chuang, PRX, 2016 (13 pages).

[3197] See the tutorial A Grand Unification of Quantum Algorithms by John M. Martyn, Zane M. Rossi, Andrew K. Tan and Isaac L. Chuang, May 2021 (39 pages).

[3198] See Quantum Signal Processing, Phase Extraction, and Proportional Sampling by Lorenzo Laneve, March 2023 (11 pages).

[3199] See Realization of quantum signal processing on a noisy quantum computer by Yuta Kikuchi et al, Quantinuum, March 2023 (15 pages).

[3200] See Generalized Quantum Signal Processing by Danial Motlagh and Nathan Wiebe, August 2023 (20 pages).

[3201] See A quantum approach for digital signal processing by Alok Shukla and Prakash Vedula, September 2023 (29 pages).

A QFT is decomposing a series of qubits computational base states complex amplitudes in frequencies[3202]. The complex amplitude data encoding sits in the prepared register of qubits $|x_i\rangle$ with i=1 to n, as shown below. These qubits contain a set of N=2^n amplitudes α_j of the computational state basis orthogonal vectors $|j\rangle$, with j=0 to N-1.

The QFT implements a Discrete Fourier Transform (DFT) on these discrete amplitudes and converts it into a new computational state vector with amplitudes being the result of the QFT. The initial vector state can be written as in the formula on the right.

$$|\psi\rangle = \sum_{j=0}^{N-1} \alpha_j |j\rangle$$

The QFT creates a new state vector $\mathrm{QFT}_N(|\psi\rangle)$ with N β_k amplitudes of the N computational basis vectors $|k\rangle$. It is a formula similar to the above starting point.

$$\mathrm{QFT}_N(|\psi\rangle) = \sum_{k=0}^{N-1} \beta_k |k\rangle$$

The amplitudes β_k are computed with a big sum using all the amplitudes α_j with the coefficient ω^{jk} which is a N^{th} root of 1, meaning that $\omega^{jkN} = 1$.

$$\beta_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} \omega^{jk} \alpha_j$$

These coefficients $\omega^{jk} = e^{\frac{-2\pi i}{N} jk}$ explain the heavy use of R_n phase rotation gates in the QFT algorithm as described on the right. You remove the minus sign to obtain a reverse QFT.

$$\beta_k = \frac{1}{\sqrt{N}} \sum_{j=0}^{N-1} e^{-2\pi i \frac{jk}{N}} \alpha_j$$

In the end, the $\mathrm{QFT_N}$ is a unitary matrix transformation $[\mathrm{QFT_N}]$ with simple coefficients $[\mathrm{QFT_N}]_{jk}$, as in a DFT.

$$[\mathrm{QFT_N}]_{jk} = \frac{1}{\sqrt{N}} \omega^{jk}$$

When n=1 and N=2, the QFT becomes a Hadamard gate transform. The QFT is indeed presented as a generalization of the Hadamard operation, applied to dimensions N>2.

Since preparing such an arbitrary vector could take an exponential time with regards to the number of qubits, it is usually done through some faster preparation mechanism like in Shor's algorithm.

What are we really getting out of a QFT? Let's say we have 4 qubits and complex amplitudes with a rotating phase by 45° steps. It means we'll have a full phase periodic rotation for each of the 8 amplitudes and 2 full rotations for the whole state vector. The QFT will then output a register with the third qubit at $|1\rangle$ and all the others at $|0\rangle$.

This third qubit corresponds to value 2, which is the frequency of the phase rotation. But we could have a more complex QFT with several added frequencies in the signal.

Getting all the β_k coefficients and frequencies still wouldn't make much sense. Indeed, recovering a whole computational basis state would require running the QFT at least one or two orders of magnitudes of 2^N. We'd lose any quantum speedup. What is usually done is to directly reuse this vector in the remainder of another quantum algorithm like Shor. Otherwise, after running the QFT a limited number of times, we can extract the computational basis state with the highest frequency. In other words, it means we'll have the main frequency extracted from the QFT, but not all of them.

The QFT relies on two types of logic gates: Hadamard gates to perform an overlay and two-qubit phase-controlled R gates whose phase is inversely proportional to 1 up to N (Figure 747). This creates a huge problem of accuracy in the calculation: the larger N is, the smaller the angle of rotation of the qubit in its Bloch sphere will be and the more impacting the phase errors will be. This requires very precise control of the activation of the qubits.

[3202] See Quantum circuit for the fast Fourier transform by Ryo Asaka et al, 2020 (20 pages) which describes a QFT variant using a faster basis encoding for the input register.

In practice, phase-controlled R gates are generated by a combination of H, Z and T gates, plus a CNOT for the entanglement of the control qubit with the target qubit. And it takes a lot! For example, for an R_{15} gate, 127 H/Z/T gates must be used to obtain an accuracy of 10^{-5}, which is enormous[3203]. This can be optimized with auxiliary qubits. And of course, we must integrate the associated error correction codes that add a good order of magnitude to the number of quantum gates in the depth of the calculation. This mainly impacts the calculation duration since the error correction codes are supposed to lengthen the duration of the qubit coherence.

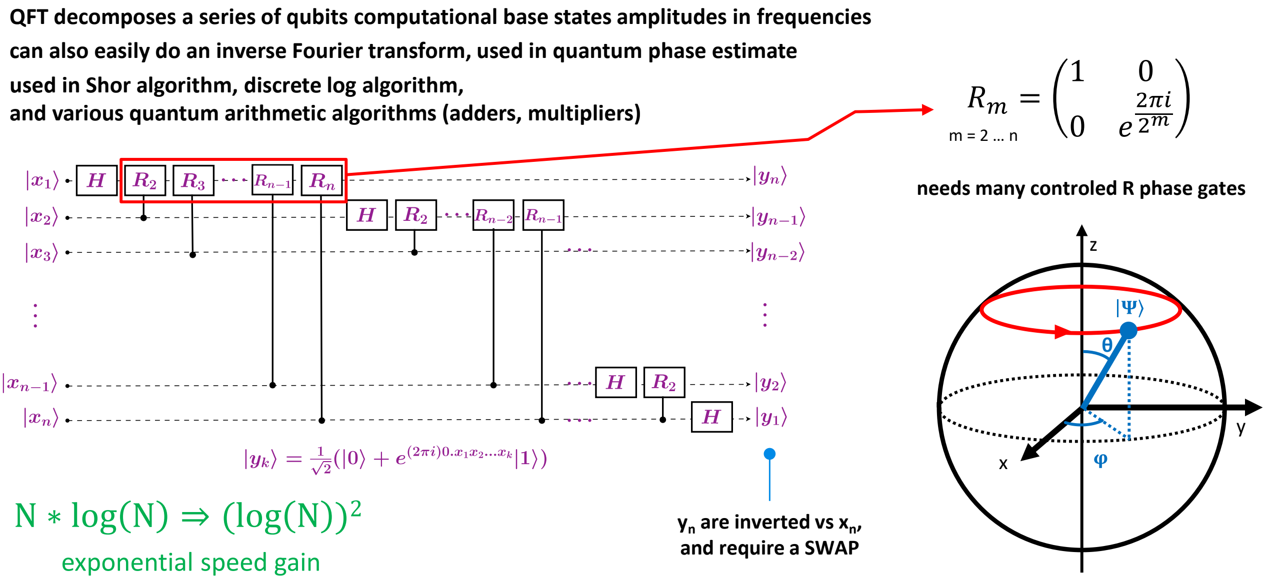

Figure 747: the quantum gates resource constraint with a QFT are enormous as its size grows. It requires controlled R phase gates that are very costly to generate, using in many cases a long combination of tens of H and T gates. (cc) Olivier Ezratty and various sources.

How about an R_{2048} gate decomposition, the last of a long series of R phase gates to break a 2048-bit RSA key? That's about the same number of gates. This comes from the Solovay-Kitaev theorem according to which this decomposition depends only on the targeted error rate[3204]. In the case of superconducting qubits, the generation of variable phase gates is achieved by sending a shorter microwave pulse.

The QFT theoretical exponential acceleration vs its classical counterpart may still be limited. A 2022 paper from Edwin Miles Stoudenmire et al conjectured that most of the QFT algorithm has low entanglement with the consequence that it could be efficiently classically emulated[3205]. Consequently, a QFT could be used as a quantum-inspired classical algorithm for computing a discrete Fourier transform using tensor networks. Before that, Alastair A. Abbott has even dequantized the QFT under some circumstances in 2010/2020[3206].

Quantum Phase Estimation (QPE)

Quantum phase estimation is an algorithm used to find the phase of an eigenvector of a unitary operator U. This operator can be implemented as an oracle function applied to a quantum state $|\psi\rangle$ that is decomposed in n controlled-unitaries operating on m qubits from $|\psi\rangle$.

[3203] See Efficient decomposition methods for controlled-R n using a single ancillary qubit by Taewan Kim et Byung-Soo Choi, 2018 (7 pages) and Approximate quantum Fourier transform with O(n log(n)) T gates by Yunseong Nam et al, 2020 (6 pages).

[3204] The main method of R_n gate decomposition is documented in Optimal ancilla-free Clifford+T approximation of z-rotations by Neil J. Ross and Peter Selinger, 2016 (40 pages). It's cotton!

[3205] See The Quantum Fourier Transform Has Small Entanglement by Jielun Chen, E.M. Stoudenmire, and Steven R. White, PRX Quantum, October 2022-October 2023 (27 pages).

[3206] See De-quantisation of the Quantum Fourier Transform by Alastair A. Abbott, June 2010-January 2020 (14 pages).

We are then looking for the phase angle θ according to $U|\psi\rangle = e^{2\pi i\theta}|\psi\rangle$. This algorithm is based on an inverse QFT. Practically speaking, the QPE can estimate the angle θ with a precision ϵ with executing U for $O(1/\epsilon)$ times (meaning, with a high probability within an error ϵ). The angle θ is encoded over n classical bits at the exit of the inverse QFT.

This algorithm was proposed by Alexei Kitaev in 1995[3207]. It is a cornerstone of quantum chemistry quantum algorithms to find the ground state Hamiltonian of a quantum many-body system. In that case, the initial preparation of the Hamiltonian must be done classically with care[3208]. Il is also used in quantum linear algebra (HHL) and quantum random walks. A QPE circuit is also very similar to Shor's integer factoring algorithm, the difference being in the way the period finding part is encoded and the circuit output exploited[3209].

As it makes use of a QFT, this algorithm requires high-fidelity qubits and FTQC quantum architectures. Still, some QPE variations exist that could run on NISQ QPUs[3210 3211 3212 3213].

Quantum Amplitude Estimation (QAE)

Quantum amplitude estimation was proposed by Gilles Brassard et al in 2000. In simple terms, it is used to evaluate the average value of a quantum oracle. The original version is a combination of a quantum phase estimation and Grover's algorithm (Figure 748). Some newer versions avoid the quantum phase estimate step and are more suitable to NISQ architectures[3214]. Its speedup is only quadratic.

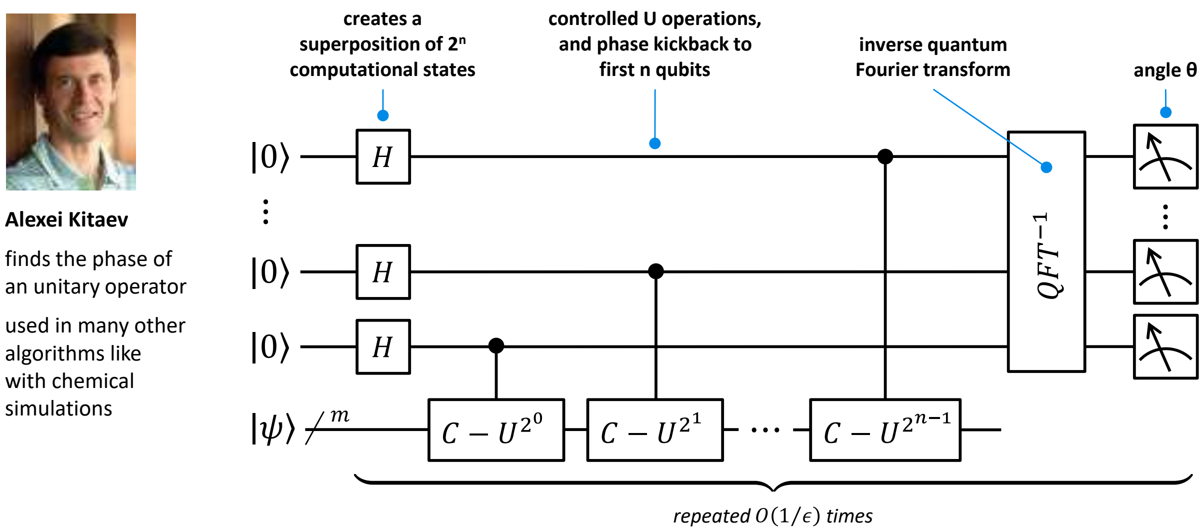

Figure 748: the quantum phase estimate algorithm explained. The probed unitary U must be decomposed beforehand into components. (cc) Olivier Ezratty with various sources. 2022-2023.

[3207] New versions appear from time to time like Quantum Algorithm for the Direct Calculations of Vertical Ionization Energies by Kenji Sugisaki et al, University of Osaka, March 2021 (6 pages).

[3208] See Initial state preparation for quantum chemistry on quantum computers by Stepan Fomichev et al, Xanadu, Volkswagen et al, October 2023 (30 pages)..

[3209] See Finding Prime Factors of Integer using Quantum Computer by Saptashwa Bhattacharyya, June 2022.

[3210] See On low-depth algorithms for quantum phase estimation by Hongkang Ni, Haoya Li and Lexing Ying, February 2023 (9 pages).

[3211] See Even shorter quantum circuit for phase estimation on early fault-tolerant quantum computers with applications to ground-state energy estimation by Zhiyan Ding and Lin Lin, Berkeley, November 2022 (46 pages).

[3212] See Quantum phase estimation of multiple eigenvalues for small-scale (noisy) experiments by Thomas E. O'Brien, Brian Tarasinski and Barbara Terhal, New Journal of Physics, 2019 (43 pages).

[3213] See An adaptive Bayesian quantum algorithm for phase estimation by Joseph G. Smith et al, March 2023 (7 pages).

[3214] See Quantum Amplitude Amplification and Estimation by Gilles Brassard, Peter Hoyer, Michele Mosca and Alain Tapp, 2000 (32 pages) and Amplitude estimation without phase estimation by Yohichi Suzuki et al, 2019-2022 (13 pages).

Uncompute trick

The uncompute trick was created by Charles Bennett in 1989. It is used to rewind some parts of an algorithm affecting ancilla or input qubits. It cleans up the state of a qubits register without requiring a qubit reset that may damage the stored values in the qubits with the algorithm results.

It is also used to disentangle the ancilla qubits from the input qubits (Figure 749). It then makes it possible to go on using these ancilla qubits for the remainder of the algorithm. In a word, it cleans up the qubits register garbage at the end of some computing. The transformation works if the unitary U_f is a reversible circuit which is the case for any combination of quantum gates, without any measurement done in between.

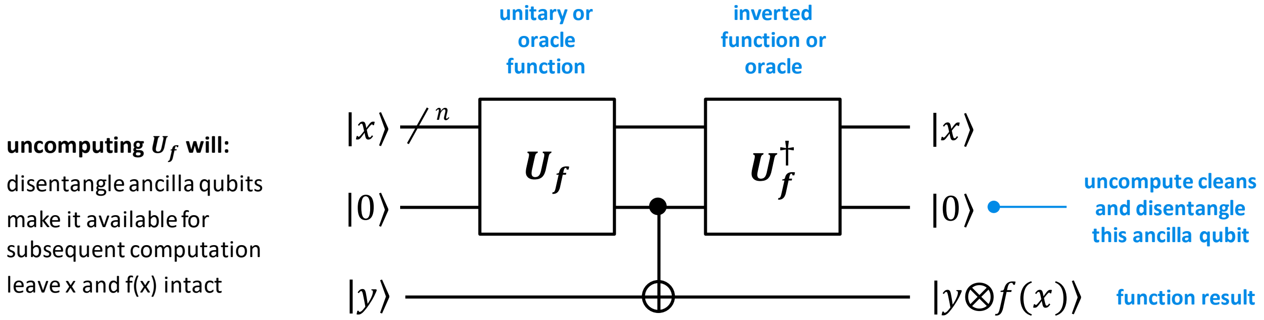

Figure 749: the uncompute trick algorithm cleans up a register and its ancilla qubits with disentangling them from the data qubits while preserving the function result from the computed algorithm. (cc) Olivier Ezratty with various sources. 2022.

The uncompute trick is often used when the algorithm is running an oracle function. But it presumes we are working with "clean" qubits with no error. In a NISQ setting, an oracle inversion would generate so many errors that the inverted state would not really correspond to the initial state.

Linear and differential equations

Many other quantum algorithms exist that allow complex mathematical operations such as solving differential equations, inverting matrices, or processing various linear algebra problems. They are then used elsewhere as in QML and in fluid dynamics simulations.

HHL is the best-known linear equation solving algorithm, named after its creators Harrow, Hassidim and Lloyd, and created in 2009. It allows to solve linear equations, with an exponential performance gain. Its input is a combining a $2^N x 2^N$ sparse Hermitian matrix A (or is prepared to be Hermitian, with some block-encoding technique as shown below in Figure 750[3215]) and an input state $|b\rangle$ with 2^N amplitudes. Its output is $|x\rangle = |A^{-1}b\rangle$. Namely, it inverts matrix A and multiplies it by $|b\rangle$. The processing is done in time O(N) with an exponential speed-up. But the state $|b\rangle$ must be prepared in some sort of qRAM that doesn't exist yet or be prepared quantumly. Also, input matrix A must follow a lot of constraints, with a having only a few nonzero values (sparsity)[3216].

There's a caveat, that Scott Aaronson explained well in 2015[3217]. The HHL output is a quantum state $|x\rangle$ that can't be read right away. The vector can be read to get some statistical information about it or, among other stuff, an evaluation of a dot product between $|x\rangle$ and another vector $|z\rangle$. If you want to know everything about $|x\rangle$, you'll need to repeat the operation 2^N times and lose any exponential advantage gained in the first place. In the end, HHL is not really inverting the matrix A with a real exponential speedup.

[3215] See Block-encoding structured matrices for data input in quantum computing by Christoph Sünderhauf, Earl Campbell and Joan Camps, Riverlane, February 2023 (26 pages) which proposes an efficient block-encoding technique.

[3216] See Quantum Resources in Harrow-Hassidim-Lloyd Algorithm by Pradeep Kumar et al, August 2023 (11 pages).

[3217] See Read the fine print by Scott Aaronson, Nature Physics, 2015 (3 pages).

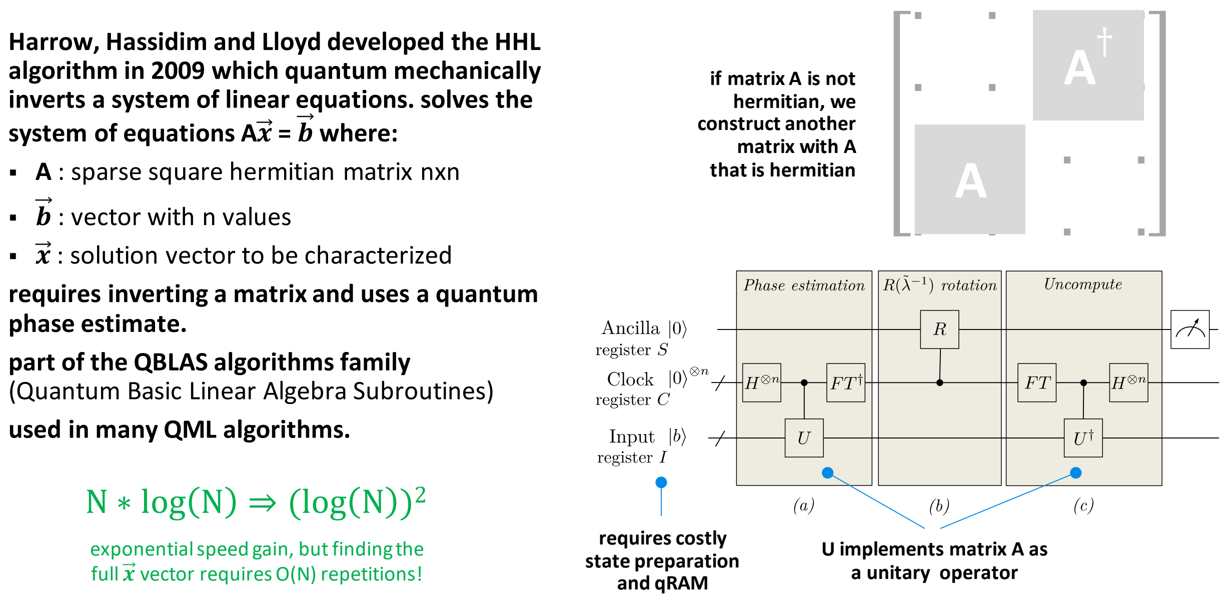

Figure 750: the HHL linear equation solving algorithm. But its output is a quantum state that is costly to decode and should ideally be used with a subsequent quantum algorithm. In circuit on the right, R is a single ancilla qubit. The computed matrix must be Hermitian. If it is not, it can be prepared to become Hermitian as explained on the top right. This preparation is named "block-encoding" or "standard-form" preparation. 2022.

Still, HHL is an algorithm that can be piggybacked by other algorithms to solve interesting problems. It is mostly used with quantum machine learning algorithms. It can also be used to solve physics and engineering problems involving Poisson's equation[3218].

Partial Differential Equations (PDE) are a type of differential equation involving multiple independent variables and their partial derivatives against some of these variables. The equation unknown is a function with several variables like time and space. They are linear or non linear. These equations are used in physics, fluid mechanics, nuclear fusion, engineering, financial services and applied mathematics. The Schrödinger wave equation is one of the most famous PDEs. Solving a PDE involves finding the unknown function that satisfies the given equation, usually using some boundary or initial conditions.

Various classical PDE equation solving methods exist, such as separation of variables, Fourier transforms, numerical techniques, and computer simulations.

Quantum algorithms are developed to solve PDEs, most of them being based on a QFT (for the frequency domain) or QPE (for the time domain[3219 3220 3221]). Most of these PDE quantum algorithms require a FTQC QPU and provide a theoretical polynomial speedup[3222 3223]. Some NISQ algorithms are proposed to solve simple PDEs using variations of QAOA[3224] and other variational algorithms,

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[3218] See Advanced Quantum Poisson Solver in the NISQ era by Walter Robson et al, September 2022 (4 pages) which shows however that such an algorithm doesn't work on an IBM 65-qubit system. A conclusion that shouldn't be generalized given future NISQ QPUs from various vendors including IBM could show up with better fidelities.

[3219] See Quantum algorithm for time-dependent differential equations using Dyson series by Dominic W. Berry and Pedro C. S. Costa, December 2022 (19 pages).

[3220] See Quantum simulation of partial differential equations via Schrodingerisation by Shi Jin and Yue Yu, December 2022 (9 pages).

[3221] See Fast quantum algorithm for differential equations by Mohsen Bagherimehrab, Kouhei Nakaji, Nathan Wiebe and Alan Aspuru-Guzik, June 2023 (14 pages).

[3222] See Quantum Algorithms for Solving Partial Differential Equations by Arthur Pesah, UCL, March 2020 (8 pages).

[3223] See High-precision quantum algorithms for partial differential equations by Andrew M. Childs et al, University of Maryland, February 2020-November 2021 (40 pages).

[3224] See Solving partial differential equations on near-term quantum computers by Anton Simen Albino et al, August 2022 (9 pages).

even for non-linear PDEs, although the current NISQ hardware is insufficient to deliver some quantum advantage[3225] [3226]. Other PDE algorithms are proposed for analog quantum simulators[3227].

Hamiltonian simulation

Literally, as we've seen when describing Schrödinger's wave equation, a Hamiltonian of a quantum system is its description and evolution of its total energy, including kinetic and potential energy, over time. It is hard to evaluate for a given single quantum object and even harder for a multi-objects system. That's what Hamiltonian simulations are all about. One of their goals is to find the total energy of a system and its approximate ground state configuration which usually corresponds to its natural lowest-energy equilibrium state. It is particularly important in condensed matter physics and in organic chemistry.

In that later case, it makes it possible to find the way molecules are naturally organized in three dimensions, from simple peptides to large proteins. It could also theoretically help simulate the interactions between different molecules.

Simulating a Hamiltonian was at the core of Richard Feynman's idea coined in 1981 when he wondered whether a quantum system could simulate another quantum system more efficiently than a classical computer, breaking down the fatal exponential growth of computing resources required to implement this kind of simulation on classical computers.

Such a simulation problem is described by a Hamiltonian which is a Hermitian matrix H of size $2^N x 2^N$, when working with N qubits or N two-states quantum objects like spin-1/2 particles.

This is based on the hypothesis of a constant H or a slowly evolving one, as in the adiabatic theorem. The quantum system evolves over time according to (1), given e^{itH} is the exponential of H (times i and t), is a unitary matrix. It is a solution to the Schrödinger equation (2):

$$(1) \quad |\psi(t)\rangle = e^{itH}|\psi(0)\rangle \qquad (2) \quad i\hbar\frac{\partial\psi(t)}{\partial t} = E\psi(t).$$

Simulating a Hamiltonian consists in finding matrix H or some characteristics of H. That simple. Or not. The technique of the local Hamiltonian problem is a simplification of a Hamiltonian simulation. Thanks to special and general relativity, a Hamiltonian evolves according to local interactions. All Hamiltonian evolutions with only local interactions can be simplified as a combination of Hamiltonians acting on a limited space with at most ℓ of the total of N variables[3228].

Finding a system ground state of a local Hamiltonian is a QMA-complete class problem that can theoretically be efficiently solved on a quantum computer (QMA is defined later...)[3229]. Efficiently means with a polynomial instead of exponential growth in qubits. It mandates the usage of some quantum certificate or quantum proof, a validation technique used with QMA problems processing[3230].

There are of course many variations of Hamiltonian simulations depending on the type of quantum system to emulate and the characteristics we want to extract from H. It includes hybrid solutions associating classical and gates-based quantum computing including the quantum adiabatic algorithm.

[3225] See Variational quantum algorithms for nonlinear problems by Michael Lubasch et al, 2019 (15 pages).

[3226] See Quantum Variational Solving of Nonlinear and Multi-Dimensional Partial Differential Equations by Abhijat Sarma, Peter L. McMahon et al, November 2023 (23 pages).

[3227] See Analog quantum simulation of partial differential equations by Shi Jin et al, August 2023 (27 pages).

[3228] See Using Quantum Computers for Quantum Simulation by Katherine L. Brown et al, 2010 (43 pages).

[3229] See QMA-completeness: the Local Hamiltonian Problem by Paul Fermé, based on lecture notes by Umesh Vazirani and lecture notes by Thomas Vidick, 2015 (6 pages).

[3230] See Lecture 20: Local Hamiltonian ground state problems by Richard Kueng, on a course from John Preskill, December 2019 (17 pages).

If the Hamiltonian is of the family of an Ising model, it can be simulated using quantum annealers or quantum simulators. There are other families of Hamiltonians that can be simulated on quantum simulators or coherent quantum annealers with more than one degree of freedom (see Qilimanjaro).

Quantum teleportation

One of the most intriguing quantum gate-based quantum algorithms is qubit teleportation. It was created by Charles H. Bennett (USA), Gilles Brassard (Canada), Claude Crépeau (Canada), Richard Jozsa (USA), Asher Peres (Israel) and William K. Wootters (USA) in 1993[3231] .

It allows to teleport the state of a qubit from one place to another. The principle of this algorithm consists in exploiting a pre-existing quantum entanglement channel to transmit the state of a qubit from one end of this channel to the other. Teleportation involves the transmission of two classical bits in the protocol that are used to reconstitute the qubit sent on arrival. As a result, the transmission of the latter cannot be faster than light (Figure 751). Quantum teleportation is one type of feedforward operation.

Due to the quantum no-cloning theorem, this teleportation is a "move" and not a "copy" (or a "cut & paste" instead of a "copy & paste" to use an easy to understand analogy). The state of the transferred qubit is thus destroyed at its origin[3232]. The main use case of this algorithm and its many variants are in quantum cryptography and telecommunications systems that we will discover later.

It could also be used in distributed quantum computer architectures. Note that this algorithm can be tested locally in a quantum computer, as proposed by IBM in its Q Systems with Qiskit.

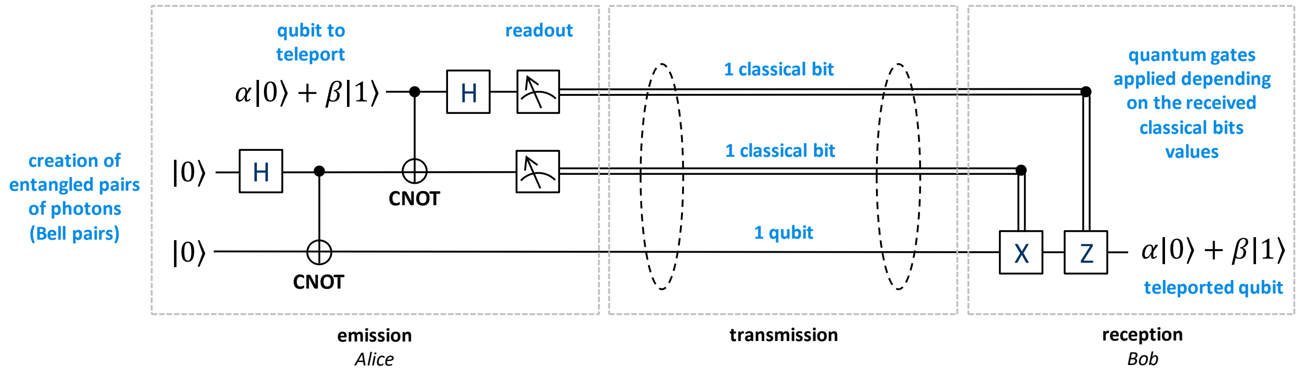


Figure 751: the quantum teleportation algorithm and its two classical channels. (cc) Olivier Ezratty, 2020, using various sources.

Higher level algorithms

We'll now cover higher level algorithms which are based on the algorithm's toolbox described in the previous part[3233]. Quantum software engineering requires three main set of skills: understanding both low and high-level algorithms, then some know-how about the way these algorithms can be assembled and also coupled with classical algorithms, and then, above all, how to find the ways to translate "business problems" into these algorithms[3234].

[3231] See Teleportation as a quantum computation by Gilles Brassard, 1996 (3 pages).

[3232] See Quantum Teleportation in a Nutshell by Fabian Kössel, 2013 (35 slides).

[3233] See Quantum Algorithms by Ashley Montanaro, July 2016 (62 slides), Quantum algorithms: an overview by Ashley Montanaro, 2015 (16 pages) and Quantum Algorithm Implementations for Beginners by Abhijith J. et al, April 2018-June 2022 (96 pages).

[3234] See the excellent review paper Quantum algorithms: A survey of applications and end-to-end complexities by Alexander M. Dalzell, Fernando G. S. L. Brandão et al, AWS, RWTH Aachen University, Imperial College London, Caltech, October 2023 (337 pages).

Oracle-based algorithms

One of the first quantum algorithms invented comes from David Deutsch, with its derivative called **Deutsch-Jozsa,** co-invented with Richard Jozsa and created in 1992. This algorithm makes it possible to characterize a function $f()$ called an "oracle" for which we know in advance that it will return for all its inputs, either always the same value, 0 or 1, or the values 0 and 1 in equal parts.

The algorithm makes it easy to determine if the function $f()$ is balanced or not. It is working to a set of qubits n. Function $f()$ is making some classical computing on each 2^N values from the computational basis of n qubits (Figure 752).

The input qubits are all initialized to $|0\rangle$ except one which is initialized to $|1\rangle$. They are then all superposed between $|0\rangle$ and $|1\rangle$ with Hadamard gates. The qubits are thus said to have simultaneously all possible 2^{N+1} combinations of values.

It is easy to understand why this quantum algorithm is much more efficient than its traditional version: in traditional computation, more than half of the possible input values would have to be scanned sequentially, whereas in the quantum version, they are all analyzed at the same time by the oracle function working on all 2^N values of the first N qubits. The result is obtained with a few series of quantum gates, almost instantaneously, and it is perfectly deterministic.

These superposed qubits are processed by the oracle which contains a set of gates implementing function $f()$ to be evaluated. The output is then measured to see if the function is balanced or not thanks to other Hadamard gates.

The initialization of the last qubit to $|1\rangle$ is used to generate an interference with the other qubits that will impact the values leaving the H gates after passing through the oracle. The function $f()$ is constant if the final measurement gives $|000 \dots 0000\rangle$ and unbalanced otherwise[3235].

What is the practical interest of such an algorithm given there are rather few functions $f()$ of this kind? This is an example of an ultra-powerful algorithm that has no known practical use to date. On top of that, there are very efficient classical probabilistic algorithms that are fast and cancel a good part of the quantum power gain coming from the Deutsch-Jozsa algorithm. This is particularly the case with the Monte Carlo search algorithm which evaluates the oracle function on a limited number of randomly selected inputs. The probability of errors depends on the number of evaluations and decreases very quickly[3236].

So, quantum computing is useless? Of course not. Other algorithms, less powerful but much more useful, have emerged since this patient zero of quantum algorithmics!

Bernstein-Vazirani's algorithm is less talked-about in textbooks. This algorithm created by Ethan Bernstein and Umesh Vazirani in 1992 is a variant of the Deutsch–Jozsa algorithm. Instead of using two different classes of functions, it tries to learn a secret string encoded in an oracle function (Figure 753).

[3235] To find out how it works in detail, you can see the associated mathematical formulas as well as Eisuke Abe's Deutsch-Jozsa Algorithm presentation, 2005 (29 slides). But it is not that obvious!

[3236] See on this subject the document Quantum Computation Models (30 pages).

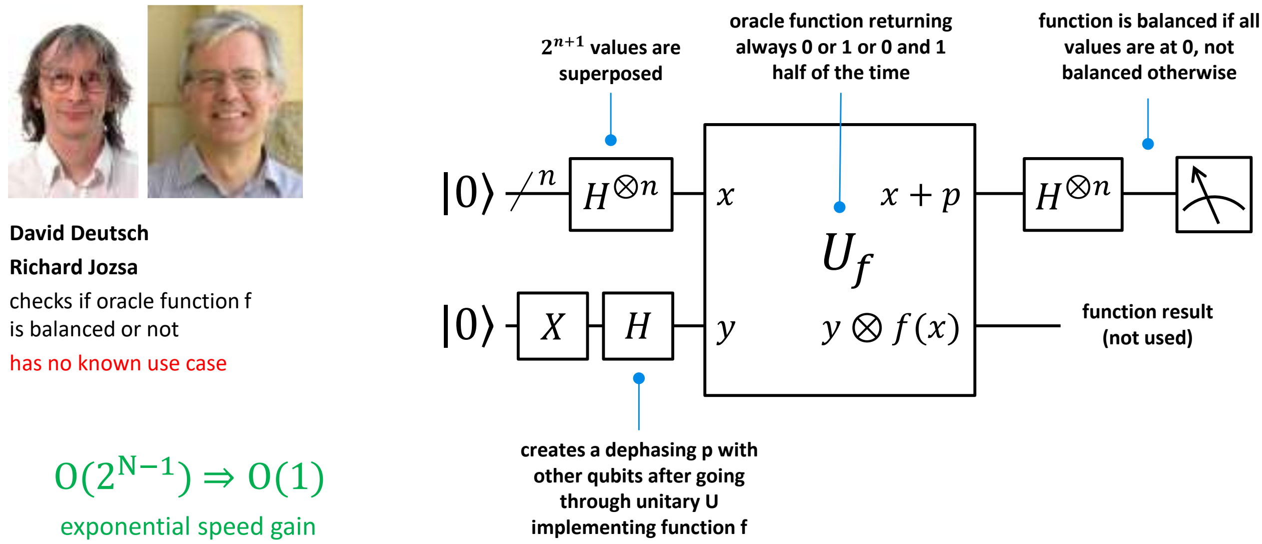


Figure 752: the famous Deutsch-Jozsa algorithm which says if a function is balanced or not and doesn't have any known practical application as far as I know. (cc) Olivier Ezratty with various sources. 2022-2023.

The algorithm was designed to prove an oracle separation between complexity classes BQP and BPP. The speedup of this algorithm is polynomial, but a derivative recursive version of the algorithm showcases an exponential speed gain. The algorithm has not much practical use cases although it could be used in some cryptography cases[3237]. It may also be relatively resilient to some forms of qubit noise[3238].

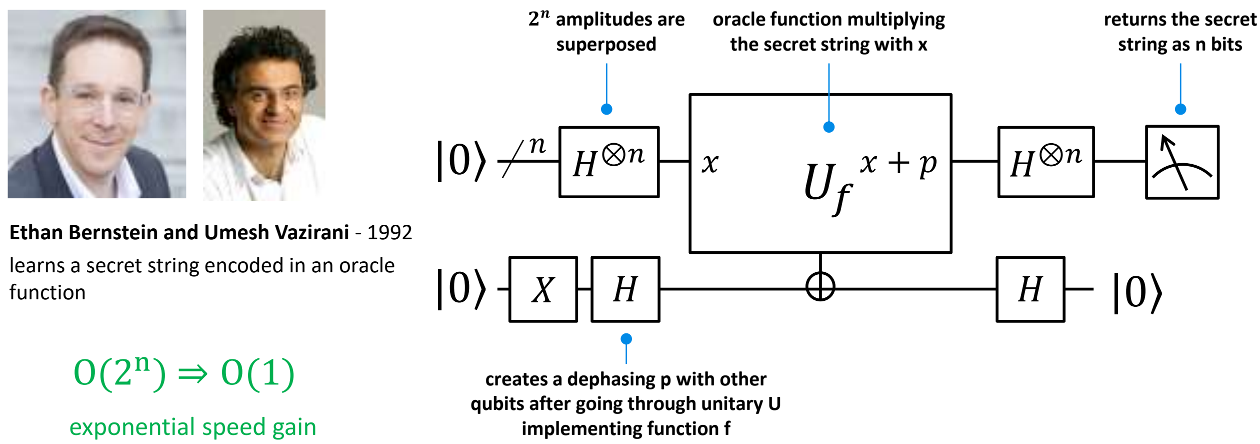


Figure 753: Bernstein-Vazirani algorithm. (cc) Olivier Ezratty with various sources. 2022-2023.

Simon's algorithm is a more sophisticated variant of the Deutsch-Jozsa algorithm with regards to its effect[3239]. It consists in finding the combinations of values that verify a condition imposed by the oracle function (Figure 754). It solves the so-called hidden subgroup problem (HSP). Its performance gain is very interesting and, this time, the algorithm is useful, particularly to solve path problems in graphs like with quantum walks. The gain in performance is typical of what quantum computing can bring: we go from a classical calculation which is in exponential time ($2^{N/2}$) to a linear time in N.

[3237] See Using Bernstein–Vazirani algorithm to attack block ciphers by Huiqin Xie et al, 2019 (22 pages).

[3238] See Effects of noise on performance of Bernstein-Vazirani algorithm by Archi Gupta et al, May 2023 (15 pages).

[3239] It is documented in On the power of quantum computation by Daniel Simon, 1997 (10 pages).

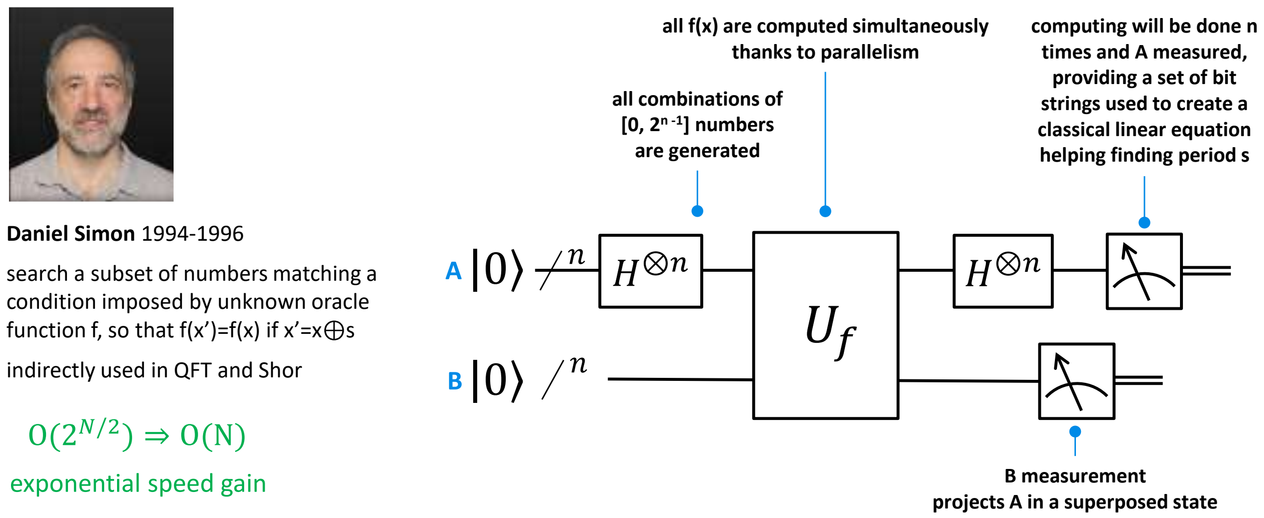


Figure 754: Simon algorithm. (cc) Olivier Ezratty with various sources. 2022-2024.

The other best-known algorithm in this category is **Grover'**s algorithm, created in 1996 by Lov Grover. It performs a fast quantum search in a database. It is however more generic: it can find an item in a long list that matches some specific criteria specified by an oracle function like finding the minimum item of an unsorted list of N integers, determining if a graph of N vertices is connected, or doing pattern matching searches, which can be useful in genomics.

The Grover oracle function is supposed to return 1 only for one combination of 0 and 1 with N bits. It also uses qubit state superposition to speed up processing compared to a traditional sequential search in an unsorted and non-indexed database. The performance improvement is significant compared to an unsorted database, except that in real life, we usually use indexed databases!

The question is to know if a 1 is yielded once and to which input combining 0 and 1s it corresponds. To do this, again with Hadamard gates, the algorithm will gradually amplify the combination of qubits of the result to an amplitude approaching 1 and make the other combinations of qubits converge to 0. This amplification operation is nicknamed the "global diffusion operator" and is repeated $\sqrt{N}$ times, N being the number of qubits. It explains why Grover's algorithm has only a quadratic speedup.

It will then be possible to measure the result and obtain the combination of qubits with the desired value (still, by repeating the algorithm several times and making an average of the results). This is well explained in Figure 755.

The computing time is proportional to the square root of the base size and the storage space required is proportional to the logarithm of the base size. A classical algorithm has a computation time proportional to the size of the base. Going from a time N to $\sqrt{\text{N}}$ is therefore an interesting gain, but it will not transform an exponential size problem into a polynomial size problem (2^N to N power M). On the other hand, this algorithm can be exploited to be integrated into other algorithms such as those that allow the discovery of the optimal path in a graph or the minimum or maximum number of a series of N numbers. Grover's algorithm is also used in some quantum machine learning algorithms like for measuring min/max/mean distances or other metrics between sets of data points and for automatic clustering[3240].

[3240] See Quantum Policy Iteration via Amplitude Estimation and Grover Search -- Towards Quantum Advantage for Reinforcement Learning by Simon Wiedemann et al, TUM and Siemens, June 2022-May 2023 (17 pages).

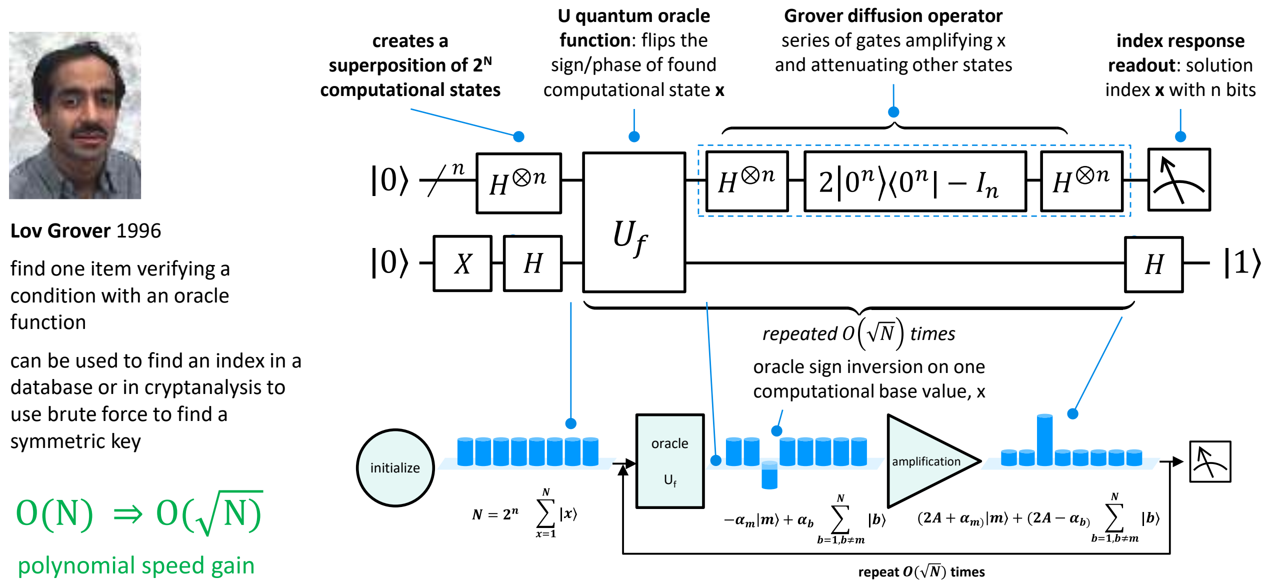


Figure 755: Grover algorithm. (cc) Olivier Ezratty with various sources, 2020-2023.

The settings are in the oracle, that encodes a constraint function or a function cost for which we are searching for a minimum. It can be used to solve traveling salesperson problems[3241] and various other optimization problems[3242].

Grover's algorithm implementation typically requires a FTQC QPU given its size grows polynomially with the number of qubit and cannot therefore be shallow. Practically speaking, a NISQ Grover implementation can't scale beyond a couple qubits, making it impractical to obtain any quantum advantage[3243 3244 3245]. When polling classing data, it also needs some quantum memory (qRAM) to "load" the related database in memory, in the oracle function[3246]! There are however some available optimization techniques available for quantum state preparation in the Grover oracle that could remove this qRAM requirement[3247].

In a 2002 lesson, **Serge Haroche** points out the known fact that these search algorithms have quantum optical interference equivalent implementations, as described in Figure 756 with 4 qubits. This has been described for a while, even trying to use only classical optical elements.

An oracle-based algorithm is efficient if the oracle itself is efficient, which depends on its implementation. If it is accessing some classical data or function, the algorithm's efficiency may be questionable in the end.

3241 See A Realizable GAS-based Quantum Algorithm for Traveling Salesman Problem by Jieao Zhu et al, December 2022 (16 pages).

3242 See Quantum algorithm for robust optimization via stochastic-gradient online learning by Debbie Lim, João F. Doriguello and Patrick Rebentrost, CQT, April 2023 (21 pages) which improves Oracle-based robust optimization via online learning by Aharon Ben-Tal, Elad Hazan, Tomer Koren and Shie Mannor, Technion, Operations Research, 2015 (21 pages).

3243 See Performance of Uncoded Implementation of Grover's Algorithm on Today's Quantum Processors by Yunos El Kaderi et al, Cergy Paris University, ENSEA, CNRS, December 2022 (5 pages).

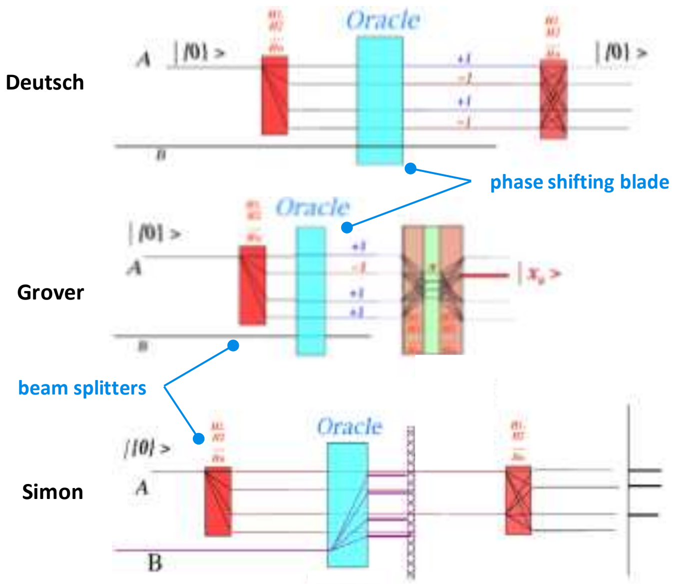
3244 See Analyses of the viability of automating the quantum circuit construction of Grover Oracle for executing wildcard searches on NISQ processors by Willie Huang, AWS, March 2023 (11 pages).

3245 See Better-than-classical Grover search via quantum error detection and suppression by Bibek Pokharel and Daniel Lidar, USC, November 2022 (21 pages).

3246 This is notably documented in Quantum algorithms for linear algebra by Anupam Prakash, 2015 (92 slides).

3247 See Black-box quantum state preparation without arithmetic by Yuval R. Sanders et al, UNSW and Microsoft Research, 2018 (5 pages).

Various papers argue that, from a practical standpoint, these implementations don't scale well with a growing number of qubits, but they remind us that quantum algorithms are toying with waves and interferences and that optical analogies are well suited to understand their underlying processes[3248].



Deutsch algorithm can be described as a quantum interference process using 2^n paths: the Hadamard gates H_1 to H_n play the role of beam splitters decomposing and recomposing the states of register A. Oracle O is an analog to a phase shifting blade that modifies the relative phase of superposed states in A.

Grover algorithm optical equivalent would combine two beam splitters with a phase shifting blade. In both Deutsch and Grover, the sought-after effect is to favor in the final state a state analog to a shiny interference fringe.

Simon algorithm uses entanglement and projective measurement. The oracle creates some entanglement betwen input registers A and B, then measurement of B projects A in a superposition of two states. Then, after a mix in the beam splitter, the final interference signature provides an information on the separation between these two states, then on the searched period.

Figure 756: quantum algorithms explained with interferences when implemented with quantum optics. Source: Serge Haroche, Chaire de Physique quantique. 2001-2002. 8th lesson, February 26th, 2002.

In March 2023, E.M. Stoudenmire (Flatiron Institute) and Xavier Waintal (CEA IRIG) questioned the very existence of any quantum advantage for Grover's algorithm, even with an entire quantum oracle. Their rationale is that the Grover diffusion operator is not using entanglement[3249]. Grover's algorithm quantumness is implemented in the oracle which is usually unspecified. The authors argue that the oracle could be efficiently classically emulated. This debate shows a typical discrepancy between theory and practice. The paper drove Scott Aaronson to write a scathing response and various responses from the authors[3250] [3251]. Like Mathias Troyer says, Grover's algorithm may bring some polynomial speedup compared to classical algorithms, but its constants create a threshold that shows up in non-human timescales even with future FTQC QPUs. A paper from China then argued that Grover's algorithm could bring some advantage, even on NISQ QPUs[3252].

Shor integer factoring

Shor's factoring allows you to decompose integers into prime numbers much faster than with a traditional computer. It works in two stages as described in Figure 758 and Figure 757 [3253]:

- A **classical part** which reduces the factoring problem to an order-finding problem and produces a function f(x) encoded as a unitary block in the quantum part of the algorithm.

[3248] See Grover's search algorithm: An optical approach by P. G. Kwiat et al, 1999 (6 pages), Implementation of quantum search algorithm using classical Fourier optics by N. Bhattacharya and al, 2002 (4 pages) and Classical wave-optics analogy of quantum information processing by Robert J. C. Spreeuw, 2001 (9 pages).

[3249] See Grover's Algorithm Offers No Quantum Advantage by E.M. Stoudenmire and Xavier Waintal, March 2023 (16 pages) also debated on SciRate.

[3250] See Of course Grover's algorithm offers a quantum advantage!, Scott Aaronson, March 2023.

[3251] See Xavier Waintal responds (tl;dr Grover is still quadratically faster), Xavier Waintal's response in Scott Aaronson's blog, March 2023.

[3252] See Quantum Advantage of Noisy Grover's Algorithm by Jian Leng et al, June 2023 (11 pages).

[3253] See On Shor's algorithms, the various derivatives, their implementation and their applications by Martin Ekera, 2019 (135 slides) which describes in detail how Shor's algorithm works.

- A **quantum part** implementing a quantum phase estimate, made of three sub-stages with a set of Hadamard gates, repeated squaring for a modular exponentiation transformation and an inverse QFT. The period finding and inverse QFT are responsible for the exponential speedup of Shor's factoring algorithm. The quantum part of the algorithm generates an intermediate result in the form of some periods in the computational basis. The QPE needs at least n qubits for inverse QFT, n being the smallest integer with $N \leq 2^n$, N being the integer to factorize and n qubits for the period finding part. It is executed several times and scales as log(x), x being the order of a as shown in Figure 757.
- Another **classical part** that extracts a period from the results consolidating the results from several shots of the quantum part, using the continued-fractions algorithm, then, yields a prime factor of the factorized integer.

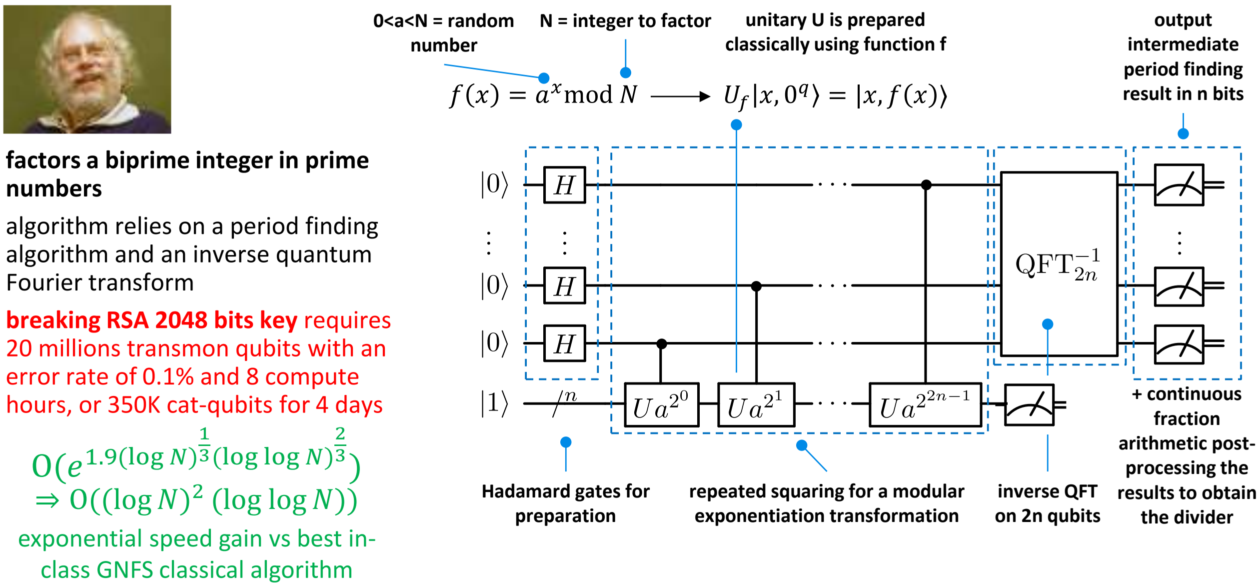


Figure 757: Shor's algorithm with all its qubits. Source: Wikipedia description of Shor's factoring algorithm. 2021-2023.

What is the gain in speed generated by Shor's algorithm compared to conventional calculation? The computation time scale goes from N*log(N) for the best simple Fourier transforms to $\log_2(N)$ for the QFT. We thus go from a linear order of magnitude to a logarithmic order of magnitude. But the state of the art of classical integers factoring is much better than the usual $O(\sqrt{N}/2)$ divisions (itself in $O(N^2)$) pointed out in textbooks, like:

$$\exp((1.923 + o(1))(\log N)^{1/3}(\log \log N)^{2/3})$$

One of the first implementations of Shor's algorithm took place in 2001 at IBM with an experimental quantum computer of 7 qubits, to factorize the number 15. Since then, we have just moved to a 5-digit number, 56,153[3254], but with a different factoring algorithm than Shor's algorithm.

It is in fact an optimization algorithm that was running on a D-Wave quantum annealer! A record was reached in 2016 with the factorization of 200,099 with 897 qubits on a D-Wave but with yet another algorithm than Peter Shor's[3255].

[3254] This is documented in Quantum factorization of 56153 with only 4 qubits by Nikesh S. Dattani and Nathaniel Bryans, 2014 (6 pages).

[3255] The record was beaten in 2019, it was beaten by engineers from Zapata Computing and IBM with the factoring of 1,099,551,473,989 into 1,048,589 * 1,048,601, but using a variational hybrid algorithm on a few qubits, and with an undocumented speedup. See Analyzing the Performance of Variational Quantum Factoring on a Superconducting Quantum Processor by Amir H. Karamlou et al, 2019 (14 pages).

It is important to remember that Shor's algorithm theoretically allows to break the public keys of the RSA cryptography that is commonly used in Internet security. Public keys work by sending a very long integer number to a recipient who already has its divisor.

It must just divide the large number received by his divisor to retrieve the other divisor and use it to decipher the encrypted message. Whoever does not have the divisor cannot exploit the complete key unless he has enormous traditional computing power to find his divisors.

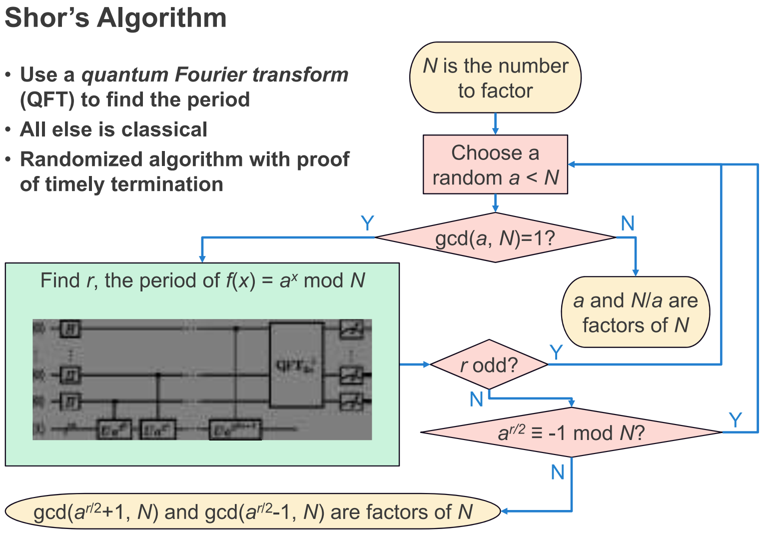


Figure 758: Shor's algorithm high-level components. Source: Quantum Annealing by Scott Pakin, NSF/DOE Quantum Science Summer School June 2017 (59 slides).

Until now, only NSA supercomputers have officially been able to break reasonably sized keys in the 256 to 800 bits range. But at 1,024 bits and beyond, the task is inaccessible in a reasonable amount of time for these supercomputers. As far as we know!

In theory, this would become accessible someday to large fault-tolerant quantum computers[3256]. To break a good 2048-bit RSA public key, one will still have to be patient because it requires to create quantum computers with a very large number of corrected qubits. It takes about twice as many logical qubits as there are bits in an RSA key. To factorize a 2048-bit RSA key, a minimum of 4098 logical qubits are required[3257]. Because of qubit noise, it is estimated that hundreds to tens of thousands of physical qubits per logical qubit would be needed.

Thus, such an RSA key break would require about 22 million qubits according to a famous Google algorithm from 2019. This algorithm based on a distance-27 surface code with about 14,000 logical qubits would require a bandwidth of 7.3 terabit/s per logical qubit for error processing[3258]. Another option would be to use some addressable quantum memory and reduce the qubits count to 13,436[3259].

Yet, another option would be to rely on qubits like cat-qubits for which the physical/logical qubits ratio would be much lower, in the 10-100 range.

Note that Shor's algorithm also allows to break cryptography using elliptic curves, which competes with RSA cryptography. By the way, some of the cryptography used in the Bitcoin protocol would also be broken by Shor integer factoring, which we will see page 709.

[3256] See Shor's Algorithm Does Not Factor Large Integers in the Presence of Noise by Jin-Yi Cai, University of Wisconsin-Madison, June 2023 (21 pages).

[3257] The formula is 2xN+2 qubits. See Factoring using 2n + 2 qubits with Toffoli based modular multiplication by Thomas Haner et al, 2017 (12 pages) and Circuit for Shor's algorithm using 2n+3 qubits by Stephane Beauregard, 2013 (14 pages).

[3258] See Hierarchical decoding to reduce hardware requirements for quantum computing by Nicolas Delfosse, January 2020 (8 pages).

[3259] See Factoring 2048 RSA integers in 177 days with 13436 qubits and a multimode memory by Élie Gouzien and Nicolas Sangouard, March 2021 (18 pages). It requires some quantum memory of 2 hours storage time and qubits with a 10^{-3} error rate. The authors suggest realizing such an architecture with a microwave interface between a superconducting qubits processor and some multiplexed addressable quantum memory using the principle of photon echo in solids doped with rare-earth ions like Erbium or NV centers. Their physical qubits would use some 3D gauge color error correction codes.

In any case, Shor's algorithm has been terrorizing security specialists for a couple decades. This explains the interest in exploiting quantum keys distribution, which are supposed to be tamper-proof because their interception can be detected by their legitimate recipient, as well as post quantum cryptography, consisting in classical cryptographic algorithms and methods to make them (theoretically) tamper-proof by quantum computers using Shor's or any other algorithm.

But Shor's factoring is not the only quantum factoring algorithm created so far. Several other options are investigated like the **Variational Quantum Factoring** algorithm alternative that maps the factoring problem to the ground state of an Ising Hamiltonian that could be solved in a hybrid manner using the quantum approximate optimization algorithm (QAOA), running on a gate based NISQ processor[3260]. But so far, the scalability of this algorithm with a large number of qubits and integer numbers is not proved.

Shor dlog

Peter Shor did create his quantum dlog (*aka* discrete logarithm) algorithm simultaneously with his factoring algorithm in 1994 and solves another classically intractable problem. The discrete logarithm $k=\log_b(a)$ (or logarithm of a in base b) is an integer k such that $b^k = a$, where a and b are given integer numbers. You understand that this problem is intractable with digging in group isomorphisms logic, which I won't cover.

The dlog algorithm could help break Diffie-Hellman signatures, including those using elliptic curves.

Integer factoring and finding a dlog are both special cases of the hidden subgroup problem for finite Abelian groups (as seen just below).

Hidden Subgroup Problems (HSP)

The hidden subgroup problem is a generic problem which encompasses Shor's order finding, Simon's, the discrete log and the graph isomorphism problems. The definition of this problem is the following: let G be a group and $H \subseteq G$ one of its subgroup. Let S be any set and $f: G \rightarrow S$ a function that distinguishes cosets of H, meaning that for all g_1 and g_2 in G, $f(g_1)=f(g_2)$ means $g_1H=g_2H$ (left cosets of H are equal). The hidden subgroup problem (HSP) is about determining the subgroup H using calls to function f with any combinations of g's in G.

Verstanden? Well, not really if you have no idea of what is a group, a subgroup, a set and a coset. So let's define these:

- **Set**: arbitrary ensemble of elements.
- **Subset**: ensemble of some elements from a set.
- **Group**: a set coupled with an operation on the elements in the set, where any combination of two elements with this operation gives another item from the group. One example is the group $\mathbb{Z}$ of all integers associated with the addition. Any addition of integers yields an integer. A group also has an identity element (0 for integers) and all elements have an inverse element (inverse of integer a is -a). Operations are also associative: the order in which the operation is done is not important. For example, with integers: a+(b+c)=(a+b)+c.
- **Subgroup**: subset of the group G being also a group with regards to its associated operation. For example, with integers, the even set is a subgroup with addition since adding even numbers always give even numbers. It is however not true with uneven numbers given adding two uneven numbers gives an even number.

[3260] See Variational Quantum Factoring by Eric R. Anschuetz, Alán Aspuru-Guzik et al, 2018 (18 pages).

* **Coset**: set or ensemble of elements from G that contains all elements of H multiplied by a given item g from G. If you multiply all elements of H on the left by one element g of G, the set of products is a left coset. If multiplied by the right, it is a right coset (these operations may be non-commutative with some non-integer elements like matrices). A subgroup H of a group G may be used to decompose G into disjoint equal-size cosets. H cosets have the same number of elements as H.

Another definition of the hidden subgroup problem is: given a function f that is constant with all cosets of some subgroup H, find the subgroup H.

In its quantum version, the function f is usually implemented as an oracle. Solving HSP takes an exponential time classically with the size of log(|G|) whereas it can be solved efficiently for certain types of groups with quantum versions if done in a polynomial time of log(|G|), given log(|G|) is the logarithm of the number of elements in the group G[3261].

There are HSPs for Abelian and non-abelian groups given a group G is Abelian if xy = yx for all x, y in G. There is actually not a single quantum HSP algorithm but many of these that are applicable to different classes of groups and subgroups. It is a whole specialized field in itself.

One famous HSP problem is Pell's equation, a quadratic Diophantine equation of the form $x^2 - ny^2 = 1$ with n being a positive nonsquare integer, and x, y being integer solutions to the equation. A quantum algorithm to Pell's equation was created by **Sean Hallgren** at Princeton in 2002. It is based on a QFT[3262]. It has the particularity to be applied to an infinite group given we don't know in advance what are the bounds for x and y.

Is solving that equation useful? It may be for some cryptographic purposes. The **Hallgren algorithm** finds one solution to the Pell equation, who has many. It has a (roughly) polynomial time vs an exponential time for its classical version, so we're in for some exponential speedup.

Fluid mechanics

Fluid mechanics simulations are mostly based on solving Navier-Stokes equations (Figure 759). These are nonlinear partial differential equations, whose solution is essential to the aerospace industry, weather forecasting, plasma magneto-hydrodynamics and astrophysics. The problem with Navier-Stokes is nonlinear and quantum computing is implementing linear algebra.

Some tricks are available to turn nonlinear equations into linear ones[3263]. Various quantum algorithms have been designed to solve Navier-Stokes equations, mostly in the FTQC regime:

* **Hybrid methods** making use of a quantum nonlinear processing unit (QNPU) that is a unitary transformation implementing nonlinear operations[3264] [3265].
* **Continuous variable qubits** implementing nonlinearities with using multiple copies of the vector representing the state of the system to be investigated[3266]. Polynomial values are obtained by creating tensor products of all or part of these multiple copies of the vector state. It can simulate the dynamics of the nonlinear Schrödinger equation with quantum linear differential equation solvers.

[3261] See a good overview of various HSP algorithms in The Hidden Subgroup Problem Master's Project by Frédéric Wang, 2010 (99 pages).

[3262] See Polynomial-Time Quantum Algorithms for Pell's Equation and the Principal Ideal Problem by Sean Hallgren, 2006 (21 pages).

[3263] See Quantum computing for fluids: where do we stand? by Sauro Succi et al, July 2023 (7 pages).

[3264] See Variational quantum algorithms for nonlinear problems by Michael Lubasch et al, 2019 (15 pages).

[3265] See Hybrid quantum algorithms for flow problems by Sachin S. Bharadwaj and Katepalli R. Sreenivasan, July 2023 (19 pages).

[3266] See Quantum algorithm for nonlinear differential equations by Seth Lloyd et al, December 2020 (17 pages).

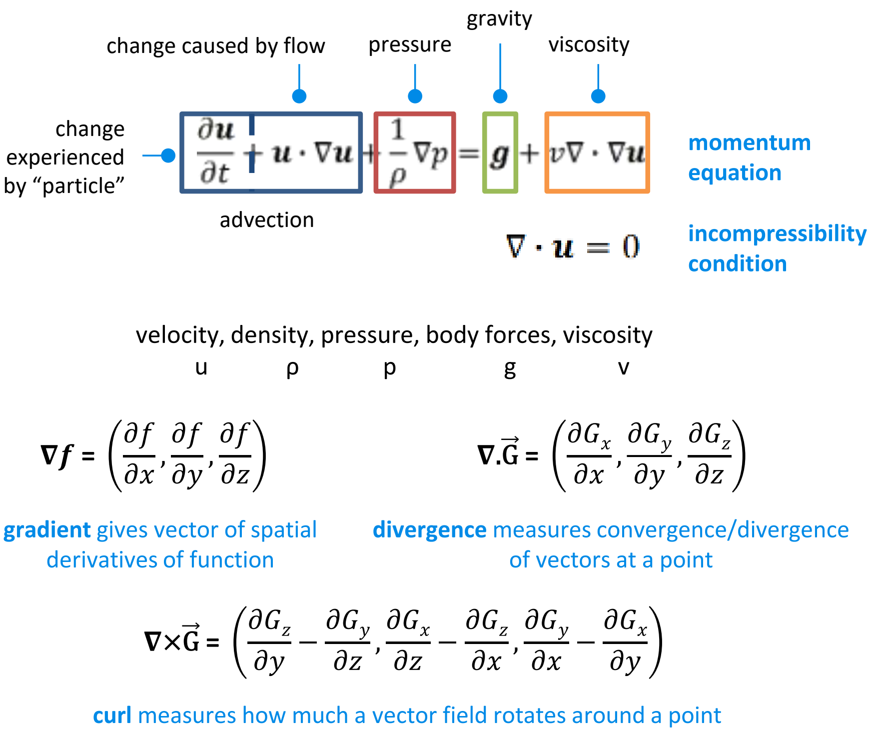


Figure 759: Navier-Stoke equation explained. (cc) Olivier Ezratty with various sources. 2021.

- **Differentiable quantum circuits** to solve differential equations[3267].
- **Converting a nonlinear system into a linear one** with transforming nonlinear problems into an array of linear equations[3268], which however seems to have strong limitations[3269].
- **Quantum annealing for laminar plane channel flow problem** with a solution using a D-Wave quantum annealer[3270].

Various other ad-hoc algorithms are proposed with up to some exponential speedups[3271] [3272] [3273] [3274].

Worth mentioning, **Vorticity** (2001, USA) is developing custom (classical) DSA (domain specific accelerators) to solve Navier-Stokes equations with a 10^5 speed gain over classical methods. They don't provide any technical information on their technology (FPGA, ASIC?).

[3267] See Solving nonlinear differential equations with differentiable quantum circuits by Oleksandr Kyriienko et al, 2020 (22 pages).

[3268] See Efficient quantum algorithm for dissipative nonlinear differential equations by Jin-Peng Liu, Andrew M. Childs et al, March 2021 (36 pages) and New Quantum Algorithms Finally Crack Nonlinear Equations by Max G Levy, Quanta Magazine, January 2021.

[3269] See Limitations for Quantum Algorithms to Solve Turbulent and Chaotic Systems by Dylan Lewis et al, July 2023 (10 pages).

[3270] See Towards Solving the Navier-Stokes Equation on Quantum Computers by N. Ray et al, April 2019 (16 pages).

[3271] See Potential quantum advantage for simulation of fluid dynamics by Xiangyu Li, Nathan Wiebe et al, March 2023 (23 pages).

[3272] See Quantum computing of fluid dynamics using the hydrodynamic Schrödinger equation by Zhaoyuan Meng and Yue Yang, February 2023 (37 pages).

[3273] See Quantum Algorithm for Lattice Boltzmann (QALB) Simulation of Incompressible Fluids with a Nonlinear Collision Term by Wael Itani, Katepalli R. Sreenivasan and Sauro Succi, New York University, April 2023 (62 pages).

[3274] See Ensemble Fluid Simulations on Quantum Computers by Sauro Succi, Wael Itani, Katepalli R. Sreenivasan and Rene Steijl, April 2023 (8 pages).

Quantum Machine Learning (QML)

What if quantum computing could accelerate machine learning and deep learning training and inferences? This is one of its potential domains of applications, but it is not that obvious. First, quantum computing does not seem to enable machine learning tasks that are impossible to implement with classical computing, including the many specialized AI-bound classical hardware (tensors processing units, spiking neurons and other neuromorphic circuits).

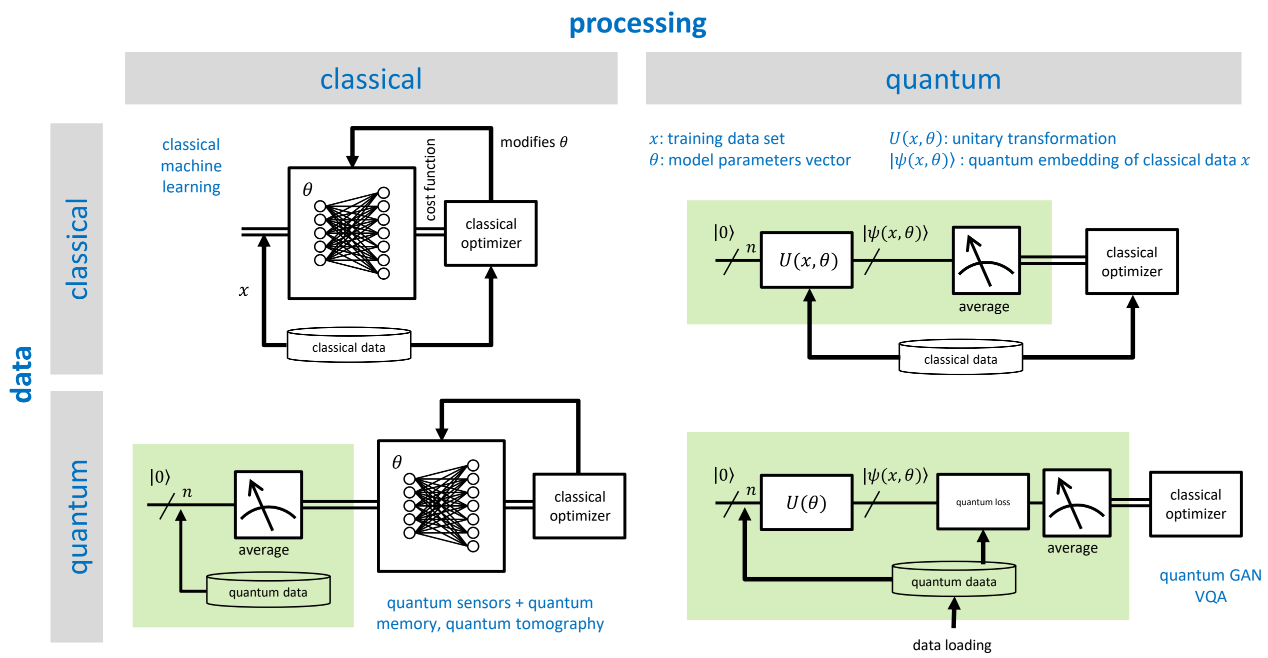


Figure 760: the four main types of QML depending on whether data loading is classical or quantum and part of the processing is classical or quantum.
Source: inspired by An Introduction to Quantum Machine Learning for Engineers by Osvaldo Simeone, July 2022 (229 pages).

Second, their benefits are still hard to evaluate, particularly given all quantum machine learning are hybrid in nature[3275].

In many cases, benchmarks tend to favor a comparison in the quality of the results like minimizing an error function and error rates more than proving a quantum speedup[3276].

Various quantum algorithms have been created in the last decades that cover the field of classical machine learning and with a lot of variations in neural networks and deep learning[3277]. Quantum Machine Learning algorithms are either targeting NISQ platforms with variational varieties or FTQC with using linear algebra algorithms like the foundational HHL algorithm[3278 3279].

3275 See Reliable AI: Does the Next Generation Require Quantum Computing? by Aras Bacho et al, July 2023 (26 pages).

3276 See Is quantum advantage the right goal for quantum machine learning? by Maria Schuld and Nathan Killoran, March 2022 (10 pages) and Why measuring performance is our biggest blind spot in quantum machine learning by Maria Schuld, Xanadu, March 2022, which provides an interesting perspective on why QML is not an obvious near-term candidate for some quantum advantage vs classical methods. And Quantum machine learning: a classical perspective by Ciliberto et al, 2020 (26 pages). It concludes with: "*Despite a number of promising results, the theoretical evidence presented in the current literature does not yet allow us to conclude that quantum techniques can obtain an exponential advantage in a realistic learning setting*".

3277 See Machine Learning in the Quantum Era - Machine Learning unlocks the potential of emerging quantum computers by Loïc Henriet (Pasqal), Christophe Jurczak (Quantonation) and Leonard Wossnig (Rahko), November 2019. It highlights the potential of cold atom-based qubits for QML.

3278 See Quantum Machine Learning on Near-Term Quantum Devices: Current State of Supervised and Unsupervised Techniques for Real-World Applications by Yaswitha Gujju et al, IBM Quantum, July 2023 (40 pages).

3279 See this series of MOOC on QML: Quantum Machine Learning MOOC, 2018-2021, created by the late Peter Wittek from the University of Toronto in Spring 2019 who died in the Himalayas in September 2019.

The literature on QML defines four models that connect how the data is fed into the model (classically, quantumly) and how the process is handled (classically or at least partially quantumly) as shown in Figure 760 [3280]:

CC with classical data that are processed by classical algorithms. This is classical machine learning.

CQ with classical data that is encoded in quantum states and processed by quantum algorithms, which may need the use of a quantum RAM (qRAM). This is the most common method in NISQ systems.

QC with quantum data that is converted in a classical form and processed by classical algorithms, a solution implemented to analyze quantum physics and sensors measurement statistics, like for doing a qubit tomography.

QQ with quantum data that is processed by quantum algorithms which could be implemented with feeding a QML algorithm directly with quantum data coming from a quantum sensor.

Generically, QML algorithms from the CQ category rely on **variational circuits**, a family of hybrid algorithms that combine a quantum algorithm and a traditional algorithm that drives the latter[3281]. VQE and VQA are some of them[3282]. It allows finding global minimums. These algorithms are adapted to NISQ QPUs but their true quantum acceleration is not proven yet[3283]. These algorithms are characterized by a cost function defining the problem, the quantum ansatz, how is the cost function optimized, the data encoded as the input to the VQA and the desired output.

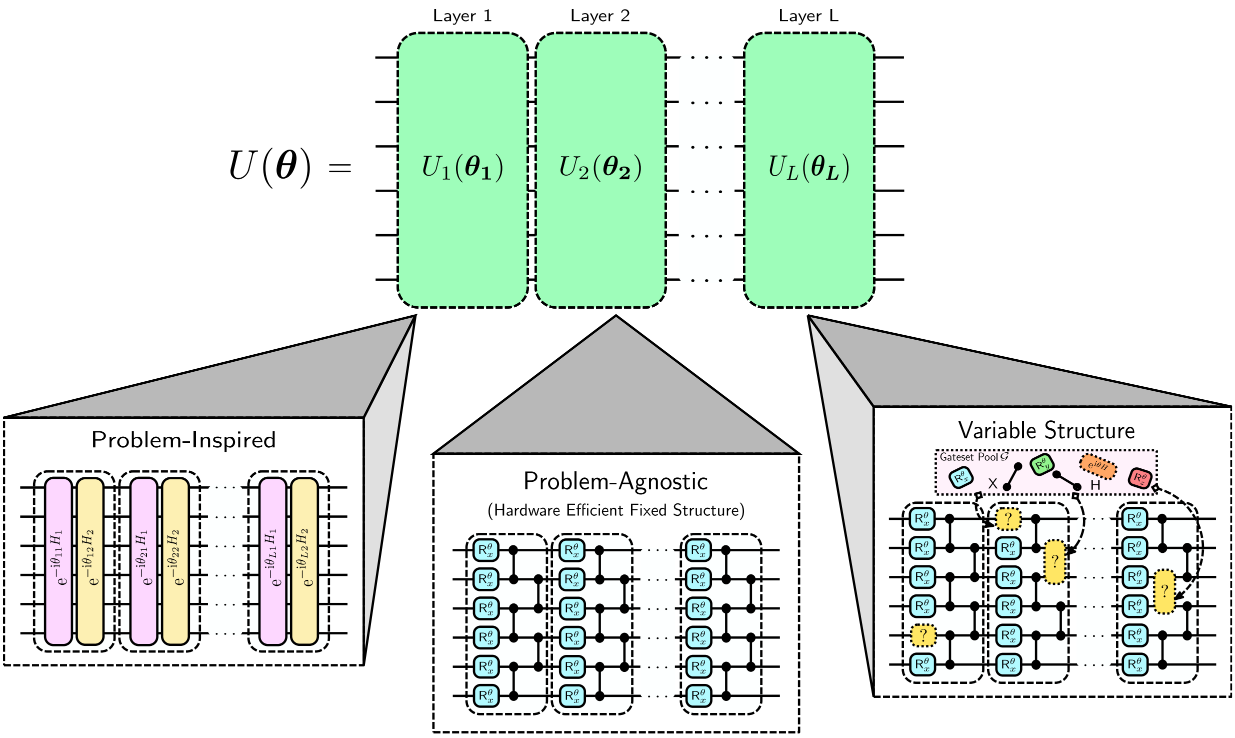


Figure 761: various ways of preparing a QML ansatz or model, problem inspired, problem agnostic and with a variable structure. Source: Machine learning applications for noisy intermediate-scale quantum computers by Brian Coyle, University of Edinburgh, May 2022 (263 pages).

[3280] See An Introduction to Quantum Machine Learning for Engineers by Osvaldo Simeone, July 2022 (229 pages).

[3281] See Universal Variational Quantum Computation by Jacob Biamonte, 2019 (5 pages).

[3282] See Accelerated Variational Quantum Eigensolver by Daochen Wang, Oscar Higgott, and Stephen Brierley, 2019 (11 pages) which proposes a machine learning method to reduce the depth of the quantum circuits used (number of quantum gates to be executed). See also Quantum advantage with shallow circuits by Robert König et al, 2018 (97 slides). This list of quantum machine learning algorithms can be found in Quantum Machine Learning What Quantum Computing Means to Data Mining by Peter Wittek, 2014 (178 pages).

[3283] See Classically Approximating Variational Quantum Machine Learning with Random Fourier Features by Jonas Landman, Elham Kashefi et al, October 2022 (17 pages) which shows that some variational circuits could be efficiently emulated classically.

Variational circuits use **parametrized quantum circuits** or parametric quantum circuits (not to be confused with post-quantum cryptography)[3284]. These are also labelled as "ansatz". These circuits usually contain a series of single qubit rotation and two-qubit entanglement gates that contain the parameters of the problem to encode. A PQC model is defined by a classical optimizer. There are several types of data preparation ansatzes: with only single-qubit gates ("mean field ansatz") which require only N gates for N qubits and is simple and fast, then "hardware efficient ansatz" that also use a fixed number of entangling gates and then parametrized 2-qubit gates that are entirely tailored for the problem. These three models are presented above in Figure 761[3285].

The breadth of QML algorithms is vast, covering all categories of classical machine learning and deep learning with supervised machine learning (to classify items or make predictions on time series), unsupervised machine learning (for automatic clustering), up to all sorts of neural networks (convolutional networks, recurrent networks, generative networks)[3286]:

- **SVM** (Support Vector Machine), a traditional method of segmentation that often relies on matrix inversions, based on its use of HHL[3287]. It can be used for text sentiment analysis.
- **PCA** (Principal Component Analysis) is used to determine the key variables in a data set[3288]. This is similar to searching for eigenvectors of a data set. Again, HHL is behind it.
- **Quantum nonlinear regression** algorithm, one of the basic quantitative value prediction methods of the learning machine[3289].
- **Recommendation** systems useful in marketing or content[3290].
- **Decision tree classification** algorithms, enabling regression and binary classification tree construction and retraining[3291].
- **Automatic data clustering** using unsupervised learning algorithms[3292].

3284 See the review paper on PQCs Parameterized quantum circuits as machine learning by Parameterized quantum circuits as machine learning models by Marcello Benedetti, Erika Lloyd, Stefan Sack and Mattia Fiorentini, Quantum Science and Technology, 2019 (18 pages).

3285 See the thesis Machine learning applications for noisy intermediate-scale quantum computers by Brian Coyle, University of Edinburgh, May 2022 (263 pages), done under the supervision of Elham Kashefi.

3286 See the review paper Quantum Machine Learning - State of the Art and Future Directions by Christian Bauckhage et al, Fraunhofer and Capgemini, 2023 (122 pages).

3287 See Support Vector Machines on Noisy Intermediate-Scale Quantum Computers by Jiaying Yang, 2019 (79 pages) which discusses the use of SVM on NISQ computers, Quantum Machine Learning with Support Vector Machines by Anisha Musti, April 2020 and a practical example with Quantum support vector machines for aerodynamic classification by Xi-Jun Yuan et al, August 2022 (12 pages).

3288 See Quantum principal component analysis by Seth Lloyd, Masoud Mohseni and Patrick Rebentrost, from MIT and Google, July 2013 (9 pages) which lays the groundwork on the matter.

3289 See Nonlinear regression based on a hybrid quantum computer, 2018 (7 pages), from researchers in several laboratories in China.

3290 See Quantum Recommendation Systems by Iordanis Kerenidis and Anupam Prakash, 2016 (22 pages, and video) is a proposed quantum machine learning algorithm for recommendation. The quantum algorithm of Iordanis Kerenidis had been challenged by a classical algorithm proposal by Ewin Tang in 2018. She "dequantized" Kerenidis's algorithm, meaning, she found a classical efficient equivalent. But Iordanis pointed out that with certain recommendation parameters, the quantum algorithm was still clearly superior. Always, as long as a machine is there to execute it. Both these algorithms are investigated in Exponential Advantages in Quantum Machine Learning through Feature Mapping by Andrew Nader et al, December 2020 (16 pages).

3291 See *Des-q*: a quantum algorithm to construct and efficiently retrain decision trees for regression and binary classification by Niraj Kumar, Marco Pistoia et al, JPMorgan Chase, September 2023 (48 pages). It may need some QRAM which is far from being ready.

3292 See Quantum spectral clustering by Iordanis Kerenidis and Jonas Landman, April 2021 (20 pages). The method named spectral clustering consists in building a similarity graph with using distances between data vectors, extracting the eigenvectors from a matrix built with this graph and projecting the data onto this new orthogonal space and applying a classical k-means clustering method. But as seen frequently with QML algorithms, the best acceleration requires using some qRAM.

- **Gradient descent and backpropagation** used during training phase of neural network[3293]. So far, it has been tested at a very limited scale[3294] and doesn't scale well unless some conditions are met[3295 3296].

Now onto the various breeds of deep learning models which nearly all have quantum equivalents:

- **Perceptrons** implementation, related to the original neural network architecture designed by Franck Rosenblatt in 1957[3297 3298].
- **Quantum Graph Neural Networks** have many applications, particularly in chemistry and biology[3299 3300].
- **Quantum Convolutional Neural Networks** (QCNN), still modest in size for the moment[3301]. These are also named **Quanvolutional Neural Network** algorithms[3302]. They seem to have the advantage of avoiding the ill-fated barren plateaus that make it difficult to converge a network[3303]. Variations of QCNN algorithms also exist for D-Wave quantum annealers[3304].
- **Feature Mapping** in deep learning and convolutional neural networks, to detect patterns efficiently[3305].
- **Equivariant Neural Networks** (ENN) with better geometrical robustness and a better resistance to adversarial attacks[3306].

[3293] See Quantum algorithms for feedforward neural networks by Jonathan Allcock, Iordanis Kerenidis et al, 2018 (18 pages) and Quantum Circuit Parameters Learning with Gradient Descent Using Backpropagation by M Watabe et al, 2020 (15 pages).

[3294] See Deep quantum neural networks equipped with backpropagation on a superconducting processor by Xiaoxuan Pan et al, December 2022 (18 pages) with a system trained using 6 superconducting qubits, way below any quantum advantage and with no indications on feasibility either in a quantum advantage NISQ or FTQC regime.

[3295] See On quantum backpropagation, information reuse, and cheating measurement collapse by Amira Abbas et al, Google and Caltech, May 2023 (29 pages) which found that gradients based optimization will not scale as efficiently as deep neural networks equipped with backpropagation. The researchers propose a way forward by reducing the task of estimating gradients from shadow tomography with a "quantum-efficient" protocol for computing gradients, but highlight the unavoidable classical costs involved.

[3296] See Quantum Machine Learning: from physics to software engineering by Alexey Melnikov et al, January 2023 (45 pages).

[3297] See An Artificial Neuron Implemented on an Actual Quantum Processor by Francesco Tacchino et al, 2018 (8 pages).

[3298] See Training Multilayer Perceptrons by Sampling with Quantum Annealers by Frances Fengyi Yang et al, March 2023 (22 pages).

[3299] See Quantum Graph Neural Networks by Guillaume Verdon et al, 2019 (10 pages).

[3300] See the thesis Quantum neural networks by Kerstin Beer, May 2022 (189 pages).

[3301] See Quantum Convolutional Neural Networks by Iris Cong et al, May 2019 (12 pages), Quantum Neurons: analyzing the building blocks of quantum deep learning algorithms by Zachary Cetinic et al, December 2019 (12 pages) and Quantum Algorithms for Deep Convolutional Neural Networks by Iordanis Kerenidis, Jonas Landman and Anupam Prakash, 2019 (31 pages). Also, Advances in Quantum Deep Learning: An Overview by Siddhant Garg and Goutham Ramakrishnan, May 2020 (17 pages) is focused on quantum neural networks including quantum convolutional neural networks and contains a good introduction to classical neural networks. And Realizing quantum convolutional neural networks on a superconducting quantum processor to recognize quantum phases by Johannes Herrmann et al, Nature Communications, 2022 (7 pages) which implements a QCNN algorithm on a 7-qubit QPU from IBM for a narrow problem (recognizing quantum phases) with some superiority in the results quality (and of course, not with a speedup given the low number of qubits).

[3302] See Predict better with less training data using a QNN by Barry D. Reese, Marek Kowalik, Christian Metzl, Christian Bauckhage, and Eldar Sultanow, Capgemini, June 2022 (23 pages).

[3303] See Absence of Barren Plateaus in Quantum Convolutional Neural Networks by Arthur Pesah et al, PRX, DoE Los Alamos Lab and UCL, November 2021 (26 pages).

[3304] See Adiabatic Quantum Computation Applied to Deep Learning Networks by Jeremy Liu et al, May 2018 (28 pages).

[3305] See Supervised learning with quantum enhanced feature spaces by Aram Harrow et al, 2018 (22 pages) which describes the use of quantum to detect complex shapes, far beyond what convolutional neural networks ("feature mapping") can do.

[3306] See Introduction to Robust Machine Learning with Geometric Methods for Defense Applications by Pierre-Yves Lagrave and Frédéric Barbaresco, Thales, July 2021 (9 pages).

- **Recurrent Neural Networks** that detect patterns in sequential data or time series data[3307]. It is used for MNIST handwriting recognition, an existing common task for classical OCR (optical character recognition)[3308], and with potential use cases in weather forecasts[3309].
- **Reinforcement Learning** networks where a computing agent learns how to make decisions and take actions in an environment in order to maximize a cumulative reward. It is inspired by the way humans and animals learn from interaction with their surroundings[3310] [3311].
- **Hybrid transfer learning** which trains a quantum neural network using an already trained classical network[3312].
- **Generative Learning Models** (QGLMs) [3313], including the models based on so-called Quantum Circuit Born machines (QCBM), Quantum Variational Autoencoders (QVAE), Quantum Generative Adversarial Networks (QGAN) which generate synthetic content from existing content by checking its plausibility via a network of recognition neurons[3314] [3315] [3316] (Figure 762) and Quantum Invertible Neural Networks[3317] (QINN)[3318]. QGLMs can be used to create (quantumly generated) synthetic training data sets used in classical machine learning models[3319] as well as for option derivatives pricing in financial services[3320]. Their quantum advantage may however be limited in scope compared to classical generative models[3321].

3307 See Learning Quantum Processes with Memory - Quantum Recurrent Neural Networks by Dmytro Bondarenko et al, January 2023 (46 pages).

3308 See Recurrent Quantum Neural Networks by Johannes Bausch (12 pages) and Quantum reservoir computing using arrays of Rydberg atoms by Rodrigo Araiza Bravo et al, Harvard and IBM Research, November 2021-July 2022 (10 pages).

3309 See Quantum Recurrent Neural Networks for Sequential Learning by Yanan Li et al, February 2023 (31 pages).

3310 See the review paper A Survey on Quantum Reinforcement Learning by Nico Meyer et al, November 2022 (62 pages).

3311 See Experimental quantum speed-up in reinforcement learning agents by V. Saggio et al, Nature, January 2021 (8 pages) and arXiv.

3312 See Classical-to-quantum convolutional neural network transfer learning by Juhyeon Kim et al, August 2022 (15 pages).

3313 See the review paper Recent Advances for Quantum Neural Networks in Generative Learning by Jinkai Tian et al, June 2022 (30 pages).

3314 This is well documented in Quantum generative adversarial learning by Seth Lloyd and Christian Weedbrook, 2018 (5 pages), Quantum generative adversarial learning in a superconducting quantum circuit, 2018 (5 pages) and Synthetic weather radar using hybrid quantum-classical machine learning by Graham R. Enos, Chad Rigetti et al, Rigetti, November 2021 (8 pages).

3315 See A Hybrid Quantum-Classical Generative Adversarial Network for Near-Term Quantum Processors by Albha O'Dwyer Boyle et al, July 2023 (13 pages).

3316 See MosaiQ: Quantum Generative Adversarial Networks for Image Generation on NISQ Computers by Daniel Silver et al, Northwestern University, August 2023 (10 pages).

3317 Invertible Neural Networks are designed to be reversible and bijective. They can perform both forward computation (mapping inputs to outputs) and inverse computation (mapping outputs back to inputs). They can transform data so that the original input can be recovered from the transformed output. Their applications include image and signal processing, including image denoising, image super-resolution and image synthesis.

3318 See Generative Invertible Quantum Neural Networks by Armand Rousselot and Michael Spannowsky, SciPost Physics, February 2023 (19 pages).

3319 See Quantum versus Classical Generative Modelling in Finance by Brian Coyle, Elham Kashefi et al, August 2020 (17 pages) and The Born Supremacy: Quantum Advantage and Training of an Ising Born Machine by Brian Coyle, Daniel Mills, Vincent Danos and Elham Kashefi, April 2021 (47 pages).

3320 See Conditional Generative Models for Learning Stochastic Processes by Salvatore Certo et al, Deloitte Consulting and IBM Research, April 2023 (10 pages).

3321 See A Framework for Demonstrating Practical Quantum Advantage: Racing Quantum against Classical Generative Models by Mohamed Hibat-Allah et al, Zapata Computing, March 2023 (17 pages) which shows that generative models could demonstrate a quantum advantage only with limited training data.

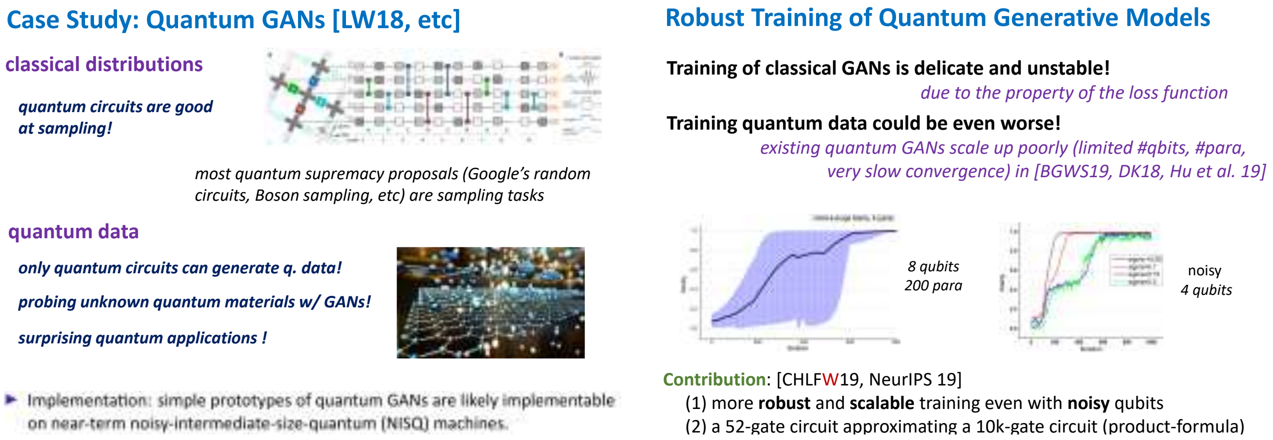


Figure 762: quantum generative neural networks.
Source: [Applications and training of quantum generative models](#) by Xiaodi Wu, UMD, Q2B, December 2019 (11 slides).

- **Capsule Networks**, which recognizes images features with considering their relative position[3322].
- **Reservoir Computing Networks** which map input signals to a high-dimensional space having complex quantum superposition and connects to the desired output through a linear regression model or a relatively simple neural network. It has various use cases like in foreign exchange market applications, including, potentially, in NISQ platforms[3323] [3324].
- **Self-Attention Mechanisms** are used in transformer neural networks. They detect dependencies between different portions of sequences of inputs, like in text and enable models to assign different weights to different parts of sequences, making it possible to focus on relevant information and capture long-range dependencies[3325] [3326].
- **Active Learning** algorithms which select the training data set to make learning more efficient and save up to 85% of training time. It consists in labeling unlabeled data using an iterative supervised learning[3327].
- **Geometric Quantum Machine Learning** (GQML) which takes account group symmetries abstract representation of training data. For various reasons like the need to avoid barren plateaus (described page 929) or reducing the number of dimensions and parameters, some strong inductive biases must be embedded in QML models. Imposing an a priori or an inductive bias leads models to only explore a subset of the function space due to its definition hypotheses, keeping the optimal functions in the target space. It can sometimes considerably reduce the parameter space without altering the search for solutions. Geometric QML is based on this desire to build QML models that respect the underlying invariances in the data, called symmetries. The most successful neural networks are models with inductive biases respecting the underlying structure and symmetries of the domain on which they act (translational invariance for a CNN, invariance by permutation for a transformer, invariance by time deformation for an LSTM). Geometric QML implements systematic methods for creating such models having better generalization performance, smaller training data requirements, as well as favorable optimization landscapes. The underlying

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3322 See Quantum Capsule Networks by Zidu Liu et al, January-December 2022 (18 pages).

3323 See the review paper Opportunities in Quantum Reservoir Computing and Extreme Learning Machines by Pere Mujal et al, February-July 2021 (14 pages) and Time Series Quantum Reservoir Computing with Weak and Projective Measurements by Pere Mujal et al, May 2022 (19 pages).

3324 See Configured Quantum Reservoir Computing for Multi-Task Machine Learning by Wei Xia et al, March 2023 (15 pages).

3325 See A natural NISQ model of quantum self-attention mechanism by Shangshang Shi et al, May 2023 (13 pages).

3326 See Quantum Self-Attention Neural Networks for Text Classification by Guangxi Li et al, May 2022-September 2023 (13 pages).

3327 See Active Learning on a Programmable Photonic Quantum Processor by Chen Ding et al, August 2022 (17 pages).

concepts are quite complicated and involve Lie groups, group invariant QML and the likes[3328] [3329] [3330].

- **Binary Neural Networks** with parameters and activation functions constrained by only two possible values[3331].
- **Spiking neurons** emulation although its speedup is not obvious[3332].
- **Image classification and analysis**[3333] [3334] with different use cases like improve satellite imaging interpretations[3335] [3336] [3337], edge detection improvements[3338], generic image classification[3339] [3340], medical image classification[3341] [3342], similarity detection[3343], and defects detection in manufacturing[3344]. Image classification can also be based on inverse Quantum Fourier Transforms[3345]. There are also quantum versions of video analysis algorithms although they are usually tested with small resolution videos and without addressing the data loading question[3346].

3328 See Representation Theory for Geometric Quantum Machine Learning by Michael Ragone, Marco Cerezo et al, October 2022-February 2023 (43 pages).

3329 See Theoretical Guarantees for Permutation-Equivariant Quantum Neural Networks by Louis Schatzki et al, October 2022 (36 pages).

3330 See Group-Invariant Quantum Machine Learning by Martin Larocca, Marco Cerezo et al, Google Brain, University of Waterloo, May 2022 (28 pages).

3331 See Quantum HyperNetworks: Training Binary Neural Networks in Quantum Superposition by Juan Carrasquilla et al, January 2023 (10 pages).

3332 See An artificial spiking quantum neuron by Lasse Bjørn Kristensen, Alán Aspuru-Guzik et al, April 2011 (7 pages).

3333 See the review paper Quantum Image Processing by Alok Anand et al, Carnegie Mellon, March 2022 (10 pages).

3334 See Processing Images in Entangled Quantum Systems by S.E. Venegas-Andraca and J.L. Ball, 2010 (11 pages).

3335 See Towards Bundle Adjustment for Satellite Imaging via Quantum Machine Learning by Nico Piatkowski, Thore Gerlach, Romain Hugues, Rafet Sifa, Christian Bauckhage and Frederic Barbaresco, Thales and Fraunhofer IAIS, April 2022 (8 pages) which proposes keypoints extraction and objects alignment in pattern recognition applied to satellite imaging.

3336 See also Hyperbolic Equivariant Convolutional Neural Networks for Fish-Eye Image Processing by Pierre-Yves Lagrave and Frédéric Barbaresco, Thales, February 2022 (9 pages).

3337 See Exploiting the Quantum Advantage for Satellite Image Processing: Review and Assessment by Soronzonbold Otgonbaatar and Dieter Kranzlmüller, August-November 2023 (9 pages) which makes some resource estimations in the FTQC regime showing that a quantum computing advantage in satellite imagery is quite remote.

3338 See A hybrid quantum image edge detector for the NISQ era by Alexander Geng et al, Fraunhofer ITWM, March 2022 (19 pages).

3339 See Quantum machine learning for image classification by Arsenii Senokosov et al, April 2023 (9 pages).

3340 See A practical overview of image classification with variational tensor-network quantum circuits by Diego Guala et al, October 2022 (11 pages).

3341 See Quantum Methods for Neural Networks and Application to Medical Image Classification by Jonas Landman, Anupam Prakash, Iordanis Kerenidis et al, December 2022 (30 pages).

3342 See Hybrid quantum image classification and federated learning for hepatic steatosis diagnosis by Luca Lusnig et al, Terra Quantum, November 2023 (10 pages) which proposes n hybrid algorithm using 5 qubits. I would label this as a “classical-classical” algorithm given 5 qubits run faster and cheaper in a classical emulator.

3343 See SLIQ: Quantum Image Similarity Networks on Noisy Quantum Computers by Daniel Silver et al, Northeastern University, September 2023 (9 pages).

3344 See Quantum Kernel for Image Classification of Real World Manufacturing Defects by Daniel Beaulieu et al, Deloitte Consulting and Strangeworks, December 2022 (8 pages).

3345 See Inverse Quantum Fourier Transform Inspired Algorithm for Unsupervised Image Segmentation by Taoreed Akinola et al, January 2023 (9 pages).

3346 See A Quantum Moving Target Segmentation Algorithm for Grayscale Video by Wenjie Liu, Lu Wang and Qingshan Wu, Advanced Quantum Technologies, October 2023 (15 pages).

- **Quantum Natural Language Processing** (NLP) is making some inroads as well, in various limited cases. It can help with classification tasks[3347] [3348] and simple text summaries. Quantum computers capacities and data loading constraints won't however make it easy to implement Large Language Models (LLMs, *ala* ChatGPT)[3349]. One algorithm was proposed to use Grover's search algorithm to invert a sparse attention computation matrix efficiently to fasten a hybrid LLM but its resources estimated are not provided and the potential polynomial speedup from Grover algorithm is suspicious and highly dependent on how the matrix, sparse or not, is loaded[3350].
- **Federated Machine Learning** algorithms, which distributes training on several quantum computers to improve the training time while preserving privacy, with distributing and sharing the learned model instead of the training data[3351] [3352] [3353] [3354], including ways to create secured communication with Quantum Secure Aggregation (QSA)[3355].

In all these cases, the proposed algorithms are not specific to some qubit hardware architecture beyond the nuances between gate-based and analog quantum computers. There are still some proposals that are very hardware specific like some which are using continuous variable photonic QPUs[3356].

The table in Figure 763 positions the different quantum accelerations associated with various algorithms used in machine learning and deep learning[3357].

Accelerations in log(N) are more important than those expressed as the square root of N[3358].

Note the need for quantum memory for many of these algorithms, a type of memory that doesn't exist yet. None of these algorithms have been tested on a large scale, due to the absence of a quantum processor with more than fifty qubits.

3347 See Quantum Text Encoding for Classification Tasks by Aaranya Alexander and Dominic Widdows, January 2023 (7 pages).

3348 See Quantum Text Classifier -- A Synchronistic Approach Towards Classical and Quantum Machine Learning by Dr. Prabhat Santi, Kamakhya Mishra and Sibabrata Mohanty, Tata Consultancy Services, May 2023 (7 pages).

3349 See Quantum Natural Language Generation on Near-Term Devices by Amin Karamlou et al, IBM Quantum, November 2022 (11 pages).

3350 See Fast Quantum Algorithm for Attention Computation by Yeqi Gao et al, Adobe, University of Washington and University of Texas, July 2023 (21 pages).

3351 See the review paper Towards Quantum Federated Learning by Chao Ren, Leong Chuan Kwek et al, June 2023 (19 pages).

3352 See Federated Quantum Machine Learning by Samuel Yen-Chi Chen and Shinjae Yoo, Brookhaven Lab, March 2021 (25 pages) and Federated Quantum Natural Gradient Descent for Quantum Federated Learning by Jun Qi, GeorgiaTech, August 2022 (9 pages).

3353 See Quantum federated learning based on gradient descent by Kai Yu et al, December 2022 (13 pages).

3354 See Quantum Federated Learning: Analysis, Design and Implementation Challenges by Dev Gurung et al, June 2023 (7 pages).

3355 See Federated Learning with Quantum Secure Aggregation by Yichi Zhang et al, July 2022-September 2023 (31 pages).

3356 See Experimentally Realizable Continuous-variable Quantum Neural Networks by Shikha Bangar et al, June 2023 (13 pages).

3357 The table is from The prospects of quantum computing in computational molecular biology by Carlos Outeiral, April 2020 (23 pages) which covers both QML algorithms and quantum simulation ones. It also mentions protein structures predictions. See also Quantum Machine Learning by Jacob Biamonte et al, May 2018 (24 pages).

3358 Also see Application of Quantum Annealing to Training of Deep Neural Networks (2015), Machine learning &... artificial intelligence in the quantum domain, 2017 (106 pages), On the Challenges of Physical Implementations of RBMs, 2014, with Yoshua Bengio and Ian Goodfellow among the authors, illustrating the interest of AI specialists for quantum and Quantum Deep Learning, 2014, all extracted from Near-Term Applications of Quantum Annealing, 2016, Lockheed Martin (34 slides). See also Quantum machine learning for data scientists, 2018 (46 pages).

TABLE 1 Overview of the main quantum machine learning algorithms that have been reported in the literature, and complexities

Algorithm	Classical	Quantum	QRAM
Linear regression	$\mathcal{O}(N)$	$\mathcal{O}(\log N)^*$	Yes
Gaussian process regression	$\mathcal{O}(N^3)$	$\mathcal{O}(\log N)^\dagger$	Yes
Decision trees	$\mathcal{O}(N \log N)$	Unclear	No
Ensemble methods	$\mathcal{O}(N)$	$\mathcal{O}(\sqrt{N})$	No
Support vector machines	$\approx\mathcal{O}(N^2)$-$\mathcal{O}(N^3)$	$\mathcal{O}(\log N)$	Yes
Hidden Markov models	$\mathcal{O}(N)$	Unclear	No
Bayesian networks	$\mathcal{O}(N)$	$\mathcal{O}(\sqrt{N})$	No
Graphical models	$\mathcal{O}(N)$	Unclear	No
k-Means clustering	$\mathcal{O}(kN)$	$\mathcal{O}(\log kN)$	Yes
Principal component analysys	$\mathcal{O}(N)$	$\mathcal{O}(\log N)$	No
Persistent homology	$\mathcal{O}(\exp N)$	$\mathcal{O}(N^5)$	No
Gaussian mixture models	$\mathcal{O}(\log N)$	$\mathcal{O}(\text{polylog } N)$	Yes
Variational autoencoder	$\mathcal{O}(\exp N)$	Unclear	No
Multilayer perceptrons	$\mathcal{O}(N)$	Unclear	No
Convolutional neural networks	$\mathcal{O}(N)$	$\mathcal{O}(\log N)$	No
Bayesian deep learning	$\mathcal{O}(N)$	$\mathcal{O}(\sqrt{N})$	No
Generative adversarial networks	$\mathcal{O}(N)$	$\mathcal{O}(\text{polylog } N)$	No
Boltzmann machines	$\mathcal{O}(N)$	$\mathcal{O}(\sqrt{N})$	No
Reinforcement learning	$\mathcal{O}(N)$	$\mathcal{O}(\sqrt{N})$	No

Figure 763: main quantum machine learning algorithms given it considers only the training set size N and not other parameters like the number of features and dimensionality.
Source: The prospects of quantum computing in computational molecular biology by Carlos Outeiral, April 2020 (23 pages).

QML is also one of the fields of application of D-Wave's quantum annealers. Annealers work well to find minimum energy of complex systems which is equivalent to searching for a minimum level of errors in the adjustment of the weight of neurons in a neural network[3359]. So far, they have tested an RBM (Restricted Boltzmann Machine) model[3360]. They also did it with a hybrid algorithm for image recognition in a neural network, based on a variational circuit and a hybrid algorithm. But with very low-resolution images[3361]! D-Wave offers machine learning services in its Leap quantum cloud computing offering. But they are not the only ones. Many startups are specialized in Quantum Machine Learning, such as **QC Ware**.

Many challenges remain to be addressed to operationalize QML beyond the emergence of sufficiently powerful and reliable quantum processors[3362] [3363]:

[3359] Examples source: D-Wave Quantum Computing - Access & application via cloud deployment by Colin Williams, 2017 (43 slides).

[3360] See Benchmarking Quantum Hardware for Training of Fully Visible Boltzmann Machines by Dmytro Korenkevych et al, Kindred AI et D-Wave, 2016 (22 pages).

[3361] See also An analog quantum variational embedding classifier by Rui Yang, Samuel Bosch, Bobak Kiani, Seth Lloyd and Adrian Lupascu, November 2022 (10 pages) that explored the same path.

[3362] See Quantum machine learning: a classical perspective by Ciliberto et al, 2020 (26 pages). It concludes with: "*Despite a number of promising results, the theoretical evidence presented in the current literature does not yet allow us to conclude that quantum techniques can obtain an exponential advantage in a realistic learning setting*".

[3363] See Challenges and Opportunities in Quantum Machine Learning by Marco Cerezo et al, March 2023 (16 pages).

- **Loading training data** may take time and have a negative impact on the acceleration provided by QML. It also requires quantum random access memory (qRAM) which does not yet exist, even if some Quantum Data Loaders are proposed to circumvent this need[3364]. Some specific methods have to be created to load images efficiently[3365]. Other methods are developed that could require fewer training data than with classical machine learning[3366]. Various papers tend to show that a quantum acceleration would be warranted only with having training data already provided as a quantum state[3367] [3368]. Other propose standardized machine learning training datasets that are encoded in quantum gates and usable classically, to benchmark QML and classical solutions[3369].
- **Nonlinear activation functions** such as sigmoids used in classical neural networks are difficult to implement in quantum algorithms since quantum gates all apply linear transformations[3370]. There are workarounds, on which Iordanis Kerenidis has worked[3371] and some others[3372] [3373] [3374]. For example, quantum measurement can create the sough-after activation function nonlinearity in neural networks. There are also suggestions to use continuous variables qubits architectures to handle neural networks with nonlinearity provided by non-Gaussian qubit gates[3375], photonic quantum neural networks using Kerr nonlinearities[3376] and even other techniques not requiring measurement[3377].
- **Avoiding the barren plateau** phenomenon which limits the efficiency of QML algorithms as the problem size scales[3378].

3364 See Quantum embeddings for machine learning by Seth Lloyd, January 2020 (11 pages). In Nearest Centroid Classification on a Trapped Ion Quantum Computer by Sonika Johri, Iordanis Kerenidis et al, December 2020 (15 pages), the QML algorithm "Nearest Centroid Classification" is implemented on a 11 trapped ions IonQ processor with an efficient amplitude data loader. See also Data compression for quantum machine learning by Rohit Dilip et al, April 2022 (8 pages) and the thesis Quantum Algorithms for Unsupervised Machine Learning and Neural Networks, by Jonas Landman, November 2021 (192 pages).

3365 See A Novel Quantum Image Compression Method Based on JPEG by Jian Wang et al, 2017 (30 pages).

3366 See Generalization in quantum machine learning from few training data by Matthias C. Caro et al, Nature Communications, 2022 (11 pages).

3367 See Quantum algorithms for group convolution, cross-correlation, and equivariant transformations by Grecia Castelazo, Dirk Englund, Seth Lloyd et al, September 2021-September 2022 (27 pages).

3368 See Quantum advantage in learning from experiments by Hsin-Yuan Huang, Hartmut Neven, John Preskill et al, December 2021 (52 pages).

3369 See MNISQ: A Large-Scale Quantum Circuit Dataset for Machine Learning on/for Quantum Computers in the NISQ era by Leonardo Placidi et al, June 2023 (24 pages).

3370 The trick is explained in Quantum Neuron: an elementary building block for machine learning on quantum computers by Yudong Cao, Gian Giacomo Guerreschi and Alán Aspuru-Guzik in 2017 (30 pages).

3371 See Quantum Algorithms for Deep Convolutional Neural Network by Iordanis Kerenidis et al, 2020 (36 pages) which is discussed in Deep Convolutional Neural Networks for Quantum Computers by Jonas Landman, 2020.

3372 See for example Continuous-variable quantum neural networks by Nathan Killoran et Al, June 2018 (21 pages).

3373 See A Novel Spatial-Temporal Variational Quantum Circuit to Enable Deep Learning on NISQ Devices by Jinyang Li et al, George Mason University, Oak Ridge National Laboratory and Pacific Northwest National Laboratory, July 2023 (11 pages).

3374 See Realization of a quantum neural network using repeat-until-success circuits in a superconducting quantum processor by M. S. Moreira et al, Qutech and Intel, December 2022 (24 pages).

3375 See Continuous-variable quantum neural networks by Nathan Killoran, Maria Schuld, Seth Lloyd et al, 2018 (21 pages).

3376 See Realistic quantum photonic neural networks by Jacob Ewaniuk et al, August 2022 (20 pages).

3377 See Quantum activation functions for quantum neural networks by Marco Maronese et al, January 2022 (28 pages).

3378 See Quark: A Gradient-Free Quantum Learning Framework for Classification Tasks by Zhihao Zhang et al, October 2022 (19 pages) which circumvents this problem with a quantum model and quantum optimizer.

- **Classical surrogates** are classical implementations of QML models for classical inferences. They indirectly question the nature of the quantum advantage of QML[3379].
- **Compressing** the neural networks to save quantum resources[3380].
- **Implementing inferences classically** to save QPU resources and improve response times[3381].
- **Distributing Quantum Machine Learning** processes across multiple QPUs could be interesting for handling large problems[3382].
- **Speedup** must be brought by QML compared to today's most advanced processors[3383]. We still lack resource estimates to assess these speedups which are very theoretical at this point. IBM published in 2021 a mathematical proof of a potential quantum advantage for a quantum machine learning classification task done with a quantum kernel method based on the Shor dlog algorithm. There was no actual experiment done due to the inexistence of sufficiently powerful quantum computers[3384]. We'll probably be stuck in this situation for several years from now.
- **Algorithms explainability** must also be addressed. The decomposition of the training and inference process of these quantum neural networks will probably be different from their implementation in more traditional processors[3385] [3386] [3387].

On the other hand, QML's algorithm developments have served as a source of inspiration to improve algorithms that work with classical computation. As we will see in the section on quantum software and tools vendors, starting page 1094, there is no shortage of those who have specialized in QML. In general, they provide development tools and means to create QML proofs of concept.

In the AI domain, the European project H2020 **Quromorphic** launched in July 2019 aims to create a quantum processor dedicated to the execution of neural networks inspired by the brain[3388].

3379 See Classical surrogates for quantum learning models by Franz J. Schreiber, Jens Eisert, and Johannes Jakob Meyer, June 2022 (4 pages).

3380 See Quantum Neural Network Compression by Zhirui Hu et al, July 2022 (11 pages) and Experimental Quantum End-to-End Learning on a Superconducting Processor by Xiaoxuan Pan et al, March 2022 (10 pages).

3381 See Shadows of quantum machine learning by Sofiene Jerbi et al, May 2023 (23 pages).

3382 See the review An Invitation to Distributed Quantum Neural Networks by Lirandë Pira and Chris Ferrie, November 2022 (34 pages).

3383 See Quantum Machine Learning: Algorithms and Practical Applications by Iordanis Kerenidis, QC Ware, Q2B Conference, December 2019 (34 slides) which makes an inventory of some potential gains with QML algorithms.

3384 See IBM shows quantum computers can solve these problems that classical computers find hard by Daphne Leprince-Ringuet, ZDNet, July 2021 that refers to Quantum kernels can solve machine learning problems that are hard for all classical methods, IBM Research, July 2021, itself referring A rigorous and robust quantum speed-up in supervised machine learning by Yunchao Liu, Srinivasan Arunachalam and Kristan Temme, Nature Physics, July 2021 (27 pages).

3385 These techniques will be challenged by future memristor-based neuromorphic processors that will allow networks to converge more rapidly with backpropagation. Memristors will make it possible to place the neuron's computational functions and the associated memory in the same location in a semiconductor circuit, accelerating access to memory by several orders of magnitude during computations. This is another area of research, operated notably by Julie Grollier of the CNRS laboratory located at Thales TRT in Palaiseau.

3386 See eXplainable AI for Quantum Machine Learning by Patrick Steinmüller et al, November 2022 (14 pages).

3387 See Explaining Quantum Circuits with Shapley Values: Towards Explainable Quantum Machine Learning by Raoul Heese et al, January-March 2023 (36 pages).

3388 See Quantum computer: We're planning to create one that acts like a brain by Michael Hartmann and Heriot-Watt leads on next-gen computers, November 2018. The project is led by Michael Hartmann of IPaQS (Institute of Photonics and Quantum Sciences) at Heriot-Watt University in the UK, together with ETH Zurich, Delft University (the Netherlands), Basque University (Spain), IBM Zurich and Volkswagen (Germany). 2.2M€ from the FET Open program were allocated to the project by the European Commission (details). My interpretation? The objective of the project has been adapted to the sauce of science fiction in order to recover community funding. The rest is about photonics.

It reminds us of the very controversial European flagship Human Brain Project led by Henri Markram from Switzerland. Quromorphic involves IBM Zurich, ETH Zurich, TU Delft, Volkswagen and Spanish and German Universities. Given the participants, we can guess that this will be based on superconducting qubits. The project got a funding of 2.9M€ in 2019 and is scheduled to end in 2022[3389]. This is quite reasonable.

Let us note finally the existence of an association promoting the field of AI and quantum computing, the **IAIQT** foundation based in Switzerland.

Classical machine learning can be useful for quantum physics and quantum computing. We saw that Google used a deep learning algorithm to optimize the microwave frequency plan of the Sycamore processor's qubit control. Machine learning can also be used to model and simulate condensed matter, with an impact on the development of various qubits, especially superconducting qubits[3390].

We will probably be exposed to various fancy claims combining artificial intelligence and quantum computing and the devil will always be in complicated details[3391]. For instance, how about using these QML algorithms in robotics? Not so fast[3392]! It is still science fiction and *click-bait*.

Quantum physics simulation

Quantum simulation algorithms are used to reproduce matter at the quantum level in a computer. It can be used to simulate the interaction between atoms in molecules for the creation of new materials.

They can also simulate physical phenomena related to magnetism or the interaction between photons and matter. This amounts to solving "N-body problems", i.e., calculating the interaction between several particles according to the physical laws governing their interaction. Quantum simulation also helps studying how superconducting materials behave, particularly at (relatively) high temperature, superfluids at low temperature, the temperature-dependent magnetism of certain materials and the interactions between graphene and light[3393].

These algorithms run in gate-based universal quantum computers as well as on quantum simulators and quantum annealers although we still lack data to compare their respective performance.

Starting with 50 electron orbitals in a molecule, classical computers can no longer simulate their dynamics, which corresponds to just a few atoms. For simple molecules, the applications are in the field of materials physics: carbon or nitrogen capture, new batteries, discovery of superconducting mechanisms that can then be used in medical scanners, ideally operating at room temperature.

[3389] See Quantum computer: we're planning to create one that acts like a brain, January 2019.

[3390] See some review papers like Machine learning & artificial intelligence in the quantum domain by Vedran Dunjko and Hans J. Briegel, 2017 (106 pages), Modern applications of machine learning in quantum sciences by Anna Dawid et al, April 2022-November 2023 (288 pages), Artificial Intelligence and Machine Learning for Quantum Technologies by Mario Krenn, Jonas Landgraf, Thomas Foesel, Florian Marquardt, August 2022 (23 pages) and Synergy of machine learning with quantum computing and communication by Debasmita Bhoumik et al, October 2023 (34 pages).

[3391] Here is one good example with Using Pioneering Quantum Machine Learning Methods, CQC Scientists Offer Bright Forecast For Quantum Computers That Can Reason par Matt Swayne, 2021 referring to Variational inference with a quantum computer by Marcello Benedetti, April 2021 (17 pages) which is about apply some quantum version of MCMC (Markov-Chain Monte Carlo) algorithm using Born machines. These are described in The Born Supremacy: Quantum Advantage and Training of an Ising Born Machine by Brian Coyle, Elham Kashefi et al, April 2021 (10 pages).

[3392] As described in Daniel Manzano's The Rise of Quantum Robots, April 2018. And with Qubit or Qubot? Quantum Technology May Help Robots Learn Faster par Matt Swayne, 2021, Robots learn faster with quantum technology by University of Vienna, March 2021 pointing to Experimental quantum speed-up in reinforcement learning agents by V. Saggio et al, Nature, March 2021 (10 pages).

[3393] See this interesting lecture by Jacqueline Bloch at the Academy of Sciences which makes an excellent overview: Quantum Simulators: Solving Difficult Problems, May 2018 (29 mn).

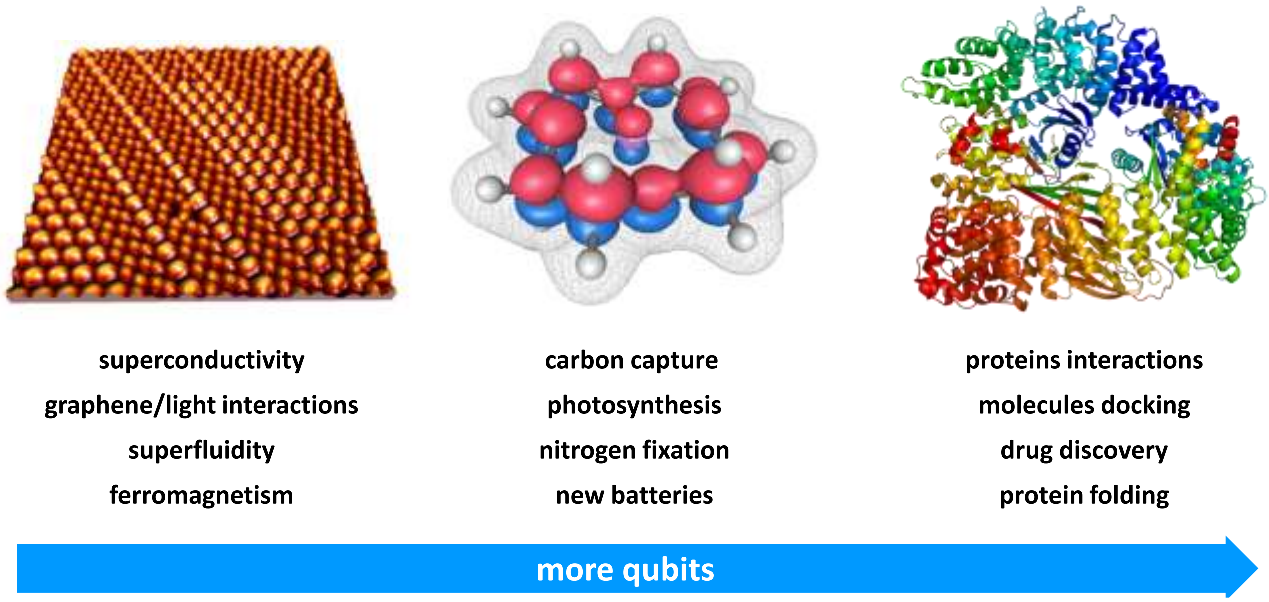

Figure 764: quantum physics simulation applications and a fuzzy grade of complexity given that very quickly these simulations require thousands of logical qubits in the FTQC regime. (cc) Olivier Ezratty, 2020.

This could be accessible with universal quantum computers with 50 to a few hundred corrected logical qubits. For molecular biology simulations, it will probably take much longer before this is possible. We may need thousands or even hundreds of thousands of logical qubits with tiny error rates (smaller than 10^{-14}), which is far away in time (Figure 764).

Figure 765 positions the number of logical qubits needed to simulate the functioning of a mitochondrial protein, MRC2, in an optimistic way. On a practical level, most quantum chemistry proposed algorithms are about computing the ground state of the electronic Hamiltonian of a molecule. It deals with finding the smallest eigenvalue of this Hamiltonian. With this starting point, quantum chemists can determine chemical reaction energies, potential energy surfaces in molecules to better understand reaction dynamics and find "molecular docking" sites and at last, undertake virtual spectroscopy of the studies molecules.

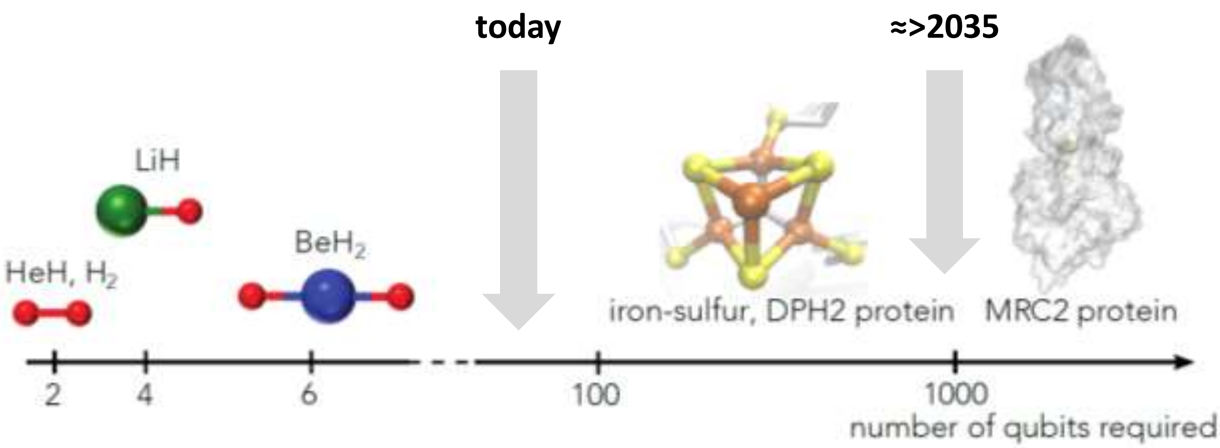

Figure 765: another grade of complexity, for molecular ground state Hamiltonian simulations, in logical qubits. It seems quite optimistic since many cases studies are mentioning logical qubit counts in the thousands and tens of thousands even for rather simple molecules. The number of logical qubits is dependent of the number of relevant electron orbitals in the molecule and there are many such orbitals per atom. And these are just resource estimates for ground state Hamiltonian estimation, not for molecular dynamics, molecular docking or chemical reaction simulations.
Source: from Quantum optimization using variational algorithms on near-term quantum devices by IBM researchers in 2017 (30 pages).

Here are some examples of quantum simulation algorithms tested at very small scales to compute energy ground states of small molecules or materials, that you could run in a second on your own smartphone with Quirk:

- Simulation of **quantum field theory** as reviewed by John Preskill[3394].
- Simulation of **hydrogen atoms** using two superconducting qubits computers[3395].
- Hybrid simulation of a **simple CaH^+ ion** also using two superconducting qubits[3396].
- Simulation of **simple molecules** by Alán Aspuru-Guzik, one of the world's leading authorities in the field[3397]. It is centered around the H_2 molecule. It was tested with 5 qubits.
- Simulation of a **three atoms molecule**, beryllium hydride (BeH_2) with 6 qubits by IBM[3398].
- Determination of the **equilibrium state of simple molecules**, also with 5 qubits [3399].
- Hybrid **molecular simulations** combining classical and quantum algorithms, with 6 qubits[3400].
- Simulation of **semiconductor dynamics** using 6 qubits[3401].

And then, some resource estimates for more complicated quantum chemistry simulations:

- Simulation of the **cytochrome molecule**. With error rates of 0.1%, the run time would be 73 hours and require 4.6M physical qubits. With an error rate of 0.001% which is way out of reach, computing time could be down to 25 hours with 500K physical qubits. The equivalent classical computing time is 4 years and requires 348 GB RAM and 2TB of storage[3402].
- Simulation of **catalytic chemical processes** proposed by researchers from Microsoft and ETH Zurich with a requirement of about 4,000 logical qubits[3403].
- Simulating organic molecules of medium complexity such as **cholesterol** would require about 1,500 logical qubits and, above all, the ability to use billions of quantum gates[3404] (Figure 767). VQE algorithms could also be used there with a universal gate-based quantum computer using a reasonable depth of quantum gates (number of steps in the algorithm) [3405].
- Simulating **periodic solids**, which would require tens to hundred million of qubits[3406]!

3394 See Simulating a quantum field theory with a quantum computer by John Preskill, 2018 (22 pages).

3395 See Computation of Molecular Spectra on a Quantum Processor with an Error-Resilient Algorithm, 2018 (7 pages).

3396 See Researchers succeed in the quantum control of a molecule by Román Ikonicoff, May 2017 (38 pages), pointing to Preparation and coherent manipulation of pure quantum states of a single molecular ion, 2017 (38 pages).

3397 See Simulation of Electronic Structure Hamiltonians Using Quantum Computers by James Whitfield, Jacob Biamonte and Alán Aspuru-Guzik, 2010 (22 pages)

3398 See Tiny Quantum Computer Simulates Complex Molecules by Katherine Bourzac, IEEE Spectrum, 201

3399 See Simulated Quantum Computation of Molecular Energies by Wiebe, Wecker and Troyer, 2006 (21 pages).

3400 See Quantum Machine Learning for Electronic Structure Calculations, October 2018 (16 pages).

3401 See Solving strongly correlated electron models on a quantum computer by Matthias Troyer et al, 2015 (27 pages).

3402 See Reliably assessing the electronic structure of cytochrome P450 on today's classical computers and tomorrow's quantum computers by Joshua J. Goings, Craig Gidney et al, Google, PNAS, February 2022 (24 pages).

3403 See Quantum computing enhanced computational catalysis by Vera von Burg, Matthias Troyer et al, July 2020 (104 pages).

3404 See Quantum Computation for Chemistry and Materials by Jarrod McClean, Google 2018 (36 slides).

3405 See An adaptive variational algorithm for exact molecular simulations on a quantum computer by Sophia Economou et al, 2019 (9 pages) which indicates in particular that "*VQE is much more suitable for NISQ devices, trading in the long circuit depths for shorter state preparation circuits, at the expense of a much higher number of measurements*".

3406 See Quantum Computation for Periodic Solids in Second Quantization by Aleksei V. Ivanov et al, Riverlane and Johnson Matthey, PRR, Oct 2022-March 2023 (29 pages).

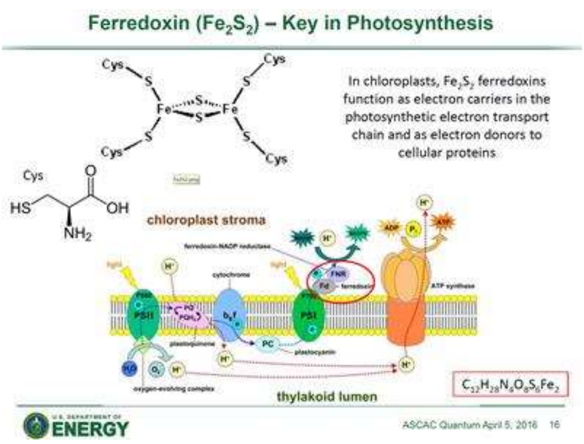

Figure 766: Source: [Quantum Computing (and Quantum Information Science)](#) by Steve Binkley, US Department of Energy, 2016 (23 slides).

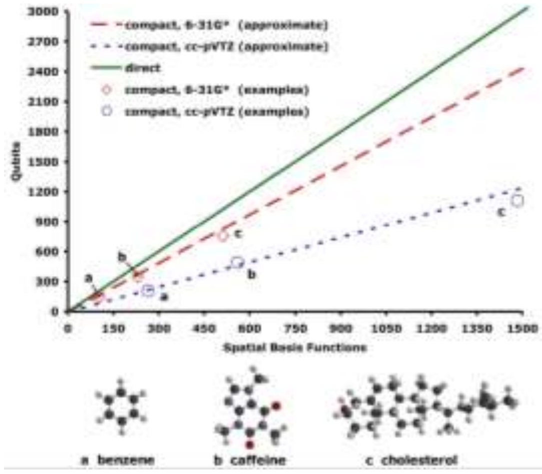

Figure 767: how many logical qubits are actually necessary to simulate relatively simple molecules. Source: [Simulated Quantum Computation of Molecular Energies](#) by Wiebe, Wecker and Troyer, 2006 (21 pages).

One of the applications of molecular quantum simulation is to better understand how photosynthesis works in order to improve or imitate it, the involvement of different forms of ferredoxin, relatively simple iron and sulfur-based molecules that serve to transport electrons from the photoelectric effect used in photosynthesis in plants (Figure 766). Algorithmic research on this molecule simulation has downsized the duration of quantum theoretical simulation from 24 billion years to one hour in a few years, but, yet, on some future quantum computers! The simulation of photosynthesis could pave the way for better carbon capture, among others to produce synthetic fuel. Research is also advancing in this field, without quantum computation for the moment[3407]. Matthias Troyer explains how this algorithm was optimized[3408].

At a higher abstraction level sits the simulation of atomic interactions in organic chemistry and molecular biology, going progressively from the smallest to the largest molecules: amino acids, peptides, polypeptides, proteins and perhaps much later, ultra-complex molecules such as ribosomes that fabricates proteins with amino acids using messenger RNA code.

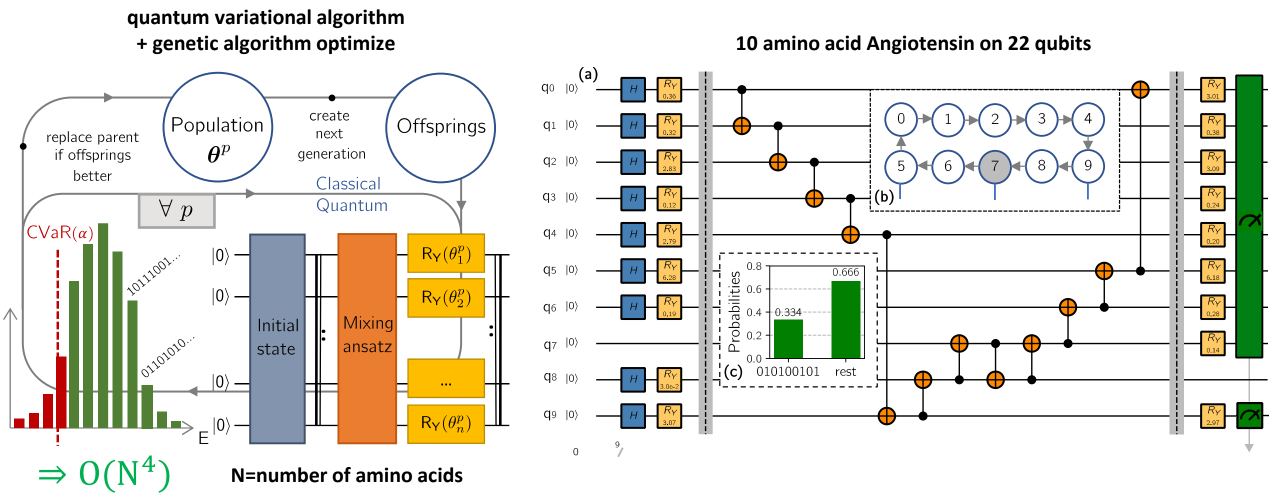

Figure 768: a hybrid classical and quantum algorithm to fold proteins.
Source: [Resource-efficient quantum algorithm for protein folding](#), Anton Robert et al, 2020 (5 pages).

[3407] As seen in [Semi-Artificial Photosynthesis Method Produces Fuel More Efficiently Than Nature](#), September 2018.

[3408] In [What Can We Do with a Quantum Computer](#), Matthias Troyer, ETH Zurich, 2016 (41 slides), source for the illustration on the right.

Like classical algorithms, quantum simulation algorithms use approximation models based on known molecules, like what AlphaFold 3 from DeepMind does to predict the 3D structure of folded proteins. It works well for proteins which are close to those proteins used to train the model. For entirely new proteins (*aka* "de-novo proteins"), quantum simulation seems to be required[3409].

Various quantum algorithms have already been created for this purpose, including the Aspuru-Guzik algorithm in 2012, which was tested at a small scale on a D-Wave One[3410]. In 2020, researchers from IBM Zurich and from Institut Pasteur in France created an algorithm able to predict the 3D structure of a peptide, angiotensin, made of 10 amino acids and on just 22 qubits as shown above in Figure 768. Algorithms even exists to simulate RNA folding[3411].

The orders of magnitude of the quantum computers needed to solve these organic chemistry problems for large proteins have yet to be evaluated. Computing times can be extremely long even with the various optimizations that are proposed[3412]!

Quantum computing can also help simulate classical physics more efficiently than classical computing. For example, a Google AI team created a complex classical oscillators quantum algorithm providing an exponential speedup compared with the best possible classical algorithms[3413].

Optimization algorithms

Optimization algorithms are a key domain for quantum computing, whatever the paradigm that is being used. We'll describe in the NISQ section the QAOA algorithms classes that are proposed for gate-based QPUs and QUBO models that are mostly suitable for analog QPUs, both quantum annealers and analog quantum simulators, although QUBO problems can be reformulated into QAOA problems and be run on gate-based QPUs.

What about optimization algorithms that require an FTQC QPU and would scale well?

Quantum least squares fitting solves the least squares problem, minimizing the sum of the squares of differences between a set of data points and the fitted function[3414]. It is based on the HHL algorithm.

Quantum semidefinite programming (SDP) deals with optimizing a linear objective function. Also, semidefinite programming can be useful to solve some problems in quantum information science[3415].

Oracle based-search algorithms that we already covered, including Grover's search algorithm.

Quantum Integer Programming which deals with finding optimal solutions within a discrete set of possible values. The applications are various, like in supply chain optimization and resource allocation. These problems can be converted into QUBO and QAOA problems.

[3409] See Evolution, energy landscapes and the paradoxes of protein folding by Peter Wolynes, 2015 (13 pages).

[3410] See D-Wave quantum computer solves protein folding problem by Geoffrey Brumfiel, 2012.

[3411] See A QUBO model of the RNA folding problem optimized by variational hybrid quantum annealing by Tristan Zaborniak et al, University of Victoria, August 2022 (12 pages).

[3412] See Quantum Information and Computation for Chemistry, 2016 (60 pages), which provides a good inventory of the various algorithmic works on quantum simulation of organic chemistry, A Comparison of the Bravyi–Kitaev and Jordan–Wigner Transformations for the Quantum Simulation of Quantum Chemistry by Andrew Tranter et al, 2018 (14 pages) that provides some solutions to reduce the gates count for quantum chemistry simulation with gate-based quantum computers and Creating and Manipulating a Laughlin-Type ν=1/3 Fractional Quantum Hall State on a Quantum Computer with Linear Depth Circuits by Armin Rahmani et al, November 2020 (7 pages).

[3413] See Exponential quantum speedup in simulating coupled classical oscillators by Ryan Babbush, Nathan Wiebe et al, Google AI, April 2023 (40 pages).

[3414] See Quantum algorithm for total least squares data fitting by Hefeng Wang and Hua Xiang, China, June 2019 (7 pages).

[3415] See Semidefinite Programming in Quantum Information Science by Paul Skrzypczyk and Daniel Cavalcanti, University of Bristol and ICFO, June 2023 (31 pages).

Quadratically Constrained Quadratic Programs (QCQPs) is a generic class of algorithms used to optimize a quadratic objective function subject to quadratic constraints[3416]. It contains the Max-Cut problems and various Boolean optimizations, optimal power flow (OPF) problems used in electrical engineering and various financial portfolio optimizations.

Quantum Monte Carlo Methods which are used in operational research for solving complex problems involving large sampling using random processes, such as in financial risk and modeling assessments[3417]. They require FTQC QPUs to solve large scale problems in a quantum advantage regime.

Quantum walks are search algorithms in graphs introduced in 1993 by Yakir Aharonov et al[3418], with many applications like searching a triangle in a graph or even Hamiltonian simulations[3419]. Andrew Childs demonstrated that quantum walks can be viewed as a universal quantum programming primitive, showing that an arbitrary set of qubit gates could be reduced to solving a quantum walk, which could be interesting with quantum systems implementing quantum walks at the hardware level one with photonic settings or even superconducting qubits[3420] (Figure 769).

That was the case with a 62 qubits system presented in China in 2021 that was dedicated to implementing a random quantum walk. The project was driven by Jian-Wei Pan. The processor is based on a 8x8 matrix of transmon superconducting qubits. It simulates a Mach-Zehnder interferometer. The matrix uses a nearest-neighbor connectivity like with Google Sycamore. Like Google's processor, two qubits were malfunctioning and deactivated, thus we have 62 instead of 64 qubits plus a non-functioning coupler.

It runs at 10 mK and used 186 control lines, including 16 readout output lines with lines shared by 4 qubits[3421]. Quantum walks can also be implemented with photon qubits[3422].

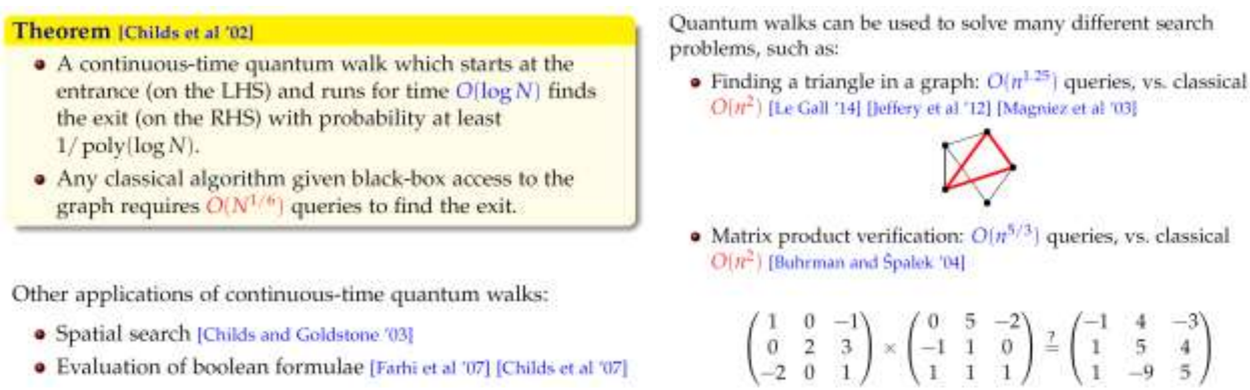

Figure 769: quantum walks and their applications.

In classical computer science, random walk or Markov chain are algorithmic tools applied to search and sampling problems. Their quantum walks equivalent provide a framework for creating fast quantum algorithms. Quantum walks are based on the simulated coherent quantum evolution of a particle moving on a graph.

3416 See A hybrid algorithm for quadratically constrained quadratic optimization problems by Hongyi Zhou et al, September 2023 (8 pages).

3417 See the thesis Quantum Algorithms for the Monte Carlo Method by Yassine Hamoudi, 2021 (175 pages).

3418 See Quantum random walks by Yakir Aharonov (father of Dorit Aharonov), L. Davidovich and N. Zagury, 1993 (4 pages). See also this overview in Quantum walks by Martin Štefaňák, 2020 (44 slides).

3419 See On the relationship between continuous- and discrete-time quantum walk by Andrew M. Childs, 2008 (22 pages).

3420 See Universal computation by quantum walk by Andrew M. Childs, 2008 (9 pages).

3421 See Quantum walks on a programmable two-dimensional 62-qubit superconducting processor by Ming Gong et al, 2021 (18 pages).

3422 See Two-dimensional quantum walks of correlated photons by Zhi-Qiang Jiao et al, 2021 (22 pages).

Quantum walk algorithms use faster hitting (the time it takes to find a target vertex from a source vertex) and faster mixing (the time it takes to spread-out over all vertices after starting from one source vertex) [3423]. The quantum time gain can be exponential for hitting and quadratic for mixing. Since quantum walks are efficient ways to evaluate Boolean formulae, it can be used to solve satisfaction problems (Max-Cut, SAT, 3-SAT).

In gate-based systems, quantum walks can be solved using a Grover search with an oracle function using an adjacency matrix for the searched walk. It can help find the shortest path in a graph (we're back at the traveling salesperson's problem) (Figure 770), finding if a graph is bipartite (with all edges in one vertex connected to the edges in the other), finding subgraphs such as a triangle and solving maximal clique problems (used for example in social networks to find groups of people who know each other).

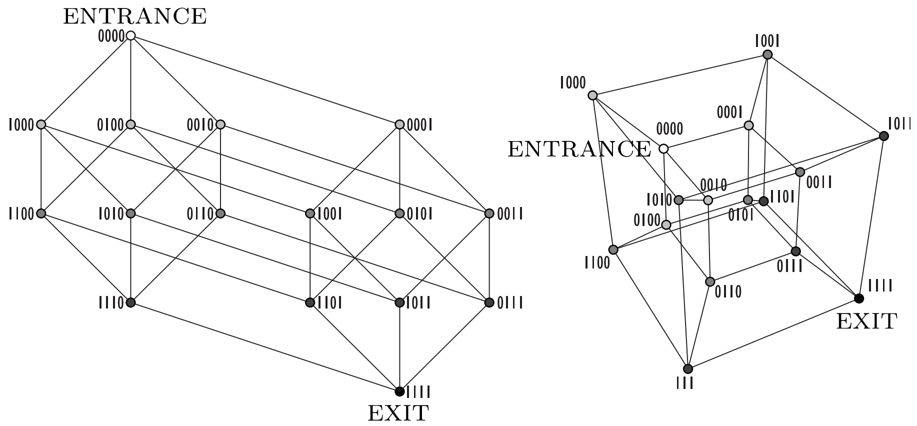

Figure 770: Source: Quantum Walks by Daniel Reitzner, Daniel Nagaj and Vladimir Buzek, 2012 (124 pages), page 13

Then, you have quantum random walks that help reduce quantum walks query complexity to search and find graph properties, with the discrete time and continuous time variations[3424].

These are equivalents of the famous Galton's board.

On top of Andrew M. Childs, let's mention three great contributors to the quantum walk domain: **Stacey Jeffery**[3425], **Julia Kempe** and **Frédéric Magniez**.

NISQ algorithms

The Noisy Intermediate Scale Quantum computing era was defined by John Preskill in his keynote address at the first Q2B conference from QC Ware in California in December 2017 and laid out in a paper published in Quantum in 2018[3426]. He conjectured that it may be possible to use quantum computers with at least 50 qubits that could perform tasks which surpass the capabilities of today's classical digital computers but would be limited to about 1,000 quantum gates. As a result, NISQ algorithms are shallow and should be tolerant to the noise generated during qubit initialization, qubit gates and qubit measurement. John Preskill also included analog quantum computing in the NISQ era[3427].

[3423] I'm summarizing here the quantum walks description from Quantum algorithms an overview by Ashley Montanaro 2015 (16 pages).

[3424] See Quantum Algorithm Implementations for Beginners by Abhijith J. et al, April 2018-June 2022 (96 pages) which provides many references related to quantum walks and quantum random walks algorithms.

[3425] See her thesis: Frameworks for Quantum Algorithms by Stacey Jeffery, 2014 (166 pages) and a lot of subsequent work in quantum walks algorithms.

[3426] See Quantum Computing in the NISQ era and beyond by John Preskill, Quantum Journal, January-July 2018 (20 pages).

[3427] This part is reusing some text from Where are we heading with NISQ? by Olivier Ezratty, April 2023 (47 pages).

Algorithms classes

The best known quantum algorithms suitable for NISQ systems belong to the broad variational quantum algorithms (VQA) class[3428 3429] (Figure 772). Given existing and near future qubit gate fidelities, these algorithms quantum circuits should have a shallow depth, meaning a small number of qubit gate cycles, preferably under 10. This class includes VQE (variational quantum eigensolvers [3430 3431]) for quantum physics simulations, QAOA (quantum approximate optimization algorithm[3432]) for various optimizations tasks, VQLS (variational quantum linear solvers[3433]) to solve linear equations and QML (quantum machine learning) for various machine learning and deep learning use cases.

Many other species of NISQ VQA algorithms are also proposed, particularly in chemical simulations[3434 3435 3436] and for search[3437]. These are most of the time heuristic algorithms that determine near-optimal solutions to various forms of optimization problems, VQE, QAOA and QML being all various breeds of optimization problems to find energy or cost function minima.

Variational algorithms are hybrid by design with a very significant part being implemented in a classical computer, a part that is itself a NP-hard class problem that scales exponentially with the input size[3438 3439].

In the space and speed domains, a quantum advantage requires at least from 50 to 100 physical qubits. The space and speed domains advantages are however distinct. There are situations where some speedup could be obtained with qubits in the 30-50 range, at least when comparing a QPU with perfect qubits, fast gates and a classical server cluster executing the same code in emulation mode, which is usually not the best-in-class equivalent classical solution. Under 18 qubits, it is even recommended to use a local quantum code emulator[3440]. It is not only cheaper, but faster and convenient since your computing job is not placed on a potentially long waiting list and you do not have to pay for expensive cloud QPU (quantum processing unit) resources access.

[3428] These algorithms use variational quantum circuit (VQC) also labelled as parametrized quantum circuits.

[3429] See the review paper Variational Quantum Algorithms by Marco Cerezo, Ryan Babbush, Simon C. Benjamin, Jarrod R. McClean et al, December 2020-Octobre 2021 (33 pages).

[3430] See A Variational Eigenvalue Solver on a Photonic Quantum Processor by Alberto Peruzzo, Jarrod McClean, Peter Shadbolt, Man-Hong Yung, Xiao-Qi Zhou, Peter J. Love, Alán Aspuru-Guzik & Jeremy L. O'Brien, Nature Communications, 2014 (7 pages).

[3431] See Quantum Chemistry in the Age of Quantum Computing by Yudong Cao, Alan Aspuru-Guzik et al, 2018 (194 pages) is a good review paper with a section describing how VQE works.

[3432] See A Quantum Approximate Optimization Algorithm by Edward Farhi, Jeffrey Goldstone and Sam Gutmann, 2014 (16 pages).

[3433] See Variational Quantum Linear Solver by Carlos Bravo-Prieto et al, 2020 (21 pages).

[3434] See Variational Gibbs State Preparation on NISQ devices by Mirko Consiglio et al, March 2023 (12 pages) which is a VQA variant for simulating the thermodynamics of a many-body quantum system.

[3435] See Verifiably Exact Solution of the Electronic Schrödinger Equation on Quantum Devices by Scott E. Smart and David A. Mazziotti, University of Chicago, March 2023 (8 pages).

[3436] See Orders of magnitude reduction in the computational overhead for quantum many-body problems on quantum computers via an exact transcorrelated method by Igor O. Sokolov et al, January 2022-March 2023 (21 pages).

[3437] See Variational Quantum Search with Exponential Speedup by Junpeng Zhan, December 2022 (13 pages).

[3438] See Training variational quantum algorithms is NP-hard — even for logarithmically many qubits and free fermionic systems by Lennart Bittel and Martin Kliesch, PRL, January 2021 (12 pages).

[3439] The classical part of variational algorithms can use tensor networks as described in Tensor Network Assisted Variational Quantum Algorithm by Junxiang Huang et al, December 2022 (9 pages) and Combining Matrix Product States and Noisy Quantum Computers for Quantum Simulation by Baptiste Anselme Martin, Thomas Ayral, François Jamet, Marko J. Rančić and Pascal Simon, May 2023 (13 pages).

[3440] Mohammad Kordzanganeh, Benchmarking simulated and physical quantum processing units using quantum and hybrid algorithms, November 2022 (17 pages).

A laptop, a single cloud server or server cluster is always cheaper than a quantum computer in that case. As shown in Figure 771, most published use cases in arXiv are in the "laptop equivalent" realm.

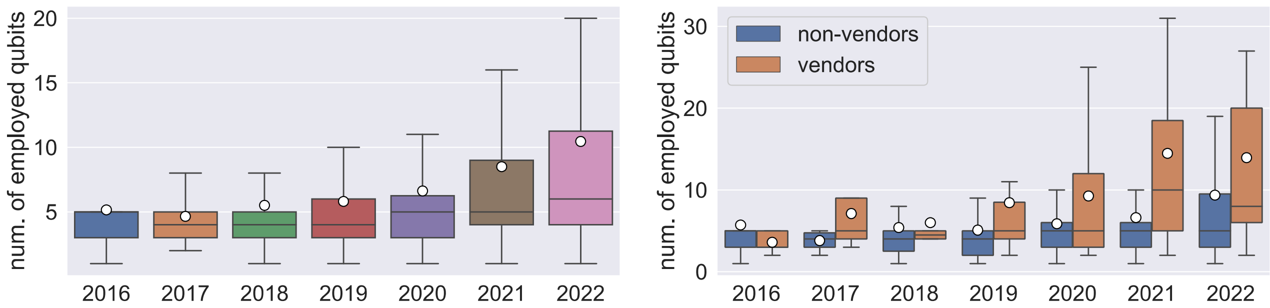

Figure 771: how many qubits were used in the algorithms and case studies preprints on arXiv? Source: [A comprehensive survey on quantum computer usage: How many qubits are employed for what purposes?]() by Tsubasa Ichikawa et al, July 2023 (14 pages) which analyzed 750 preprints on arXiv between 2016 and 2022. Added in 2023.

Thus far, most NISQ experiments have been run with fewer than 30 qubits. While they are elegant proofs of concepts, they do not yet demonstrate any speed up over classical computing, meaning they are not yet in the NISQ regime as defined by John Preskill and listed in Figure 773.

The aim of these experiments is mainly to verify that a small scale noisy quantum computer can generate some useful results compared to a classical computing system emulating perfect qubits. They are not yet proof of a quantum advantage on a larger scale.

Another concern is that is very hard to identify the best-in-class classical solutions which makes it difficult to create apple-to-apple comparisons and well documented quantum speedup assessments, particularly given classical and quantum algorithm don't yield similar results, like a full solution in classical computing versus a value of interest in its quantum equivalent.

When trying to obtain some quantum speedup advantage, existing variational algorithms breadth and depth seem too large for existing NISQ qubit qualities and architectures[3441].

There is some hope that quantum error suppression and mitigation techniques may alleviate this requirement but not on a very large scale.

On the other hand, noisy qubits and shallow algorithms can be efficiently emulated with tensor network-based techniques. It can be done efficiently, which means "at most in polynomial time", but not necessarily faster than a quantum computer. And there are only a few benchmarks yet done in that regime[3442].

Hardware resource and time estimation is a key quantum computing discipline. It creates a bridge between practical use cases, their related algorithms and their required physical resources and computing time. Late 2022, Microsoft released a resource estimator software tool that does for fault-tolerant quantum computing algorithms[3443]. PsiQuantum created such a tool, dedicated to linear differential equation solving[3444].

[3441] See Quantum advantage with shallow circuits by Sergey Bravyi, David Gosset and Robert König, April 2017 (23 pages) that demonstrates that shallow quantum algorithms can showcase a quantum advantage compared to their classical equivalent, but with perfect qubits. In that case, we'd be in the FTQC and not in the NISQ regime.

[3442] Marcel Niedermeier, Jose L. Lado and Christian Flindt, Tensor-Network Simulations of Noisy Quantum Computers, April 2023 (15 pages).

[3443] See Assessing requirements to scale to practical quantum advantage by Michael E. Beverland et al, Microsoft Research, November 2022 (41 pages).

[3444] See The cost of solving linear differential equations on a quantum computer: fast-forwarding to explicit resource counts by David Jennings et al, PsiQuantum, September 2023 (30 pages).

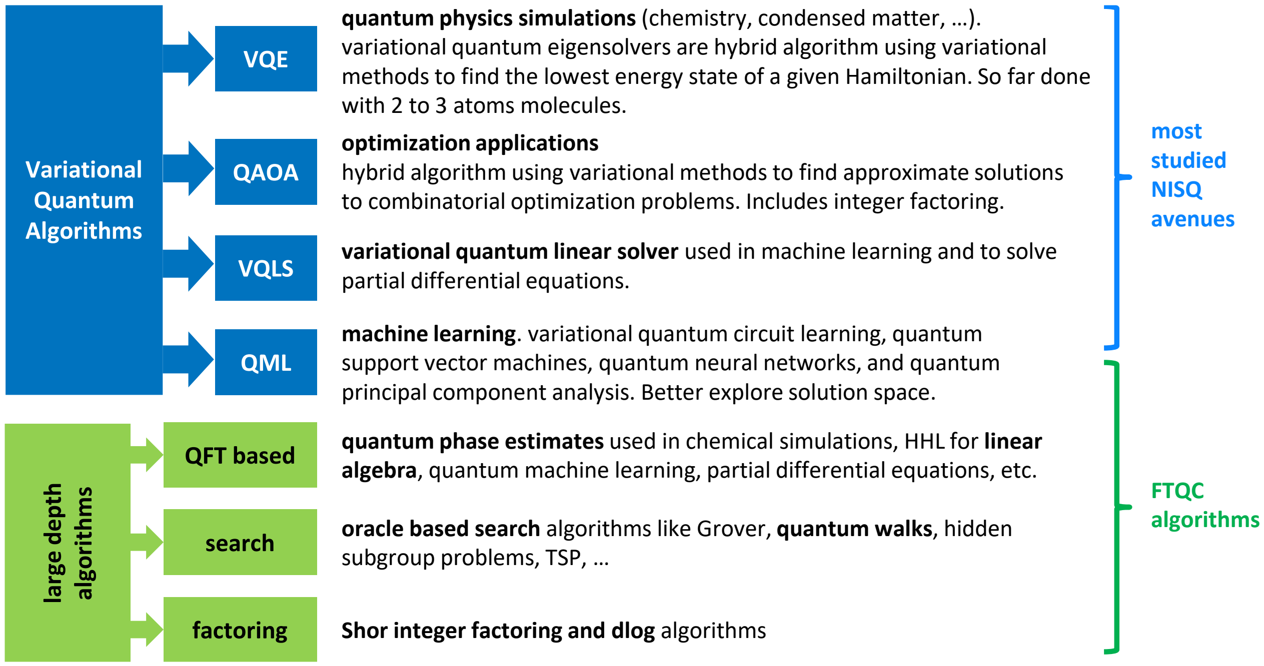

Figure 772 : the main NISQ algorithms classes as proposed by researchers and industry vendors, in blue, when those in green are specific to fault-tolerant quantum computers relying on quantum error correction. These NISQ variational algorithms should be theoretically resilient to noise and shallow but so far, they are not, particularly in a quantum advantage regime with over 50 qubits and with their current algorithm depth actual requirements. VQE algorithms scale in depth as N^6, N being their qubit numbers. Some proposed NISQ algorithms, not shown here, do not use a variational mechanism and may work better than variational algorithms. (cc) Olivier Ezratty, 2023.

No such generic tool seems to exist for NISQ quantum computing with regards to the number of quantum circuit to run to obtain the expected value of the observable on a given ansatz, the number of ansatzes to run and the cost of the classical part of variational algorithms as described in details in Figure 775[3445].

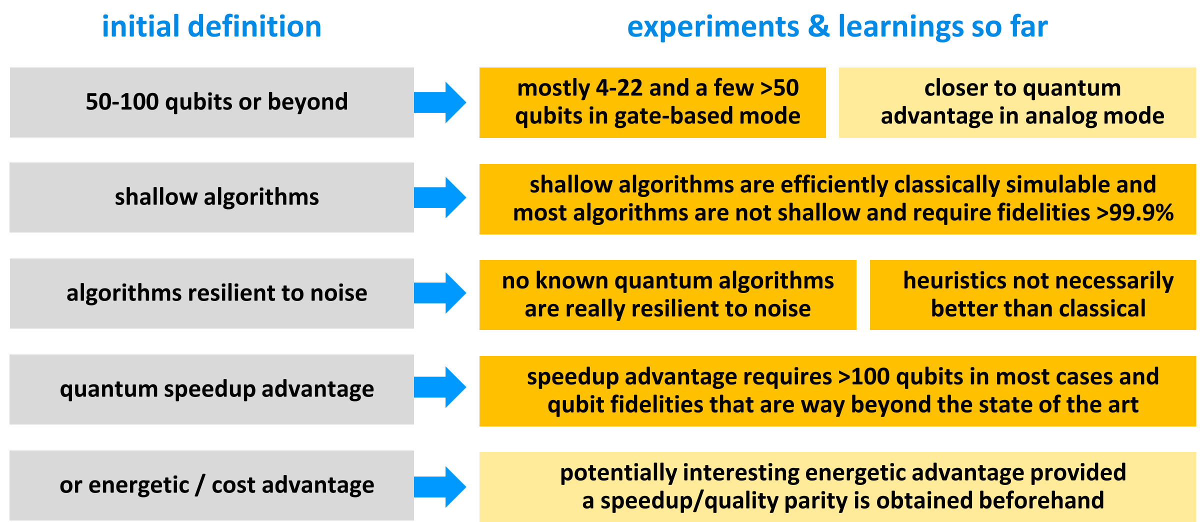

Figure 773: from John Preskill's NISQ definition to actual experiments and learnings. Analog computing refers to digital annealing (D-Wave) and quantum simulators (Pasqal, QuEra, …). Shallow algorithms have only a few gate cycles, preferably under 10. Most variational algorithms have a much larger number of gate cycles in a potential quantum advantage speedup regime. Source: (cc) Olivier Ezratty, 2023.

[3445] See Noisy intermediate-scale quantum computers by Bin Cheng et al, March 2023 (50 pages) is a review paper from researchers in China and Japan that doesn't really address the resource constraints of NISQ relevant algorithms. It's more of a (good) review of current qubit technologies than a NISQ perspective.

There are however some very interesting review papers documenting well this aspect, at least for VQE algorithms[3446] [3487].

At the same time, any estimation of NISQ resources should be compared to an estimation of the classical computing resources required to solve the same problem. At present, there is a lack of estimators for such best-in-class classical algorithms computing resources. This is always done on a case-by-case analysis, and comparing things with a moving classical target, often in different circumstances, with or without heuristic approaches.

Qubit requirements

One general rule of thumb that determines physical resource requirements. It links the physical qubit error rate, and the breadth and depth of a given algorithm[3447] [3448]. The considered error rate corresponds to the gates with the lowest fidelity, which corresponds with most qubit technologies to two-qubit gates like a CNOT[3449].

$$\text{qubit gates error rate} \ll \frac{1}{\text{algo breadth} * \text{algo depth}}$$

The breadth corresponds to the number of qubits used in the algorithms and its depth, to the number of quantum gate cycles. It is a sort of quantum algorithm quantum volume. You could make some trade-offs here between these two dimensions and run either a very shallow algorithm with more qubits or a deeper algorithm with fewer qubits.

This qubit error rate must be below the inverse of the computing breadth x depth as shown in the above formula[3450].

When you compute these numbers with existing quantum hardware, you discover that things don't add up very well, as highlighted in Figure 774. On one hand, to obtain some quantum advantage and match NISQ constraints, you'd need at least 50 physical qubits. On the other hand, the shallowest algorithms have a depth of 8 quantum gate cycles. You end up in that very lower bound case with needing physical qubit gate fidelities of about 99.7%, applicable mainly to two qubit gates and also qubit readout[3451]. Today, no single available QPU has such two-qubit gate fidelities with over 50 qubits. Google Sycamore "2022 edition" with 72 qubits has two qubit gates fidelities of 99.4%[3452]. IBM's 2020 Prague/Egret system is closer to this threshold with 99.66% fidelities obtained with 33 qubits. IBM expects to reach 99.9% two-qubit gate fidelities with its future Heron 133 qubit processors to be unleashed in 2023. Looking at all vendor roadmaps, IBM is the only vendor expecting to exceed 99% qubit fidelities with over 100 qubits, and possibly even 99.99%.

[3446] See Identifying challenges towards practical quantum advantage through resource estimation: the measurement roadblock in the variational quantum eigensolver by Jérôme F. Gonthier et al, Zapata Computing and BP, December 2020-August 2022 (49 pages) provides an evaluation of the number of qubits and circuit shots for the calculation of combustion energies for small organic molecules within chemical accuracy with VQE algorithms. They found that finding the energy of ethanol would last 71 days.

[3447] See The bitter truth about gate-based quantum algorithms in the NISQ era by Frank Leymann and Johanna Barzen, Quantum Science and Technology, 2020 (29 pages).

[3448] See Special Session: Noisy Intermediate-Scale Quantum (NISQ) Computers -- How They Work, How They Fail, How to Test Them? by Sebastian Brandhofer et al, January 2023 (10 pages).

[3449] See Error per single-qubit gate below 10-4 in a superconducting qubit by Zhiyuan Li et al, February 2023 (7 pages).

[3450] See Limitations of Noisy Quantum Devices in Computational and Entangling Power by Yuxuan Yan et al, June 2023 (18 pages) which determines a more constrained limitation in circuit depth that couldn't scale above O(log(n)), n being the number of qubits.

[3451] See NISQ computing: where are we and where do we go? by Jonathan Wei Zhong Lau, Leong Chuan Kwek et al, September 2022 (30 pages).

[3452] See Suppressing quantum errors by scaling a surface code logical qubit by Rajeev Acharya et al, Google AI, Nature, February 2023 (44 pages in arXiv, 7 pages in Nature).

As another example, Rigetti plans to create a 84 qubits QPU with only 99% two-qubit gate fidelities and, later, a 336 qubits version barely reaching 99.5% fidelities, which is clearly insufficient for running any NISQ algorithm with that number of qubits (see Figure 342 page 399).

Most two-qubit gate fidelities provided by industry vendors are median or average fidelities. What is important is their standard deviation and minimum values[3453]. Good median fidelities with high standard deviation are not at all practical, particularly for the first gates of a given algorithm.

High error rates can irreversibly damage early on most running algorithm[3454]. One solution is to deactivate, after calibration, the adjacent qubits for which hardware defects create "stable" faulty two qubit gates. Still, even with these precautions, the publicized median two-qubit gate fidelities are still not good enough to run NISQ algorithms successfully. It is also the case with ion-trap qubits which have very good fidelities but are seemingly hard to scale beyond a couple dozen qubits preventing developers to obtain a space-related computing advantage. These qubits are also too slow to drive, damaging their potential to generate a speedup in a quantum advantage regime. This doesn't show up with experiments implemented with fewer than 25 qubits.

In their current plans and roadmaps, most QPUs are not expected to scale-in beyond a couple of hundred qubits with supporting the high two-qubit gate fidelities required for either NISQ or FTQC, in excess of 99.9%.

The most frequently retained option is a scale-out approach with connecting several QPUs together, a bit like with distributed and parallel computing used in high-performance computing (HPC)[3455]. These connections must preserve qubits overall entanglement and fidelities.

Only a few quantum computing companies have started to work on this next stage which could be explored in a parallel way to the development of their QPU. Scale-out architectures can use multiple techniques like microwave guides between qubits or entangled photons-based connections.

Specialized quantum information network startups like Welinq (France) and QPhoX (the Netherlands) have started to build quantum links based on entangled photon-based connections, also providing quantum memories capabilities for computing and intermediate communication buffers.

With several hundred or a thousand qubits, you end up needing gate fidelities between 99.9% and 99.9999% which are clearly out of reach for today's quantum computers even in lab settings with a few qubits as previously shown page 247 in Figure 237 [3456]. And this ignores the fact that many NISQ algorithms requiring such many qubits are not necessarily as shallow as those requiring only fewer than 10 gate cycles.

There is another notable difference between NISQ and FTQC hardware architectures. As we'll see later, NISQ variational algorithms make a lot of use of R gates with arbitrary rotation angles around the X, Y and Z Bloch sphere axis in their "ansatz" that are prepared classically.

[3453] See Not All Qubits Are Created Equal - A Case for Variability-Aware Policies for NISQ-Era Quantum Computers by Swamit S. Tannu and Moinuddin K. Qureshi, Georgia Institute of Technology, 2018 (12 pages).

[3454] See Bounding quantum gate error rate based on reported average fidelity by Yuval R. Sanders, Joel J. Wallman and Barry C. Sanders, New Journal of Physics, 2015 (14 pages).

[3455] See Variational Quantum Eigensolvers in the Era of Distributed Quantum Computers by Ilia Khait et al, Entangled Networks, February 2023 (9 pages).

[3456] See Quantum information phases in space-time: measurement-induced entanglement and teleportation on a noisy quantum processor by Jesse C. Hoke et al, Google AI, March 2023 (26 pages) posits that it will be very difficult to create useful NISQ platform beyond 12x12 (144) qubits due to their current and planned fidelities.

NISQ gate-based hardware resource requirements

resources	initial estimates	realistic estimates and constraints
qubit number	50 qubits for a computational advantage (Preskill)	100s to 1000s qubits for many practical NISQ algorithms to obtain a speedup advantage (Guerreschi, Albino).
computing depth	use shallow algorithms with under 10-gate cycles	most NISQ algorithms in the quantum advantage regime have >100s gate cycles
available fidelities	NISQ is to use currently available qubit fidelities that are in the 99.9% to 99% range	current QPUs either have low fidelities and >30 qubits (transmons) or better fidelities and <30 qubits (trapped ions)
required fidelities	error rate $\ll \frac{1}{\# \, qubits * algo \, depth}$ *for QAOA, but seemingly for other NISQ algorithms as well* *https://iopscience.iop.org/article/10.1088/2058-9565/abae7d* *error rate usually relates to the two-qubit error rate, which should ideally be its minimum error rate and not median/average rate.*	the fidelities requirements are not matched by actual hardware even for the shallowest computing depth 1/(1121 q * 8 d) => 99,99% possible? 1/(127 q * 8 d) => 99,9% IBM Heron's 133 qubit QPU in 2024? 1/(65 q * 8 d) => 99,8% not available. 1/(53 q * 8 d) => 99,7% Google Sycamore is at 98,6%. *minimum ansatz depth of 8 gate cycles*
qubit gates set	variational algorithms use single qubit gates R_x, R_y, R_z with arbitrary angles, on top of Hadamard and CNOT gates	arbitrary rotations gates require very high precision and finely tuned calibration, it is a key difference with FTQC which relies on T and Toffoli gates which can be transversally corrected.

Figure 774 : table showing the qubits requirements for NISQ algorithms as estimated and with realistic estimates and constraints. It is showing some inconsistencies between the need for more than 50 qubits and their required two-qubit gate fidelities with even the shallowest algorithms. Even with the shallowest NISQ algorithms in the quantum advantage regime, the required fidelities are way above the current state of the art and its expected evolution in the next few years. On top of that, 8-depth cycles algorithms are easy to emulate with tensor networks on classical computing systems. Arbitrary rotation single qubit gates are another difference with NISQ, which at first glance is an advantage for NISQ implementation as compared to the large overhead of arbitrary rotation gates generations based on fault-tolerant sets including gates like T and Toffoli gates. Source: (cc) Olivier Ezratty, 2023.

These R arbitrary rotations gates must be implemented with very high precision, which is constrained, among other aspects, by the quality of their electronic drive[3457] [3458].

With FTQC, these gates are avoided since it is hard to correct their errors in a fault-tolerant manner. They are replaced by a universal gate set usually containing Clifford group gates (Pauli X, Y and Z gates, Hadamard gate, a CNOT gate for entanglement) and a gate enabling the generation of all rotations in the famous Bloch sphere representing the state of a single qubit, usually a T gate (Z angle of 45%) or a Toffoli 3-qubit gate.

Then, arbitrary rotation gates are constructed with long assemblies of these primary gates, depending on the needed angle precision, according to the famous Solovay-Kitaev theorem[3459]. These gates are used since they can be error-corrected in a fault-tolerant manner which seems not to be the case for arbitrary rotations gates.

[3457] It also leads to the notion of parameterized gates, in the case for example of superconducting qubits, where some optimization is implemented in the ansatz preparation to bypass the very notion of gates and drive the qubits directly at the pulse level controlling the rotations within the Bloch sphere. There are various implementation proposals like Towards Advantages of Parameterized Quantum Pulses by Zhiding Liang et al, April 2023 (15 pages), NAPA: Intermediate-level Variational Native-pulse Ansatz for Variational Quantum Algorithms by Zhiding Liang et al, August 2022-February 2023 (13 pages), PANSATZ: Pulse-based Ansatz for Variational Quantum Algorithms by Dekel Meirom and Steven H. Frankel, Technion, December 2022-March 2023 (11 pages), and ctrl-VQE in Gate-free state preparation for fast variational quantum eigensolver simulations by Oinam Romesh Meitei, Sophia E. Economou et al, npj Quantum Information, October 2021 (11 pages), improved in Minimizing state preparation times in pulse-level variational molecular simulations by Ayush Asthana, Sophia E. Economou et al, March 2022 (12 pages). Tools like AWS Braket Pulse enable pulse control as well.

[3458] See Probabilistic Interpolation of Quantum Rotation Angles by Bálint Koczor, John Morton and Simon Benjamin, May 2023 (12 pages).

[3459] See The Solovay-Kitaev algorithm by Christopher M. Dawson and Michael A. Nielsen, 2006 (15 pages) demonstrates the theorem.

Computing time

Another resource to estimate is the total NISQ algorithm computing time, including its classical portion. After all, we're looking for some computing speedup, but with reasonable computing times related to our patience. Its scaling must be carefully estimated in the quantum advantage regime due to various costs: the number of Pauli strings, the sought precision and the exponential cost of quantum error mitigation, as shown in Figure 775.

Whatever the use cases and speedup, NISQ computing times should be reasonable. We will see that it is not necessarily the case in a quantum advantage regime, when it fares better than classical computing. Most NISQ variational algorithms have a computing time with a lot of variables, equal to $N_i * I_t$ with $I_t = (C_t + S * Q_t)$, with:

N_i = **number of iterations** of the variational algorithm to converge on an acceptable value. It is case dependent and depends on the way the variational algorithm converges to the expected solution. Various optimization techniques are proposed here to reduce the number of iterations[3460] [3461].

I_t = **iteration time** to classically prepare an ansatz[3462] (C_t) and to run it on the quantum computer ($S * Q_t$), representing one iteration, Q_t being the time to run a single shot. C_t also contains the time it takes to handle the classical post-processing of the data coming from the quantum computing shots to generate the expectation value of the Hamiltonian observable from the ansatz. It is highly dependent on the number of shots described below.

S = **number of quantum circuit shots** corresponding to the number of times the ansatz must be executed on the quantum computer to compute the expected value of the observable of the ansatz in order to reach a given precision. This number of shots can scale as high as $O(n^4/\epsilon^2)$, n being the number of useful data qubits and ϵ the target error rate, with typical VQE algorithms to find the ground state of a molecule[3472].

The $O(n^4)$ scale corresponds to the number of Pauli strings, which applies a basis change with series of single qubit gates, changing the computational basis before qubits readouts[3431]. It is like doing a partial state tomography of the data qubits[3463]. For example, determining the ground state of a molecule such as benzene with its 12 atoms (C_6H_6), would require 72 qubits and running 330,816 Pauli strings in a VQE algorithm[3464] [3465], although this could be optimized to use only 8 qubits[3466]. Some Pauli strings can however be regrouped with some preprocessing, reducing the number of Pauli strings to $O(n)$, but at the cost of longer circuits that may be prohibitive in NISQ regime[3487].

With QAOA algorithms, the number of Pauli strings scales as $O(2L)$[3467], L being the number of rotations/entanglement cycles in the quantum circuit, *aka* layers.

[3460] See WEPRO: Weight Prediction for Efficient Optimization of Hybrid Quantum-Classical Algorithms by Satwik Kundu et al, Pennsylvania State University, July 2023 (12 pages).

[3461] See DISQ: Dynamic Iteration Skipping for Variational Quantum Algorithms by Junyao Zhang et al, Duke University, MIT, University of Chicago, University of Michigan and North Carolina State University, August 2023 (12 pages).

[3462] The ansatz preparation usually converts the problem specific ansatz to a hardware efficient ansatz with these series of rotation and entanglement gates. It minimizes the circuit depth and maximizes the circuit accuracy.

[3463] See Measurements as a roadblock to near-term practical quantum advantage in chemistry: resource analysis by Jérôme F. Gonthier et al, December 2020-August 2022 (49 pages).

[3464] See Large-Scale Simulation of Quantum Computational Chemistry on a New Sunway Supercomputer by Honghui Shang et al, July 2022 (13 pages).

[3465] See Calculating the ground state energy of benzene under spatial deformations with noisy quantum computing by Wassil Sennane, Jean-Philip Piquemal and Marko J. Rancic, March-November 2022 (24 pages).

[3466] See Molecular Symmetry in VQE: A Dual Approach for Trapped-Ion Simulations of Benzene by Joshua Goings et al, IonQ, August 2023 (7 pages).

[3467] See Evaluation of QAOA based on the approximation ratio of individual samples by Jason Larkin et al, December 2020 (13 pages).

Measurements are done for each Pauli string with $O(1/\epsilon^2)$ circuit executions shots[3468], corresponding to an outcome precision $1-\epsilon$, which by the way does not correspond to the chemical accuracy obtained as a result, that is also damaged by the qubit noise[3469]. Then, some classical postprocessing generates the ansatz objective function result. Some other tactics are proposed to optimize the number of required shots[3470].

The target error rate ϵ can be very low for VQE algorithms used in quantum chemical simulations, increasing as a result the number of required shots to astronomical levels. In 2015, it was estimated that finding the ground energy state of ferredoxin (Fe_2S_2) with 112 spin-orbitals with VQE would require 10^{19} circuit shots and 10^{26} gate operations[3471]. Various optimizations are proposed to remove the polynomial or exponential curse against the number of data qubits, and they are algorithm dependent[3472] [3473]. It can otherwise become a key showstopper of NISQ implementations beyond n=40 qubits, and to reach some practical quantum advantage.

You can then complement this list with the various quantum error mitigation techniques overhead which further increases the number of shots and adds some more classical processing burden. This overhead scales exponentially with the circuit depth or qubit number depending on the used mitigation techniques. With qubits having sufficient fidelities, making rather simple chemical computations with optimized VQE algorithms could last several decades if not centuries with superconducting qubits[3487]. How about the better trapped ions qubits with their high fidelities? These qubits are completely out of the game here, due to their quantum gates that can be about 1,000 times slower than with superconducting qubits[3474]. A theoretical speedup compared to classical computing is of no value if it practically happens at non-human time scales!

Again, a practical full-stack evaluation of all these time costs would be useful when discussing potential NISQ quantum advantages. It is not always studied in many NISQ algorithms papers which mostly deal with sub-NISQ scaling regimes with fewer than 30 qubits. It still drives some interesting architecture designs where many of these numerous shots would be run in parallel either on different QPUs or even, within a single QPU that would be logically divided in several small qubit zones running the same circuit[3475].

[3468] This comes from statistical mathematics. A standard error $\epsilon = \sigma/\sqrt{S}$, with S being the number of shots and σ the measurement standard deviation, which is 1 in the case of qubit measurement which values can only be 0 and 1. This is explained in A modern introduction to probability and statistics: understanding why and how by F.M. Dekking et al, Springer, 2005 (483 pages) starting page 96 with the 7.4 variance chapter.

[3469] See The Cost of Improving the Precision of the Variational Quantum Eigensolver for Quantum Chemistry by Ivana Miháliková et al, 2021 (25 pages). Chemical accuracy is measured in mHa (milli-Hartree) and must be below 1.6 mHa, corresponding to 4 kJ/mol.

[3470] See Resource-efficient utilization of quantum computers by Ijaz Ahamed Mohammad, Matej Pivoluska and Martin Plesch, May 2023 (12 pages).

[3471] See Progress towards practical quantum variational algorithms by Dave Wecker, Matthew B. Hastings, and Matthias Troyer, PRA, 2015 (10 pages).

[3472] See Quantum expectation-value estimation by computational basis sampling by Masaya Kohda et al, Physical Review Research, September 2022 (19 pages).

[3473] See Pauli String Partitioning Algorithm with the Ising Model for Simultaneous Measurement by Tomochika Kurita et al, May-September 2022 (25 pages).

[3474] Some parallelization could be envisioned here are is being studied. But it would bring its own QPU hardware, interconnect and classical computing overhead.

[3475] See How Parallel Circuit Execution Can Be Useful for NISQ Computing? by Siyuan Niu and Aida Todri-Sanial, LIRMM Montpellier, December 2021 (6 pages).

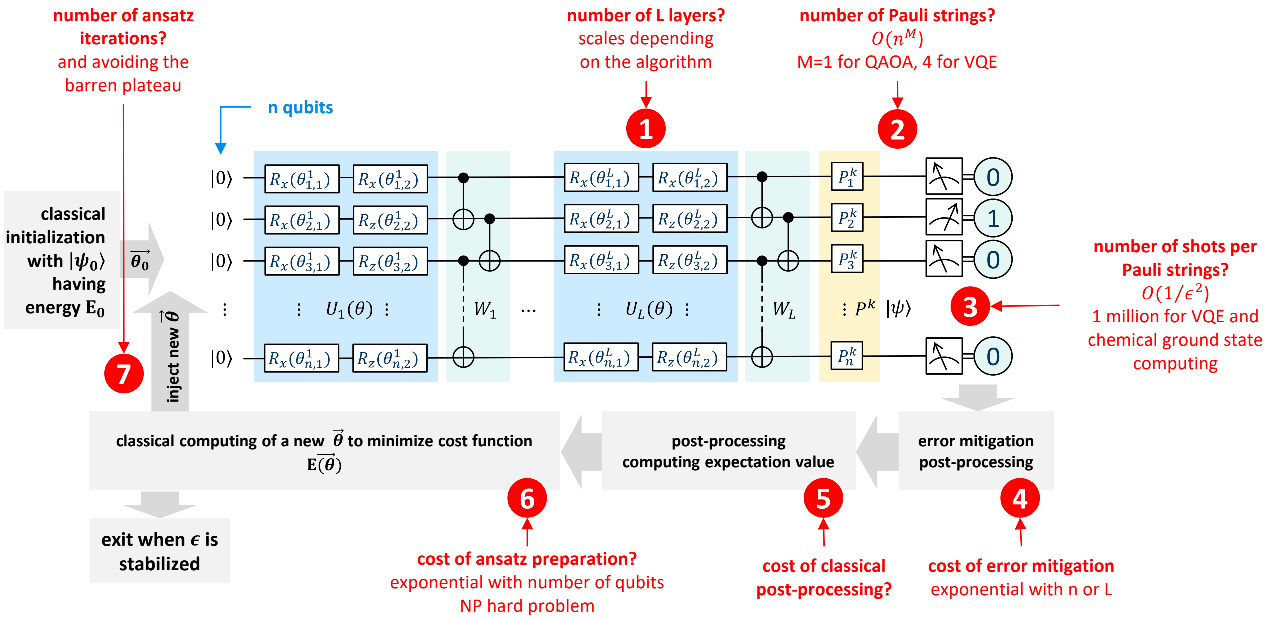

Figure 775: chart describing how variational quantum algorithms (VQA) operate and their scaling parameters. The grey part corresponds to the classical components of these algorithms. An ansatz contains a Hamiltonian encoded with single rotation and two-qubit CNOT gates cycles or layers[3476]. It is prepared classically to generate an expected value of the Hamiltonian after computing several runs. Additional ancilla qubits and operations can be added to the ansatz and are not shown here for simplification. The key scaling parameters here are: (1) the number of qubits and of phase and mixing operators in the ansatz (which in that case is labelled a "Quantum Alternating Operator Ansatz", another QAOA, also used in VQEs[3477]) determining the circuit depth and conditions the required qubit fidelities, given the computing depth is at least equal to the number of qubits due to the number of entangling gates to execute and the SWAP gates used with nearest neighbor qubits topologies[3489], (2) the number of Pauli strings for the measurement of the expected values from observables of the computed Hamiltonian which can scale polynomially with the number of qubits for VQE algorithms[3478] but scales linearly with QAOA optimization algorithms, (3) the number of shots to obtain a given precision for each Pauli string which scales as high as $O(1/\epsilon^2)$, meaning one million shots for typical chemical simulation precisions of one per thousand with VQE algorithms, (4) the additional cost of quantum error mitigation which can scale exponentially with the circuit depth or qubit number, (5) the classical post-processing to compute the cost function value, (6) the classical cost to prepare each ansatz, which is usually an NP complete problem, and (7) the number of ansatzes to converge on a satisfying cost function value, avoiding the barren plateau syndrome. The number of quantum circuit shots can become gigantic in the quantum advantage regime, particularly with VQE algorithms which require very high accuracy for chemical simulations. Source: (cc) Olivier Ezratty, 2023 and Jules Tilly et al[3487].

Variational Quantum Eigensolvers

One of the most pre-eminent NISQ algorithm class is the VQE (**Variational Quantum Eigensolver**), invented in 2013 by Alán Aspuru-Guzik[3479 3480]. Its main usage is to determine the ground state of a many-body quantum system[3481].

It is typically used to simulate the structures of molecules in inorganic and organic chemistry. It combines a classical part that determines an approximate starting point and a quantum part that refines

[3476] CNOT entangling gates could be replaced by RZX gates with a significan gain in fidelity and computing time as proposed in Toward Consistent High-fidelity Quantum Learning on Unstable Devices via Efficient In-situ Calibration by Zhirui Hu, Travis Humble, Weiwen Jiang et al, September 2023 (12 pages).

[3477] Other ansatzes can be used in VQE like the unitary coupled cluster ansatz, the quantum alternating operator ansatz, the variational Hamiltonian ansatz, and the hardware-efficient ansatz. See Implementing Jastrow--Gutzwiller operators on a quantum computer using the cascaded variational quantum eigensolver algorithm by John P. T. Stenger et al, May 2023 (10 pages).

[3478] See Near- and long-term quantum algorithmic approaches for vibrational spectroscopy by Nicolas P. D. Sawaya et al, September 2020-February 2021 (49 pages) gives an indication of the number of Pauli strings for a variational algorithm. In the quantum advantage regime above 40 qubits, it scales between 10^4 and 10^7.

[3479] VQE now belongs to a broader category of hybrid algorithms, Variable Quantum Algorithms (VQA). See Variational Quantum Algorithms by Marco Cerezo et al, Nature Reviews Physics, August 2021 (29 pages).

[3480] See Quantum chemistry calculations on a trapped-ion quantum simulator by Cornelius Hempel, Jarrod McClean, Thomas Monz, Ryan Babbush, Alan Aspuru-Guzik, Rainer Blatt, Christian Roos et al, March-August 2018 (22 pages).

[3481] See an history timeline on Towards an experimentally viable variational quantum eigensolver with superconducting qubits, 2016 (18 slides). See also Variational Quantum Eigensolver explained, November 2019, the result. The classical part prepares a so-called ansatz which is a set of parameters defining a quantum state, with using some nonlinear optimization techniques[3482].

The quantum side of the algorithm is used to compute a cost function outputting a real number that we seek to minimize with varying the parameters of the ansatz in the classical part[3483]. VQE can also be used in quantum machine learning[3484].

To date, most VQE experiments were implemented with a few qubits, way under the quantum advantage threshold, nearly always way under the 50 qubits mark. There are several reasons for these experiments being done in a pre-NISQ regime, way below 50 qubits. First, many projects by PhD candidates last between one and three years. Second, while several QPUs are available with over 50 qubits, particularly from IBM and Google, these have qubit gate fidelities too low to enable larger scale VQE (and VQA) noisy-resilient algorithms. The real usable QPUs quantum volumes are very low, with a record of 2^{22} obtained with Quantinuum trapped ions QPUs[3485]. These experiments are useful to test algorithms whereabouts before QPUs can scale and accommodate a larger number of qubits[3486].

These experiments most often deal with condensed matter physics, nuclear physics, high-energy particles physics, vibrational and vibronic spectroscopy, photochemical reaction properties predictions, to name a few, as described in the excellent Tilly et al VQE review paper[3487].

In the chemical simulation realm, VQE experiments are usually limited to finding the ground state energy of the Hamiltonian of simple two to three atoms molecules like LiH, BeH_2 or H_2O[3488 3489].

As we've seen before, finding the ground state of a slightly more complicated molecule as benzene drives NISQ systems in uncharted territory and very long computing times and requirements for very high-fidelity physical qubits [3465 3490]. VQE can also help compute the excited states of molecules[3491 3492].

3482 See Obtaining Unobtainium - Variational Quantum Eigensolvers 101, by Dickson Wu, March 2021 and its good strawman explanation of VQE algorithms.

3483 It is now possible to get rid of the classical part of the algorithm as explained in An adaptive variational algorithm for exact molecular simulations on a quantum computer, by Sophia Economou et al, 2019 (9 pages).

3484 See VQE-generated Quantum Circuit Dataset for Machine Learning by Akimoto Nakayama et al, February 2023 (9 pages).

3485 See Quantum Volume in Practice: What Users Can Expect from NISQ Devices by Elijah Pelofske, Andreas Bärtschi and Stephan Eidenbenz, DoE Los Alamos Research Laboratory, June 2022 (27 pages).

3486 See VQE Method: A Short Survey and Recent Developments by Dmitry A. Fedorov et al, 2021 (23 pages) which confirms that current hardware can't accommodate useful VQE implementations.

3487 See The Variational Quantum Eigensolver: a review of methods and best practices by Jules Tilly et al, Physics Reports, August-November 2022 (156 pages) is an excellent and thorough review of VQE and its resources constraints.

3488 See Ground-state energy estimation of the water molecule on a trapped-ion quantum computer by Yunseong Nam et al, npj Quantum Information, 2020 (6 pages).

3489 See How will quantum computers provide an industrially relevant computational advantage in quantum chemistry by Vincent E. Elfving et al, 2020 (20 pages) which among other things provide a clarification on the difference between accuracy (an end-goal with chemical simulations) and precision (related to some computing task).

3490 See Quantifying the effect of gate errors on variational quantum eigensolvers for quantum chemistry by Kieran Dalton et al, University of Cambridge and Hitachi, npj Quantum Information, November 2022-January 2024 (11 pages), which states: "*Our results show that, for a wide range of molecules, even the best-performing VQE algorithms require gate-error probabilities on the order of 10^{-6} to 10^{-4} to reach chemical accuracy. This is significantly below the fault-tolerance thresholds of most error-correction protocols. Further, we estimate that the maximum allowed gate-error probability scales inversely with the number of noisy (two-qubit) gates. Our results indicate that useful chemistry calculations with current gate-based VQEs are unlikely to be successful on near-term hardware without error correction*".

3491 See Many-Body Excited States with a Contracted Quantum Eigensolver by Scott E. Smart et al, UCLA, May 2023 (13 pages).

3492 See Folded Spectrum VQE : A quantum computing method for the calculation of molecular excited states by Lila Cadi Tazi and Alex J.W. Thom, Cambridge University and ENS Paris-Saclay, May 2023 (14 pages).

VQE is not yet addressing more pressing computational chemistry needs like determining large molecular structures, finding complex vibrational and rotational spectra, and molecular docking that are all useful in drugs design and in the chemical industry. These use cases belong generally to the FTQC regime, and in most cases, in extreme situations with very large numbers of logical qubits. For example, estimating the ground state of a complex molecule Hamiltonian in the FTQC domain is to be based on the quantum phase estimate (QPE) algorithm.

Its precision depends on the number of ancilla qubits in which the eigenvalue result is encoded.

VQE is sometimes described as the most appropriate VQA subset of algorithms that are suitable for NISQ QPUs. According to Sebastian Brandhofer et al[3493], VQE chemistry simulation algorithms do not scale in the quantum advantage regime unless qubit gate fidelities are very good with error rates below 0.18%. These gate fidelities are not available yet, particularly over 50 qubits, whatever the qubit technology[3494]. Other researchers point to chemical simulations requiring very high precision, which is hard to obtain in NISQ regimes, up to a point that FTQC versions of VQE algorithms are proposed[3495], but their computing time is totally prohibitive, even for small molecules[3496].

At last, a June 2023 preprint from Thibaud Louvet, Thomas Ayral and Xavier Waintal finds that qubit noise prevents VQE from providing sufficient chemical accuracy in chemical simulations[3497]. It was confirmed in another paper from LG Electronics researchers[3498].

Various VQE optimization techniques are proposed:

- Improve the efficiency of the ansatz, or its expressivity, with using (classical) tensor networks for their preparation[3499].
- Implement VQE-specific error mitigation techniques using machine learning[3500].
- Using ansatz with only Clifford gates, which may reduce its expressivity[3501].
- Embed measurement based elements in the VQE ansatz circuit to form a hybrid VQE, improving the ansatz tuning[3502].

3493 See Error Analysis of the Variational Quantum Eigensolver Algorithm by Sebastian Brandhofer et al, University of Stuttgart and UNSW, January 2023 (6 pages).

3494 See Benchmarking the Variational Quantum Eigensolver using different quantum hardware by Amine Bentellis et al, May 2023 (6 pages) that shows better VQE results on a H_2 molecule simulation with trapped ions from AQT than with superconducting qubits from IBM, but at a very small scale of only 5 qubits despite using QPUs with respectively 16 and 27 qubits.

3495 See A fault-tolerant variational quantum algorithm with limited T-depth by Hasan Sayginel et al, March 2023 (10 pages).

3496 See Reducing the cost of energy estimation in the variational quantum eigensolver algorithm with robust amplitude estimation by Peter D. Johnson et al, Zapata Computing, March 2022 (15 pages) proposes an optimization method to run VQE for simple molecules. The results range from 1,300 to 634,915 years of computing, provided you have between 120,000 and 352,000 physical qubits with 99.99% fidelities!

3497 See Go-No go criteria for performing quantum chemistry calculations on quantum computers by Thibaud Louvet, Thomas Ayral and Xavier Waintal, June 2023 (6 pages). They land on the same conclusion even with QPE (quantum phase estimations) used in a FTQC regime.

3498 See Practical limitations of quantum data propagation on noisy quantum processors by Gaurav Saxena et al, LG Electronics, June 2023 (14 pages).

3499 See Tensor Network Assisted Variational Quantum Algorithm by Junxiang Huang et al, December 2022 (9 pages).

3500 See Variational Denoising for Variational Quantum Eigensolver by Quoc Hoan Tran et al, Fujitsu, April 2023 (19 pages).

3501 See Towards chemical accuracy with shallow quantum circuits: A Clifford-based Hamiltonian engineering approach by Jiace Sun et al, Tencent, June 2023 (17 pages).

3502 See Hybrid variational quantum eigensolvers: merging computational models by Albie Chan et al, May 2023 (24 pages).

- The ADAPT-VQE derivative which uses fewer qubits and its variations[3503] [3504] [3505] [3506] [3507]. The ADAPT-VQE-SCF variation proposed by Algorithmiq and Trinity College to optimize molecular simulations is using a "self-consistent field approach" within the Adaptive Derivative-Assembled Problem-Tailored Ansatz Variational Quantum Eigensolver (ADAPT-VQE) framework. They expected these techniques to yield useful quantum advantages in 2023. ADAPT-VQE can also be used to simulate atom nuclear shell models[3508]. Other variations of ADAPT-VQE significantly reduce the number of measurement Paul strings and thus the number of circuit shots[3509] [3510].
- Other methods to reduce the number of measurement strings[3511] or shots using ancilla qubits[3512].
- Use a pre-trained neural network to adjust the ansatz at each iteration, enabling faster convergence and reducing the cost of this preparation[3513].

[3503] See Comparative study of adaptive variational quantum eigensolvers for multi-orbital impurity models by Anirban Mukherjee et al, Ames National Laboratory and Maryland University, Nature Communications Physics, January 2023 (15 pages).

[3504] See A self-consistent field approach for the variational quantum eigensolver: orbital optimization goes adaptive, Algorithmiq and Trinity College, December 2022 (21 pages).

[3505] See An adaptive variational algorithm for exact molecular simulations on a quantum computer by Harper R. Grimsley, Sophia E. Economou, Edwin Barnes and Nicholas J. Mayhall, Nature Communications, 2019 (8 pages).

[3506] See Overlap-ADAPT-VQE: practical quantum chemistry on quantum computers via overlap-guided compact Ansätze by César Feniou, Jean-Philip Piquemal et al, Nature Communication Physics, July 2023 (11 pages).

[3507] See Greedy gradient-free adaptative variational quantum algorithm on a noisy intermediate scale quantum computer by César Feniou, Jean-Philip Piquemal et al, July 2023 (25 pages) which dealt with a shallow 25-body Ising model circuit running on 25 qubits with an Overlap-ADAPT-VQE method running in a GPU-based quantum simulator.

[3508] See Nuclear shell-model simulation in digital quantum computers by A. Pérez-Obiol et al, February 2023 (16 pages).

[3509] See Adaptive variational quantum algorithms on a noisy intermediate scale quantum computer by César Feniou, Jean-Philip Piquemal et al, June 2023 (24 pages).

[3510] See Benchmarking Adaptative Variational Quantum Algorithms on QUBO Instances by Gloria Turati et al, August 2023 (7 pages).

[3511] See Accelerated variational quantum eigensolver with joint Bell measurement by Chengfeng Cao et al, July 2023 (12 pages).

[3512] See Low-depth Gaussian State Energy Estimation by Gumaro Rendon et al, Zapata Computing and Error Corp, September 2023 (42 pages).

[3513] See Neural network encoded variational quantum algorithms by Jiaqi Miao et al, Zhejiang University and Tencent, August 2023 (15 pages).

Quantum Approximate Optimization Algorithms

The second key NISQ hybrid algorithm class is the Quantum Approximate Optimization Algorithm **(QAOA)**, created by Edward Farhi in 2014. It is a combinatorial optimization algorithm finding approximate solutions with graph and slice management problems (Max-Cut)[3514] [3515] [3516], various tasks and jobs scheduling problems like the Binary Paint Shop Problem (BPSP), TSP (traveling salesperson problem)[3517], Maximum Independent Set problems[3518], QUBO problems (the same problems that are also solved in analog quantum computers) as well as for solving 3SAT Boolean satisfiability problems[3519].

A QAOA algorithm often relies on a QAOA component. This acronym strangeness comes from the Quantum Alternating Operator Ansätze (QAOA), the ansatz circuit that is used within a variational algorithm, alternating single qubit rotation gates and CNOT gates, as shown in Figure 775[3520].

Despite it requires fewer circuit shots than VQE algorithms, it seems that QAOA doesn't scale well and requires a larger number of higher quality qubits to bring some quantum advantage with practical use case in the enterprise operations domain[3521] [3522] [3523].

According to Anton Simen Albino et al, "*thousands of qubits will be needed before QAOA and its variants can be used to solve these problems, due to the linear relationship between the dimensionality of the problem and the number of qubits. However, the qubits used will not necessarily be error-corrected due to the characteristics of the heuristic itself, which requires low-depth circuits and few measurements of the final state*", in a paper dealing with solving partial derivative equations (PDEs) in fluid mechanics[3524]. Johannes Weidenfeller et al provides a lot of clues on QAOA running on NISQ systems, highlighting some obstacles to overcome to "*improve to make QAOA competitive, such as gate fidelity, gate speed, and the large number of shots needed*" [3525].

[3514] See Quantum Approximate Optimization Algorithm explained, May 2020, An Introduction to Quantum Optimization Approximation Algorithm by Qingfeng Wang and Tauqir Abdullah, December 2018 (16 pages).

[3515] See QAOA: Quantum Approximate Optimization Algorithm by Peter Shor (25 slides), Quantum Approximate Optimization Algorithm: Performance, Mechanism, and Implementation on Near-Term Devices, by Leo Zhou, Mikhail Lukin et al, 2019 (23 pages).

[3516] See Quantum approximate optimization of non-planar graph problems on a planar superconducting processor by Matthew P. Harrigan et al, 2021 (19 pages) which uses a QAOA algorithm on Google's 53 qubits Sycamore.

[3517] See Comparative study of variations in quantum approximate optimization algorithms for the Traveling Salesman Problem by Wenyang Qian et al, July 2023 (18 pages) which solves the TSP problem with up to 5 cities! That not a lot and far from any quantum advantage. See also Indirect Quantum Approximate Optimization Algorithms: application to the TSP by Eric Bourreau, Gerard Fleury and Philippe Lacomme, November 2023 (8 pages).

[3518] See Iterative Quantum Algorithms for Maximum Independent Set: A Tale of Low-Depth Quantum Algorithms by Lucas T. Brady et al, QAIL, September 2023 (15 pages).

[3519] See Amplitude amplification-inspired QAOA: Improving the success probability for solving 3SAT by Alexander Mandl et al, University of Stuttgart, March 2023 (25 pages).

[3520] See From the Quantum Approximate Optimization Algorithm to a Quantum Alternating Operator Ansatz by Stuart Hadfield et al, September 2017-February 2019 (51 pages).

[3521] QAOA is an algorithm providing approximate results as its name implies. It can't be compared to exact solutions classical algorithms like those from the Integer Linear Programming (ILP) class. It must be compared to approximate solutions algorithms like those from the PTAS class (Polynomial Time Approximated Solution). These are the ones that require thousands of qubits to exceed the capacities of classical systems. Thus, the need for much higher quality qubits than are available.

[3522] See A Practitioner's Guide to Quantum Algorithms for Optimisation Problems by Benjamin C. B. Symons et al, May 2023 (20 pages).

[3523] See QAOA with fewer qubits: a coupling framework to solve larger-scale Max-Cut problem by Yiren Lu et al, China, July 2023 (25 pages) which runs with 18 qubits and therefore, is far away from any quantum advantage.

[3524] See Solving partial differential equations on near-term quantum computers by Anton Simen Albino et al, August 2022 (9 pages).

[3525] See Scaling of the quantum approximate optimization algorithm on superconducting qubit based hardware by Johannes Weidenfeller et al, February-December 2022 (25 pages).

Their paper covers transpiler optimizations techniques and how QAOA works with the IBM heavy-hex qubit connectivity. It also provides an estimation of the number of shots to $O(n^2/\epsilon)$, n being the number of qubits and ϵ the expected algorithm precision. This adds a significant overhead to QAOA runtimes, although being lower than most VQE implementations.

In a paper dealing with using QAOA to solve a graph partitioning Max-Cut problem, G. G. Guerreschi and A. Y. Matsuura conclude that "*quantum speedup will not be attainable, at least for a representative combinatorial problem, until several hundreds of qubits are available*"[3526]. In their work, they make a classical comparison using a single Intel Xeon Phi processor. Such a single CPU would beat a QPU until it reaches about 900 qubits. 900 qubits and even a shallow algorithm would indeed land us in the high-fidelity qubit requirement territory zone with 1/(900*8) error rate, so 99.9986% (see Figure 776). Meanwhile, most QAOA experiments are done with only a few qubits[3527 3528]. A Max-Cut problem may be even more demanding in precision than a VQE used for some chemical simulation[3529].

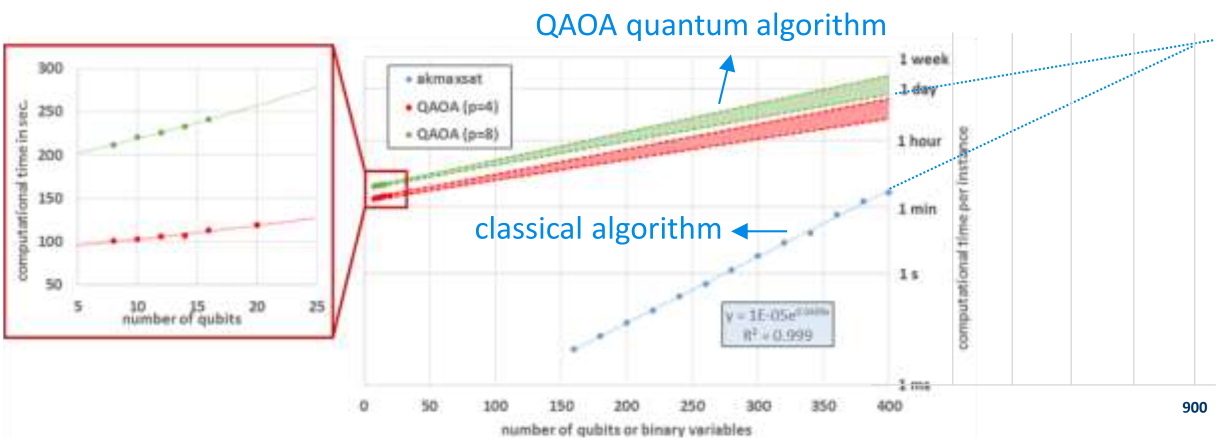

Figure 776 : QAOA hardware requirements showing a need for at least 900 qubits to reach some speedup quantum advantage. p corresponds to the number of times the QAOA circuit blocks are repeated in the algorithm ansatz. It means that p=8 has a depth that is twice as large as for p=4. This would require physical qubit fidelities in the 99.9986% range, which is far out of scope for NISQ architectures. Source: G. G. Guerreschi and Anne Y. Matsuura [3526]. Added in 2023.

Guillermo González-García et al land with a similar conclusion[3530]: "*We find that, even with a small noise rate, the quality of the obtained optima implies that a single-qubit error rate of 1/(nD) (where n is the number of qubits and D is the circuit depth) is needed for the possibility of a quantum advantage [...]. We estimate that this translates to an error rate lower than 10^{-6} using the QAOA for classical optimization problems with two-dimensional circuits*".

[3526] See QAOA for Max-Cut requires hundreds of qubits for quantum speed-up by G. G. Guerreschi and A. Y. Matsuura, Nature Scientific Reports, May 2019 (7 pages).

[3527] See Comparing Quantum Service Offerings: A Case Study of QAOA for Max-Cut by Julian Obst et al, 2023 (10 pages) with a comparison of a QAOA implementation on superconducting and trapped ions QPUs, with a depth of just 16 gate cycles. It generates erroneous results with 5 qubits on superconducting qubits, and satisfying results with trapped ions, although of course not in a quantum advantage regime.

[3528] See QAOA with N.p ≥ 200 by Ruslan Shaydulin and Marco Pistoia, JPMorgan Chase, March 2023 (5 pages). A trial of Max-Cut problem solving done as part of DARPA ONISQ challenge with a Quantinuum H1-1 QPU using 10 qubits.

[3529] The need for deeper circuits to solve QAOA in a quantum advantage regime is also detailed in the thesis On the performance of Quantum Approximate Optimization by Vishwanathan Akshay, 2023 (110 pages).

[3530] See Error Propagation in NISQ Devices for Solving Classical Optimization Problems by Guillermo González-García et al, PRX Quantum, December 2022 (17 pages).

And with 1,000 qubits! As a direct consequence, FTQC and over a million physical qubits seem to be required for implementing QAOA algorithms in the quantum advantage regime! One workaround would be to build relatively large scale NISQ systems with high qubit connectivity[3531].

A 2023 paper from Quantinuum & co, shows some evidence of a scaling advantage of QAOA, with a classical emulation with 40 qubits but concludes that "*Our results provide evidence for the utility of QAOA as an algorithmic component when executed on an idealized quantum computer*"[3532]. We're back on the same road: "*it's the hardware, stupid*"!

The excellent 2023 QAOA review paper by Kostas Blekos et al is thorough and more nuanced. It provides guidelines on the way to select the right approach, ansatz variant, parameter optimizations and how to best embed error correction mitigation techniques[3533]. Various tricks are indeed proposed to reduce the circuit depth of QAOA ansatzes like ADAPT-QAOA or FALQON. They are slightly moving the needle in the direction of a potential quantum advantage[3534] [3535] [3536] [3537] [3538] [3539], including some which are hardware dependent[3540] [3541] [3542]. These techniques, however, can have a significant classical overhead preventing their scaling to a large number of qubits[3543]. Other methods deal with improving global optimizations of the ansatz[3544].

There is also a more efficient QAOA variation that is based on Grover algorithm and uses twice as many qubits[3545].

3531 See Deep-Circuit QAOA by Gereon Koßmann et al, October 2022-February 2023 (19 pages) which studies how QAOA would perform with deep circuits, using higher fidelities qubits and/or FTQC architectures. They conclude that there wouldn't be a generic QAOA applicability with deep circuits.

3532 See Evidence of Scaling Advantage for the Quantum Approximate Optimization Algorithm on a Classically Intractable Problem by Ruslan Shaydulin et al, JPMorgan Chase, Quantinuum and Argonne National Laboratory, August (31 pages).

3533 See the review paper A Review on Quantum Approximate Optimization Algorithm and its Variants by Kostas Blekos et al, June 2023 (85 pages).

3534 See A Review on Quantum Approximate Optimization Algorithm and its Variants by Kostas Blekos et al, June 2023 (85 pages), page 14.

3535 See Quantum Alternating Operator Ansatz (QAOA) beyond low depth with gradually changing unitaries by Vladimir Kremenetski et al, May 2023 (40 pages).

3536 See Enhancing Quantum Algorithms for Maximum Cut via Integer Programming by Friedrich Wagner et al, February 2023 (24 pages).

3537 See Solution of SAT Problems with the Adaptive-Bias Quantum Approximate Optimization Algorithm by Yunlong Yu et al, October 2022-April 2023 (18 pages).

3538 See Investigating the effect of circuit cutting in QAOA for the Max-Cut problem on NISQ devices by Marvin Bechtold et al, February 2023 (31 pages).

3539 See Parallel circuit implementation of variational quantum algorithms by Michele Cattelan and Sheir Yarkoni, April 2023 (12 pages).

3540 See Scaling of the quantum approximate optimization algorithm on superconducting qubit based hardware by Johannes Weidenfeller et al, February-December 2022 (25 pages) which deals with circuit optimizations to minimize the need for SWAP gates in relation to limited qubits connectivity.

3541 See Quantum Approximate Optimization Algorithm with Cat Qubits by Pontus Vikstål, Laura García-Álvarez, Shruti Puri and Giulia Ferrini, Chalmers University and Yale University, May 2023 (14 pages).

3542 See the tutorial Theory and Implementation of the Quantum Approximate Optimization Algorithm: A Comprehensible Introduction and Case Study Using Qiskit and IBM Quantum Computers by Andreas Sturm, Fraunhofer, January 2023 (114 pages). These experiments were conducted on IBM's Ehningen 27-qubit QPU installed in Germany.

3543 See Dynamic-ADAPT-QAOA: An algorithm with shallow and noise-resilient circuits by Nikola Yanakiev et al, August 2023 (15 pages).

3544 See Restricted Global Optimization for QAOA by Peter Gleißner et al, September 2023 (17 pages).

3545 See Quantum Dueling: an Efficient Solution for Combinatorial Optimization by Letian Tang et al, February-May 2023 (18 pages).

Quantum Machine Learning

NISQ-based machine learning algorithms are usually based on Variational Quantum Algorithms (VQA)[3546]. These algorithms are plagued with about the same problems than QAOA algorithms with regards to the way they could practically scale[3547]. In November 2022, Lucas Slattery et al estimated that there is "*no quantum advantage with NISQ on QML with classical data*". Even worse, "*the geometric difference between "well-behaved" quantum models and classical ones is small and goes down with the number of qubits*"[3548].

On existing QPUs, Mohammad Kordzanganeh et al found that the precision of a shallow quantum neural network training algorithm is below 10% when run with over 8 qubits for IBM and Rigetti as shown in Figure 777 [3440]. It is better with OQC and IonQ but limited in number of qubits since these don't scale yet beyond 20 qubits and 20 qubits are cheaper to emulate classically whatever the scenario. So, we are very far from any quantum advantage, let alone doing something that cannot run on a simple laptop.

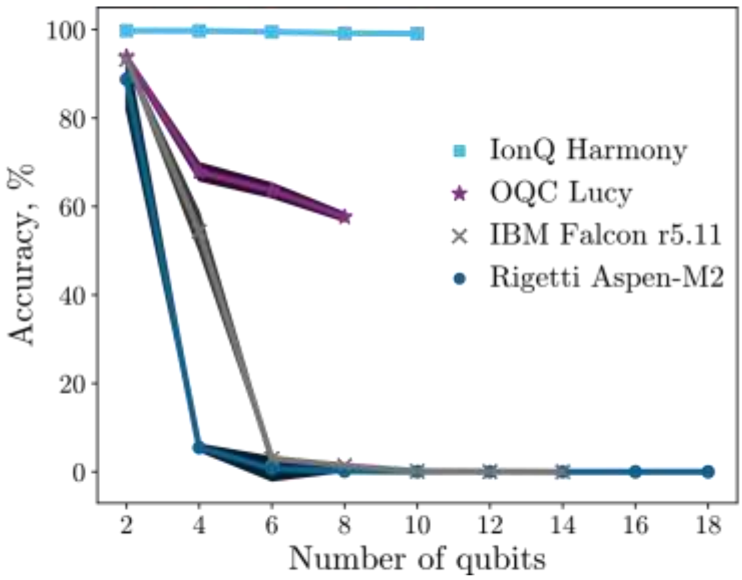

Figure 777: NISQ's actual algorithm depth with some current QPUs available with IBM and AWS cloud services when running some hybrid quantum neural network algorithm inferences (HQNN, proposed by Terra Quantum). It shows that the accuracy of the neural network predictions is trending to zero after 8 qubits for superconducting qubit platforms and is very good but capped at 20 qubits with trapped ion systems. We have here the illustration of the difficulty to have both breadth (number of qubits) and depth (number of gate cycles linked to qubit fidelities) in current NISQ platforms. Source: Kordzanganeh et al [3440]. Added in 2023.

Other advances in QML algorithms are tested on QPUs with a very low number of qubits, like in the work of Diego H. Useche et al which "*presents a novel classical-quantum density estimation strategy for current noisy quantum computers, which combines quantum algorithms to compute the expectation values of density matrices with a new quantum variational representation of data called quantum adaptive Fourier features (QAFF)*". It was tested on an IBM Oslo QPU with 7 qubits and the discussion about its scalability seems absent in regards of these systems qubit gate fidelities[3549].

[3546] See the well documented thesis Variational quantum algorithms for machine learning: theory and applications by Stefano Mangini, University of Pavia, June 2023 (220 pages).

[3547] See Challenges and Opportunities in Quantum Machine Learning by Marco Cerezo et al, March 2023 (16 pages).

[3548] See Numerical evidence against advantage with quantum fidelity kernels on classical data by Lucas Slattery et al, November 2022 (9 pages).

[3549] See Quantum Density Estimation with Density Matrices in NISQ Devices: Application to Quantum Anomaly Detection by Diego H. Useche et al, January 2022-February 2023 (15 pages).

Thanks to quantum algorithms dequantization, Jordan Cotler et al show "*that classical algorithms with sample and query (SQ) access can sometimes be exponentially more powerful than quantum algorithms with quantum state inputs*"[3550]. For them, the only QML advantage can be obtained when the QPU has direct access to quantum data as input. Quantum algorithm dequantization consists in converting a quantum algorithm into a classical algorithm, with decomposing it into subsets of tensor matrix operations that can be executed efficiently on a classical computer. The purpose of dequantization is to run a given quantum algorithm more efficiently on a classical computer. Pioneering work in dequantization work was done by Ewin Tang in her thesis supervised by Scott Aaronson when she dequantized a recommendation algorithm under certain conditions in 2018[3551].

Pradeep Niroula et al created a deep learning algorithm enabling the creation of documents summaries[3552]. This hybrid algorithm had a classical part doing a lot of classical data preparation. It analyzed a dataset of 300,000 news articles from CNN and the Daily Mail and precomputed it with a BERT NLP (natural language processing) classical deep learning model that handled sentences extraction and their conversion into vectors. The quantum part managed the text summarizing from respectively 20 to 8 and 14 to 8 sentences, with Quantinuum QPUs H1-1 and H1-2 QPUs using respectively 20 and 14 qubits, and with a 100 qubit gates depth which is excellent. But we are not yet in the quantum advantage regime with this number of qubits which, again, can be emulated on a simple laptop, and probably faster on a server cluster!

The paper doesn't provide resources requirements estimates for a larger summary set for, say, 100 or 1,000 sentences.

In another work, Robin Lorenz, Bob Coecke et al implemented some natural language processing algorithm using over 100 sentences as entry with using only 5 qubits on an IBM QPUs[3553]. Likewise, Wei Xia et al presented in March 2023 an improved quantum reservoir computing algorithm that could run on up to 7 qubits with some precision improvement over classical reservoir computing methods for forex forecasts.

But these 7 qubits run faster on a classical software emulator than on any existing QPU and the paper doesn't mention any qubit fidelities and number requirements to reach some quantum advantage[3554].

Late 2022, Ismail Yunus Akhalwaya et al touted that NISQ systems would soon be able to solve topological data analysis problems[3555]. TDA is used for extracting complex and shape-related summaries of high-dimensional data[3556 3557]. NISQ-TDA was presented as the "*first fully implemented end to-end quantum machine learning algorithm needing only a linear circuit-depth, that is*

3550 See Revisiting dequantization and quantum advantage in learning tasks by Jordan Cotler et al, December 2021 (6 pages).

3551 See A quantum-inspired classical algorithm for recommendation systems by Ewin Tang, July 2018 (32 pages).

3552 See Constrained Quantum Optimization for Extractive Summarization on a Trapped-ion Quantum Computer by Pradeep Niroula et al, June-October 2022 (16 pages).

3553 See QNLP in Practice: Running Compositional Models of Meaning on a Quantum Computer by Robin Lorenz, Anna Pearson, Konstantinos Meichanetzidis, Dimitri Kartsaklis and Bob Coecke, February 2021 (15 pages).

3554 See Configured Quantum Reservoir Computing for Multi-Task Machine Learning by Wei Xia et al, March 2023 (15 pages). Their conclusion states "*We attribute the superior computation power of our approach to the quantum coherence embedded in the quantum reservoir dynamics*".

3555 See Towards Quantum Advantage on Noisy Quantum Computers by Ismail Yunus Akhalwaya et al, IBM et al, ICLR 2024, September-December 2022-March 2024 (32 pages).

3556 See Topological data analysis and machine learning by Daniel Leykam and Dimitris G. Angelakis, CQT, Technical University of Crete and AngelQ Quantum Computing, June-July 2022 (15 pages).

3557 See Higher-order topological kernels via quantum computation by Massimiliano Incudini et al, University of Verona, July 2023 (9 pages) which implements the LGZ algorithm, from Quantum algorithms for topological and geometric analysis of data by Seth Lloyd, Silvano Garnerone and Paolo Zanardi, Nature Communications, 2016 (7 pages). It targets a FTQC QPU, all simulations being done so far with noiseless qubits.

applicable to non-handcrafted high-dimensional classical data, with potential speedup under stringent conditions". Practically speaking, TDA can identify clusters in high-dimensional data. It serves to estimate a "Betti number" which measures the connectivity of a topological space.

But we are far from being able to implement this algorithm in a NISQ regime[3558] [3559]. It is a narrow implementation of the TDA class of problems, and it imposes stringent data conditions to generate any computing advantage. On top of this, a NISQ computing advantage would require over 96 qubits with 99.99% two-qubit gate fidelities which are not in the radar yet. Again, with even a fidelity of 99.9%, we would need at least about 9,600 such physical qubits.

Alexander Schmidhuber and Seth Lloyd "*argue that quantum algorithms for TDA run in exponential time for almost all inputs by showing that (under widely believed complexity theoretic conjectures) the central problem of TDA - estimating Betti numbers - is intractable even for quantum computers* […] *Our results imply that quantum algorithms for TDA offer only a polynomial advantage*"[3560], which, if implementable in a real NISQ regime would make sense. But given the overhead of FTQC that would be mandated to solve this class of problem, we'd have to look at the constants and other fixed costs to check that a quantum advantage would show up in a reasonable regime.

Another older paper is more optimistic on TDA resource requirements. It states that a quantum TDA algorithm can have a guaranteed superpolynomial quantum speedup vs classical computing[3561]. It says a quantum advantage would require at least 80 physical qubits but gives no precise indication on the algorithm depth. With the shallowest algorithm possible of 8 gate cycles, we still would need two-qubit gate fidelities in the 99.8% range.

On the other hand, quantum machine learning speedups are not the sole potential quantum advantage attribute but, as Maria Schuld and Nathan Killoran pinpoint, the comparisons are complicated between classical and quantum machine learning algorithms[3562]. It deals with classifications quality, generalization capability on unseen training data, training data requirements and the likes, with few benchmarking references. On top of this, training data ingestion is mostly done by the classical part to prepare the algorithm quantum ansatz, and it scales linearly with the data size, so with no foreseeable quantum advantage.

At last, like VQE algorithms, QML algorithms must fight the famous barren plateau problem, which prevents training convergence unless the ansatz circuit is shallow[3563]. It is the equivalent of avoiding local minima traps in classical machine learning when a global minimum is searched but difficult to reach[3564].

[3558] See Quantum-Enhanced Topological Data Analysis: A Peep from an Implementation Perspective by Ankit Khandelwal and M Girish Chandra, February 2023 (10 pages).

[3559] See Quantifying Quantum Advantage in Topological Data Analysis by Dominic W. Berry, Ryan Babbush, et al, September 2022-January 2023 (42 pages).

[3560] See Complexity-Theoretic Limitations on Quantum Algorithms for Topological Data Analysis by Alexander Schmidhuber and Seth Lloyd, September 2022 (24 pages).

[3561] See Towards quantum advantage via topological data analysis by Casper Gyurik et al, Quantum Journal, May 2020-October 2022 (37 pages).

[3562] See Is quantum advantage the right goal for quantum machine learning by Maria Schuld and Nathan Killoran, PRX Quantum, March 2022 (13 pages) and Maria Schuld, Why measuring performance is our biggest blind spot in quantum machine learning, March 2022.

[3563] See Noise-induced barren plateaus in variational quantum algorithms by Samson Wang et al, Nature Communications, 2021 (11 pages).

[3564] See Barren plateaus in quantum neural network training landscapes by Jarrod R. McClean, Sergio Boixo, Vadim N. Smelyanskiy, Ryan Babbush and Hartmut Neven, Google AI, Nature Communications, 2018 (6 pages).

Variational Quantum Linear Solver

VQLS (Variational Quantum Linear Solver) is a NISQ algorithm proposal to solve linear equations aka Quantum Linear Systems Problems (QLSP)[3565]. It is used in machine learning[3566] and to solve partial differential equations and has been tested on existing QPUs with relatively large data space of $2^{50} \times 2^{50}$. It can also be used to compute excited states of molecules[3567].

Quantum Singular Value Decomposition

QSVD (Quantum Singular Value Decomposition) is another variational NISQ algorithm that generalizes the classical singular value decomposition (SVD) to the quantum setting.

It allows an efficient decomposition of quantum states into a set of singular values and corresponding orthonormal bases[3568]. Some interesting algorithms make use of QSVD like a proposal for a quantum page rank algorithm[3569].

Barren plateaus

Variational NISQ algorithms must fight the famous barren plateau problem, which prevents convergence unless the ansatz circuit is shallow[3570] [3571]. It is the equivalent of avoiding local minima traps in classical machine learning when a global minimum is searched but difficult to reach[3572]. Barren plateaus are induced by various factors including the number of qubits, the gate types, the circuit depth, the types of measurements and of course, qubit noise[3573].

Research is very active to fix this problem like with adding additional parameters and constraints to improve gradients in the variational training loop without resorting to inefficient overfitting[3574].

It also seems that the barren plateau syndrome can be avoided in VQE algorithms[3575] [3576]. It is a matter of balancing circuit expressivity and trainability[3577].

3565 See Variational Quantum Linear Solver by Carlos Bravo-Prieto, Ryan LaRose, Marco Cerezo, Yigit Subasi, Lukasz Cincio and Patrick J. Coles, September 201-June 2020 (21 pages).

3566 See Variational Quantum Linear Solver enhanced Quantum Support Vector Machine by Jianming Yi et al, Fraunhofer Institute for Industrial Mathematics ITWM, September 2023 (14 pages).

3567 See Scalable Quantum Computation of Highly Excited Eigenstates with Spectral Transforms by Shao-Hen Chiew and Leong-Chuan Kwek, February 2023 (16 pages) which is a variation of VQE.

3568 See Quantum Singular Value Decomposer by Carlos Bravo-Prieto, Diego García-Martín and José I. Latorre, May 2019-June 2020 (7 pages).

3569 See Variational Quantum PageRank by Christopher Sims, April 2023 (9 pages). QSVD based.

3570 See A Unified Theory of Barren Plateaus for Deep Parametrized Quantum Circuits by Michael Ragone, M. Cerezo et al, September 2023 (20 pages).

3571 See Noise-induced barren plateaus in variational quantum algorithms by Samson Wang et al, Nature Communications, 2021 (11 pages).

3572 See Barren plateaus in quantum neural network training landscapes by Jarrod R. McClean, Sergio Boixo, Vadim N. Smelyanskiy, Ryan Babbush and Hartmut Neven, Google AI, Nature Communications, 2018 (6 pages).

3573 See Cost function dependent barren plateaus in shallow parametrized quantum circuits by M. Cerezo et al, Nature Communications, 2021 (12 pages).

3574 See Hamiltonian variational ansatz without barren plateaus by Chae-Yeun Park and Nathan Killoran, Xanadu, February 2023 (17 pages).

3575 See Universal effectiveness of high-depth circuits in variational eigenproblems by Joonho Kim, Jaedeok Kim, and Dario Rosa, PRR, June 2021 (12 pages).

3576 See Theory of overparametrization in quantum neural networks by Martin Larocca, Nathan Ju, Diego García-Martín, Patrick J. Coles, Marco Cerezo, September 2021 (30 pages).

3577 See Connecting Ansatz Expressibility to Gradient Magnitudes and Barren Plateaus by Zoë Holmes, Kunal Sharma, M. Cerezo, and Patrick J. Coles, PRX Quantum, January 2022 (23 pages).

It also seems that the barren plateau syndrome can be avoided in VQE algorithms[3578] [3579] [3580].

Analog quantum computing and simulations

Quantum annealing and analog quantum computing are not darlings of the quantum computing industry. On one hand, in quantum annealing, D-Wave has been criticized for a long time for "not being quantum" nor being in position to bring any computing advantage. On the other hand, analog quantum computers (programmable Hamiltonian simulation or programmable quantum simulators) are developed and commercialized by a very small number of vendors such as PASQAL and QuEra and said to have their own scalability challenges.

Still, you find many solutions running on these systems that seem closed to some quantum advantage than existing gate based QPUs[3581]. Recent benchmarks also show that analog quantum computers currently have greater computing capacity than gate-based noisy quantum computers as discussed in the Q-Score section.

Most of these solutions rely on formulating a problem into a QUBO (Quadratic Unconstrained Binary Optimization), a generic form of binary optimization problem. It is supported on analog quantum computers including quantum annealers from D-Wave and quantum simulators from QuEra and Pasqal[3582].

In quantum annealers and simulators, QUBO problems are converted in a problem manageable by the hardware through the process of graph embedding that is then natively processed in the QPU. There is some more flexibility in the graph shape and form with quantum simulators than with quantum annealers.

Various optimization problems, in turn, can be converted in QUBO models, including chemical simulation problems. QUBO algorithms can for example solve TSP problems (traveling salesperson)[3583], 3-SAT combinatorial problems[3584] [3585] and job-shop scheduling problems in manufacturing where multiple jobs must be processed on different machines, each job having a specific sequence of operations and each operation requiring a specific machine for its execution.

3578 See Universal effectiveness of high-depth circuits in variational eigenproblems by Joonho Kim, Jaedeok Kim, and Dario Rosa, PRR, June 2021 (12 pages).

3579 See Theory of overparametrization in quantum neural networks by Martin Larocca, Nathan Ju, Diego García-Martín, Patrick J. Coles and Marco Cerezo, September 2021 (30 pages).

3580 See Hamiltonian variational ansatz without barren plateaus by Chae-Yeun Park and Nathan Killoran, Xanadu, February 2023 (17 pages).

3581 See Practical quantum advantage in quantum simulation by Andrew J Daley, Matthias Troyer, Peter Zoller et al, Nature, July 2022 (14 pages).

3582 QUBO can also be implemented on gate based QPUs. See Combinatorial Optimization on Gate Model Quantum Computers: A Survey by Ehsan Zahedinejad et al, 2017 (17 pages).

3583 See Improving Performance in Combinatorial Optimization Problems with Inequality Constraints: An Evaluation of the Unbalanced Penalization Method on D-Wave Advantage by J. A. Montanez-Barrera, Pim van den Heuvel, Dennis Willsch and Kristel Michielsen, May 2023 (8 pages).

3584 See Solving (Max) 3-SAT via Quadratic Unconstrained Binary Optimization by Jonas Nüßlein et al, February 2023 (14 pages).

3585 See Influence of Different 3SAT-to-QUBO Transformations on the Solution Quality of Quantum Annealing: A Benchmark Study by Sebastian Zielinski et al, May 2023 (9 pages).

The goal is to determine an optimal schedule that minimizes the time required to complete all jobs or other performance criteria and resource constraints[3586] [3587] [3588] [3589].

Sheir Yarkoni et al's review paper on quantum annealing provides an up-to-date status of the D-Wave platform usability[3590]. It inventories a broad set of algorithms and trials related to mobility traffic flow optimization and vehicle routing problem, scheduling and logistics problems, finance portfolio optimization, quantum simulation, chemistry and material design, physics, biology, machine learning (classification, reinforcement learning, cluster analysis), matrix factorization and other finite-element design. All these algorithms are hybrid like most NISQ known algorithms. Compared to the various known gate based NISQ algorithms, quantum annealers are more generic than usually thought.

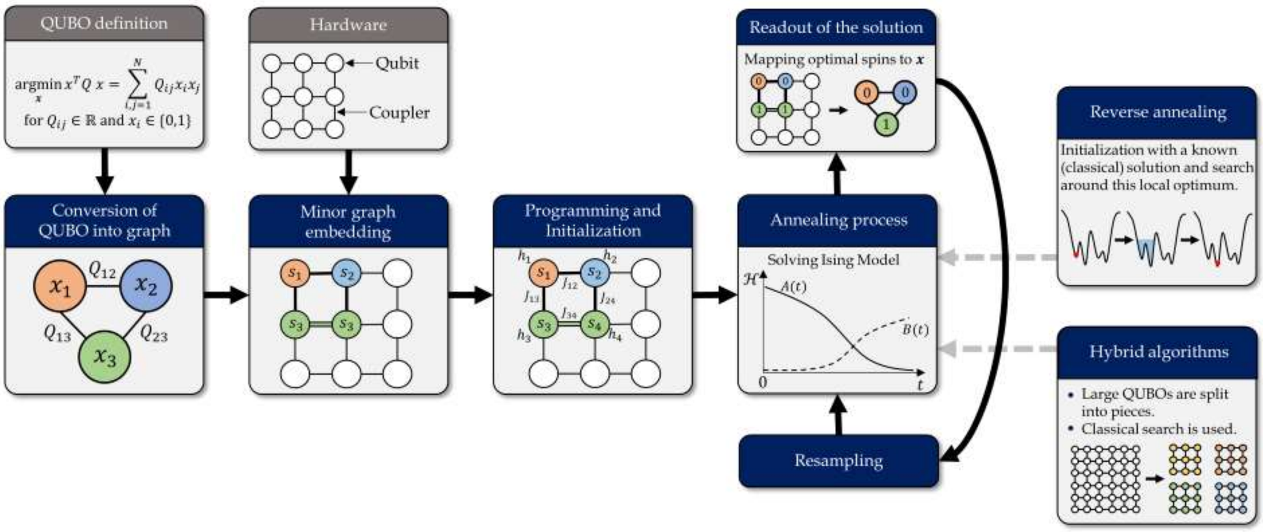

Figure 778: quantum annealing process taking the example of an optimization problem encoded in a QUBO problem and graph (quadratic unconstrained binary optimization). The graph is then automatically converted in a graph corresponding to the superconducting topology of a D-Wave QPU through the process of "minor graph embedding". The graph is then encoded in the system with an initialization of the qubit weights and connections. The annealing process takes place with setting a transverse magnetic field that sets the qubits value in a superposed state and progressively removing this field, which implements the annealing process, converging the qubit towards their optimum value minimizing the total system energy. Then the qubits are read out. The process is repeated several times and the result values averaged. Variations involved reverse annealing when the graph initialization uses a known classical solution and the annealing process helps find a better solution around it, and hybrid algorithms where large QUBO problems are split classically into smaller QUBO problems. Source: Sheir Yarkoni et al [3590]. Added in 2023.

Yarkoni et al highlight that quantum annealing can only address specific problem formulations and implements metaheuristic quantum optimizations, meaning approximate solutions (Figure 778). Also, the number of needed physical qubits scales polynomially with the number of logical variables from the problem formulation.

[3586] See Solving job-shop scheduling problems with QUBO-based specialized hardware by Jiachen Zhang et al, ICAPS, 2022, (9 pages).

[3587] See Quantum algorithm for process parallel flexible job shop scheduling by Berend Denkena et al, 2021 (15 pages).

[3588] See Solving flexible job shop scheduling problems in manufacturing with Quantum Annealing by Philipp Schworm, Production Management, 2023 (11 pages). It concludes that "*the presented approach demonstrated its ability to fnd high-quality solutions in a short time and can be used to generate diferent schedule variants that can be evaluated against each other.Nevertheless, efficient algorithms for solving FJSSP are still quite far away from industrial application*".

[3589] See Solving the Job Shop Scheduling Problem: QUBO model and Quantum Annealing by Riad Aggoune and Samuel Deleplanque, Luxembourg Institute of Science and Technology and Centrale Lille, 2023 (4 pages).

[3590] See Quantum Annealing for Industry Applications: Introduction and Review by Sheir Yarkoni et al, Reports on Progress in Physics, December 2021-June 2022 (43 pages).

Other sources found gate based QAOA to be potentially more efficient than quantum annealing based QUBO[3591].

The review paper lists some improvements required at the hardware level to generate some quantum advantage such as additional qubit control, driver Hamiltonians and operators, and higher connectivity.

QUBO has also a variant, Higher-order Unconstrained Binary Optimization (HUBO), that is involved in solving more generic search and optimization problems. For example, a new variation of HUBO can reduce the number of qubits needed to run a Grover adaptive search[3592].

In detail, many algorithms tested on quantum annealers are able to solve sizeable problems, but usually, still under the demanding quantitative requirements levels of real-life scenarios[3593]. Some examples come mostly from the financial sector with portfolio optimizations[3594 3595 3596]. Then we have also financial index tracking[3597] and foreign exchange optimizations[3598].

Analog quantum computers bring more flexibility on paper with the ability to define arbitrary graph trees with better connectivity. It can be used for simulating the physics of some topological spin liquid[3599], to predict which companies could fail in loans reimbursements[3600] and solve various graph problems[3601].

When reviewing all these case studies, both in the gate-based and analog quantum computing categories, one thing is striking: the most powerful solutions available are in the analog space rather than in the gate-based space. Quantum inspired classical solutions implementing linear algebra and tensor networks computing are also making classical computing more competitive in several areas[3602].

3591 See Quantum Approximate Optimization Algorithm: Performance, Mechanism, and Implementation on Near-Term Devices by Leo Zhou, Sheng-Tao Wang, Soonwon Choi, Hannes Pichler and Mikhail D. Lukin, PRX, 2020 (23 pages). This is still a theoretical paper that makes resource projections using this QAOA algorithm on a gate-based neutral atom QPU using several hundreds of atoms.

3592 See Accelerating Grover Adaptive Search: Qubit and Gate Count Reduction Strategies with Higher-Order Formulations by Yuki Sano et al, August 2023 (11 pages).

3593 See Neural Networks for Programming Quantum Annealers by Samuel Bosch, Bobak Kiani, Rui Yang, Adrian Lupascu and Seth Lloyd, MIT and University of Waterloo, August 2023 (15 pages).

3594 See Dynamic Portfolio Optimization with Real Datasets Using Quantum Processors and Quantum-Inspired Tensor Networks by Samuel Mugel et al, Multiverse, June 2020-December 2021 (13 pages) that compares implementations of portfolio optimizations with classical tensor networks, hybrid quantum annealing and a NISQ VQE algorithm running on IBM gate based QPUs. They got the best results and largest calculations with the two first solutions, handling 55 assets over 8 years.

3595 See Hybrid Quantum Investment Optimization with Minimal Holding Period by Samuel Mugel et al, Nature, December 2020-December 2021 (6 pages) proposing a variant for investment optimization with a minimal holding period constraint with handling 50 assets over a one year period, all using a D-Wave 2000Q. It requires a few minutes of computing per day.

3596 See Comparing Classical-Quantum Portfolio Optimization with Enhanced Constraints by Salvatore Certo et al, Deloitte, March 2022 (6 pages) handled a SP500 portfolio optimization with comparing CPLEX (classical optimization), BQM (a QUBO binary quadratic model) and CQM (QUBO Constrained Quadratic Model).

3597 See Financial Index Tracking via Quantum Computing with Cardinality Constraints by Samuel Palmer et al, August 2022 (8 pages).

3598 See Finding the Optimal Currency Composition of Foreign Exchange Reserves with a Quantum Computer by Martin Vesely et al, March 2023 (30 pages).

3599 See Probing topological spin liquids on a programmable quantum simulator by G. Semechin et al, Science, April-December 2021 (21 pages).

3600 See Financial Risk Management on a Neutral Atom Quantum Processor by Lucas Leclerc et al, Multiverse, PASQAL and CACIB, December 2022 (17 pages).

3601 See Quantum evolution kernel: Machine learning on graphs with programmable arrays of qubits by Louis-Paul Henry, Slimane Thabet, Constantin Dalyac, and Loïc Henriet, PRA, September 2021 (19 pages).

3602 See Mean-Field Approximate Optimization Algorithm by Aditi Misra-Spieldenner et al, March 2023 (17 pages) is about a quantum inspired QAOA classical algorithm that seems to better perform than QAOA.

These are not quantum at all. Then, other use cases directly put you in the FTQC zone, requiring thousands of logical qubits and thus, millions to hundred million physical qubits.

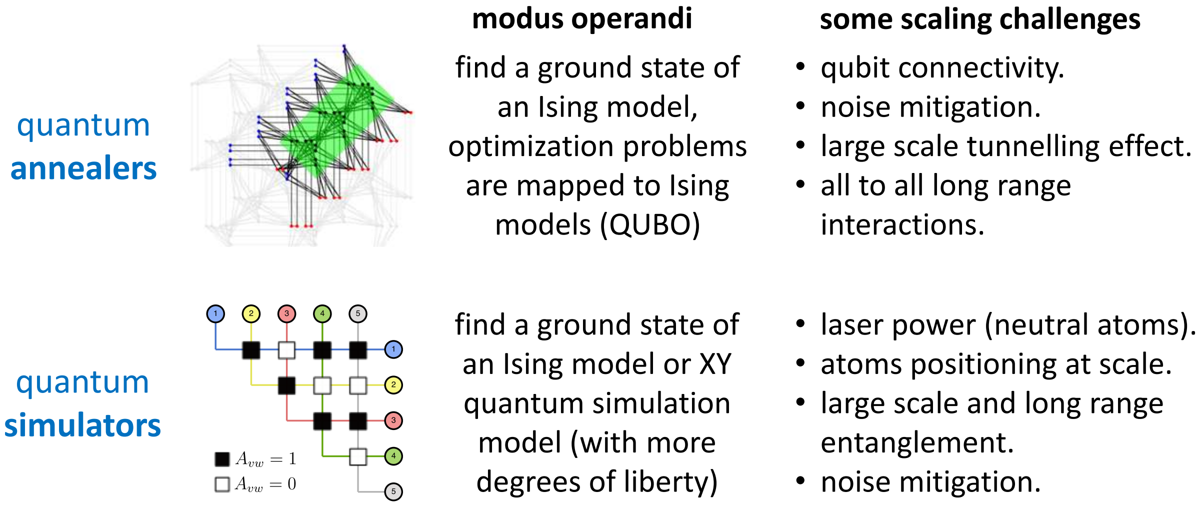

Figure 779: some scaling challenges for quantum annealers and analog quantum computers. Source: (cc) Olivier Ezratty, 2023.

However, even if analog quantum computing existing use cases are closer to real-life production grade levels than the gate-based equivalents, there are still some challenges to overcome in generating a quantum advantage with analog quantum computers as summarized in Figure 779[3603]. Quantum annealers require more tunability of qubit connections and better qubits connectivity.

Noise mitigation must also be handled [3604] [3605] [3606]. Some techniques are used to benchmark the quality of entanglement of these systems[3607]. And there's a remaining question similar as the one with NISQ systems: how far large scale quantum effects can operate, particularly, based on the tunnel effect that is at the core of quantum annealing.

With neutral atoms, their scaling is linked to the ability to control large chunks of well-positioned entangled atoms in ultra-vacuum. The related tools are made of more powerful and stable lasers, and their related control electronics.

At last, let's mention **DAQC** (Digital-Analog Quantum Computing), a proposal to implement a hybrid gate-based and analog quantum computing model[3608]. DAQC is supposed to make a more efficient use of quantum computing resources and enable NISQ algorithms with fewer qubits and to run faster than regular NISQ QPUs. It is adapted to optimization and machine learning. It is proposed by Kipu Quantum (Germany) and Qilimanjaro (Spain). Kipu Quantum is investigating the use of superconducting, trapped ion and neutral atoms qubits.

QPU chips would have custom designs to handle global entangled states for the annealing part.

[3603] See Going Beyond Gadgets: The Importance of Scalability for Analogue Quantum Simulators by Dylan Harley et al, University of Copenhagen, Stanford University, ENS Lyon, UGA June 2023 (53 pages).

[3604] See Analog errors in quantum annealing: doom and hope by Adam Pearson et al, 2019 (16 pages).

[3605] See Post-Error Correction for Quantum Annealing Processor using Reinforcement Learning by Tomasz Śmierzchalski et al, March 2022 (14 pages).

[3606] See Boosting the Performance of Quantum Annealers using Machine Learning by Jure Brence et al, March 2022 (14 pages).

[3607] See Solving optimization problems with local light shift encoding on Rydberg quantum annealers by Kapil Goswami et al, August 2023 (18 pages).

[3608] See Digital-Analog Quantum Computation by Adrian Parra-Rodriguez et al, December 2018-July 2020 (12 pages).

An implementation proposal using superconducting qubits would use SQUIDs to connect qubits in 2D matrices[3609].

It can improve computing fidelities to some extent[3610]. Questions abound on the speedups obtained with this architecture, its dependance on algorithms classes[3611] and its impact on control electronics and energetics. Also, it is more complicated to debug algorithms and few development tools are supporting it. In a recent paper, Narendra N. Hegade and Enrique Solano could factorize a 48-bit integer on 10 Quantinuum qubits and asserted that a DAQC NISQ platform could enable a factorization of RSA-2048 keys[3612].

Another paradigm consists in mixing quantum annealing and gate-based quantum computing. It is proposed by a team of Taiwan researchers with their large-system sampling approximation (LSSA) algorithm that solves Ising problems more efficiently than quantum annealers alone[3613].

NISQ distributed computing

Another sought option is to distribute some problems on several QPUs that are interconnected classically, using another form of hybrid algorithm. Such algorithms are designed for problems that can be split to run on several QPUs, with many constraints. This won't necessarily bring any exponential computing advantage but is an interesting path to work with NISQ QPUs[3614]. Among other scenarios, it is proposed to distribute some separable quantum neural network classification tasks to several QPUs[3615] and even more generic tasks[3616].

Quantum inspired algorithms

Quantum inspired algorithms are classical algorithms whose design is inspired by quantum algorithms and interference management, but not programmed as quantum algorithms run through a classical emulator[3617].

Quantum inspired algorithms can be helpful for solving linear algebra problems[3618], simulating quantum systems[3619], in fluid dynamics simulations[3620], with portfolio optimization, options pricing[3621],

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3609 See Superconducting Circuit Architecture for Digital-Analog Quantum Computing by J. Yu et al, EPJ Quantum Technology, March 2021-May 2022 (37 pages).

3610 See Noise in Digital and Digital-Analog Quantum Computation by Paula García-Molina et al, July 2021-December 2022 (10 pages).

3611 See Enhancing Quantum Annealing in Digital-Analog Quantum Computing by Tadashi Kadowaki, Denso, June 2023 (9 pages).

3612 See Digitized-counterdiabatic quantum factorization by Narendra N. Hegade and Enrique Solano, January 2023 (3 pages).

3613 See Hybrid Gate-Based and Annealing Quantum Computing for Large-Size Ising Problems by Chen-Yu Liu et al, August 2022 (14 pages).

3614 See Quantum Divide and Conquer for Combinatorial Optimization and Distributed Computing by Zain H. Saleem et al, Argonne Lab, Princeton, University of Colorado Boulder and SuperTech/ColdQuanta, July 2021 (13 pages).

3615 See Scalable Quantum Neural Networks for Classification by Jindi Wu, Zeyi Tao and Qun Li, Department of Computer Science William & Mary, Williamsburg, August 2022 (11 pages).

3616 See Enabling multi-programming mechanism for quantum computing in the NISQ era by Siyuan Niu and Aida Todri-Sanial, LIRMM, March 2022 (23 pages).

3617 See the review paper Quantum inspired algorithms in practice by Juan Miguel Arrazola, Seth Lloyd et al, 2020 (24 pages).

3618 See An improved quantum-inspired algorithm for linear regression by András Gilyén et al, January 2022 (23 pages).

3619 See one example in Classical algorithms for many-body quantum systems at finite energies by Yilun Yang, J. Ignacio Cirac and Mari Carmen Banuls, April 2022 (11 pages).

3620 See Complete quantum-inspired framework for computational fluid dynamics by Raghavendra D. Peddinti et al, August 2023 (14 pages).

3621 See Application of Tensor Neural Networks to Pricing Bermudan Swaptions by Raj G. Patel et al, Multiverse, April 2023 (15 pages).

recommendation systems (like with the famous solution from Ewin Tang), images classification[3622] and machine learning [3623] [3624] (Figure 780).

Creating a quantum inspired algorithm is sometimes said to rely on "dequantizing" or the "de-quantization" of a quantum algorithm[3625]. There is even a quantum inspired version of the NISQ QAOA algorithm[3626].

As Alastair Abbott and Cristian S. Calude wrote in 2010, "*de-quantization helps formulate conditions to determine if a quantum algorithm provides a real speed-up over classical algorithms. These conditions can be used to develop new quantum algorithms more effectively (by avoiding features that could allow the algorithm to be efficiently classically simulated), as well as providing the potential to create new classical algorithms (by using features which have proved valuable for quantum algorithms)*". In their paper, they also found that any algorithm in which entanglement is bounded is dequantizable[3627].

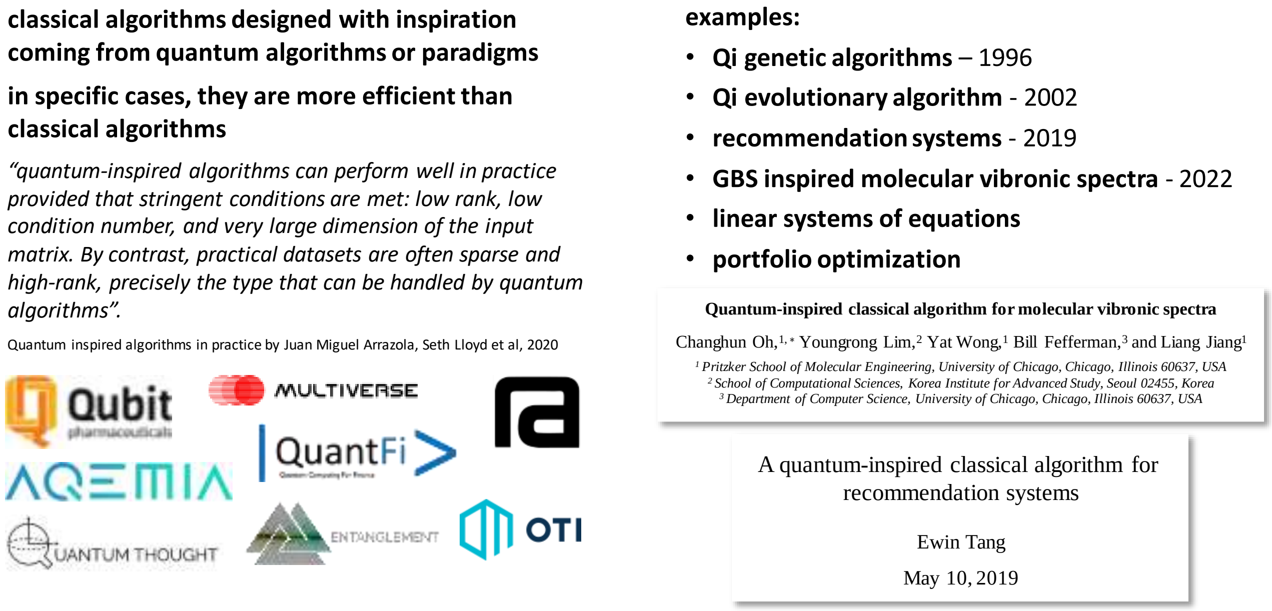

Figure 780: quantum inspired algorithms examples. (cc) Olivier Ezratty and various sources.

Quantum inspired algorithms even exist that transpose the complicated gaussian boson sampling photonic based technique to simulate molecular vibronic spectra. Molecular vibronic spectra come from its light absorption that depends on the transitions between its different electronic states and changes in its vibrational energy[3628].

3622 See AutoQML: Automatic Generation and Training of Robust Quantum-Inspired Classifiers by Using Genetic Algorithms on Grayscale Images by Sergio Altares-López et al, August 2022 (13 pages) which improves medical imaging grey images classification using a quantum inspired machine learning algorithm.

3623 See Robust Dequantization of the Quantum Singular value Transformation and Quantum Machine Learning Algorithms by François Le Gall, April 2023 (55 pages).

3624 See Quantum-Inspired Machine Learning: a Survey by Larry Huynh et al, August 2023 (56 pages).

3625 See one example in Sampling-based sublinear low-rank matrix arithmetic framework for dequantizing quantum machine learning by Nai-Hui Chia, Ewin Tang et al, October 2019 (79 pages).

3626 See Mean-Field Approximate Optimization Algorithm by Aditi Misra-Spieldenner et al, March 2023 (17 pages).

3627 See Understanding the Quantum Computational Speed-up via De-quantisation by Alastair Abbott and Cristian S. Calude, 2010 (12 pages).

3628 See Quantum-inspired classical algorithm for molecular vibronic spectra by Changhun Oh, Liang Jiang et al, University of Chicago, February 2022 (19 pages).

Quantum inspired algorithms are used in finance, healthcare and by many startups like Qubit Pharmaceuticals, Aqemia and Rahko in chemical simulations or QuantFi and Multiverse Computing in financial optimization. Quantum software startups find in quantum inspired algorithms a way to monetize their know-how while waiting for sufficiently powerful quantum computers.

Quantum inspired algorithms can also be parallelized in CPU and GPU architectures[3629]. And digital quantum annealers can also be considered as implementing quantum inspired techniques, but at the hardware level.

Tensor networks

In our presentation of basic linear algebra, we have described the notion of tensor products to mathematically represent a qubit register (page 168). It is a multiplication of matrices and a qubit register is represented by an exponentially growing vector state of 2^N complex numbers. Tensor products are also heavily used with neural networks programming in classical deep learning, including with the famous TensorFlow SDK from Google.

Describing a many-body quantum system is making use of these exponentially large tensor products and they are hard to compute classically. So here comes the notion of tensor networks which helps factorize very large tensors into networks of smaller tensors. It can be viewed as techniques that "zips" the tensor representation of these many-body systems, providing up to an exponential gain in the number of computing parameters and operations[3630].

One of the reason these systems can be compressed relies on the so-called area-law that says that entangled quantum systems are connected only to their neighborhood, thus enabling the split of many-body systems into separable smaller body systems[3631] [3632].

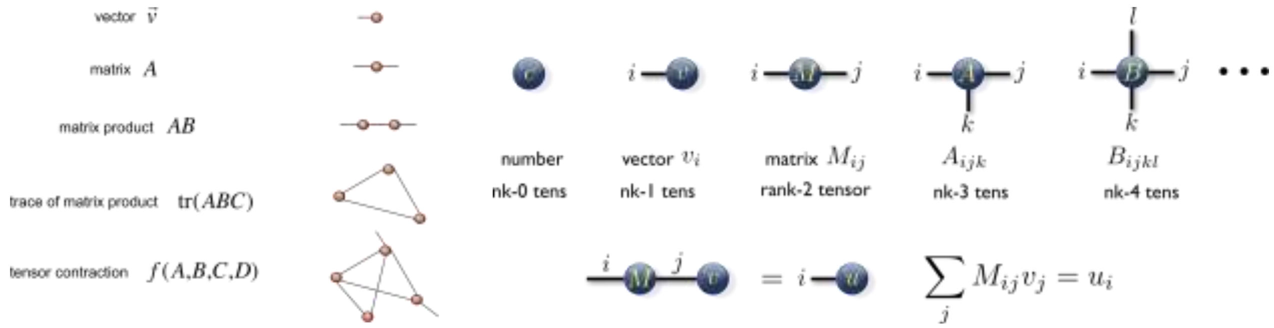

Figure 781: Sources: Introduction to Tensor Network States and Methods by Román Orús, DIPC & Multiverse Computing, 2020 (229 slides) and Lecture 1: tensor network states by Philippe Corboz, Institute for Theoretical Physics, University of Amsterdam (56 slides).

Tensor networks have various use-cases in many-body quantum physics digital simulations, for classical simulations of quantum computers, in chemistry, as well as in machine learning and applied mathematics.

This is an entirely new world of mathematics.

[3629] See Massively Parallel Tensor Network State Algorithms on Hybrid CPU-GPU Based Architectures by Andor Menczer and Örs Legeza, May 2023 (18 pages).

[3630] I found the zip analogy in Tensor network states to compress the many body problem by Antoine Tilloy, Inria, November 2021 (15 slides).

[3631] For more on the area law, see Colloquium: Area laws for the entanglement entropy by J. Eisert, M. Cramer and M. B. Plenio, 2010 (28 pages) and Area Laws for Entanglement by Fernando G.S.L. Brandão and Michal Horodecki, Stanford University, 2014 (56 slides). To some extent, this law has indirect implications on the real scalability of quantum computers since on one hand, it seems to depend on the size of maximally entangled systems and in practice, these systems mays not exist due to the area law… !!!

[3632] See The resource theory of tensor networks by Matthias Christandl et al, July 2023 (37 pages).

How is it connected to quantum computing? The frontier is fuzzy. Most of the tensor network literature deals with classical computing but tensor flow optimization techniques are also applicable to quantum computing. Some companies like Multiverse Computing make a lot of use of tensor networks techniques in quantum inspired algorithms.

In typical graphical representations shown in Figure 781, tensor networks use graph notation connecting matrices (mid-points in graphs), vectors (line endpoints), traces of matrix products (triangles), and tensor contraction.

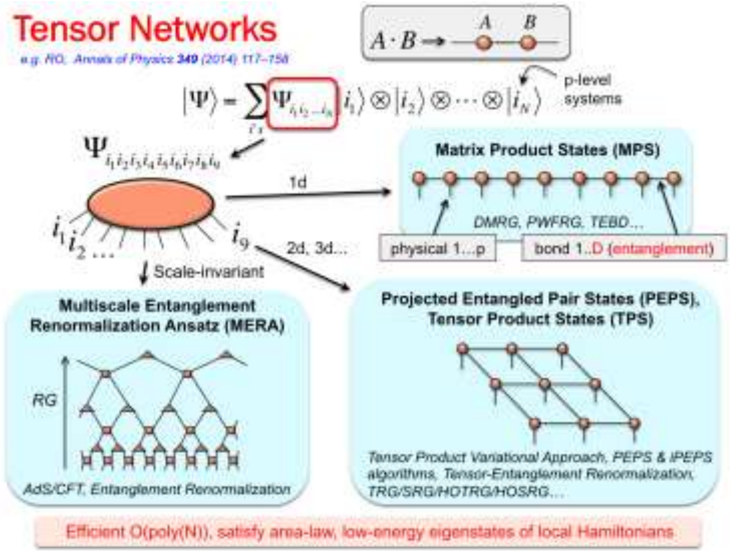

Figure 782: how tensor networks are graphically represented using the above notation. Source: Same as above.

Then, tensor network techniques are represented with these graphical views. As shown in Figure 782, the three main techniques are **MPS** (matrix product state represented in a 1D series of link with the preeminent **DMRG** variant[3633 3634 3635]), **PEPS** (Projected Entangled Pair States, used with graphs), **TPS** (Tensor Product States), and **MERA** (Multiscale Entanglement Renormalization Ansatz).

The graphical representation of gate-based quantum circuits happens to be a special case of tensor networks[3636]! The ZX Calculus graphical language is also derived from the tensor network formalism[3637].

About 80 classical tools implement Tensor networks like **Google TensorNetwork** which relies on TensorFlow and was released in 2019[3638] or the **QTensor** library from the Argonne National Laboratory that is used for quantum circuits emulations[3639]. Before quantum computers bring some quantum advantage, tensor network based techniques remain key contenders to enable classical many-body simulations, particularly thanks to many architectural and implementation improvements, particularly around DMRG based techniques[3640].

[3633] Here are some key researchers behind tensor networks: Steven R. White, who created DMRG in 1992 and who's group at UCI in California maintains the ITensor software library for tensor network, Edwin Miles Stoudenmire who was his PhD and Ulrich Schollwöck from LMU München who also contributed to the development of DMRG.

[3634] See Density matrix renormalization group, 30 years on by Frank Verstraete, Miles E. Stoudenmire et al, Nature Review Physics, April 2023.

[3635] See Density-matrix renormalization group: a pedagogical introduction by G. Catarina and Bruno Murta, April 2023 (30 pages).

[3636] The Basics of Tensor Network - An overview of tensors and renormalization by Samuel Desrosiers, Glen B. Evenbly and Thomas E. Baker (5 pages) explain how we build tensor network algorithms using these representations. This graphical representation was created by Roger Penrose in 1971.

[3637] See a use-case of ZX Calculus in the creation of a tensor network for the classical part of a variational circuit Barren plateaus in quantum tensor network optimization by Enrique Cervero Martín, September 2022 (26 pages).

[3638] See The landscape of software for tensor computations by Christos Psarras et al, March 2021-June 2022 (16 pages) and Google TensorNetwork Library Dramatically Accelerates ML & Physics Tasks, 2019.

[3639] See Performance Evaluation and Acceleration of the QTensor Quantum Circuit Simulator on GPUs by Danylo Lykov et al, Argonne National Laboratory, April 2022 (8 pages) and a QAOA-based use case in QArchSearch: A Scalable Quantum Architecture Search Package by Ankit Kulshrestha, Danylo Lykov, Ilya Safro, Yuri Alexeev, October 2023 (10 pages).

[3640] See Boosting the effective performance of massively parallel tensor network state algorithms on hybrid CPU-GPU based architectures via non-Abelian symmetries by Andor Menczer et al, September 2023 (17 pages).

After some time spent understanding how tensor networks work[3641], you may wonder where quantum computing plays a role here.

It seems mainly used as design tools to create quantum error correction codes, topological computing, solve condense matter physics problems[3642] and quantum machine learning algorithms[3643]. Tensor networks are also used in the creation of quantum code emulation software since these need various tensor contraction tools to save memory in storing the large quantum vector states if not density matrices of a number of qubits as large as possible[3644]. Xanadu's PennyLane framework contains tensor network circuit templates[3645]. Zapata Computing is also working on using tensor networks to prepare parametrized quantum circuits used in hybrid algorithms[3646]. Tensor networks may soon benefit from optimization techniques like the one developed by DeepMind, AlphaTensor[3647].

Complexity classes

So far, we have reviewed a lot of the most common quantum algorithms and their theoretical acceleration.

Quantum computing is sometimes presented as a miracle solution to extend computing capacities beyond the limits of classical supercomputing. It allows solving so-called "intractable" problems on conventional computers.

But what is the nature of the problems that can be solved with a quantum computer and that cannot be solved with classical computers? And above all, what are the limits of quantum computers? Do we still have computational limits?

We will see that these limits are rather blurred and shifting over time. This deals with **complexity theories**, a rather cryptic field of science and mathematics. It is a very abstract world involving a cryptic semantic made of P, NP, BQP and other complexities classes. Mathematicians have been discussing for half a century whether **P = NP** or not.

This is the science of problem complexity classes. Behind mathematics of complexity lie key technical but also philosophical considerations that are fundamental to Man and his omnipotence and control desires.

Problem complexity classes are Russian dolls more or less nested with each other that describe the range of time it takes to solve a problem, to verify given solutions and also on the associated required memory space, with regards to the problem size. The size of a problem is often formulated as an integer N, giving the number of items in the problem.

[3641] See Tensor Networks in a Nutshell by Jacob Biamonte et al, 2017 (34 pages), Lectures on Quantum Tensor Networks by Jacob Biamonte, January 2020 (178 pages), Hand-waving and Interpretive Dance: An Introductory Course on Tensor Networks Lecture Notes by Jacob C. Bridgeman and Christopher T. Chubb, 2017 (62 pages), Tensor Network Contractions by Maciej Lewenstein et al, 2020 (160 pages), the review paper Tensor Network Algorithms: a Route Map by Mari Carmen Bañuls, May 2022 (14 pages) and Hand-waving and Interpretive Dance: An Introductory Course on Tensor Networks by Jacob C. Bridgeman and Christopher T. Chubb, 2017 (62 pages).

[3642] See Applications of Tensor Networks: Machine Learning & Quantum Computing by Edwin Miles Stoudenmire, 2018 (142 slides).

[3643] See Towards Quantum Machine Learning with Tensor Networks by William Huggins, Edwin Miles Stoudenmire et al, Berkeley and Flat Iron Institute, July 2018 (12 pages).

[3644] See TensorCircuit: a Quantum Software Framework for the NISQ Era by Shi-Xin Zhang et al, May 2022 (43 pages).

[3645] See Tensor-Network Quantum Circuits, June 2022.

[3646] See Synergy Between Quantum Circuits and Tensor Networks: Short-cutting the Race to Practical Quantum Advantage by Manuel S. Rudolph et al, August 2022 (12 pages).

[3647] See Discovering faster matrix multiplication algorithms with reinforcement learning by Alhussein Fawzi et al, Google DeepMind, Nature, October 2022 (17 pages).

As far as time scales are concerned, there are many ways in which this problem solving time scale can grow with N. The key ones are constant, logarithmic, linear, polynomial and exponential. In this time scale, a time is considered reasonable if it is polynomial or below polynomial in the growing scale. A polynomial time is proportional to a given power of N.

Quantum computing allows under certain conditions to solve certain exponential problems in polynomial time. It must be translated in: a given problem that would require an exponential time to be solved on a classical computer would require a polynomial time to be solved on a quantum computer.

But what lies beyond exponential time? There are still various inaccessible problems with, for example, exponential of exponential time scales. And we have also exponential memory space which can add another complexity dimension. Quantum computers will not be able to solve all these problems, even when we will be able to align gazillions of logical qubits.

These limitations have an indirect impact on predictions about the creation of some omniscient artificial intelligence capable of transcending human reasoning and solving all problems. This hypothetical AGI (Artificial General Intelligence) will be limited by the data and concepts that feed it and by the impossibility of solving all complex problems.

Mankind will continue to confront impossible computing tasks. It will not be able to solve all the complex problems around! Quantum computing does not allow us to dominate nature, to put the whole Universe in equations and to predict how it will run with quantum precision. Chance and the unexpected will continue to play a role in a very indeterministic world, and for the better. It is a small lesson in humility for Mankind.

Problem Complexity Classes

To dive into complexity classes, you need to define the main classes of problems by level of complexity. Here I am trying to simplify complexity, this time in the literal sense of the word.

Complexity classes often describe problems that are solved by using brute force with testing several combinations to find the ones matching some criteria (like with the so-called SAT problems) or with using mathematical equations defining complex systems (differential equations, Schrödinger's equation, ...).

Problem classes use the notion of Turing's deterministic and non-deterministic machines. Turing machines are conceptual models of computers created by Alan Turing before the Second World War (Figure 783).

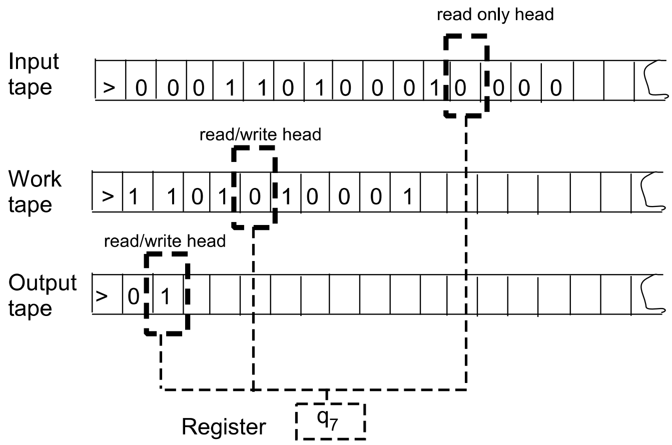

Figure 783: the famous Turing machine. Source: Computational Complexity: A Modern Approach by Sanjeev Arora and Boaz Barak, 2007 (489 pages).

They model computer processing based on the notion of programs and data, embodied by continuous and infinite rolls of paper, the first for the program, the second for the input data and the third for generating the results. Turing's theoretical model has long been used to define classes of problems that can or cannot be solved by a computer[3648].

[3648] See Computational Complexity: A Modern Approach by Sanjeev Arora and Boaz Barak, 2007 (489 pages) which is a good reference document on complexity theories. Students of a master's degree from ENS Lyon made a Turing machine in Lego in 2012 to celebrate the centenary of Alan Turing's birth (video) and it wasn't the only one of its kind (video)! Another one was made with wood in 2015 (video).

Computers are all metaphorically Turing machines, with limited space and memory, reproducing this logic by reading program instructions and managing data in random access memory (RAM) or persistent storage (hard disk, SSD, ...).

Associated with the notion of Turing machine is the notion of **Church-Turing's thesis**, named after Alonzo Church and Alan Turing, according to which there is an equivalence between computational problems that can be solved manually and with unlimited resources, those that can be handled with a Turing machine and those that can be solved with so-called recursive functions.

In a deterministic machine, the sequence of actions to be performed is predetermined and sequential. In the non-deterministic Turing conceptual machine model, computational rules can lead to execute several different operations for each evaluated situation (Figure 784). Basically, by exploring several paths in parallel and looking for a positive response to an algorithm component and closing parallel test loops once the sub-solutions are found.

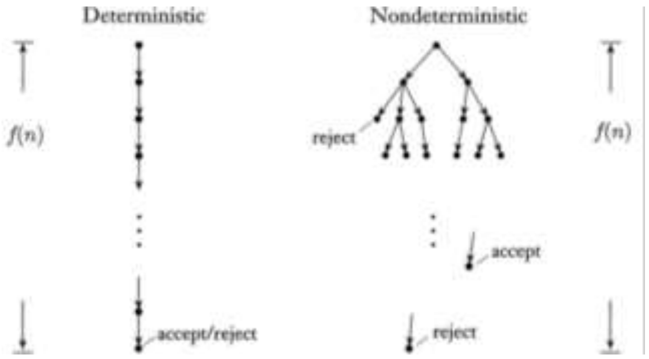

Figure 784: deterministic and non-deterministic Turing machines.

A non-deterministic machine increases the computational combinatorics compared to a deterministic machine. And this combinatorics usually jumps from polynomial to exponential.

Generic complexity classes

The level of complexity refers to the computing time and memory space required for these calculations. We are usually limited by computing time long before being limited by the available memory. However, some problems such as scheduling reach memory limits before computation limits.

The association of a problem to a complexity class is related to the performance of the best-known algorithms to solve problems in that class.

Problem class levels in complexity theories are often based on black box or oracle models to which a system asks questions and gets answers based on the data provided. This is a logic of "brute force" and hypothesis scanning. The scale of the tested combinations depends on the problem class.

So here are these classes by increasing levels of complexity knowing that we will spend more time on NP related classes (Figure 785).

L or LSPACE, or DLOGSPACE, defines the class of problems that can be solved on a logarithmic scale of consumed memory and on a deterministic Turing machine, that is, on a traditional computer. Computational complexity increases slowly with the size of the problem. Unfortunately, very few complex problems sit in this class. These include queries in previously indexed relational databases, searches for DNA sequences, and generally speaking, search techniques that use pointers and optimize the use of computer memory.

NL is the class of problems solved on a logarithmic time scale on a non-deterministic machine. Complexity theory specialists are still trying to figure out whether L=NL or not! But they are less busy here than on determining whether P<>NP.

P covers problems that can be solved with a time growing polynomially with data to process and on a deterministic machine. If N is the size of the problem, the processing time is proportional to N^M, with M being an integer, preferably 2. It is an easy problem to solve and said to be "tractable". This includes sorting lists, validating the existence of a path in a graph, searching for a minimum path in a graph, multiplying matrices or evaluating a number to see if it is prime.

BPP is a class of problems that can be solved by random approaches ("Bounded-Error Probabilistic Polynomial-Time"). It would seem that BPP=P but this has not yet been demonstrated.

NP describes the class of problems for which it is easy to check the validity of a solution, i.e., that it can be realized in polynomial time by a deterministic machine. The other definition of the class is that it contains problems whose solution time is polynomial on a non-deterministic machine. These more complex problems have a computing time that is at least exponential when the method used is said to be naive, testing all possible combinations. These are "intractable" problems. In practice, these are problems particularly suited to quantum computers because of their ability to evaluate in parallel 2^N combinations of some classical computation.

Some examples of NP problems are the Steiner tree to determine whether an electrical network can connect a number of houses at a certain price, checking that a DNA sequence is found in several genes and the distribution of tasks to different agents to minimize the time it takes to complete them.

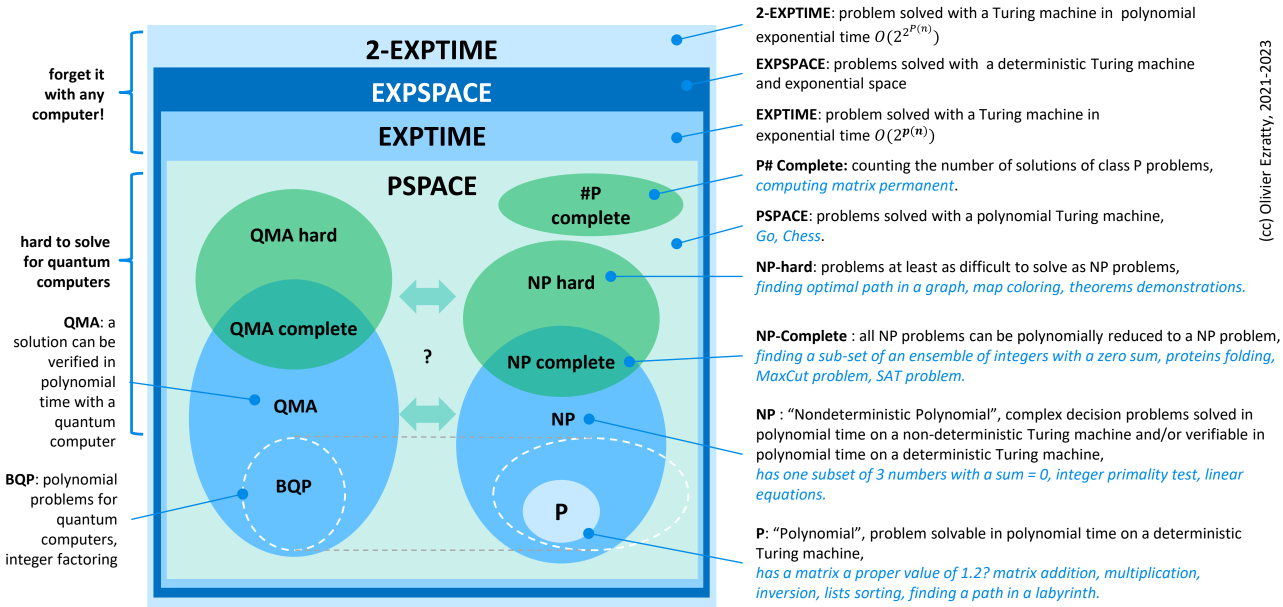

Figure 785: quantum and classical complexity classes, compilation (cc) Olivier Ezratty, 2021-2024.

These problems are omnipresent in logistics, planning, production, transportation, telecom, utilities, finance and cryptography. Note that a "decidable" problem, i.e., one that requires exploring a finite space of options, is not necessarily feasible from a practical point of view. Even if it can be solved in a finite amount of time, its resolution may take too long. An exponential problem has an elegant solution if one can find one solution that has a polynomial or, at best, linear duration. Polynomial times scale better than exponential times!

A big debate has been going on since 1971 (Stephen Cook et Leonid Levin) as to whether class P equals class NP. If P = NP, it would be as simple to find a result when one can also simply verify it[3649].

The general consensus is that P is not equal to NP[3650].

[3649] NP and NP-complete classes were defined in [The complexity of theorem-proving procedures]() by Stephen Cook of the University of Toronto, 1971 (8 pages), best popularized in [An overview of computational complexity]() (8 pages), [Reducibility among combinatorial problems]() by Richard Karp, 1972 (19 pages) and in [Complexity and calculability]() by Anca Muscholl of the LaBRI, 2017 (128 slides).

[3650] See [Fifty Years of P vs. NP and the Possibility of the Impossible]() by Lance Fortnow, 2021 (11 pages).

The demonstration of whether or not P<>NP is part of one of the seven Clay Mathematics Institute mathematical challenges launched in 2000, each with a prize of $1M (Figure 786)[3651].

Among these challenges are the demonstration of the Navier-Stokes fluid mechanics equations and of Riemann's hypothesis on the distribution of prime numbers.

On the P vs NP side, the wording of the challenge provides an example of such a problem: you have to allocate 50 rooms of two students to 400 candidates but some candidates do not need to live in the same room[3652].

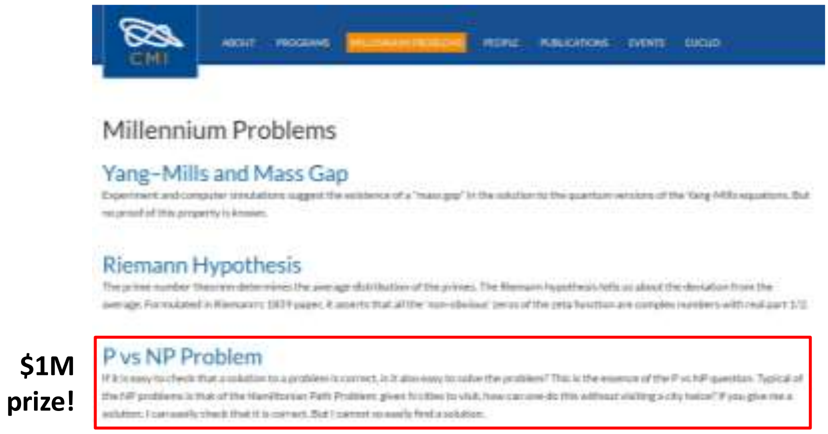

Figure 786: the Millennium challenge and P vs NP problem. Source: Clay Mathematics Institute mathematical challenges.

The combinatorial choice of 100 students among 400 is huge, so the problem is not easily handled with a supercomputer and brute force. It is indeed an NP problem because a given solution is easy to verify since it is simple to check that none of the rooms contains a forbidden pair of individuals. It is a bit like an all-or-nothing theory because if P = NP, all NP problems would have an efficient polynomial solution. If P ≠ NP, none of the NP problems have a "pure" efficient solution[3653]. Also, worth mentioning, a P problem could have a solution in n^k, with n being the size of the problem and k a very large integer. It would still be intractable on a practical basis, rending the N vs NP class discussion moot.

Complete NP corresponds according to Richard Karp to problems in which other NP problems can be polynomially reduced. They have no known P (polynomial) solution. They are not accessible to quantum computers. It is in this class that we find the SAT or 3SAT type Boolean logic problems! More than 3000 NP-Complete problems are identified to date (list).

One of the typical problems is filling up the trunk of a car when you go on vacation or when you come back from Christmas with a bunch of presents for your family (Figure 787). And then the **bin packing** problem consists in filling a backpack in an optimal way with a set of objects, to obtain the largest load and without exceeding a maximum weight (*aka* "bin packing" or “knapsack” problems) with at least three constraints. There are three classes of knapsack problems: the 0-1 knapsack problem, where only full items can be used[3654] [3655], the bounded knapsack problem (BKP) where you can use fractional items and the unbounded knapsack problem (UKP) with no upper bound on the number of each kind of item that can be used, and which can be solved with a greedy classical algorithm. With two constraints, an ϵ (error rate) approximation classical solution is available.

[3651] A Brazilian researcher, André Luiz Barbosa, published in 2010 his P ≠ NP Proof (25 pages) as well as a paper invalidating Cook's theorem that a Boolean SAT problem is NP-Complete, The Cook-Levin Theorem Is False, 2010 (11 pages). But this work seems ignored by specialists.

[3652] See The P versus NP problem by Stephen Cook (12 pages).

[3653] The classical method for solving these problems is to use heuristics allowing to obtain a satisfiable approximate solution, therefore not necessarily optimal, and in particular via probabilistic approaches.

[3654] See A quantum algorithm for the solution of the 0-1 Knapsack problem by Sören Wilkening et al, Leibniz Universität and Volkswagen, October 2023 (15 pages).

[3655] See Hybrid classical-quantum branch-and-bound algorithm for solving integer linear problems by Claudio Sanavio et al, November 2023 (11 pages).

Complete NP problems also include the **subset sum problem** of finding a subset of a set of integers whose sum is equal to an arbitrary integer.

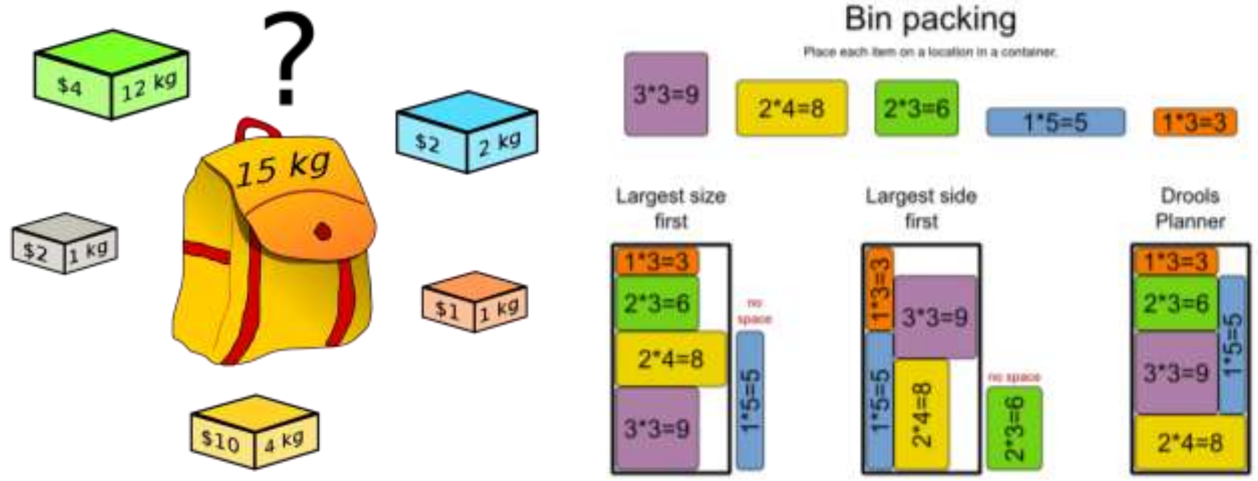

Figure 787: the famous bin-packing problems. Ever filled your car's trunk when going to vacation? Sources: Wikipedia and Stackoverflow.

The **deminer's problem** consists in locating hidden mines in a field with only the number of mines in adjacent areas and the total number of mines in the field as an indication. All this without detonating them. It is a game well known by Windows users, launched in 1989 (Figure 788)!

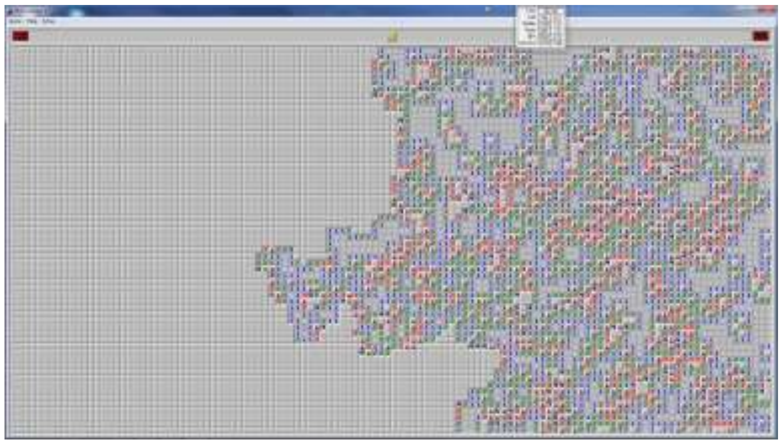

Figure 788: the deminer's problem is also an NP complete problem. Source of the illustration.

The simulation of complex protein folding is also a NP-Complete problem[3656].

So, this would be a potentially very difficult problem to solve with a quantum computer with large proteins. It is demonstrated that if an optimal solution to an NP-Complete problem could be found, all the solutions to problems in this class would be found. This is the important notion of problem reduction.

Graphs coloring with different colors for knots, branches or surfaces is part of the NP, NP-Complete and NP-Hard problems[3657] (Figure 789). The first two cases requiring a number of colors depending on the maximum number of connections between elements of the graph and the last case, relating to the coloring of maps in different adjacent colors which requires a maximum of four, thanks to the computer demonstration of the four-color theorem done in 1976 by Kenneth Appel and Wolfgang Haken.

- **Graph nodes** coloring has applications in the placement of mobile antennas and in the allocation of memory registers for a compiler. The problem is NP-Complete for its resolution and NP-Difficult to find its optimal solution.

[3656] See Is protein folding problem really a NP-complete one? First investigations, 2013 (31 pages).

[3657] See Graph Coloring with Physics-Inspired Graph Neural Networks by Martin J. A. Schuetz et al, AWS, February-November 2022 (12 pages) which is a state of the art classical algorithm for graph coloring.

- **Branch** coloring has applications in the frequency allocation of multimode fiber optic networks. It also allows us to optimize the placement of objects or people according to their compatibility or incompatibility (*aka* the famous “wedding tables problem”). Optimum coloring is an NP-Hard problem.
- **Area map** coloring is used to define the coverage areas of mobile radio antennas or telecommunications satellites. It can even be used to allocate microwave frequencies for the activation of superconducting qubits. The coloring with three colors is an NP-Complete problem.

In general, many C problem classes have a subclass C-Complete and C-Hard. A problem is C-Hard if there is a type of reduction of problems from class C to this problem. If the problem C-Hard is part of class C, then it is said to be "C-Complete"[3658].

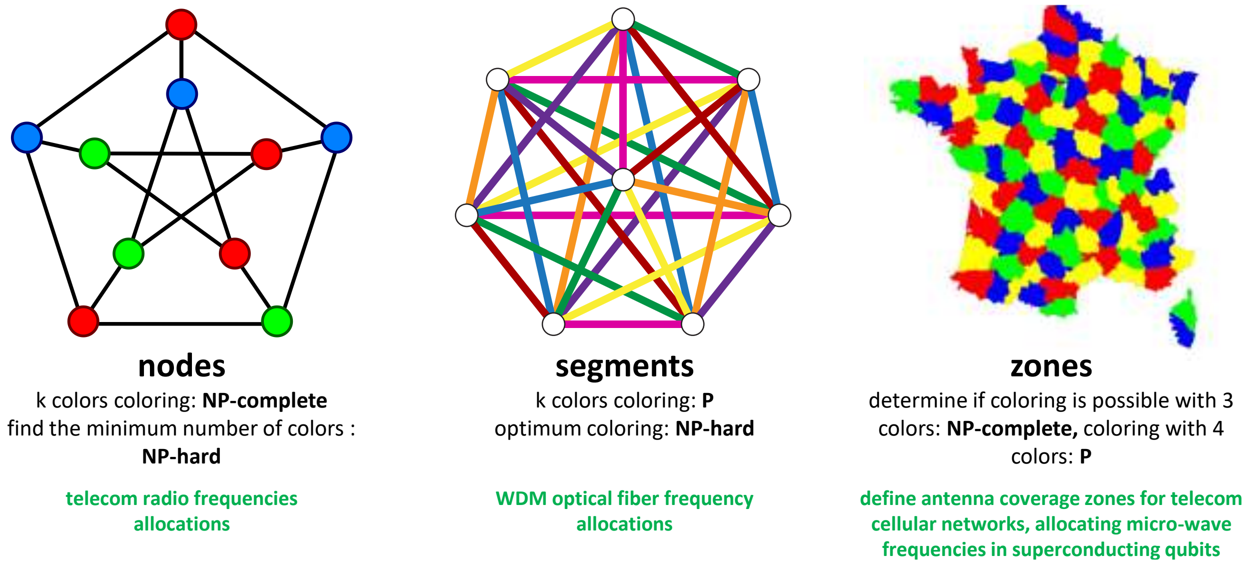

Figure 789: graph problems with nodes, segments and zones coloring.

NP Hard covers optimization problems where a minimum or a maximum is sought with a large combinatorics.

A problem is NP-Hard if all NP-Complete problems can be reduced by polynomial simplification to this problem.

It is the case of the solution of the **traveling salesperson problem** where one must test a large combination of routes to find the quickest one passing through a fixed number of cities. In this case, all solutions must be tested.

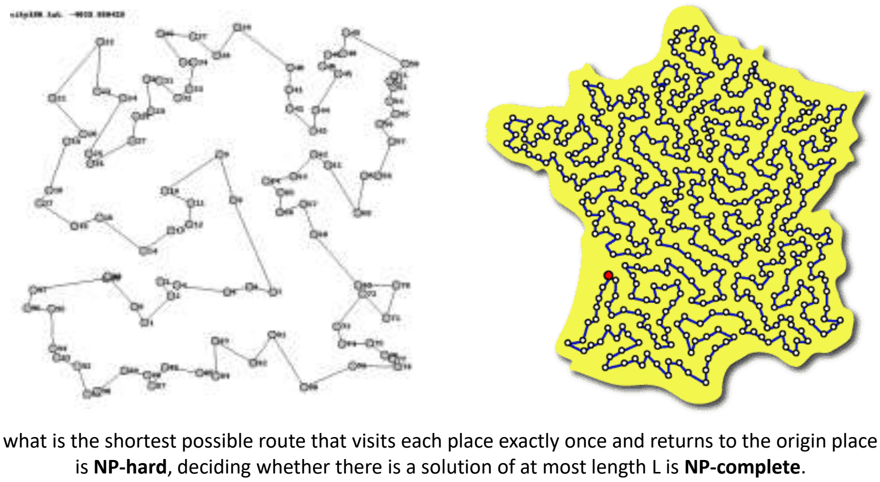

Figure 790: the TSP (traveling salesperson problem).

[3658] For more information, see in particular Complexity Theory Part I (81 slides) and Part II (83 slides), which is part of a Stanford course on Complexity Theories, Calculability and Complexity- Some Results I Know by Etienne Grandjean of the University of Caen, 2017 (43 slides) as well as this video by Olivier Bailleux (2017, 20 minutes).

If a traveler must go through 125 cities in less than 30 days, if there is a solution that works in that time frame, then the problem is NP (Figure 790).

But nothing says that all the solutions have been found. Solving the problem below an arbitrary travel time with a return to the starting point is an NP-complete problem. This is called a Hamiltonian circuit: a path running a graph passing once and only once through each of the nodes and returning to its starting point. The determination of the shortest travel time is NP Hard.

The brute force algorithm to solve it has a time that depends on N! where N is the number of nodes in the network. The known optimum time is $N^2 2^N$. The problem is difficult to solve beyond 20 steps[3659]! There is no known efficient quantum algorithm able to solve a TSP problem.

The NP-Hard problem class also contains a number of **Nintendo** games like Super Mario Bros, The Legend of Zelda and Pokemon[3660]. Quantum computing would not be able to solve the most complex NP-Hard problems.

PSPACE is the class of problems that can be solved in polynomial space on a deterministic machine. NPSPACE is the class of problems that can be solved in polynomial space on a non-deterministic machine. And NPSPACE = PSPACE according to Savitch's theorem.

EXPTIME is the class of problems decidable in exponential time by a deterministic machine. Precisely, the computation time of these problems is $2^{p(N)}$ where p(N) is a polynomial of N, N being the level of complexity of the problem. They are intractable with traditional machines. Some of these problems can be converted into problems that can be treated polynomially by quantum computers. Chess and Go games on arbitrarily sized grids belong to this category. In size-limited grids, the exponential effect has limits. These were exceeded for Deep Blue's chess game in 1996 and for DeepMind's AlphaGo game of Go in 2016 and 2017.

NEXPTIME is the class of problems decided in exponential time by a non-deterministic machine with unlimited memory space.

EXPSPACE is the class of problems that can be solved in exponential space. In other words, they are difficult to access on today's and even tomorrow's machines.

2-EXPTIME is a class including the previous ones that covers decision problems that can be solved by a deterministic Turing machine in double exponential time with an order of magnitude of $O(2^{2^{P(n)}})$where P(n) is a polynomial of n. In other words, it is an exponential of an exponential problem. For example, computing the complement of a regular expression[3661].

We should add the class **#P** of problems for counting the number of solutions of class P problems, which are solved in polynomial time. Proposed in 1979 by Leslie G. Valiant, it obviously has its associated classes **#P Hard** and **#P Complete**.

The computation of the permanent of a square matrix filled with 0 and 1 is a complete #P problem according to Ben-Dor and Halevi's theorem demonstrated in 1993. In 2011, Scott Aaronson demonstrated that the calculation of the permanent of a matrix is a #P Difficult problem[3662]. All this is related to the numerical simulation of the boson sampling which is compared to its resolution by photon-based systems that we study in a section on photon qubits, page 526.

[3659] See The Traveling Salesman Problem site which provides some examples of such problems such as the itinerary of all 49,687 English pubs or 49,603 tourist places in the USA.

[3660] See Classic Nintendo Games are (Computationally) Hard, 2012 (36 pages).

[3661] See Finite Automata, Digraph Connectivity, and Regular Expression Size by Gruber, Hermann, Holzer, Markus, 2008 (13 pages).

[3662] In A Linear-Optical Proof that the Permanent is #P-Hard par Scott Aaronson, 2011 (11 pages).

The classes PSPACE, EXPTIME, NEXPTIME, EXPSPACE and 2-EXPTIME do not correspond to practical problems that are easy to identify in everyday life. They cover the problems of predicting the behavior of ultra-complex systems with strong interactions. If it is possible that modeling the folding of a protein is an NP problem, what would be the class of problem to simulate the functioning of a whole living cell, or even a multicellular organism? There are so many interactions at the atomic, molecular, and cellular level that the class of this kind of problem is probably well beyond NP-Hard level.

There are many other problem complexity classes that won't be described here: EXP, IP, MIP, BPP, RP, ZPP, SL, NC, AC0, TC0, MA, AM and SZK! You can find them in the [Complexity Zoo]site which inventories the zoo of problem complexity classes (Figure 791). There seems to be over a hundred of them[3663].

Figure 791: there is even a zoo website for complexity classes! Source: the Complexity Zoo.

Quantum complexity classes

Let's now discuss the classification of problems that are theoretically addressable by quantum computers, the correspondence with the *above* classes being still a problem that is not entirely solved!

The classification is different because quantum computers can parallelize processing in an exponential way while classical computers like Turing machines cannot do it.

This is still very theorical since it doesn't take into account known constraints of quantum computers: their short coherence time creates constraints on the number of quantum gates that can be chained together to solve a problem. This is a constraint that traditional computers do not have. But again, theoretically, this computation time constraint could be removed by using correction error codes.

PH is a class of problems that generalizes the NP class. PH means "Polynomial Hierarchy".

BQP defines a class of problem that is solvable in polynomial time on a quantum computer with a constrained error rate of 1/3. This may in some cases correspond to P problems. The class was defined in 1993, when the first quantum algorithms appeared (Figure 792).

[3663] To learn more about the subject of complexity theories, you can read the well-documented Computational Complexity A Modern Approach by Sanjeev Arora and Boaz Barak of Princeton University, 2007 (489 pages).

Whether the BQP class is different from the P class is an ongoing debate. It has already been shown that the P class of polynomial problems is in BQP. But is NP in BQP? Seems not. It is, however, difficult to prove it in a generic way. The exact relationship between BQP and NP is still unknown.

The key point is to find algorithms that are part of BQP (processable in quantum) and that are not in PH (processable with any present and future classical architecture). This uncertainty has been removed only very recently[3664]. Oracle-based algorithms were found that are in BQP but not in PH.

NISQ is a class proposed by researchers from UC Berkeley, Harvard, Caltech and Microsoft in a paper published in 2022[3665]. It describes the class of problems that could be processed by a hybrid system using a NISQ quantum computer. It is showing that this class is in between BPP and BQP but it seems closer to BPP (problems accessible to classical computers) than BQP (problems accessible only to quantum computers) for Grover algorithm and farther for Bernstein-Vazirani algorithm. It does not include problems that are solvable by quantum annealers and simulators.

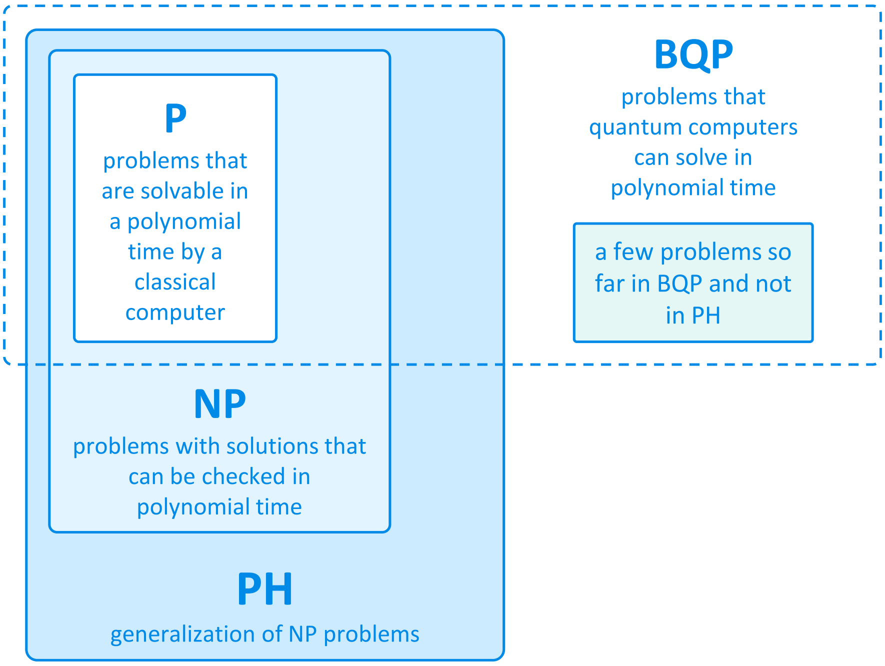

Figure 792: how BQP relates to the P and NP complexity classes. Inspired from Finally, a Problem That Only Quantum Computers Will Ever Be Able to Solve by Kevin Hartnett, 2018. (cc) Olivier Ezratty, 2023.

Therefore, these algorithms have a polynomial resolution time on quantum computers which remains exponential on their equivalent crafted for classical computers.

By the way, what about a possible complexity difference for problems manageable with universal gate quantum accelerators vs. quantum annealing accelerators? According to several researchers, there is no difference[3666]. Various theorems show that a problem that can be solved with universal quantum gates can also be solved with a D-Wave quantum annealing architecture and vice versa with only a polynomial time difference.

[3664] See Finally, a Problem That Only Quantum Computers Will Ever Be Able to Solve by Kevin Hartnett, 2018, referring to Oracle Separation of BQP and PH by Ran Raz and Avishay Tal, May 2018 (22 pages), presented in the Electronic Colloquium on Computational Complexity conference. This is the source of the illustration on this page.

[3665] See The Complexity of NISQ by Sitan Chen, Jordan Cotler, Hsin-Yuan Huang, Jerry Li, October 2022 (52 pages).

[3666] See Adiabatic Quantum Computation is Equivalent to Standard Quantum Computation by Dorit Aharonov, Wim van Dam and Julia Kempe (from CNRS), 2008 (30 pages).

QMA (for Quantum Merlin Arthur) defines a class of problems that is verifiable in polynomial time on a quantum computer with a probability greater than 2/3. It is the quantum analog of the "traditional" NP complexity class. The QMA class contains the classes P, BQP and NP[3667]. Like the NP class, the QMA class has two derived subclasses, QMA Complete and QMA Hard. In practice, these are difficult problems to solve with quantum computers. Unfortunately, the literature on the subject does not describe its nature or provide examples. This is a pity for those who appreciate a practical sense of things!

QCMA is a hybrid problems class situated between QMA and NP. The proof is provided in classical polynomial time, but the resolution is at the QMA level and is performed in a quantum way.

Many publications point out the limitations of quantum algorithms and computers. A BQP problem that is not in PH gives the advantage to quantum computing.

But exponential intractable problems for which the improvement brought by quantum computing is only a square root of classical time do not change their exponential nature. This is what Scott Aaronson points out[3668]. Complete NP problems and beyond remain inaccessible to quantum computers. Brute force has limits that even quantum computing cannot overcome in theory! This partly explains the difficulty of creating efficient quantum algorithms.

Finally, **NEEXP** is a class of problems that requires a double exponential computation time for its verification. Recent work shows that a result can be verified with several quantum computers with entangled qubits (Figure 793). This does not however enable us to solve this type of problems[3669]!

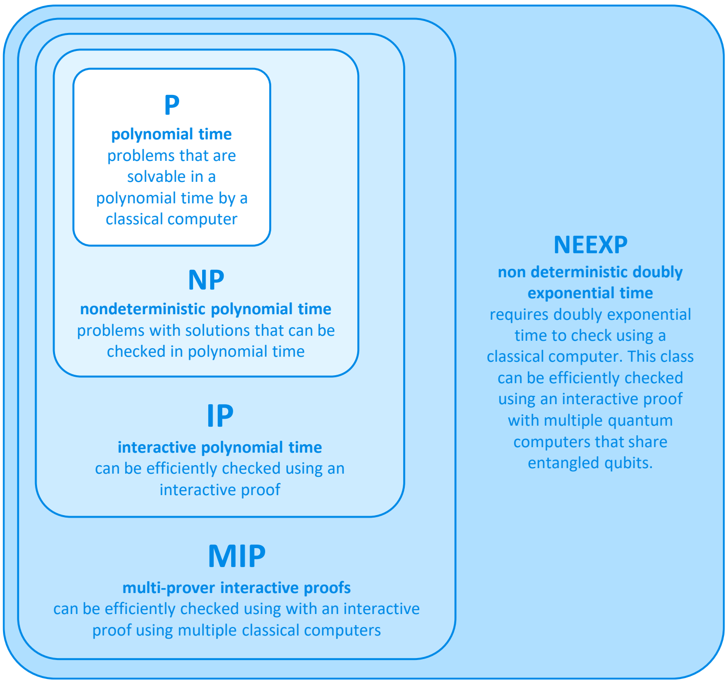

Figure 793: the NEEXP complexity class. (cc) Olivier Ezratty, 2023, inspired by Computer Scientists Expand the Frontier of Verifiable Knowledge, 2019.

Some problems are undecidable, i.e., they cannot be solved by an algorithm, no matter how much time you have. This is the case for the determination of the end of a Turing machine program.

In other words, there is no program that can determine whether any program written in a common programming language will stop or loop for an infinite amount of time.

However, in 2020, there was some progress with a demonstration that the classes MIP* and RE were identical [3670].

3667 See QMA-complete problems by Adam Bookatz, 2013 (18 pages).

3668 See The Limits of Quantum Computers (16 pages) and NP-complete Problems and Physical Reality (23 pages).

3669 See NEEXP in MIP* by Anand Natarajan and John Wright, 2019 (122 pages) and Computer Scientists Expand the Frontier of Verifiable Knowledge, 2019.

3670 The MIP* class of problems that can be verified by quantum entanglement is equal to the RE class of problems that are no more difficult than the problems of program termination. See A quantum strategy could verify the solutions to unsolvable problems - in theory by Emily Conover, 2020 which refers to MIP*=RE by Zhengfeng Ji et al, January 2020 (165 pages) and seen in Mathematicians Are Studying Planet-Sized Computers With God-Like Powers by Mordechai Rorvig, 2020. See Landmark Computer Science Proof Cascades Through Physics and Math by Kevin Hartnett, March 2020.

In the same order, **Rice's theorem** demonstrates that no non-trivial property of a program can be decided algorithmically. All this is to say that there is no automatic method to detect bugs in a program or to certify that it runs well. There are, however, formal methods of proof that can be used to certify the execution of specific programs. This involves the use of formal program specifications that serve as a reference for assessing how well a program is running. This is already widely used, without quantum, in industrial information technology and in critical systems such as aerospace.

Quantum speedups

The chart in Figure 794 summarizes the theorical performance gains of some of the deterministic algorithms we have just seen. Complexity levels (exponential, polynomial, linear, ...) are generic.

The precise levels of complexity of each algorithm are roughly associated with these classes. Nlog(N) is the complexity of a classical Fourier transform and is nearly linear since N grows much faster than log(N) and $\log(N)^3$ is a log level complexity for the Shor algorithm and a QFT.

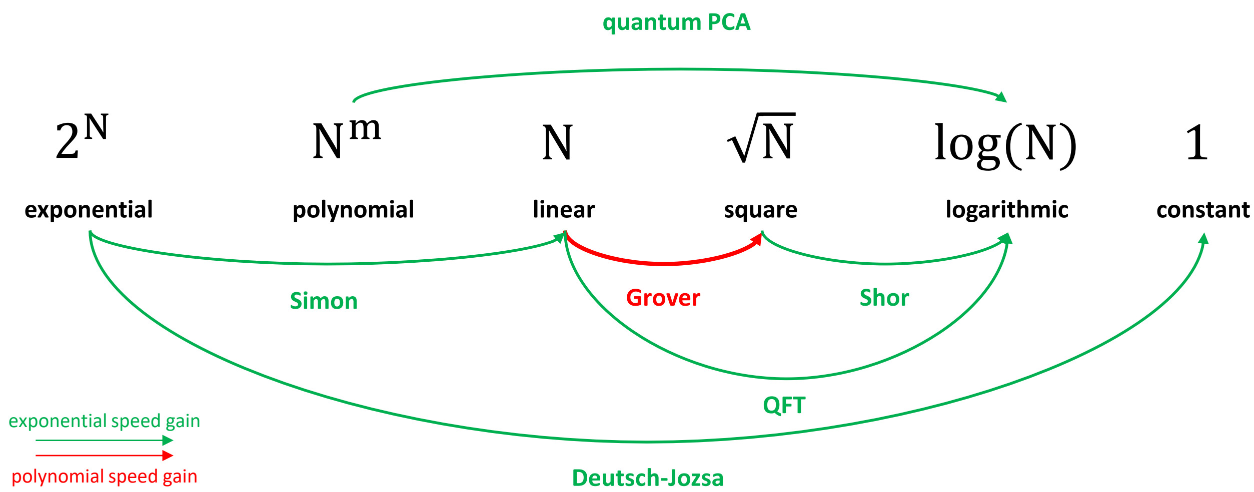

Figure 794: how do O() compare for complexity classes in quantum computing. The arrows show how their classical and quantum solution compare. (cc) Olivier Ezratty, 2021-2023.

You can also visualize graphically the way computing time grows according to the big O() scale of problems in Figure 795[3671]. A quantum algorithm's main goal is to move your problem's computation time from the red zone to the orange, or better, the yellow and green zones.

The factorial complexity class $O(n!)$ is equivalent to $O(\sqrt{2\pi n}\left(\frac{n}{e}\right)^n)$ when n is very large, according to Stirling's formula. It is beyond the scale of the speedup gains that could be brought by quantum computers, even in the far distant future.

An exponential gain is also obtained when we move from N or $\sqrt{N}$ to log(N). A QFT thus generates a theoretical exponential gain. The time scales are more meaningful in the table from Figure 796.

The ideal performance gain consists in traversing several complexity scales, and particularly for an exponential problem. In practice, the main algorithms skip one or two complexity classes, but not necessarily from the exponential problem class. But my scheme is misleading. N can also grow exponentially depending on the size of a problem. The classic example is Shor's algorithm.

[3671] Big O() notation corresponds to an upper bound on the time or space complexity of an algorithm in the worst-case scenario. Small o() notation, not much used here, corresponds to upper bounds on the time or space complexity of an algorithm that is not asymptotically tight.

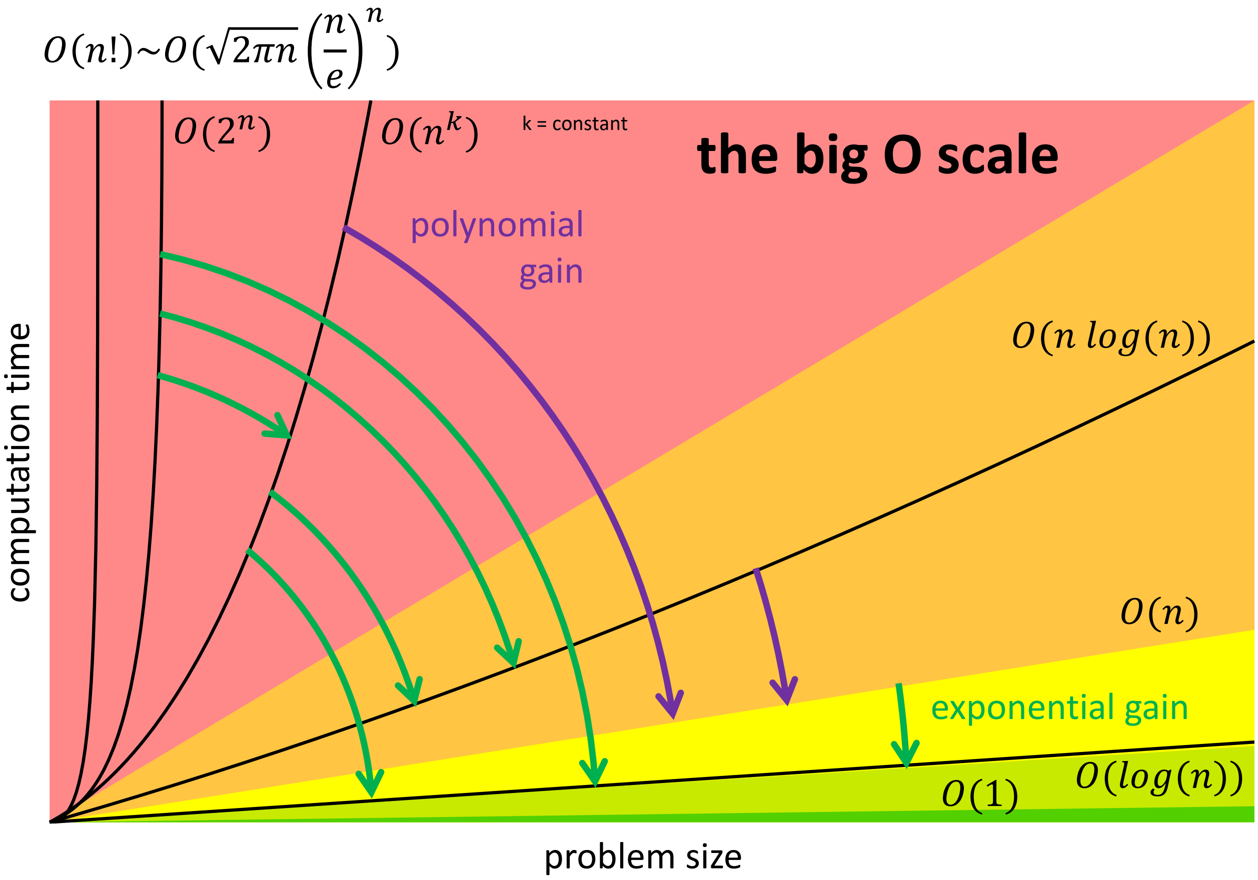

Figure 795: another view of the big O() scale. Source: Wikipedia, reformatted add additions by Olivier Ezratty, (cc) 2022-2023.

The starting point is an N which is actually a RSA key size which itself is evaluated in power of 2. A 1024-bit key is 2^{1024}. If we move from 2^{256} to 2^{1024}, the growth of the key size is exponential. With Shor's algorithm, we get an exponential performance gain by going from a square root of 2^{1024} to $\log(2^{1024})$, that is to say 1,024 (in log base 2)! So, the time scale moves from 2^{512} to 1,024, which is a perfectly exponential gain. Deutsch-Jozsa's algorithm has the particularity of traversing all levels of this scale, from an exponential time to a fixed time. We have unfortunately seen that it has no known practical application.

Complexité	n	$n \log_2 n$	n^2	n^3	1.5^n	2^n	$n!$
$n = 10$	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	4 s
$n = 30$	< 1 s	< 1 s	< 1 s	< 1 s	< 1 s	18 min	10^{25} ans
$n = 50$	< 1 s	< 1 s	< 1 s	< 1 s	11 min	36 ans	∞
$n = 100$	< 1 s	< 1 s	< 1 s	1s	12, 9 ans	10^{17} ans	∞
$n = 1000$	< 1 s	< 1 s	1s	18 min	∞	∞	∞
$n = 10000$	< 1 s	< 1 s	2 min	12 jours	∞	∞	∞
$n = 100000$	< 1 s	2 s	3 heures	32 ans	∞	∞	∞
$n = 1000000$	1s	20s	12 jours	31, 710 ans	∞	∞	∞

Figure 796: complexity classes and times scales. Heures = hours. Jours = days. Ans = years.
Source: Complexity in time, Ecole Polytechnique (25 pages).

The speedup gain of an algorithm depends on the gates it is using (Figure 797). Shor's algorithm and any QFT based algorithm provide an exponential gain since it uses phase-controlled R gates (Figure 798). Grover's algorithm provides a polynomial gain since it uses only Hadamard gates. But Deutsch-

Jozsa's algorithm has an exponential gain although it is using only Clifford's group gates like the Hadamard gate. Why? Because it uses an Oracle function that may use non-Clifford's group gates[3672].

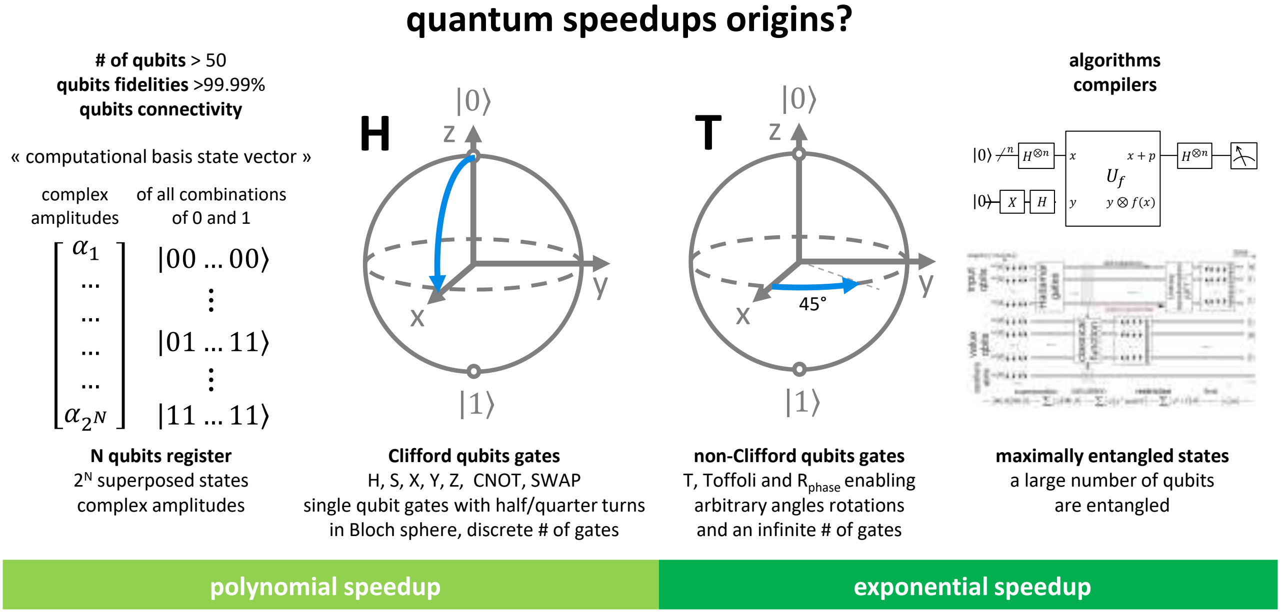

Figure 797: the origins of quantum speedups are not obvious. It may be counter-intuitive but the exponential size of the computational vector space of N qubits doesn't explain any potential exponential gain in quantum computing. You need to have at least two other conditions: use non-Clifford qubit gates and have a N-qubit maximally entangled space. (cc) Olivier Ezratty, 2022-2024.

On top of using non-Clifford quantum gates, a quantum algorithm is also having an exponential speedup if it handles **maximally entangled states**[3673]. It means that there's a correlation of states between a maximum number of qubits in the register until the end of computing. If an algorithm handles islands of disconnected sets of qubits in the register, the speed-up will be constrained by the size of these islands. The bigger the island, the bigger the Hilbert space managed by the algorithm. Let's explain this simply with a register of N qubits. If the algorithm is split into 4 different subspaces of N/4 qubits, you'll end-up managing a space with $4x2^{N/4}$ amplitudes, or $2^{2+N/4}$. That number is way smaller than 2^N, starting with N=3. For N=50, the difference is between $2.3x10^5$ and 10^{15}!

Studies on maximally entangled states and multipartite entanglement are numerous and accessible only to specialists[3674]. They deal with the theory, like with absolutely maximally entangled states (AME) that are multipartite quantum states and carry absolute maximum entanglement for all possible subsystem partitions[3675], with how much entanglement is required for quantum algorithms[3676] and with how it can be checked with quantum measurement[3677]. As of 2023, the largest set of entangled

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[3672] See Focus beyond quadratic speedups for error-corrected quantum advantage, by Hartmut Neven et al , 2021 (11 pages) that also explain why quadratic speedups are not efficient due to the error correction overhead.

[3673] On maximally entangled states, see On the role of entanglement in quantum computational speed-up by Richard Jozsa and Noah Linden, 2003 (22 pages), Review on the study of entanglement in quantum computation speedup by ShengChao Ding and Zhi Jin, 2007 (6 pages) and Necessity of Superposition of Macroscopically Distinct States for Quantum Computational Speedup by Akira Shimizu et al, University of Tokyo and NTT, 2013 (16 pages). It refers to an indice p defined in Macroscopic entanglement of many-magnon states by Tomoyuki Morimae et al, 2005 (12 pages).

[3674] See A brief introduction to multipartite entanglement by Ingemar Bengtsson and Karol Zyczkowski, 2016 (38 pages).

[3675] See the thesis Symmetry and Classification of Multipartite Entangled States by Adam Burchardt, September 2021 (126 pages).

[3676] See How Much Entanglement Do Quantum Optimization Algorithms Require? by Yanzhu Chen, Linghua Zhu, Chenxu Liu, Nicholas J. Mayhall, Edwin Barnes and Sophia E. Economou, May 2022 (12 pages).

[3677] See Quantifying multiparticle entanglement with randomized measurements by Sophia Ohnemus, Heinz-Peter Breuer and Andreas Ketterer, July 2022 (17 pages) and Scalable estimation of pure multi-qubit states by Luciano Pereira, Leonardo Zambrano and Aldo Delgado, npj, May 2022 (12 pages).

quantum object was achieved with 3,000 atoms, but not for doing any computing and without a characterized quality of their entanglement[3678].

Entanglement nonlocality is another interpretation for quantum computing speedups, but it is still widely disputed among quantum physicists. While we can't use entanglement and the teleportation algorithm to transmit classical data faster than light, it appears that entanglement is interconnecting qubits nearly instantaneously during computing while they are affected by qubit gates.

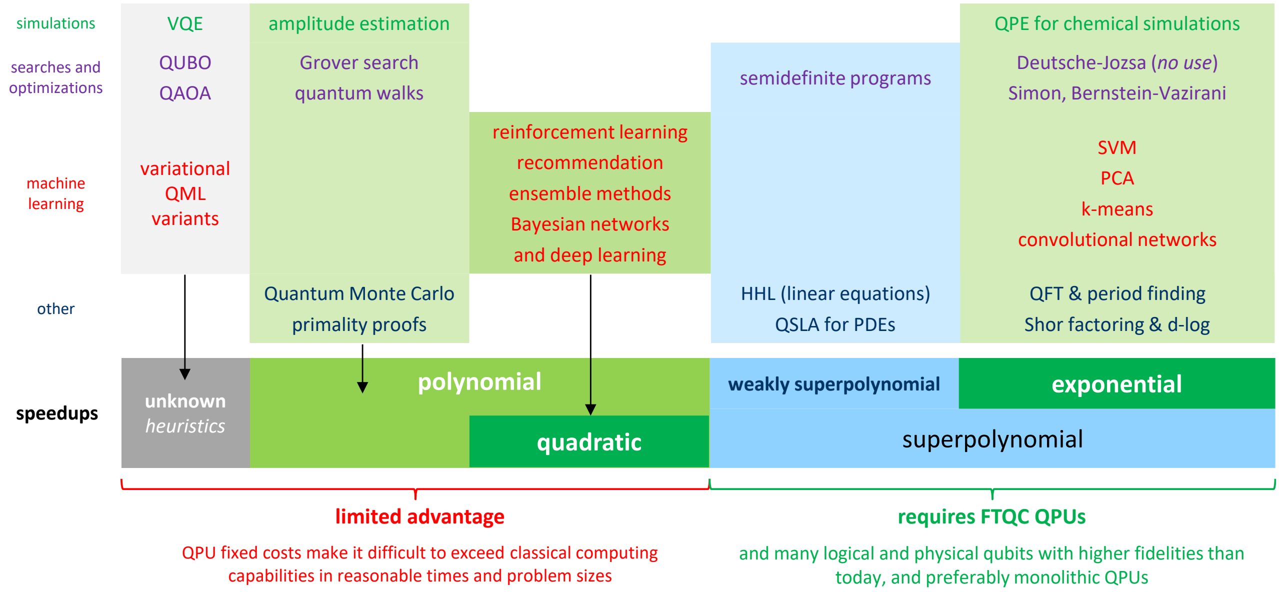

Figure 798: polynomial, superpolynomial and exponential speedups and their corresponding most common quantum algorithms. (cc) Olivier Ezratty, 2022-2024.

We should also introduce the notion of superpolynomial speedup. A superpolynomial time is not bounded above by any polynomial. An exponential speedup is naturally superpolynomial but some superpolynomial speedups are not exponential and said "weakly superpolynomial". Most common quantum algorithms showcase either a polynomial or an exponential speedup (*aka* strong superpolynomial speedup).

Other exponential speedups have been found for various algebraic algorithms like **estimating Gauss sums**[3679], **approximating Jones polynomial** that is not based on a QFT and still brings some exponential speedup, with some applications in topological quantum computing[3680] or counting solutions of **finite field equations**[3681].

It is also necessary to boil in the fact that the complexity of some problems can be addressed on conventional computers with probabilistic or heuristic approaches that also allow a significant reduction of the computing time of exponential problems.

Practically, when moving from this kind of solution to a quantum algorithm, we replace one probabilistic approach with another since quantum computing is also usually highly probabilistic and prone to many computing errors.

[3678] See Entanglement with negative Wigner function of almost 3,000 atoms heralded by one photon by Robert McConnell, Hao Zhang, Jiazhong Hu, Senka Ćuk and Vladan Vuletić, Nature, March 2015 (11 pages).

[3679] See Efficient Quantum Algorithms for Estimating Gauss Sums by Wim van Dam and Gadiel Seroussi, 2008 (11 pages).

[3680] See A Polynomial Quantum Algorithm for Approximating the Jones Polynomial by Dorit Aharonov, Vaughan Jones and Zeph Landau, 2006 (19 pages).

[3681] See Quantum computing and polynomial equations over the finite field by Christopher M. Dawson et al, 2004 (7 pages).

All in all, quantum algorithms are interesting, but they are not always the only solution to cleverly solve a complex problem. This is amplified by the emulation going on between algorithms designers. Each and every new quantum performance challenges the classical supercomputers algorithms designers to improve the performance of their own tools.

This is what Toshiba did in 2019 with a classic optimization algorithm that was 10 times more powerful than the state of the art. That was fine even though a linear x10 gain is still not an exponential progress[3682].

This also explains why some are proposing to create a notion of "strong quantum speedup" using complexity classes instead of algorithms speeds[3683]. This being said, even if a polynomial gain is considered as a minor gain in complexity theories, it can still have a non-negligible practical value and make quantum computing attractive, without going through the Holy Grail of some fancy exponential acceleration.

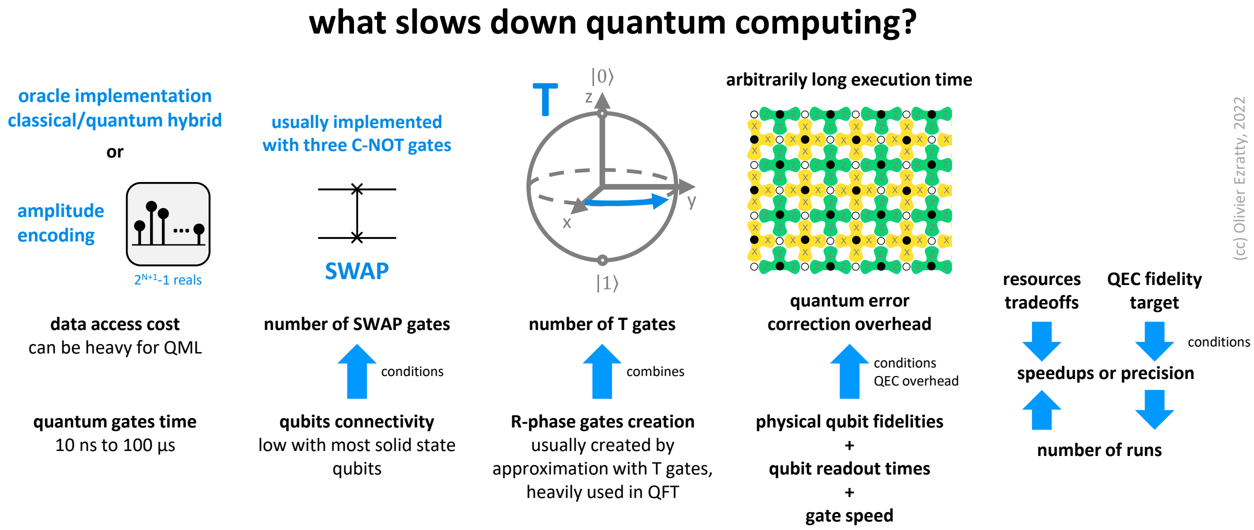

Figure 799: quantum computing speedup must also include faces sources of slowdowns, which have to be known by algorithms developers. (cc) Olivier Ezratty, 2022.

But the devils in the details must be cared about[3684] (Figure 799):

- The quantum vs classical comparison must be using real-life scenarios and not worst-case scenarios. Worst-case scenarios can be both unrealistic and unfavorable to classical solutions and overselling the quantum advantage[3685 3686].

[3682] See Toshiba Promises Quantum-Like Advantage on Standard Hardware by Tiffany Trader, 2020, which references Combinatorial optimization by simulating adiabatic bifurcations in nonlinear Hamiltonian systems by Hayato Goto et al, April 2019 (9 pages).

[3683] See Measures of quantum computing speedup by Anargyros Papageorgiou and Joseph F. Traub, 2013 (4 pages). Proposing that "strong quantum speedup" is measured by a ratio between quantum complexity class and classical complexity class for a particular problem instead of comparing just cost of best-in-class classical algorithm vs cost of quantum algorithm "quantum speedup".

[3684] This is well explained in What Is the Quantum Advantage of Your Quantum Algorithm? by Jack Krupansky, February 2020.

[3685] See Quantifying Grover speed-ups beyond asymptotic analysis by Chris Cade et al, March 2022-September 2023 (57 pages).

[3686] See Realistic Runtime Analysis for Quantum Simplex Computation by Sabrina Ammann et al, TU Braunschweig, November 2023 (39 pages) which concludes that "*the asymptotic advantage of a quantum method like the one of Nannicini appears to be unlikely to play out in the practical dimensions that are relevant for realistic applications (reflected by the considered large-scale benchmark instances): Even under very benevolent assumptions (e.g., ignoring error correction and other physical aspects of quantum computers) and for purposefully constructed sparse and well conditioned linear programs that are better suited for such a quantum algorithm, the required gate efficiency seems beyond what is physically possible. This will not fundamentally change for even larger instances of practical dimensions*".

- The quantum speedup will be affected by the number of times the quantum circuit must be run. These repetitions are also named shot count or shots. It depends on the problem, the number of qubits, the algorithm output (integers, real numbers, vector state) and the paradigm (NISQ, FTQC). With Google's supremacy and its 53 qubits, it was 3 million shots. With IBM Quantum System One using from 5 to 28 qubits, the typical proposed shots number ranges from 1,000 to 8,000 shots. With FTQC systems, in most cases, the number of shots is not that large and depends on the used algorithm and the algorithm output being a single value in the computational basis (Grover's algorithm) or a superposed quantum state (Shor algorithm, HHL).
- The quantum advantage also depends on the number of qubits in your register that are put in superposition using Hadamard gates. Other qubits might be used elsewhere in the algorithm such as ancillas.
- Data preparation must also be handled, which is of particular importance for quantum machine learning algorithms[3687].
- The same should be said of oracle-based algorithms, particularly when they rely on some classical data access using (not yet available) qRAM.
- Then, with large scale quantum computing, quantum error correction will add some additional burden. Using concatenation codes, you may lose polynomially on your speedup. Meaning that an initial polynomial speedup may be lost at this stage[3688].

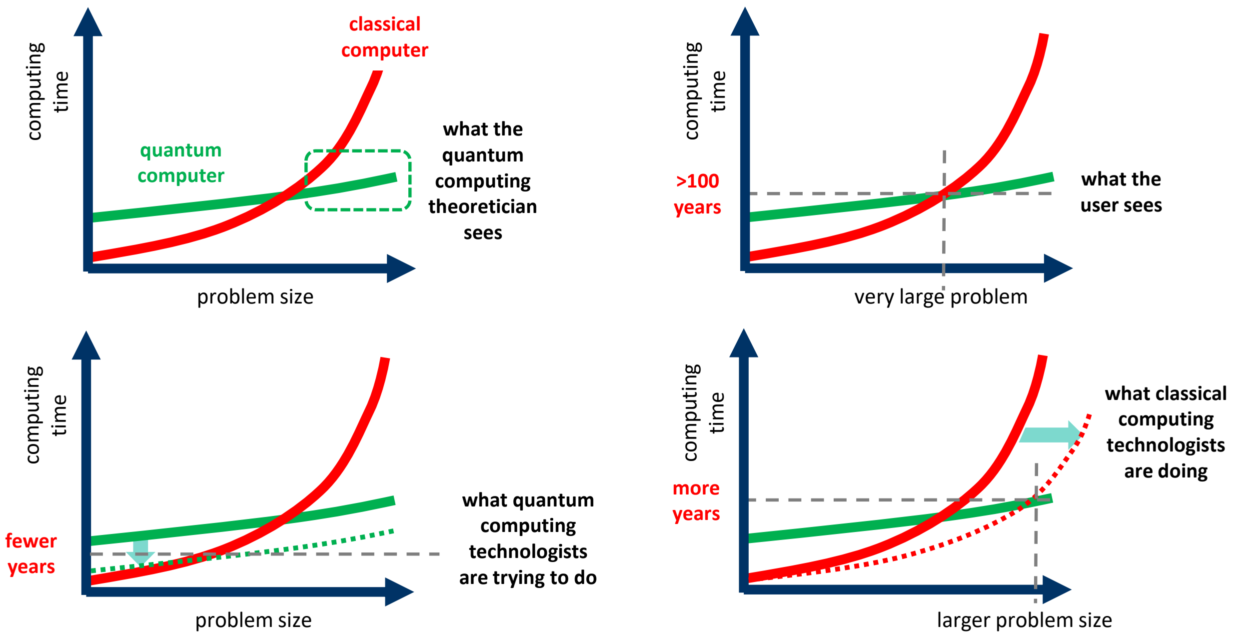

Figure 800: a theoretical quantum speedup might be impractical if the quantum algorithm is faster than its classical counterpart only when reaching an unreasonable time with a large problem. But the target is a moving one since on one hand, quantum technologists may improve the speed of QPUs with various techniques (faster gates, parallelizing computing ono several QPUs, faster algorithms, etc) and on the other hand, classical algorithms and hardware may also improve. Source: Olivier Ezratty, 2023, inspired by Disentangling Hype from Practicality: On Realistically Achieving Quantum Advantage by Torsten Hoefler, Thomas Häner, Matthias Troyer, May 2023.

- You must be careful to assess when the speedup will occur regarding the problem size and corresponding timing. If the quantum algorithm is faster than its best classical counterpart only with

3687 See Information-theoretic bounds on quantum advantage in machine learning by Hsin-Yuan Huang, Richard Kueng and John Preskill, April 2021 (34 pages) which describes the conditions when QML algorithms provide a real speedup.

3688 See Focus beyond Quadratic Speedups for Error-Corrected Quantum Advantage by Ryan Babbush, Craig Gidney, Hartmut Neven et al, March 2021 (11 pages). Quadratic speedups requires more than 1M physical qubits!

timings exceeding a human lifetime, you can forget about using quantum computing[3689]. See Figure 800 which lays out the discrepancy.

So, as mentioned before, appreciating the real speedup of any quantum algorithm requires adopting an end-to-end approach considering all the parameters of the quantum algorithm execution time.

I would summarize the kinds of benefit coming from quantum computing over time like this:

Better. Some algorithms running on NISQ systems may bring a quality advantage in the solution they provide, like with quantum machine learning or various optimization tasks. This is due to the ability of quantum systems to better explore the problem computational space. For a QML problem, better is expressed in precision[3690].

Faster. We may then create solutions that are running faster than their best equivalent on classical computers. This may happen between the NISQ and the FTQC/LSQ era.

Beyond. In the longer term, and pending unlocking many showstoppers, large scale quantum computing with over 100 logical qubits may then be able to solve problems that are entirely not accessible by classical supercomputers, like in the quantum physics simulation realm.

Quantum algorithms key takeaways

- Quantum algorithms have been created since the early 1990s, over ten years before any quantum computer was working out of a research laboratory.
- Quantum algorithms use very different concepts than those used in classical programming, even including artificial intelligence development tools or object oriented programming. They are based on the manipulation of large matrices and using interferences.
- The main algorithms classes are oracle based and search algorithms, optimization algorithms, quantum physics simulation algorithms and quantum machine learning algorithms.
- A quantum algorithm is interesting if it provides some quantum speedup compared to their equivalent best-in-class classical version, including those that are heuristics based. These problems are said to be intractable on classical hardware. Most of the time, quantum speedups are theorical and do not incorporate the costs of quantum error corrections and of creating non-Clifford quantum gates. These gates are implementing small phase changes and are used in quantum Fourier transforms and implemented in many other algorithms. A quantum speedup that is not exponential is highly questionable. All of this requires some understanding of complexity classes like P, NP and BQP.
- Another key aspect of quantum algorithms is data loading and/or preparation. It is often overlooked and can have a significant time cost, on top of frequently requiring some form of not-yet-available quantum memory hardware. As a consequence, quantum computing is not adequate for big-data computations.
- Gate-based computers, quantum annealers and quantum simulators algorithms are all hybrid, combining a classical (preparation) part and a quantum part. A special breed of quantum algorithms are the variational quantum algorithms (VQA) and their variants for optimizations (QAOA), chemical simulations (VQE) and machine learning (QML) that targets the NISQ systems (noisy intermediate quantum computers). These combine the classical preparation and adjustment of a Hamiltonian that computes a cost function with several classical optimization steps and quantum computing circuit executions. All algorithms requiring a Quantum Fourier Transform (QFT) require a fault-tolerant quantum computer with quantum error correction. They are thus for the long term.
- Quantum inspired algorithms are running on classical computers and are using some form of quantum mathematical models and are based on tensor networks classical computing methods (MPS, DMRG). They can drive performance improvements in classical computing.

[3689] See Disentangling Hype from Practicality: On Realistically Achieving Quantum Advantage by Torsten Hoefler, Thomas Häner and Matthias Troyer, Communications of the ACM, May 2023.

[3690] See The Complexity of NISQ by Sitan Chen, Jordan Cotler, Hsin-Yuan Huang, Jerry Li, October 2022 (52 pages) which casts serious doubts on real quantum speedups that could be obtained with NISQ QPUs. In that case, we are left with better quality results as a potential quantum advantage.

Quantum software development tools

We now need to explore quantum computing software to implement the various algorithms we've just uncovered. It is a completely new world with very different paradigms. Quantum algorithms still require programming, programming languages and development environments.

As shown in Figure 801, quantum software is organized in layers with, starting from the bottom, the physical qubits followed by low-level machine language to drive them at the physical level (microwaves length and frequencies, readouts, etc).

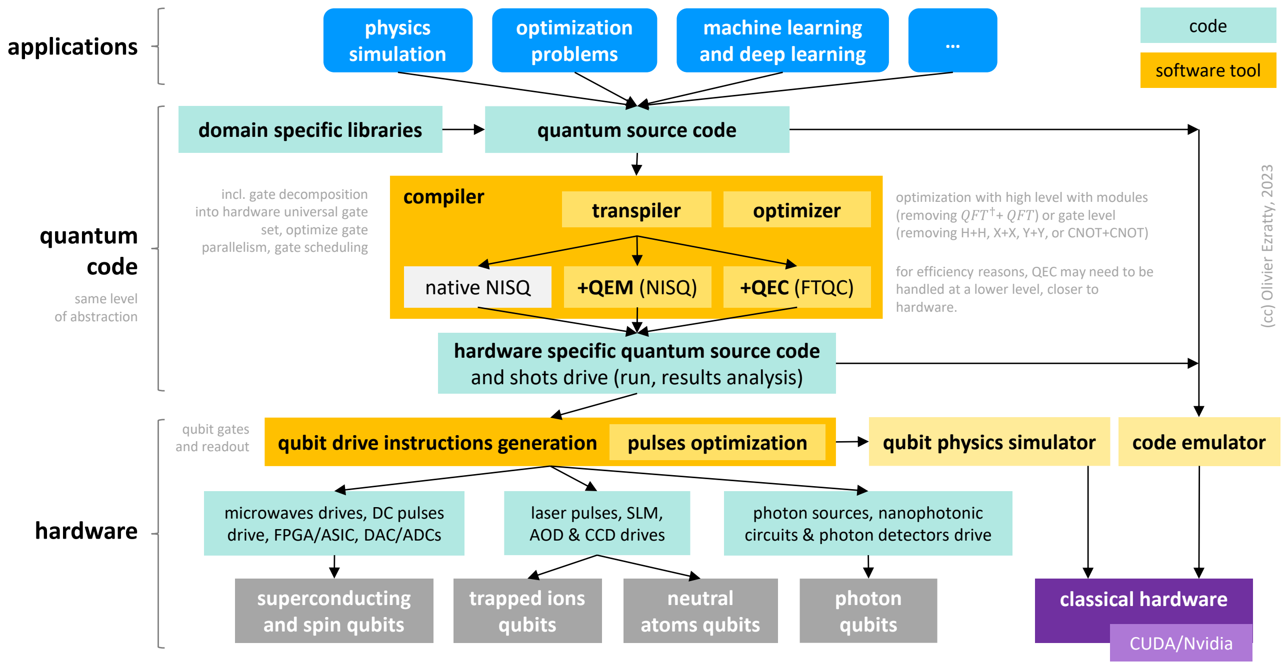

Figure 801: classification of quantum software engineering tools. (cc) Olivier Ezratty, 2023.

As shown in Figure 802, the software stacks are a bit different with analog computing (quantum annealing, analog quantum simulation) with the various tools used to convert the problem into some Ising or QUBO model and its graph embedding onto the hardware qubit layout that is more or less flexible.

Next comes high level quantum source code which is in fact a kind of macro-assembler, able to take advantage of function libraries with ready-to-use algorithms (quantum Fourier transforms, phase estimates, etc.) and finally, high-level languages or libraries tailored to specific business needs.

In the lower layers between machine language and macro-assembler are functions for converting quantum gates into a set of universal quantum gates supported by the quantum hardware as well as error correction code systems that may require the execution of a large number of quantum gates.

A quantum compiler also implements many optimizations by for example removing quantum gate sequences that do not change the state of a qubit, such as two consecutive Hadamard or X (NOT) gates.

It also arranges these quantum gates to minimize the number of quantum gate steps in the solution. Quantum software architectures are generally hybrid. They manage side by side the execution of classical and quantum code .

At the very least, the classical computer is used to control the execution of quantum algorithms, if only to trigger the quantum gates at the right time, sequentially.

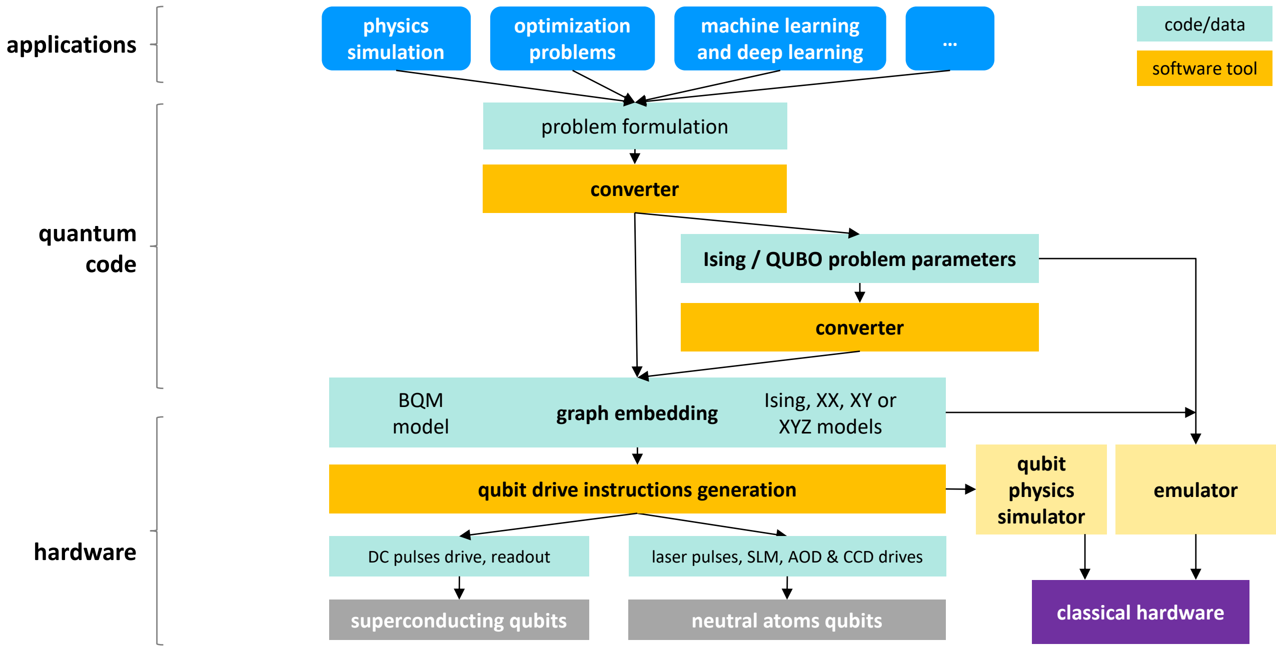

Figure 802: software tools for analog computing. (cc) Olivier Ezratty, 2023.

Development tool classes

We can identify some major classes of tools for creating quantum software: graphic programming tools, scripting languages, intermediate languages, machine languages, compilers and application libraries[3691].

Graphical programming tools

They allow us to visually define the sequence of quantum gates to create algorithms and execute it on quantum accelerators.

These tools can also emulate, run and visualize the status of qubits when their number is reasonable with various methods: Bloch sphere, register state and density matrix.

One example of such tools is the IBM Quantum Experience Composer that is available online since 2016 (Figure 803). Code can be executed on an IBM emulator or on one of the many IBM quantum systems available in the cloud, and for free up to 15 operational qubits.

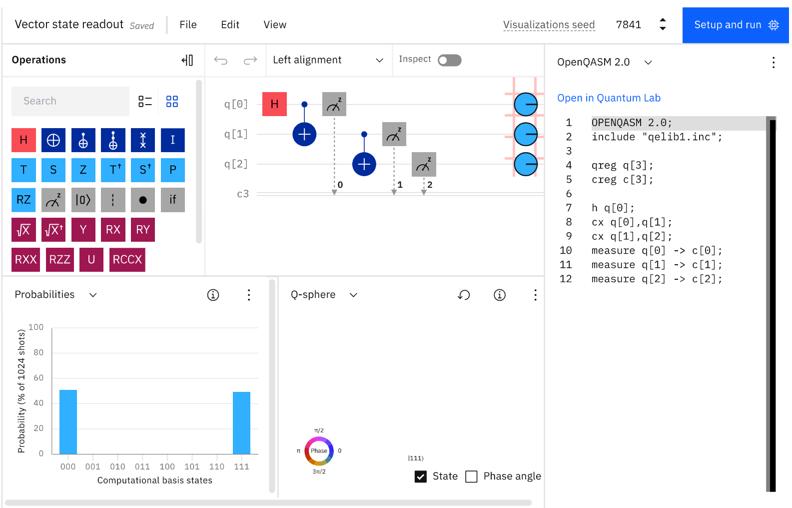

Figure 803: IBM Quantum Experience visual interface. Source: IBM.

There are also graphical simulators of qubits that can be used to understand how to chain quantum gates on a few qubits and visualize the result visually. The most open one is the open source tool Quirk which can simulate up to 16 qubits.

[3691] See the quantum software tools review Quantum Computing Toolkit From Nuts and Bolts to Sack of Tools by Himanshu Sahu and Hari Prabhat Gupta, February 2023 (27 pages).

It works online and can be downloaded to run on your own computer locally. Below is an example of a quantum Fourier transform performed in Quirk. It was developed by **Craig Gidney**, now a Google engineer specialized in algorithms and error correction codes (Figure 804).

Figure 804: Quirk's visual open source quantum programming tool, working in any browser. Source: Quirk Algassert.

Quantum circuits graphical representation can quickly become complicated with over 30 qubits and a large number of gate cycles. It can be thus interesting to visualize code in a simplified manner with a higher level of abstraction with gate block functions. Such a tool named Quantivine has been proposed by a team of researchers in 2023[3692] (Figure 805).

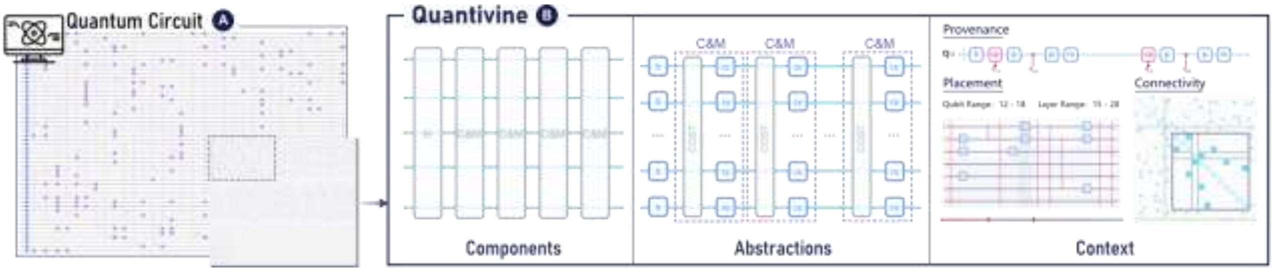

Figure 805: Quantivine is a graphical tool to simplify a large circuit representation with circuit blocks, here with an example using 50 qubits. Source: Quantivine: A Visualization Approach for Large-scale Quantum Circuit Representation and Analysis by Zhen Wen et al, Zhejiang University, July 2023 (11 pages).

ZX-calculus is a graphical programming language that uses topological composition rules. It was created in 2008 by Bob Coecke and Ross Duncan[3693]. It visualizes the modifications made to a set of qubits and is based on transformations applicable to the geometric representation of quantum gates that simplify models (Figure 806).

It is particularly useful for programming a quantum computer in MBQC (measurement base quantum computing), for visually model error correction codes and design braiding models in the topological quantum computing paradigm[3694]. It can also help optimize quantum code compiling[3695].

[3692] See Quantivine: A Visualization Approach for Large-scale Quantum Circuit Representation and Analysis by Zhen Wen et al, July 2023 (11 pages).

[3693] See Interacting Quantum Observables: Categorical Algebra and Diagrammatics by Bob Coecke and Ross Duncan, 2009 (80 pages).

[3694] See The ZX-calculus as a Language for Topological Quantum Computation by Fatimah Rita Ahmadi and Aleks Kissinger, University of Oxford, November 2022-August 2023 (14 pages).

[3695] See Effective Compression of Quantum Braided Circuits Aided by ZX-Calculus by Michael Hank et al, November 2020 (13 pages).

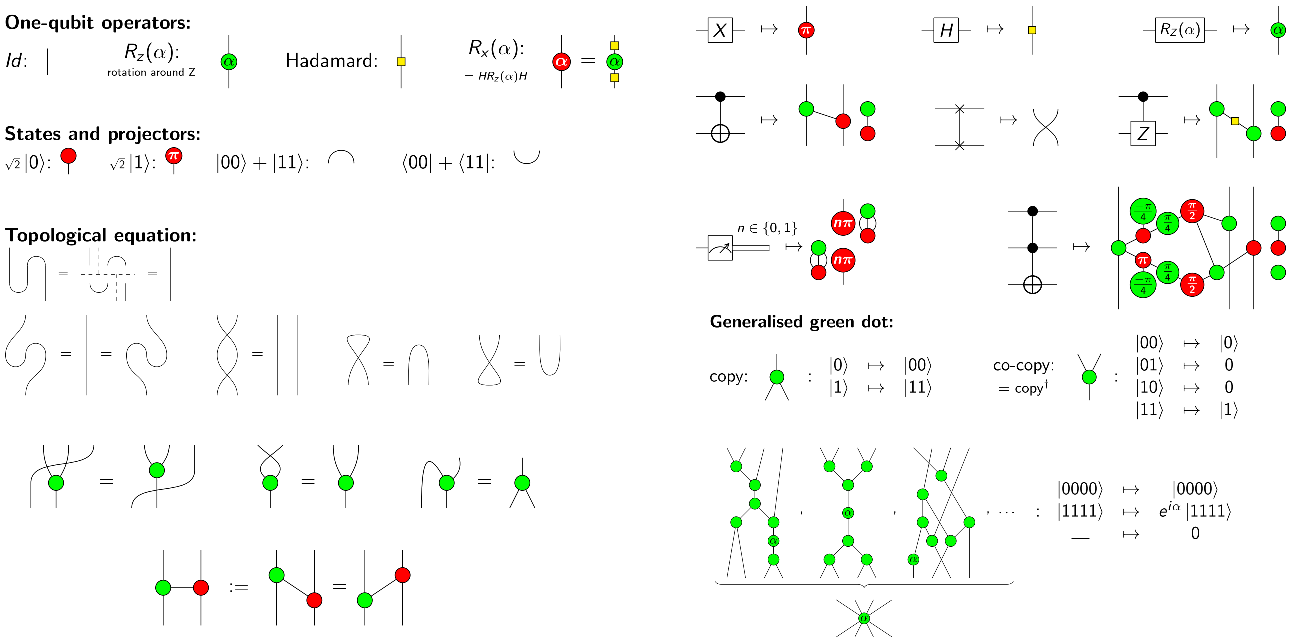

Figure 806: ZX calculus graphical language key operations.
Source: Completeness of the ZX-Calculus by Renaud Vilmart, 2018 (123 slides).

Contributors to the ZX-Calculus work include researchers from Loria under the responsibility of Simon Perdrix, a research laboratory located in Nancy and Dominic Horsman then at UGA LIG in Grenoble and now at Oxford University[3696].

ZX has its own zoo of extensions:

- **PyZX** is a Python tool created in 2019 that implements ZX-Calculus principles for the creation, visualization, and automated rewriting of large-scale quantum circuits.
- **SZC-calculus** (Scalable ZX-calculus) is a high-level extension of ZX-calculus for the design and verification of quantum computations with qubits registers. Among other things, it can be used to describe graph states used in MBQC and error correcting codes[3697].
- **ZXH-calculus** is a graphical language based on ZX-calculus helps modelize many-body states[3698].
- **ZW-calculus** is a variant that allows better entanglement modeling and **ZH-calculus** is used for the generalization of MBQC programming models thanks to the addition of a box implementing the Hadamard gate and an easier integration of Toffoli gates in its diagrams. It is associated with a development tool, **Quantomatic**, created by Aleks Kissinger and Vladimir Zamdzhiev then at of Oxford University and now at Inria in France [3699].

  They organize their own international conference, the **QPL** (Quantum Physics and Logic) but it covers broader topics than ZX calculus.[3700]

[3696] See Completeness of the ZX-Calculus by Renaud Vilmart, 2018 (123 slides) and Completeness of the ZX-Calculus by Emmanuel Jeandel, Simon Perdrix and Renaud Vilmart (73 pages) which explains them.

[3697] See SZX-calculus: Scalable Graphical Quantum Reasoning by Titouan Carette, Dominic Horsman and Simon Perdrix, April 2019 (29 pages).

[3698] See AKLT-states as ZX-diagrams: diagrammatic reasoning for quantum states by Richard D. P. East et al, December 2020 (22 pages).

[3699] See Quantomatic: A Proof Assistant for Diagrammatic Reasoning, 2015 (11 pages).

[3700] See SZX-calculus: Scalable Graphical Quantum Reasoning by Titouan Carette, Dominic Horsman and Simon Perdrix, April 2019 (29 pages).

- **DisCoPy** (Distributional Compositional Python) is an open source toolbox for computing with string diagrams and functors[3701]. Its diagram data structure allows to encode various kinds of quantum processes, with functors for classical simulation and optimization, as well as compilation and evaluation on quantum hardware. It supports ZX calculus and its variants and linear optical quantum computing. Its photonics module that was under development as of May 2022 will allow to build linear optical circuits and interface with the quantum simulator **Perceval** from Quandela. DisCoPy encodes arbitrary string diagrams and interprets these for computation on various classical (NumPy, JAX, TensorFlow, PyTorch, Sympy) and quantum systems (Perceval, PyZX). DisCoPy is of particular use with quantum natural language processing (QNLP) and quantum machine translation applications[3702].

Scripting languages

They are used to program quantum algorithms in text mode. These tools allow us to associate classical programming with the chaining of quantum functions conditioned by the state of variables in classical memory or coming from mid-circuit measurement. Such code can contain repetitive sequences that would be cumbersome to input in a graphical representation of the algorithm.

There are two main types of quantum scripting languages: imperative and functional languages (Figure 807):

- **Imperative languages** are procedural programming languages where step-by-step algorithms are described. They include the usual languages such as C, C++, PHP or Java.
- **Functional languages** are used by defining various functions that are called on an ad-hoc basis by the program. The loops (for, while) are replaced by recursive functions and there are no modifiable variables. It uses high-level abstract data types that are manipulated by functions. It creates more concise code.

Many traditional programming languages can be used for imperative or functional programming, especially if they use function pointers or support event-driven logic. To some extent, JavaScript and jQuery can be used as functional languages via their call-back functions. This is also the case with C++.

With quantum computer vendors such as IBM or Rigetti, two types of languages are sometimes offered: an intermediate language (Quil at Rigetti, openQASM with IBM) and a higher-level language in the form of extensions to the Python programming language (pyQuil at Rigetti, IBM Qiskit). A conversion tool converts the second one into the first one.

The most common programming language used with quantum libraries is Python. It provides language constructs, data types, for-loops code branching, modularity, and classes. It can help build repetitive code structures, which would be harder than with graphical circuit design. It can also be used to create automatic testing tools.

Beyond gate base quantum computing, there are also some domain specific languages devised to drive quantum simulators[3703].

[3701] See DisCoPy for the quantum computer scientist by Alexis Toumi et al, CQC and University of Oxford, May 2022 (6 pages). A functor is "*a design pattern inspired by the definition from category theory, that allows for a generic type to apply a function inside without changing the structure of the generic type*" (Wikipedia).

[3702] See Towards Machine Translation with Quantum Computers by Irene Vicente Nieto, 2021 (48 pages).

[3703] See SimuQ: A Domain-Specific Language For Quantum Simulation With Analog Compilation by Yuxiang Peng et al, University of Maryland, March 2023 (26 pages).

Table 1: A selection of some quantum programming languages.

Name	Style	Notes
QCL	Imperative	Has classical sublanguage, multiple high-level programming features.
qGCL	Imperative	Emphasis on algorithm derivation and verification.
LanQ	Imperative	Full operational semantics, proven type soundness.
Quipper	Functional	Focus on scalability, plans to include linear types for static checks (currently done at run-time).
QPL	Functional	Statically typed, denotational semantics in terms of CPOs of superoperators.
QML	Functional	Linearly typed, focused on weakening - not contraction. Quantum control and quantum data.
Qumin	Functional	Two sublanguages (untyped and linearly typed). Focus on ase of use and clean, functional style of programming.

Figure 807: imperative and functional quantum programming languages.
Source: Qumin, a minimalist quantum programming language, 2017 (34 pages).

Machine languages

These are the lowest level programming languages of the quantum computer, which program the initialization of qubits and drive the physical signals sent to the qubits to implement universal gates and qubit readouts. They are generally specific to each type of quantum computer, or even to each quantum computer. Most quantum algorithms developers never use this type of low-level language.

Compilers

Quantum compilers translate your code into the low-level sequences of qubit electronic controls for the target quantum accelerator (Figure 808). It can also integrate quantum error correction codes (QEC). These compilers transform the program gates into universal physical gates operated by the quantum computer and then into control pulses of the qubits, this part being labelled transpiling. A quantum compiler is mostly a transpiler and an optimizer which reduces the number of qubit gates through various optimization techniques. Transpiling quantum code can scale badly with a large number of qubit and gates and requires its own optimizations[3704].

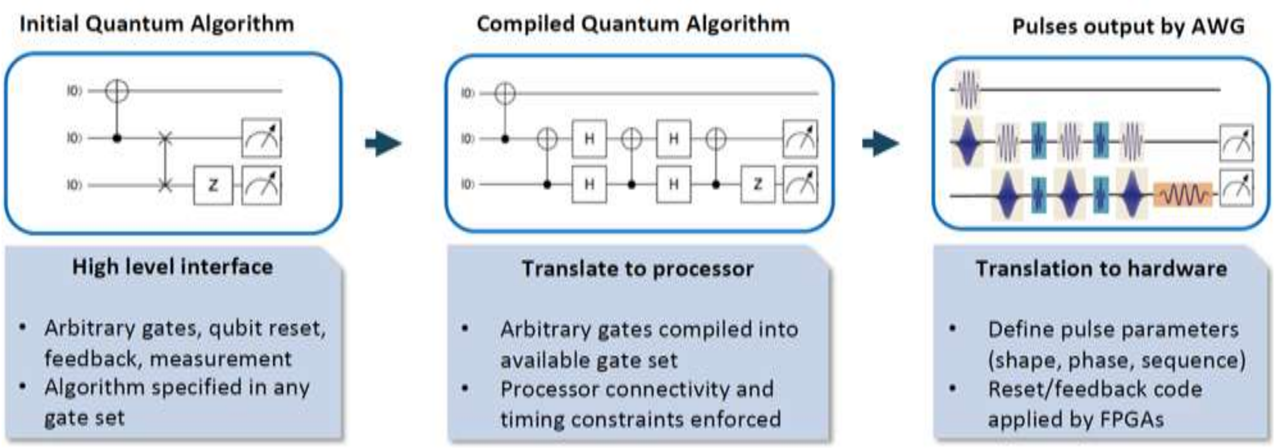

Figure 808: the various roles of a quantum code compiler, first to translate high-level gate codes into primitive gates supported by the quantum processor, and then to turn these gates into low-level electronic controls driving qubit gates and readout. Source: How about quantum computing? by Bert de Jong, June 2019 (47 slides).

[3704] See QASMTrans: A QASM based Quantum Transpiler Framework for NISQ Devices by Fei Hua et al, August 2023 (10 pages).

It will also compute the gates activation times and verify that the accumulation of these activation times is within the range of the target accelerator's qubits coherence time.

Compilers can also carry out optimizations specific to certain types of algorithms[3705] [3706] [3707] and also hardware specifics.

Many advances can reduce the number of gates used and create shallower circuits. It can be done with qubit mapping which optimizes SWAP insertion techniques that are used heavily when qubits connectivity is poor, like with converting SWAP gates into 3 CNOTs and removing redundant combinations of CNOTs[3708], replacing CNOTs by $\sqrt{iSWAP}$ gates[3709], optimizing CNOT circuits generation[3710] [3711] [3712] [3713], transpiling taking to account qubit connectivity[3714], and optimizing the balance between CNOTs and T gates, particularly for fault-tolerant schemes that are very costly with T gates. It is about minimizing the T-count, the number of T gates used in the algorithm or its constituent parts like with quantum Fourier transforms[3715] [3716] [3717] [3718] and recycling unused qubits with automated uncompute set of gates[3719]. There are even strategies available to optimize the compilation of a VQE algorithm for NISQ QPUs[3720], to distribute computing across multiple quantum computers[3721],

[3705] This is the case of Partial Compilation of Variational Algorithms for Noisy Intermediate-Scale Quantum Machines by Pranav Gokhale et al, 2019 (13 pages) which deals with a two-pass compiler optimized for Variational Algorithms (VQE).

[3706] See Spacetime tradeoffs when optimizing large quantum computations by Craig Gidney (Google AI Quantum, USA), IQFA 2020, December 2020 (60mn) and slides. He is turning serialized circuits into parallelized ones. With T state distillation, he gains three orders of magnitude in fidelities.

[3707] See Full-stack quantum computing systems in the NISQ era: algorithm-driven and hardware-aware compilation techniques by Medina Bandic et al, QuTech, April 2022 (6 pages) which summarizes well the full-stack architecture thinking influencing compilers design.

[3708] See Architecture aware compilation of quantum circuits via lazy synthesis by Simon Martiel and Timothée Goubault de Brugière, Atos, December 2020 (31 pages), Not All SWAPs Have the Same Cost: A Case for Optimization-Aware Qubit Routing by Ji Liu et al, North Carolina State University, May 2022 (17 pages) and Qubit Mapping Toward Quantum Advantage by Chin-Yi Cheng et al, October 2022 (14 pages).

[3709] See MIRAGE: Quantum Circuit Decomposition and Routing Collaborative Design using Mirror Gates by Evan McKinney et al, August 2023 (13 pages).

[3710] And Decoding techniques applied to the compilation of CNOT circuits for NISQ architectures by Timothée Goubault de Brugière, Marc Baboulin, Benoît Valiron, Simon Martiel and Cyril Allouche, January 2022 (31 pages).

[3711] See Energy risk analysis with Dynamic Amplitude Estimation and Piecewise Approximate Quantum Compiling by Kumar J. B. Ghosh et al, IBM Research, E.On and University of Helsinki, May 2023 (25 pages).

[3712] See Shallower CNOT circuits on realistic quantum hardware by Timothée Goubault de Brugière and Simon Martiel, March 2023 (28 pages). Compiler optimization of CNOT implementation.

[3713] See Algorithmic Theory of Qubit Routing by Takehiro Ito et al, May 2023 (14 pages).

[3714] See QContext: Context-Aware Decomposition for Quantum Gates by Ji Liu et al, February 2023 (10 pages).

[3715] See Gaussian Elimination versus Greedy Methods for the Synthesis of Linear Reversible Circuits by Timothée Goubault de Brugière, Marc Baboulin, Benoît Valiron, Simon Martiel and Cyril Allouche, January 2022 (27 pages).

[3716] See Reducing the Depth of Linear Reversible Quantum Circuits by Timothée Goubault de Brugière, Marc Baboulin, Benoît Valiron, Simon Martiel and Cyril Allouche, January 2022 (22 pages).

[3717] See Halving the cost of quantum Fourier transform by Byeongyong Park et al, July 2022 (19 pages).

[3718] See Optimal Hadamard gate count for Clifford+T synthesis of Pauli rotations sequences by Vivien Vandaele, Simon Martiel, Simon Perdrix and Christophe Vuillot, February 2023 (30 pages).

[3719] See Recycling qubits in near-term quantum computers by Galit Anikeeva et al, PRA, April2021 (11 pages).

[3720] See Policy Gradient Approach to Compilation of Variational Quantum Circuits by David A. Herrera-Martí, November 2021-September 2022 (17 pages).

[3721] See Qurzon: A Prototype for a Divide and Conquer Based Quantum Compiler by Turbasu Chatterjee et al, November 2021 (11 pages).

compiling techniques optimization using precompilation and predicting schemes[3722] and precomputation methods to optimize circuits execution timing and making use of the gate teleportation mechanism[3723].

Like Eviden/Atos' aQASM, these compilation tools can be cross-platform and support different gate-based quantum computer architectures. Quantum programming languages are generally able to combine classical procedural programming with quantum registers and gates programming. They allow parallel management of classical memory with quantum registers.

All these programming tools come from either research labs or from quantum computer vendors such as IBM, Rigetti and D-Wave[3724].

Application specific frameworks

On top of scripting languages used to program quantum computers, annealers or simulators sit a wealth of application specific frameworks aimed at solving particular problems. The bulk of these frameworks are proposed by the many software vendors we inventory in a special part of this book, starting page 1094. They usually support various underlying hardware architectures from classical digital emulators to QPUs running on-premises or in the cloud, the best ones being able to support heterogeneous classical and quantum platforms for the test and implementation of hybrid algorithms.

The most common proposed frameworks cover generic optimization problems (mostly using variational techniques mixing classical and quantum computing like QAOA and VQE/VQA), and others for chemical simulations and financial optimizations, which have been identified as the first go-to-markets for quantum computing so far.

Some application specific frameworks are also proposed by the large vertically integrated quantum computing vendors like IBM (with their Qiskit frameworks). Others can also be proposed by research labs like Q^2 Chemistry which comes from USTC in China[3725].

Emulators and simulators

Emulators are software and/or hardware tools that emulate the execution of quantum algorithms on classical computers. Their qubits emulation capacity is closely related to the amount of memory available and the emulation mode that is implemented, but also on pure computing power[3726].

These emulators are also labelled "simulators" or "quantum classical simulators", which creates some confusion with quantum simulators which are computing solutions simulating quantum physics systems, working either classically or quantumly with so-called analog quantum simulators. In a 2023 paper, I tried to clarify the situation between these different labels[3727]. It is framed in Figure 809 which creation a distinction between quantum code emulators which run at a high level and their potential underlying layers implementing a quantum simulation of the QPU qubit physics and noise models.

3722 See Reducing the Compilation Time of Quantum Circuits Using Pre-Compilation on the Gate Level by Nils Quetschlich et al, May 2023 (10 pages).

3723 See Accelerating Quantum Algorithms with Precomputation by William J. Huggins and Jarrod R. McClean, Google AI, May 2023 (18 pages).

3724 See this presentation which describes some of the tasks performed by quantum compilers: Opportunities and Challenges in Intermediate-Scale Quantum Computing by Fred Chong, 2018 (34 slides).

3725 See Q^2 Chemistry: A quantum computation platform for quantum chemistry by Yi Fan et al, August 2022 (32 pages).

3726 See the list of quantum algorithm simulation tools at https://quantiki.org/wiki/list-qc-simulators.

3727 See Disentangling quantum emulation and quantum simulation by Olivier Ezratty, January 2023, with a clear separation between a quantum emulation (quantum code executed on a classical computer) from a quantum simulation that is a simulation of a quantum many-body system implemented either classically or on a digital or analog quantum computer.

Then, you have analog quantum simulators ala Pasqal which use many body quantum systems to solve various problems, usually through a QUBO formulation, and quantum digital quantum simulators that do the same on gate based QPUs using algorithms like VQE in NISQ QPUs and QPE in future FTQC QPUs. A classical digital quantum simulator is also a very commonplace usage of classical computers to digitally simulate many body and correlated quantum systems like with DFTs and the likes to find for example the ground state Hamiltonian of some simple molecules.

Quantum code emulation serves multiple purposes: learning how to code, visualize a quantum algorithm internal data (which can't be done with a real quantum computer), develop quantum algorithms and test them at a low scale. It can also be used to simulate how quantum hardware behaves at a low level, reproducing digitally their defects like noise and other imperfections. Some emulators are even able to simulate the inner physics of the underlying physical qubits, like Perceval from Quandela (for photon qubits) or Callisto from C12 (for carbon nanotubes spin qubits).

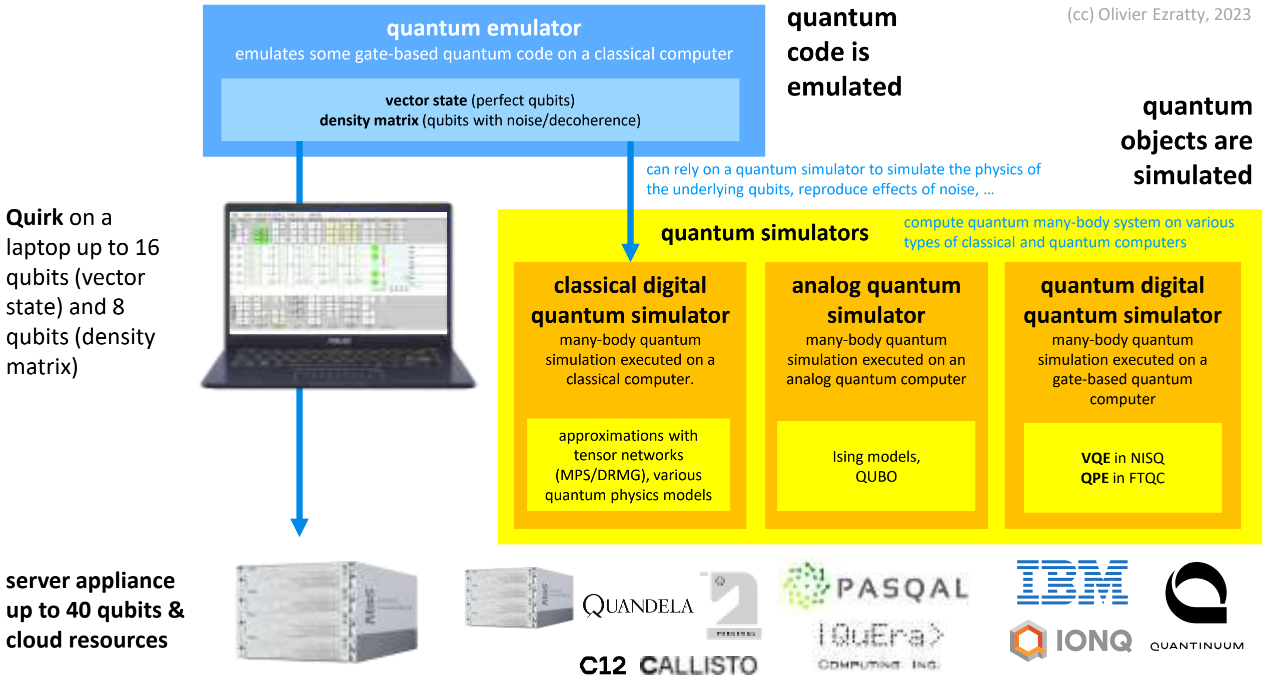

Figure 809: separating emulation from simulation in quantum computing lingua. Source: Olivier Ezratty, 2023.

The required classical computing capacity grows exponentially with the number of supported qubits and depends on the type of emulation (Figure 810). On a laptop equipped with 16 GB of memory, you can simulate up to about 20 qubits, and 16 with Quirk, including showing density matrices of up to 8 qubits. You can emulate a couple qubits in simple microcomputers from the early 1980's like 2 qubits on a Commodore 64 with 200 lines of BASIC language[3728]. With a C language compiler, you can probably easily emulate up to 10 qubits on an IBM PC from 1981 using 640 KB of memory and with its 4.77 MHz Intel 8088 CPU. And these would be perfect qubits. The same could be said for a Raspberry Pi kit or a regular Smartwatch. So, when you see some "quantum advantage" case study experimented on less than 12-16 qubits, remind yourself that it can be run faster and way cheaper on such devices and on your own laptop.

Specialized server appliances such as Eviden/Atos' QLM fits in a datacenter rack, are designed to manage a very large amount of memory and can support the emulation of up to 40 qubits. More than 40 qubits can be emulated on massively parallel architectures and supercomputers or on a large number of clusters.

[3728] See Quantum Computing on a Commodore 64 in 200 Lines of BASIC by Davide Gessa (dakk), July 2023.

Table I. Comparison between different simulation methods, where N is the number of qubits, m is the number of gates, χ is the bond dimension of MPS, m_T is the number of T gates in the circuit, and W denotes the tree-width of the graph associated with the quantum circuit.

Methods		Memory	Run time	Approx. or exact	Noiseless or noisy	Application regime
Full state	State-vector	worst $O(2^N)$	worst $O(m2^N)$	Exact	Noiseless#	General, good for small circuits*
	Density-matrix	worst $O(2^{2N})$	worst $O(m2^{2N})$	Exact	Both	General, good for small circuits+
MPS state/MPO		$O(N\chi^2)$	$O(N\chi^6)$	Approx.	Noisy	General, good for shallow circuits
Tensor network		On demand	$O(e^W)$	Both	Both	General, good for shallow circuits
Stabilizer		$O(e^{m_T})$	$O(e^{m_T})$	Approx.	Both	Circuits dominated with Clifford gates, particularly in QEC

State-vector simulators can also be used to simulate noisy circuits to get an approximate result with the Monte Carlo method.
* Circuits with $N > 32$ with the state-vector simulator should generally run on an HPC server.
+ Circuits with $N > 16$ with the density-matrix simulator should generally run on an HPC server.

Figure 810: table showing the classical resources in memory and run time for the various gate based quantum emulation modes; full state with state vector or density matrix, MPS and tensor networks and stabilizers which are used to quantum error correction design using Clifford quantum gates. The less demanding is the state vector emulation and can emulate perfect qubits. Density matrix emulation can take qubit noise into account but is way more costly and thus limited in number of qubits. MPS and tensor networks are used to emulate shallow circuits but can accommodate a larger number of qubits. Source: A Herculean task: Classical simulation of quantum computers by Xiaosi Xu et al, February 2023 (14 pages).

There are various methods for emulating a circuit of N qubits and a certain level of depth of quantum gate sequences which will resonate with what we've learned about registers computational basis and density matrices[3729]. We'll cover some of them from the hardest to the simplest regarding their computational resource requirements.

- **Quantum state vector** handles the complex amplitudes of all the Hilbert space managed by N qubits in memory. It requires 2^N complex numbers representing 2^{N+1} floating point numbers and 2^{N+5} bytes with double precision floating point numbers using 16 bytes. The action of quantum gates on this large vector consists in applying to it the quantum gates unitary matrices to one, two or three qubits which are respectively made of 2x2, 4x4 or 8x8 complex numbers. This method is implemented on supercomputers with huge memory capacities of the order of several Po. It is currently limited to about 50 qubits.
- **Density matrix** computing requires 2^Nx2^N complex numbers, so 2^{2N+1} floating point numbers. It is the most memory-hungry method and is not used with a large number of qubits. It can be necessary if you need to emulate imperfect qubits with their noise and decoherence and their impact on quantum algorithm's execution.

[3729] See Classical Simulation of Intermediate-Size Quantum Circuits, Alibaba, 2018 (12 pages). See also What limits the simulation of quantum computers? by Yiqing Zhou, Edwin Miles Stoudenmire and Xavier Waintal, March 2020 (14 pages) which provides a theoretical and practical framework for the optimization of quantum code emulation. Noteworthy is the work on the emulation of superconducting qubit modules with ... superconducting qubits. See Quantum computer-aided design: digital quantum simulation of quantum processors by Thi Ha Kyaw et al, 2020 (23 pages).

- **Tensor network** compression techniques are used to simplify emulation and ease its distribution across multiple classical computing nodes[3730]. It was used for example in September 2019 by Alibaba on a cluster of 10,000 servers with 96 CPUs. They simulated Google Bristlecone's 70 qubits (which never really run practically) over a depth of 34 quantum gates with 1,449 instances of their Cloud Elastic Computing Service (ECS), each comprising 88 Intel Xeon chips with 160 GB of memory. So, a total of 127,512 processors[3731]! This method can be implemented with many compression techniques providing a lower accuracy[3732]. It was improved in October 2021, again by Alibaba, to classically simulate the Google Sycamore on the new Sunway supercomputer in 304 seconds[3733]. The chart in Figure 811 from this team shows the extent of tensor network compression capabilities as compared to state vector emulation.

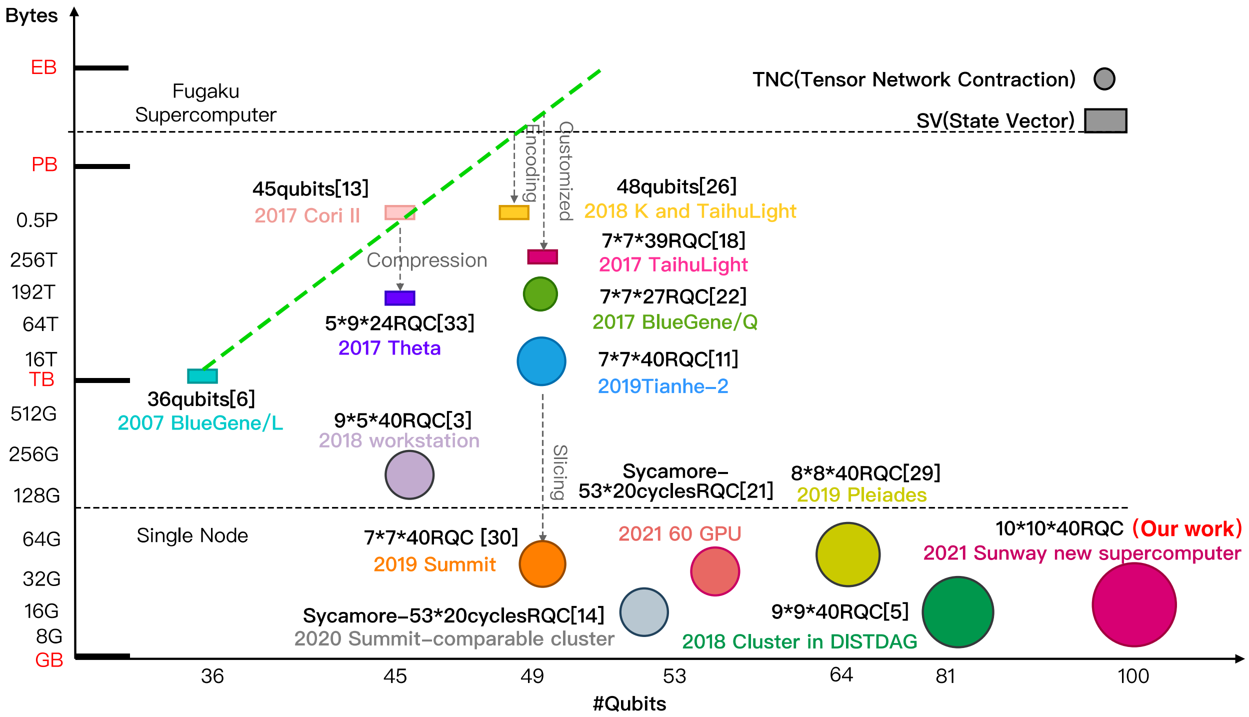

Figure 811: the capacities of various quantum emulators in number of qubits (X) and memory capacity (Y). Source: [Closing the "Quantum Supremacy" Gap: Achieving Real-Time Simulation of a Random Quantum Circuit Using a New Sunway Supercomputer](#) by Yong (Alexander) Liu et al, October 2021 (18 pages).

- **Weak simulation** which manages quantum state vector amplitudes without the phase, with 2^N floating point numbers and thus 2^{N+4} bytes, representing output measurement probabilities in the $|0\rangle$ and $|1\rangle$ computational basis[3734]. The method is easier to distribute over several servers. It is, however, not widely used.

[3730] Partitioning methods for quantum simulation are well described in Distributed Memory Techniques for Classical Simulation of Quantum Circuits, Ryan LaRose of the University of Michigan, June 2018 (11 pages).

[3731] See Alibaba Cloud Quantum Development Platform: Large-Scale Classical Simulation of Quantum Circuits, September 2019 (5 pages).

[3732] See Full-State Quantum Circuit Simulation by Using Data Compression by Xin-Chuan Wu et al, 2020 (29 slides).

[3733] See Closing the "Quantum Supremacy" Gap: Achieving Real-Time Simulation of a Random Quantum Circuit Using a New Sunway Supercomputer by Yong (Alexander) Liu et al, October 2021 (18 pages). The China team behind this was awarded the 2021 ACM Gordon Bell Prize. Their work was later contradicted by ORNL researchers who had developed Google's supremacy classical simulation in 2019. See China's exascale quantum simulation not all it appears by Nicole Hemsoth, NextPlatform, November 2021.

[3734] See Just Like the Real Thing: Fast Weak Simulation of Quantum Computation by Stefan Hillmich et al, July 2020 (6 pages).

- **Hybrid quantum computing emulation** which emulates the quantum part, and executes the classical part as developed in China on a Sunway supercomputer[3735].

The main limitations of supercomputers for emulating quantum algorithms are more related to their memory (RAM) than to their processing capacity. It would take 16 Po of memory to fully simulate 50 qubits. How about 96 qubits? The memory requirement would be multiplied by 2^{46}*2. Moore's law with memory cannot therefore keep pace with a linear increase in the number of used qubits in a quantum computer.

Nevertheless, the number of emulated qubits on supercomputers is still constantly increasing. China research teams have been the most active in this emulation race, particularly at Alibaba and Huawei, with several records set in 2018 up to 2021.

Origin Quantum, a Chinese multi-role (hardware, software) startup in partnership with the Guang-Can Guo team from the **University of Science and Technology of China**, emulated 64 qubits with a 22-depth algorithm on a cluster of 128 nodes in 2018[3736]. They used a method to transform combinations of CZ gates (conditional Pauli Z gates) and single-qubit gates into simpler sub-circuits that do not need to be interleaved. They also thought they could simulate 72 qubits over a depth of 23 gates on a supercomputer running for 16 hours. This work shows that two key parameters condition the emulation capabilities in classical computers: not only the number of qubits but also the number of quantum gate sequences. The larger the number of qubits emulated, the fewer quantum gate sequences we can simulate.

A second 2018 record coming from **Alibaba** was achieved with 81 qubits and 40 quantum gate sequences[3737]. Their Taizhang simulation exploited a method created by Igor Markov and Shi Yaoyun in 2005 that allows a quantum algorithm to be distributed over a farm of thousands of servers[3738]. The Alibaba Quantum Laboratory is managed by the same Shi Yaoyun, a professor at the University of Michigan. Their simulations included 100 qubits over 35 layers (10x10x35), 121 qubits over 31 layers (11x11x31) and 144 qubits over 27 layers (12x12x27). Another 2018 record came from **Huawei** and its "HiQ Cloud" service, capable of emulating 42 to 169 qubits[3739]. The method was similar to the one used by **Alibaba**. The 42 qubits were simulated in "full amplitude" mode. 81 qubits were simulated with "a single amplitude" and 169 qubits with a single amplitude and a small number of quantum gates. Other records have been broken in the USA in 2019, such as **Google'**s record with NASA, the University of Illinois and the Oak Ridge laboratory with 49 to 121 qubits on the IBM Summit[3740].

IBM broke a record of 56 qubits emulation in 2017 on a classic supercomputer of their own, the Vulcan BlueGene installed at the Lawrence Livermore National Laboratory in California. The same Oak Ridge laboratory is at the origin of **XAAC** (eXtreme-scale ACCelerator programming framework), a framework for Eclipse that manages hybrid calculations combining quantum computers and supercomputers such as the Titan equipped with Nvidia GPUs installed in Oak Ridge[3741] .

[3735] See Large-Scale Simulation of Quantum Computational Chemistry on a New Sunway Supercomputer by Honghui Shang et al, July 2022 (13 pages).

[3736] See Researchers successfully simulate a 64-qubit circuit, Science China Press, June 2018.

[3737] See Alibaba Says Its New "Tai Zhang" Is the World's Most Powerful Quantum Circuit Simulator, May 2018 et Alibaba announced that it has developed the world's strongest quantum circuit simulator "Taizhang", May 2018.

[3738] See Simulating quantum computation by contracting tensor networks by Igor Markov et Shi Yaoyun, 2005 (21 pages).

[3739] See Huawei Unveils Quantum Computing Simulation HiQ Cloud Service Platform, October 2018.

[3740] See Establishing the Quantum Supremacy Frontier with a 281 Pflop/s Simulation, May 2019 (11 pages). This Summit must have consumed a good part of the production of Nvidia V100! Here is also the list of qubit and qubit simulation records in https://quantumcomputingreport.com/scorecards/qubit-count/.

[3741] See Eclipse Science and Open Source Software for Quantum Computing, 2017 and the article describing XAAC: A Language and Hardware Independent Approach to Quantum-Classical Computing, July 2018 (15 pages).

It can transform quantum code for computers with quantum gates or quantum annealing models into executable code on any quantum architecture.

Many other IT players want to jump on the quantum emulation bandwagon. It was the case with **Dell** which announced in 2021 its hybrid solution combining classical computing and quantum emulation using its Dell EMC PowerEdge R740xd server appliance and IBM's Qiskit Runtime.

Eviden, formerly Atos (France) sells since 2017 an Intel-based quantum emulator appliance, the Atos Quantum Learning Machine (QLM).

It has been widely adopted worldwide, such as by the US DoE's ORNL, by the CEA, at the University of Reims, at the cybersecurity research department of the University of Applied Sciences of Upper Austria in Hagenberg, by the Hartree Science and Technology Facility Centre (STFC) in the UK, at the C-DAC (Centre for Development of Advanced Computing) in India, in its Quantum Computing Experience Center[3742], in Japan, in Finland at the CSC IT Center for Science Kvasi in collaboration with IQM, in the new Quantum Integration Centre (QIC) from LRZ (Leibniz Supercomputing Centre) of the Bavarian Academy of Sciences and in Spain (CESGA). A QLM was also sold to CERN in 2021 (Figure 812).

In June 2020, Atos launched the QLM E, a new version of this emulator integrating from 2 to 32 Nvidia V100 GPUs and multiplying computing power by 12 compared to the Intel-based initial version. This system was first delivered in December 2020 to the Irish HPC center (ICHEC). This was completed in early July 2020 with the support of a limited form of quantum annealing emulation.

Figure 812: Atos QLM customers. Source: Atos. 2022.

In May 2019, Atos launched myQLM, a quantum programming tools for researchers, students, and developers. It is a Python-based development environment that allows users to simulate quantum programs on their own computer. Programming is carried out in AQASM (Atos Quantum Assembly Language) and pyAQSM. To access a number of qubits that exceeds the current capacity of PCs, i.e., more than 20 qubits, developers can run their code on an Atos Quantum Learning Machine simulator in the cloud, but at a charge. Atos also enables the sharing of quantum practices, libraries and application codes. Atos offers one of the open source translators of myQLM code to other quantum programming environments. In September 2020, this software offer became free of charge to all audiences[3743]. myQLM supports the emulation of the three quantum computing paradigms: quantum annealing, quantum simulation and gate-based programming.

It supports several modes: state vectors emulation, with using various compression methods (stabilizers, binary decision diagrams, tensor networks MPS), and with simulating qubit noise.

[3742] See Atos and C-DAC sign a cooperation agreement to accelerate the development of quantum and exascale computing and Artificial Intelligence in India, August 2019.

[3743] See Atos roadmap in The Atos Quantum Program - Paving the way to quantum-accelerated HPC by Jean-Pierre Panziera, June 2021 (10 slides).

Atos announced in 2020 that they would launch a NISQ quantum accelerator by 2023. They are looking at several tracks such as superconductors (with IQM), trapped ions (with the University of Innsbruck and AQT), cold atoms (with Pasqal) and in the longer-term silicon qubits (with CEA-Leti). They participate in the European Flagship projects **AQTION** (quantum accelerator), **PASQuanS** (analog quantum simulator) and **NEASQC** (NExt ApplicationS of Quantum Computing, which they coordinate).

Atos is also heavily involved in the EuroHPC project, which includes the **European Processor Initiative**, an initiative to develop a processor adapted to the needs of supercomputers and on-board as well as autonomous vehicles[3744]. And, of course, in the $\langle \boldsymbol{HPC}|\boldsymbol{QS}\rangle$ project which will deploy in Finland, Germany and France three hybrid quantum solutions combining a supercomputer and a quantum computer.

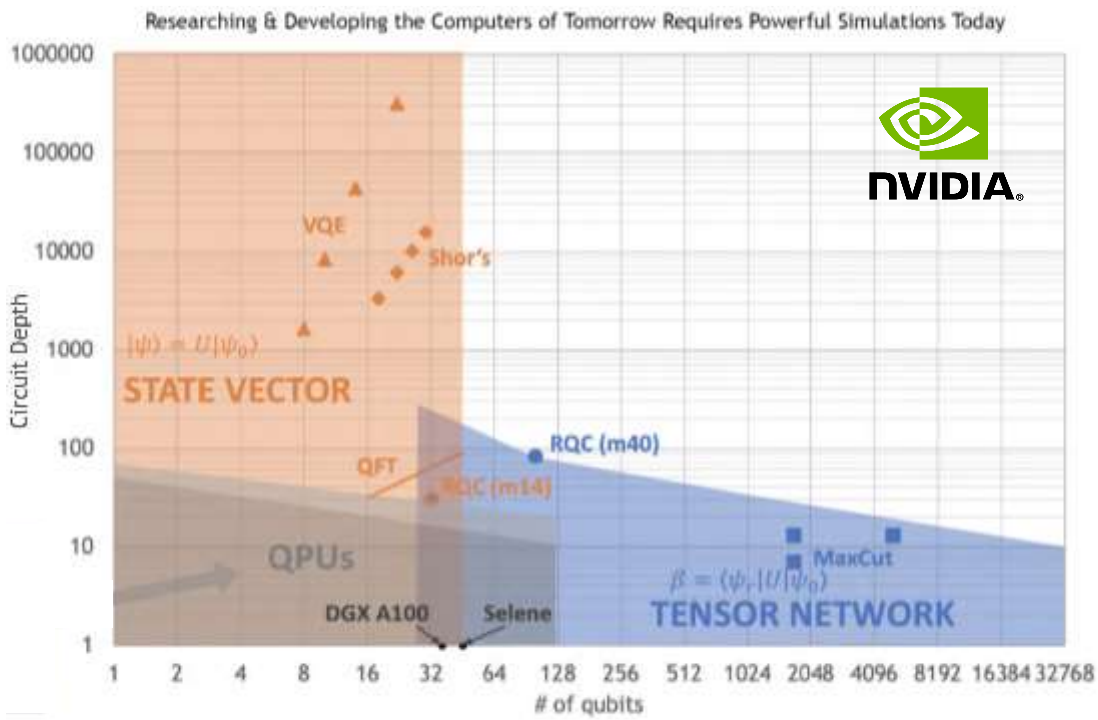

Figure 813: Nvidia positioning the scope of state vector quantum emulation in a regime with fewer than 32 qubits but no limitation in the circuit depth (in the Y axis), and tensor networks emulation which can scale with hundreds of qubits with a shallow algorithm. This last solution is adequate to emulate NISQ algorithms with not many limitations. The figure shows that classical emulation has a broader scope than existing NISQ quantum computers (in grey). The Shor point in the scatter plot corresponds to running Shor integer factoring algorithm on very small RSA keys, not the sought-after RSA-2048 key. It is the same with the VQE point which corresponds to rather small chemical simulation requirements. Source: Nvidia[3745]. Added in 2023.

Nvidia (USA) developed the cuQuantum SDK running on top of their GPGPUs. It implements gates-based programming emulation, announced in April 2021, beta in November 2021[3746] and released in March 2022[3747].

[3744] In July 2018, Atos also acquired Syntel for $3.4B in the USA, a $923M service provider specializing in the development and deployment of applications in the cloud with 22,500 employees, created in 1980 by Indo-Americans. This does not seem to have anything to do with quantum.

[3745] François Courteille and Sam Stanwyck, Bringing GPU acceleration to Hybrid Quantum-Classical Computing, Nvidia, presentation at the EDF TQCI Seminar on Quantum Hybridization and Integration, January 2023 (27 slides).

[3746] See NVIDIA Teams With Google Quantum AI, IBM and Other Leaders to Speed Research in Quantum Computing by Sam Stanwyck, Nvidia, November 2021.

[3747] See Nvidia Unveils Onramp to Hybrid Quantum Computing by Timothy Costa, March 2022.

Thanks to the GPGPU tensors implementing matrix multiplications and fast HBM memory, the acceleration provided is clear, making it possible to emulate Google Sycamore processor with a depth of 20 gates in less than 10 minutes[3748]. It is supported by various cloud offerings from JUQCS-G (Julich), Qgate (NVAITC), Qiskit-AER (IBM), QuEST (Oxford), SV1 (Amazon Web Services) and Vulcan (QC Ware).

The cuQuantum SDK supports both state-vector emulation with tens of qubits (**cuStateVec**) and a less resources-hungry tensor-network based emulation (**cuTensorNet**) that supports up to thousands of qubits (Figure 813). They integrated cuStateVec into qsim, Google Quantum AI's state vector simulator that can be used through Cirq. cuStateVec can also be used with Qiskit Aer, IBM's emulation framework[3749].

Nvidia also proposes its quantum compiler **NVQ++** that targets the Quantum Intermediate Representation (QIR), a low-level machine language specification covering hybrid classical/quantum computing needs. It is supported by the Linux Foundation led **QIR Alliance** with contributions from ORNL, Rigetti, Quantinuum, Microsoft and Quantum Circuits Inc[3750].

Nvidia's software tools adopters include QC Ware (for quantum chemistry and QML using cuQuantum on the Nvidia A100-based Lawrence Berkeley National Laboratory Perlmutter supercomputer launched in 2021), ORNL (using cuQuantum in TNQVM, a framework for tensor network quantum circuit simulations), Xanadu (using cuQuantum in their PennyLane framework for QML and quantum chemistry), Classiq (in their Quantum Algorithm Design platform) and Zapata Computing (in Orquestra). Nvidia also works with Google Quantum AI, IBM, IonQ and Pasqal.

In 2022, Pasqal (France) deployed an on-premises Nvidia DGX POD to run digital simulations of its quantum simulator using cuQuantum. In July 2022, Nvidia announced its quantum software emulation and hybrid computing, the Quantum Optimized Device Architecture (**QODA**) platform[3751]. It helps develop software that can run on both GPU-based classical emulation and on QPUs, including hybrid quantum/classical solutions (Figure 814).

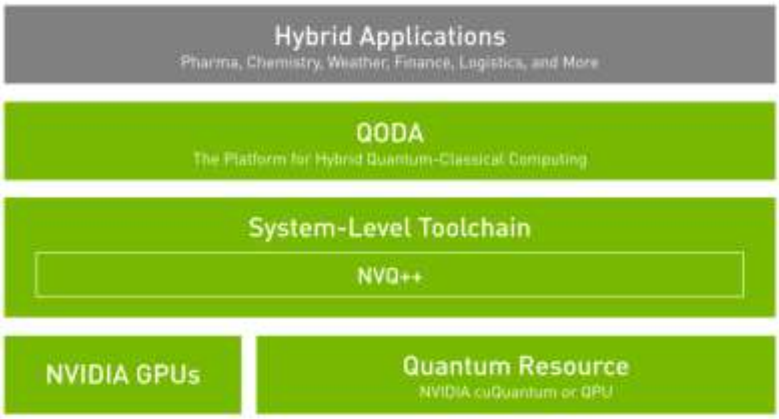

Figure 814: Nvidia QODA architecture. Source: Nvidia.

In March 2023, Nvidia formed a partnership with Quantum Machines to combine their DGX Quantum server appliance with Quantum Machines' OPX+ quantum qubit control hardware. The new DGX is using the Grace Hopper GH200 powerful chip including both custom arm cores Grace CPUs and GH100 GPUs introduced in May 2023[3752]. Both systems are connected at the PCIe bus level enabling very low sub-microsecond latency between Nvidia GPUs and QPUs. In the future, it will enable fast implementation of quantum error correction codes.

[3748] See What Is Quantum Computing? by Dion Harris, April 2021 and Nvidia entangled in quantum simulators by Nicole Hemsoth, April 2021.

[3749] See cuQuantum SDK: A High-Performance Library for Accelerating Quantum Science by Harun Bayraktar et al, Nvidia, August 2023 (37 pages).

[3750] See https://github.com/qir-alliance.

[3751] See Bringing GPU acceleration to Quantum-Classical Computing by François Courteille, Nvidia, January 2023.

[3752] See Nvidia's grace-hopper hybrid systems bring huge memory to bear by Timothy Prickett Morgan, TheNextPlatform, May 2023.

Beyond the solutions mentioned above, many software emulation tools are available and are mostly always open source [3753]:

Quirk runs in your browser and even on your smartphone. It was developed in 2016 by Craig Gidney before he joined Google in 2017. It is very useful to learn quantum circuit encoding and understanding what's happening on the data in qubits. Then, you move to learning how to code with scripting languages like Python for larger codes since Quirk is limited to supporting 16 qubits.

Quantum Circuit Simulator runs under Android and was developed in 2013.

QX Quantum Computer Simulator is a universal quantum computer simulator developed at QuTech by Nader Khammassi.

SimulaQron is another quantum circuit simulator developed by Stephanie Wehner et al at QuTech supporting up to 34 qubits.

Qrack is a C++ quantum bit and gate simulator, with the ability to support arbitrary numbers of entangled qubits—up to system limitations[3754]. Suitable for embedding in other projects, the Qrack QInterface contains a full and performant collection of standard quantum gates, as well as variations suitable for register operations and arbitrary rotations. The developers of Qrack maintain a fork of the ProjectQ quantum computer compiler which can use Qrack as the simulator, generally. This stack is also compatible with the SimulaQron quantum network simulator. Further, it maintains a QrackProvider for Qiskit. Both ProjectQ and Qiskit integrations for Qrack support the PennyLane stack. For Qiskit, a fork of the Qiskit plugin provides support for a "QrackDevice".

Psitrum is a software emulator developed in Matlab in Saudi Arabia. It computes the algorithm matrix, the density matrix of the simulated qubit register and the output state vector. It can simulate quantum noise and has a graphical circuit designer. It provides density matrices visualization tools[3755].

Qiskit Aer is IBM's emulation software solution launched in December 2018. It supports simulations in state vector mode and density matrix mode as well as in the more exotic matrix product state (adapted to weakly entangled states) and stabilizer modes (supporting only Clifford group gates).

Google's state vector based emulator qsim is written in C++ and can simulate 30 qubits on a laptop and up to 40 qubits in Google Cloud or on a 90 core Intel Xeon workstation.

Sandbox AQ created a DMRG based quantum circuit emulator, along with Google. They broke some records in 2023 with using a TPU-v3 pod supercomputer, thanks to its fast distributed matrix multiplications capacity, originally built to train large machine learning models. This code computes the ground state of a local quantum many-body Hamiltonian, a classical equivalent of a NISQ solution that would be implemented with a VQE algorithm. In that case, Google could support a bond dimension of $2^{16} = 65{,}536$, that sizes the number of entanglements in the simulated many-body system[3756].

Amazon Braket emulator supports state vector, tensor network and density matrix emulations with respectively SV1, TN1, and DM1[3757].

[3753] See this long list of emulators.

[3754] See a QRack benchmark in Exact and approximate simulation of large quantum circuits on a single GPU by Daniel Strano, Benn Bollay, Aryan Blaauw, Nathan Shammah, William J. Zeng and Andrea Mari, April 2023 (9 pages).

[3755] See Psitrum: An Open Source Simulator for Universal Quantum Computers by Mohammed Alghadeer et al, Saudi Arabia, March 2022 (27 pages).

[3756] See Density Matrix Renormalization Group with Tensor Processing Units by Martin Ganah et al, Sandbox AQ and Google AI, February 2023 (17 pages).

[3757] See Simulating quantum circuits with Amazon Braket by Katharine Hyatt and Eric Kessler, September 2021.

LIQUi|> is a software architecture and tool suite for quantum computing developed by the Quantum Architectures and Computation Group (QuArC) at Microsoft Research starting back in 2016. It includes a programming language, optimization and scheduling algorithms, and quantum simulators. LIQUi|> can be used to translate a quantum algorithm written in the form of a high-level program into the low-level machine instructions for a quantum device.

Intel IQS (formerly qHiPSTER) created by Intel in 2016[3758]. It supports up to 42 qubits pure states in state vector simulation mode. As of 2023, Intel released its Quantum SDK 1.0. with a new emulator supporting C++ language[3759].

Fujitsu has a 39-qubit CPU-based state vector quantum simulator hardware that is based on 512 PrimeHPC FX700 2U server nodes using the arm based A64FX Fujitsu CPUs. This hardware used the Qulacs software emulator, created by Osaka University and QunaSys. This hardware and software offering was initially launched in March 2022.

QuTech (2014, the Netherlands) is the quantum hardware spin off from TU Delft University. It collaborates with Intel in the development of superconducting qubits and with Microsoft in topological quantum. The company is an applied contract research laboratory. It also develops software, such as the **Quantum Inspire** development platform, which enables quantum algorithms to be run on conventional computers in emulation mode. It provides a graphical programming interface in the QASM language. The code can then be executed in emulation mode in the cloud on a classic machine, the Dutch national supercomputer Cartesius, with 5, 26 and 32 qubits, depending on the chosen package. Cartesius is equipped with thousands of Intel Xeon and Xeon Phi CPUs and a few dozen Nvidia Tesla K40m GPUs with 130 TB of memory delivering 1.84 PFLOPS. The equipment comes from Atos. Quantum Inspire also provides cloud access to QuTech 5 superconducting qubits and 2 electron spin qubits since April 2020.

staq is another emulator available as a GitHub repository, staq is a modern C++17 library for the synthesis, transformation, optimization and compilation of quantum circuits authored by softwareQ Inc. under the MIT License. It is usable either through the provided binary tools or as a header-only library that can be included to provide direct support for parsing & manipulating circuits written in the OpenQASM circuit description language. Inspired by Clang, staq is designed to manipulate OpenQASM syntax trees directly, rather than through an intermediate representation which makes retrieving the original source code impossible. OpenQASM circuits can be inspected and transformed (in most cases) without losing the original source structure. This makes staq ideally suited for source-to-source transformations, where only specific changes are desired. Likewise, this allows translations to other common circuit description languages and libraries to closely follow the OpenQASM source.

QuIDDPro is a fast, scalable, and easy-to-use computational interface for generic quantum circuit simulation. It supports state vectors, density matrices, and related operations using the Quantum Information Decision Diagram (QuIDD) data structure. Software packages including Matlab, Octave, QCSim, and libquantum, have also been used to simulate quantum circuits. However, unlike these packages, QuIDDPro does not always suffer from the exponential blow-up in size of the matrices required to simulate quantum circuits. As a result, we have found that QuIDDPro is significantly faster and uses significantly less memory as compared to other generic simulation methods for some useful circuits with many more than ten qubits.

Wolfram Quantum Framework is a “paclet”, a quantum code emulator package for the Wolfram Language Paclet Repository. It contains a library of pre-built quantum functions like the Grover search algorithm and a QFT. It is integrated with the Wolfram Cloud offering and Amazon Braket.

[3758] qHiPSTER stands for quantum High Performance Software Testing Environment. See [qHiPSTER: The Quantum High Performance Software Testing Environment]() by Mikhail Smelyanskiy et al, 2016 (9 pages).

[3759] See [Quantum Researchers Use Intel Quantum SDK to Explore Complex Problem Solving]() by Scott Bair, February 2023.

Quantum Programming Studio is a web-based graphical user interface designed to allow users to construct quantum algorithms and obtain results by simulating directly in the browser or by executing on real quantum computers. The circuit can be exported to multiple quantum programming languages/frameworks and can be executed on various simulators and quantum computers.

Quantum Computer Emulator (QCE) is a software tool that emulates various hardware designs of quantum computers[3760]. QCE simulates the physical processes that govern the operation of a hardware quantum processor, strictly according to the laws of quantum mechanics. QCE also provides an environment to debug and execute quantum algorithms under realistic experimental conditions. The software consists of a Graphical User Interface (GUI) and the simulator itself.

SimQubit is a GUI quantum circuit simulator, written on top of the Q++ (sourceforge.net/projects/qplusplus) quantum templates. It allows editing of quantum circuits and applying them to quantum states, with multiple ways to view the output probabilities.

Qubit101 simulator is a user-friendly quantum circuit editor and simulator. The tool helps users to create, modify and save quantum circuits. Along with this, users can simulate its effect over a predefined quantum state, watch the evolution of the state stage by stage, together with the possible measurements results, use other quantum circuits as gates, so complex circuits can be easily created and finally, simulate an almost arbitrary number of qubits. Supported platforms include Rigetti Forest, IBM Qiskit, Google Cirq and TensorFlow Quantum, Microsoft Quantum Development Kit, Amazon Braket and more.

TornadoQSim is a quantum circuit emulator written in Java that can leverage various hardware acceleration architectures and backends[3761].

Hyperion is a state vector emulator developed by Qubit Pharmaceuticals running on GPUs and used to evaluate chemical simulation algorithms at small scale[3762].

ScaffCC is a compiler and scheduler adapted to the Scaffold programming language supporting the LLVM infrastructure. It supports QASM.

QuEST (Quantum Exact Simulation Toolkit) is a quantum emulator developed in C and C++ and supporting QUDA APIs (not CUDA) and Nvidia's GPUs, created in 2017 by Simon Benjamin's Quantum Technology Theory Group from Oxford University and distributed in open source. The system can simulate 26 to 45 qubits depending on the available memory, respectively 2 GB and 256 GB. It was extended in 2023 with a Virtual Quantum Device (FQD) architecture supporting many types of qubit noise models, connectivity, and gate sets (trapped ions, neutral atoms, NV centers, silicon qubits and superconducting qubits)[3763].

QCLAB+ is a quantum circuit emulator from the Lawrence Berkeley National Laboratory that works on Nvidia GPUs[3764].

[3760] See QCE: A Simulator for Quantum Computer Hardware, by Kristel Michielsen and Hans de Raedt, Jülich Supercomputing Center, February 2022.

[3761] See TornadoQSim: An Open-source High-Performance and Modular Quantum Circuit Simulation Framework by Ales Kubicek et al, University of Manchester, May 2023 (29 pages).

[3762] See Sparse Quantum State Preparation for Strongly Correlated Systems by C. Feniou, Jean-Philip Piquemal et al, November 2023-January 2024 (22 pages).

[3763] See The Virtual Quantum Device (VQD): A tool for detailed emulation of quantum computers by Cica Gustiani, Simon Benjamin et al, Oxford University and Quantum Motion, June 2023 (33 pages).

[3764] See QCLAB++: Simulating Quantum Circuits on GPUs by Roel Van Beeumen et al, Lawrence Berkeley National Laboratory, February 2023 (13 pages). x40 with Nvidia A100 GPU.

QOkit is a QAOA circuit emulator designed by JPMorgan and Argonne Lab with significant speedup compared to classical state-vector emulators[3765].

Then you can find QPU emulators and/or simulators that support specific architectures, like **Perceval**, a photon qubits emulator and simulator developed by Quandela, **SOQCS** also for photon qubits simulations[3766], and **Pulser**, a cold atoms simulator developed by Pasqal[3767] and another one created by the University of Illinois at Urbana-Champaign and HRL Laboratories[3768], hybrid quantum computing simulations[3769] and even simulators of Boson sampling and Gaussian boson sampling experiments[3770 3771 3772]. In that case, these are indeed simulators since they simulate the lower-level operations of Boson sampling physics. Other proposed emulators, originating from research labs, are dedicated to algorithms like the quantum linear solver applied to sparse data, expanding the scope of emulation capacities on classical hardware[3773] or the combinatorial NchooseK model[3774].

There are fewer packaged emulators supporting tensor networks. Some are adapted to NISQ QPU emulations[3775 3776] and others can emulate up to 1,000 qubits[3777]. Others are designed to specifically simulate the defects of qubit types[3778].

The lower a quantum emulator supports the quantum simulation of its underlying qubits, the closer it resembles a qubit physical circuit EDA (Electronic Design Automation) software tool, like the ones proposed by Cadence and Siemens/Mentor Graphics. Solid state qubit EDA are still poorly integrated and many qubit designers are still relying on custom EDA tools like for this physics simulator of bosonic qubits inner dynamics[3779].

Resource estimators

Resource estimators are software tools designed to estimate the quantum computing hardware resources using as inputs a given algorithm and the various hardware characteristics. There are only a few of these tools nowadays. A **QuRE** tool was proposed in 2013 by USC, UCSB and Berkeley[3780].

3765 See Fast Simulation of High-Depth QAOA Circuits by Danylo Lykov et al, September 2023 (10 pages).

3766 See Implementation of a Stochastic Optical Quantum Circuit Simulator (SOQCS) by Javier Osc and Jiri Vala, July 2023 (25 pages).

3767 See Cloud on-demand emulation of quantum dynamics with tensor networks by Kemal Bidzhiev et al, Pasqal, February 2023 (12 pages).

3768 See Simulating Neutral Atom Quantum Systems with Tensor Network States by James Allen et al, September 2023 (14 pages).

3769 See iQuantum: A Case for Modeling and Simulation of Quantum Computing Environments by Hoa T. Nguyen et al, March 2023 (10 pages).

3770 See Tensor network algorithm for simulating experimental Gaussian boson sampling by Changhun Oh et al, June 2023 (20 pages).

3771 See On classical simulation algorithms for noisy Boson Sampling by Changhun Oh et al, January 2023 (29 pages).

3772 See Simulating Gaussian boson sampling quantum computers by Alexander S. Dellios et al, University of Melbourne, August 2023 (16 pages).

3773 See Scalable Program Implementation and Simulation of the Large-Scale Quantum Algorithm: 1024x1024 Quantum Linear Solver and Beyond by Zhao-Yun Chen et al, March 2023 (17 pages).

3774 See Implementing NChooseK on IBM Q Quantum Computer Systems by Harsh Khetawat et al, May 2019 (15 pages).

3775 See Tensor-Network Simulations of Noisy Quantum Computers by Marcel Niedermeier et al, Aalto University, April 2023 (15 pages).

3776 See Classical simulations of noisy variational quantum circuits by Enrico Fontana, Ross Duncan et al, June 2023 (13 pages).

3777 See Towards practical and massively parallel quantum computing emulation for quantum chemistry by Honghui Shang et al, March 2023 (13 pages).

3778 See Fault Simulation for Superconducting Quantum Circuits by Mingyu Huang et al, November 2022 (7 pages).

3779 See Adiabatic elimination for composite open quantum systems: Heisenberg formulation and numerical simulations by François-Marie Le Régent and Pierre Rouchon, Mar 2023 (22 pages).

3780 See QuRE: The Quantum Resource Estimator Toolbox by Martin Suchara et al, October 2013 (8 pages).

Another resource estimator was launched by Microsoft in November 2022 to estimate the qubit numbers, T gates count and execution time for a given algorithm targeting an FTQC platform[3781].

It takes as input, on top of the algorithm circuit, various QPU architecture specifications: qubit fidelities, connectivity, and the used quantum error correction code. It is related to the **rQOPS** metric, or reliable Quantum Operations Per Second which measures how many reliable operations can be executed in a second[3782].

The Microsoft resource estimator was used by Alice&Bob, showcasing the need for FTQC architectures to run commonplace algorithms[3783]. No such generic tool seems to exist for NISQ architectures, one of the reasons being the difficulty to assess the classical part of these algorithms which are usually variational, and the number of circuit shots to run to converge toward an acceptable result.

Research-originated quantum development tools

Here is an overview of the main quantum languages created to date, starting with languages that are independent of hardware architectures and that often originate from research laboratories.

They have the disadvantage that they are not generally linked to cloud quantum computer offerings. The related researchers are the equivalents of the Kernighan and Richie (creators of the C language) and Bjarne Stroustrup (creator of C++) in the quantum realm! A good number of these languages come from Europe.

- **QCL** or Quantum Computation Language has syntax and data types close to those of the C language. This language is one of the first for quantum programming, created in 1998 by **Bernhard Ömer** from the Austrian Institute of Technology in Vienna.

  It is described in a review paper which positions very well the conceptual differences between classical and quantum programming languages and methods (Figure 815).

Classical concept	Quantum analogue
classical machine model variables variable assignments classical input	hybrid quantum architecture quantum registers elementary gates quantum measurement
subroutines argument and return types local variables dynamic memory	operators quantum data types scratch registers scratch space management
boolean expressions conditional execution selection conditional loops	quantum conditions conditional operators quantum if-statement quantum forking

Figure 815: classical and quantum programming concepts. Source: Structured Quantum Programming, 2009 (130 pages).

- **Q Language** is an extension of the C++ language that provides classes for programming quantum gates (Hadamard, CNOT, SWAP, QFT for quantum Fourier transform)[3784].
- **QFC** and **QPL** are two functional languages defined by Peter Selinger, from Canada, the first one using a graphical syntax and the second one using a textual syntax[3785].

[3781] See Assessing requirements to scale to practical quantum advantage by Michael E. Beverland et al, Microsoft Research, November 2022 (41 pages), Introduction to resource estimation - Azure Quantum | Microsoft Learn and Quickstart: Submit a quantum program to the Resource Estimator - Azure Quantum | Microsoft Learn.

[3782] See Using Azure Quantum Resource Estimator for Assessing Performance of Fault Tolerant Quantum Computation by Wim van Dam et al, Microsoft, November 2023 (6 pages) which uses the Microsoft Resource Estimator to compute the resources of three arithmetic algorithms.

[3783] See Alice&Bob tests azure quantum resource estimator, highlighting the need for fault-tolerant qubits by Blaise Vignon, Alice&Bob, November 2022.

[3784] It is documented in Toward an architecture for quantum programming, 2003 (23 pages), with as co-author, Stefano Bettelli from the Laboratory of Quantum Physics of the Paul Sabatier University of Toulouse.

[3785] They are described in Towards a Quantum Programming Language, 2003 (56 pages).

- **QML** is a functional programming language created by Thorsten Altenkirch and Jonathan Grattage (UK)[3786].
- **qGCL** or Quantum Guarded Command Language was created by Paolo Zuliani of the University of Newcastle[3787].
- **ProjectQ** is a scripting language from ETH Zurich that takes the form of an open source Python framework, released on GitHub since 2016. It includes a compiler that converts quantum code into C++ language for execution in a quantum simulator with a traditional processor[3788]. Launched in early 2017, it supports IBM's quantum computers via their OpenQASM language, which is normal since ETH Zurich is a partner of the latter, as well as simulation on a traditional computer via a C++ implementation that supports up to 28 qubits (Figure 816). ProjectQ is compatible with OpenFermion from Rigetti and Google.

Figure 5: Individual stages of compiling an entangling operation for the IBM back-end. The high-level Entangle-gate is decomposed into its definition (Hadamard gate on the first qubit, followed by a sequence of controlled NOT gates on all other qubits). Then, the CNOT gates are remapped to satisfy the logical constraint that controlled NOT gates are allowed to act on one qubit only, followed by optimizing and mapping the circuit to the actual hardware.

Figure 816: ProjectQ compiler entangling gates decomposition. Source: ProjectQ: An Open Source Software Framework for Quantum Computing by Damian Steiger, Thomas Häner and Matthias Troyer, 2018 (13 pages).

- **Q.js** is a graphical quantum emulator launched in 2019, running in JavaScript and thus running in a browser[3789].
- **QuTiP** (Quantum Toolbox in Python) is another open source quantum code emulation tool developed by Paul Nation of IBM, Robert Johansson of Rakuten and Franco Nori of RIKEN (Japan) and the University of Michigan. The project started in 2011. It targets superconducting qubits.
- **QNET** is a language from Stanford University created in 2012, which allows to simulate the operation of quantum networks.
- **OpenQL** is an open-source quantum programming language created by TU Delft in 2020. It includes a high-level quantum programming language, its associated quantum compiler and a low-level assembly language, cQASM[3790].
- **Silq** is a concise and static quantum programming language proposed by a ETH Zurich team[3791].

[3786] See A functional quantum programming language, 2004 (15 pages). The principles are well described in the presentation Functional Quantum Programming, (151 slides).

[3787] See Compiling quantum programs, 2005 (39 pages).

[3788] See ProjectQ: An Open Source Software Framework for Quantum Computing by Damian Steiger, Thomas Häner and Matthias Troyer, 2018 (13 pages) which explains how the compiler optimizes the code according to the gates available in the quantum computer.

[3789] See Quantum Programming: JavaScript (Q.js) - a drag and drop circuit editor by Stewart, 2020. And https://quantumjavascript.app/.

[3790] See OpenQL : A Portable Quantum Programming Framework for Quantum Accelerators by N. Khammassi et al, 2020 (13 pages).

[3791] See Swiss scientists launch high-level quantum computing language by ETH Zurich, June 2020.

- **Quipper** is a language created in 2013 that builds on the classic [Haskell](#) language, created in 1990, to which it provides extensions in the form of data types and function libraries[3792]. It manipulates a software version of qRAM, an addressable quantum memory register, that is essential for the execution of algorithms such as Grover and QMLs (Figure 817). The language does not seem to have evolved since 2016. One of its creators is Benoît Valiron who teaches quantum programming at CentraleSupelec in France[3793].

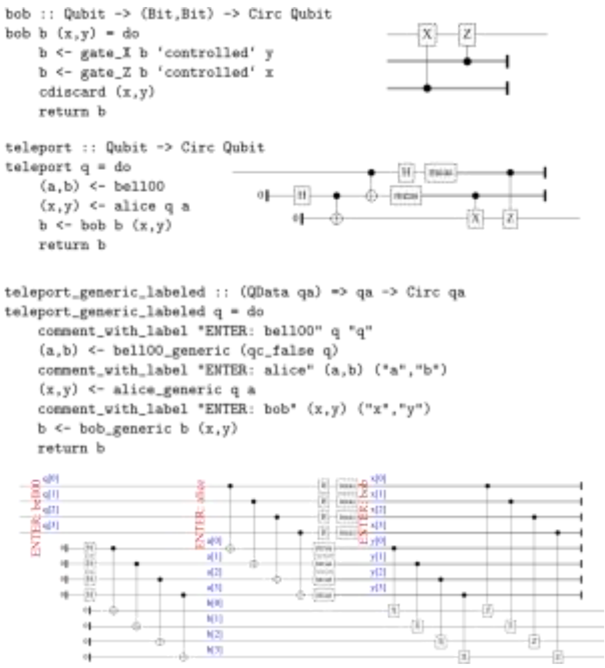

Figure 817: example of Quipper code. Source: [An Introduction to Quantum Programming in Quipper](#), 2013 (15 pages).

- **QWire** is another quantum programming language close to Quipper, launched in 2018, from the University of Pennsylvania[3794]. It is associated with a formal proof solution.

- **Qubiter** is an open-source language developed in Python that can be used on top of IBM's OpenQASM and Google's OpenFermion. It was created in 2017.

- **Scaffold** is a language developed at Princeton University[3795]. It is used to program traditional code which is then automatically transformed into quantum gates via its C2QG (Classical code to Quantum Gates) function. In particular, Scaffold can generate QASM. It can be interesting to develop oracles for search algorithms. Figure 818 contains a sample Scaffold code, almost easy to understand! Its development was also funded by IARPA.

- **Qumin** is a minimalist open-source quantum language designed in Greece in 2017[3796].

```
// Pauli X, Pauli Y, Pauli Z, Hadamard, S, and T gates
gate X(qreg input[1]);
gate Y(qreg input[1]);
gate Z(qreg input[1]);
gate H(qreg input[1]);
gate S(qreg input[1]);
gate T(qreg input[1]);

// Daggered gates
gate Tdag(qreg input[1]);
gate Sdag(qreg input[1]);

// CNOT gate defined on two 1-qubit registers
gate CNOT(qreg target[1], qreg control[1]);

// Toffoli (CCNOT) gate
gate Toffoli(qreg target[1], qreg control1[1], qreg control2[1]);

// Rotation gates
gate Rz(qreg target[1], float angle);        //Arbitrary Rotation

// Controlled rotation
gate controlledRz(qreg target[1], qubit control[1], float angle);

// One-qubit measurement gates
gate measZ(qreg input[1], bit data);
gate measX(qreg input[1], bit data);

//One-qubit prepare gates: initializes to 0
gate prepZ(qreg input[1]);
gate prepX(qreg input[1]);

//Fredkin (controlled swap) gate
gate fredkin(qreg targ[1], qreg control1[1], qreg control2[1])
```

Figure 818: Scaffold code example. Source: [Scaffold: Quantum Programming Language](#) by Ali Javadi Abhari et al, 2012 (43 pages) page 15.

[3792] It is documented in [An Introduction to Quantum Programming in Quipper](#), *2013 (15 pages)*. Its creation was funded by IARPA.

[3793] See his presentation [Programming a Quantum Computer](#), 2017 (38 slides) and [Quantum Computation Model and Programming Paradigm](#), 2018 (67 slides).

[3794] See [QWIRE: A Core Language for Quantum Circuits](#) (13 pages) and [A core language for quantum circuits](#) by Jennifer Paykin et al, 2017 (97 slides).

[3795] See [Scaffold: Quantum Programming Language](#) by Ali Javadi Abhari et al, 2012 (43 pages).

[3796] See [Qumin, a minimalist quantum programming language](#), 2017 (34 pages).

- Quantum implementation languages of **lambda calculus**, conceptualized by Alonzo Church and Stephen Cole Kleene during the 1930s, followed. This type of computation makes it possible to solve very complex and NP-complete problems, the class of problems that can be verified in polynomial time and whose resolution requires exponential time on classical computers and potentially polynomial time on quantum computers[3797]!
- Researchers at the University of Chicago's Enabling Practical-scale Quantum Computation **(EPiQC)** laboratory proposed a compiler that can improve the speed and reliability of quantum computers by a factor of 10. Here again, the compiler has to adapt to the underlying hardware architecture[3798]. Their video explains the process. The team used Google's TensorFlow library to optimize the physical control parameters of the qubits.
- **MCBeth** is a language created at Yale University and Chicago tailored for MBQC (measurement based quantum computing) programming[3799]. It can represent, program and simulate measurement-based and cluster state computation. Its compiled code can be executed directly on MBQC hardware as well as on traditional gate based QPUs. This language is based on the initial work by Vincent Danos, Elham Kashefi and Prakash Panangaden in 2007[3800].
- **eQASM** is an intermediate quantum machine language from Delft University and its subsidiary QuTech. It sits in between high-level programming tools (QASM) and the quantum accelerator. It is a compiled language, hence the "e" for executable. The compiler manages the dependencies with hardware implementation specifics. Tests have been carried out with a 7-qubit superconducting chip (**Error! Reference source not found.**).
- **QIRO** (Quantum Intermediate Representation for Optimization) is a two-dialect language proposal to enable quantum-classical co-optimizations[3801].
- **Yao.jl** is a package for the Julia language used for creating quantum circuits.

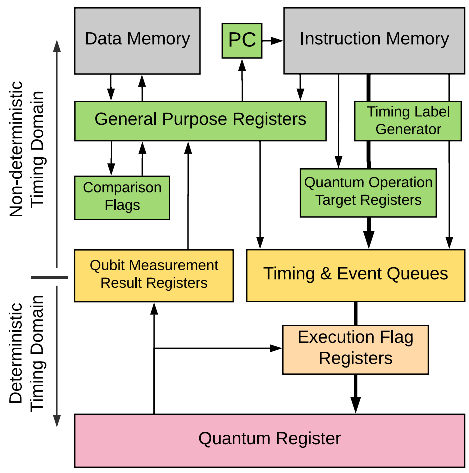

Fig. 2. Architectural state of eQASM. Arrows indicates the possible information flow. The thick arrows represent quantum operations, which read information from the modules passed through.

Figure 819: eQASM architecture, from Delft University. Source: eQASM: An Executable Quantum Instruction Set Architecture, March 2019 (14 pages).

- **Qunity** is a language created in 2022 at the Universities of Maryland and Chicago, and at AWS[3802]. Its goal is to unify quantum and classical programming concepts in a single language. Its syntax uses familiar programming constructs that can have both quantum and classical effects like summing linear operators, using exception handling syntax with projective measurements and using aliasing to induce entanglement. It can also automatically construct reversible subroutines from

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[3797] See A lambda calculus for quantum computation with classical control by Peter Selinger and Benoît Valiron, 2004 (15 pages).

[3798] See Research provides speed boost to quantum computers, April 2019.

[3799] See MCBeth: A Measurement Based Quantum Programming Language by Aidan Evans et al, April 2022 (27 pages).

[3800] See The measurement calculus by Vincent Danos, Elham Kashefi and Prakash Panangaden, 2007 (46 pages).

[3801] See Enabling Dataflow Optimization for Quantum Programs by David Ittah et al, Microsoft Research and ETH Zurich, January-August 2021 (15 pages).

[3802] See Qunity: A Unified Language for Quantum and Classical Computing by Finn Voichick et al, April 2022 (34 pages).

irreversible quantum algorithms through the uncomputation of “garbage” outputs. It can for example create full quantum oracle functions for algorithms like Grover, Deutsch-Jozsa and Simon. Qunity is still being developed and will be compiled to generate OpenQasm lower level code.

- **TWIST** is a language created in 2021 by MIT’s CSAIL lab that enforces how qubits are entangled or not, handles the notion of purity (a set of qubits not influenced by others) and enables the creation of safer programs. It introduces μQ, a small functional quantum language. Its creators plan to devise a higher-level abstraction language using TWIST[3803]. Although this project was jointly funded by IBM Research, it remains unclear whether IBM could reuse it in its quantum software toolbox.
- **ScaleQC** is a framework developed by Princeton researchers for hybrid quantum and classical computing[3804].
- **BQSKit** aka Berkeley Quantum Synthesis Toolkit is a quantum code compiler, transpiler and optimizer. Its QFactor circuit optimizer is using tensor networks[3805].

Most quantum programming software tools are open source (Figure 821, Figure 822). Their differentiation is mainly concentrated on documentation and tutorials[3806] (Figure 820). However, in practice, few commercial application developers use the languages discussed in this section. Instead, they are hooked to the languages and toolkits provided by commercial quantum computer vendors listed afterwards. They are easily locked into "full stack" approaches that are proprietary in practice although also open sourced in principle.

The most interesting thing about all this is that many development tools allow us to get our hands on small-scale quantum algorithms before scalable and usable quantum computers are available. And most of them are also open sourced and free to install and use[3807].

Some optimization tools can also be mentioned here like **CutQC** which distributes in an optimized way a large quantum circuit onto several (non-connected) QPUs and classical platforms (CPU or GPU) for co-processing[3808]. It enables using NISQ QPUs at their optimum regime, when a small number of qubits have a sufficient fidelity. Obviously, it doesn’t generate an equivalent system to the sum of the qubits of the used QPUs.

3803 See Twist: Sound Reasoning for Purity and Entanglement in Quantum Programs by Charles Yuan, Christopher McNally and Michael Carbin, January 2022 (32 pages).

3804 See ScaleQC: A Scalable Framework for Hybrid Computation on Quantum and Classical Processors by Wei Tang and Margaret Martonosi, Princeton, July 2022 (12 pages).

3805 See Berkeley Quantum Synthesis Toolkit, Lawrence Berkeley Lab.

3806 As described in Open-source software in quantum computing, by Mark Fingerhuth, Thomas Babej and Peter Wittek, December 2018 (28 pages). It makes a detailed inventory of these different tools and gauges them against classical open source software features like source code documentation.

3807 See on this subject the presentations of FOSDEM 2019 conference.

3808 See Cutting Quantum Circuits to Run on Quantum and Classical Platforms by Wei Tang and Margaret Martonosi, Princeton University, May 2022 (11 pages).

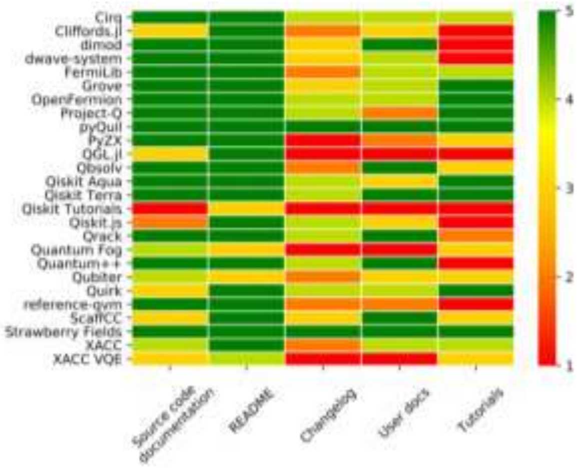


Figure 820: heatmap of various quantum coding tools and their quality figures of merits. Source: [Open-source software in quantum computing](), by Mark Fingerhuth, Thomas Babej and Peter Wittek, December 2018 (28 pages).

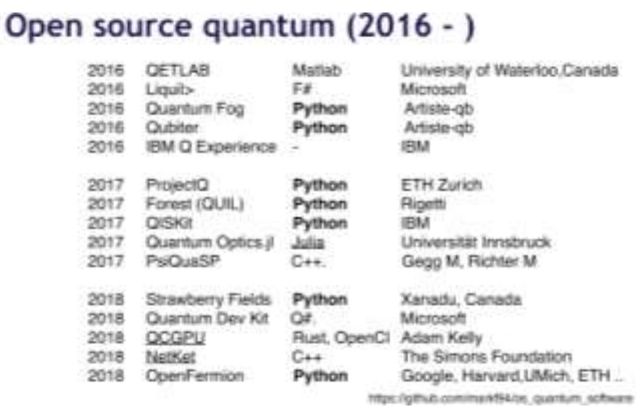


Figure 821: a summary timeline of the appearance of various quantum development tools. Source: [Quantum Software Engineering Landscapes and Horizons]() by Jianjun Zhao, July 2020-December 2021 (31 pages) which provides an excellent overview of development tools covering the entire quantum software creation cycle, including the thorny issues of debugging and testing.

Year	Language	Reference(s)	Semantics	Host Language	Paradigm
1996	Quantum Lambda Calculi	[188]	Denotational	lambda Calculus	Functional
1998	QCL	[214–217]		C	Imperative
2000	qGCL	[252, 329–331]	Operational	Pascal	Imperative
2003	λ_q	[295, 296]	Operational	Lambda Calculus	Functional
2003	Q language	[31, 32]		C++	Imperative
2004	QFC (QPL)	[256–258]	Denotational	Flowchart syntax (Textual syntax)	Functional
2005	QPAlg	[143, 164]		Process calculus	Other
2005	QML	[10, 11, 115]	Denotational	Syntax similar to Haskell	Functional
2004	CQP	[103–105]	Operational	Process calculus	Other
2005	cQPL	[187]	Denotational		Functional
2006	LanQ	[196–199]	Operational	C	Imperative
2008	NDQJava	[312]		Java	Imperative
2009	Cove	[237]		C#	Imperative
2011	QuECT	[47]		Java	Circuit
2012	Scaffold	[1, 140]		C (C++)	Imperative
2013	QuaFL	[166]		Haskell	Functional
2013	Quipper	[116, 117]	Operational	Haskell	Functional
2013	Chisel-Q	[182]		Scala	Imperative, functional
2014	LIQUi|⟩	[306]	Denotational	F#	Functional
2015	Proto-Quipper	[244, 247]		Haskell	Functional
2016	QASM	[220]		Assembly language	Imperative
2016	FJQuantum	[81]		Feather-weight Java	Imperative
2016	ProjectQ	[124, 279, 285]		Python	Imperative, functional
2016	pyQuil (Quil)	[272]		Python	Imperative
2017	Forest	[61, 272]		Python	Declarative
2017	OpenQASM	[66]		Assembly language	Imperative
2017	qPCF	[222, 224]		Lambda calculus	Functional
2017	QWIRE	[225]	Denotational	Coq proof assistant	Circuit
2017	cQASM	[149]		Assembly language	Imperative
2017	Qiskit	[4, 242]		Python	Imperative, functional
2018	IQu	[223]		Idealized Algol	Imperative
2018	Strawberry Fields	[150, 151]		Python	Imperative, functional
2018	Blackbird	[150, 151]		Python	Imperative, functional
2018	QuantumOptics.jl	[161]		Julia	Imperative
2018	Cirq	[284]		Python	Imperative, functional
2018	Q#	[282]		C#	Imperative
2018	$Q\|SI\rangle$	[181]		.Net language	Imperative
2020	Silq	[34]	Operational		Imperative, functional
2020	Quingo	[286]		Python	Imperative

Figure 822: a timeline of quantum programming tools.
. Source: [Quantum Software Engineering Landscapes and Horizons]() by Jianjun Zhao, July 2020-December 2021 (31 pages).

Quantum vendors development tools

Even before general-purpose quantum computers are operational on an exploitable scale, the software platforms battle has already begun. The major quantum computing players have almost all adopted an end-to-end vertical integration approach from quantum processors to development tools. This is particularly the case at IBM, Microsoft, Rigetti and D-Wave. This is well illustrated in the chart in Figure 823, which also describes the main development environments for quantum applications from Rigetti and IBM.

The vertical offering of the above-mentioned vendors often integrates a low-level quantum language, then a higher-level language similar to the macro-assembler of traditional computers, then an open sourced framework that can be most often used in Python with ready-to-use functions, a development environment, possibly a quantum gates graphical coding tool, and often some access to their cloud based quantum accelerators and simulators.

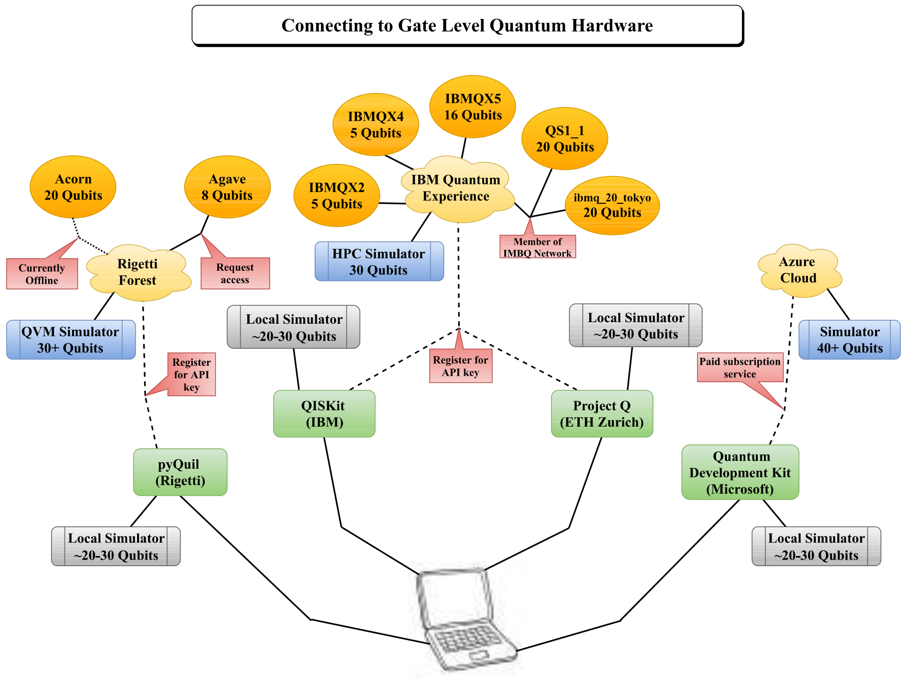


Figure 823: Source: Overview and Comparison of Gate Level Quantum Software Platforms by Ryan LaRose, March 2019 (24 pages).

One remaining tool to invent would be a higher level of abstraction tool to free developers from understanding the intricacies of quantum gates and interferences. Most recent supposed "higher level" languages are classical gate-based programming tools.

Another consolidation of these proprietary - although also open sourced - quantum software development platforms is shown in Figure 824.

	IBM	rigetti	D-Wave	Xanadu	Google	Microsoft	aws	EVIDEN
visual programming and integrated development environments	Quantum Experience	Forest	OCEAN		Quantum Playground	Visual Studio		
thematic quantum libraries (chemistry, finance, machine learning, …)	QisKit Aqua	OpenFermion	Ising model QUBO	PennyLane	OpenFermion	Quantum Chemisty PNNL	PennyLane	Qaptiva
generic quantum libraries / full-stack	QisKit	Grove QAOA	BQM	Strawberry Fields	Cirq	Quantum Developer Kit	Braket SDK	pyAQASM
high level machine language (quantum circuits)	QisKit Terra	PyQuil	QMASM			Q#		AQASM
low level machine language	Open QASM	QUIL Quil-T	QMI	Blackbird	many machine languages		rigetti IONQ OQC	Qiskit Cirq, …
qubits and quantum gates	super-conducting	super-conducting	quantum annealing	qumodes photons, GBS	super-conducting	topologic, IonQ, Quantinuum		any

Figure 824: the various software stacks from large quantum vendors. (cc) Olivier Ezratty, 2020-2023. Based on a schema found in Quantum Computing languages landscape by Alba Cervera-Lierta of the Quantum World Association, September 2018.

When all software tools are open source, it is not anymore a differentiating factor, or it is when you look at the fine print. Is the open-source software controlled by the vendor or by an independent third party?

Are all software tools really in open source or just the lower layers with additional proprietary layers? Who are the main contributors to the open-source tool? What is the exact open-source license used?

D-Wave

D-Wave proposes a complete range of software tools that have evolved a lot since its creation[3809]. The latest iteration of D-Wave's software platform is called **Ocean**. It includes low- and high-level building blocks for the development of quantum applications[3810] (Figure 825).

Their lowest level language is **QMI**, a sort of machine language for defining the links between the qubits and prepare the related Hamiltonian. QMI is usable from C, C++ Python and even Matlab, via the SAPI (Solver API) interface. Problems to be solved on D-Wave annealers are formulated as BQM, for binary quadratic models.

QUBO (Quadratic Unconstrained Binary Optimization) and Ising model are converted by Ocean into BQM models to be processed by D-Wave annealers or even a classical computer in digital annealing mode.

Developers can also use the open source **QMASM** (Quantum Macro Assembler) language, which is a low-level language suitable for programming on a D-Wave annealer. It is a third-party tool coming from a D-Wave partner. QMASM is used to describe a Hamiltonian made of coupler-based qubit relationships. This method has a drawback: it is preferable to initialize the system in a state close to the search solution and this state can only be determined by classical calculations. It is in any case a very different programming model from the universal quantum gate model, even if there is a theoretical equivalence between quantum annealing and gate-based models. QMASM is also part of **Quadrant**, a comprehensive platform for the development of D-Wave's cloud-based solutions for machine learning launched by D-Wave in 2018[3811].

The D-Wave Ocean SDK also includes **Hybrid**, an open-source framework for creating hybrid algorithms, usually, the most efficient solution to solve a problem with a D-Wave annealer. We can add third party tools such as **QSage**, an optimization problems framework and **ToQ**, another framework for solving constraint satisfaction problems, as well as the SDK from **1Qbit**.

As of spring 2021, D-Wave, its partners and customers had prototyped over 250 algorithms and solutions. They have not necessarily generated any definite quantum advantage, but they do allow customers to learn quantum programming.

[3809] D-Wave software offering was centered for a while around QBSolv that handled an intermediate QUBO representation for problems to be sent on the QPU using their low level QMI instruction set. Il was discontinued at the end of 2022 and replaced by dwave-hybrid, an hybrid classical/annealing framework iterating sets of samples through samplers to solve arbitrary-sized QUBOs. It includes Kerberos, a reference example sampler that finds best samples by running in parallel classical tabu search, classical simulated annealing and D-Wave subproblem sampling.

[3810] D-Wave provides a very good document describing the problems that can be solved with their computers: D-Wave Problem - Solving Handbook, October 2018 (114 pages).

[3811] See D-Wave Announces Quadrant Machine Learning Business Unit, May 2018.

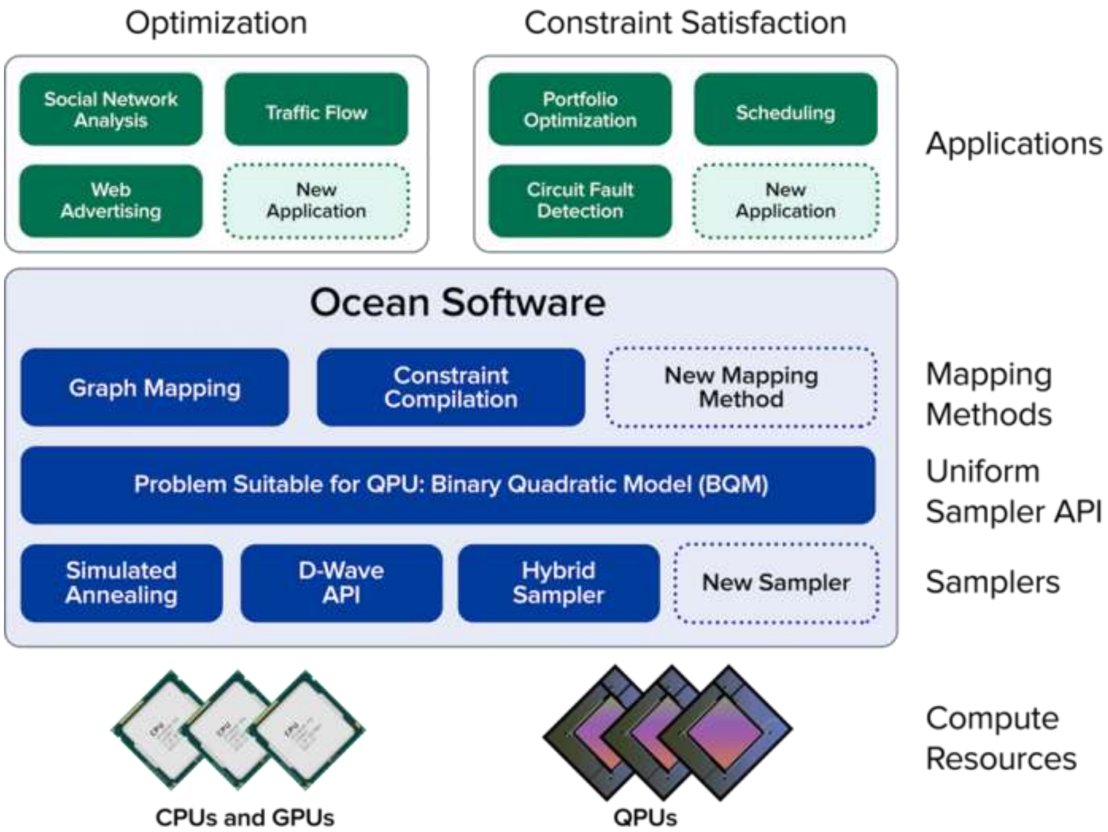


Figure 825: D-Wave's software architecture components around the Ocean platform. Source: D-Wave.

D-Wave's offering is mainly offered as a cloud-based resource, under the name **Leap**.

Leap V2 was launched in February 2020[3812].

It includes a new hybrid solver service that can handle optimization problems with up to 10,000 variables and a new interactive development environment using Python.

Prices ranged from $335 to $3,000 per month for access to 10 to 90 minutes of quantum computing time (Figure 826). A 12-month access costs $70K.

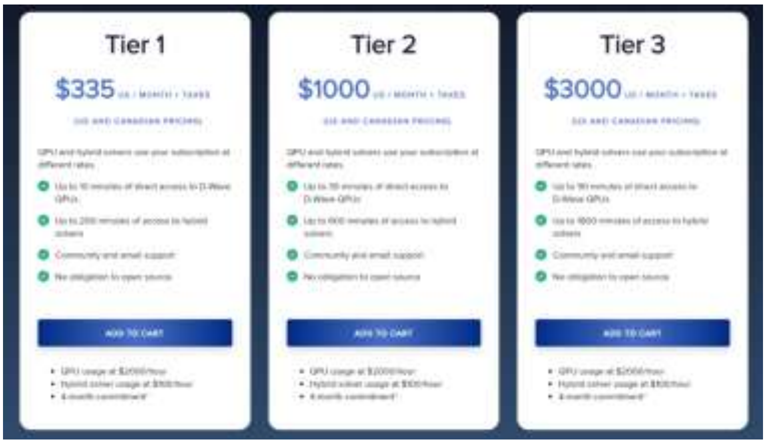


Figure 826: D-Wave's Leap pricing as of 2021. It has probably changed since then, given D-Wave is also available through intermediate vendors and intermediates depending on the country.

[3812] See D-Wave announces Leap2, its cloud service for quantum computing applications by Emil Protalinski, February 2020.

IBM

IBM's quantum software development platform is built around Qiskit and OpenQASM.

OpenQASM is an open-source programming language introduced in 2017 that complements IBM's online graphical programming tool Q Experience Composer[3813]. The current version of OpenQASM is v3 and was codeveloped with AWS and the University of Sussex in the UK[3814].

It added support for arbitrary control flow, calling external classical functions, a description of quantum circuits at multiple levels of specificity, and extensions to drive gates timing, modifiers and even pulse control (Figure 827).

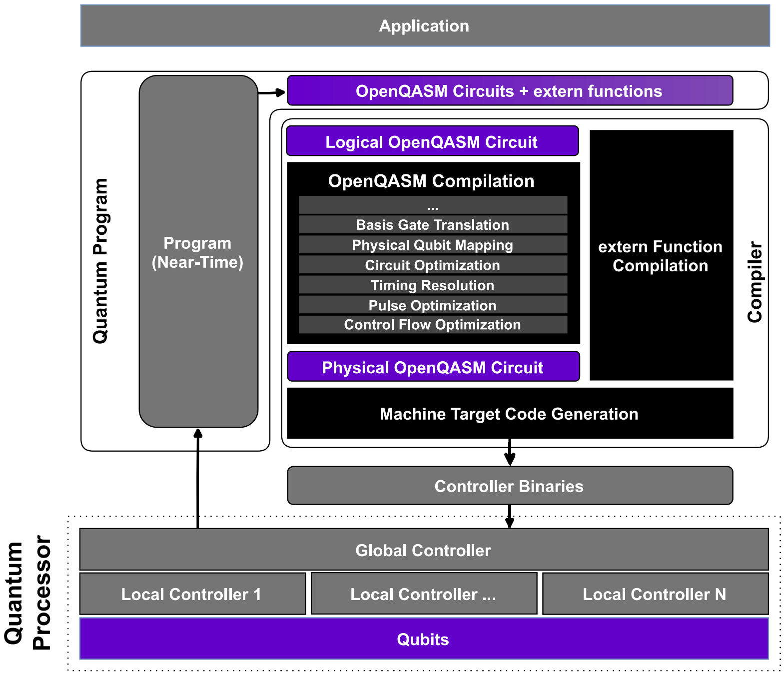


Figure 827: IBM software architecture. Source: IBM.

Qiskit is a high-level scripting library associated with OpenQASM. It can be used with Python, JavaScript and Swift (a general-purpose language from Apple) and on Windows, Linux and MacOS. It was launched in early 2017 and is also published in open source. It is of course associated with a compiler targeting both classical emulation and IBM QPU (Figure 828).

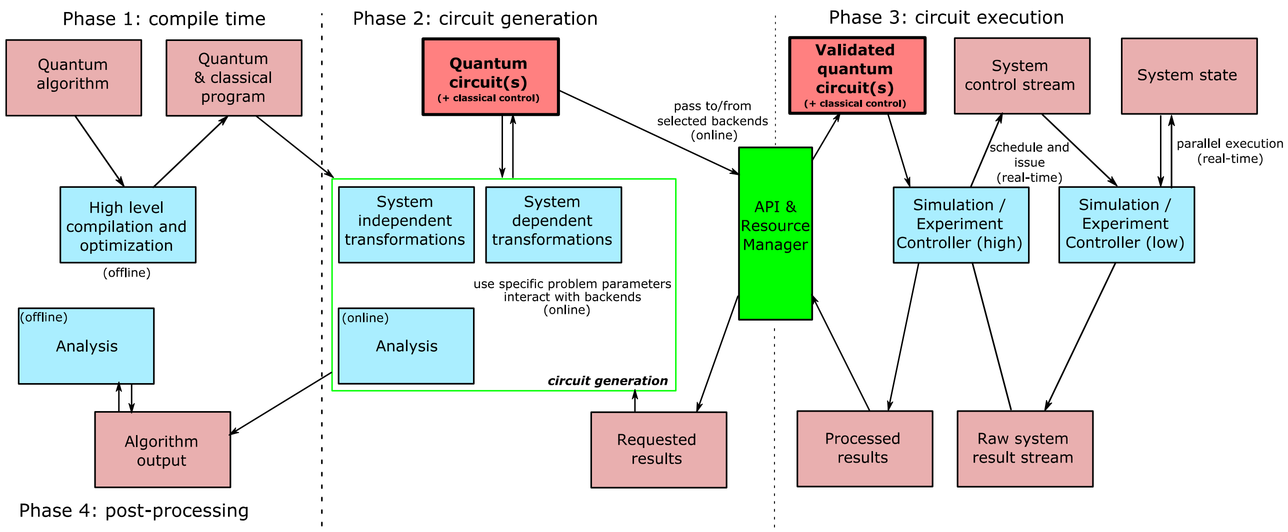


Figure 828: Qiskit block-diagram of processes (blue) and abstractions (red) to transform and execute a quantum algorithm. Source: Open Quantum Assembly Language, 2017 (24 pages).

[3813] It is specified in Open Quantum Assembly Language, 2017 (24 pages), this document describing the many tasks performed by the associated compiler.

[3814] See OpenQASM 3: A broader and deeper quantum assembly language by Andrew W. Cross, Jay Gambetta et al, March 2022 (60 pages).

Qiskit comes with numerous templates and sample codes to exploit a wide range of known quantum algorithms. These can be found in Qiskit documentation and Qiskit textbook on qiskit.org. It includes a graphical circuit-drawing function that generates a graphical visualization of quantum circuits using the open source document composition language LaTeX. Qiskit is or will be supported by other quantum computers vendors such as **IonQ** (USA) and **AQT** (Austria), both with trapped ions qubits, and **ColdQuanta** (USA) with cold atoms.

Qiskit is organized with modules around software building blocks (Figure 829):

- **Qiskit Terra** provides the circuit building and optimization functionalities and manages execution on the different backends like IBM's Qiskit Aer quantum simulator, and QPUs devices from various hardware providers including of course IBM.
- **Qiskit Aqua** is where use cases applications stood. It was deprecated to increase the library portfolio, with more specialized modules like Qiskit Finance, Qiskit Optimization, Qiskit Machine Learning and Qiskit Nature (for chemistry and materials science).
- **Qiskit Metal** is an open source EECAD (Electronic and Electrical Computer-Aided Design) used to design custom superconducting qubits chips and simulate their behavior and performance. It was launched in March 2021.
- **Qiskit Cold Atoms** supports quantum simulation models with fermionic modes and spins[3815].
- **Bosonic Qiskit** is a third-party extension which simulate bosonic qubits at the physical level, either purely photon based or in the quantum electrodynamic field, like with cat-qubits [3816].

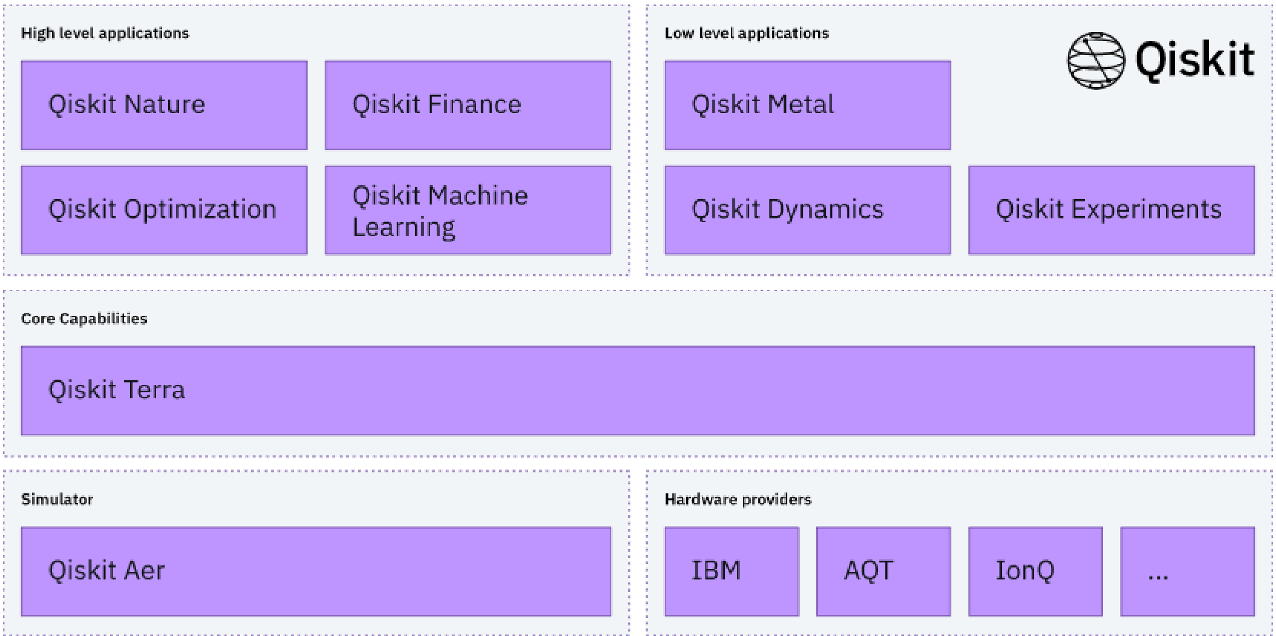


Figure 829. Qiskit components, source qiskit.org. 2022.

Quantum code compilation takes place either on IBM's classic cloud based HPC simulator or on a single quantum computer such as those from IBM that are available in the cloud with 5 and 7 qubits (free access), followed by 16, 27, 65 and 127 qubit versions (charged access) launched between 2019 and 2021.

[3815] See You Can Use Qiskit to Control Cold Atom Systems, May 2022.

[3816] See Bosonic Qiskit by Timothy J Stavenger, Eleanor Crane, Kevin Smith, Christopher T Kang, Steven M Girvin and Nathan Wiebe, DoE PNNL, NIST, Yale University, University of Toronto and University of Washington, September 2022 (8 pages).

The graphical **IBM Quantum Composer**, shown in Figure 830, is used to create quantum code graphically online and run it on a quantum emulator or on the various IBM quantum systems available online.

It allows you to interact indifferently with the text code on the right or with its graphical version in the middle. It shows vector states after running the code. The IBM qasm simulator emulates up to 32 qubits, including measurement pseudo-randomness, and also supports noise models injection. Circuits can be executed from 1 to 8,192 shot(s). The local simulator included in Qiskit does not have this limitation but usually does not run efficiently above circa 15 qubits algorithms.

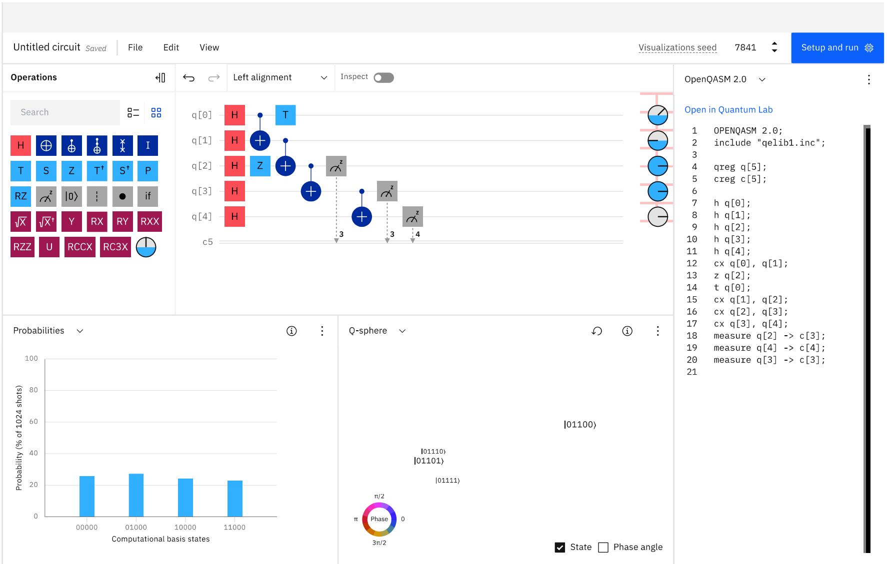


Figure 830: IBM Quantum Composer, the graphical tool to design your quantum circuit, interacting with the QASM scripted version on the left. Source: IBM Quantum Experience. October 2023.

Qiskit Aer also provides access to a 32 qubits state vector simulator, as well as 63, 100, 5,000 qubit simulators, with some restrictions on the quantum gate sets. Since the batches are submitted one after the other, on the 9 open devices, and 400,000+ registered users, one can wait up a long time, more than one hour for your code to be executed[3817].

Over the years since 2016, IBM has been building a user community, not only within the IBM Quantum Network with over 180 various partners, industry participants, startups and universities accessing premium devices and support, but also with a broad public, particularly students.

Beyond free access to the quantum devices and simulators, IBM organizes Quantum Challenges, public and free Quantum Global Summer Schools in August since 2020 with two weeks of lectures and workshops, as an online and worldwide event, provides online Qiskit documentation in many languages including Japanese, Spanish, German and French, and online textbooks for learning quantum computing, a Qiskit channel on YouTube with learning content and weekly “seminar series” featuring scientists and technologists of this field. Setting the tone, a mobile application (“Hello Quantum”) was launched for learning and playing with qubits and gates, as shown in Figure 831.

In February 2021, IBM complemented its hardware roadmap announced in September 2020 with a five-year software roadmap[3818]. Its main item was **Qiskit Runtime** bringing a 120x improvement on the time needed to run variational algorithms (such as VQE, which uses a classical optimizer iteratively with a call to the quantum processor until an exit condition is reached). With Qiskit Runtime both parts are run in the cloud, within the same job submission, avoiding returning to the queue for each iteration.

[3817] There’s a legal caveat in the tool terms of use: *"You may not use IBM Q in any application or situation where failure could lead to death or serious bodily injury of any person, or to severe physical or environmental damage, such as aircraft, motor vehicles or mass transport, nuclear or chemical facilities, life support or medical equipment, or weaponry systems"*. Given that with the few qubits offered, you can wonder how you could risk doing any of these nasty things.

[3818] See [IBM’s roadmap for building an open quantum software ecosystem]() by Karl Wehden, Ismael Faro and Jay Gambetta, February 2021.

This x120 improvement can be broken down as follows: x4 from Qiskit itself, then x1.8 for the algorithm, x1.5 for system software, x4.2 with electronics control systems enabling faster readouts and, at last, x2.8 thanks to device fidelities. The rest of the announcement covered the willingness to address vertical markets with partners. They plan to package off-the-shelf libraries for natural science, optimization, machine learning, and finance with partners like **Strangeworks**.

Figure 831: Hello Quantum mobile app. Source: IBM.

IBM launched in March 2021 a certification program and test for Qiskit developers based on 60-question exam running on the Pearson VUE electronic testing solution[3819]. This is a typical tactic used to build technical communities. It was implemented a long time ago by Novell (Certified Novell Engineers) and Microsoft (Certified Professionals and Most Valuable Professionals programs).

Other software announcements were made in May 2022[3820]. They cover further improvements with Qiskit Runtime thanks to using "dynamic circuits" enabling a reduction of circuit depth and adding "threads", allowing the control of parallelized quantum processors (Figure 832).

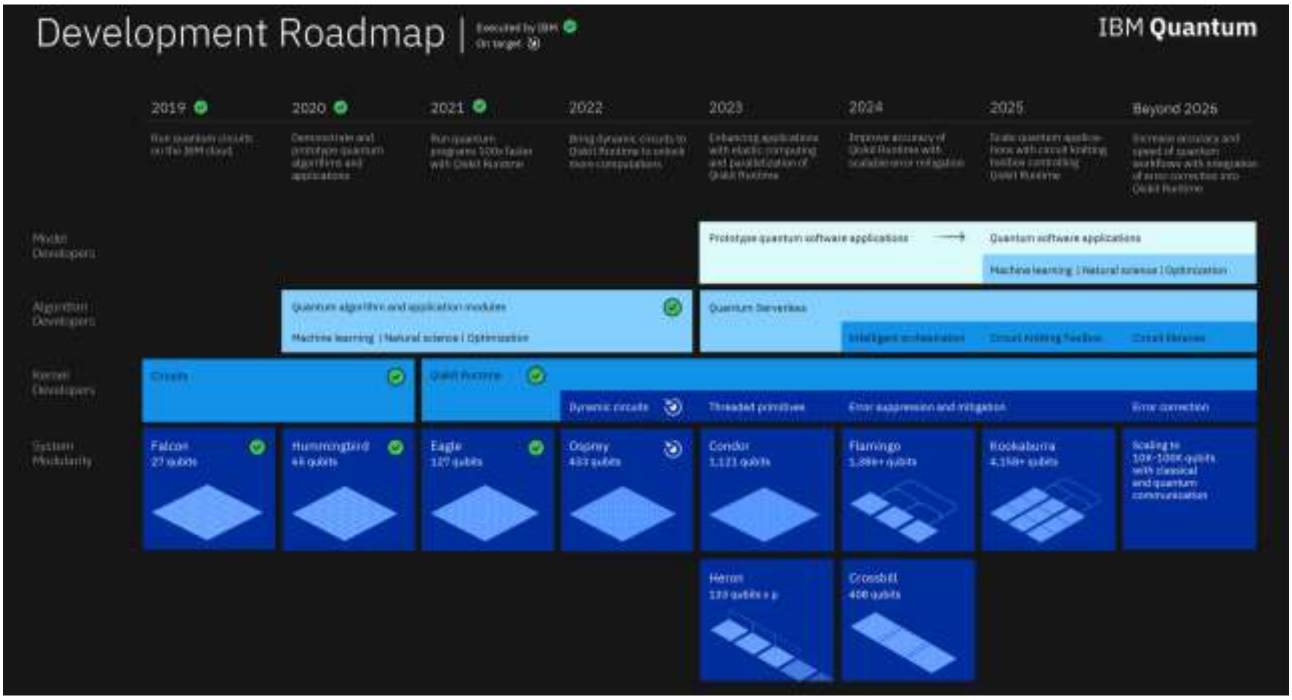


Figure 832: IBM software and hardware roadmap as of May 2022. Source: Expanding the IBM Quantum roadmap to anticipate the future of quantum-centric supercomputing by Jay Gambetta, May 2022.

IBM is to introduce "Quantum Serverless" in 2023 which transparently allocates classical and quantum processor resources to the developer. It will also enable "circuit knitting" and "entanglement forging", their techniques distributing large quantum circuits onto smaller circuits and reconstructing the results with consolidating their respected results. One of these techniques is "entanglement forging".

[3819] See IBM offers quantum industry's first developer certification by Abe Asfaw, Kallie Ferguson, and James Weaver, IBM, March 2021.

[3820] See Expanding the IBM Quantum roadmap to anticipate the future of quantum-centric supercomputing by Jay Gambetta, May 2022.

It is used to double the size of the quantum systems we could address with the same number of qubits, but it was used just in a particular case for the simulation of a single water molecule on 5 qubits[3821]. And it works only for weakly entangled states, those who do not bring a real exponential speedup!

In 2024 and 2025, IBM will introduce error mitigation and suppression techniques. IBM will also improve Qiskit Runtime Service's primitives and process distribution across classical and quantum processors.

Rigetti

Rigetti proposes an integrated software development platform with the low-level language **Quil** that supports a mixed classical and quantum memory model[3822]. It runs on Windows, Linux and MacOS. The language uses the gates class to describe operations to be performed on qubits, indexed from 0 to n-1, for n qubits and with quantum gates.

The language allows you to create conditional programming based on the qubits state. It is completed by the open-source library **pyQuil** launched in 2017 which includes the Grove library of basic quantum algorithms (documentation). It can be used with the Python programming language (Figure 833). The high level pyQuil (assembler) generates the low-level Quil language (machine code). Figure 834 contains a simple example with a single qubit activated by a Hadamard gate that creates a superposition of |0⟩ and |1⟩ to create a quantum random number generator.

pyQuil generates Quil

```
from pyquil.gates import  X, CNOT, H, Z, RX, I
from pyquil.api import QVMConnection
from pyquil.quil import Program
import numpy as np

qvm = QVMConnection()

alice_register = 0
ancilla_register = 1

flip_correction_branch = Program(X(1))
phase_correction_branch = Program(Z(1))

prog = (Program()
        .inst(H(0))
        .inst(CNOT(0, 1))
        .inst(RX(0.2 * np.pi, 2))
        .inst(CNOT(2, 0))
        .inst(H(2))
        .measure(0, alice_register)
        .measure(2, ancilla_register)
        .if_then(alice_register, flip_correction_branch)
        .if_then(ancilla_register, phase_correction_branch))

qvm.run_and_measure(prog, list(prog.get_qubits()), trials=10)
```

```
H 0
CNOT 0 1
RX(pi/5) 2
CNOT 2 0
H 2
MEASURE 0 [0]
MEASURE 2 [1]
JUMP-WHEN @THEN1 [0]
JUMP @END2
LABEL @THEN1
X 1
LABEL @END2
JUMP-WHEN @THEN5 [1]
JUMP @END6
LABEL @THEN5
Z 1
LABEL @END6
```

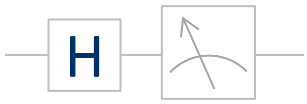
Figure 833: pyQuil language example and the lower level Quil language generated on the right. Source: Quantum Cloud Computing by Johannes Otterbach, Rigetti, January 2018 (105 slides).

Used iteratively in a classical loop, the program can generate a random series of 0s and 1s with a 50% chance of having either one allowing to create a completely random single binary code (Figure 834).

```
from pyquil.gates import H

# The Hadamard produces what is called a superposition state
coin_program = Program(H(0))
wavefunction = quantum_simulator.wavefunction(coin_program)

print("H|0> = ", wavefunction)
print("With outcome probabilities\n", wavefunction.get_outcome_probs())
```



Hadamard gate on qubit 1

```
H|0> =  (0.7071067812+0j)|0> + (0.7071067812+0j)|1>
With outcome probabilities
{'0': 0.4999999999999989, '1': 0.4999999999999989}
```

readout output

Figure 834: Hadamard gate programmed with pyQuil.
Source: pyQuil Documentation Release 1.9.1.dev0, Rigetti, July 2018 (116 pages).

[3821] See Scientists double the size of quantum simulations with entanglement forging by Robert Davis, IBM, January 2022 and Doubling the Size of Quantum Simulators by Entanglement Forging by Andrew Eddins, Sarah Sheldon et al, January 2022 (15 pages).

[3822] It is documented in A Practical Quantum Instruction Set Architecture, 2017 (15 pages).

Rigetti offers the execution of quantum programs in its cloud systems and on conventional simulators via its QVMs, for **Quantum Virtual Machines**[3823]. Since 2020, it is also available on Amazon Braket cloud services. It is usable from the **Forest** development environment proposed by Rigetti. These tools are open source, but not cross-platform (Figure 835).

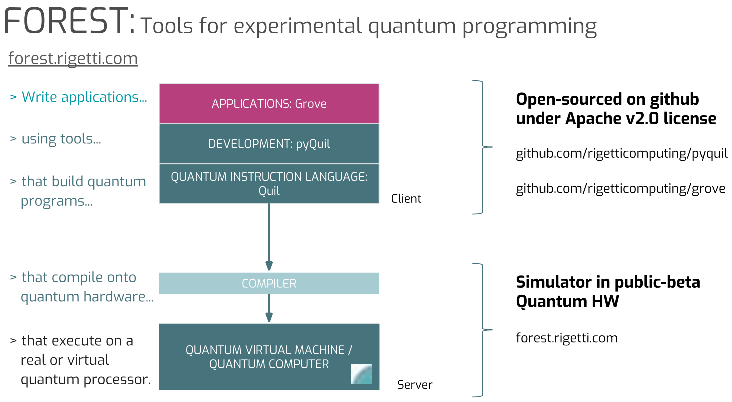


Figure 835: Rigetti Forest software platform. Source: Quantum Cloud Computing by Johannes Otterbach, Rigetti, January 2018 (105 pages).

At the end of 2017, Google and Rigetti launched the open-source initiative **OpenFermion**.

This framework developed in Python exploits research work from the Universities of Delft and Leiden in the Netherlands. It is a software solution for the creation of quantum algorithms for the simulation of chemical functions supporting any quantum computer, from Universal Quantum Computers to D-Wave annealers.

It complements Atos[3824]. In 2018, Rigetti finally launched a Quantum Algorithm Contest with a $1M prize, but with an interesting bias, comparing the creators of quantum algorithms with others seeking to create equivalents running on conventional computers.

The process could last 3 to 5 years and looks like the XPrize process[3825].

At last, Rigetti is also promoting **Quantum Programming Studio**, a web based interactive programming tool that can run your code on a Rigetti quantum computer in the cloud.

Google

In addition to OpenFermion which is a high-level framework, Google launched on July 19, 2018, its own quantum framework **Cirq**, of course also in open source. It is a Python framework.

Since Google's superconducting Sycamore systems are not available on the cloud, Cirq is mainly used on a cloud simulator provided by Google[3826]. A tool for compiling OpenFermion code in Cirq is also proposed. It also supports IonQ trapped ions qubits that are supported in Google Cloud since 2021. It supports also Pasqal cold atoms systems (Figure 836) and Rigetti superconducting qubits. As of August 2023, the current version of Cirq was v1.2.0 released in July 2023 which contains Cirq-FT, a new Fault-Tolerant algorithms package for rapid prototyping and resource estimation.

In March 2020, Google launched **TensorFlow Quantum**, an extension of the famous open source machine and deep learning framework. It provides hybrid classical/quantum computing functions for machine learning[3827]. Of course, the library supports Cirq.

[3823] This is documented in pyQuil Documentation, June 2018 (120 pages) which contains many code examples like the one *above*.

[3824] See the announcement in October 2017, OpenFermion: The Electronic Structure Package for Quantum Computers, 2018 (19 pages) and Openfermion documentation.

[3825] See Can You Make A Quantum Computer Live Up To The Hype? Then Rigetti Computing Has $1 Million For You by Alex Knapp, Forbes, October 2018.

[3826] See explanations in Google Cirq and the New World of Quantum Programming by Jesus Rodriguez, July 2018.

[3827] See TensorFlow Quantum: A Software Framework for Quantum Machine Learning by M Broughton et al, 2020 (39 pages, and associated video).

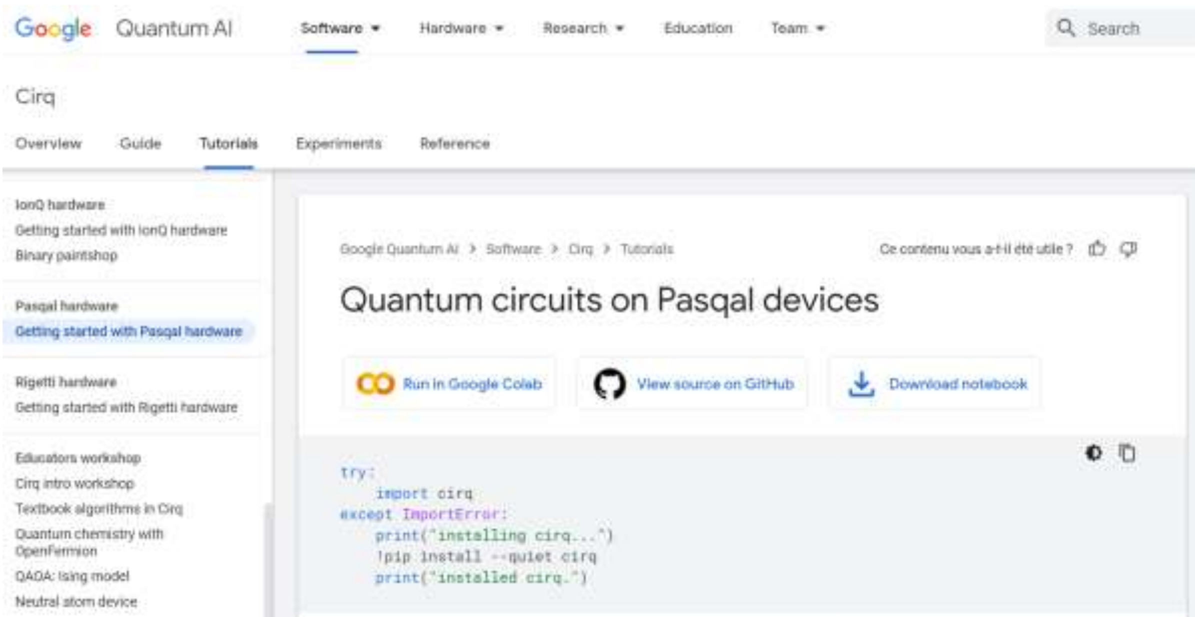


Figure 836: Cirq supports Pasqal cold atoms computer circuits. Source: Google Cirq tutorials.

It is adapted to quantum simulators running on classical computers based on CPUs, GPUs and TPUs (Tensor Processing Unit, the specialized AI processors running in Google's data centers) (Figure 837).

Eventually, QPUs (Quantum Processing Units) will be supported. Why doesn't Google use its 53-qubit quantum computer? Because it is a research object and not yet a production tool that could be integrated at this stage in a cloud offer. Meanwhile, you can access IonQ trapped ions in Google's cloud!

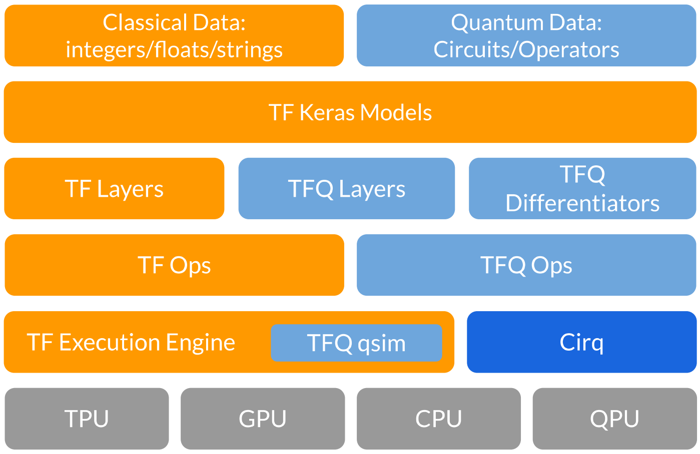


Figure 837: Google's hybrid quantum classical software architecture. Source: TensorFlow Quantum: A Software Framework for Quantum Machine Learning by M Broughton et al, 2020 (39 pages).

Microsoft

Microsoft's quantum efforts are proposed under the umbrella of Azure Quantum. It embeds various efforts ranging from fundamental hardware research to commercial offerings. Research covers Majorana fermions. Commercial offerings contain a wealth of software development solutions and a cloud offering hosting emulation software and third-party quantum hardware vendors QPUs.

Let's try to decompose the Microsoft quantum platform.

Q# language is their cornerstone open-source quantum language[3828] given Azure Quantum also supports Qiskit and Circ frameworks.

[3828] There's a long history behind Q#. Microsoft's first forays in quantum software developments started with the **LIQUi|>** extension of the **F#** scripting language which allowed to simulate quantum programs. In December 2017 was launched **Q#**, using a syntax derived from Microsoft's C# language. See Q#: Enabling scalable quantum computing and development with a high-level domain-specific language, 2018 (11 pages). It involves Alain Sarlette, Anthony Leverrier, Eric Fleury, Hélène Robak and Laurent Massoulie from Inria and Nicolas Delfosse from Microsoft Research.

Development is usually done with VScode, a lightweight and extensible integrated development environment. It is using Python as a scripting language which calls various modules developed in various languages including Python.

Quantum Development Kit is a software kit for Q# launched in 2017 that supports the development cycle and execute quantum code either on quantum hardware or on quantum simulators, all supporting Jupyter Notebooks. The QDK can run on your local computer and run with your preferred interactive development environment (Visual Studio or another). It contains several libraries supporting Hamiltonian (gate-based) simulations, amplitude amplification (for Grover search), phase estimation, arithmetic and quantum error correction codes. A new version of the QDK was released in September 2023 that was rewritten in RUST and is 100x faster. Its emulator is supposed to run 1000s of circuit shots per second for many common algorithms on a powerful laptop[3829].

Quantum Katas is an open-source project launched in July 2018 that contains examples of quantum Q# code integrated into interactive tutorials[3830]. In December 2018, Microsoft introduced a chemical simulation library co-developed with Pacific Northwest National Labs, an equivalent of OpenFermion, which is co-developed by Rigetti and Google[3831]. The library complements PNNL's NWChem quantum chemistry simulation software package. A new version of Quantum Katas was released in September 2023.

QIR (Quantum Intermediate Representation) is an intermediate representation for quantum programs launched in September 2020, serving as a layer between gate-based quantum programming languages like Q# and target quantum computers. It is based on **LLVM** open-source intermediate language that was created in 2000 at the University of Illinois and is handled by the LLVM Foundation run by Tanya Lattner. It can also be used to run code on an emulator. The support is done with a compiler extension supporting that QIR with Q# but it can support other languages and frameworks like Qiskit. It is used by Azure Quantum to support the various hardware platforms it serves (IonQ, Honeywell, QCI). But it seems, not yet by any other vendor. It became the **QIR Alliance** in November 2021, to promote the adoption of QIR. It is now part of the Linux Foundation. The Alliance founding members are Honeywell, Microsoft, the DoE Oak Ridge National Laboratory, Quantum Circuits Inc. and Rigetti Computing[3832].

Quantum simulation is proposed in several fashions, with full state simulation (limited to 30 qubits, operating the full quantum state vector), sparse simulation (sparse matrices and quantum states), a Toffoli simulator limited to X, CNOT and multicontrol X operations) and a noise simulator. They also have a resources estimator that computes the quantum resources necessary to run some quantum code (number of qubits, gates, CNOT, T and R gates, measurements, code depth).

Azure Quantum | Quantum coding with Copilot in Azure Quantum was launched in 2023. It is an interactive web UX to emulate your quantum code.

Quantum hardware access is provided in Azure Quantum with an ever increasing breath of QPUs, starting with IonQ, Quantinuum, Rigetti and Pasqal. Access pricing is calculated at the quantum gate level with a different formula for each hardware. A single qubit gate costs $0.00003 and two-qubit gates costing $0.0003 on IonQ systems.

3829 See Introducing the Azure Quantum Development Kit Preview by Bill Ticehurst, Microsoft, September 2023.

3830 See Learn at your own pace with Microsoft Quantum Katas, July 2018.

3831 See Simulating nature with the new Microsoft Quantum Development Kit chemistry library, December 2018. PNNL is a research laboratory co-funded by the US Department of Energy and operated by the non-profit foundation Battelle Memorial Institute. Battelle operates numerous US laboratories such as Lawrence Livermore National Laboratory, Los Alamos National Laboratory and Oak Ridge National Laboratory.

3832 See New Quantum Intermediate Representation Alliance Serves as Common Interface for Quantum Computing Development, November 2021.

Quantum Inspired software models are also proposed in Azure Quantum. They are implemented classically. It contains various software libraries: Parallel Tempering (sort of digital annealing), Simulated Annealing (stochastic simulation method that mimics annealing), Population Annealing (walker simulation), Quantum Monte Carlo (a quantum-inspired optimization) and Tabu Search (an heuristic search algorithm used to solve QUBO optimization problems).

Azure Quantum Elements is another tool launched in 2023 to improve chemistry and materials science research like for screening candidates, study mechanisms, and design molecules and materials. It is based on classical implementations of quantum chemistry.

All that is summarized below in Figure 838 in an outdated slide from 2021. Microsoft has a lot to improve in it marketing of the Azure Quantum Platform.

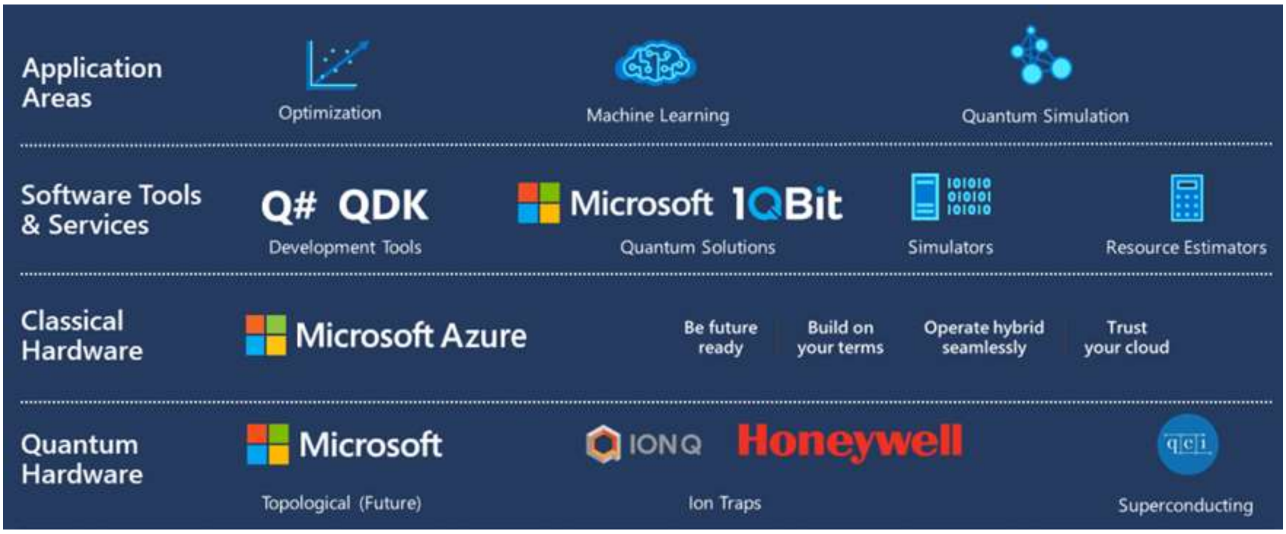


Figure 838: Microsoft Azure Quantum overview. Since then, some new hardware vendors have been added or announced like Rigetti and Pasqal. QCI that is in this slide was announced in 2019 but never delivered a functional QPU. Source: Microsoft, 2021.

Amazon

Amazon's quantum software offering is organized in their Braket platform. It contains both a custom hardware independent development framework as well as the PennyLane framework from Xanadu, and all the tools to submit quantum code to the various AWS supported systems (IonQ, Rigetti and OQC) as well as their own classical computing emulators for testing and learning purpose.

IonQ

Like Rigetti, IonQ also has its own "full stack" software offering adapted to their trapped ions quantum computer architecture and proposed in the cloud. It is also offered in Amazon and Microsoft's quantum cloud services.

Intel

At this stage, Intel is not very advanced in the development of quantum software. They have created a quantum emulation software for classical computers, IQS, the first two authors working at Intel and the last one at Harvard. It can simulate up to forty qubits[3833]. In 2023, Intel released its Quantum SDK 1.0. containing a classical emulator supporting C++ language[3834]. It seems they did that to attract

[3833] Documented in qHiPSTER: The Quantum High Performance Software Testing Environment by Mikhail Smelyanskiy, Nicolas Sawaya, and Alán Aspuru-Guzik, 2016 (9 pages).

[3834] See Quantum Researchers Use Intel Quantum SDK to Explore Complex Problem Solving by Scott Bair, February 2023.

serious developers, Python being not serious enough, although being used by mostly all machine learning and deep learning developers using frameworks like TensorFlow and PyTorch.

Huawei

At the end of 2018, Huawei launched its own quantum application development framework, compatible with ProjectQ, and including a graphical interface for algorithm creation. All this is integrated into their HiQ cloud-based quantum emulation service [3835]. It is provided free of charge for simulating up to 38 qubits. It can also simulate up to 81 qubits with a processing depth of 30 and 169 qubits with a computing depth of 20.

Eviden

Eviden (a branch of Atos to become independent late 2023) is not a manufacturer of quantum computers. In May 2023, they launched Qaptiva the new name for its quantum emulation and computing platform. It adopts a full-stack approach with emulation and execution tools targeted at the main quantum computing paradigms (gate-based, annealing, simulations). The platform contains emulators, compilers, libraries and various connectors to program, optimize, compile, emulate code on classical computers and execute code on QPUs. Their emulation hardware appliance becomes the Qaptiva 800, replacing what was formerly known as the Atos QLM. It simulates 30 to 40 qubits in state vector mode and up to 20 qubits in density matrix mode, to model quantum noise. depending on the configuration. Since July 2020, they have a version using Nvidia GPGPUs. In November, the emulation capacity of Qaptiva 800 was extended to over 100 qubits and with a tensor-network based emulator for NISQ algorithms.

The software platform welcomes third party solutions coming, as a starter, from ColibrITD, QuantFi, QubitSoft, Qubit Pharmaceuticals, QuRISK and Multiverse Computing. On the QPU side, Qaptiva will support IQM, Quandela and Pasqal systems.

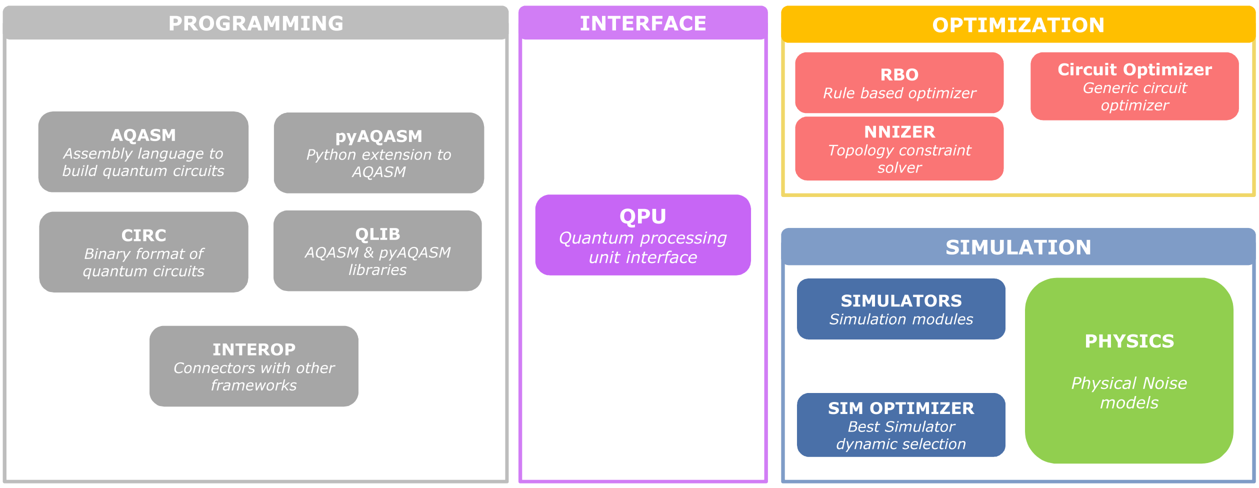


Figure 839: Eviden/Atos software platform around pyAQSM.

Their lower layers contain aQASM (Atos Quantum Assembly Programming Language), a programming language that complements Python to create quantum algorithms executable on QLM emulator or on any physical quantum computer architecture with universal gates. The language allows to define quantum gates using other quantum gates, equivalents of objects, functions or macros in traditional programming[3836]. aQASM is based on the OpenQASM standard language. It is completed by the

[3835] See Huawei Unveils Quantum Computing Simulation HiQ Cloud Service Platform, October 2018.

[3836] Source of the diagram: Atos QLM, a future-proof approach to quantum computing by Christelle Piechurski, Atos, March 2018 (26 slides).

PyAQASM Python library used to generate aQUASM files (Figure 839). The language helps programing the repetitive execution of looped gates and create reusable functions.

Their compiler optimizes code and removes useless combinations of quantum gates due to the addition of many SWAP and CNOT gates in relation to qubits connectivity limitations with their Lazy Synthesis feature[3837]. They can also interoperate with Qiskit, PyQuil and Cirq. A Qiskit code can be used to drive a QLM emulation backend and be used as a QPU by a QLM appliance.

The aQASM code compiler generates CIRC binary code that is the low-level pivot language, which is then converted into the control language for specific universal quantum computers or for simulation supercomputers via the Quantum Processing Unit Interface (QPU). It is complemented by various optimization plugins that eliminate useless gates and tune the generated low-level code for the targeted quantum accelerator hardware architecture.

Cloud quantum computing

A large share of quantum computers is intended to be offered through cloud services. It is even got a specific nickname: **QCaaS** (Quantum Computing as a Service). There are various estimates for the cloud quantum computing market including a very optimistic one of $26B by 2030 by The Quantum Daily[3838].

There are a few technical and economic reasons behind the ineluctable cloudification of quantum computing access. Most quantum computers are relatively expensive devices, costing at least a couple $M. D-Wave systems are sold at a price of about $14M. These are rapidly changing devices with one generation cancelling the previous one about every year. Also, many quantum algorithms are hybrid in nature, requiring a nearby classical computer, if not an HPC or supercomputer. All this mandates some mutualization. Making a quantum computer accessible in the cloud requires putting in place a software infrastructure reminiscent of the old mainframe days. Indeed, QPUs are not “multitasking” machines. They are fed by classical computers through a queueing system in “batch mode”. The batches first compile the code, execute it several times, usually a couple thousand times (at least, in the NISQ regime), then the result is sent in asynchronous mode to the user[3839].

Work is being done to craft heterogeneous classical/quantum cloud platforms[3840] [3841] and optimize resource allocation models in distributed architectures[3842] [3843] [3844]. Another proposal for creating Quantum Data Centers deals with sharing QRAM, T gates and oracle resources[3845].

3837 See Architecture aware compilation of quantum circuits via lazy synthesis by Simon Martiel and Timothée Goubault de Brugière, December 2020 (32 pages).

3838 See Report: Quantum Computing as a Service Market to Hit $26 Billion by End of Decade by The Quantum Daily, August 2021 and Quantum Computing as a Service Market Sizing - How We Did It by The Quantum Daily, August 2021. These forecasts are fairly inconsistent with other quantum computing forecasts mentioned in this document, page 526, planning a total $2B in 2030.

3839 See Quantum Computing in the Cloud: Analyzing job and machine characteristics by Gokul Subramanian Ravi et al, University of Chicago and Super.tech, March 2022 (13 pages).

3840 See Architectural Vision for Quantum Computing in the Edge-Cloud Continuum by Alireza Furutanpey et al, TU Vienna and University of Stuttgart, May 2023 (16 pages).

3841 See Enabling Multi-threading in Heterogeneous Quantum-Classical Programming Models by Akihiro Hayashi et al, January 2023 (8 pages).

3842 See Service Differentiation and Fair Sharing in Distributed Quantum Computing by Claudio Cicconetti et al, January 2023 (30 pages).

3843 See Hypergraphic partitioning of quantum circuits for distributed quantum computing by Waldemir Cambiucci et al, January-February 2023 (14 pages).

3844 See Stochastic Qubit Resource Allocation for Quantum Cloud Computing by Rakpong Kaewpuang et al, October 2022 (8 pages).

3845 See Quantum Data Center: Perspectives by Junyu Liu and Liang Jiang, University of Chicago, September 2023 (9 pages).

There are even proposals to enable the simultaneous execution of multiple circuits from different users on the same QPU, which of course, bears its own security issues[3846].

In the FTQC regime, things might get complicated as some algorithms will have very long execution times, potentially in the month domain. Cloud resource allocation will require specific flexible architectures like with the distribution of quantum processing on several QPUs connected through entanglement resource sharing using photonic links. This distribution will have to be dynamic with flexible routing like on the Internet so that virtual QPU provisioning can be done on an ad-hoc basis.

Cloud quantum solutions usually also provide access to emulation solutions. This can be found at IBM, Microsoft, Google, Alibaba and, Huawei to name a few. If the cloud is to run a hybrid algorithm, it must also provision classical datacenters or HPC resources and synchronize their availability with the related QPU (and right now, we're using only one QPU at a time). It will require its own software stacks and middleware[3847].

When would customers prefer to purchase a QPU and run it on-premises in their own datacenters? It would make sense when economies of scale are large within the customer or if he handles very sensitive data and processes. The first on-premises installations will probably be deployed for military and intelligence use cases.

The first company to launch a quantum cloud offering was **IBM**, which started to make its first QPUs online in 2016. It now proposes cloud access to over 24 QPUs with 1 to 433 superconducting qubits as of September 2023. Their QPUs with fewer than 7 qubits are accessible for free (Figure 840).

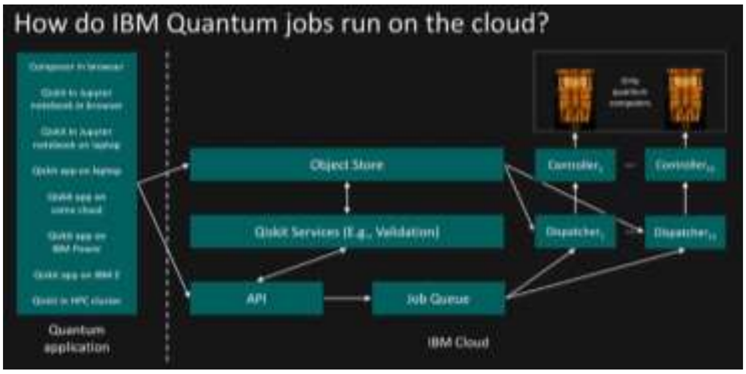


Figure 840: how IBM is running a quantum job in the cloud. Source: IBM.

D-Wave launched its Leap cloud solution in 2018 with its quantum annealers.

Rigetti followed with its Cloud Services launched in 2019.

Alibaba has a similar offer in China . We are here in the context of vertically integrated offers, the operator of the cloud service being the designer of the quantum computers.

Quantum cloud offering also started to be proposed in 2020 by cloud vendors selling access to third-party quantum computers, mixed with quantum emulation resources on conventional servers. This is what Amazon and Microsoft announced almost simultaneously at the end of 2019 and made available in 2020. Google followed-on in 2021.

Microsoft announced in November 2019 that it was integrating a quantum computing offering into the Azure cloud, and relying on **IonQ** and **Honeywell** (trapped ions qubits) as well as **QCI** (superconducting qubits). As of spring 2022, no QCI QPU was available online, seemingly since their qubits are not yet operational[3848].

[3846] See Enabling Multi-programming Mechanism for Quantum Computing in the NISQ Era by Siyuan Niu and Aida Todri-Sanial, February 2021-February 2023 (34 pages).

[3847] See A Conceptual Architecture for a Quantum-HPC Middleware by Nishant Saurabh et al, Utrecht University, Rutgers University, Brookhaven National Lab, Ludwig Maximilian University Munich and BMW, August 2023 (12 pages).

[3848] Microsoft Azure Quantum was introduced step by step: announced in December 2019, released in limited preview in May 2020 and then in public preview in February 2021.

Rigetti (superconducting qubits) was added to Azure Quantum in December 2021 (in "private previews" as of March 2022) and **Pasqal**'s quantum simulator addition was announced in May 2022. Microsoft is associated among others with **1QBit** (Canada) to propose quantum software application layers[3849].

They promote quantum inspired algorithms that rely on traditional cloud resources, as in this case study of MRI scanner optimization at Case Western Reserve University[3850]. Microsoft and **KPMG** announced a partnership related to the development of quantum inspired algorithms in December 2022.

Microsoft Azure Quantum also supports Qiskit and Cirq Python-based quantum code since October 2021. You can test Azure Quantum for free with a credit of $500 and even win a credit of $10K after submitting your applications. Microsoft is also selling the access to **Toshiba**'s Simulated Quantum Bifurcation Machine+ (SQBM+), a classical Ising model solver using quantum inspired models[3851]. In March 2023, Microsoft launched its hybrid quantum computing platform starting with working with Quantinuum[3852].

Amazon made its entrance in the quantum cloud market at the end of 2019 with the announcement of three components: Amazon Braket cloud services, AWS Center for Quantum Computing at Caltech University[3853], and Amazon Quantum Solutions Lab, a customer evangelization program reminiscent of IBM's Q initiative[3854]. Amazon also uses the brand Quantum Compute Cloud (QC2) for its offering. Braket is their whole quantum software infrastructure. It provides access to quantum computers from D-Wave (the 2000Q and Advantage annealers with respectively 2,048 qubits and 5,000 qubits), IonQ (it started with 11 qubits), Rigetti (16Q Aspen-4 with 16 superconducting qubits and their 31 qubits Aspen-9 version, and later, their 80 qubits system) and OQC (with 8 coaxmon qubits, in 2022). IonQ is thus proposed by both Microsoft and Amazon. Amazon announced in November 2021 that QuEra's cold atoms systems running in quantum simulation mode (*aka* "Hamiltonian simulation") would be added to AWS Braket in 2022. Late 2021, Amazon Braket Hybrid Jobs was added to their software portfolio with the capacity to run hybrid algorithms associating classical and quantum computation[3855]. Amazon also proposes software emulation of quantum algorithms on classical servers.

Amazon Braket is associated with an in-house SDK based on the classic Python language. Development is supported in the integrated open source environment Jupyter. It also includes support for the OCL (Object Constraint Language) constraint programming language.

[3849] See Experience quantum impact with Azure Quantum, November 2019 and Microsoft Announces Azure Quantum with Partners IonQ, Honeywell, QCI, and 1QBit by Doug Finke, 2019. At the same time, Microsoft also announced that it has brought together many other quantum software partners: ProteinQure, Entropica Labs, Jij, Multiverse Computing, Qu&Co, QC Ware, OTI, Qubit Engineering, Qulab, QunaSys, Rahko, Riverlane, SolidStateAI, StrangeWorks, Xanadu, Zapata Computing. See the list here : Quantum Network- A community of pioneers by Microsoft, 2019.

[3850] See How the quest for a scalable quantum computer is helping fight cancer by Jennifer Langston, July 2019.

[3851] See Toshiba launches new SQBM+ quantum-inspired optimization provider on Azure Quantum, June 2022.

[3852] See Azure Quantum unlocks the next generation of Hybrid Quantum Computing by Fabrice Frachon, Microsoft, March 2023.

[3853] The AWS Center for Quantum Computing is headed by Brazilian Fernando Brandao (1983), who is both a professor at Caltech and director of this Amazon AWS laboratory. He was previously a researcher at Microsoft Research. He is a good generalist, initially a physicist and now a specialist in quantum algorithms. In June 2020, John Preskill, also a professor at Caltech, announced that he would spend one day a week at the research center.

[3854] See Amazon Braket-Get Started with Quantum Computing by Jeff Barr, December 2019 and the presentation of the announcement Introducing Quantum Computing with AWS by Fernando Brandao and Eric Kessler (video and slides, featuring the Eiffel Tower of Rydberg atoms from French startup Pasqal in slide 15). I discovered in the hundreds of presentations at the Amazon Reinvent conference in December 2019 where this Braket announcement took place that Amazon was also presenting the QLDB, or Quantum Ledger Database, a blockchain management software component. But that doesn't seem to have anything quantum at all.

[3855] See Introducing Amazon Braket Hybrid Jobs – Set Up, Monitor, and Efficiently Run Hybrid Quantum-Classical Workloads, Amazon, November 2021. See also PennyLane on Braket + Progress Toward Fault-Tolerant Quantum Computing + Tensor Network Simulator by Jeff Barr, December 2020.

Like Microsoft, Amazon is also a partner of quantum software vendors. We find almost the same players with Xanadu, Zapata Computing , Rahko, QC Ware, 1Qbit and Qsimulate.

Google do not offer any public access to its own Sycamore quantum computers in its cloud offering. Its Sycamore QPUs are only available to a handful of academic institutions in a sort of "private cloud" access. Google still has a digital emulation offering supporting up to 40 qubits. In June 2021, it also announced the integration of IonQ's 11 qubits processor in its Cloud Marketplace. IonQ becomes de-facto the most distributed solution in the cloud with Amazon, Google and Microsoft.

In the European Union, hybrid classical-quantum offerings are launched that target the academic and industry communities. These are associating QPUs and supercomputers. The European project **HPC-QS** (for quantum simulation) is deployed in three sites: in Finland (CSC LUMI), Germany (Munich at the Jülich Supercomputing Centre) and France (Bruyères-le-Châtel at CEA, as part of the HQI initiative). It will start with attaching an Atos QLM appliance to these sites' supercomputers, themselves all running classical CPUs and GPGPUs from Nvidia.

Pasqal quantum simulators will be first installed in Jülich and at CEA. Then, in 2023, they will be completed by a European QPU. Eviden's QLM will both run quantum code emulation in the three paradigms (annealing, simulations, gates) and drive QPUs quantum code execution. In Germany and France, the first installed QPUs will be quantum simulators from Pasqal. In Finland, it will probably be superconducting qubits systems from IQM. In Germany, an IQM system will also be deployed with an Eviden QLM at the Leibniz Supercomputing-LRZ center, as part of the project Q-EXA. These hybrid computing centers will mainly serve the needs of academic researchers and large companies.

France's **OVHcloud** entered the cloud quantum space in May 2022 with announcing that its datacenter was handling the classical part of **Pasqal**'s first entry in the cloud with its 100-qubits cold atoms-based quantum simulator. This is to be extended with the support of other European QPU solutions. They also host Perceval, **Quandela**'s photon qubits classical emulator. In early 2023, OVHcloud also announced having ordered a Quandela QPU with a couple qubits to launch a quantum random number generator service. OVHcloud plan to offer these early emulators and QPU cloud services to the industry, researchers and students.

In Germany, cloud provider and web hoster **IONIS** announced in November 2022 a partnership with QMWare, the University of Stuttgart and Fraunhofer FOKUS to create the **SeQenC** cloud platform for the German industry with over 10M€ three-year project funding by the German government (Germany's Ministry of Economic Affairs and Climate Action, BMWK)[3856].

Always in the European union, **Quantum Inspire** in the Netherlands proposes an access to small superconducting and silicon spin qubit platforms and a code emulator.

In China, **Baidu** promotes its Quantum Leaf cloud offering that is provided with **Paddle Quantum**, a quantum machine learning development toolkit based on the PaddlePaddle programming language, **QCompute**, a Python-based open source SDK and **Quanlse**, a machine-level programming tools controlling the pulse sent to emulated superconducting qubits.

In May 2023, a new **China** quantum computing cloud platform was announced as a result of a collaboration between the Beijing Academy of Quantum Information Sciences (BAQIS), the Institute of Physics (IOP) under the Chinese Academy of Sciences, and Tsinghua University[3857].

[3856] See Germany to Create Its First Quantum Computing Business Cloud, November 2022.

[3857] See China Unveils New Quantum Computing Cloud Platform by Jia Liu, CAS, May 2023.

The platform that was actually launched in November 2022 to early beta testers is providing access to three superconducting qubit QPUs with respectively 10, 18 and 136 qubits[3858] [3859].

QuantumCTek is also providing some interconnect infrastructure between these three QPUs that are located in different places. No qubit fidelities or cloud access links seem to be available.

Figure 841 provides a summary view of these cloud-based quantum computing offerings, distinguishing between emulating quantum code on classical computers and executing quantum code on quantum computers and focusing on the Western world offerings.

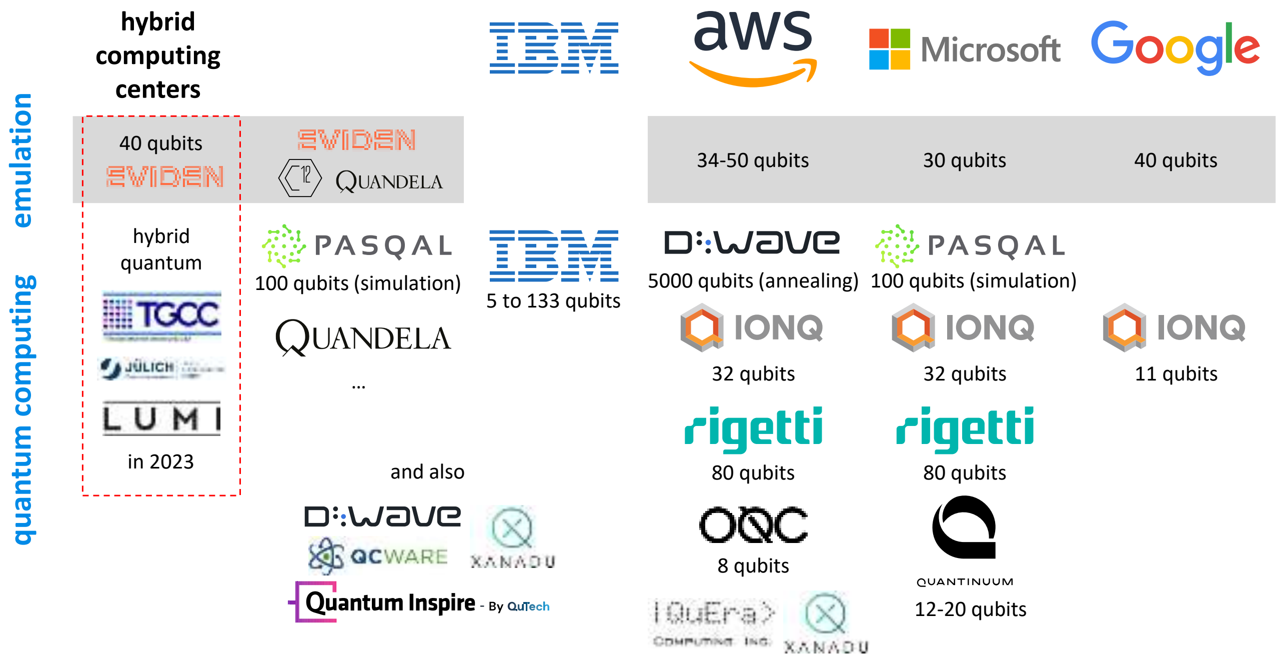


Figure 841: main quantum cloud emulation and QPU offerings worldwide. (cc) Olivier Ezratty, 2021-2024.

Quantum software engineering

Certification and verification

The verification and certification of quantum algorithms and the result of their use is an important new topic. The factorization of integer numbers is obviously easy to verify. But when a quantum algorithm is used to simulate physical interactions such as those of atoms in molecules, it is less obvious.

Theoretical work shows that it is possible to prove polynomially that a result of a quantum algorithm is accurate[3860]. Unfortunately, we cannot explain in detail the origin of the result by breaking it down. Nor can we prove that the result found, however valid it may be, is the best of all if there are several good ones[3861].

[3858] See Quafu-RL: The Cloud Quantum Computers based Quantum Reinforcement Learning by BAQIS Quafu Group, Beijing, May 2023 (14 pages).

[3859] See Quafu-Qcover: Explore Combinatorial Optimization Problems on Cloud-based Quantum Computers by BAQIS Quafu Group, Beijing, May 2023 (7 pages).

[3860] See How to Verify a Quantum Computation by Anne Broadbent, 2016 (37 pages) which demonstrates that all quantum algorithm results can be verified with classical polynomial algorithms by performing several tests and encrypting the input data. See also Verification of quantum computation: An overview of existing approaches by Alexandru Gheorghiu, Theodoros Kapourniotis and Elham Kashefi, 2018 (65 pages).

[3861] See also Quantum cloud computing with self-check by Rainer Blatt et al, May 2019, which discusses quantum simulation calculations on 20 qubits of trapped ions with results controlled on the quantum computer as fast as on the PC.

On top of that, we must make a distinction between error corrected hardware (FTQC) and noisy systems (NISQ). Surprisingly, while FTQC regimes will be inaccessible to classical hardware emulation and make verification difficult, verification is also complicated for noisy systems, particularly when they repeat some sequence of code iteratively.

At this point, quantum verification is out of feasibility, requiring either a very large overhead in qubit numbers[3862] or some quantum network connecting two QPUs[3863]. The other key point is to make sure, in the case of the use of a remote quantum computer, that the recovered result corresponds to the submitted calculation and that an intruder did not interfere in the history nor was able to alter the calculation on the quantum computer side. Many research inroads have been made in the last 15 years for that respect. One of the methods consists in relying on the concept of **blind computing** devised in 2009 by Anne Broadbent, Joseph. Fitzsimons and Elham Kashefi[3864]. Another method consists in using QPU fingerprinting to make sure that the QPU you are using is the right one[3865].

CEA LIST announced in June 2020 that it had created **QBRICKS**, an environment for the specification, programming and formal verification of quantum algorithms. They used to do this for critical embedded systems where certification by formal proof is particularly important. They are now entering the field of quantum programming and have experimented their model with QPE, the quantum phase algorithm (QPE= that fits into Shor's model for integer factorization, the full Shor algorithm and Grover's algorithm. This work involves the joint LRI laboratory at the University of Paris-Saclay and CentraleSupelec[3866].

One of the major advances in the explicability of quantum algorithms comes from researcher **Urmila Mahadev,** whose work between 2012 and 2018 has led to the creation of a method for verifying quantum computer processing. She was a postdoc at Berkeley and supported by Scott Aaronson and Umesh Vazirani, two eminent researchers in quantum algorithmic research. Her work aims at proving that a quantum computer has indeed performed the treatments it has been asked to do. She shows that a classical computer coupled to a simple quantum computer can verify in a polynomial way the results of a quantum computer[3867] . The method exploits a technique of post-quantum cryptography that the verifier cannot break (LWE: Learning With Errors). LWEs are part of the Lattice-based cryptography (EN) or Euclidean networks (FR) class[3868]. The method was recently improved by an Austria team to work with an untrusted quantum computer[3869].

[3862] See Classical Verification of Quantum Computations in Linear Time by Jiayu Zhang, Caltech, September 2022 (122 pages) that tries to optimize the computing time of classical verification. It turns is from $O(poly(\kappa)|C|^3)$ to $O(poly(\kappa)|C|)$ with κ being a security parameter and $|C|$ the size the of the verified circuit. κ can be as large as 1,000, creating a significant overhead in qubit on the server side.

[3863] See Verifying BQP Computations on Noisy Devices with Minimal Overhead by Dominik Leichtle, Luka Music, Elham Kashefi, and Harold Ollivier, PRX Quantum, October 2021 (16 pages) is an alternative without some qubit overhead but using a quantum communication between a client QPU and a server QPU.

[3864] See Universal blind quantum computation by Anne Broadbent, Joseph Fitzsimons and Elham Kashefi, 2008 (20 pages) and the associated presentation (25 slides) and A Framework for Verifiable Blind Quantum Computation by Theodoros Kapourniotis, Harold Ollivier, Elham Kashefi et al, June 2022 (33 pages) which shows a mathematical link between code verification and error correction.

[3865] See Fast Fingerprinting of Cloud-based NISQ Quantum Computers by Kaitlin N. Smith et al, Duke University and Super.tech, November 2022 (12 pages).

[3866] See Toward certified quantum programming by Sébastien Bardin, François Bobot, Valentin Perelle, Christophe Chareton and Benoît Valiron, 2018 (4 pages) and An Automated Deductive Verification Framework for Circuit-building Quantum Programs by Christophe Chareton, Sébastien Bardin, François Bobot, Valentin Perelle and Benoît Valiron, 2021 (30 pages).

[3867] See a description of the method in near-natural language in Graduate Student Solves Quantum Verification Problem, October 2018 and two reference publications: Classical Verification of Quantum Computations, September 2018 (53 pages) and Interactive Proofs For Quantum Computations, April 2017 (75 pages).

[3868] See this presentation describing the LWE protocol: An Introduction to the Learning with Errors Problem in 3 Hours (76 slides).

[3869] See Towards experimental classical verification of quantum computation by Roman Stricker et al, AQT and Innsbruck University, March 2022 (19 pages).

Other quantum programs verifiers[3870] from research laboratories include the **Path-sum** framework from the University of Waterloo[3871], **VOQC** (Verified Optimizer for Quantum Circuits) from the University of Maryland, itself based on **SQIR** (Small Quantum Intermediate Representation) supporting verification[3872] and QHL from Tsinghua University[3873].

Debugging

Like any computer software, quantum software requires a set of quality control processes. Like most human-originated creations, they are prone to bugs and errors. Some classical computing methods can be reused for this respect, but many require some adaptation to the specifics of quantum computing, whether done on gate-based systems or on analog quantum simulators.

A quantum circuit is not easy to debug! It will certainly require new debugging tools and approaches. And a majority of quantum code bugs are "quantum" in nature and not easy to spot with traditional methods[3874].

For the moment, simple circuits can be analyzed and debugged with a quantum emulator running on a classical computer, to understand how the qubit register vector state evolves step-by-step. But when quantum circuits in the advantage regime, beyond any classical emulation capacity, other means will have to be used.

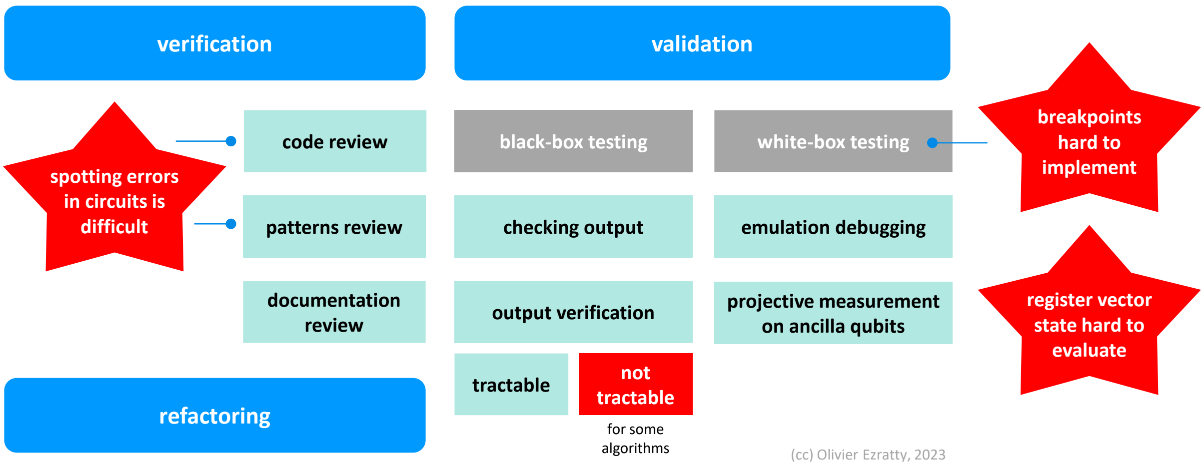


Figure 842: some of the challenges with quantum software engineering. (cc) Olivier Ezratty, 2022-2023.

Software quality control usually goes through two main steps: verification and validation (Figure 842).

Verification deals with verifying that the code will run as expected. It includes checking code documentation, designs, circuits and the various software components or patterns that are used. Verification also deals with making sure that the specifications are correctly implemented by the system. It responds to the question: are we building the product right?

[3870] See the review paper Formal Methods for Quantum Programs: A Survey by Christophe Chareton, Sebastien Bardin, Dongho Lee, Benoit Valiron, Renaud Vilmart and Zhaowei Xu, September 2021 (66 pages).

[3871] See Towards Large-scale Functional Verification of Universal Quantum Circuits by Matthew Amy, University of Waterloo, 2018 (21 pages).

[3872] See A Verified Optimizer for Quantum Circuits by Kesha Hietala et al, University of Maryland (36 pages).

[3873] See Quantum Hoare logic with classical variables by Yuan Feng et al, University of Technology Sydney, Australia, Chinese Academy of Sciences and Tsinghua University, China, April 2021 (44 pages).

[3874] As studied in A Comprehensive Study of Bug Fixes in Quantum Programs by Junjie Luo et al, January 2022 (8 pages).

In classical programming, good programmers and code reviewers can spot an error with just looking at the code. Code inspection tools can also detect undeclared variables or variables used in the wrong context.

These errors are way more difficult to detect visually on a quantum circuit, particularly with large ones. It may require the use of code decomposition in modules or patterns, like in object-oriented programming.

Validation concerns the program output and making sure it works as planned. It includes testing and validating the code against the user's needs. It responds to the question: are we building the right product?

Quantum computing results validation is usually fast like with integer factoring (it requires a simple classical multiplication) or a Grover search (it requires checking the Oracle once in a classical way).

But some circuit validations may need to be done on a quantum computer, like with a boson sampling or with a QMA prover (Quantum-Merlin-Arthur). On top of that, contrarily to classical computers, quantum computations errors can also come from hardware imperfections and the fateful quantum noise[3875]. Compilers, code optimizers and even error correction codes can also generate software bugs and amplify some errors.

Quantum software bugs can have various sources: errors in the data preparation (which is itself based on quantum gates), incorrect operations and transformations, incorrect compositions and iterations and also incorrect qubits deallocations (or "uncomputations").

During validation, testing uses the white-box and black-box approaches.

White box testing tests internal data structures and program flow and may include some interactive debugging. One solution is to decompose manually or automatically the quantum code into accumulated slices of code with its incremental different parts like data preparation, oracle execution and amplitude amplifications in the case of a Grover algorithm, as pictured in Figure 843. The debugging tool with run each accumulated slice followed by a measurement one after the other[3876]. It is not a classical pause-play like in classical interpreters like JavaScript but "pause" and "play again from the start", probably with several shots being run and their results averaged to get a sampled output.

Black-box testing looks at the functionality, ignoring the inner workings of the software, making sure the expected output is obtained with a given input? How about using interactive debugging in the white-box approach? Right now, it can be done on quantum emulators but is limited by their computing/memory capacity. It can't exceed 40 qubits and practically 16 qubits. A state vector representation is quite difficult to visualize beyond 8 qubits. On a real machine, implementing interactive breaking points in a quantum circuit is difficult due to the impact of measurement on the qubits register vector state and on the probabilistic nature of quantum computing. Let's say we'd like to implement a breaking point and line by line code execution. We'd need to run the quantum algorithm and stop it at the breaking point then make some measurements.

But good measurement, just to get a state in the computational basis would require running the code many times. And even way more times if we'd need to reconstitute the full vector state. Then, to move to the next series of gates, the circuit would have to be re-run the same number of times. And again and again and again. It would be worse if we were to check the entanglement within the register. Deciding if a register is separable is in itself an NP-hard problem. One way to proceed is to implement unit testing with splitting the code in trusted blocks and patterns.

[3875] See Formal Verification vs. Quantum Uncertainty by Robert Rand et al, University of Maryland, 2019 (12 pages) that pinpoints the role of hardware errors in quantum programs.

[3876] See A Tool For Debugging Quantum Circuits by Sara Ayman Metwalli and Rodney Van Meter, Keio University, May 2022 (11 pages).

Other debugging tools can involve projective measurements on ancilla qubits or even gentle measurement techniques[3877]. And this deals just with classic gate-based quantum computing. Analog quantum computing and special techniques like MBQC or FBQC (from PsiQuantum) will mandate specific debugging techniques and tools.

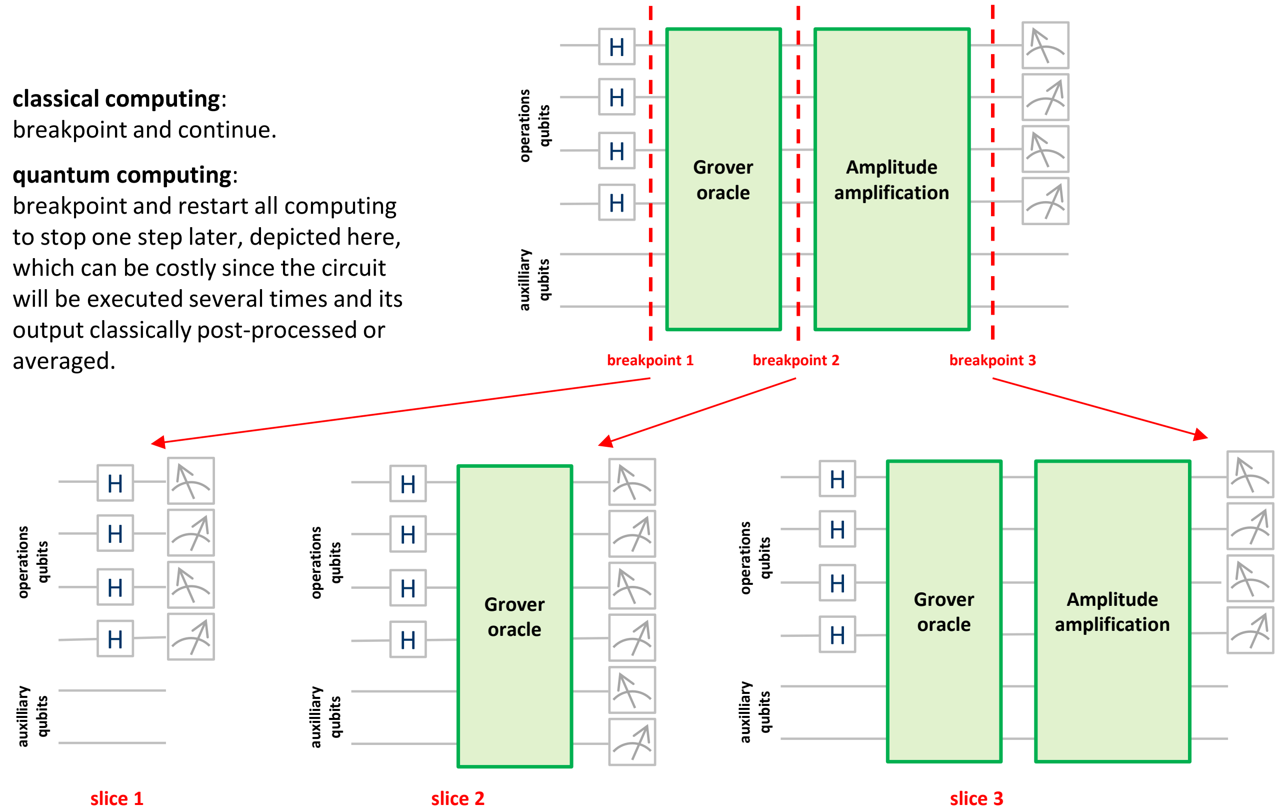


Figure 843: a quantum code debugging approach with code slicing. Source: A Tool For Debugging Quantum Circuits by Sara Ayman Metwalli and Rodney Van Meter, Keio University, May 2022 (11 pages).

Finally, let's mention that many quantum algorithms are hybrid and aggregate classical and quantum algorithms, which requires another set of discipline and tools.

Research is going on in all these dimensions around the world. These are strategic components for quantum computing[3878].

Code porting

Some developers are asking whether classical computing code could be ported to run on a quantum computer. I suspect it does not make any sense at the programming language and framework levels given the handles objects are quite different. But why not at a high abstraction level, when using libraries that could be implemented in both classical and quantum implementations. In that case, we would be close in both cases to the notion of no code programming. So, … no code porting!

Code refactoring

Code refactoring is a technique consisting in modifying a program with making it more readable, better structured, removing clutter and making it less prone to bugs.

[3877] See Debugging Quantum Processes Using Monitoring Measurements by Yangjia Li and Mingsheng Ying, 2014 (7 pages) describes the process of interim measurement process within code and Projection-Based Runtime Assertions for Testing and Debugging Quantum Programs by Gushu Li et al, 2020 (29 pages) proposes to use some ancilla qubits to indirectly detect vector state characteristics. It uses projective measurements on a different basis than the computational basis of each qubit.

[3878] It includes the study Program Verification, Debugging, and QC Simulation — EPiQC, a IARPA funded project on quantum program verification and debugging.

The technique is used in classical programming and could also be used in quantum code development. The underlying optimizations are different from the ones generated by compilers, transpilers and optimizers since it modifies the code itself instead of transforming it into a code more readily usable for the QPU itself in a one-way process[3879] (Figure 844).

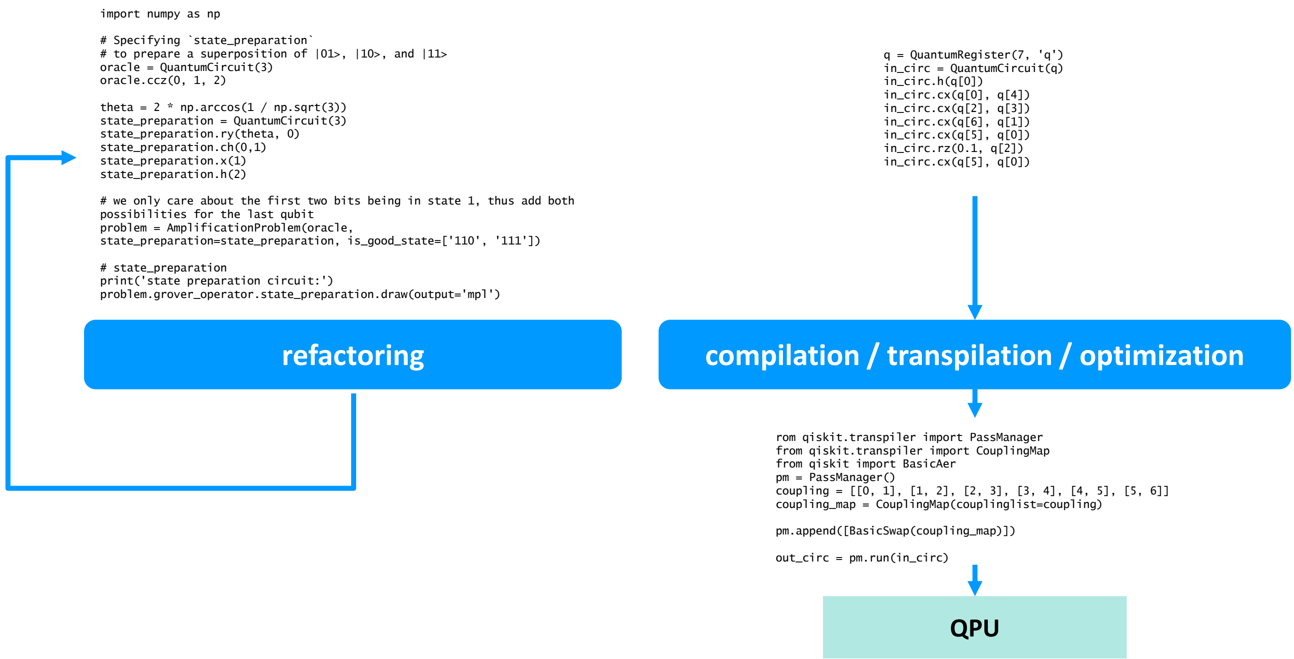


Figure 844: the difference between code refactoring and code compilation, transpilation and optimization. (cc) Olivier Ezratty, 2023.

Securing quantum computing

Securing quantum computing infrastructures is an emerging discipline. But what are the threats? They seem numerous, at least, as soon as a QPU is a shared resource in the cloud, as shown in Figure 845. They are still very theoretical. For example, side channel attacks and fault injection techniques could be physically perpetrated on a QPU with analyzing the pulse control signals send to a quantum processor and reverse engineered to capture the related quantum code[3880] [3881]. This would require some malicious capacity to hook hardware devices on the various parts of a QPU. Seemingly, these QPUs will not be easy to access in cloud infrastructures.

Spying could be perpetrated in a much simpler way by capturing the quantum code itself while being transmitted to the cloud operator. Other attacks can involve some malicious untrusted compiler or malicious attacks of QML models[3882] [3883] [3884] [3885].

[3879] See On Refactoring Quantum Programs by Jianjun Zhao, [Submitted on 18 Jun 2023].

[3880] See Exploration of Quantum Computer Power Side-Channels by Chuanqi Xu, Ferhat Erata and Jakub Szefer, April 2023 (17 pages).

[3881] See Classification of Quantum Computer Fault Injection Attacks by Chuanqi Xu et al, September 2023 (7 pages).

[3882] See Exploring the Vulnerabilities of Machine Learning and Quantum Machine Learning to Adversarial Attacks using a Malware Dataset: A Comparative Analysis by Mst Shapna Akter et al, Kennesaw State University, USA, May 2023 (10 pages).

[3883] See TrojanNet: Detecting Trojans in Quantum Circuits using Machine Learning by Subrata Das and Swaroop Ghosh, June 2023 (9 pages).

[3884] See QDoor: Exploiting Approximate Synthesis for Backdoor Attacks in Quantum Neural Networks by Cheng Chu et al, July 2023 (9 pages).

[3885] See Quantum Neural Networks under Depolarization Noise: Exploring White-Box Attacks and Defenses by David Winderl et al, November 2023 (9 pages).

Other weird techniques could allow fingerprinting cloud QPUs and enable their identification for subsequent malicious usage[3886]. Spying techniques could try to measure the value of qubits that would be improperly reset after a shot run by another customer, or when several shots are mutualized in a single QPU[3887].

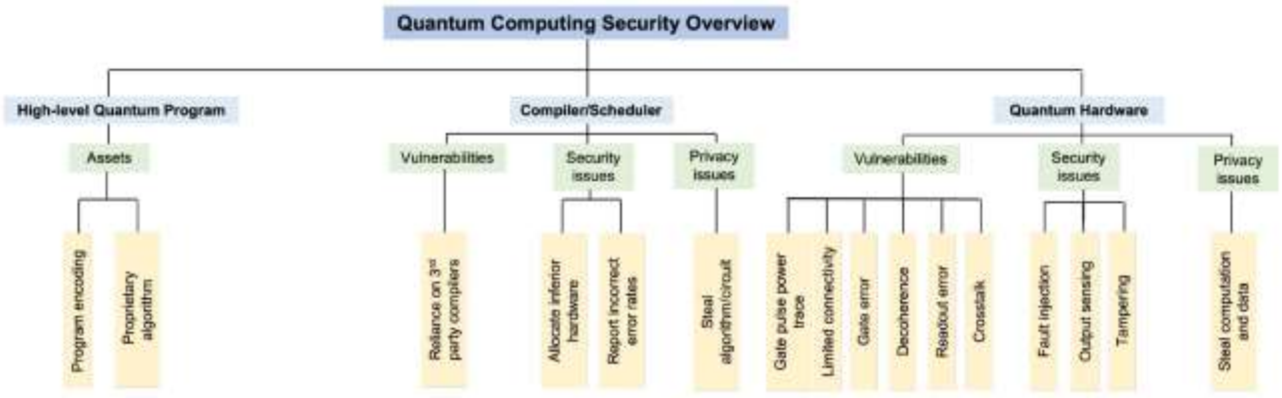


Figure 845: an overview of various threats affecting the security of your quantum code. Source: A Primer on Security of Quantum Computing by Swaroop Ghosh et al, May 2023 (19 pages).

Other concerns exist on the way to secure quantum machine learning algorithms against data-based attacks[3888] [3889] including adversarial attacks[3890] [3891].

Malicious attacks could even potentially happen on your code in compilers, particularly when these are used through various third party services. Thus, proposals like code obfuscation techniques using random circuits and their uncomputation[3892] [3893] and even related to pulse controls[3894].

These are fundamental research attacks which may not be practical at all. One key defense can rely on quantum blind computing as we'll see later in the quantum telecom part of this book.

[3886] See Short Paper: Device- and Locality-Specific Fingerprinting of Shared NISQ Quantum Computers by Allen Mi, February 2022 (5 pages).

[3887] See Securing Reset Operations in NISQ Quantum Computers by Allen Mi et al, 2022 (15 pages).

[3888] See QuMoS: A Framework for Preserving Security of Quantum Machine Learning Model by Zhepeng Wang et al, April 2023 (8 pages).

[3889] See QTrojan: A Circuit Backdoor Against Quantum Neural Networks by Cheng Chu et al, Indiana University, February 2023 (5 pages).

[3890] See the perspective Towards quantum enhanced adversarial robustness in machine learning by Maxwell T. West et al, Nature Machine Intelligence and arXiv, May 2023 (10 pages), described in From self-driving cars to military surveillance: Quantum computing can help secure the future of AI systems by Muhammad Usman, The Conversation, May 2023.

[3891] See Adversarial attacks on hybrid classical-quantum Deep Learning models for Histopathological Cancer Detection by Biswaraj Baral et al, Qausal AI, Clemson University, Mississippi State University and SVQC, September 2023(7 pages).

[3892] See Randomized Reversible Gate-Based Obfuscation for Secured Compilation of Quantum Circuit by Subrata Das and Swaroop Ghosh, May 2023 (11 pages).

[3893] See Toward Privacy in Quantum Program Execution On Untrusted Quantum Cloud Computing Machines for Business-sensitive Quantum Needs by Tirthak Patel et al, Rice University and Northeastern University, July 2023 (14 pages).

[3894] See Hardware Architecture for a Quantum Computer Trusted Execution Environment by Theodoros Trochatos et al, Yale, August 2023 (13 pages).

Benchmarking

Benchmarking quantum computing is becoming a strategic tool for both vendors and users. In the IT space, vendors have continuously relied on benchmarking to showcase new hardware advances and customers rely on it to assess the cost/benefit ratio of emerging technologies. Benchmarks participate to fostering innovation with driving the competition between vendors and technologies[3895].

In classical computing, raw computing power is measured in **FLOPs** (floating point operations per seconds). **Linpack** is used in the HPC TOP500 ranking and is based on a linear equation solving task. **MLPerf** is used in machine learning and for comparing GPGPUs like those from Nvidia and their competitors. Personal computers processors can be compared with **Passmark Software** benchmarks or alternatives from other vendors.

Quantum computing benchmarking can have several purposes:

- Comparing quantum computing with classical computing, which involves coupling best-in-class algorithms and software in both cases, all being moving targets given the steady progress of classical computing, particularly with so-called "domain specific architectures", the most famous one being the tensor-based GPGPUs and ASICs. This sort of benchmarking can be exploited to assess a potential quantum advantage, where a quantum solution is either solving a problem in a shorter time than classical computers or able to solve a problem that can't be solved with existing classical computers. The archetypal such benchmark is the cross-entropy benchmark (XEB) used by Google in 2019 to showcase its quantum supremacy with Sycamore[3896].
- Comparing different quantum computers competing with each other to solve particular tasks. It can be done with quantum computers using the same programming paradigm like gate-based systems (e.g. IBM's quantum volume), or even, using different paradigms (annealing, simulation, gate-based; e.g. Eviden/Atos Qscore). An objective comparison can be made with solving one or a broader set of specific problems and assessing the maximum problem size addressable by competing solutions (e.g., Eviden/Atos Qscore, QED-C and IonQ's algorithmic qubits).
- Comparing different characteristics. The most common is computing time, but other metrics will become important as well such as precision, energy spent, weight, environmental footprint and total cost (e.g. DARPA's benchmarking project RFP won by Raytheon BBN and the Quantum Energy Initiative's Green 500 benchmark proposal).
- Comparing different algorithms solving a given problem on similar or different quantum hardware. These are not yet there.

Benchmarking tools already abound and are very diverse. Following a bottom to top system approach, there are benchmarks for spare system level features (number of qubits, qubits gates and readout fidelities, connectivity, gates speed aka CLOPS with IBM, all of which are not benchmark results per se, entanglement quality, in Figure 846). These are the quantum equivalent of number of cores, CPU clock, RAM, storage size and speed and network specs in classical computing.

Then, some are benchmarking hardware capabilities at a higher level and regardless of the use case (quantum volume from IBM). At last, others are assessing the capability to solve one or several typical use cases, and usually, their maximum size (QED-C, Atos Q-score, IonQ algorithmic qubits, SupermarQ) (see Figure 847 and Figure 848). At some point, benchmarking will be even more tricky when mixing quantum and classical computers.

[3895] See The Race to Quantum Advantage Depends on Benchmarking by Matt Langione, Jean-François Bobier, Lisa Krayer, Hanl Park and Amit Kumar, February 2022.

[3896] This benchmarking technique like the quantum volume is limited by the capacities of classical emulation. One proposed way to overcome this problem is to run only Clifford gates, which reduces the resources requirements in the classical computer. See Linear Cross Entropy Benchmarking with Clifford Circuits by Jianxin Chen et al, June 2022 (30 pages).

what and whom	what	pros	cons	timing / adoption
IBM quantum volume	breath/depth computing capacity, 2^#qubits	simple qualifier of qubits quality	doesn't work in advantage regime due to emulation needs requirements	published in 2019 IBM, Quantinuum
Cisco MBQC quantum volume	computing capacity for MBQC CV photon qubits	adapted to photon qubit using a different model than circuit based models	to be adapted to direct variable photons MBQC model	proposed in 2022 by Cisco
IBM CLOPS	circuit layers operations per seconds	complements QV for speed	N/A	announced in November 2021
cycle benchmarking	qubits entanglement evaluation	useful to benchmark qubits quality	limited to one low-level feature	2019, Canada, Denmark and Austria universities
scalable benchmarks for gate-based QC	six low-level structured circuits tests	tested 21 configurations from IBM, IonQ and Rigetti	low-level benchmark not usage based	published in 2021 QuSoft, Cambridge, Caltech
PQF (photonic quality factor)	assess performance of linear optics single photons multimode QPUs	covers many NISQ photon qubit implementations	limited to a specific photonic qubit configuration	published in 2022 by Quandela
entanglement-based volumetric benchmark	estimate size of maximum entangled qubit state	entanglement is a key feature of quantum acceleration	narrow and not usage oriented	proposed in 2022 par DoE Oak Ridge et al

Figure 846: low level benchmarking proposals. (cc) Olivier Ezratty, 2022.

These tools are created by different kinds of players, sometimes working together: research laboratories (e.g. DoE Sandia Labs), software and hardware vendors (IBM, IonQ, Atos, Zapata Computing, …), industry consortium usually working with a combination of these players (USA's QED-C) and then standards bodies (e.g. IEEE). The combination of these players is important.

	what and whom	what	pros	cons	timing / adoption
multiple use cases	**scalable benchmarks for gate-based QC**	six low-level structured circuits tests	tested 21 configurations from IBM, IonQ and Rigetti	low-level benchmark not usage based	published in 2021 QuSoft, Cambridge, Caltech
multiple use cases	**QED-C supported benchmark**	set of low-level algorithms benchmarks	breadth of use cases	complicated visualization	published in 2021 QED-C, Princeton, HQS, QCI, IonQ, D-Wave, Sandia Labs
multiple use cases	**IonQ Algorithmic Qubits**	$\min(\#\text{qubits}, \sqrt{\#gates})$	run on different use cases	a bit complicated	published in 2020 and refined in 2022, IonQ
multiple use cases	**SupermarQ from Super.tech**	suite of applications benchmark	also handles error correction benchmarking		published in March 2022, Intel and Amazon
multiple use cases	**Qpack by TU Delft**	three sets of problems (Max-Cut, TSP, DSP)	measure differents metrics	Adoptions	proposed in April 2022
single use cases	**Atos Q-score**	maximum size of solvable MAXCUT problem size	application need oriented works in advantage regime hardware independant	limited to MAXCUT problems marketing & adoption	published in 2020 Atos, be be expanded to other algos
single use cases	**DoE ORNL**	chemical simulation	works on existing superconducting hardware	limited to three 2-atom molecules simulations	published in 2020 DoE
single use cases	**Zapata benchmark for fermionic quantum simulations**	one-dimensional Fermi Hubbard model (FHM) VQE running on NISQ	tested on Google Sycamore with its tunable couplers	narrow use case	proposed in March 2020

Figure 847: application level benchmarking proposals, either multiple or singe cases. (cc) Olivier Ezratty, 2022.

Industry vendors have some biased interest. They don't want to favor the adoption of benchmarks that would be unfavorable to their offerings. This is amplified by the wide spectrum and diversity of quantum computing technologies and qubits including aspects like gate-times, fidelities and topologies.

what and whom	what	pros	cons	timing / adoption
Unitary Fund Metriq	repository of benchmark results	N/A	N/A	announced in May 2022
DARPA project	SWAP (size-weight-application-power)	hardware-agnostic and resource estimates	N/A at this point	awarded in 2022 to Raytheon BBN
IEEE QC Perf Metrics & Perf Benchmarking PAR	gate-based QC benchmarking	ongoing standardization project		submission in Oct 2023 completion in Oct 2024
Quantum Energy Initiative	QC energetics benchmarking consolidated approach, QGreen500 proposal could consolidate cryogeny benchmarks	methodology (MNR) to optimize QC energetics, first analysis done with superconducting qubits	research and industry must build coordination around this goal	joint research/industry Quantum Energy Initiative launched in 2022. IEEE Working Group P3329 launched in 2023.
BACQ	application and low-level full-stack benchmarking proposal.	covers many use cases and figures of merit. Includes energetics performance.	participants are so far only from the France quantum ecosystem.	project launched in 2023 by CEA, CNRS, Thales, Teratec and LNE.

Figure 848: other benchmarks proposals. (cc) Olivier Ezratty, 2022-2023.

Some benchmarks can also be misleading if improperly extrapolated in the future, given all technologies don't have the same upscaling potential. An equivalent to Moore's law can't be applied in a simplistic way to all these technologies roadmaps. In all cases, quantum computing benchmarking will be difficult to interpret without the right technical background. You can be easily fooled by vendors given the complexity of the matter.

Quantum Volume and CLOPs

Since late 2021, IBM uses three systems-level metrics and benchmarks to characterize its quantum computers: the scale with the number of qubits, the speed measured in CLOPS (circuit layer operations per second) and qubits quality measured with their homemade quantum volume metric.

The quantum volume was introduced in 2017 and was adopted by **Honeywell** in March 2020 and afterwards by **IonQ** in October 2020, although they later tweaked it to create their own custom algorithmic qubit benchmark.

Quantum volume is an integer that associates the quantity of qubits and the number of quantum gates that can be executed consecutively with a reasonable error rate. Indeed, having N qubits but being limited by the number of quantum gates that can be used can be detrimental to the execution of many quantum algorithms. Some are greedy for quantum gates, others are not[3897].

This quantum volume number is supposed to aggregate four performance factors:

- The **number of physical qubits of** the processor.
- The **number of quantum gates** that can be chained consecutively without the error rate being detrimental to the results.
- The **connectivity between these qubits**, which will impact the length of execution of an algorithm and potentially improve quantum volume for qubits with high connectivity such as with trapped ions. It reduces the number of SWAP gates that are required when connectivity is limited.
- The **number of quantum gates** that can be executed in parallel[3898].

The QV is evaluated using a random calculation benchmark consisting of chaining random quantum gates and which must give a correct result in two thirds of the cases. Why two thirds? Because quantum computing provides a probabilistic result.

[3897] Some algorithms can thus be satisfied with a limited number of quantum gates, such as Deutsch-Jozsa's and is satisfied with only four series of quantum gates. Peter Shor's integer factoring algorithm requires a depth of quantum gates equal to the cube of the number of qubits used.

[3898] Trapped ions quantum computers can't do that with two-qubit gates.

To obtain a deterministic result, the calculation is executed several times and the average of the results is evaluated, up to thousands of times as proposed by IBM in its cloud system. With an average of two-thirds good results, one can therefore statistically converge to a good result after a few measurements. The accuracy of the result will depend on this number, which is usually a few thousand.

In the first version in 2017, the quantum volume was the square of the maximum number of qubits on which the processor could perform this calculation[3899]. The definition then was changed in 2019 to become 2 to the power of this number of qubits[3900]. The following illustration explains how the 2017 and 2019 quantum volumes are evaluated (Figure 849).

The diagram in Figure 850 from a paper by Robin Blume-Kohout and Kevin Young specifies how the m (number of qubits) and the d (computational depth) are evaluated[3901].

$$d \simeq 1/(n\epsilon_{eff})$$

d = maximum computing depth
n = number of qubits
ε_{eff} = % error rate of 2 qubits gates

$$V_Q = dn = 1/\epsilon_{eff}$$

base quantum volume = qubits # * computing depth

$$V_Q = \min(n,d)^2$$

quantum volume becomes min(n, depth)² to avoid tweaking the system with a low n=2 and a very high fidelity

$$V_Q = \max_{n' \leq n} \min\left[n', \frac{1}{n'^{\epsilon_{eff}(n')}}\right]^2$$

2017 QV : scans all combinations of n qubits below the available number of qubits, to run a random algorithm generating 2/3 good results.

$$\log_2 V_Q = \operatorname*{argmax}_m \min[m, d(m)]$$

2019 QV : QV is a power of 2 of the number of qubits

Figure 849: how is/was IBM's quantum volume calculated. (cc) Olivier Ezratty, 2021.

The number of qubits obtained to evaluate the quantum volume is much lower than the total number of qubits available: 8 for 16 in this case. The benchmark allows only 8 series of quantum gates in a row over 8 qubits, for 38 with only two qubits. In its 2017 version, the quantum volume was the grey square area containing the red lined squares.

In its 2019 version, it became $2^{8,}$ or 256 instead of 64 (8^2). In the end, it is the dimension of Hilbert's vector space, i.e., the number of different superposed states that it is able to manage from a practical point of view with a depth of computation equal to the number of corresponding qubits.

When IBM states that their 27-qubit processor has a quantum volume equal to 128, it means that they only managed to validate their benchmark with 7 qubits among these 27 qubits.

IonQ announced in 2020 a quantum volume greater than four million corresponding to a QV of 4,194,304, representing 2^{22}. So, with the ability to run 22 sets of quantum gates on 22 of their 32 qubits, with two-thirds correct results on the used random benchmark. This record seems to be related to the good connectivity of trapped ion qubits. These can all be directly entangled with each other, unlike superconducting qubits, which are at best entangled with their immediate neighbors.

This allows the benchmark to be achieved in fewer series of quantum gates than on superconducting qubits, which require a lot of SWAP gates generating rapidly accumulating errors.

[3899] See Quantum Volume by Lev Bishop, Sergey Bravyi, Andrew Cross, Jay Gambetta and John Smolin, 2017 (5 pages).

[3900] See Validating quantum computers using randomized model circuits by Andrew W. Cross et al, 2019 (12 pages).

[3901] In A volumetric framework for quantum computer benchmarks, February 2019 (24 pages), Robin Blume-Kohout and Kevin Young propose volumetric benchmarks to evaluate the performance of quantum computers based on IBM's quantum volume. The latter also proposes its own quantum volume evaluation code.

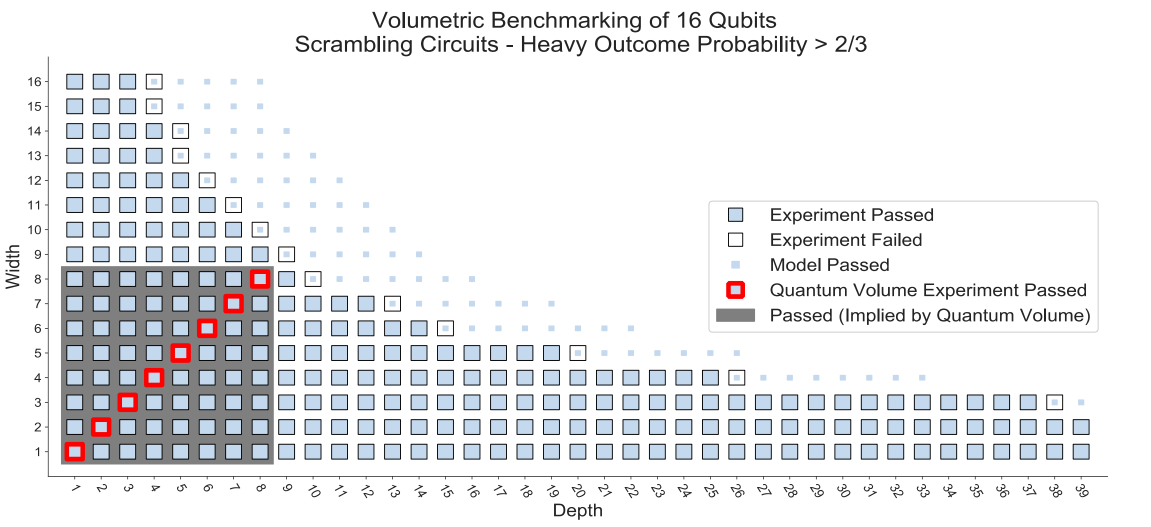


Figure 8(a). Volumetric benchmarking of a 16 qubit device using scrambling circuits. If at least 2/3 of the measurement results are heavy for a given width/depth pair, then the pair passes the test and is marked with a large, solid blue box. Using linear axes, the quantum volume experiments appear along the diagonal and are outlined with heavy, red lines. For this example, $\log_2(V_Q) = 8$. It is expected that scrambling circuits with both width and depth less than or equal to the quantum volume should succeed, and we highlight these with a gray background.

Figure 850: a better visualization of how a quantum volume is evaluated. Source: [A volumetric framework for quantum computer benchmarks]() by Robin Blume-Kohout and Kevin Young, February 2019 (24 pages).

When you look at the relative progress of QV between IBM, IonQ and Honeywell/Quantinuum systems, you see a clear difference: IBM has a tougher time to use all its qubits while trapped ions do it better, although with a rather limited number of qubits[3902] (Figure 851).

But if and when IBM fixes some scalability issues with their qubits, like with reducing qubit crosstalk, they can potentially increase their QV to higher levels than those from trapped ions vendors.

Year	Brand	Version	Hw Qubits	Log2(QV)	%
2017	IBM	Tenerife	5	2	40%
2018	IBM	Tokyo	20	3	15%
2019	IBM	Johannesburg	20	4	20%
2020	Honeywell		4	4	100%
2020	IBM	Raleigh	28	5	18%
2020	IBM	Montreal	27	6	22%
2020	Honeywell	H0	6	6	100%
2021	IBM	Montreal	27	7	26%
2020	Honeywell	H1-1	10	7	70%
2021	Honeywell	H1-1	10	9	90%
2021	Honeywell	H1-1	10	10	100%
2022	IBM	Manhattan	127	6	5%
2020	IonQ	Aria	32	22	69%
2022	Quantinuum	H1-2	12	12	100%
2022	Quantinuum	H1-1	22	13	59%
2023	Quantinuum	H1-1	22	15	68%
2023	Quantinuum	H1-1	20	19	95%

Figure 851: evolution of systems quantum volumes over time. (cc) Olivier Ezratty, 2023.

QV is limited to about 50 operational qubits because it can only be evaluated with a benchmark comparing the qubits with their simulation on a conventional computer.

This emulation is constrained by memory size, which reaches its limits between 50 and 55 qubits[3903]. As a consequence, a QV is only applicable for a pre-NISQ QPU that is below any quantum advantage threshold[3904].

[3902] See [Quantum Volume in Practice: What Users Can Expect from NISQ Devices]() by Elijah Pelofske et al, March 2022 (19 pages).

[3903] See [Why Is IBM's Notion of Quantum Volume Only Valid up to About 50 Qubits?]() by Jack Krupansky, October 2020.

[3904] See [Quantum Volume in Practice: What Users Can Expect from NISQ Devices]() by Elijah Pelofske et al, March-June 2022 (27 pages) provides some fidelities of major QPU systems.

Quantum computing scientists are circumspect about the interest of this indicator which is too simplistic[3905]. This use is contested by Scott Aaronson, a specialist in complexity theories and quantum algorithms [3906]. He reminds us that current QVs are more than easily emulable in a simple classical computer, if not on an Apple Watch! This does not make it particularly powerful. And when the QV will get significant, beyond the threshold of 50, we won't be able to measure it!

Scott Aaronson believed that this quantum volume indicator, which is a marketing simplification tool from IBM, should be avoided. He prefers a description of systems characteristics like the number of qubits, their connectivity, their coherence time (T_1, T_2), their reset, one and two-qubit gates and readout fidelities. With most vendors, these indicators are generally found in the scientific publications of researchers but not always in the vendors marketing literature. However, IBM publishes most of this data on their quantum systems available on Q Experience so you can have it all.

In August 2022, a team from **Cisco** proposed a way to evaluate a quantum volume for a MBQC-based photonic quantum processor. It is based on using a continuous-variable cluster state and the Gottesman-Kitaev-Preskill (GKP) encoding. It computes the system quantum volume based on the logical gate error channels of cluster state GKP squeezing and photon loss rate[3907] [3908]. At a lower level, the DoE is proposing a volumetric benchmark for entangled states aka cluster states or graph states, which evaluates the quality of entanglement in a set of qubits[3909]. It may be useful to quality both gate-based systems and MBQC QPUs.

In November 2021, **IBM** added the CLOPS speed metric (circuit layers operations per seconds), an equivalent to the clock of a classical CPU, given the numbers are different for resetting qubits, operating quantum gates and measuring qubits[3910]. As of 2021, IBM's systems CLOPS where between 1.5 and 2.4K, so about 2,000 circuit layers of qubit gates per second. CLOPS are calculated as M * K * S * D / time taken, where M is the number of templates, K the number of parameter updates, S the number of shots and D, the number of quantum volume layers, or $\log_2(QV)$. IBM published a methodology where M=100, K=10, S=100 and D was dependent on the quantum volume of each of the benchmarked systems[3911].

IBM planned to reach 10K to 15K CLOPS in 2022 with its 433 qubits Osprey processor, thanks to using "dynamic circuits" that handle feedback and feedforward of quantum measurements, enable long range SWAPs and help fasten quantum error corrections.

In 2023, an IBM team proposed a definition of what is a good benchmark insisting on its randomness and platform independence, and what are the appropriate and non-appropriate benchmark optimizations techniques for obtaining a good QV[3912].

3905 Imagine an indicator of the power of your laptop aggregating the processor clock frequency, its number of cores, the power of its CPU, the RAM memory, the storage capacity, its type (hard disk, SSD) etc? And there, to ask yourself if you will be able to efficiently use your video editing, photo derush or video game software on augmented reality headphones!

3906 In Turn down the quantum volume, Scott Aaronson, published just after Honeywell's February 2020 announcement.

3907 See Quantum Volume for Photonic Quantum Processors by Yuxuan Zhang et al, August 2022-April 2023 (22 pages).

3908 See also the other proposal Measurement-based interleaved randomised benchmarking using IBM processors by Conrad Strydom et al, Stellenbosch University, March 2022 (17 pages).

3909 See An entanglement-based volumetric benchmark for near-term quantum hardware by Kathleen E. Hamilton et al, DoE, September 2022 (21 pages).

3910 See Scale, Quality, and Speed: three key attributes to measure the performance of near-term quantum computers by Andrew Wack, Hanhee Paik, Ali Javadi-Abhari, Petar Jurcevic, Ismael Faro, Jay M. Gambetta and Blake R. Johnson, October 2021 (8 pages). With their Falcon R5 processor, qubit reset takes 450 ns while qubits readout takes 750 ns. It's much longer than gates.

3911 See Driving quantum performance: more qubits, higher Quantum Volume, and now a proper measure of speed by Jay Gambetta, Ali Javadi-Abhari, Blake Johnson, Petar Jurcevic, Hanhee Paik and Andrew Wack, November 2021.

3912 See Defining Standard Strategies for Quantum Benchmarks by Mirko Amico et al, IBM, March 2023 (12 pages).

Another team, led by the Unitary Fund, did a benchmark evaluating the impact of a quantum mitigation technique, ZNE, on the quantum volume of four IBM QPU. It increased it by a power of 2, of in log2() parlance by only one qubit[3913].

IBM published a preprint on a new benchmarking technique named Layer Fidelity (LF), adapted to >100 qubits QPUs, different from their quantum volume. It takes into account the effect of crosstalk between qubits and can help assess QPUs capabilities when using quantum error mitigation techniques[3914].

Algorithmic qubits

IonQ was initially supportive of IBM's quantum volume metric but now uses a QV variation denominated "algorithmic qubits". They wanted to create a more relevant single-number metric that is usage oriented and more telling for customers. They wanted to avoid using random circuits benchmarking as in IBM's QV.

It is not far from $\log_2$(IBM's QV) but not exactly. It was initially defined as the size of the largest circuit that could run with N qubits and N^2 two-qubit gates but was then refined as $\min(\#\text{qubits}, \sqrt{\#gates})$ with algorithm success probability ≥50% (not the 66% from IBM's QV), assuming the algorithm requires N^2 two-qubit gates [3915].

Practically speaking, the #AQ benchmark is run on different algorithms, like the ones defined in the QED-C benchmark. You'll then get several numbers, one for each algorithm and for each tested machine. The #AQ must be represented in a 2D chart as shown in Figure 852, with these various algorithms' success probability represented as colored circles and two axis: on X, the 'depth' of the circuit represented by a log scale of number of two qubits entangling gates, and on Y, the number of qubits used.

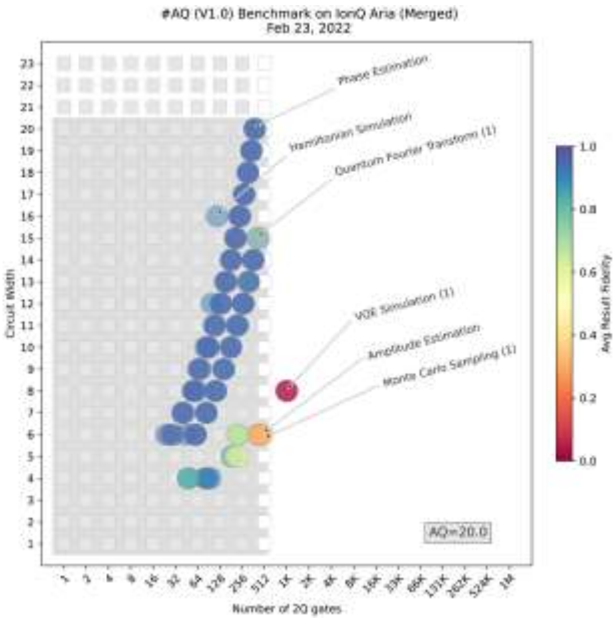


Figure 852: Source: [Algorithmic Qubits: A Better Single-Number Metric](#) by IonQ, February 2022.

IonQ touted in March 2022 having reached an AQ record of 20 with its 32 bits Aria system. It was achieved with one of these algorithms, surprisingly, the most successful one, quantum phase estimation, which by the way, is very far from a practical "customer need". Other algorithms had #AQs of 4 to 16.

With higher fidelities qubits, you could run an algorithm with a greater number of two-qubit gates and improve X. But in that case, your #AQ would still be constrained by the number of qubits used. However, with poor fidelities qubits, your #AQ could become much lower than this number of qubits, a bit like with IBM's quantum volume, but on a real algorithm and not with a randomized benchmark.

Since the suite of tested algorithms will change over time, IonQ will define release numbers for its #AQs.

[3913] See Increasing the Measured Effective Quantum Volume with Zero Noise Extrapolation by Elijah Pelofske et al, June 2023 (16 pages).

[3914] See Benchmarking Quantum Processor Performance at Scale by David C. McKay et al, IBM, November 2023 (15 pages).

[3915] The algorithmic qubits benchmark is described in details in Algorithmic Qubits: A Better Single-Number Metric by IonQ, February 2022 and in QIP 2022 | The application oriented benchmarks for quantum computing (Luming Zhao, IonQ), (1h video).

Other systems level benchmarks and metrics

Other various low-level systems benchmarks are worth mentioning:

Cycle benchmarking from a team involving Canada, Denmark and Austria which assesses the low-level quality of qubits entanglement, created in 2019[3916].

A proposal made by a team from QuSoft (the Netherlands), the University of Cambridge (UK) and Caltech (USA) in April 2021 is bound to measure the performance of universal quantum computers in a hardware-agnostic way with six structured circuits tests (Bell test, Schrödinger's microscope, Mandelbrot, line drawing, matrix inversion and platonic fractals). It is quite complex to interpret and reading out the graphical results is not straightforward, nor connected to an application need[3917].

In 2021, **DARPA** launched a research RFP for the creation of benchmarks in two categories: application-specific hardware-agnostic benchmarks (TA1, with $1.45M for 18 months) for quantum computing and hardware resource estimates for quantum computers (TA2, with a funding of $1.5M over 18 months). In the end, the project was awarded in March 2022 to Raytheon BBN and University of Southern California. Their benchmark targets all sorts of quantum technologies, both computing and sensing, and are summarized with the **SWAP** nickname corresponding to (size, weight, application and power)[3918].

In July 2022, as part of these benchmarking programs, DARPA awarded a three-year contract of $2.9M to Rigetti, the University of Technology Sydney, Aalto University and the University of Southern California to create benchmarks for large-scale quantum computers.

Another similar approach, but a narrower one, was proposed by researchers from **Brookhaven** and **Pacific Northwest** National Laboratories from the DoE which estimates hardware resources needed for key algorithms[3919]. Another DoE lab, the **Sandia Labs**, proposed a variation of randomized benchmarking that works in the quantum advantage regime[3920]. In 2022, DoE's **Oak Ridge** lab and several US universities proposed a volumetric benchmark qualifying the quality of qubit entanglement, that was tested first on IBM QPUs[3921].

And to be complete, another team from Berkeley, HRL Labs and University of Chicago devised a randomized benchmark measuring noise in non-Clifford quantum gates (those gates that are needed for a QFT and for generating an exponential speedup), extending the work from Google on their 2019 supremacy experiment[3922]. Other extensions are proposed by Alibaba USA with their Universal randomized benchmarking (URB) that scales better than the cross-entropy benchmark used by Google[3923] and with a randomized benchmark supporting a universal gate set[3924].

3916 Presented in Characterizing large-scale quantum computers via cycle benchmarking par Alexander Erhard et al., 2019 (7 pages).

3917 See Scalable Benchmarks for Gate-Based Quantum Computers by Arjan Cornelissen et al, April 2021 (54 pages).

3918 See DARPA asks Raytheon BBN and USC researchers to test limits of quantum computing for military applications by John Keller, Military+Aerospace Electronics, March 2022.

3919 See On the importance of scalability and resource estimation of quantum algorithms for domain sciences by Vincent R. Pascuzzi, Ning Bao and Ang Li, May 2022 (5 pages). They notices that for a QFT, the number of gates and CNOT gates scale exponentially with an increased number of qubits

3920 See Measuring the Capabilities of Quantum Computers by Timothy Proctor et al, Sandia Labs, August 2020/January 2022 (4 pages) and Scalable randomized benchmarking of quantum computers using mirror circuits by Timothy Proctor et al, Sandia Labs, December 2021 (8 pages).

3921 See An entanglement-based volumetric benchmark for near-term quantum hardware by Kathleen E. Hamilton, Sophia Economou et al, September 2022 (21 pages).

3922 See Benchmarking near-term quantum computers via random circuit sampling by Yunchao Liu et al, April 2022 (43 pages).

3923 See Randomized Benchmarking Beyond Groups by Jianxin Chen et al, Alibaba USA, March 2022 (35 pages).

3924 See Demonstrating scalable randomized benchmarking of universal gate sets by Jordan Hines, Robin Blume-Kohout, Irfan Siddiqi, Birgitta Whaley, Timothy Proctor et al, August 2022 (31 pages).

V-score is a benchmark for many-body problems simulation proposed by a broad international research team[3925]. The smaller the V-score, the better (like 10^{-16}).

As part of the proposal, the researchers found that a quantum advantage would be obtained with at least two-dimensional many-body problem geometries, while with one-dimensional problems, the classical DMRG technique is very effective.

In the NISQ realm, **PyQBench** is a benchmarking tool proposed by the Institute of Theoretical and Applied Informatics from the Polish Academy of Sciences[3926]. Let's also mention a framework for constructing predictive models from benchmarking data, and two case studies demonstrating how to do so with error models and a machine learning network[3927] and convolutional neural networks based model to predict NISQ QPU capabilities[3928].

Q-score

Atos proposed its Q-score benchmark in December 2020. It is based on determining the maximum size of a standardized problem that can be solved on a given hardware on any quantum programming paradigm[3929]. The first selected problem is the classical combinatorial **Max-Cut** (Figure 853). Its variations are used to solve the traveling salesperson problem or various graphs problems with applications in logistic, industry and finance. It can also be used to handle clustering in quantum machine learning. The Q-score benchmark is evaluated with using a hybrid classical+quantum algorithm like with QAOA (Quantum Approximate Optimization Algorithm).

This benchmark creates a simple metric (the number of variables that can be used in the optimization problem) and is independent from the computing paradigm used (gate-based or other) and it doesn't require a quantum computing emulation capacity like with the IBM Quantum Volume. And the algorithm solutions can be verified polynomially on a classical computer. The Q-Score software tools are also open source and published on Github.

In August 2022, a Dutch team evaluated **D-Wave** 2000Q and Advantage annealers and obtained a record of 70Q and 140Q, which is much better than existing gate-based QPUs[3930] [3931]. That same team proposed a Q-score extension with a Max-Clique combinatorial problem[3932] [3933].

Hybrid solutions may under certain conditions outperform supercomputers. Another team, from **Pasqal**, implemented a Q-score benchmark on their neutral atoms simulator and obtained a Q-score of 80Q using a digital simulation of their system[3934].

[3925] See Variational Benchmarks for Quantum Many-Body Problems by Dian Wu et al, February 2023 (25 pages).

[3926] See PyQBench: a Python library for benchmarking gate-based quantum computers by Konrad Jałowiecki et al, March 2023 (49 pages).

[3927] See Predictive Models from Quantum Computer Benchmarks by Daniel Hothem, [Submitted on 15 May 2023].

[3928] See Learning a quantum computer's capability using convolutional neural networks by Daniel Hothem et al, Sandia Labs, April 2023 (27 pages).

[3929] See Benchmarking quantum co-processors in an application-centric, hardware-agnostic and scalable way by Simon Martiel, Thomas Ayral and Cyril Allouche, IEEE Transactions on Quantum Engineering, February 2021 (11 pages).

[3930] See Evaluating the Q-score of Quantum Annealers by Ward van der Schoot et al, the Netherlands Organisation for Applied Scientific Research, August 2022 (8 pages).

[3931] See also another favorable benchmark for D-Wave annealers: Milestones on the Quantum Utility Highway by Catherine C. McGeoch et al, D-Wave, May 2023 (34 pages).

[3932] See Q-score Max-Clique: The First Quantum Metric Evaluation on Multiple Computational Paradigms by Ward van der Schoot et al, February 2023 (17 pages).

[3933] See New developments around Atos Q-score by Philippe Duluc, Atos, August 2022.

[3934] See Efficient protocol for solving combinatorial graph problems on neutral-atom quantum processors by Wesley da Silva Coelho, Mauro D'Arcangelo and Louis-Paul Henry, August 2022 (23 pages).

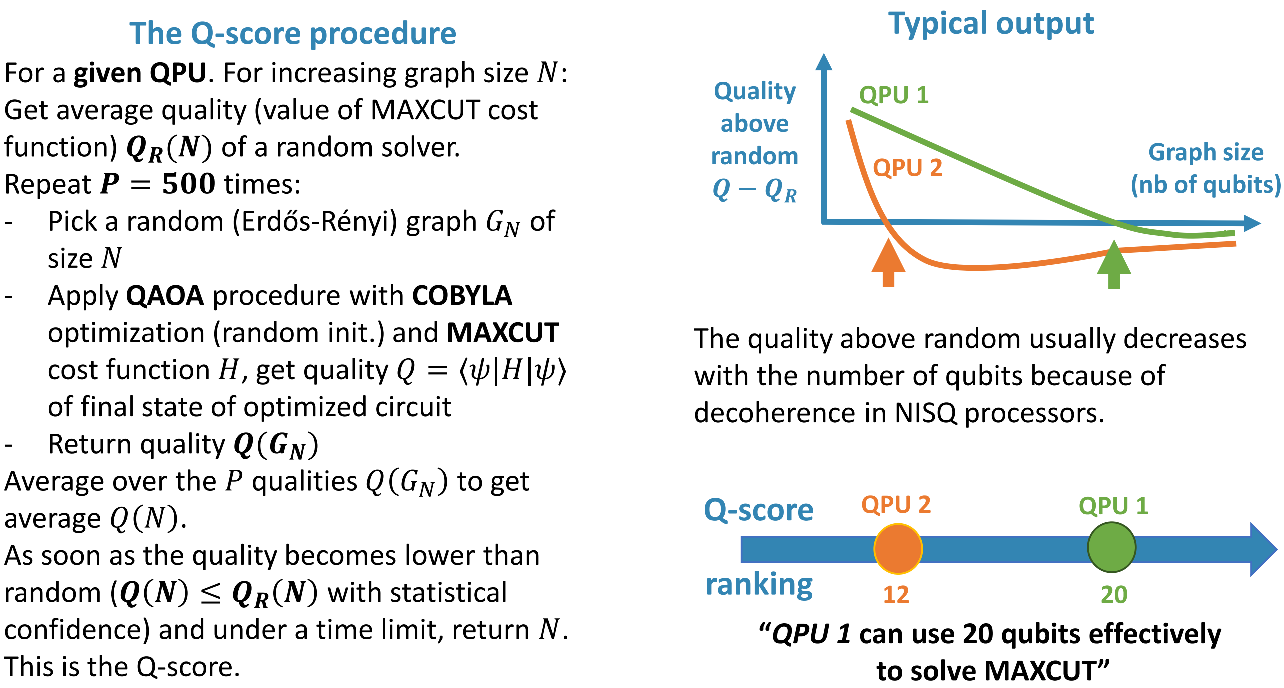


Figure 853: Eviden/Atos Qscore calculation method. Source: Atos.

They solved the benchmark Max-Cut problem using a classical machine learning technique to reduce the number of runs on their QPU.

In October 2023, the first Q-Score was published by a gate-based quantum computer vendors, IQM, with 8Q obtained with their latest 20 qubit QPU[3935].

The Q-score would need to be completed by other benchmarks such as one for quantum chemistry simulation (number of atoms in molecule) and another on the size of the maximum number that can be factorized (in power of 2). It may happen as part of the BACQ project mentioned later.

Other use case benchmarks

Many applications oriented benchmark have been proposed in the last few years (Figure 854).

QED-C is supporting a series of application oriented benchmarks proposed by researchers from Princeton, HQS, QCI, IonQ, D-Wave and Sandia Labs in the USA[3936]. It mixes the volumetric benchmarking method from IBM and a comparison of performance with various standard algorithms. They did some comparison on actual quantum hardware from IBM, Rigetti, HQS and IonQ. It was launched in October 2021. In 2023, they extended their standard proposal with an optimization algorithms benchmark supporting NISQ QAOA and quantum annealing[3937].

SupermarQ is a benchmark proposed in 2022 by Super.tech (acquired in May 2022 by Infleqtion) and supported by Amazon and Intel (both of which have no functional quantum computer yet). It is a suite of application benchmarks that covers use cases in finance, pharmaceuticals, energy, chemistry and other verticals. SupermarQ also contains an error correction benchmark[3938].

[3935] See Finland launches a 20-qubit quantum computer – development towards more powerful quantum computers continues, IQM, October 2023.

[3936] See Application-Oriented Performance Benchmarks for Quantum Computing by Thomas Lubinski et al, October 2021-January 2023 (35 pages).

[3937] See Optimization Applications as Quantum Performance Benchmarks by Thomas Lubinski et al, QED-C, February 2023 (30 pages).

[3938] See SupermarQ: A Scalable Quantum Benchmark Suite by Teague Tomesh et al, Princeton, University of Chicago, Super.tech and Intel, February 2022 (15 pages) and Applying classical benchmarking methodologies to create a principled quantum benchmark suite by Teague Tomesh et al, March 2022.

QPack is a benchmark based on the Max-Cut problem (like Atos's Q-score), the dominating set problem (DSP) and the traveling salesperson problem (TSP), and using the QAOA algorithm as a benchmarking tool, proposed by TU Delft researchers. It measures the maximum problem size a quantum computer can solve, the required computing runtime and the achieved accuracy[3939] [3940] [3941].

The DoE **ORNL** (Oak Ridge National Laboratory) proposed a benchmark for chemical simulation[3942]. It deals with the simulation of three 2-atoms molecules (NaH, KH et RbH) which can be simulated on existing IBM and Rigetti 20 and 16 qubits superconducting systems. It is not generic and can't go beyond these molecule sizes.

Other single use-cases benchmarks have also been created: **Agnostiq** created a benchmark dedicated to optimizing financial portfolio using the Quantum Approximate Optimization Algorithm (QAOA)[3943], scientists from New-York created a benchmark related to Grover's search algorithm[3944] and Zapata Computing created one benchmarking on fermionic simulations[3945].

HamLib, created by Intel et al in 2023 is a library of ready to use circuits (Hamiltonians) solving problems in various areas (Max-Cut, TSP, condensed matter physics, chemistry)[3946].

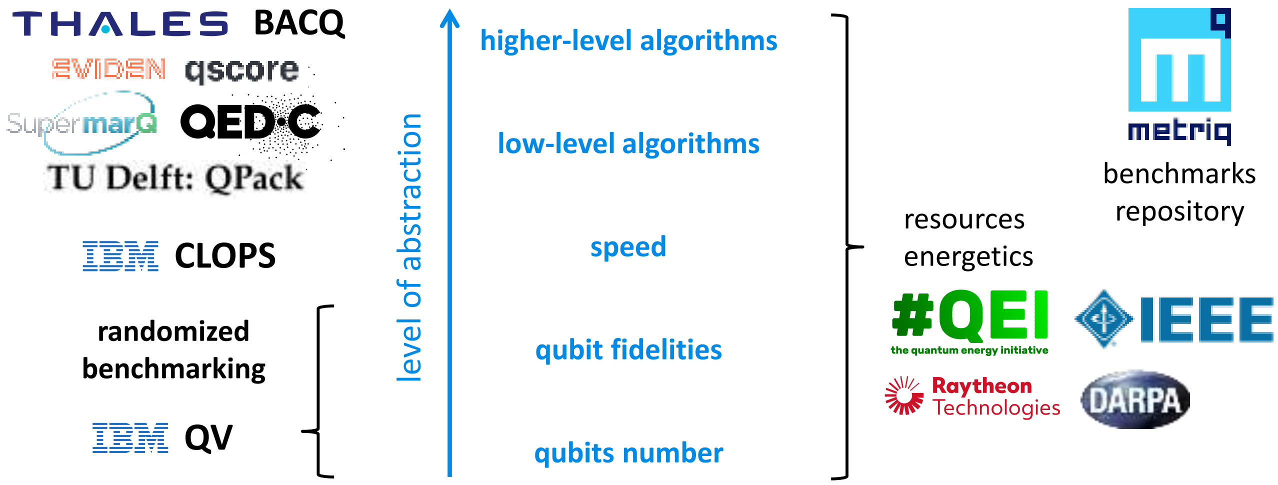


Figure 854: the various levels of abstraction in quantum computing benchmarking. (cc) Olivier Ezratty, 2023.

[3939] See QPack: Quantum Approximate Optimization Algorithms as universal benchmark for quantum computers by Koen Mesman et al, April 2022 (28 pages).

[3940] See QPack Scores: Quantitative performance metrics for application-oriented quantum computer benchmarking by Huub Donkers et al, May 2022 (23 pages).

[3941] See QPack: A cross-platform quantum benchmark-suite, Quantitative performance metrics for application oriented quantum computer benchmarking by Huub Donkers, 2022 (67 pages.

[3942] See ORNL researchers advance performance benchmark for quantum computers, January 2020.

[3943] See Wasserstein Solution Quality and the Quantum Approximate Optimization Algorithm: A Portfolio Optimization Case Study by Jack S. Baker et al, February 2022 (21 pages).

[3944] See Quantum search on noisy intermediate-scale quantum devices by Kun Zhang, Kwangmin Yu and Vladimir Korepin, February 2022 (12 pages).

[3945] See An application benchmark for fermionic quantum simulations by Pierre-Luc Dallaire-Demers et al, Zapata Computing, March 2020 (14 pages).

[3946] See HamLib: A library of Hamiltonians for benchmarking quantum algorithms and hardware by Nicolas PD Sawaya et al, Intel, NASA, June 2023 (33 pages).

The **BACQ project** launched in France in 2023 is also a high-level applications oriented benchmark, but still using low-level technical figures of merits[3947]. The considered classes of problems are optimization, linear systems solving, quantum physics simulations and prime numbers factorization. The benchmark will look at various figures of merit like computation time, latency, problem size, approximation rate, resolution probability, accuracy, fidelity and also power consumption. It will cover all QPU architectures, from NISQ and FTQC gate based to analog machines. The project is run by Eviden, CEA, CNRS[3948], Thales, Teratec and LNE.

There are other application specific benchmarks like on quantum generative learning[3949], or for the DAQC hybrid analog/gate-based quantum computing paradigm[3950].

International standard organizations

Many standard organizations are engaged in quantum technologies, and it goes beyond benchmarking. They have various forms. International standards bodies like ISO have representatives from countries and their own standard bodies whereas IEEE has representatives from organizations (business or research organizations). There are also regional bodies like ETSI and CEN/CENELEC who form the European Union system for standardization[3951], in relation with ISO.

ISO has working group working on quantum computing (ISO/IEC JTC 1/WG 14).

IEEE has launched several benchmarking initiatives on its own with a standard to be submitted in 2024[3952].

They entertain several working groups with participating industry vendors and academic institutions.

- Trial-Use Standard for a Quantum Algorithm Design and Development (P2995, details).
- Standard for Quantum Computing Architecture (P3120, details).
- Software-Defined Quantum Communication (P1913).
- Standard for Quantum Computing Definitions (P7130).
- Standard for Quantum Computing Performance Metrics & Performance Benchmarking (P7131).
- Standard for Quantum Computing Energy Efficiency (P3329).

CEN/CENELEC has a focus group working on quantum technologies (FGQT).

UIT has a study group 13 (FG-QIT4N) dedicated to quantum networks and QKD.

ETSI is also working on QKD.

Benchmarking tools

Metriq from **Unitary Fund** announced in May 2022 is a repository of benchmarking results.

[3947] See BACQ: Delivering an Application-oriented Benchmark Suite For Evaluating Quantum Computing Performance by Matt Swayne, May 2023.

[3948] The participating CNRS team is the |QET> (Quantum Energy Team) from Alexia Auffèves (CNRS MajuLab Singapore) and Robert Whitney (CNRS LPMMC Grenoble), also two of the cofounders of the Quantum Energy Initiative.

[3949] See Application-Oriented Benchmarking of Quantum Generative Learning Using QUARK by Florian J. Kiwit et al, August 2023 (8 pages).

[3950] See Benchmarking Digital-Analog Quantum Computation by Vicente Pina Canelles et al, IQM, July 2023 (21 pages).

[3951] See Towards European Standards for Quantum Technologies by O. van Deventer, March 2022 (39 pages).

[3952] See P7131 - Standard for Quantum Computing Performance Metrics & Performance Benchmarking. It covers gate-based quantum computing. See also Metrics & Benchmarks for Digital Quantum Computing by Robin Blume-Kohout (18 slides) and Summary of the IEEE Workshop on Benchmarking Quantum Computational Devices and Systems, 2019. Also, see P2995 - Trial-Use Standard for a Quantum Algorithm Design and Development and P3120 - Standard for Quantum Computing Architecture.

MQT Bench aka the Munich Quantum Toolkit is a multiple-abstraction level suite of benchmarking tools covering both low-level abstraction building blocks (QFT, QPE, amplitude estimation) and higher-level ones (Grover, Shor, HHL)[3953].

Arline (Germany) is also proposing its own benchmarking tools suite that is used to compare compiler optimizers[3954].

QUARK (Germany) is another industry application-centric benchmarking proposal based on an opt-source framework[3955].

Quantum supremacy and advantage

Quantum supremacy, advantage and utility are various terms used to qualify the superiority of quantum computers as compared with the most powerful supercomputers. They must be defined carefully.

Quantum supremacy was coined by John Preskill in a paper presented at the Solvay Congress in 2011[3956]. It is achieved when a problem, useful or not, is only solvable with a quantum algorithm running on a quantum computer, and there is no known classical algorithm for the most powerful supercomputers that could solve it in a reasonable human scale time[3957]. Quantum supremacy was a goal for researchers and vendors like Google and it became claims staring in October 2019.

Quantum supremacy does not mean that a given quantum computer is supremely more powerful than all its contemporary supercomputers. The term is applicable to a combination of a specific problem and related quantum algorithm, a given quantum computer, and the best-in-class available classical algorithms adapted to the most powerful available supercomputers. The criteria are moving targets. Supercomputers have not said their last word and are classical algorithms are also improved[3958].

Robert König (Technical University of Munich), David Gosset (University of Waterloo, Canada) and Sergey Bravyi (IBM) demonstrated in October 2018 that quantum computers could perform operations inaccessible to conventional computers but based only on the case of a particular algorithm[3959].

Others are devising "*proofs of quantumness*", which are methods to demonstrate to a classical verifier that a quantum computer can perform some computational tasks that a classical computer with comparable resources cannot (meaning, the classical computer must achieve things in a tractable way, i.e. less than polynomial time)[3960].

Some D-Wave and Google benchmarks carried out in 2015 and showing the superiority of the quantum solution were then contradicted by the creation of algorithms optimized for supercomputers under certain conditions. In a few years' time, it will certainly come into play for a few algorithms that cannot have optimized supercomputer equivalents.

[3953] See MQT Bench: Benchmarking Software and Design Automation Tools for Quantum Computing by Nils Quetschlich et al, TUM Germany, Johannes Kepler University Linz, Austria and Hagenberg GmbH (SCCH), Austria, April 2022 (7 pages).

[3954] See Arline Benchmarks: Automated Benchmarking Platform for Quantum Compilers by Y. Kharkov et al, February 2022 (27 pages).

[3955] See QUARK: A Framework for Quantum Computing Application Benchmarking by Jernej Rudi Finžgar, Philipp Ross, Leonhard Hölscher, Johannes Klepsch and Andre Luckow, August 2022 (12 pages).

[3956] It is described in Quantum Computing and the Entanglement Frontier by John Preskill, 2011 (27 slides).

[3957] See Quantum supremacy: Some fundamental concepts by Man-Hong Yung, January 2019 (2 pages) according to which there are three ways to demonstrate quantum supremacy: boson sampling, PQI and chaotic quantum circuits.

[3958] See Quantum Algorithms Struggle Against Old Foe: Clever Computers by Ariel Bleicher, February 2018. This mentions the discovery of classical algorithms that are as powerful as their quantum equivalents, such as the one from Ewin Tang, already mentioned on page 61.

[3959] See First proof of quantum computer advantage, October 2018 and Quantum advantage with shallow circuits, April 2017 (23 pages).

[3960] See Simpler Proofs of Quantumness by Zvika Brakerski et al, May 2020 (12 pages).

Google's quantum supremacy announced in October 2019 was the first one of its kind, using Google's Sycamore 53 qubits experimental QPU[3961]. It was based on running a *randomized benchmark sampling* algorithm using 53 qubits, aka a cross-entropy randomized benchmark[3962]. There was only a 0.15% chance of getting a good result, thus the need to run the algorithm 3 million times to compute an average. What took 3 minutes to run on Sycamore was supposed to last 10K years on the most powerful supercomputer in 2019, IBM's Summit, installed at the DoE Oak Ridge national laboratory in Tennessee. IBM quickly responded that with adding 64 PB of SSD storage to the supercomputer, the classical sampling equivalent could run in 2.5 days[3963]. It now runs in 6 seconds on the DoE Frontier Aurora supercomputer thanks to the use of improved tensor networks methods. The huge caveat here is that it is running a random algorithm with no useful input data. It is not "computing" anything per se.

Cristian and Elena Calude of the University of Auckland in New Zealand then argued that a high-performance limit, that of a precise quantum computer, is compared to a low limit which is the best performance in solving the same problem in a supercomputer[3964].

Quantum supremacy is thus a comparable between the existence of a quantum performance and the assumption of the non-existence of an equivalent performance in classical computing. On top of that, the supremacy claim was of course misunderstood for a quantum advantage, up to leading some to think that quantum computers were not far from breaking RSA keys. The disappointment came in 2020 when Google published several papers experimenting with Sycamore for real use cases like in computational chemistry. These cases couldn't use more than 20 qubits and were as a result far off any quantum practical advantage as compared with best-in-class classical solutions.

Then, many researchers tried to simulate Google's supremacy classically using various breeds of tensor networks and compression techniques.

A 2020 paper from Yiqing Zhou (University of Illinois), Edwin Miles Stoudenmire (Flatiron Institute) and Xavier Waintal (CEA-IRIG) stated that emulating Sycamore's processes in a classical computing could use some compression technique to take into account the qubit's noise. It could even be done on a simple microcomputer[3965]. This corresponded however to a 95% emulated gates fidelity. With a 99% fidelity matching Sycamore's capability, it would still require a couple hundred cores and some TB of memory, fitting in a datacenter rack. There would be an energy advantage for Sycamore but going down from x500,000 vs the IBM Summit to about only x325 for a 500 core cluster server.

A research team from China published an even better performance in 2021, using an improved tensor contraction technique and running for 15 hours on a cluster of 512 GPUs[3966]. In 2021, a China team classically simulated the Google Sycamore cross-entropy benchmark with a single Nvidia A100 GPGPU running for 149 days with a fidelity of 73.9% while Sycamore's fidelity was only 0.2%[3967].

3961 See The power of random quantum circuits by Bill Fefferman, 2019 (25 slides) that explains the power behind the randomized benchmarking technique chose by Google.

3962 See the review paper Computational advantage of quantum random sampling by Dominik Hangleiter and Jens Eisert, University of Maryland, Freie Universität Berlin, Review of Modern Physics, July 2023 (87 pages).

3963 See Leveraging Secondary Storage to Simulate Deep 54-qubit Sycamore Circuits by Edwin Pednault et al, October 2019 (39 pages).

3964 In The road to quantum computing supremacy, 2017.

3965 See What limits the simulation of quantum computers? by Yiqing Zhou, Edwin Miles Stoudenmire and Xavier Waintal, PRX, November 2020 (14 pages).

3966 See Solving the sampling problem of the Sycamore quantum circuits by Feng Pan, Keyang Chen, and Pan Zhang, PRL, July 2022 (9 pages).

3967 See Simulating the Sycamore quantum supremacy circuits by Feng Pan and Pan Zhang, March 2021 (9 pages). The authors improved their work in Solving the sampling problem of the Sycamore quantum supremacy circuits by Feng Pan et al, November 2021 (9 pages).

Another team, from Alibaba, found a way to optimize Sycamore's emulation in September 2021 to reach only 20 days of computing on a system equivalent of the IBM Summit[3968]. And yet another one in October 2021 could simulate it on the new Sunway supercomputer in 304 seconds, using a tensor compression technique[3969]. Late 2022, another compression technique was tested using 41,932,800 cores during 8.5 days[3970].

In summer 2022, Xavier Waintal, Edwin Miles Stoudenmire and a team of Eviden/Atos researchers including Thomas Ayral improved their classical simulation of Google's supremacy with a density-matrix tensor networks renormalization group (DMRG) algorithm. It runs on a few classical CPU cores from an Atos QLM and in a couple hours. The algorithm has a simulation cost scaling polynomially with the number of qubits and the depth of the circuit. A consequence of this work is that to reach an exponential quantum computing advantage, it is more important to increase qubit fidelities than their number[3971] [3972]. Dorit Aharonov, Umesh Vazirani et al landed a similar conclusion a little later, and that Google supremacy was not a "*an experimental violation of the extended Church-Turing thesis*", given it could potentially be polynomially simulated classically[3973] [3974].

Likewise, and the other way around, a Google team did show that the 2021 Chinese gaussian boson sampling (GBS) supremacy experiment could be efficiently simulated classically (still, quadratically)[3975]. Another team, from the UK, achieved a similar feat in 2022, in which they did simulate a 100-mode and up to 60 click detection events GBS on a ~100,000-core supercomputer[3976].

In October 2022 and May 2023, Gil Kalai et al produced an in-depth and well-crafted detailed analysis of Google's supremacy experiment and related protocols raising doubts on some statistical data[3977] [3978].

Google AI's team up the ante in 2023 with another convoluted variation of their 2019 claim, using a random circuit sampling method implemented with 70 qubits[3979].

[3968] See Efficient parallelization of tensor network contraction for simulating quantum computation by Cupjin Huang et al, Alibaba, September 2021 (10 pages).

[3969] See Closing the "Quantum Supremacy" Gap: Achieving Real-Time Simulation of a Random Quantum Circuit Using a New Sunway Supercomputer by Yong (Alexander) Liu et al, October 2021 (18 pages). The China team behind this was awarded the 2021 ACM Gordon Bell Prize. Their work was later contradicted by ORNL researchers who had developed Google's supremacy classical simulation in 2019. See China's exascale quantum simulation not all it appears by Nicole Hemsoth, NextPlatform, November 2021.

[3970] See Validating quantum-supremacy experiments with exact and fast tensor network contraction by Yong Liu et al, December 2022 (7 pages).

[3971] See Density-Matrix Renormalization Group Algorithm for Simulating Quantum Circuits with a Finite Fidelity by Thomas Ayral, Thibaud Louvet, Yiqing Zhou, Cyprien Lambert, E. Miles Stoudenmire, and Xavier Waintal, PRX Quantum, August 2022-April 2023 (26 pages).

[3972] See Quantum supremacy: Is it true that current quantum computers can beat classical computers? by Thomas Ayral, Atos Quantum Lab, Eviden, May 2023.

[3973] See A polynomial-time classical algorithm for noisy random circuit sampling by Dorit Aharonov, Umesh Vazirani et al, November 2022 (27 pages).

[3974] See New Algorithm Closes Quantum Supremacy Window by Ben Brubaker, Quanta Magazine, January 2023.

[3975] See Efficient approximation of experimental Gaussian boson sampling by Benjamin Villalonga, Hartmut Neven et al, September 2021 (15 pages).

[3976] See The boundary for quantum advantage in Gaussian boson sampling by Jacob F.F. Bulmer et al, 2022 (8 pages).

[3977] See Google's 2019 "Quantum Supremacy" Claims: Data, Documentation, and Discussion by Gil Kalai, Yosef Rinott and Tomer Shoham, October 2022 (32 pages).

[3978] See Questions and Concerns About Google's Quantum Supremacy Claim by Gil Kalai, Yosef Rinott and Tomer Shoham, May 2023 (49 pages).

[3979] See Phase transition in Random Circuit Sampling by A. Morvan et al, Google AI, April 2023 (39 pages). With a paper containing 196 names, providing some clue about Google's quantum team.

In 2018, IBM researchers demonstrated that quantum supremacy was assured in the long run, even with quantum computers that can chain a finite and constrained number of quantum gates[3980]. In December 2020, they published a theorical model that could prove some quantum advantage, solving binary function problems, and tested on a low scale on a 27 qubits superconducting system[3981]. These various, sometimes convoluted, performances are very hard to compare and evaluate.

One key aspect of all supremacy claims is that they implement some random benchmark that is difficult to simulate digitally. Like with boson samplings, these are physical processes that are difficult to simulate. Supremacies are obtained with algorithms using no input data. Thus, they don't solve any useful problem. An advantage is supposed to solve a useful problem with some input and output data.

We cannot avoid mentioning the debates around the term supremacy with its bad social and political meaning.

So far, I have seen only one replacement proposal, that hasn't been adopted yet, which consists in using the simpler term **primacy** with the same meaning[3982]. Another interesting fringe phenomenon is the usage of some quantum supremacy for things that are not at all related to quantum computing. It's borderline click baiting[3983]!

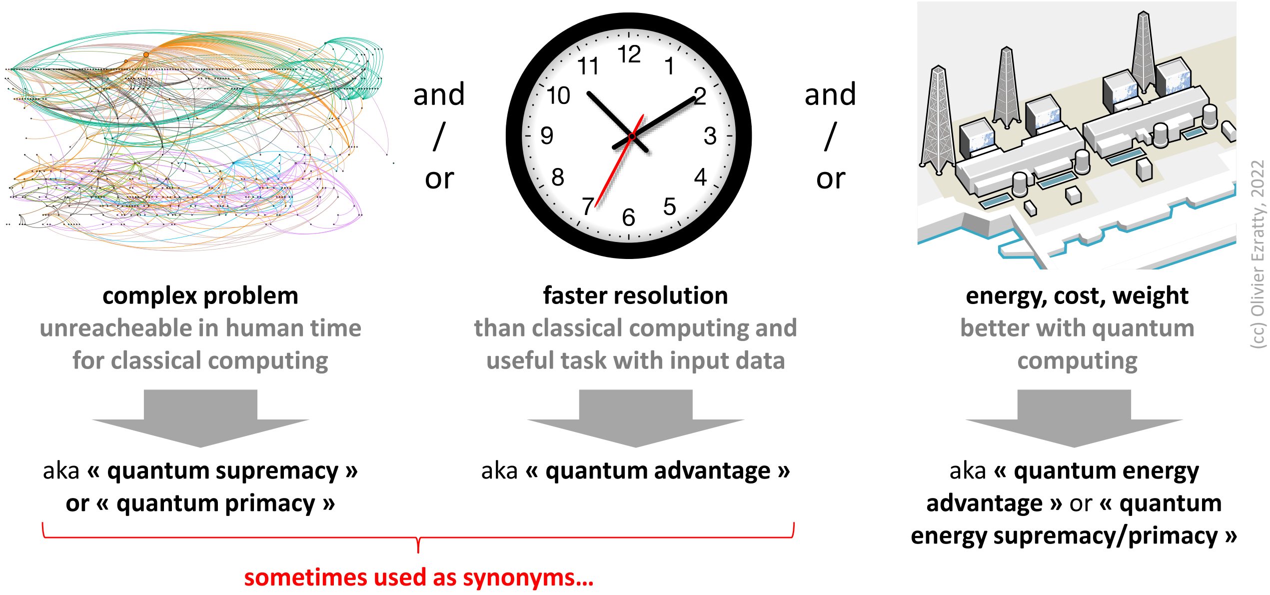


Figure 855: trying to define quantum supremacy (or primacy) and quantum advantage. (cc) Olivier Ezratty, 2022.

Quantum advantage is a different concept that corresponds to a situation where a quantum computer executes a useful algorithm faster than on the most powerful supercomputers. It seems at first glance not as strong an argument as with quantum supremacy, but it happens that it is more difficult to reach a quantum advantage than a quantum supremacy. But some are pushing various definitions for quantum advantage. Sometimes, it even has the same meaning than quantum supremacy but with a more politically correct terminology and for others, it is a stronger statement than quantum supremacy, meaning the same but for a useful algorithm.

[3980] See Scientists Prove a Quantum Computing Advantage over Classical by Bob Sutor, October 2018, Quantum advantage with shallow circuits, Sergey Bravyi, David Gosset and Robert Koenig, 2017 (23 pages) and the video Quantum advantage with shallow circuits, IBM Research, December 2017 (44 minutes).

[3981] See Quantum advantage for computations with limited space by Dmitri Maslov, Sarah Sheldon et al, IBM Research, December 2020 (12 pages). Also published in Nature Physics in June 2021.

[3982] See Quantum Computing 2022 by James D. Whitfield et al, January 2022 (13 pages).

[3983] See Quantum supremacy in mechanical tasks: projectiles, rockets and quantum backflow by David Trillo et al, IQOQI Vienna, September 2022 (18 pages). It deals with some relativistic physics phenomenon.

One equivalent term is "quantum utility" and it can relate to many other dimensions as presented later in Figure 860, page 1027, with a variable mix of speedup, result quality, energetics and cost benefits compared to some given full-stack classical solution[3984].

There are currently much fewer advantage claims than supremacy claims. And sometimes, they are misnomers or applied to very specific cases, even beyond quantum computing[3985]. And they have no more input data than with quantum supremacy claims.

One first example comes from an interesting work from a team of researchers from France and Edinburgh announced in February 2021, including Eleni Diamanti and Iordanis Kerenidis[3986]. It involved a complicated photonics-based experiment that didn't do any real calculation. It was about putting in place a QMA (Quantum Merlin Arthur) verification protocol.

The implemented protocol is an interactive test that requires, through a network, the verification of the solution of a complex NP-complete optimization problem without having to communicate the whole solution. The breakthrough that made this possible was the creation of a system encoding the solution result with partial information about the solution to be verified from one network node to another. The protocol compresses a large vector state describing the partial information on the solution, involving some entanglement and multi-mode photons quantum communications. This compression protocol would make it possible to verify the results in much less time. No actual verification was done on the other end of the system.

We have here a quantum advantage coming from the way to connect a quantum computer solving an NP-complete SAT problem and another quantum computer verifying the solution with partial information. Both computers do not exist yet. Another view on this would be that it proposes an architecture to verify a solution to an NP-problem on an end-to-end solution.

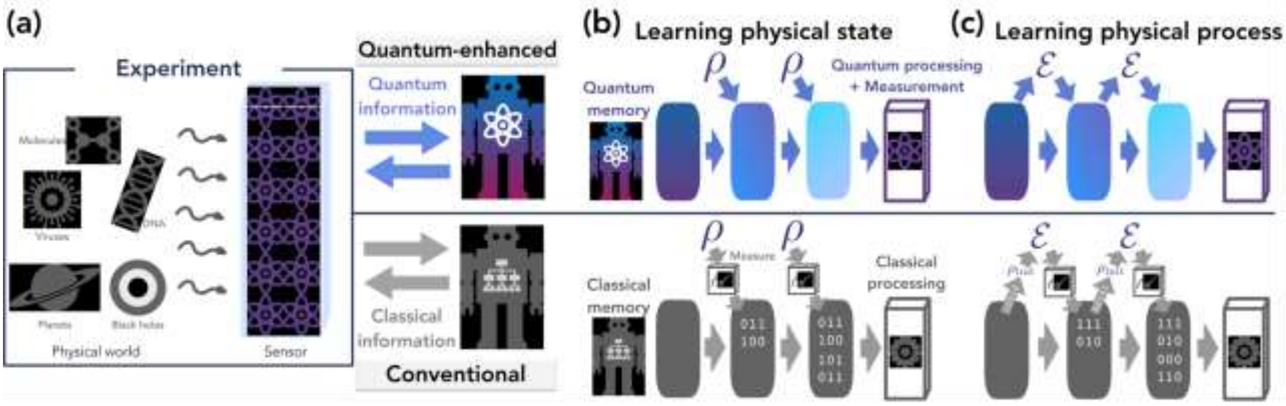


Figure 856: a quantum advantage can come from connecting quantum sensors and quantum computers, avoid the tedious steps of quantum-to-classical and classical-to-quantum data conversions. Source: Quantum advantage in learning from experiments by Hsin-Yuan Huang, Hartmut Neven, John Preskill et al, December 2021 (52 pages) with 40 Sycamore qubits.

Another example of quantum computing advantage could be reached with feeding a quantum computer with data coming from quantum sensors data and transmitted "quantumly" instead of classically.

[3984] See What is quantum utility? by Robert Davis, IBM, November 2023 which also defines quantum utility as "*what we get when a quantum computer is able to perform reliable computations at a scale beyond brute force classical computing methods that provide exact solutions to computational problems*".

[3985] See Quantum Advantage of Threshold Changeable Secret Sharing Scheme by Xiaogang Cheng et al, September 2022 (11 pages) which deals with secret key exchanges using quantum computing but not about solving an NPish problem.

[3986] See Experimental demonstration of quantum advantage for NP verification with limited information by Federico Centrone, Niraj Kumar, Eleni Diamanti, and Iordanis Kerenidis, published in Nature Communications, February 2021 (13 pages). This was a follow-up of Quantum superiority for verifying NP-complete problems with linear optics by Juan Miguel Arrazola, Eleni Diamanti & Iordanis Kerenidis, Nature, 2018 (8 pages).

That's what demonstrated a team of Google and other researchers in 2021 and shown in Figure 856[3987]. This requires some form of quantum memory that is still to be created.

Many experts estimate that the threshold of 50-ish quality qubits, with a low error rate and a long coherence time, will be needed to achieve any real quantum advantage. These will probably be logical qubits, assembling physical qubits and some quantum error correction codes. However, it may be needed to reach a higher level of qubit number and quality to obtain some significant quantum advantage. Researchers from Google AI indeed found that it will be difficult to benefit from a quadratic speedup on early FTQC architectures, with a dependence on the QEC used. It is due to the large computing time constant factors coming from surface code based QEC[3988].

The quantum advantage label is also used in non-computing domains, like with quantum sensing, when some technique is generating better accuracy or precision than classical sensors.

Quantum dominance corresponds to a (future) quantum computer able to execute a task that is entirely inaccessible to any classical computer[3989].

Quantum energy advantage is another threshold that may arise someday when on top of some computing time benefits, we could highlight the fact that quantum computers consume much less energy than supercomputers for solving similar problems (Figure 855). This is still a subject of research and dealt with as part of the Quantum Energy Initiative that is described starting page 286[3990].

Figure 857 presents a tabulated consolidation of the various quantum supremacies and advantages announced since 2019[3991]. It shows that between 2019 and 2021, none of these achieved real useful computing with some application input data. And in 2022, we start to see appearing some real or potential interesting and narrow quantum advantages with input data.

The IBM et al example below was touting a quantum advantage, but it would require a NISQ QPU with 96-qubits and 99.99% 2-qubit gates and measurement fidelities which is extrapolated from a 12-qubit Quantinuum QPU having such fidelities. One key problem is that these trapped ion QPUs are quite hard to scale.

Advantage genericity. Many quantum advantage claims are positioned in the future, in scientific papers title starting with "Toward". The claims are unfortunately never generic. It must always relate to some specific problem, algorithm, (future) hardware configuration and to many data constraints. For example, the problem data must be sparse or show some symmetry. This lack of advantage genericity is problematic. We can hope that in the future, such advantages will become more generic, when we can say that, in general, quantum computers can solve a broad spectrum of chemical simulation or optimization problems in a reasonable time, that is not at all accessible to supercomputers.

Time scales. Finally, speedup related quantum advantages must be practical and not theoretical. It does not matter much that you obtain some polynomial or exponential speedup to run a chemical simulation of a useful molecule if it lasts 1,500 years! If and when we are able to build scalable FTQC computers with thousands of refined logical qubits, the practical quantum computing time scales for real useful problems will become a key concern and generate new scientific challenges.

[3987] See Quantum advantage in learning from experiments by Hsin-Yuan Huang, Hartmut Neven, John Preskill et al, December 2021 (52 pages) with 40 Sycamore qubits.

[3988] See Focus beyond Quadratic Speedups for Error-Corrected Quantum Advantage by Ryan Babbush, Jarrod R. McClean, Michael Newman, Craig Gidney, Sergio Boixo, and Hartmut Neven, PRX Quantum, March 2021 (11 pages).

[3989] See Quantum utility -- definition and assessment of a practical quantum advantage by Nils Herrmann et al, Quantum Brilliance, March 2023 (15 pages).

[3990] See the Quantum Energy Initiative already discussed elsewhere in this book starting page 259.

[3991] The Arizona performance is documented in Researchers demonstrate a quantum advantage by University of Arizona, June 2021, referring to Quantum-Enhanced Data Classification with a Variational Entangled Sensor Network by Yi Xia et al, June 2021 (17 pages). Their setting used variational quantum circuits for a classification of multidimensional radio-frequency signals using entangled sensors.

who and when	architecture	algorithm	input data	comment
Google, Oct 2019	Sycamore, 53 superconducting qubits	cross entropy benchmarking	**none**	running a random gates algorithm
China, December 2020	70 photons modes GBS (Gaussian Boson Sampling)	interferometer photons mixing	**none**	running a random physical process
IBM Research, December 2020	IBM 27 superconducting qubits	symmetric Boolean functions	SLSB3 function parameters	**theoretical demonstration of quantum advantage**
Kerenidis, Diamanti et al, March 2021	multi-mode photon dense encoding of verified solution	Quentin Merlin Arthur based verification	output from some quantum computation (not implemented)	**no actual computing done in the experiment**
China, April 2021	Quantum walk on 62 superconducting qubits	simple quantum walk	simulating a 2-photons Mach-Zehnder interferometer	**no quantum advantage at all**
University of Arizona, May 2021	supervised learning assisted by an entangled sensor network	variational algorithm, classical computing	data extracted from three entangled squeezed light photonic sensors	**not a quantum « computing » advantage per se**
China, June 2021	66 superconducting qubits and 110 couplers, Zuchongzhi 1, then 2.1	cross entropy benchmarking	**none**	56 used qubits
China, September 2021				60 used qubits
China, June 2021	144 photons modes GBS and up to 113 detected events	interferometer photons mixing	**none**	**parametrizable photon phases could lead to a programmable system**
Google, AWS, Harvard et al, December 2021	quantum sensors feeding a quantum computer	learning about the principal component of a noisy state	**quantum output from quantum sensors**	**requires some quantum memory**
Xanadu June 2022	216 squeezed photons modes GBS (Gaussian Boson Sampling)	time domain multiplexing and interferometer photons mixing	**programmable GBS with 1,296 parameters**	**first programmable GBS**
IBM et al, September 2022	hybrid algorithm that could run on NISQ QPUs	QML-TDA unsupervised machine learning technique for extracting valuable shape-related data features	**small data sets related to cosmic microwave background**	**exponential speedup, resilient to noise, requires 96-qubit QPU with 2Q gate and measurement fidelity of 99.99%...**
IBM, June 2023	127 qubits	trotterized time evolution of a 2D transverse-field Ising model	**none**	**presented as a utility and not an advantage, quickly classically emulated.**
Google, June 2023	70 qubits on 72 Sycamore chipset	cross entropy benchmarking	**none**	running a random gates algorithm

Figure 857: an inventory of past quantum advantages/supremacies announcements and their underlying characteristics. (cc) Olivier Ezratty, 2021-2023. Sources: Google 2019: Quantum supremacy using a programmable superconducting processor by Frank Arute, John Martinis et al, October 2019 (12 pages). China 2020: Quantum computational advantage using photons by Han-Sen Zhong et al, December 2020 (23 pages). IBM 2020 : Quantum advantage for computations with limited space by Dmitri Maslov et al, December 2020 (12 pages). Kerenidis / Diamanti 2021: Experimental demonstration of quantum advantage for NP verification with limited information by Federico Centrone, Niraj Kumar, Eleni Diamanti, and Iordanis Kerenidis, published in Nature Communications, February 2021 (13 pages). China April 2021: See Quantum walks on a programmable two-dimensional 62-qubit superconducting processor by Ming Gong, Science, May 2021 (34 pages). Arizona 2021: Quantum-Enhanced Data Classification with a Variational Entangled Sensor Network by Yi Xia et al, June 2021 (19 pages). China June 2021: Strong quantum computational advantage using a superconducting quantum processor by Yulin Wu, Jian-Wei Pan et al, June 2021 (22 pages). China September 2021: Quantum Computational Advantage via 60-Qubit 24-Cycle Random Circuit Sampling by Qingling Zhu, Jian-Wei Pan et al, September 2021 (15 pages). China June 2021: Phase-Programmable Gaussian Boson Sampling Using Stimulated Squeezed Light by Han-Sen Zhong, Chao-Yang Lu, Jian-Wei Pan et al, June 2021 (9 pages). Google, AWS, Harvard: Quantum advantage in learning from experiments by Hsin-Yuan Huang, Hartmut Neven, John Preskill et al, December 2021 (52 pages) with 40 Sycamore qubits. Xanadu: Quantum computational advantage with a programmable photonic processor by Lars S. Madsen et al, Xanadu, June 2022 (11 pages). IBM: Towards Quantum Advantage on Noisy Quantum Computers by Ismail Yunus Akhalwaya et al, September 2022 (32 pages) also discussed in Quantifying Quantum Advantage in Topological Data Analysis by Dominic W. Berry, Ryan Babbush et al, September 2022 (41 pages) and contested in Complexity-Theoretic Limitations on Quantum Algorithms for Topological Data Analysis by Alexander Schmidhuber and Seth Lloyd, September 2022 (24 pages). IBM: Evidence for the utility of quantum computing before fault tolerance by Youngseok Kim et al, Nature, June 2023 (8 pages). Google: Phase transition in Random Circuit Sampling by A. Morvan et al, Google AI, April 2023 (39 pages).

Quantum software development tools key takeaways

- Gate-based programming involves either graphical circuit design (mostly for training purposes) and (usually) Python based programming when qubit gates structures must be designed in an automated way.
- Python based programming relies on libraries like IBM's Qiskit or Google's Cirq. There are however many development tools coming from universities and research labs like Quipper. Some tools like ZX Calculus are highly specialized and used to create quantum error correction codes or low-level systems.
- Currently, NISQ quantum computing is based on running algorithms multiple (thousands if not more...) times and averaging the results. A single individual run yields a probabilistic outcome while many run averages will converge into deterministic ones. It is even more complicated with VQE (variational quantum eigensolvers) which require millions of circuit shots to reconstitute a many-body system Hamiltonian to measure its ground state energy level.
- Most quantum computers are used in the cloud, through offerings coming from the computer vendors themselves like IBM or D-Wave or from cloud service providers like Amazon, Microsoft, Google and OVHcloud. In the long term, cloud quantum computing services architectures will need to be distributed and require dynamic routing of photonic entanglement resources to enable the creation of large virtual QPUs.
- Quantum emulators are very useful to learn programming, test it until it reaches the limits of classical emulation (about 40-50 qubits) and also help debug small-scale quantum algorithms. When these emulators include physical simulators of the underlying qubit physics like with Bosonic Qiskit and Quandela Perceval, they help create algorithms that are error-resilient and also design new quantum error correction codes. Quantum emulation is an indispensable part of any quantum cloud offering.
- Gate-based programs debugging is a significant challenge as it is difficult to implement equivalents of classical code breaking points. As a result, quantum code certification and verification is a new key discipline, particularly for distributed computing architectures such as the ones relying on the concept of blind quantum computing.
- Benchmarking quantum computers is an unsettled technique with many competing approaches. It includes the various techniques used to qualify so-called quantum supremacies and quantum advantages. Not a single of them, as of 2023, did show a real computing advantage compared to classical computing. The reasons were multiple, the main ones being that these experiments usually do not implement any algorithm using some input data. But starting in 2022, we see appearing some relevant quantum advantage with actual data and useful algorithms running on NISQ hardware, including with the boson sampling method used with photon qubits.

Quantum computing applications

Most algorithms mentioned in the previous parts are generally very low-level. How about assembling them into business solutions, market by market? We are still far from having things settled for that respect. The quantum software industry is still very immature and for good reasons, since quantum computers are very limited at this stage. NISQ computers are not yet ready for prime time, at least in their gate-based paradigms. We are in a stage equivalent where the computer industry was in the mid-1950s, when the software industry was in its infancy, if not even before, during the mid-1940s with the ENIAC which was powerful compared with contemporary standards made of tabulating machines, mechanical calculators and slide rules, but not really practically programmable.

Still, you can discover here and there a lot of so-called case studies, coming from various sources: vendors like D-Wave and IBM, their customers or partners, and academic researchers (Figure 858). It creates a strong discrepancy between the generated perception of a business ready quantum computing scene and the harsh reality. Most of the time, these case studies are small scale proof-of-concepts and software prototypes. They have not yet reached production grade scale nor brought any practical benefit compared to classical computing due to the limitations of existing quantum hardware.

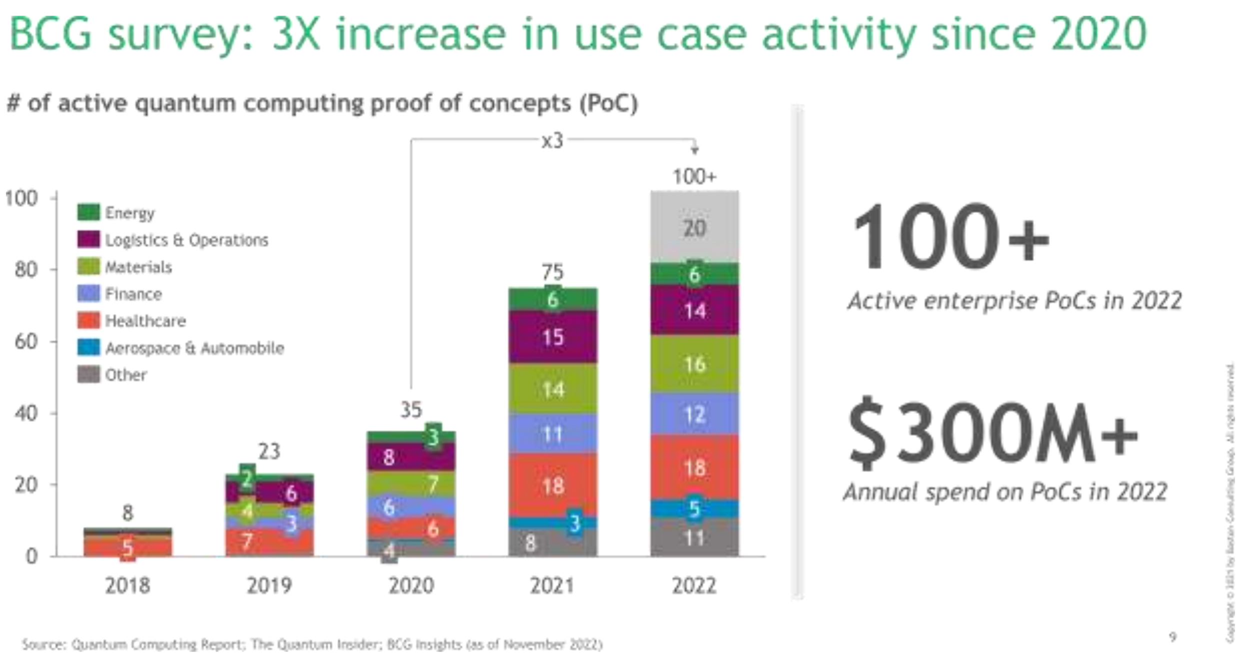


Figure 858: Source: Make the case for quantum by Jean-François Bobier, BCG, April 2023 (16 slides) which shows the growth in the number of case studies per verticals over the recent years although none of these case studies are showcasing some quantum advantage..

Still, these are useful as prospective tools. They correspond to an indispensable learning phase for research, startups, and industry. It is part of a readiness process that will speed things up if and when hardware will ramp up. And this ramp-up may happen progressively. Analysts like to say that the overarching question is more “when” than “if”. You have seen that nuance may be required here. The “if” is still open. We really don’t know yet whether NISQ can deliver its promises, if analog computing can scale and if FTQC is possible to implement.

Case studies evaluation

Let’s say you discover an analyst or IT vendor white paper or an academic review paper showcasing how quantum computing will transform your industry. It is usually written using the present tense, showcasing many practical case studies, sometimes with their scientific references.

Meanwhile, so far, no corporation has deployed these solutions. What explains this discrepancy? How can you assess these case studies' readiness for business grade applications? It is a new challenge that I will try to address here. We'll look first at the conditions that could enable quantum computing to add some competitive value compared with classical computing and then, at the ways to classify and assess case studies. In Figure 859 and in the below text, I propose a framework of questions to ask when reviewing case studies and color codes showing potential red flags making the case study moot and green flag making it relevant[3992]. More than two red flags should turn a "case study" into something labelled differently like a new algorithm. We'll call this the **PReQaCAQD framework** :). You pronounce it the way you want!

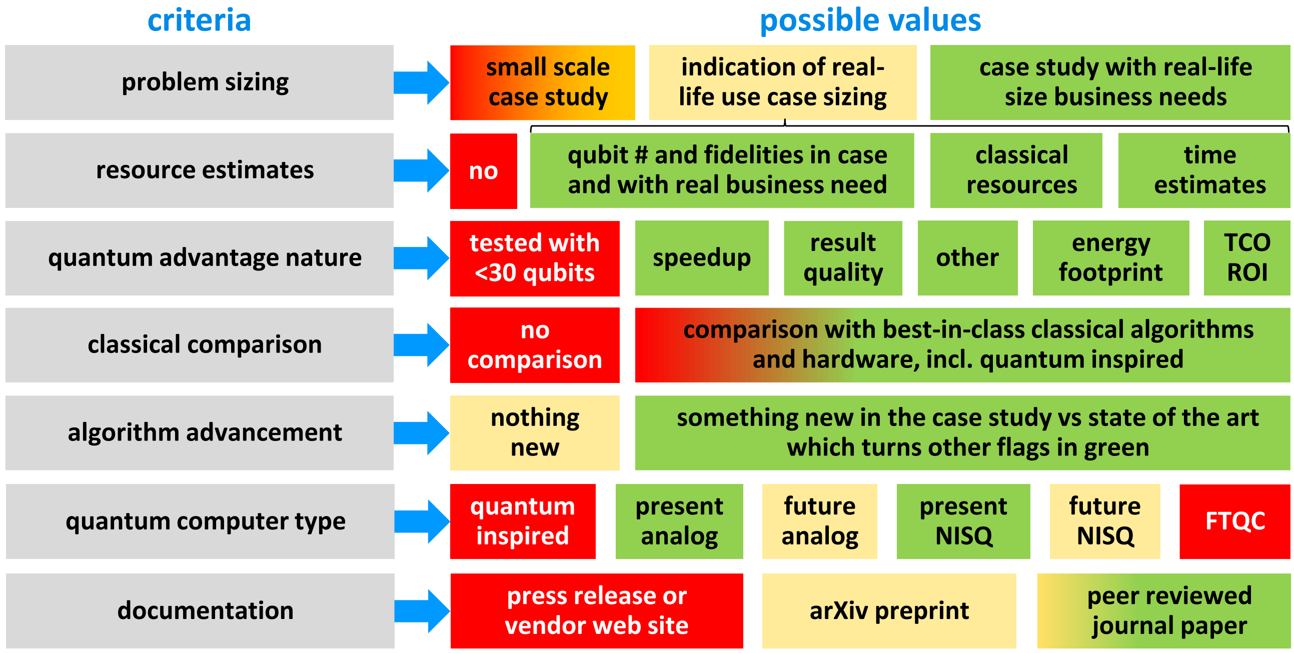


Figure 859: how to evaluate a quantum computing case study. Red flags indicate key missing points in the case study making it irrelevant to show any advantage for quantum computing for the given business problem. Orange flags are intermediate situations but not real showstoppers. Green flags are indications of a serious use case. Quantum inspired use cases are not using quantum computing and therefore should not be presented as "quantum computing use cases". TCO means total cost of ownership, an economic concept coming from the classical information technology world and ROI, return on investment, all being compared to 100% classical solutions. (cc) Olivier Ezratty, 2023.

Problem sizing

One first key aspect of a case study is the sizing of the problem. Is the conducted experiment toying with real life data and matching usual business needs or just a small scale prototype? How about the scale of the problems businesses are trying to solve more efficiently with quantum computers? It deals with the size of the data and the number of parameters or variables of a given problem, like a number of assets in a financial portfolio. Unfortunately, most existing case studies are using small scale data and should be complemented with some indications on the real size quantitative scenario needs[3993].

Resource estimates

A good case study presents estimates of the classical and quantum computing resources needed to solve a given real-world problem and its relation to the problem sizing defined above. It is even more

[3992] See Biology and medicine in the landscape of quantum advantages by Benjamin A. Cordier, Nicolas P. D. Sawaya, Gian Giacomo Guerreschi and Shannon K. McWeeney, Journal of The Royal Society Interface, November 2022 (31 pages) which provides some guidance to assess case studies in the healthcare market.

[3993] See Quantum Algorithm for Maximum Biclique Problem by Xiaofan Li et al, September 2023 (14 pages) which proposes to find a maximum biclique within a given bipartite graph, showing a polynomial speedup, with potential use cases in ecommerce, social network recommendations and biology, but with no indication of the qubit and time resources needed to solve practical problems.

important when the solution was only tested with a small sized problem. You need to have some clues on the size and type of quantum computers which could be able to solve a real-life problem.

When looking at the details:

- **NISQ case studies** should evaluate the algorithm depth and breadth, the number of shots and the cost of classical computing for all variational algorithms. Are qubit numbers and fidelities numbers mentioned in case study and for its extension to a real business need sizing?
- **FTQC case studies** should highlight the number of T gates which correspond to the bulk of quantum error correction overhead and to the logical qubit fidelities target. Coupled with the physical qubit quality and connectivity, it leads to assessing the number of physical qubits. It may be different if the target platform uses the MBQC paradigm, using photonic cluster states.
- **Analog quantum computing case studies** should also be associated with some data on the hardware requirement to obtain some quantum advantage like a number of qubits and their topologies for a quantum annealer and a quantum simulator.

Whatever, if the solution was tested on fewer than 20 qubits, you can consider it as a classical solution that could run unchanged, faster and cheaper on your own laptop! This is the case for most of the use cases presented in this part. Under 30 qubits, it can run on a classical server cluster.

Quantum advantage nature

What is a quantum advantage? For a long time, it has been defined as a situation where a computing speedup can be observed between a quantum hardware and software combination, and a best-in-class classical solution. And preferably, this speedup was to be exponential, leaving in the dust of uselessness any classical computing solution for solving large scale problems. That is the speed advantage. But a speed advantage must be practical and not just theoretical. Do not confuse speedup and actual computing time. Complexity theory classifications are interesting for theoretical purposes but is useless if the actual computing time to solve a given problem exceeds a human lifetime! And any comparison should include all the classical computing involved in any quantum computing solution[3994].

The quantum computing speed advantage is also usually confused with the exponential space advantage of quantum computing. It is generated by the large quantum state space of N qubits that has an exponential size of 2^N complex numbers. As we have learned, this large "data handling space" is not sufficient to generate a polynomial or exponential speedup even though, when N>55, this data space cannot be fully emulated on a classical computer due to memory constraints. If the case study has been tested with fewer than 30 qubits, you can be sure that it does not bring any speed advantage with classical computers, even a simple server.

A quantum computing advantage can also be qualitative when the generated result is somewhat better than its classical counterpart, without necessarily having been produced faster. This can happen for example with machine learning when it needs less training data or produce classification and prediction models with fewer errors. Here, you should also make sure that this qualitative result is not accessible to classical computers. For example, if the case study quantum code runs on fewer than 20 qubits, it is the equivalent of a classical algorithm improvement that could run faster on your own laptop. It would be a red flag.

You may also trade quality for speedup, typically with the typical heuristic based approaches used in quantum computing, particularly in NISQ regime with variational algorithms. Comparisons may be touchy when you compare apples and oranges, like the best solution on one hand and one non optimal solution on the other hand.

[3994] This corresponds to the notion of quantum utility as presented in Milestones on the Quantum Utility Highway by Catherine C. McGeoch and Pau Farré, D-Wave, May 2023 (34 pages).

Then, we have the energetics of the solution. It is becoming impossible to avoid considering this aspect when deploying new technologies. A quantum computing solution may provide some energetic advantage compared to classical solutions, but it remains to be proven on a case by case basis.

Finally, the total cost of ownership (TCO) of the new quantum solution must be evaluated. This cost includes all incurred expenses to deploy a solution: classical and quantum hardware, software, cloud, services, and training related costs. Studies are not yet available at this point given the immaturity of the technology. Sometimes, you find out that some quantum computers have ridiculously high catalog prices up to tens of millions of dollars. In that case, you are better off with using cloud computing resources. You can then mix TCO with ROI (return on investment) considerations, given all cost and value considerations must be compared to classical legacy solutions.

The quantum utility terminology can be used to mix these various combinations of quantum benefits.

advantage	definition proposal
space	when the qubit register data space - scaling in 2^N complex numbers with N qubits - exceeds the memory capacity of classical computers.
speed	when a fully-burdened quantum algorithm, including its classical part, runs faster than an equivalent best-in-class entirely classical algorithms running on either the largest supercomputers or a given HPC configuration.
quality	when the quality of the results of a quantum algorithm is better for some respect than the best-in-class classical algorithms. It can relate for example to the error rate of some machine learning classification, to its explainability or to the precision of a chemical simulation to find the ground state energy of a many-body system.
energetic	when a fully-burdened quantum computer and algorithm configuration consumes less energy than the best-in-class classical equivalent. It becomes a sort of energetic supremacy if no classical computing configuration can solve the given problem.
cost	when the total cost of the quantum solution is lower than the total cost of a best-in-class classical solution. There are many ways to calculate this cost. It can be just about hardware and software or also include other incurred costs like people training and cost of software development.

Figure 860: a proposal set of definition for the various advantages when comparing quantum and classical quantum settings. When making any comparisons, quantum settings should include all their surrounding classical computing environments. Also, a comparison can be made with either the largest supercomputer in the world or with a smaller classical computing setting, like a mid-size HPC system. In the end, the business benefit will come from a given balance of cost-speed-quality benefits and trade-offs. Source: (cc) Olivier Ezratty, 2023.

Classical comparisons

Is there an honest and up-to-date comparison between the proposed quantum solution and classical equivalent solutions? It must rely on using the state-of-the-art hardware (HPC, GPUs, …) and software (algorithms, heuristics, tensor networks, …). The comparison must not mix apples and oranges between exact solutions and heuristic approximate solutions that would favor one side of the comparison. In the case where the quantum computing solution can be easily emulated on a classical system or was only emulated, it will mean there is no quantum advantage.

Algorithm advancement

Was there something new in the case study vs the state of the art in both classical and quantum computing? Where was the progress? Does it have some business consequences?

Quantum computer type

Quantum case studies belong usually to one of these categories from being implementable in the short term to the very long term:

- **Classical emulation** (not shown in the graph) does not demonstrate any quantum advantage whatsoever if it is not complemented with resource and computing time estimations for a real quantum computer execution. It is also an indication, if this comes out of some hardware vendor, of the immaturity of their hardware platform.
- **Quantum inspired solutions**. These solutions may improve the state of the art of existing classical solutions, but they are not quantum per se. They benefit from advances in tensor network algorithms (MPS, DMRG) and from CPU/GPU and HPC hardware performance improvements.
- **Present Analog quantum computing solutions** which can run on existing analog systems like a D-Wave quantum annealer of a Pasqal/QuEra quantum simulator. At this point in time, these are the most powerful available solutions, but none have yet reached some quantum advantage.
- **Future Analog quantum computing solutions** which require a future analog computer with a larger number of qubits and/or better connectivity. These hardware offerings are presumed to show up in the short term of less than 5 years to support over 300 workable qubits.
- **Present NISQ solutions** which are being tested on an existing quantum computer and can usually be also tested in emulation mode on a classical computer. Their business value is usually minimal, and they usually showcase no quantum advantage in whatever category. Cautiousness is required here.
- **Future NISQ solution for a future quantum computer** which may bring some quantum advantage. The related hardware may show up in the short term (IBM's Heron with 133 qubits with 99.9% fidelities) or in a longer term (higher fidelities and even more qubits).
- **FTQC solutions** with the capacity to support from a hundred to thousands of logical qubits and the related fidelities matching the algorithm breadth and depth. These QPUs may be available some day in the longer term, probably beyond 10 to 20 years. It is also important to have an idea of the requirement FTQC hardware with documented resource estimates. There's a big gap between having 100 and 4,000 logical qubits, particularly given the required physical resources are scaling nearly polynomially here to ensure adequate logical qubits fidelities.

Documentation

The last criterion for validating a case study is its documentation.

Is it provided only with a vendor or customer press release and web site page (red flag), with an arXiv preprint (better) or with a peer-reviewed journal paper (much better, although not necessarily safe, read the paper conclusions carefully)? The more scientific details and data you have on the case study, the better. It may take time from the vendor/customer press release being published and some related scientific publication. So, patience will be a virtue here.

Example

We will use an example to assess a case study from Quantinuum (Figure 862). It was presented in The Quantum Insider in September 2023 as showing "*early stage quantum advantage*"[3995]. It is not clear what that means exactly. When looking at the corresponding arXiv paper, we clearly see that we are not yet in any form of quantum advantage (Figure 861). Indeed, the tests of this new multipurpose Monte Carlo engine were implemented with 6 physical qubits, far from any quantum advantage since it can be emulated on your smartphone and laptop, probably faster than on any quantum computer. The engine contains a resource estimation tool which shows that an early quantum advantage would require at least 100 qubits with 99.99% two-qubit gates fidelities, far from what is available today.

[3995] See Quantinuum's Quantum Monte Carlo Integration Engine Shows Early Stage Quantum Advantage by Matt Swayne, TQI, September 2023.

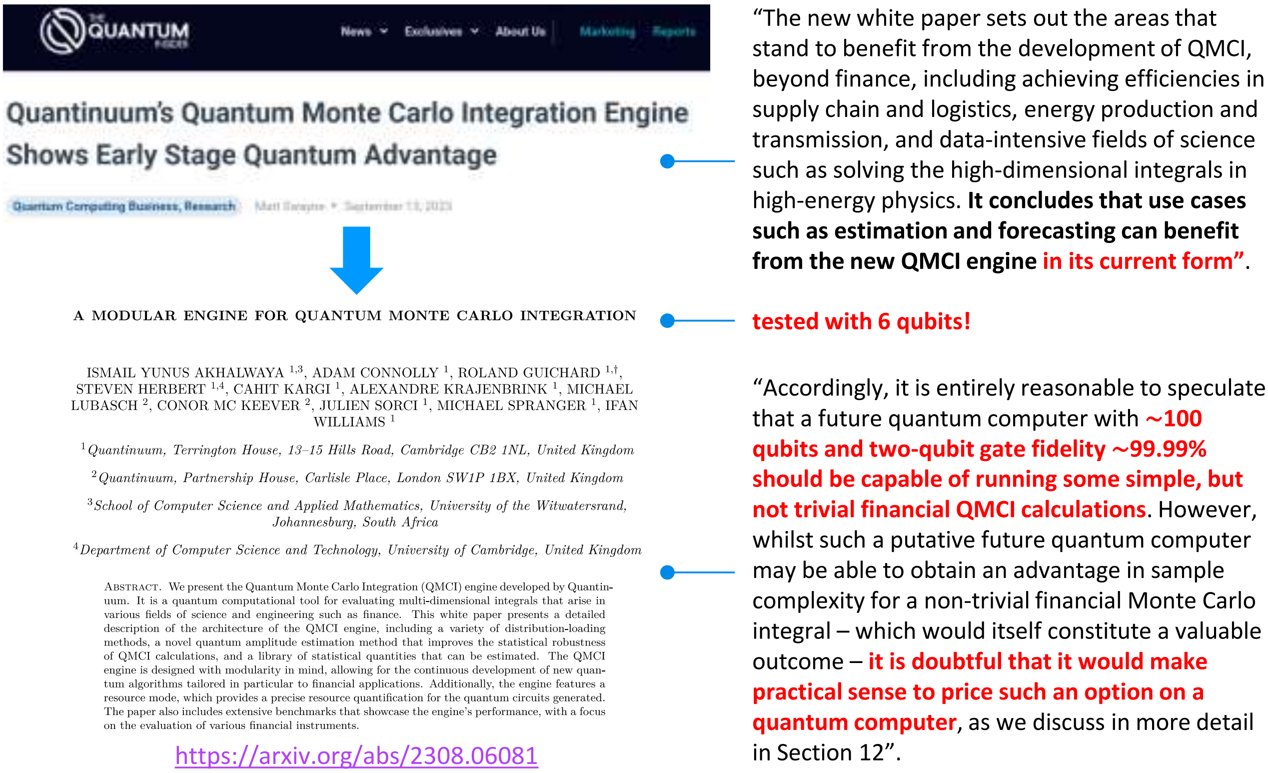


Figure 861: Quantinuum Quantum Monte Carlo Integration Engine as presented in a misleading way in The Quantum Daily, mentioning "early stage quantum advantage". When looking at the actual arXiv paper, there is no such early stage quantum advantage given it would require at least 100 qubits with 99.99% fidelities. Source: [A Modular Engine for Quantum Monte Carlo Integration](https://arxiv.org/abs/2308.06081) by Ismail Yunus Akhalwaya et al, Quantinuum, August 2023 (87 pages).

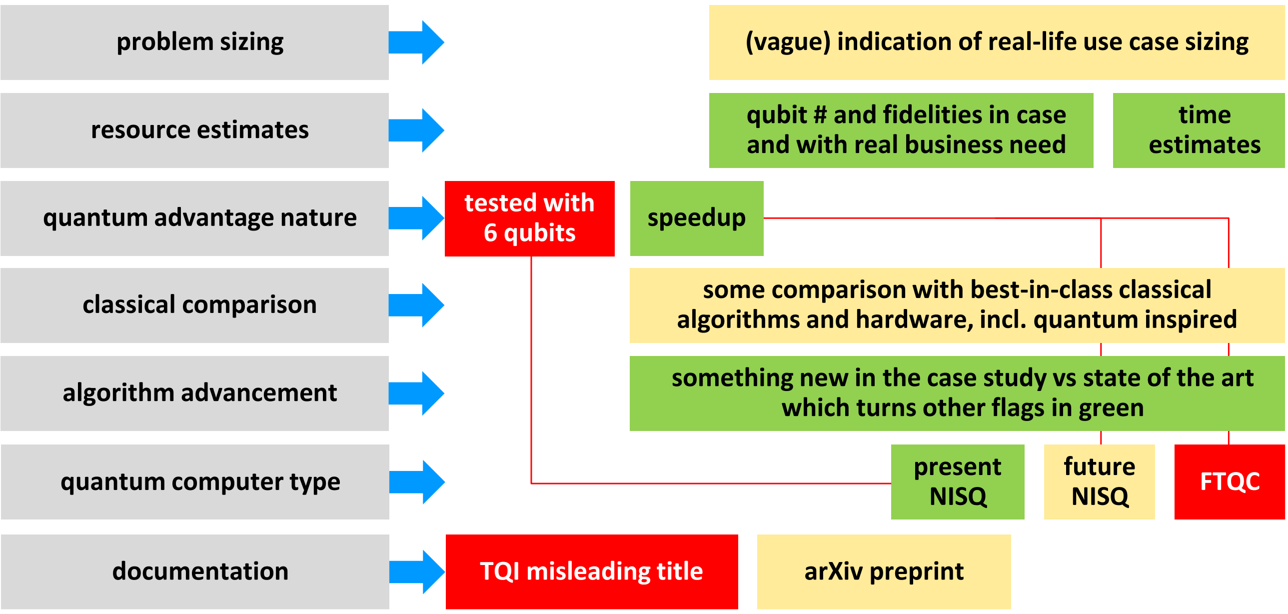


Figure 862: assessing the Quantinuum QMCI engine paper using our case studies evaluation framework. All in all, this is a new engine that may provide some quantum advantage but in future NISQ and FTQC QPUs, not those that are available as of 2023. (cc) Olivier Ezratty, 2023. It is much better than Characterizing a non-equilibrium phase transition on a quantum computer by Eli Chertkov et al, Quantinuum, November 2022 (25 pages) which is a 20-qubit use case, with no real quantum advantage.

Market

You could add a last parameter for the use case, its targeted market or domain. As shown in Figure 863, you can separate these use cases in two broad categories: research use cases and operations use cases. Research use case correspond to R&D work to design new materials, processes and products which, once industrialized, are running with classical physical resources.

These use cases will demand a smaller number of quantum computers than the operations categories which corresponds to uses cases running on a continuous basis and with potentially some scale and volume. Chemical simulation use cases will sit in the first category while most optimization problems solving will be in the second part.

All this leads to adding a few necessary items to the framework. Any use case should contain an identity card with a customer name, or customer segment, and optionally, an industry partners (quantum hardware or software) and optionally some academic research partner. and at last, a date.

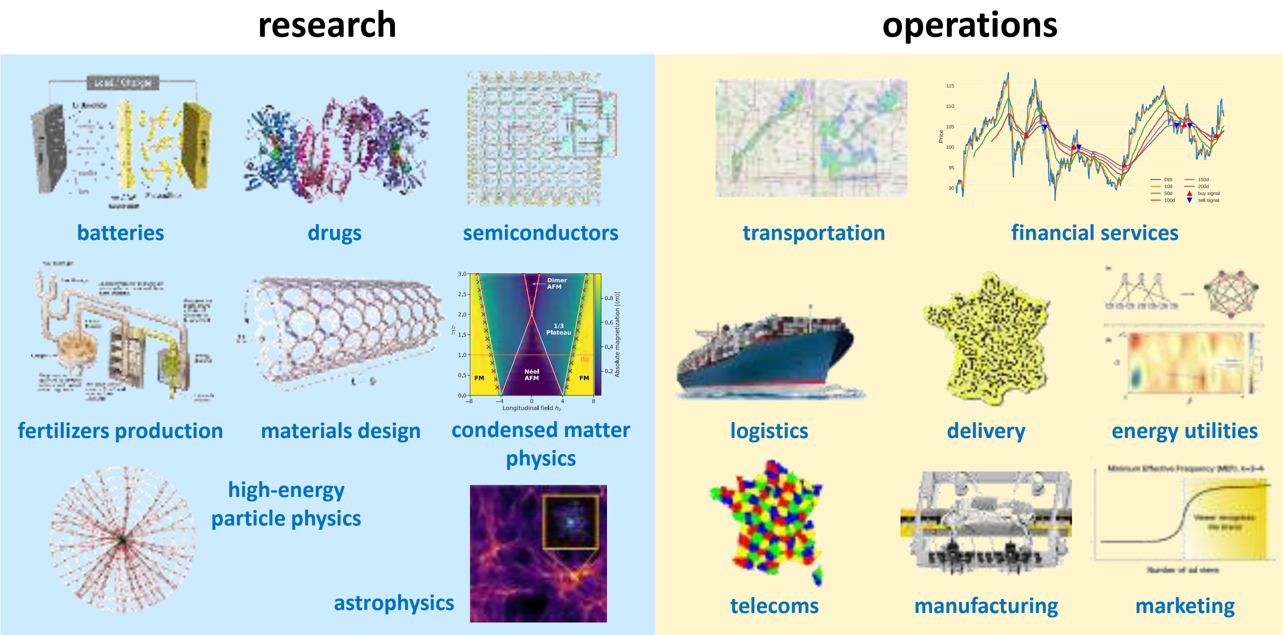


Figure 863: a map of the main solution domains for quantum computing applications separating research oriented use cases and operation level use cases. (cc) Olivier Ezratty, 2022-2023.

Market forecasts

Any new technology wave brings its market forecasts data born out of analysts and market survey companies. They have a very traditional closed-loop system in place: vendors want to get some ideas of customer demand or positive confirmation of their own biases, analysts poll large customers to get some understanding about their plans, and *voila*, you get your nice market predictions. It often looks like linear or simple nonlinear regressions. These predictions can become either self-fulfilling prophecies or total failures. The Gartner Group has turned its simplistic hype curve into a kind of Schrödinger's time and topic-independent wave equation of technology trends. But nobody really checked it, particularly when this curve was highly dependent on complicated scientific and technological challenges. It is more about probabilities than simplistic curves.

Quantitative assessments

So, how could we predict the size and shape of the quantum hardware and software markets, vertical per vertical, when you have no idea of when actual useful quantum computer will show up? Will it follow an exponential market growth rate worthy of those of the microcomputer and smartphones industries? Let's look at what we have in store.

BCG's quantum computing growth forecasts illustrate this strong uncertainty. They showcase predictions with an optimistic scenario, which starts seeing growth around 2030, and a very conservative one, which only takes off after 2040[3996]. In both cases, the quantum computing market grows linearly.

[3996] See The coming quantum leap in computing, BCG, May 2018 (19 pages).

Predicting the size of the quantum computing market is indeed highly probabilistic. It was supposed to reach $553M in 2023 (in 2017) and $4.4B by 2028 (in 2023[3997]) according to **Markets and Markets**, then $1.9B in 2023 for CIR and $2.64B in 2022 for **Market Research Future** (in 2018).

A **Global Market Insights** report from April 2023 estimated that the quantum computing market size was of $15B in 2022 and would reach $65B by 2032[3998]. I'm wondering what they are smoking here since it is probably 20 times off track of reality. And this survey is sold for $5,150! In 2021, **Hyperion Research**, a company created by former analysts from the Garner Group, forecasted a $830M market for 2024[3999], and, in 2022, upgraded it to $964M for the same year and to $1.2B in 2028[4000].

At the end of 2022 at the Q2B conference in Santa Clara, Bob Sorensen from Hyperion said wisely that it was too early to make long-term projections beyond 2025[4001] (see Figure 864). It doesn't prevent some analysts to make quantum computing forecasts per vertical market [4002].

Then we reached $8.45B in 2024 for **Homeland Security** (in 2018), $10B in 2028 for **Morgan Stanley** (as of 2017), $15B by 2028 for **ABI Research** (2018) and $64B by 2030 for **P&S Intelligence** (in 2020). **ResearchAndMarkets** predicted in May 2021 that the global quantum technology market would even reach $31.57B by 2026, including $14.25B for quantum computing [4003]. **IDC** planned for a 6-year compound annual growth rate (CAGR) of 50.9% over the 2021-2027 period with a market reaching $8.6B in 2027[4004].

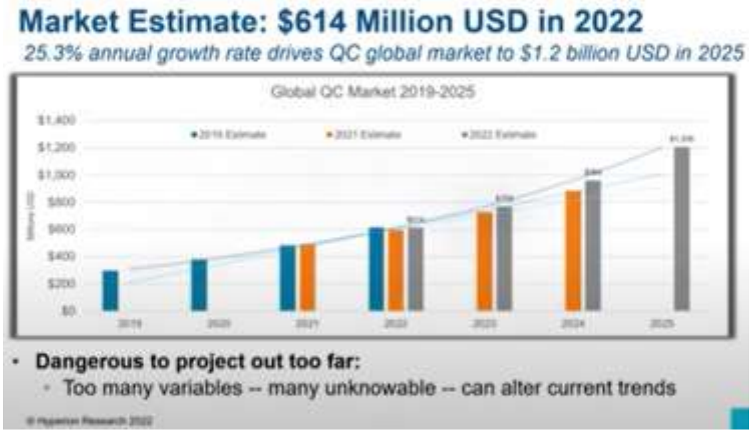


Figure 864: Hyperion Research cautious market estimates for quantum computing that stops in 2025 due to the level of uncertainty. Source: Global Quantum Computing Market Size and Forecast by Bob Sorensen, Hyperion, Q2B Silicon Valley, December 2022 (20 mn).

[3997] See Quantum Computing Market Size, Share and Industry Growth Analysis Report by Offering, Deployment (on-Premises and Cloud), Application (Optimization, Simulation, Machine Learning), Technology (Trapped Ions, Quantum Annealing, Superconducting Qubits), End User and Region- Global Growth Driver and Industry Forecast to 2028, MarketAndMarkets, February 2023.

[3998] Global Market Insights, April 2023.

[3999] See Quantum Computer Market Headed to $830M in 2024 by John Russell, HPC Wire, September 2021. Hyperion's forecasts have ups and downs. See New Study Estimates More Than 20 Percent Annual Growth of Global Quantum Computing Marketplace Through 2024, Hyperion Research, February 2022. The forecast is based on polling 112 quantum computing vendors from around the world and was funded by QED-C and QC Ware. A previous 2020 forecast did plan for a 27% annual increase and now, we are at 21,9%!

[4000] See Hyperion: QC Market Headed to $1.2B in 2025; Lots of Momentum but also Uncertainty by John Russell, December 2022. Their chart on major algorithms by revenue with 16% in cybersecurity algorithms shows that the polled customers don't understand well what quantum computing is all about. The report also says that "*The key factors contributing to the growth of the quantum computing market include the rising adoption of quantum computing technology in various industries and sectors, increasing investments in quantum computing technology, and a surge in the number of strategic partnerships and collaborations for advancements in quantum computing technology are among the factors driving the growth of the quantum computing market*" without mentioning the usefulness of these systems.

[4001] See Global Quantum Computing Market Size and Forecast by Bob Sorensen, Hyperion, Q2B Silicon Valley, December 2022 (20 mn).

[4002] GRD Survey predicted in April 2023 that the healthcare quantum computing market could reach $25B by 2032 and Markets&Markets did a similar prediction in September 2023 of $5.2B for the automotive industry by 2035.

[4003] See The Worldwide Quantum Technology Industry will Reach $31.57 Billion by 2026 - North America to be the Biggest Region, May 2021.

[4004] See Quantum computing market landscape by TBR Research which is reviewing only 26 vendors, including two who have either nothing to sell (Intel) or doing nothing (Nokia) in quantum computing.

In August 2023, it reduced its forecast from $1.1B in 2022 to $7.6B in 2027 with a 5Y CAGR of 48.1%[4005]. Why such a drop? Their estimates were negatively impacted by slower than expected advances in quantum hardware development, generative AI and macroeconomic factors. But $7.8B may still be overly optimistic at the speed of progress in the domain.

At last, **The Quantum Insider** has its own predictions with a total quantum computing market between $300M and $1.3B in 2021 that could grow to between $3.5B and $10B by 2025 and between $18B and $65B by 2030 with a CAGR of between 70% and 80% from 2021 to 2025, to slow down between 39% to 45% between 2025 to 2030[4006].

Some forecasts can reach other crazy heights. For **Bank of America**, quantum technologies will be as important as smartphones. The main reason? Its potential applications in healthcare. To make sure, the point is made, Haim Israel from this bank also touted that quantum computing will be more important that the invention of fire which is a bit stretch[4007].

The only problem: many analyses behind these predictions gets confused between big data and quantum computing[4008]. Some are based on vendors' expectations, others on customers fuzzy plans to adopt quantum computing, given nobody has a real clue of when and how it will work.

As of early 2020, **McKinsey** even predicted that quantum computing would be worth $1T by 2035[4009]. It is easy to identify the forecast bias used here. It is based on a trick that was used a few years ago to evaluate the size of Internet of things and artificial intelligence markets.

It is not the market estimation for quantum technologies as such, but the incremental revenue it could generate for businesses, such as in pharmaceuticals, financial services, or transportation.

It is a bit like evaluating the software market (which was around $593B in 2021, including $237B in enterprise software, source Statista) by summing up the total revenue of the companies who use some software! This would be quite a large number and a significant share of worldwide GDP. In 2022, they reused the same methodology to forecast that the Metaverse would create $5T of value by 2030[4010]. It failed miserably in about a year after the Metaverse was entirely out fashioned by LLMs (Language Learning Models ala ChatGPT).

Market predictions should focus on IT products, software and services and should be compared with existing reference markets. For example, the 2020 worldwide servers market size was $85.7B according to IDC[4011]. The HPC market size was estimated to be $43.6B in 2023 by GlobalData, with an expected future compound growth rate of 7.5%[4012]. Expecting that at some point, the quantum computing market would exceed one of these two markets is preposterous, even considering the discrepancies between hardware and hardware plus software and services markets accounting.

4005 See IDC Forecasts Worldwide Quantum Computing Market to Grow to $7.6 Billion in 2027, IDC, August 2023.

4006 See Quantum Computing Market Size Expects Double-Digit Growth by Matt Swayne, December 2021.

4007 See Quantum Computing Will Be Bigger Than the Discovery of Fire! By Luke Lango, August 2022.

4008 See Quantum computing will be the smartphone of the 2020s, says Bank of America strategist by Chris Matthews, December 2019.

4009 See McKinsey Forecasts Quantum Computing Market Could Reach $1 trillion by 2035, April 2020.

4010 See On the road to change Value creation in the metaverse, McKinsey, June 2022 (77 slides). Il mentions "$120B invested in 2022" but this includes M&As like the $69B acquisition of Activision by Microsoft and only $6B to $8B real investments in startups through venture capital.

4011 IDC quarterly 2020 server market estimates: Q1, Q2, Q3 and Q4 with respectively $18.6B, $18.7B, $22.6B and $25.8B.

4012 See High Performance Computing (HPC) Market Size, Share and Trend Analysis by Region, Component (Server, Storage, Network, Software, Services, Cloud), Deployment, Application and Segment Forecast to 2026, GlobalData, February 2023.

In 2023, McKinsey published an updated version of it market estimates with the same dubious bias[4013]. They forecast a quantum computing size of $9B to $93B in 2040, not far from what a random number generator would create. They upgrade their "value creation" to $1.3T in 2035.

The market value fallacy has a side effect in the quantum hype. Some vendors play on the confusion when they are sizing their market when talking to investors. The fallacy propagates easily like in the innocuous paper on EDAs[4014].

In a 2021 publication[4015], **BCG** estimated the size of the quantum computing market as 20% of its estimated generated value with customers of $850B, ending with a $90B to $170B market captured by technology providers, including software and services... some day after 2040, and a more reasonable $1B to $2B before 2030 and $15B to $30 after 2030. So, we have here an uncertainty based on an unknown estimated with some fuzzy technology capability predictions. The problem is some vendors take these data for a market size in their investor pitch presentations, not as a generated value size[4016]. It is highly misleading.

On its end, The Quantum Daily forecasted in 2021 that the "Quantum Cloud as a Service" (QCaaS) market would reach $26B by 2030 and tried to document its methodology with reminding us that it was based on questionable vendor roadmaps[4017].

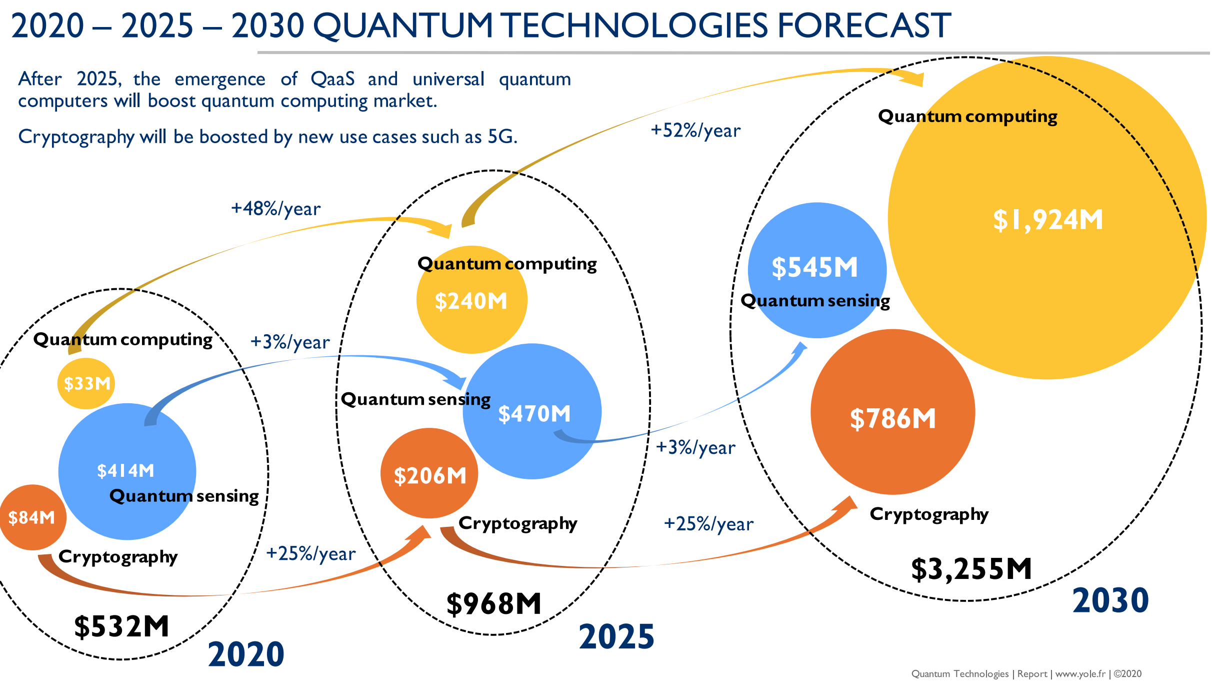


Figure 865: Yole Development's sizing of the quantum technology market by 2030. Source: Quantum Technologies Market and Technology Report 2020 -Sample, Yole Development, 2020 (22 slides).

4013 Quantum Technology Monitor April 2023, McKinsey Quantum-Black team.

4014 See EDA Tools For Quantum Chips by Marie C. Baca, SemiEngineering, November 2022 who writes "*McKinsey & Co. estimates the quantum computing market could reach $700 billion as early as 2035*" showing a clear confusion between a market and value creation.

4015 See What Happens When 'If' Turns to 'When' in Quantum Computing, BCG, July 2021 (20 pages).

4016 Two examples here with the Q2 Rigetti Quarterly report in August 2022 which describes these $850B as a highly ambiguous "*forecasted quantum computing generated operating income*" and Atlantic Quantum Emerges from MIT's Engineering Quantum Systems Lab, Raises $9M Seed Funding to Make Large-Scale Quantum Computing a Reality by James Dargan, from The Quantum Insider that is reusing a press release from Atlantic Quantum, July 2022, saying "*The enterprise quantum computing market could grow to a $450—$850 billion market in the next 15-to-30 years, according to Boston Consulting Group (BCG)*". It says it all about the confusion generated by these market value market data.

4017 See Quantum Computing as a Service Market Sizing - How we dit it, The Quantum Daily, August 2021.

A more detailed market size assessment was made in 2020 by **Yole Development** as shown in Figure 865 with seemingly more reasonable predictions for quantum technologies, with an increase to $3.2B per year by 2030, with 17% average annual growth, including $650M for hardware, $1.37B for cloud-based software and $785M for quantum cryptography (QKD)[4018]. The sensor market would grow from $400M in 2019 to $545M in 2030. This moderate growth seems a bit bearish since quantum sensors are the quantum objects with the lowest technological uncertainties and it is a market in its infancy.

Qualitative assessments

BCG created in 2018 a good inventory of the current potential qualitative use cases of quantum computing per vertical market, in Figure 866.

This covers both case studies coming from D-Wave and prospective applications devises by research labs and with large industry companies including the usual suspects from the aerospace, chemistry, energy, pharmaceuticals and financial sectors.

In another chart, in Figure 867, BCG positions these vertical markets along two dimensions: business value and expected time of quantum advantage. This is more gut feeling than any real rationale thinking since there are too many variables to have any idea of where each of these industries sit in this fancy chart. We are currently at a too early stage of the quantum computing innovation cycle to make such predictions.

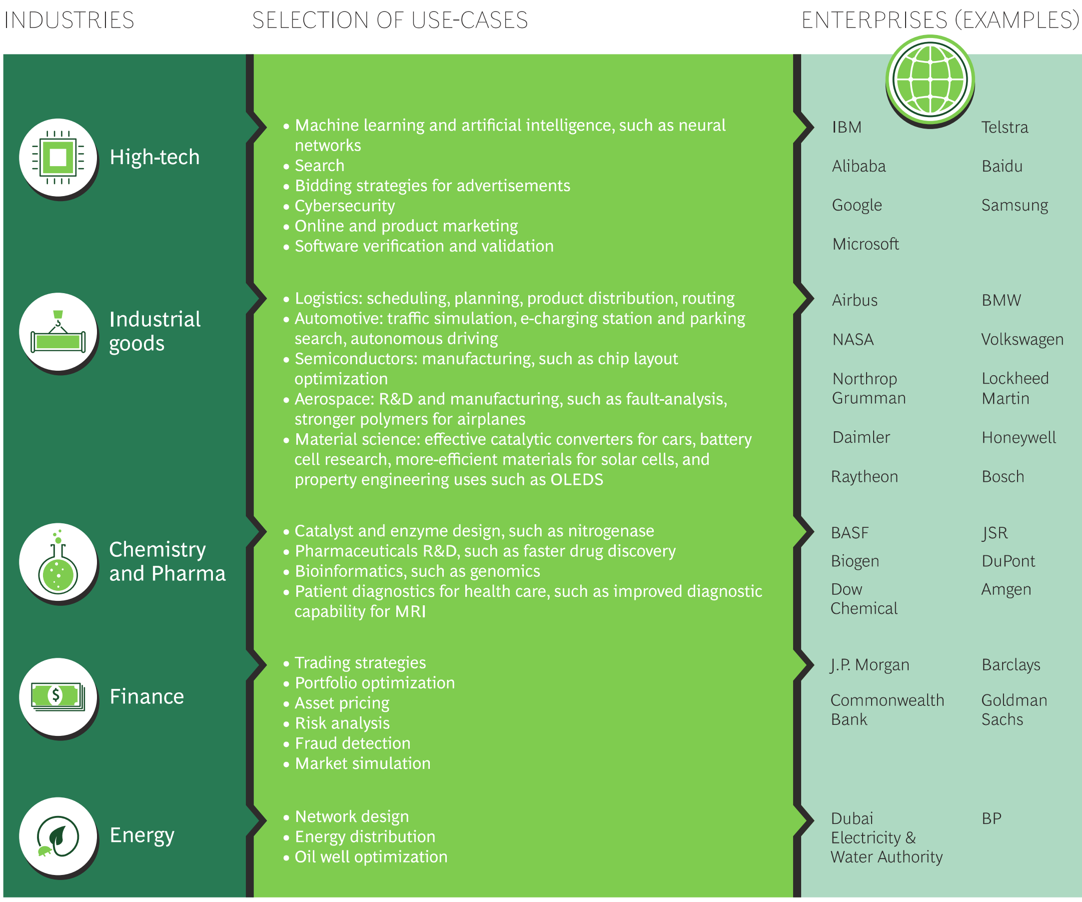


Figure 866: quantum computing use-case scenarios per vertical.
Source: The Next Decade in Quantum Computing and How to Play by Philippe Gerbert and Frank Ruess, BCG, 2018 (30 pages).

[4018] See Quantum technologies: a jump to a commercial state, Yole Development, 2020 and their sample Quantum Technologies Market and Technology Report 2020 -Sample, 2020 (22 slides).

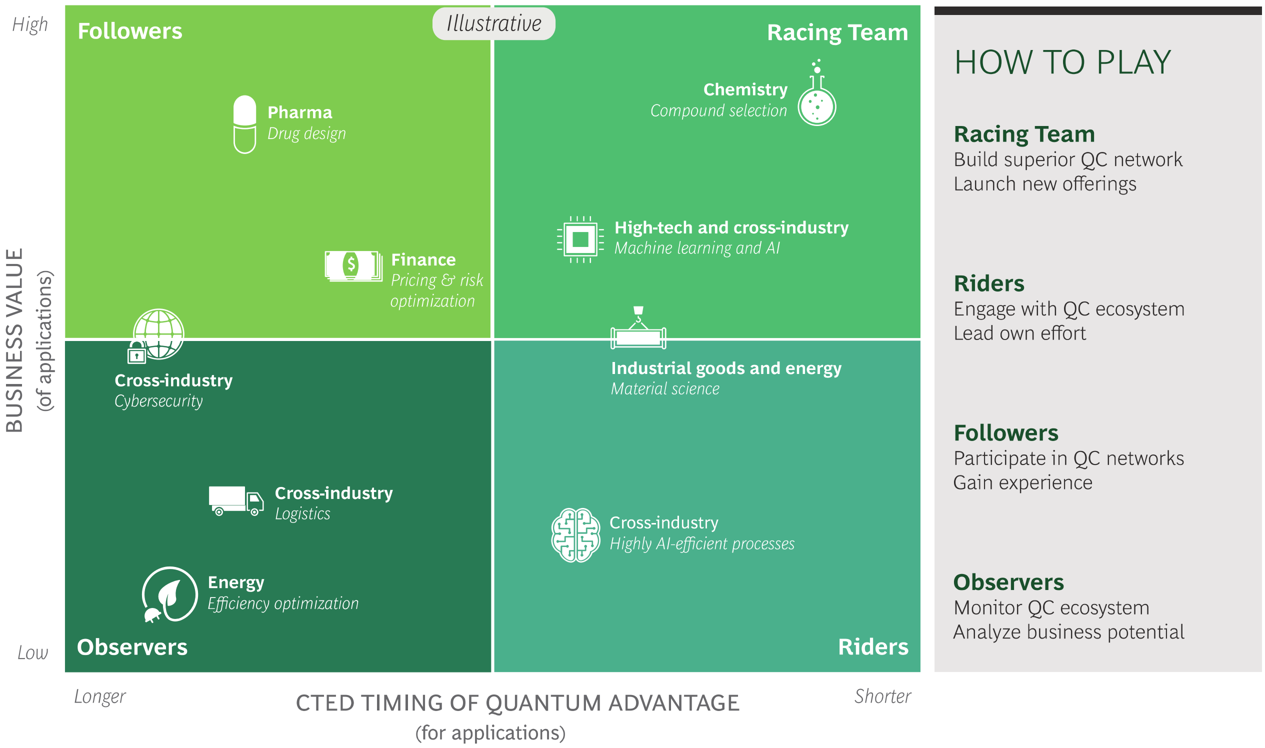


Figure 867: correlation between use cases business value and expected timing for a quantum advantage. Four years later, this raw classification remains valid. Source: The Next Decade in Quantum Computing and How to Play by Philippe Gerbert and Frank Ruess, BCG, 2018 (30 pages).

Market variables

Since we have no real idea of when scalable quantum computing will really work, let's try another exercise to determine the critical factors enabling some sort of technology commoditization for quantum computing:

Technology. The first factor is a mix of where and when we'll have first, useful quantum annealers and analog quantum computers, then useful NISQ systems, and then scalable **universal quantum computers** with more than a hundred logical qubits.

Software Tools. The second factor may be the consolidation of software development tools. These tools will continue to mature, and may raise their level of abstraction, and adapt to hardware evolutions. Libraries adapted to the needs of specific markets will consolidate, as in molecular simulation or finance. As the market matures, there will be some consolidation in this market.

Skills. One critical path to market growth as it has been the case for most previous major technology wave will be the availability of skilled workforce, particularly with developers. They will have, at least at the beginning, to handle abstractions levels that have nothing to do with the different forms of programming techniques that dominate today's computing, even in its event-driven programming variants that are common in the creation of websites and graphic applications. It is more an extension of the existing scientific computing community. A new generation of algorithm designers and developers will emerge. These will probably be young professionals who will have been able to digest new quantum computing concepts with a clean state mind.

Startups. The market will rely mostly on the fabric of startups, probably slightly ahead of traditional software publishers and IT services companies that may not necessarily venture first into this new world of quantum.

Experience. The first feedback from pilot projects, already underway, particularly with D-Wave, will be important. Most recent projects bring interesting learnings on the actual accelerations that quantum computing can provide. We will have to learn to make objective comparisons between quantum algorithms, quantum hardware architectures and their equivalents running (or not) on supercomputers. It will also be necessary to sort out into "proof of concepts" and projects actually deployed.

Mass impact. Finally, quantum computing commoditization will depend on the potential emergence of solutions that will have an impact on our daily lives. So, mostly consumer applications. It could come from healthcare and transportation. Uses cases may shift gradually from the research community to the corporate world, and then to consumer applications.

Healthcare

The healthcare market is one of the most sought after by quantum computing players with the largest number of dedicated startups. It covers of course drug discovery and pharmaceutical industries but also diagnostics, all sorts of treatments optimizations like in radiotherapy and various healthcare systems operations. We'll look at these various fields in this section, showcasing review papers and various case studies, and their related applicability[4019].

Drug discovery

Most major pharmaceutical companies have been exploring and evaluating the potential of quantum computing for a few years, starting by conducting pilot projects with D-Wave quantum annealers and, later on, gate-based quantum computers[4020]. Their dream is to extend the capabilities of current supercomputers to simulate living organisms' molecules "in silico", mainly to create or discover new treatments. This is the field of "*in-silico drug discovery*"[4021].

This quest is linked to the pharmaceuticals industry worrisome situation, that is discovering fewer new treatments and seeing diminished portfolio of commercial patented drugs. The drugs development cycle from discovery to market is becoming increasingly expensive, particularly during clinical trials. It costs up to $1B, if not more, and failure rates are numerous. 45% of cancer therapies clinical trials fail in phase III in the USA, and 97% of the new therapies tested are not approved by the FDA in the USA. If we could better digitally simulate the effects of new treatments before clinical trials, we might be able to increase these success rates. Also, quantum computing could be a critical tool to create digital twins of molecular complexes, used to find the right combinations and optimize their efficiency.

On top of news drugs discovery, pharmaceutical companies are also trying to leverage their existing portfolio with drugs re-targeting. It can speed up clinical trials since their adverse effects are already known. Even though this has not prevented the long controversy surrounding hydroxychloroquine in 2020! In any case, pharmaceutical players need simulation tools and in particular molecular simulation tools: to create molecules, from the simplest (peptides) to the most complicated (proteins, antibodies, vaccines), to model them in 3D, to analyze their interactions between their active sites and targets like cell surface proteins (transmembrane glycoproteins)[4022], and to identify contraindications. Such treatments can be created ex-nihilo, but most often, they are derived from existing ones (known protein, enzyme, bio-inspiration, ...).

[4019] See the general review paper The state of quantum computing applications in health and medicine by Frederik F. Flöther, IBM Quantum, January 2023 (15 pages).

[4020] Like Abbvie, Amgen, AstraZeneca, Bayer, Biogen, Bristol-Myers Squibb, Johnson&Johnson, Merck, Roche, Sanofi and Taleda.

[4021] See A perspective on the current state-of-the-art of quantum computing for drug discovery applications by Nick S. Blunt et al, Riverlane et al, June 2022-March 2023 (75 pages).

[4022] We could try to digitally simulate an entire cell with all its organelles. This would become quite complicated since a living cell comprises about 100 trillion atoms!

Drug design could leverage quantum computing in the future in various areas like with molecular simulations, molecular docking, drugs retargeting and genomics (Figure 868).

Several review papers on drug design using quantum computers point to the enormous time scales for computing even small molecules using either NISQ or FTQC platforms, both in the future tense. Santagati et al explain that NISQ algorithms are way too slow due to the number of requirements shots and measurement[4023] and thus focus only on FTQC architecture[4024]. Baiardi et al mentions the capability to compute vibrational structure[4025].

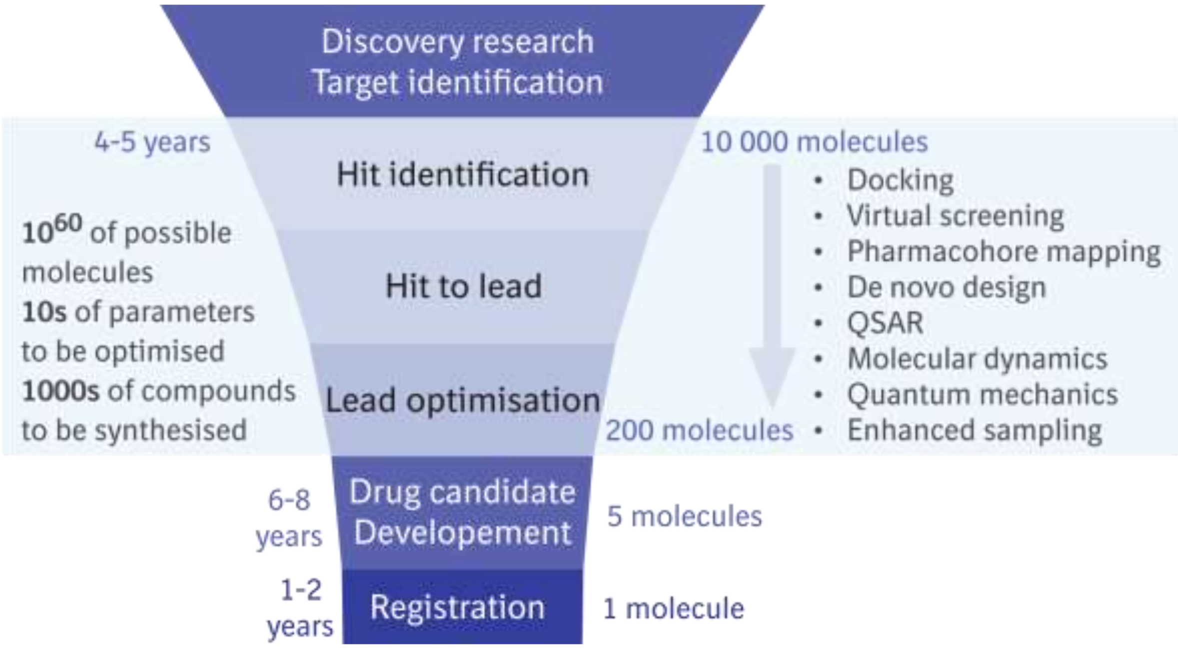


Figure 868: drug design timeline. Source: Drug design on quantum computers by Raffaele Santagati, Alan Aspuru-Guzik, Nathan Wiebe et al, January 2023 (9 pages).

Blunt et al describe the large number of physical qubits required for molecular simulations and provides a good estimation for VQE and QPE qubit and shots requirements for a given example using large-size complete active space configuration interaction (CASCI) calculations. The example relates to the drug Ibrutinib bound covalently to Brutons tyrosine kinase[4026]. The paper mentions Johnson et al[4027] which is proposing an optimization method to run VQE for simple molecules. The results range from 1,300 to 634,915 years of computing, provided you have between 120,000 and 352,000 physical qubits in store. With some optimization, computing time becomes 7.6 days with 3.6 million physical qubits having 99.99% fidelities which currently is science-fiction technology.

Whatever, quantum computing runtime scales in $O(n_o^4)$, n_o being the number of orbitals in the simulated molecule. Even if it doesn't scale exponentially like in classical computing, it can quickly become unreasonable humanlike time.

[4023] See Using Quantum Computers and Simulators in the Life Sciences, Current Trends and Future Prospects, Salil Gunashekar et al, RAND Europe, funded by Novo Nordisk Foundation, 2022 (132 pages) which points to the limitation of NISQ VQE based algorithms.

[4024] See Drug design on quantum computers by Raffaele Santagati, Alan Aspuru-Guzik, Nathan Wiebe et al, January 2023 (9 pages).

[4025] See Quantum Computing for Molecular Biology by Alberto Baiardi et al, ETH Zurich and University of Copenhagen, December 2022 (68 pages) which mentions vibrational structure computation.

[4026] See Perspective on the Current State-of-the-Art of Quantum Computing for Drug Discovery Applications by Nick S. Blunt et al, Riverlane, Journal of Chemical Theory and Computation, November 2022 (23 pages).

[4027] See Reducing the cost of energy estimation in the variational quantum eigensolver algorithm with robust amplitude estimation by Peter D. Johnson et al, Zapata Computing, March 2022 (15 pages).

In 2022, a photonic-based Boson Gaussian sampler from China was used to implement discovery with molecular docking and RNA folding but "*unequivocal quantum computational advantage has not been realized in our experiments due to photon loss*"[4028].

In 2023, a paper from Po-Yu Kao, Alán Aspuru-Guzik et al tried to demonstrate a quantum advantage on small drug discovery task using a Quantum GAN running on 4 and 8 qubits. It was probably tested on a classical emulator that, with 4 or 8 qubits, would be faster and better than any existing QPU. There is no mention of the impact of physical qubit noise on the model. In other words, they found a classical algorithm that beats another classical algorithm and at a very small scale. It is not bad per se but there's no practical scale and no quantum advantage was proven in any way since it is entirely theoretical. The paper doesn't contain any resource estimation for reaching such a practical quantum advantage either in a NISQ or FTQC regime. I'm wondering about the scale of the model and data loading with the qubit number will grow. All of this is misleading on the obtained quantum computing advantage[4029].

In 2023, a team in Russia built a compact discrete variational autoencoder (DVAE) with a Restricted Boltzmann Machine (RBM) of reduced size in its latent layer to implement generative chemistry and drug design on a D-Wave Advantage annealer, generating 2,331 novel chemical structures with potential drug function[4030].

In June 2019, **Merck** announced a three-year partnership with the Karlsruhe, Germany-based startup **HQS Quantum Simulations** for the development of quantum algorithms for chemical simulation. As already mentioned in the part related to SEEQC, **Merck** and SEEQC created together a consortium in 2021 to build a "*commercially scalable application-specific quantum computer designed to tackle prohibitively high costs within pharmaceutical drug development*". The project aw due for completion "*in 18 months*"! We are here in the pure overselling realm.

In September 2022, **Novo Nordisk** (Denmark) made an announcement that was unique in shape and form. It is launching the Novo Nordisk Foundation Quantum Computing Programme with the goal to build Denmark's generic quantum computer in 2034 with a funding of $200M, and obviously to become a platform for chemical simulations. It will fund the Niels Bohr Institute but also embed partners from the USA, the Netherlands and Canada. They will invest first in three different qubit platforms and select the best one after a first 7-year period. Also, in 2023, **Moderna** and **IBM** had a partnership agreement announced in April 2023 also for molecular design, but we'll have to wait some years before IBM quantum computers are useful for such tasks.

The pharmaceutical industry is organized in a couple consortia which help it share best practices, including in quantum computing adoption. The **Pistoia Alliance** was created in Pistoia, Italy, in 2007 by AstraZeneca, GSK, Novartis and Pfizer in 2007. It entertains a quantum computing community. **QuPharm** is a consortium of 17 pharmaceutical companies, including AbbVie, Bayer, GSK, Takeda, and Pfizer that is dedicated to quantum computing.

SandboxAQ announced in June 2023 that it was creating its Bio-Pharma Molecular Simulation Division that will work on drugs discovery using a mix of AI and quantum solutions.

[4028] See A universal programmable Gaussian Boson Sampler for drug discovery by Shang Yu et al, October 2022-March 2023 (15 pages). It is about graph-based clique finding and using a time-bin GBS reminiscent of Xanadu' 2022 experiment.

[4029] See Exploring the Advantages of Quantum Generative Adversarial Networks in Generative Chemistry by Po-Yu Kao, Alán Aspuru-Guzik et al, Journal of Chemical Information and Modelling, May 2023 (12 pages).

[4030] See Hybrid quantum-classical machine learning for generative chemistry and drug design by A.I. Gircha et al, Russian Quantum Center, August 2021-July 2023 (8 pages).

In practice, conventional HPCs helped molecules screening for therapies and to create 3D models of the covid virus and, in particular, of its glycoproteins that cling to the membranes of human cells in order to attack them and enable virus reproducing within cells[4031]. In 2022, a new SARS-CoV-2 main protease (Mpro) inhibitors was discovered using a classical GPU-based HPC using Atlas's platform from Qubit Pharmaceuticals[4032]. In the more or less distant future, quantum computing may have its say in similar pandemics[4033].

As with any new technology, quantum computing specialists must learn to interact with bioinformatics specialists. Fortunately, bioinformaticians are already bridging the gap between molecular biology and computer science and are well positioned to learn quantum methods[4034].

Molecular simulations

Molecular simulations are used to determine various properties of organic molecules based on their atomic description (at the lowest level), nucleotide structure (for DNA and RNA) and amino acid sequences (for enzymes, polypeptides, and proteins). These properties include their energetic ground state, 3D structures and potential interactions[4035].

Molecular simulations rely on the broad field of computational chemistry. The description of the nature of chemical bonds by **Linus Pauling** in 1928 really launched the vast field of quantum chemistry. Chemical bonds describe the way electrons of covalent liaisons are shared between atoms and the shape of their related orbitals. Pauling's work came just after the creation of the **Born-Oppenheimer** approximation in 1927[4036] which simplified Schrödinger's equation for a molecule by separating the nuclei of the atoms from their electrons. The same year, **Llewellyn Thomas** (1903-1992, English) and **Enrico Fermi** (1901-1954, Italian American) created the later-called **Thomas-Fermi model** which describes the electronic structure of multi-atoms systems. The **Hartree method** (1927) and **Hartree-Fock method** (1935) are also used to approximate the Schrodinger's wave function and the energy of a quantum many-body system in a stationary state.

The field of computational chemistry began much later, in 1964, with the creation of the two Hohenberg-Kohn theorems by **Walter Kohn** (1923-2016, Austrian then American) and **Pierre Hohenberg** (1934-2017, French American). This was closely followed by **Kohn-Sham**'s equations from **Lu Jeu Sham** (1938, Chinese) in 1965. They are the basis of **DFT** (Density Functional Theory), a mathematical model that describes the structure of molecules at rest as a function of inter-atomic interactions and the structure of their electron clouds, and in a simpler way than with Schrödinger's equation which manipulates too many variables. Walter Kohn was awarded the Nobel Prize in Chemistry in 1998 for this work, along with **John Pople** (1925-2004, English) who had contributed to the modeling of electronic orbitals in molecules.

DFT was followed by the work of **Martin Karplus** (1930, American), **Michael Levitt (**1947, Israeli-American) and **Arieh Warshel** (1940, Israeli-American) who contributed to the digital modeling of chemical reactions in the 1970s.

[4031] See an example in TACC Supercomputers Run Simulations Illuminating COVID-19, DNA Replication, March 2020.

[4032] See Computationally driven discovery of SARS-CoV-2 Mpro inhibitors: from design to experimental validation by Léa El Khoury, Jean-Philip Piquemal et al, Chemical Science, 2022 (14 pages).

[4033] See Covid-19: Quantum computing could someday find cures for coronaviruses and other diseases by Todd R. Weiss, April 2020.

[4034] See Thirteen tips for engaging with physicists, as told by a biologist by Ken Kosik, January 2020 which describes how to bring physicists and biologists together.

[4035] See Quantum Computing for Molecular Biology by Alberto Baiardi et al, ETH Zurich, December 2022-June 2023 (76 pages).

[4036] The Max Born from the probabilistic explanation of Schrödinger's equation and the Robert Oppenheimer from the atomic bomb.

They were awarded the Nobel Prize in Chemistry in 2013 for their work. The DFT model was also simplified by **Axel Becke** (1953, Canadian) in 1993 with the hybrid DFT. Some quantum versions of DFT are developed for quantum computing and with some progress, even for NISQ QPUs[4037].

Molecular simulation faces quasi-quantum effects related to the continuous vibrations of molecules in their aqueous medium. Chemical bonds oscillate at a femto-second rate, atoms vibrate collectively at a one picosecond rate. On the other hand, more complex chemical processes such as the production and folding of proteins occur on scales ranging from micro-seconds to seconds.

The first small-scale tests of molecular simulation using quantum algorithms were done on D-Wave and superconducting qubits QPUs, using various breeds of variational algorithms. Analog quantum simulators are also machines suitable for simulating the interaction of atoms within molecules.

One approach consists in relying on generic frameworks that can be distributed over classical computing in massively parallel architecture, and then progressively over quantum computing. This is the case of the **Tinker-HP** framework co-created by Jean-Philip Piquemal, cofounder of **Qubit Pharmaceuticals** and that the company plans to extend with hybrid quantum algorithms[4038]. Most other quantum startups like **ApexQubit**, **HQS Quantum Simulations**, **MentenAI**, **ProteinQure** and **Qulab** are indeed adopting hybrid computing models, if only to have something practical to market[4039].

The most common hybrid method is the VQE (Variational Quantum Eigensolver) that is using NISQ quantum hardware, but with various scalability challenges and long execution times[4040].

At this point in time, however, the priority is with quantum inspired algorithms[4041], *aka* classical algorithms based on quantum algorithms[4042]. Quantum and quantum inspired computation complete the vast field of machine learning which is already very common in the discovery of therapeutic molecules[4043]. Today, most molecular simulation calculations using various forms of tensor networks are carried out using algorithms running on classical supercomputers, increasingly using GPGPUs such as those from Nvidia or Google's TPUs.

Molecules simulation can start with simple organic molecules like cholesterol up to protein folding which is many orders of magnitude more complex[4044]. This last feat is therefore bound to be a very long-term one.

Today, we can simulate peptides with about ten amino acids. The best algorithms require a number of qubits that scales with N_a^4, N_a being the number of amino acids[4045].

4037 See Toward Density Functional Theory on Quantum Computers? by Bruno Senjean, Saad Yalouz and Matthieu Saubanère, University of Montpellier and University of Strasbourg, April-October 2022 (19 pages).

4038 See Computational Drug Design & Molecular Dynamics by Jean-Philip Piquemal, April 2020 (28 slides) and Tinker-HP: a massively parallel molecular dynamics package for multiscale simulations of large complex systems with advanced point dipole polarizable force fields by Louis Lagardère, Jean-Philip Piquemal et al, 2018 (17 pages). The company plans to rely first on cold atoms simulators like those from Pasqal.

4039 See Can Quantum Computing Play a Role in Drug Discovery? At least one Startup Thinks so by James Dargan, 2020, which mentions Menten AI.

4040 See Quantum Chemistry and the Variational Quantum Eigensolver by S Kokkelmans et al, December 2019 (56 pages).

4041 See Development of the Quantum Inspired SIBFA Many-Body Polarizable Force Field: I. Enabling Condensed Phase Molecular Dynamics Simulations by Sehr Naseem-Khan, Jean-Philip Piquemal et al, January 2022 (50 pages).

4042 See Quantum and Quantum-inspired Methods for de novo Discovery of Altered Cancer Pathways by Hedayat Alghassi et al, 2019 (27 pages).

4043 See Concepts of Artificial Intelligence for Computer-Assisted Drug Discovery by Xin Yang et al, 2019 (75 pages). A good review paper with 879 bibliographical references!

4044 See Designing Peptides on a Quantum Computer by Vikram Khipple Mulligan, September 2019 (20 pages) which presents Rosetta, a protein quantum design tool running on D-Wave.

4045 See Resource-Efficient Quantum Algorithm for Protein Folding by Anton Robert et al, August 2019.

This simulation is also at the limit of feasibility in terms of complexity because it is in the class of NP-complete problems as seen in the section dedicated to complexity theories, starting on page 935.

Docking

Docking is a method used to predict how one molecule will bind to another one and form a stable complex. It is used to predict the function of a drug on a target. It can be used for de-novo drug design or for drug retargeting purpose, when existing drugs are tested on various targets.

78% of the search for therapies is focused on light molecules of less than 900 Daltons, i.e. about a hundred atoms. Its function is to associate itself with a target in the cells, often a specific protein that controls a metabolism that we want to attenuate or amplify[4046] (Figure 869). The discovery of small molecules of a few dozen atoms could be within the scope of the NISQ quantum computers within a few years. The first molecular simulation experiments were carried out on D-Wave. They work with the search for energy minima, which can be suitable in theory for the simulation of the organization of molecules.

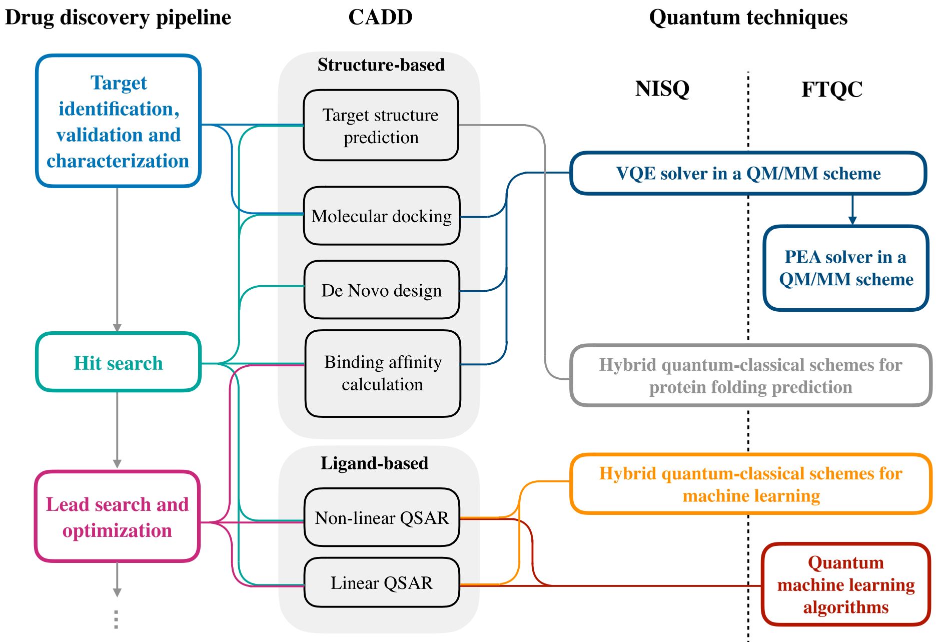


Figure 869: a process flow for drug discovery. CADD = Computer Aided Drug Design.
Source: Potential of quantum computing for drug discovery by Alán Aspuru-Guzik et al, 2018 (18 pages).

A collaboration was launched in June 2017 between **Biogen,** the Canadian quantum software company **1QBit,** and **Accenture** for the creation of new molecules. **Biogen** (1978, USA) is a mid-size biotech company with 7,300 employees specialized in the treatment of neurodegenerative diseases and leukemia.

Their use of quantum computing was aimed at retargeting therapeutic molecules, looking for matching between existing treatments and therapeutic targets in neurodegenerative or inflammatory diseases. **Amgen** is also active in the search for new therapies and is working since 2020 with **QSimulate** (2018, USA, $4M).

A similar project was launched in Spain with the consortium "QHealth: Quantum Pharmacogenomics applied to aging" launched in August 2020 with **aQuantum** (alhambraIT, Prologue Group) with **Gloin**, **Madrija** and various Spanish Universities.

[4046] See Potential of quantum computing for drug discovery by Alán Aspuru-Guzik et al, 2018 (18 pages).

Its goals are to find correlations between physiological and genetic variables, drug usage history, side effects and/or potential lack of response of new drugs to fight aging. They plan to do simulations using quantum algorithms. The project totals 5.1M€ including a grant of 3.7M€ from CDTI awarded in November 2022 by the **Center for the Development of Industrial Technology** (CDTI) of the Ministry of Science and Innovation of Spain.

Molecular docking was tested in gate-based computing with digitized-counterdiabatic QAOA which maps a molecular docking problem onto the maximum vertex weight clique problem, a typical optimization problem. This was however only tested on 6, 8 and 12 qubits, way under any quantum advantage or scalability proof threshold[4047].

A hybrid quantum/classical machine learning algorithm was used for drug retargeting using Ligand Based Virtual Screening (LB-VS). It was developed in 2022 by **IBM** and **The Hartree Center** in the UK[4048].

Genomics

In genomics, the first potential application of quantum computing is with DNA sequencing and sequences alignment. It is about reconstructing a giant puzzle with small parts of DNA sequences which come out of sequencing[4049]. The current sequence alignment quantum algorithms are not yet in the realm of current QPU solutions[4050]. Or they are tested at very small scales like in Sappington et al who tested a RNA folding and dynamics simulation algorithm, but on a 7-qubit IBM quantum computer that can be emulated faster on a simple laptop[4051] and Varsanis et al which tested a DNA sequencing on 5 qubits[4052].

New solutions are proposed to optimize data loading for these use cases[4053]. Quantum computing could also be used to process measurement data coming from electrodes that detect nucleotides during sequencing. But no qubit count provided in the related paper which is probably not in the quantum advantage domain, and works in emulation mode with being probably too slow compared to classical computing[4054].

The **DNA-Seq Alliance** combines the startup DNA-Seq and D-Wave, which also does molecular retargeting by combining genomics, protein kinase crystallography, quantum computing and the search for effective cancer treatments.

[4047] See Molecular docking via quantum approximate optimization algorithm by Qi-Ming Ding et al, August 2023 (10 pages).

[4048] See Quantum Machine Learning Framework for Virtual Screening in Drug Discovery: a Prospective Quantum Advantage by Stefano Mensa et al, The Hartree Centre, STFC and IBM Research, April 2022 (16 pages).

[4049] See QuASeR - Quantum Accelerated De Novo DNA Sequence Reconstruction by Aritra Sarkar et al, TU Delft, April 2020 (24 pages).

[4050] See A Quantum Dot Plot Generation Algorithm for Pairwise Sequence Alignment by Joseph B. Clapis, May 2021 (29 pages) mentioned as "*the quantum Fourier transform (QFT) was applied to pairwise sequence alignment*". But it can't work unless some qRAM enables fast data loading[4050]. Many other references to Grover derived algorithms that won't scale well. Data encoding could be optimized. See Classical-to-Quantum Sequence Encoding in Genomics by Nouhaila Innan and Muhammad Al-Zafar Khan, April 2023 (58 pages).

[4051] See Probabilistic Genotype-Phenotype Maps Reveal Mutational Robustness of RNA Folding, Spin Glasses, and Quantum Circuits by Anna Sappington et al, January 2023 (26 pages).

[4052] See Quantum gate algorithm for reference-guided DNA sequence alignment by G. D. Varsamis et al, August 2023 (19 pages) which was tested on IBM QPUs and Qiskit emulators.

[4053] See Quantum-parallel vectorized data encodings and computations on trapped-ions and transmons QPUs by Jan Balewski et al, Lawrence Berkeley National Laboratory, January 2023 (27 pages).

[4054] See Single-Molecule Identification of Nucleotides Using a Quantum Computer by Masateru Taniguchi, Takahito Ohshiro and Tomofumi Tada, J. Phys. Chem. B, July 2023 (no free access).

Proteins and RNA folding

Proteins folding is the gold standard of molecular simulation. It can help predict the 3D structure and function of proteins based on their amino acids sequence, itself determined by the nucleotide sequence of their RNA and DNA origins. These solutions are competing with AlphaFold, the classical solution from Google DeepMind which can fold larger proteins but with some limitations, particularly for entirely de-novo designs[4055]. Protein folding has four parts: primary, secondary, ternary or tertiary, and quaternary folding mechanisms (Figure 870). The primary is the sequence of amino acids or nucleotides, the second is the first 3D structure level, the third is the subunit 3D assembly structure and the last one is the assembled subunits.

One protein folding algorithm was able to simulate the secondary folding of the 10 amino acid Angiotensin on 22 qubits and a 7 amino acid neuropeptide using 9 qubits, all on an IBM quantum computer and in 2019[4056]. This hybrid protein folding algorithm uses a variational quantum algorithm and a classical genetic algorithm. It scales in polynomial time and computing depth as $O(N^4)$ with N being the number of monomers in a tetrahedral lattice. The paper does not mention the performance of the classical DeepMind AlphaFold that was then available in its first version launched in 2018[4057] nor results from the various CASP (Critical Assessment of Structure Prediction) programs that serves as a benchmark in this field.

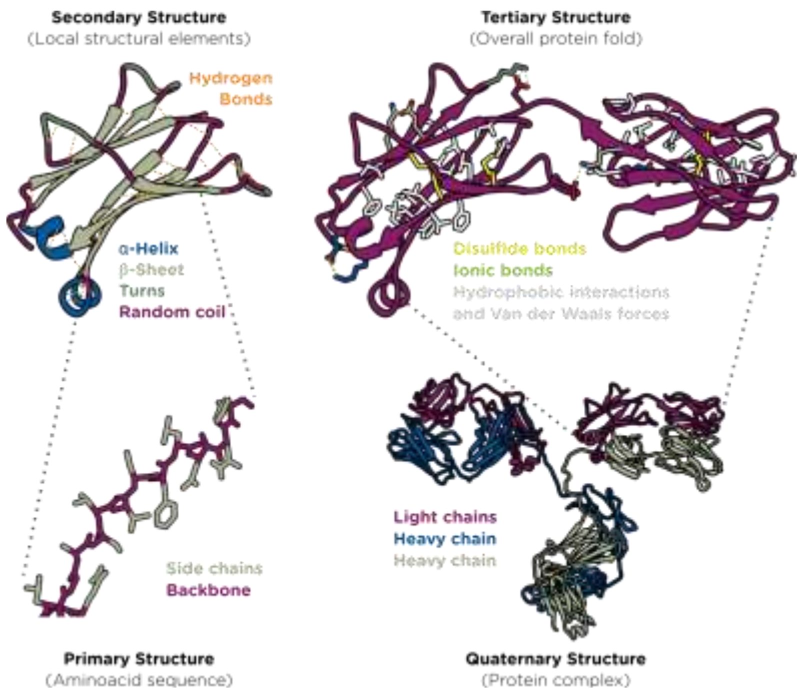


Figure 870: primary, secondary, tertiary and quarternary structure of a polypeptide or protein.
Source: FAQS Higher Order Structure and HOS Comparisons Explained. Added in 2023.

Also, since the algorithm depth scales polynomially with the size of the proteins to fold, it quickly exceeds the capacities of noisy qubit systems as the number of amino acids grows. AlphaFold can fold in ternary structures proteins with up to 450 amino acids way above what the NISQ algorithm mentioned above could implement.

Many other projects have been launched to improve quantum-based protein folding and there are still theoretical since no existing quantum computer is powerful enough to execute them[4058].

[4055] See Hype versus reality: What you can't do with DeepMind's AlphaFold in drug discovery by Katyanna Quach, The Register, September 2022.

[4056] See Resource-efficient quantum algorithm for protein folding by Anton Robert et al, 2021 (5 pages).

[4057] See Highly accurate protein structure prediction with AlphaFold by John Jumper et al, DeepMind, Nature, July 2021 (12 pages).

[4058] See QFold: Quantum Walks and Deep Learning to Solve Protein Folding by P A M Casares, Roberto Campos and M A Martin-Delgado, January 2021-March 2022 (22 pages), Folding lattice proteins with quantum annealing by Anders Irbäck et al, Lund University and Forschungszentrum Jülich, May 2022 (21 pages) which runs on a D-Wave Advantage and Protein Folding Neural Networks Are Not Robust by Sumit Kumar Jha et al, September 2021 (8 pages).

One common algorithm used here is QAOA but its applicability is questioned[4059].

In another 2023 pre-print paper, researchers from Chalmers University of Technology estimated the resources needed to fold proteins using a simplified coarse-grained model. They end up with estimates in the thousand qubits and over 10^8 two-qubit gates which mandates some FTQC architecture with millions of physical qubits at best[4060].

Another test was published in 2021 by GSK[4061]. It was about solving a mRNA codon optimization problem. Each amino acid in a protein sequence can be encoded by as many as six different codons, these series of three DNA/RNA bases encoding one amino acid. The goal was to find the right combination of these codons. The codon selection in mRNA impacts protein folding and functions. The main task is to balance G and C bases in mRNA to optimize gene expression. This was one of the first published case studies using D-Wave Advantage annealer and its 5,000 qubits and their Leap Hybrid Solver. It worked well with 30 amino acids and could scale up to 1,000 amino acids. The codon optimization problem is formulated as a Binary Quadratic Model that is itself close to an Ising model adapted to D-Wave annealers. It did fare well when compared to genetic and machine learning algorithms running on classical computers.

Khatami et al propose a protein design algorithm based on a FTQC Grover algorithm, but "*the results of real quantum computers indicate the need for devices with much lower noise to implement our circuits*"[4062]. Again, what is required are FTQC computers. Chandarana et al propose a hybrid analog-gate-based algorithm but tested only with 9 amino acids, using up to 17 qubits on Quantinuum quantum hardware[4063].

Malone et al tested a VQE protein–ligand docking algorithm but only with 8 qubits, a 19 depth circuit and with no resource estimation for a quantum advantage regime[4064]. Domingo et al tested a hybrid neural network using some quantum error mitigation to improve molecular protein binding affinity predictions, showing only a 40% gain in the training process with 9 qubits. It would require 99.99% gate fidelities and circuits with 300 gates[4065].

RNA folding is also a field of research for quantum algorithms. At this point it covers only the secondary structure and could be implemented in FTQC platforms[4066].

The Holy Grail would be to understand how the assembly and then operations of ribosomes work. These molecular complexes are made of 73 proteins and 4 large RNA molecules. Ribosomes produce all proteins in our cells using messenger RNA code, which is itself synthesized from DNA through

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[4059] See Peptide conformational sampling using the Quantum Approximate Optimization Algorithm by Sami Boulebnane et al, April 2022 (30 pages). Conclusion: "*these results cast serious doubt on the ability of QAOA to address the protein folding problem in the near term, even in an extremely simplified setting*".

[4060] See Resource analysis of quantum algorithms for coarse-grained protein folding models by Hanna Linn et al, November 2023 (18 pages).

[4061] See GlaxoSmithKline Marks Quantum Progress with D-Wave by Nicole Hemsoth, February 2021 pointing to mRNA codon optimization on quantum computers by Dillion M. Fox et al, February 2021 (35 pages).

[4062] See Gate-based Quantum Computing for Protein Design by Mohammad Hassan Khatami et al, January-November 2022 (45 pages). It contains bad faith comparison in page 41, with number of gates for classical and quantum systems, without taking into account gate durations (which are faster in classical) and the number of runs (which is large in quantum versions).

[4063] See Digitized-Counterdiabatic Quantum Algorithm for Protein Folding by Pranav Chandarana, Enrique Solano et al, December 2022 (14 pages).

[4064] See Towards the simulation of large scale protein–ligand interactions on NISQ-era quantum computers by Fionn D. Malone et al, Chemical Science, March 2022 (16 pages).

[4065] See Hybrid quantum-classical convolutional neural networks to improve molecular protein binding affinity predictions by L. Domingo et al, January 2023 (14 pages).

[4066] See Predicting RNA Secondary Structure on Universal Quantum Computer by Ji Jiang et al, May 2023 (19 pages).

an also amazing biochemical process involving many complex molecules. Thousands of ribosomes operate in every living cell and each ribosome is made of about 250,000 atoms[4067].

Diagnostics

Using quantum machine learning algorithms to do image recognition is also investigated. It may help train these systems with less data. However, existing algorithms are currently just on par with their classical peers or with serious limitations[4068 4069].

Landman et al proposed to accelerate the training of classical neural networks for medical images classification with a NISQ algorithm with a similar quality than with classical algorithms and no indication of time and cost[4070]. Toussi Kiani et al showcase a tomography enhanced image reconstruction QFT-based algorithm with an exponential speedup, provided the imaging data is provided as a quantum state, which is not the case in most if not all situations[4071]. Deep learning methods were tested to improve the classification of breast cancer by Moradi et al, using QSVCs and quantum kernel Gaussian process methods and transfer learning based QNNs, but only with 11 and 5 qubits with no indication of the conditions to obtain some quantum advantage[4072].

Another method of cancer detection proposed by Azevedo et al uses angle embedding which is simpler than a full vector state amplitudes embedding and provides better accuracy than classical methods at small scale. The speed was divided by two with only 7 qubits which means it could be even faster with a classical quantum code emulator. And there's no word in the paper on image resolution[4073]. Another paper, from Majumdar et al is using a variety of QML algorithm for histopathology classification with tests done on a PennyLane emulator with up to 7 qubits. Of course, then, with no quantum advantage[4074].

Some Turkish scientists used D-Wave and IBM QPUs to classify CT scans images related to covid diagnosis[4075]. Their quantum software generated 94% to 100% successful classifications while its classical counterpart did achieve 90% successful results. It was using only 4 qubits from 5-qubits systems (IBM Q-Rome and Q-London). It was a hybrid computing using the quantum transfer learning method.

Quantum computing was used only for the classification part at the end of a convolutional network, not for the convolutions that remain classical, using 224x224 pixels versions of the CT scans, using a training set made of 2658 lung CT images with 1,296 COVID-19 and 1,362 Normal CT images.

4067 The number of 2.5 or 3.5 million atoms is often mentioned, but this is not true. These are "Daltons" which are equivalent to one twelfth of the mass of carbon 12, or about the mass of a hydrogen atom. However, these organic molecules contain, in addition to hydrogen, a lot of carbon, nitrogen, phosphorus and oxygen. The latter contribute to a large part of the mass of the molecule, hence the fact that the number of Daltons must be divided by 10 to obtain the number of atoms of an organic molecule.

4068 See Quantum-classical convolutional neural networks in radiological image classification by Andrea Matic et al, April 2022 (12 pages).

4069 See Quantum Biotechnology by Nicolas P. Mauranyapin et al, November 2021 (34 pages).

4070 See Quantum Methods for Neural Networks and Application to Medical Image Classification, Jonas Landman et al, Quantum, 2022 (30 pages).

4071 See Quantum medical imaging algorithms by Bobak Toussi Kiani, Agnes Villanyi and Seth Lloyd, 2020 (12 pages).

4072 See Error mitigation for a clinical data classification on the IonQ and the IBM quantum computers by Sasan Moradi et al, 2022 (15 pages).

4073 See Quantum transfer learning for breast cancer detection by Vanda Azevedo et al, Quantum Machine Intelligence, February 2022 (14 pages).

4074 See Histopathological Cancer Detection Using Hybrid Quantum Computing by Reek Majumdar et al, February 2023 (6 pages).

4075 See COVID-19 detection on IBM quantum computer with classical-quantum transfer learning by Erdi Acar and Ihsan Yilma, November 2020 (16 pages).

They also tried emulators running with PennyLane, Qiskit and Cirq. The classification layer compressed 512 vectors into 4 vectors with a linear transformation.

A classification of covid-19 clinical data used a quantum assisted SVM classification using 16 qubits running on the IBM Q-Melbourne system and implementing feature mapping and hyperplane calculation with a variational quantum classifier[4076]. Classification was using time series of number of cases per counties in the USA. The classical part was done using the scikit-learn framework from Inria, France. In the end, the research team observed that classical methods outperformed QML in accuracy, particularly with a high number of data points (>300).

Developed in China in 2022, **DeepQuantum** is a hybrid quantum deep learning algorithm that was used to make predictions on mutant covid-19 variants[4077]. It was trained using a database of available mutated SARS-CoV-2 RNA sequences. But it used a mere 10 qubits way out of the scope of any quantum advantage.

Another paper, from Kazdaghli et al from **QcWare** and **AstraZeneca** showcases some improvements with the analysis of clinical data with "competitive results" obtained with 10 qubits on an IBM QPU. The quantum algorithm time is scaling linearly with the number of features while its best classical equivalent scales in cubic time which can indicate some potential quantum advantage[4078]. Using a real-life data set, the quantum algorithm would require 140 qubits and a computing depth of 400. It ends up corresponding to physical qubit fidelities of 99.998%, meaning some form of FTQC with at least 100 physical qubits per logical qubits.

Treatments

David Sahner, who created his own consulting firm **Eigenmed**, is one of the promoters of precision medicine based on predictive machine learning techniques using D-Wave annealers[4079].

Omnicom Healthcare did not hesitate to promote in 2017 the use of quantum computing in healthcare in a white paper containing strictly no relevant information on the subject, especially since they seem to confuse machine learning applications analyzing data from connected objects with the ability of quantum computers to manage problems that are intractable by traditional computers[4080].

Network medicine analyzes disease pathogenesis, coupling information coming from Omics databases (protein-protein interaction, genomics data) and environmental factors with Bayesian networks and machine learning tools. The idea is to identify novel disease genes and pathways, their diagnostics, and therapeutics. A 2022 paper from Algorithmiq and a Finland research team outlined how quantum computing could help there. Unfortunately, it didn't contain sizing recommendations for hardware and the problems it could solve[4081].

In 2023, IBM researchers published a good paper listing various use case in cell therapeutics mostly centered around quantum machine learning, as shown in Figure 871[4082].

[4076] Another one: Quantum-Enhanced Machine Learning for Covid-19 and Anderson Insulator Predictions by Paul-Aymeric McRae and Michael Hilke, December 2020 (25 pages).

[4077] See Quantum Deep Learning for Mutant COVID-19 Strain Prediction by Yu-Xin Jin et al, TuringQ, CAS and USTC, March 2022 (34 pages).

[4078] See Improved clinical data imputation via classical and quantum determinantal point processes by Skander Kazdaghli, Iordanis Kerenidis, Jens Kieckbusch and Philip Teare, QcWare and AstraZeneca, March 2023 (13 pages).

[4079] See Predictive Health Analytics by David Sahner, 2018 (54 slides).

[4080] See Exponential Biometrics: How Quantum Computing Will Revolutionize Health Tracking, 2017 (7 pages).

[4081] See Quantum network medicine: rethinking medicine with network science and quantum algorithms by Sabrina Maniscalco et al, June 2022 (15 pages).

[4082] See the review paper Towards quantum-enabled cell-centric therapeutics by Saugata Basu et al, IBM Research, July 2023 (28 pages).

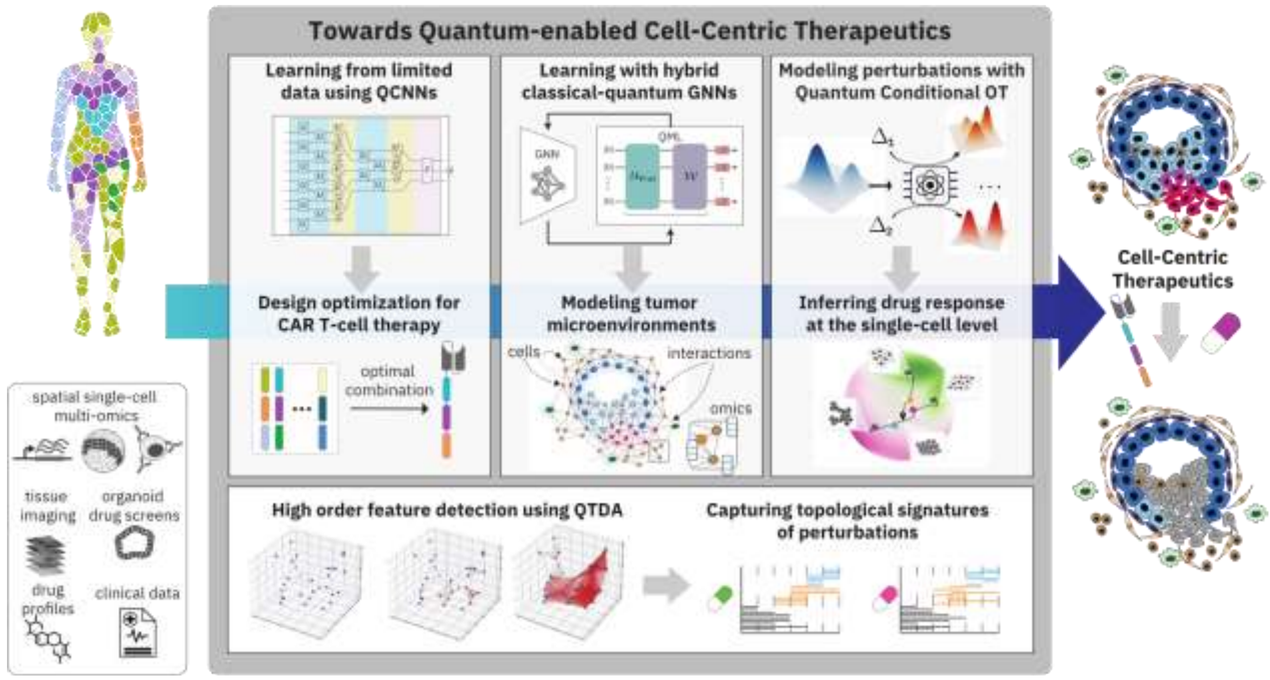


Figure 871: an overview of the various cell therapeutics applications of quantum computing.
Source: Towards quantum-enabled cell-centric therapeutics by Saugata Basu et al, IBM Research, July 2023 (28 pages).

Energy and chemistry

Energy and chemistry are gathered here due to their tight relationship.

Quantum computing could help improve the energetic efficiency of various chemical engineering processes like ammonia and fertilizers production. It could contribute to improving the chemistry of batteries or carbon capture processes.

It can feed innovative materials development research, solving complex analysis and optimization problems, in the in-silico simulation of crystalline molecules and structures[4083]. At last, it could also help energy utilities optimize power grids.

Most of these applications have been prototyped at a low scale but are not yet production-grade. The associated research can help determine the quantum computing needed resources and time to solve these various problems[4084] [4085].

Simulations can also be done with air, water and other liquid flows, and in particular their turbulence. In particular, they can exploit the famous Navier-Stokes equations[4086]. Beyond aerospace applications, it could have some applications in energy production turbines optimizations.

Quantum chemistry 101

Quantum chemistry is about studying the behavior of chemical compounds, molecules and chemical reactions, and strongly correlated systems at the atomic and electronic levels.

[4083] See Quantum hardware calculations of periodic systems: hydrogen chain and iron crystals by Kentaro Yamamoto et al, September 2021 (13 pages) with some potential applications in steel manufacturing.

[4084] See Enabling the quantum leap Quantum algorithms for chemistry and materials Report, January 2019 (115 pages) which provides a good overview of chemical simulation methods. It is a report of a workshop organized by the NSF.

[4085] See Quantum Computing: Fundamentals, Trends and Perspectives for Chemical and Biochemical Engineers by Amirhossein Nourbakhsh et al, January 2022 (28 pages).

[4086] See Quantum Navier-Stokes equations by Pina Milišić from the University of Zagreb, 2012 (12 pages) and Navier-Stokes equations using Quantum Computing, July 2020.

It is relying on the Schrodinger's wave equation and its derivatives, and even, in some circumstances, on the Dirac equation when relativistic electrons are at play. It is an incredibly complicated field that I will hereby summarize, with some usual abundant bibliographical references.

Domains

Quantum chemistry can help in various areas from lower to higher abstraction levels:

Exploring electronic structure and bonding with understanding and quantifying the nature of chemical bonds between atoms and molecular orbitals describing the distribution of electron density and energy levels in a molecule. A typical example is to understand how the water molecule H_2O works.

Understanding molecular structure and properties with predicting and explaining the arrangement of atoms in molecules in space, as well as their electronic, vibrational, rotational and spectroscopic properties. This can for example explain the shape of the water or of a benzene molecule.

Determining energy levels and reactivity with determining the energy levels of electrons in atoms and molecules, predicting chemical reactivity, chemical reaction rates and mechanisms. A molecule energy is determined by it ground state energy, its vibrational and rotational states which are usually also quantized.

Modeling and optimizing chemical reactions with learning the thermodynamics and kinetics of chemical reactions, used to study chemical reaction pathways, catalysis, and leading to chemical reaction optimization techniques. The typical example is to understand the nitrogen capture mechanism with the FeMoCo complex.

Designing and optimizing new materials, with helping researchers design and optimize strongly correlated materials with desired properties. Examples include metals, semiconductors, superconductor materials and magnets.

Simulating complex systems such as proteins or various biological macromolecules and reactions, as we've already seen in the drug design section.

While most known chemistry quantum algorithms deal with determining molecule energy ground states, studying molecules is also about looking for their electronic structure, vibrational and rotational spectrum and their space configuration. Computing these has a complexity that usually scales with the number of electron orbitals in molecules.

Classical quantum chemistry

One of the oldest quantum computational chemistry methods is the **Hartree-Fock method**, created in 1935. It solves the electronic structure problem of molecules with an approximate solution to the Schrödinger equation for a system of interacting electrons in the presence of fixed atomic nuclei. It treats electrons as indistinguishable particles that move in the average field created by all other electrons. It neglects electron correlation effects, assumes that each electron moves independently in the average field and that the many electron wavefunction can be approximated as a single Slater determinant, which is a mathematical determinant formed by filling molecular orbitals with electrons. It provides a good approximation for many molecular systems but fails to consider certain aspects of electronic correlation. Thus, the development of "post-Hartree-Fock methods" that use electron correlation effects and help understand chemical bonding, predict molecular properties, and study the electronic structure of molecules (Figure 873).

DFT is used in quantum chemistry and condensed matter physics to study the electronic structure of molecules, solids, and materials. It computes the electron density, rather than the wavefunction used in the Hartree-Fock method. It is used to describe the ground state properties of a system by determining the electron density distribution. DFT includes average electron-electron correlation effects.

However, it is not a post-Hartree-Fock method that explicitly considers electron correlation effects beyond what is accounted for in the density functional.

Post-Hartree Fock methods include electron correlation by considering multiple electron configurations or including higher-order correlation effects. The most prevalent ones being:

FCI (Full configuration interaction) takes the whole quantum system Hilbert space into account and is the most accurate method used for computing the electronic structure of molecules. In that case, the molecule wave function is expressed as a linear superposition of all electronic configurations also named Slater determinants (see an example in Figure 872).

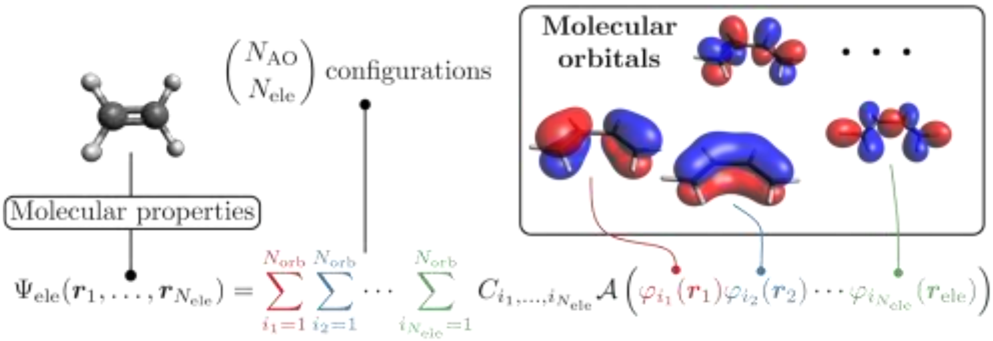


Figure 872: example of a simple butadiene molecule wave function ψ_{ele} that is decomposed with a FCI in a linear combination of antisymmetrized products of the molecule electron orbitals.
Source: [Quantum Computing for Molecular Biology](#) by Alberto Baiardi et al, Quantum for Life Center, December 2022 (68 pages).
Added in 2023.

It is currently limited to about 24 orbitals and 24 electrons due to storage space and time requirement that scale exponentially with the number of electron orbitals due to the number of Slater determinants. This number is computed using a binomial coefficient of the number of possible combinations of choosing n electron positions out of the total k available orbitals. It becomes 10^{46} for a benzene molecule with 12 atoms[4087]!

$$\binom{n}{k} = \frac{n \times (n-1) \times \cdots \times (n-k+1)}{k \times (k-1) \times \cdots \times 1}$$

MBPT (Many-body perturbation theory) incorporates electron correlation effects to improve the accuracy of electronic structure calculations. Since it is computationally demanding as higher-order terms are included, it is often used in combination with truncation schemes, such as the GW approximation or Møller-Plesset perturbation theory (MP2), which approximate the higher-order terms to achieve a balance between accuracy and computational efficiency.

CC (Coupled cluster) expresses the exact wave function in terms of an exponential form of a variational wave function ansatz. It provides a higher level of accuracy by considering electronic excitations up to doublets in CCSD or triplets in CCSD(T), which scale respectively as $O(N^6)$ and $O(N^7)$, N being the number of orbitals. UCCSD (Unitary CCSD) is a variant of CCSD that accounts for a more accurate description of electron interactions but is more demanding computing wise.

MCSCF (Multi-configuration self-consistent field) is a method used to describe molecular systems with nearly degenerate orbitals (different orbital with same energies). It introduces a small number of active orbitals, then the configuration interaction coefficients and the orbital coefficients are optimized to minimize the total energy of the MCSCF state. It can be applied to systems with around 50 active orbitals. Its computing requirement is growing exponentially with the system size.

4087 See [Quantum Chemistry on Quantum Computers](#) by Libor Veis, 2021 (31 pages) which shows how complexity scales with the size of a molecule.

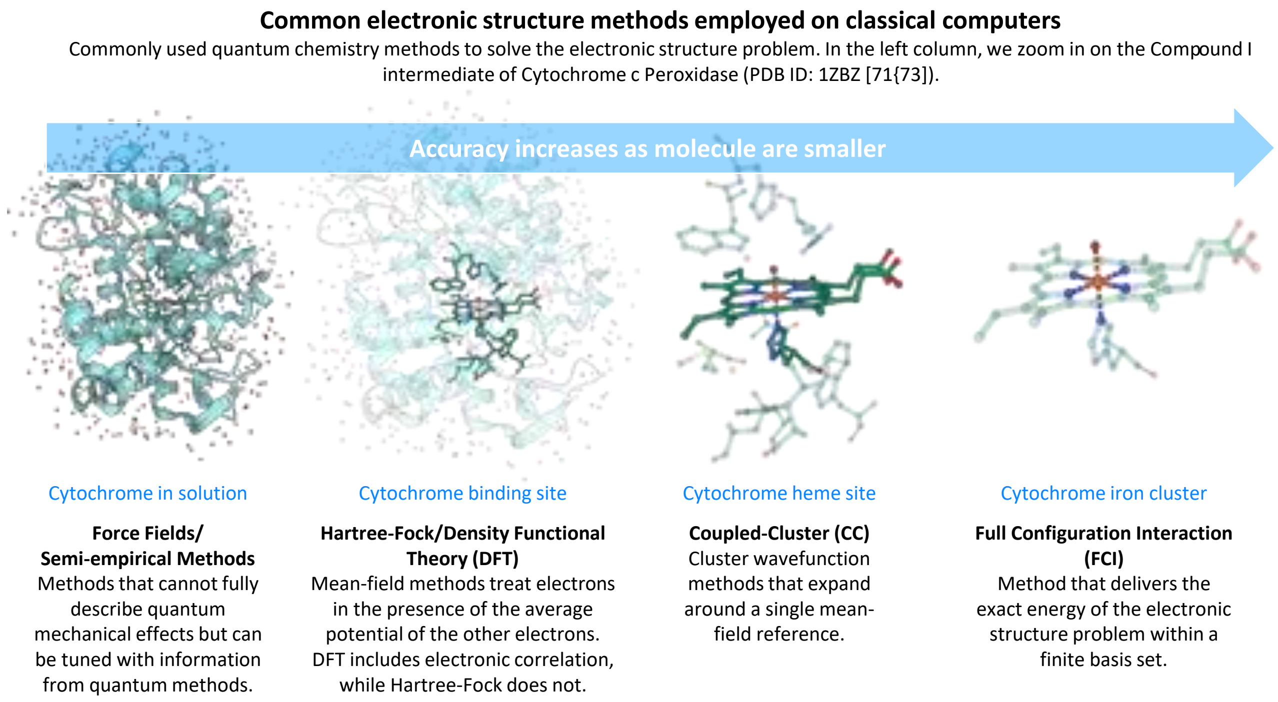


Figure 873: key classical computing electronic structure methods. Inspired from Drug design on quantum computers by Raffaele Santagati, Alan Aspuru-Guzik, Nathan Wiebe et al, January 2023 (9 pages).

CAS (Complete Active Spaces) is a classification of molecular orbitals with core, active and virtual orbitals.

CASSCF (Complete Active Space Self-Consistent Field) is a computational method used to describe the electronic structure of molecules with separating core orbitals not involved in chemical reactions and active orbitals.

DMRG (Density matrix renormalization group) is an algorithm used to solve larger chemical systems of several tens of electrons and is limited by the expressive power of its underlying variational ansatz: the matrix product state (MPS) which is a special instance of the one dimensional tensor network state.

The accuracy of computational chemistry is also improved with the **beyond Born-Oppenheimer approximation** which does not consider that the effects of nuclear motion on electronic structure are decoupled. It is particularly important for molecule vibronic coupling computations.

Trotter decomposition (or trotterization) is used to approximate the time evolution of a quantum system. It breaks down the exponential evolution operator into a series of simpler operators that are easier to compute on a quantum computer. There are various NISQ VQE and FTQC QPE algorithms implementing this technique.

All these classical algorithms are implemented in a great variety of frameworks and software tools from many vendors and widely distributed in open source[4088].

Quantum computing for quantum chemistry

Quantum computing is replacing or complementing classical alternatives, particularly post-Hartree-Fock techniques.

[4088] See Modern Software for Computer Modeling in Quantum Chemistry and Molecular Dynamics by Marina V. Malyshkina and Alexander S. Novikov, 2021 (11 pages).

It better handles molecules wave functions spanning large Hilbert spaces to estimate energies, electric polarization, magnetic dipoles, reduced density matrices and other chemical properties. Most quantum chemical algorithms deal with computing the ground and excited state energies of molecules. They are then used to compute other useful quantities, such as chemical reaction pathways, binding energies and rates of chemical reactions[4089].

As explained by Elfving et al, one must understand the difference between chemical accuracy and chemical precision in chemical simulations[4090].

Chemical accuracy is related to a computational error with respect to an experimental measurement.

Chemical precision is related to a computational error with respect to a computational reference, for example, a sufficiently accurate result obtained with a large basis set. It can be for example an error expressed in kcal/mol compared to the exact solution provided by the combination of the full configuration interaction (FCI) method and a very small basis set that is used as a reference.

The paper also defines when a quantum advantage is useless, due to the availability of accurate experimental results, to the availability of conventional computational results[4091], to an overwhelming real world complexity or to the absence of industrial applications.

Here, we can start with making a distinction with NISQ related algorithms, particularly VQE and its derivatives, and FTQC algorithms like the QPE.

VQE (variational quantum eigensolver) is a variational algorithm already described that is used to determine the energy ground state of a many-body quantum system. The starting point of VQE is an energy that is determined classically and must be close to the solution by less than 1/1,000,000 to get an energy estimate accuracy of a milli-Hartree. Its computing time overhead is significant due to the number of times ansatz must be executed, which scales as $O(N^4/\epsilon^2)$, N being the number of qubits and ϵ, the chemical accuracy[4092], thus the creation of various methods to reduce its overhead either in space (number of qubits and computing depth, like with ADAPT-VQE[4093]) or number of shots (with a clustering of the so-called Pauli measurement strings). Also, practical VQE requires much higher fidelities qubits than the ones available now and in the near future[4094].

4089 See the review paper Quantum Chemistry in the Age of Quantum Computing by Yudong Cao, Alan Aspuru-Guzik et al, 2018 (194 pages).

4090 See How will quantum computers provide an industrially relevant computational advantage in quantum chemistry by Vincent E. Elfving et al, 2020 (20 pages).

4091 For example, utilizing the density-fitting approximation, correlated 90 non-FC electrons among 1,569 orbitals and can be completed on 224 computer cores in 68 hours. They estimate that "*a state-of-the-art quantum algorithm strategy like k-UpCCGSD in a VQE approach for the same situation with chemical precision in the same basis would require over 3000 physical qubits and take completely prohibitive time, i.e. centuries per single iteration of the optimizer, even with severe approximations like the paired-electron assumption, pUCCD*".

4092 See Benchmarking the Variational Quantum Eigensolver through Simulation of the Ground State Energy of Prebiotic Molecules on High-Performance Computers by P. Lolur, October 2020-January 2021 (9 pages). It concludes that "*to utilize VQE and achieve near chemical accuracy will be extremely challenging for NISQ processors due to the large number of Pauli terms*".

4093 See An adaptive variational algorithm for exact molecular simulations on a quantum computer by Harper R. Grimsley, Sophia E. Economou, Edwin Barnes & Nicholas J. Mayhall, Nature, 2019 (9 pages). ADAPT-VQE is an optimization of VQE with a minimum ansatz, using an approximate FCI with arbitrary chemical accuracy. Its gradient optimization enables shorter circuits at the price of increased shots and measurements. See also A self-consistent field approach for the variational quantum eigensolver: orbital optimization goes adaptive by Aaron Fitzpatrick et al, Algorithmiq and Trinity College, December 2022 (21 pages).

4094 See Variational quantum chemistry requires gate-error probabilities below the fault-tolerance threshold by Kieran Dalton et al, University of Cambridge and Hitachi, November 2022 (16 pages) which states that "*for a wide range of molecules, even the best-performing VQE algorithms require gate-error probabilities on the order of 10^{-6} to 10^{-4} to reach chemical accuracy. This is significantly below the fault-tolerance thresholds of most error-correction protocols. Further, we estimate that the maximum allowed gate-error probability scales inversely with the number of noisy (two-qubit) gates. Our results indicate that useful chemistry calculations with current gate-based VQEs are unlikely to be successful on near-term hardware without error correction* ».

VQE can be combined with Monte Carlo sampling to reduce the computing workload[4095] [4096]. As a warning, most chemical simulations using VQE have been implemented with a small number of qubits, thus not demonstrating any form of quantum advantage[4097] [4098]. This is the case with this work from Boeing and IBM researchers about studying chemical reactions leading to corrosion, with the aim to create corrosion-resistant materials. It was a VQE variation tested with fewer than 20 qubits with no indication of a regime providing some quantum advantage[4099].

QPE (quantum phase estimate) is a FTQC algorithm to compute the total energy of molecule. Its scaling also requires some optimization due to the number of gates[4100]. Interestingly, the time scales of these chemical simulations are prohibitive[4101].

Vibrational energy can be computed with NISQ VQE and FTQC QPE algorithm variations. But its computing time seem prohibitive in both regimes[4102].

Scattering in chemical reactions can compute collisions between atoms and molecules using VQE, with applications in combustion simulations and gas-phase chemical reactions[4103].

Quantum Machine Learning is another technique that is studied to compute the ground state of molecule, provided some good training data is available[4104]. The benefits are unclear at this point[4105].

4095 See Quantum computing quantum Monte Carlo with hybrid tensor network toward electronic structure calculations of large-scale molecular and solid systems by Shu Kanno et al, March 2023 (27 pages).

4096 See Quantum-enhanced quantum Monte Carlo: an industrial view by Maximilian Amsler et al, QUTAC with BASF, Volkswagen, Bosch, BMW and Merck, January 2023 (16 pages). Says VQE can't work. Use Auxiliary-field quantum Monte Carlo (AFQMC) ab-initio method with $CuBr_2$, H_4 and ozone. As a molecular test case, they calculate relative energies of ozone and singlet molecular oxygen with respect to triplet molecular oxygen. Average results are obtained with only 8 qubits with no indication of how this would scale with a larger number of qubits in the quantum advantage realm.

4097 See Ab Initio Transcorrelated Method enabling accurate Quantum Chemistry on near-term Quantum Hardware by Werner Dobrautz et al, March 2023 (11 pages) which computes bond lengths, dissociation energies, and vibrational frequencies of H_2 et LiH simple molecules, with 4 and 6 qubits.

4098 See Experimental quantum computational chemistry with optimised unitary coupled cluster ansatz by Shaojun Guo, Jian-Wei Pan et al, December 2022-February 2023 (37 pages) which produces better results for the usual small test molecules (H_2, LiH, F_2) ground-state energy computation, but with only 4 to 12 qubits from a Zuchongzhi 2.0 superconducting 66 qubits processor using some QEM (quantum error mitigation).

4099 See Quantum computation of reactions on surfaces using local embedding by Tanvi P. Gujarati et al, Boeing and IBM Research, npj Quantum Information, 2023 (10 pages).

4100 See Localized Quantum Chemistry on Quantum Computers by Matthew Otten et al, JCTC, 2022 (13 pages) which provides QPE algorithms resource estimation when coupled with a variational algorithm. The number of gates scales in N instead of N^5 for QPE, N being the number of electron orbitals. But it only works with linear chains of H_2 molecules!

4101 See Fault-tolerant quantum computation of molecular observables by Mark Steudtner et al, PsiQuantum, QcWare and Quantum Lab, March 2023 (29 pages) and Fault-tolerant quantum simulation of materials using Bloch orbitals by Nicholas C. Rubin, Ryan Babbush et al, PRX Quantum, February-October 2023 (52 pages) with resource in the thousands to hundreds of thousand logical qubits and thousand days of computing time.

4102 See Near- and long-term quantum algorithmic approaches for vibrational spectroscopy by Nicolas P. D. Sawaya et al, September 2020-February 2021 (49 pages) which gives an indication of the number of Pauli strings. In the quantum advantage regime above 40 qubits, it scales between 10^4 and 10^7. See Runtime optimization for vibrational structure on quantum computers: coordinates and measurement schemes by Marco Majland et al, Kvantify Aps, November 2022 (10 pages) which provides some runtime optimizations.

4103 See A hybrid quantum-classical algorithm for multichannel quantum scattering of atoms and molecules by Xiaodong Xing et al, April 2023 (11 pages) uses a VQLS algorithm with resource estimation of 147K to 2.56M Pauli strings and 26 to 98 physical qubits. No indication of qubit fidelities requirements but we can guess it goes beyond existing qubit capabilities.

4104 See A real neural network state for quantum chemistry by Yangjun Wu et al, January 2023 (6 pages) which was successfully run on… 30 qubits.

4105 See Quantum machine learning for chemistry and physics by Manas Sajjan et al, Purdue University, Chem Soc Rev, March 2022 (99 pages) with 768 bibliographical references! It studies tomographic preparation of quantum states in the matter, classification of states and phases of matter, electronic structure of matter, force field parameterization for molecular dynamics and drug discovery pipeline. Quantum machine learning benefits are still unclear in the chemical simulation context. There is no clear differentiation between NISQ and FTQC and their resource estimates and computing times in this review paper.

Jordan-Wigner transformations map fermionic creation and annihilation operators onto spin operators, which are easier to manipulate in a quantum computer since they are constructed with Pauli operators X, Y and Z, also noted σ_x, σ_y and σ_z. This transformation is useful for the simulation of fermionic systems using quantum computers in both gate-based and annealing mode. It can for example be used by a VQE algorithm to compute a ground-state energy or molecular spectra for a given molecule[4106].

cc-pVDZ and cc-pVTZ basis are types of basis sets from the Dunning basis used in quantum chemistry calculations to approximate with high precision the electronic structure of molecules. They are among a choice of hundreds other basis sets[4107]. The weird acronyms stand for "correlation-consistent polarized valence double-zeta" or "triple zeta" basis. Why zeta? Because the basis sets are labelled with the Greek letter zeta (ζ) given the larger the zeta, the larger the width of the orbital, and the smaller, the most diffuse it is. It is called a minimal basis set with only one basis function for each atomic orbital. The cc-pVDZ/VTZ basis sets consist of Gaussian-type orbitals used to describe the spatial distribution of electrons within a molecule. These basis functions are centered around each atomic nucleus and are combined to represent the molecular orbitals. The "double-zeta" in cc-pVDZ refers to including two sets of orbitals per atomic center: one set with a larger spatial extent to capture the core electrons not involved in chemical bonding and another one that is more diffuse and polarized to describe the valence electrons involved in chemical bonding. The triple VTZ basis set adds another orbital set per atomic center which improves quantum simulations precisions. The "correlation-consistent" aspect of the cc-pVDZ/VTZ basis sets means that they have been optimized to provide accurate descriptions of electron correlation effects. The cc-pVDZ/VTZ basis sets are used in quantum chemistry calculations, such as Hartree-Fock calculations, post-Hartree-Fock methods, and density functional theory (DFT), to obtain accurate electronic structure information and molecular properties. Quantum simulations with the cc-pVTZ basis set is very demanding in number of logical qubits number and fidelities[4108].

Development tools

There are already many development tools available to run quantum chemistry computations. **OpenFermion** from Google and Rigetti and **myQLM-fermion** from Eviden[4109], among others. Plus **TenCirChem**[4110] and **ChemiQ**[4111] which both come from China.

Many other quantum computing hardware and software startups propose development frameworks for quantum simulation of matter and chemical processes. They are mentioned in a later section with software vendors. We have for example **HQS** (Germany), **Quantinuum** who launched their InQuanto computational chemistry platform coming from CQC[4112], **Pasqal** (France, thanks to their 2021 M&A

[4106] See Low-depth simulations of fermionic systems on square-grid quantum hardware by Manuel G. Algaba et al, IQM, February 2023 (18 pages) which implements Jordan-Wigner transformations with circuit optimization working with nearest neighbor qubit connectivity.

[4107] See the Basis Set Exchange web site.

[4108] See Fault-tolerant resource estimate for quantum chemical simulations: Case study on Li-ion battery electrolyte molecules by Isaac H. Kim et al, PsiQuantum, Physical Review Research, April 2021 (26 pages).

[4109] See Open source variational quantum eigensolver extension of the quantum learning machine for quantum chemistry by Mohammad Haidar, Marko J. Rančić, Thomas Ayral, Yvon Maday and Jean-Philip Piquemal, WIREs Computational Molecular Science, March 2023 (26 pages).

[4110] See TenCirChem: An Efficient Quantum Computational Chemistry Package for the NISQ Era by Weitang Li et al, March 2023 (64 pages).

[4111] See ChemiQ: A Chemistry Simulator for Quantum Computer by Qingchun Wang et al, December 2022 (7 pages). In C++.

[4112] See Quantinuum launches InQuanto, a state-of-the-art quantum computational chemistry software platform using quantum computers, 2022.

with Qu&Co), **Good Chemistry** (Canada), **Menten AI** (USA), **Q1t** (the Netherlands), **Qsimulate** (USA), **Qunasys** (Japan), **Riverlane** (UK) and **Zapata Computing** (USA).

Chemical engineering

Most quantum chemical use case applications start with simulating molecules, and then, potentially, chemical reactions. Current quantum computers can compute the ground state of the electronic Hamiltonian of molecules with a couple atoms like H_2O or LiH but without any quantum advantage[4113]. In September 2017, **IBM** simulated on a 16-qubit superconducting quantum computer a set of beryllium hydride molecules and their minimum energy balance[4114]. It went up to 12 atoms from computing the energy state of the benzene molecule under deformation, which was simulated with a 35 qubits quantum emulator, by Total and Jean-Philip Piquemal (CNRS)[4115 4116]. But determining the ground state of the same molecule with no deformation would require 72 qubits and running 330,816 Pauli strings in a VQE algorithm with prohibitive computing times in the centuries scale[4117].

Some progress is achieved with quantum annealing in chemical engineering. A 2023 paper from Mizuno et al from the Hokkaido University dealt with finding optimal chemical reaction pathways using D-Wave annealers[4118]. The conclusions were wishy-washy, indicating as usual that the necessary number of physical qubits to compute a pathway-finding problem increases quadratically as the number of logical binary variables increases and the computation time rapidly increases as well as the problem size increases. They find that computation time scaling can be reduced if a relative error in a cost value is allowable and if the maximum penalty strength to the minimum cost hardly increases with respect to the problem size.

Also, a 2023 chemical space virtual screening was implemented on a 512-qubit D-Wave annealer by an LG research team, concluding that it was effective, but without comparing it to classical methods[4119].

Some chemical industry corps are investigating quantum computing use cases. **Dow Chemical** has been a **1Qbit** partner since June 2017 for pilot projects in quantum chemical simulation. **Mitsubishi Chemical** and the **Materials Magic** subsidiary of **Hitachi Metals** are also testing quantum computing with IBM. In 2023, Mitsubishi published a paper on the design of new OLEDs with greater efficiency using a mix of VQE and QAOA algorithms, running on … 6 qubits[4120]. So, again, not at all in a quantum advantage regime.

Covestro (Germany), a polymer chemical production company and QC Ware started a 5-year collaboration in 2022 to use quantum algorithms for the discovery of new materials and catalysts on NISQ hardware.

[4113] See Is there evidence for exponential quantum advantage in quantum chemistry? by Seunghoon Lee, Ryan Babbush, John Preskill et al, August 2022 (81 pages). The study says that exponential speedups are not generically available.

[4114] See Hardware-efficient variational quantum eigensolver for small molecules and quantum magnets, October 2017 (22 pages).

[4115] See Calculating the ground state energy of benzene under spatial deformations with noisy quantum computing by Wassil Sennane, Jean-Philip Piquemal and Marko J. Rancic, March 2022 (11 pages).

[4116] See Open Source Variational Quantum Eigensolver Extension of the Quantum Learning Machine (QLM) for Quantum Chemistry by Mohammad Haidar, Jean-Philip Piquemal et al, June 2022 (39 pages)

[4117] See Creating and Manipulating a Laughlin-Type ν=1/3 Fractional Quantum Hall State on a Quantum Computer with Linear Depth Circuits by Armin Rahmani et al, PRX Quantum, 2020 (7 pages) and Probing Geometric Excitations of Fractional Quantum Hall States on Quantum Computers by Ammar Kirmani et al, PRL, July 2022 (6 pages).

[4118] See Finding Optimal Pathways in Chemical Reaction Networks Using Ising Machines by Yuta Mizuno and Tamiki Komatsuzaki, Hokkaido University, August 2023 (24 pages).

[4119] See Virtual Screening of Chemical Space based on Quantum Annealing by Takuro Tanaka et al, LG, July 2023 (11 pages).

[4120] See Quantum-Classical Computational Molecular Design of Deuterated High-Efficiency OLED Emitters by Qi Gao et al, Mitsubishi Chemical Corporation and IBM, Intelligent Computing, June 2023 (10 pages).

They started to work in 2021 and published some results on the way to reduce the quantum computing resources required for simulating and designing new materials and chemical processes and to compute energy gradients[4121]. In 2022, they used a regularized compressed double factorization (RC-DF) method to classically compute compressed representations of molecular Hamiltonians to prepare NISQ quantum simulation with improvements vs classical methods.

Their experiment did run with 12 to 20 qubits which is still way below any quantum advantage[4122].

Fertilizer production

A famous chemical process that is said to potentially benefit from quantum computing is the one involving the famous FeMoCo chemical complex that operates in bacteria nitrogenase. These ammonia producing bacteria are either independent and live in soils and water, or in legume roots nodules[4123]. This natural process fixes air nitrogen (N_2) to produce ammonia (NH_3) in a very energetic efficient manner and at ambient temperature. Ammonia is a natural fertilizer that contributes to plant growth. This natural process accounts for about 48% of global nitrogen fixation, with 14% happening in soils and 34% in oceans. It includes natural recycling with dead plants, which also happens in compost production and permaculture.

Another natural ammonia source is lightning. It converts air nitrogen into nitrous oxide, which then reacts with oxygen to form nitrogen dioxide, reacting with water in clouds to form nitrates (NO_3) that gets into soil by rain. It however accounts for only 1% of total nitrogen fixation. Another lesser known source of ammonia is combustion in various engines (cars, trucks, industry, aircraft) that fix nitrogen fixation in a pathway similar to lightning. It accounts for about 7% of total nitrogen fixation[4124].

Finally, industry chemical production of ammonia represents 29% of nitrogen fixation, using the dominant Haber-Bosch process which uses N_2 coming from the atmosphere and hydrogen coming from methane transformation (CH_4) using natural gas-based steam reforming or coal gasification[4125]. H_2 production also rarely comes from water electrolysis using fossil fuel or decarbonated renewable energy sources.

The Haber-Bosch process uses a catalyst that is usually some iron doped with potassium and fixed on silica or alumina (Figure 874).

4121 See Local, Expressive, Quantum-Number-Preserving VQE Ansatze for Fermionic Systems by Gian-Luca R. Anselmetti et al, May 2021 (26 pages) and Analytical Ground- and Excited-State Gradients for Molecular Electronic Structure Theory from Hybrid Quantum/Classical Methods by Robert M. Parrish et al, October 2021 (23 pages).

4122 See Accelerating Quantum Computations of Chemistry Through Regularized Compressed Double Factorization by Oumarou Oumarou et al, Covestro and QcWare, December 2022-May 2023 (14 pages).

4123 Legume roots nodules are a hundred times more efficient in nitrogen fixation than free bacteria. It explains why you need to plant legumes like peas or beans alternatively with tomatoes in your garden. Tomatoes are draining the nitrogen fixated in the soil while legumes are reconstituting the soil nitrogen. The alternative to this crop rotation is to renew soil with a lot of compost and artificial fertilizers.

4124 See Ammonia Technology Roadmap, IEA, October 2021 (168 pages) and The global nitrogen cycle in the twenty-first century by David Fowler et al, The Royal Society Publishing, July 2013 (13 pages).

4125 Ammonia can also be produced with other processes, but these have many disadvantages like the byproduction of acid rain or release of cyanide compounds and are thus not widely used. One consists in combining N_2 and O_2 from the atmospheric air to create nitric oxide (NO) at 3000°C and nitric dioxide (NO_2) after some sudden cooling. The result is dissolved in water to create HNO_3 which in turns helps produce fertilizers. A second one involves fixating nitrogen as calcium cyanamide ($CaCN_2$ *aka* nitrolin) using heated calcium carbide (CaC_2) at 800°C to 1,000°C. Nitrolin is added in soils and is then naturally processed to be turned into ammonia. A last one fixates atmospheric nitrogen as nitrides with using hot oxides like bore, aluminum, magnesium or silicon and coke, and are then decomposed into ammonia by water.

It is highly energy-consuming particularly for its H_2 production part and due to the heat and pressure needed in the main reactor (400°C to 650°C and 100 to 400 bars)[4126].

About 70% of industrial ammonia is used in the production of ammonia-based fertilizers[4127] with the rest being exploited in plastics, synthetic fibers and explosives production. Ammonia industrial production is currently responsible for about 6% of CO_2 global emissions[4128]. Its total energetic yield is less than 5% while the natural ammonia production yield in bacteria is said to be over 95%, using photosynthesis as primary energy sources. Fertilizers are used in agriculture throughout the world to improve production yields and feed the world.

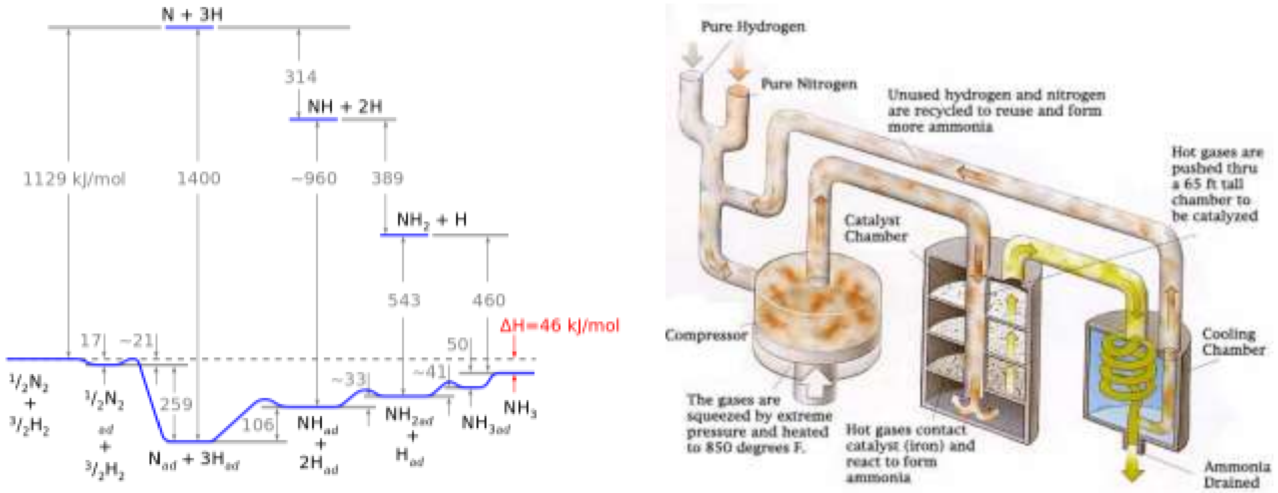


Figure 874: the usual Haber-Bosch process. Source: Catalysis How Dirt and Sand Catalyze Some of the Most Important Transformations, by Justin J. Teesdale, Harvard Energy Journal Club, September 2017.

The fertilizers industry is trying to improve the yield of existing Haber-Bosch based production and to reduce its carbon footprint. It can be based on the invention of new catalysts that could operate the process at lower temperature and pressure (Figure 875). One example are Fe/K mixtures supported on carbon nanotubes.

Other unrelated improvements deal with reducing the cost of H_2 production using for example decarbonated renewable energy sources[4129].

One talked-about option starts with better understanding how nature operates and then could rely on biomimetism to invent new more energy efficient ammonia production processes. That is where FeMoCo comes into play. It is however way more complicated than usually presented by the promoters of the benefits of quantum computing in that field.

In nature, nitrogen exists in the highly oxidized form of nitrate ions (NO_3^-) up to a highly reduced state of ammonia (NH_3), with intermediates like NO_2^-, $N_2O_2^{2-}$, N_2 and NH_2OH. Nitrates are the most degraded form of nitrogen that is said to pollute soils in intensive agriculture.

[4126] However, regular steam methane reforming (SMR) can help capture CO_2 and lead to a very low carbon footprint fertilizer production. Its CO2 intensity is of only 0.1 tons of CO2 per produced ammonia ton while coal gasification originated H_2 has a carbon footprint of 3.2 t CO_2/t. China is the first fertilizer producer and consumer in the world with 29% of global production and is mainly using coal gasification in the process. New projects to use electrolysis, pyrolysis and carbon capture and storage that have very low carbon footprint account for less than 3.5% of existing production of ammonia. Source: Ammonia Technology Roadmap, IEA, October 2021.

[4127] There are other categories of useful plant growth fertilizers like potassium and phosphorous based fertilizers.

[4128] Computed using the following basis: 450 Mt of CO_2 for ammonia production, 170 Mt CO_2 coming from electricity generation and chemical reaction taking place when fertilizers are applied to soils, and 11 Gt worldwide total CO_2 emissions per year. Source: Ammonia Technology Roadmap, IEA, October 2021.

[4129] See Towards sustainable agriculture: Fossil-free ammonia by Peter H. Pfromm, Journal of Renewable and Sustainable Energy, 2017 (12 pages).

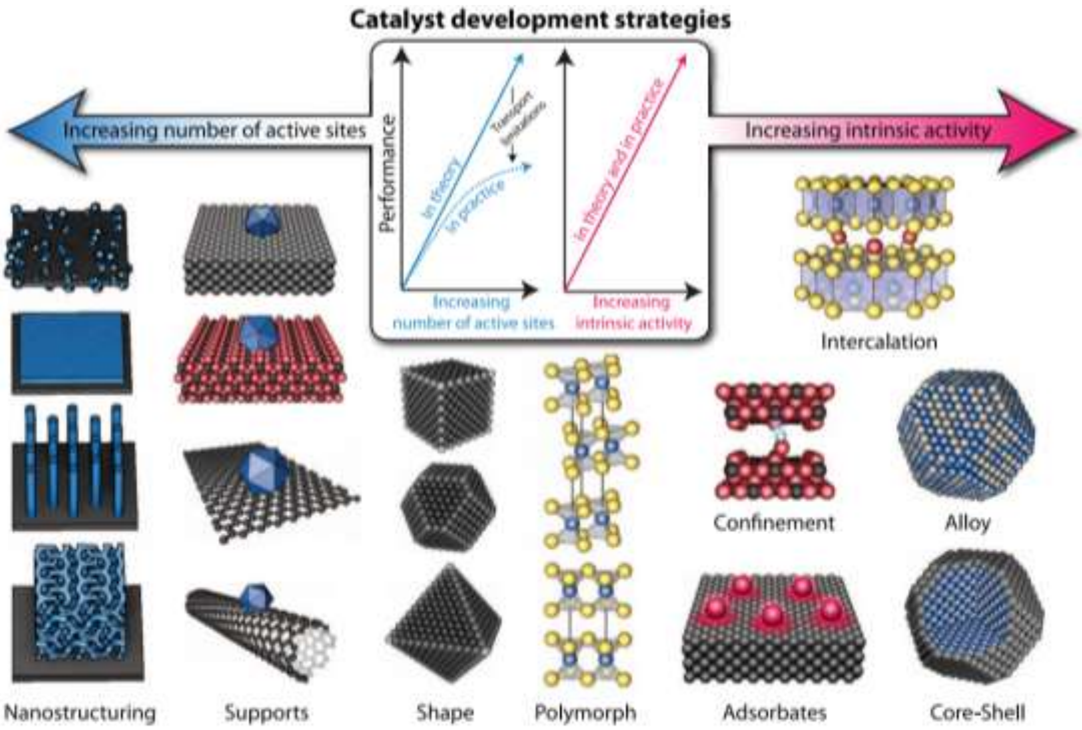


Figure 875: Source: [Combining theory and experiment in electrocatalysis: Insights into materials design](#) by Jens Jaramillo et al, Science, 2017 (33 pages).

The natural nitrogen cycle operating in nature is the third fundamental one in addition to photosynthesis that captures energy from light and uses O_2 to create glucose in chloroplasts, and respiration that uses the captured energy to power all life cycles and produced water and CO_2 through the ATP/ADP cycle happening in cell mitochondria, as shown in Figure 876.

Many constituents of living cells like amino acids, proteins and nucleic acid are reservoirs of reduced atmospheric nitrogen.

The nitrogen cycle involves nitrogen fixation (conversion of N_2 to ammonia NH_3), nitrification (conversion of ammonia to nitrite and nitrate, assimilation (conversion of inorganic NH_3, NO_2, NO_3 into organic compounds like DNA, RNA and proteins), ammonification (conversion of the amine groups of organic compounds into simpler compounds like ammonia), denitrification (conversion of NH_3, NO_2, NO_3 to N_2).

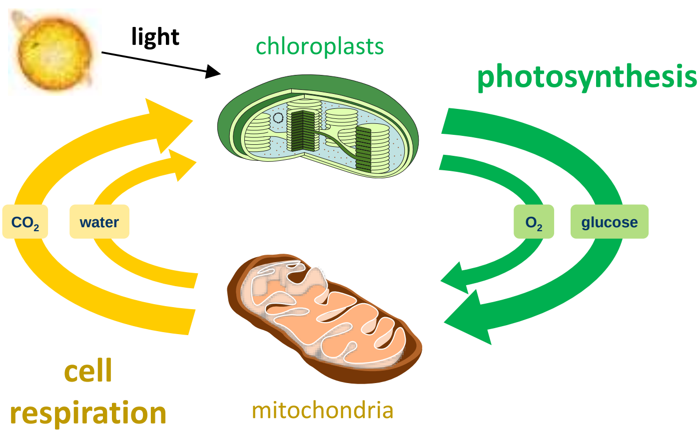
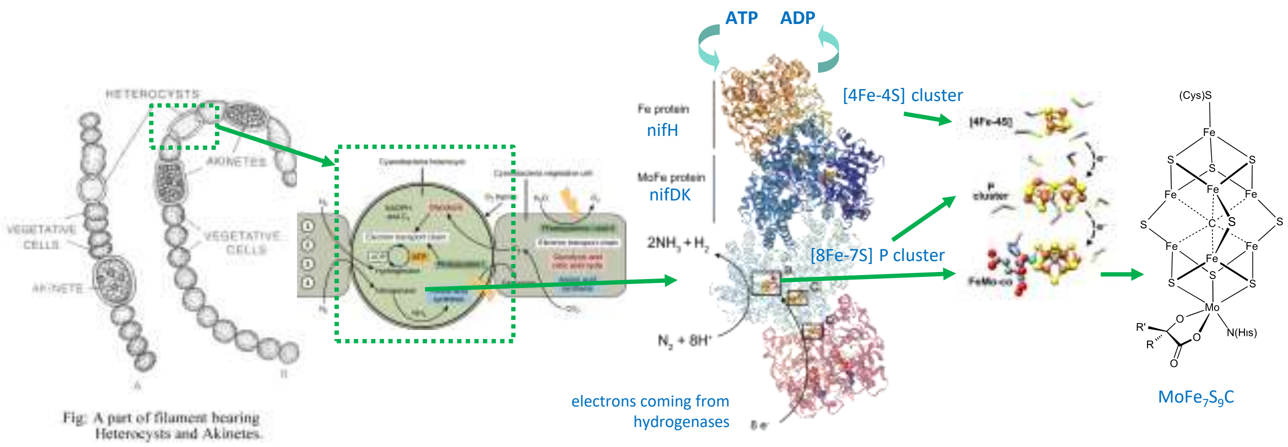


Figure 876: photosynthesis and cell respiration are the two first key cycles in life. Source: Olivier Ezratty reconstitution using open source schematics. Nitrogen fixation is the third one, happening in bacteria to feed plants. 2023.

The natural nitrogen fixation requires a lot of coordinated factors: the presence of the nitrogenase and hydrogenase enzymes, a protective mechanism against O_2 since it is an anaerobic process, an iron protein ferredoxin used as an electron carrier, a hydrogen donating system like hydrogen or glucose, a constant supply of ATP that is produced in cells mitochondria, the presence of thiamine pyrophosphate like inorganic phosphates and magnesium ions as cofactors, the presence of cobalt and molybdenum and a carbon compound for capturing the released ammonia.



cyanobacteria
bacteria producing ammonia in soil and water or legume plants root nodules

heterocyst
cell within cyanobacteria that support the chemical pathways producing ammonia using dinitrogen

nitrogenase
protein complex producing ammonia in heterocyst cells

FeMoCo complex
part of the two nidDK proteins in some nitrogenases that is involved in ammonia production

Figure 877: where FeMoCo stands in the cyanobacteria ammonia production cycle. Schematics assembled by the author using various sources including Anabaena – structure of the vegetative body and reproduction, 2018, Science Practice Challenge Questions - Prokaryotic Diversity, Biosynthesis of Nitrogenase Cofactors by Stefan Burén et al, Chemical Reviews, 2020 (48 pages), Natural and Engineered Electron Transfer of Nitrogenase by Wenyu Gu and Ross D. Milton, 2020 (24 pages), Strong Electron Correlation in Nitrogenase Cofactor, FeMoco by Jason M. Montgomery and David A. Mazziotti, May 2018 (28 pages). Added in 2023.

This process happens in cyanobacteria that are multi-cell bacteria living in moist soil and water and are living out of photosynthesis. They contain several cell types including heterocysts which contain the nitrogenase protein that is transforming nitrogen (N_2) into ammonium (NH_3). Nitrogenases usually contain two iron proteins (aka NifH) and two MoFe proteins (aka NifDK). Nitrogenase is made of thousands of atoms. In many bacteria, nitrogenase works in conjunction with hydrogenase which converts molecular hydrogen into protons and produces electrons used by nitrogenase.

The transformation process involves several complexes: the [4Fe-4S] cluster in the NifH protein, the P cluster and the FeMo cofactor *aka* FeMoCo in the two NifDK proteins. Its electron capture process is complicated and not well understood. The whole process energy comes from ATP (adenosine triphosphate) molecules which are turned into ADP (adenosine diphosphate) in the NifH protein while releasing their energy enabling electron transfers in the nitrogen fixation process. A top to bottom overall natural nitrogen fixation process is described in Figure 877 [4130].

The nitrogenase FeMoCo complex contains 8 transition metal atoms, 7 iron atoms and one molybdenum atom, interlinked by sulfur and carbon atoms. The 39-atom model is $C_7H_9Fe_7MoN_2O_3S_{10}$, with 254 active electrons which represents 2367 different orbitals[4131]. So, what quantum computing could bring on the plate? The main goal would be to understand how FeMoCo participates in the nitrogen fixation pathway, particularly the electron transport mechanism, by simulating only a few hundred of these orbitals.

The number of qubits and gates to simulate FeMoCo seems quite mind boggling, even with corrected qubits. It would require ≈ 2,142 logical qubits, 3.2 x 1,010 Toffoli gates, 4M physical qubits and 4 days run-time using an optimized version of the Quantum Phase Estimation algorithm [4132].

[4130] The whole nitrogenase chemical process is detailed in Biosynthesis of Nitrogenase Cofactors by Stefan Burén et al, Chemical Reviews, 2020 (48 pages).

[4131] See How will quantum computers provide an industrially relevant computational advantage in quantum chemistry by Vincent E. Elfving et al, 2020 (20 pages).

[4132] See Even More Efficient Quantum Computations of Chemistry Through Tensor Hypercontraction by Joonho Lee, Craig Gidney, Nathan Wiebe, Ryan Babbush et al, PRX Quantum, July 2021 (62 pages). With this method, the resource requirements become 4 million physical qubits with an error rate of 0.1%.

This would however only enable the simulation of the ground state of active-space models of FeMoCo, not the whole ammonium production chemical process happening in nitrogenase. I have not seen quantum computing resource estimates for such quantum simulations. We do not even know yet how and where N_2 binds to the FeMoCo complex. Also, most known quantum simulation algorithms do not account for the relativistic electrons that are in the non-valence layers of molybdenum[4133]. However, the simulation of the nitrogenase [4Fe-4S] Fe-S and [8Fe-7S] P clusters seem more accessible but will still require FTQC QPUs[4134] [4135]. However, a team in China could determine the ground state of the P cluster with its 73 active orbitals using a classical DMRG GPU-based classical solution[4136].

By the way, the biosynthesis of nitrogenase is not yet elucidated, meaning, how this large molecular complex is naturally assembled from its various components. It involves the assembly of many proteins generated by ribosomes and their interconnection with metal atoms compounds with iron, molybdenum, and sulfur[4137]. All in all, there is still a lot to discover with FeMoCo, beyond its molecular orbitals.

While many sources deal with simulating FeMoCo with futuristic FTQC QPUs, not many explanations are provided on what could then be done with it[4138]. Will it be enough to understand the whole chemical process and electron transport mechanisms happening in nitrogenase? Will it explain how N_2 "docks" to FeMoCo? Will it enable the creation of a biomimetic efficient artificial chemical process? Would that process be easy to simulate in a quantum computer? Would it be the equivalent of creating aircrafts with imitating birds flapping wings? The main path would be a replacement of the Haber-Bosch process using biological enzyme catalysts produced by genetically modified bacteria, synthesizing ammonia from water and air nitrogen.

But this is only one out of many other ways to decarbonate ammonia production, many of which could be put in production way before a quantum computer could just compute the FeMoCo complex ground state and how it participates to turning N_2 into NH_3 [4139].

Cement production

You may wonder why cement production that is also responsible for significant CO^2 emissions is less talked about as a quantum case. Maybe it is because nature is not producing it and biomimetics models are more difficult to implement. Still, creating innovative cement production techniques is about testing many variants of clinkers who reduce CO^2 emissions during production. Quantum computing could help simulate these various material combinations to identify a more durable solution.

Material design

New material design can use **Fermi-Hubbard models** of strongly correlated 2D systems like crystals, semiconductors and superconducting materials.

[4133] See Understanding the Electronic Structure Basis for N_2 Binding to FeMoco: A Systematic Quantum Mechanics/Molecular Mechanics by Yunjie Pang and Ragnar Bjornsson, Inorganic Chemistry, March 2023 (19 pages).

[4134] See Simulating Models of Challenging Correlated Molecules and Materials on the Sycamore Quantum Processor by Ruslan N. Tazhigulov, Ryan Babbush, Garnet Kin-Lic Chan et al, Caltech and Google AI, November 2022 (11 pages).

[4135] See Simulations Using a Quantum Computer Show the Technology's Current Limits by Philip Ball, Physics, November 2022.

[4136] See A distributed multi-GPU ab initio density matrix renormalization group algorithm with applications to the P-cluster of nitrogenase by Chunyang Xiang et al, November 2023 (32 pages).

[4137] See Biosynthesis of Nitrogenase FeMoco by Yilin Hu and Markus W. Ribbe, 2011 (14 pages).

[4138] See Elucidating Reaction Mechanisms on Quantum Computers by Markus Reiher, , Nathan Wiebe, Krysta M Svore, Dave Wecker and Matthias Troyer, May 2016 (28 pages).

[4139] See Ammonia Technology Roadmap, IEA, October 2020 that lists many of these various alternative processes starting in page 116. It also mentions that ammonia could become an interesting decarbonated energy carrier, particularly to power large ships.

These are very difficult to simulate with classical methods[4140]. One VQE based quantum algorithm was tested on Google Sycamore to do so, but only with 16 qubits, which could be run faster on a simple laptop[4141].

Sometimes, researchers indeed tout the realization of new material designs using a quantum computer when the exact same task could indeed be run on a laptop[4142].

Carbon capture

Carbon capture is another issue and researchers are simulating its molecular modus-operandi by biomimetics.

BASF is trying to simulate synthetic polymers, first on HPCs, then eventually on quantum computers. They invested in both **HQS** (Germany) and **Zapata Computing** (USA). It also announced a partnership with **SEEQC** in 2023 to "*explore applications of quantum computing in chemical reactions for industrial use*" which is probably a little premature, given SEEQC will probably provide a solution using at best about 64 noisy superconducting qubits[4143].

Since 2017, **Dow Chemicals** has been collaborating with the Canadian software publisher **1Qbit** to create new molecules, using D-Wave annealers.

TotalEnergies is working with CQC from Quantinuum to capture carbon on Metal-Organic Frameworks using a VQE based algorithm. They did simulate the use of up to 16 qubits which is way below any quantum computing advantage threshold[4144].

In another such example, researchers from the **National Energy Technology Laboratory** and the **University of Kentucky** tested an algorithm to find useful amine compounds for carbon capture. It quantifies molecular vibrational energies and reaction pathways between CO_2 and a simplified amine-based solvent model NH_3. It runs on 8 to 30 qubits on IBM QPUs and makes use of quantum error mitigation. Again, we're definitively not in any quantum advantage regime[4145].

Batteries

Research is well underway to create batteries that are more efficient in terms of energy density, charging speed and supported charging/discharging cycles[4146].

4140 See Experimental realization of an extended Fermi-Hubbard model using a 2D lattice of dopant-based quantum dots by Xiqiao Wang, et al, Nature Communications, 2022 (11 pages).

4141 See Observing ground-state properties of the Fermi-Hubbard model using a scalable algorithm on a quantum computer, by Stasja Stanisic, Ashley Montanaro et al, Nature Communications, 2022 (12 pages).

4142 See Engineers use quantum computing to develop transparent window coating that blocks heat, saves energy by University of Notre Dame, November 2022, referring to High-Performance Transparent Radiative Cooler Designed by Quantum Computing by Seongmin Kim et al, ACS Energy Letters, November 2022 (8 pages). This is a paywall paper but I bet the quantum part is some easy to emulate quantum code.

4143 See SEEQC Partners With BASF To Explore Applications of Quantum Computing in Chemical Reactions for Industrial Use by Matt Swayne, The Quantum Insider, February 2023.

4144 See Modelling Carbon Capture on Metal-Organic Frameworks with Quantum Computing by Gabriel Greene-Diniz, Elvira Shishenina, Philippe Cordier, Marko J. Rančić, David Muñoz Ramo et al, TotalEnergies, March 2022-January 2023 (21 pages), CQC and Total Announce Multi-Year Collaboration to Develop Quantum Algorithms for Carbon Capture, Utilization and Storage (CCUS), April 2020 and Total Exploring Quantum Algorithms to Improve CO2 Capture, May 2020.

4145 See Using Quantum Computing Technology to Solve a Practical Environmental Problem by Laura Thomson, AzoQuantum, March 2023, referring to Description of reaction and vibrational energetics of CO2–NH3 interaction using quantum computing algorithms by Manh Tien Nguyen et al, AVS Quantum Science, 2023 (10 pages).

4146 See The Promise and Challenges of Quantum Computing for Energy Storage (4 pages) and Can quantum computing fuel a leap in battery research? Not any time soon, but automakers and quantum companies are exploring it right now by Grace Donnelly, Emerging Tech Brew, April 2022.

Simulation is most often used to understand the operations of the chemical reactions taking place on cathodes and anodes and in the crystalline intercalation structures and to find ways to improve energy density and avoid battery wear phenomena.

Several companies are pursuing this goal. IBM and **Mitsubishi Chemical** have simulated on a superconducting qubit computer the initial steps of the reaction between lithium and oxygen in lithium-air batteries[4147].

Mercedes-Benz worked with IBM to simulate lithium-sulfur batteries[4148] and later, with PsiQuantum to estimate the computing requirements for electrolyte simulation on PsiQuantum's system. For one of the simulation needs, they would need to have about 16K logical qubits. Given, PsiQuantum plans for requiring 10,000 physical qubits to create a single logical qubit. We end up with 160M physical qubits. PsiQuantum's plan is to reach 1M physical qubits by 2030[4149].

Hyundai and IonQ are working on quantum algorithms to model lithium oxide using IonQ's existing hardware. Lithium oxide is not used in EV batteries, but its modeling could potentially help mitigate how batteries break down over time. But no IonQ computer has a computing advantage vs a classical computer! They use the Aria with 32 physical qubits and 20 "algorithmic qubits". The current project is focused on relatively small molecules containing up to two lithium and one oxygen atom. In the end, making something useful would require 2,000 logical qubits that are far away beyond 2030 in IonQ's roadmap[4150].

Volkswagen is also working on simulating lithium-ion batteries, teaming up with 1Qbit and Xanadu. They concluded that we are far off from being able to implement it. Their paper concludes that there is a need of 2,375 to 6,652 logical qubits to simulate Li_2FeSiO_4 with their 156 electrons[4151].

Ford is working with **Quantinuum** to develop chemical modeling of electric vehicles batteries[4152]. They experimented a VQE algorithm to compute the ground-state energy of $LiCoO_2$, a candidate transition metal oxide used for battery cathodes and simulated $Li_2Co_2O_4$ and Co_2O_4 gas-phase models which show up during the discharge and charge of the Li-ion battery. Computations were performed using an IBM state vector emulator with 20 qubits, sized for their active space of 10 electrons and 20 spin orbitals. We are here in the "laptop regime" zone. So, with no quantum advantage.

Finally, quantum sensing could also help develop new batteries. That's what **AMTE Power** (UK) is doing with leading the 3 years Project Quantum to use quantum sensing to develop lithium-based battery manufacturing solutions, which got a funding of £5.4M from Innovate UK. This project is about using NV center-based magnetic quantum sensing to evaluate the battery aging process[4153].

4147 See Computational Investigations of the Lithium Superoxide Dimer Rearrangement on Noisy Quantum Devices by Qi Gao et al, 2018 (6 pages).

4148 See Quantum computation of dominant products in lithium–sulfur batteries by Julia E. Rice et al, Journal of Chemical Physics, 2021 (7 pages).

4149 See PsiQuantum, Mercedes-Benz Study How Quantum Computers Can Accelerate EV Battery Design by Matt Swayne, The Quantum Insider, April 2022. Mentioning Fault-tolerant resource estimate for quantum chemical simulations: Case study on Li-ion battery electrolyte molecules by Isaac H. Kim et al, Physical Review Research, April 2022 (27 pages).

4150 See IonQ and Hyundai Motor Company Expand Quantum Computing Partnership, Continuing Pursuit of Automotive Innovation, December 2022.

4151 See Simulating key properties of lithium-ion batteries with a fault-tolerant quantum computer by Alain Delgado et al, Xanadu, Volkswagen, April 2022-February 2023 (31 pages).

4152 See Towards the simulation of transition-metal oxides of the cathode battery materials using VQE methods by Marwa H. Farag and Joydip Ghosh, Ford Motor, August 2022 (32 pages).

4153 See AMTE Power's Project Quantum Signals New Era of British Battery Manufacturing, September 2020.

Renewable energies

I have not seen much in this domain.

In 2023, the DoE simulated the linear H_4 molecule using 20 trapped ion qubits which can absorb a single photon to produce two excited states[4154]. They used a quantum solver based on the Peeters-Devreese-Soldatov approach, which was developed at Pacific Northwest National Laboratory, another DoE lab. This molecule could help new photovoltaic cells with a better yield.

On the same Quantinuum hardware, another experiment dealt with the optimization of fuel cells membranes for the oxygen reduction reaction using InQuanto Quantinuum H1-1 trapped-ion QPU.[4155].

In both cases, we were not in a quantum advantage regime and could thus consider it has classical computing advances.

In another documented case study related to the optimization of winfarm turbines layout, the classical Gurobi solution outperforms a gate-based VQE algorithm using 16 emulated qubits[4156].

Oil exploration

Well, that stinks but oil companies are still trying to optimize oil exploration and extraction.

BP is working on the design of algorithms for optimizing oil exploration. This involves using data from various sensors, particularly seismic sensors, to consolidate models for simulating geological layers under the ground. They joined in 2020 the **IBM Quantum Network** as an industry partner to get access to IBM's 65-qubit quantum computer, software and experts.

It follows **ExxonMobil** who entered the program in 2019 as one of the major companies associated with IBM.

TotalEnergies also wants to optimize oil prospection and reserves evaluation using seismic probes. They plan to address complex optimization problems such as **MINLP (**Mixed Integer NonLinear Programming[4157]) to optimize refining, planning, production and transportation[4158].

Nuclear energy

Quantum computing could be used to simulate both fission[4159] and fusion energy reactions[4160] [4161]. At this point, we are still in the testing domain for the related use cases. It relates to the capability to

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[4154] See Modeling Singlet Fission on a Quantum Computer by Daniel Claudino et al, Oak Ridge National Laboratory, J. Phys. Chem. Lett., June 2023 (6 pages).

[4155] See Applicability of Quantum Computing to Oxygen Reduction Reaction Simulations by Cono Di Paola et al, Quantinuum, Aerostack, Airbus and BMW, July 2023 (34 pages).

[4156] See Investigating techniques to optimise the layout of turbines in a windfarm using a quantum computer by James Hancock, Matthew J. Craven, Craig McNeile, and Davide Vadacchino, December 2023 (18 pages).

[4157] A version of the MINLP problem solving algorithm exists for D-Wave via their QUBO framework. See Quantum Computing and Non-Linear Integer Optimization by Sridhar Tayur February 2019 (42 slides).

[4158] Total has partnered with private players (IBM, Atos, Rigetti QC Ware, Google) and various research laboratories around the world: PCQC (Paris), LIRMM Montpellier, CERFACS, Université ParisSud, Jülich Forschungszentrum (Germany) and the University of Leiden.

[4159] See A quantum-classical co-processing protocol towards simulating nuclear reactions on contemporary quantum hardware by Francesco Turro et al, February 2023 (12 pages).

[4160] See Quantum Computing for Fusion Energy Science Applications by I. Joseph et al, Lawrence Livermore National Laboratory, December 2022 (42 pages).

[4161] See Quantum Computing Perspective for Electromagnetic Wave Propagation in Cold Magnetized Plasmas by Efstratios Koukoutsis et al, September 2023 (13 pages).

simulate complex dynamic systems such as particles behavior in plasmas. Most published use cases show requirements for large scale FTQC systems with thousands of logical qubits[4162].

One 2023 García-Ramos et al review paper makes a good inventory of what could be achieved with quantum computing and particularly with quantum machine learning in the domain[4163].

- Quantum simulations can be used to model atomic nuclei and various nuclear interactions.
- Quantum algorithms such as VQE (NISQ) and QPE (FTQC) could help solve quantum chemistry problems like with calculating nuclear energy levels or simulating nuclear reactions.
- QUBO can be used to optimize the fuel load problem in nuclear plants[4164].
- QML algorithms or quantum optimization algorithms can extract some insights from nuclear physics experimental and operations data to extract patterns, detect particles in experiments, perform classifications, and make predictions. One optimization use-case is a quantum-inspired bin-packing algorithm to process nuclear waste[4165] [4166].

Power grid

Various power grid and multi-energy optimization algorithms have been tested at low scales[4167] [4168] [4169] [4170] [4171].

In 2019, the **Dubai Electricity and Water Authority** (DEWA) was working with Microsoft to solve complex power and water distribution problems. It was just a matter of testing a few algorithms on Intel simulators running on Azure. And for good reason, Microsoft did not yet have its own quantum computer[4172]. In 2020, DEWA announced that it would train their algorithms in D-Wave annealers[4173]!

EDF is another major French company that is studying the use cases of quantum computing very closely.

4162 See Quantum computation of stopping power for inertial fusion target design by Nicholas C. Rubin et al, August 2023 (37 pages) which lists a need for 5,650 to 33,038 logical qubits for simulating various settings like the interactions between protons and deuterium in fusion reactors. The use case also shows a number of Toffoli gates in the 10^{14} to 10^{20} range.

4163 See Nuclear Physics in the Era of Quantum Computing and Quantum Machine Learning by J.E. García-Ramos et al, Universidad de Sevilla, July 2023 (21 pages).

4164 See Quantum and quantum-inspired optimization for an in-core fuel management problem by Sergey R. Usmanov et al, Russian Quantum Center, August 2023 (11 pages). It is aimed at running on quantum annealers but not yet in a quantum advantage regime.

4165 See Quantum and quantum-inspired optimization for solving the minimum bin packing problem by A. A. Bozhedarov et al, January 2023 (7 pages).

4166 See the review paper Quantum Information Science and Technology for Nuclear Physics. Input into U.S. Long-Range Planning, 2023 by Douglas Beck et al, February 2023 (26 pages).

4167 See Quantum computation: Efficient network partitioning for large scale critical infrastructures by Saikat Ray Majumder et al, GE Research, February 2023 (5 pages).

4168 See Towards optimization under uncertainty for fundamental models in energy markets using quantum computers by M.C. Braun et al, JoS Quantum, January 2023 (16 pages).

4169 See Power network optimization: a quantum approach by Giuseppe Colucci et al, December 2022 (7 pages). With grid optimization running on D-Wave and using Germany as a playground.

4170 See Noise-Resilient Quantum Power Flow by Fei Feng et al, November 2022 (6 pages).

4171 See Integrating quantum and classical computing for multi-energy system optimization using Benders decomposition by Ludger Leenders et al, ETH Zürich and RWTH Aachen University, September 2023 (13 pages).

4172 See Microsoft and DEWA bringing quantum computing to Dubai, June 2018.

4173 See DEWA organises training sessions on quantum computing in partnership with D-Wave, February 2020.

It covers quantum new materials simulations, material aging simulations particularly under radiations, safety statistics, combinatorial optimization for smart grids and battery management (teaming up with Pasqal and ParityQC[4174]) and also customer segmentation using Quantum Machine Learning.

DTU experimented in Denmark a grid optimization using an HHL algorithm with up to 7 IBM qubits QPUs. They found that they have scalability issues. Surprise[4175]!

Outside of quantum computing, QKD could be used to protect key energy production and distribution infrastructures as is being investigated for hydropower and dams in the USA[4176].

Transportation and logistics

Beyond energy matters mentioned above, transportation industries are mainly interested in algorithms for optimizing complex systems[4177]. Let's look at what can be done with road, rail, air and maritime transportation.

Road

Quantum computing could be implemented in many domains to improve both vehicle production and their operations. In the product design phase, quantum computing could help optimize vehicle fluid dynamics thanks to solving complex partial differential problems (aka Navier Stokes equations), minimizing drag and improving battery/fuel efficiency. It could also simulate the physics of various materials to improve weight/strengths vehicle ratios and design new batteries (but in the very long term).

In the operations stage, QC could handle complex optimization problems like supply-chain optimizations, warehouse robots routing, improving the accuracy of demand forecasting both with end-users and for their suppliers and inventory optimization. You have also traffic flow optimization or multimodal fleet operations[4178] and of course, the famous Traveling Salesperson Problem and its more generic Capacitated Vehicle Routing Problem where one vehicle must collect goods at various places with its limited capacity[4179].

Machine learning based solutions could under some circumstances benefit from QC, for improving pattern recognition in images and various classification tasks used in manufacturing, predictive maintenance as well as in marketing[4180].

One less credible use case is autonomous vehicle control whether it is based on some machine learning technique or not. It doesn't make much sense for quantum computing due, at least, to the data-loading problem.

[4174] See Qualifying quantum approaches for hard industrial optimization problems. A case study in the field of smart-charging of electric vehicles by Constantin Dalyac et al, June 2021 (29 pages).

[4175] See Quantum Computing for Power Flow Algorithms: Testing on real Quantum Computers by Brynjar Sævarsson et al, July 2022 (8 pages) and DTU first to use quantum computer for the power grid, 2022.

[4176] See Quantum Key Distribution for Critical Infrastructures: Towards Cyber Physical Security for Hydropower and Dams by Adrien Green et al, University of Tennessee, Oak Ridge National Laboratory and Electric Power Research Institute, October 2023 (20 pages).

[4177] See this inventory of needs, but no solutions in Quantum Applications Transportation and Manufacturing by Yianni Gamvros, IBM, 2017 (20 slides).

[4178] See Quantum computing for transport optimization by Christopher D. B. Bentley et al, Q-CTRL, June 2022 (12 pages).

[4179] See Recommending Solution Paths for Solving Optimization Problems with Quantum Computing by Benedikt Poggel et al, December 2022-October 2023 (8 pages) which proposes a framework to select the right algorithm to solve this problem.

[4180] See Will quantum computing drive the automotive future? by McKinsey, September 2020. The listed use cases are interesting but the list contains some misleading case studies that are not relevant with quantum computing like "*processing vast amounts of data to accelerate learning in autonomous-vehicle-navigation algorithms*" (QC is not good at handling big data) and "*ensure car-to-car communications in almost real time*" (no quantum technology is bound to ensure real-time whatever communication). Most use cases related to autonomous vehicles operations are also quite unrealistic.

Vehicle sensors are generating a huge stream of high-volume data that can't be ingested in a quantum computer, and particularly with one such computer sitting in the vehicle. So, let's forget this for a while.

Let's first look at the manufacturing of vehicles. And, surprise, they are mostly if not all… Germans! In 2022, **Volkswagen** Data:Lab published two papers coauthored with Terra Quantum AG related to hybrid computing associated with D-Wave quantum annealers in two areas: the optimization of assembly line work-flow scheduling[4181] and hybrid machine learning to enhance image recognition systems used for car classification[4182]. It is still not clear whether these applications really went into production.

Daimler AG is one of the leading companies working on quantum technology with IBM, with applications for logistics and planning optimization and everything to do with autonomous vehicle routing at the forefront. In 2018, they also launched an initiative with IBM to develop lithium-sulfur batteries, which improve energy density and make it possible to do without metals such as cobalt and nickel. All of this will be achieved through quantum simulation.

BMW is also willing to learn how to use quantum computing in various tasks, one being the optimization of its spare parts supply chain[4183]. They are partnering with Honeywell to do this as well as with Cambridge Quantum Computing, Zapata Computing and Entropica Labs. They started using the Honeywell trapped ions based H0 and H1 with 10 qubits. They used a Recursive Quantum Approximate Optimization Algorithm (R-QAOA) to manage their combinatorial problem. The quality of these trapped ions qubits made the trial promising although of course not usable at all given the small number of available qubits[4184]. In May 2022, BMW started a partnership with Pasqal to solve differential equations to better compute metal forming applications modeling.

Multiverse Computing and a **Bosch** Automotive Electronics plant in Madrid announced in 2022 that they were exploring the creation of a digital twin of a factory with quantum computing. They plan to capture data to assess the performance of individual equipment and optimize quality control and overall efficiencies, including energy consumption and waste management. What they call digital twins are actually optimization systems.

In 2021, the German auto industry (BMW, Mercedes-Benz, Volkswagen, and Bosch) and research organizations (DFKI who runs contract research on AI and Forschungszentrum Jülich) even launched the **Q(AI)2** project to test AI applications for quantum computers[4185]. It particularly wants to study the usage of AI to solve various manufacturing optimization problems.

The deployment of autonomous vehicles fleets is a nice target application for quantum computers. The more autonomous the vehicles are, the greater the need for automation and route coordination. The problems to be solved will be to determine step-by-step the vehicle fleet routes in order to optimize each of these vehicles and passengers the travel time.

[4181] See Solving workflow scheduling problems with QUBO modeling by A. I. Pakhomchik et al, May 2022 (10 pages). The experiment is using QUBO on a D-Wave Advantage annealer. They found that the hybrid and classical algorithms were the most successful in solving the instances, although no solver was able to solve all QUBOs at all sizes. One caveat is that the paper doesn't mention computing time but only results quality.

[4182] See Hyperparameter optimization of hybrid quantum neural networks for car classification by Asel Sagingalieva et al, May 2022 (10 pages). It's about improving the accuracy of image recognition that is used for fault detection in manufacturing. They created an hybrid QML algorithm and deployed it on the QMware cloud from Terra Quantum running on D-Wave's annealers. The benchmark compared this new method over classical machine learning and quantified performance improvements in reduced expected run times and model fitness as the problem size scales. But it's very uncertain as of a real potential quantum advantage.

[4183] See this excellent use cases inventory: Quantum Computing: Towards Industry Reference Problems by Von Andre Luckow, BMW, April 2021.

[4184] See How BMW Can Maximize Its Supply Chain Efficiency with Quantum, Honeywell, January 2021.

[4185] See Quantum AI For The Auto Industry by DFKI, June 2021.

This was an experiment done in 2017 by **Volkswagen** on D-Wave annealers[4186]. Its goal was to optimize the routes of a (virtual) cab fleet in Beijing[4187]. The experiment was using the T-Drive data set published by Microsoft in 2008 which describes the routes of 10,357 cabs. The algorithm used was QUBO (Quadratic Unconstraint Binary Optimization). The diagram below shows the result of the optimization of the route of 418 cabs making the journey from the city center to the airport taking into account the route of 10,357 vehicles[4188].

In 2023, **Honda** and Terra Quantum published a paper on how to solve the escape routing problem with a hybrid quantum computing solution involving some quantum machine learning[4189]. It was tested on 25 IonQ qubits through AWS. A team at **TotalEnergies** and LIRMM proposed an algorithm to solve the problem of "Fleet Conversion" which is about optimizing and reducing the carbon emissions and cost of operating a heterogeneous fleet of vehicles combining electric vehicles and traditional ones. It is formulated as a Maximum Weighted Independent Set (MWIS) problem which is represented in a QUBO variant implemented in gate-based mode[4190]. The prototype was tested with 7 qubits, far from any quantum advantage.

There is indeed a lack of hindsight in estimating the size of quantum computers needed to practically handle such large-scale problems. How many qubits would be needed to optimize a fleet of hundreds or even millions of autonomous vehicles? Volkswagen also experimented small-scale algorithms with D-Wave for optimizing vehicle recommendations and optimizing cars painting planning. A less ambitious version of this algorithm was used to optimize the individual travel routes of nine public-transit buses during the November 2019 Web Summit event in Lisbon.

So, we're off wondering what they really achieved or could achieve. Similar trucks routing algorithms were already explored by **Accenture** and **Denso** using D-Wave annealers. Annealers are so far in a better position to solve these problems than existing superconducting gate-based qubits systems from IBM. In an adjacent domain, **Toyota** worked with D-Wave to optimize traffic lights on a 50x50 road grid, using a D-Wave 2000Q[4191]. Small scale routing problems were also tested in gate-based bode at CQT in Singapore in 2023[4192].

At last, another work conducted by **Hyundai** with IonQ is to improve 3D object detection with quantum computing. In 2023, IonQ demonstrated a "101" image recognition solution of 43 road signs which is a very basic test for a classical convolutional network running on a simple mobile device (smartphones, laptops, whatever)[4193]. It plans to expand it to recognize pedestrians and cyclists. They believe they can do better with quantum computers than with classical NPUs (the neural processing units now embedded in most CPUs and mobile chips). They may forget the constraints of embedded systems in cars. IonQ QPUs wouldn't fit in a trunk and are not at all designed to become real-time computing systems! And forget also, by the way, the economical constraints of the automotive sector.

4186 See Volkswagen takes quantum computing from the lab to the factory by Volkswagen, August 2021.

4187 It is documented in Quantum Computing at Volkswagen Traffic Flow Optimization using the D-Wave Quantum Annealer, 2017 (23 slides).

4188 The results are published in Traffic flow optimization using a quantum annealer, August 2017 (12 pages). As with many case studies from D-Wave, this one is also contested by HPC specialists.

4189 See A supervised hybrid quantum machine learning solution to the emergency escape routing problem by Nathan Haboury et al, Terra Quantum and Honda, July 2023 (15 pages).

4190 See A Hybrid Quantum-assisted Column Generation Algorithm for the Fleet Conversion Problem by Yagnik Chatterjee et al, TotalEnergies and LIRMM, September 2023 (12 pages).

4191 See Toyota Central R&D Labs: A Quantum Approach to Transportation by D-Wave (2 pages) and Traffic Signal Optimization on a Square Lattice with Quantum Annealing by Daisuke Inoue et al, February 2021 (14 pages).

4192 See Qubit efficient quantum algorithms for the vehicle routing problem on quantum computers of the NISQ era by Ioannis D. Leonidas et al, June 2023 (9 pages).

4193 See Learn Quantum: Machine Learning Image Recognition Application, IonQ, September 2023

Rail

An experiment was done by **Ferrovie dello stato Italia** to optimize train arrivals in railways stations, also to minimize passengers' connection times, again with D-Wave. In Poland, researchers compared D-Wave annealers and classical methods to optimize train dispatch and found a potential quantum advantage in some situations[4194].

ALPHARAIL **AlphaRail** (2000, USA) is a railways software company using machine learning and quantum computing to improve railways operations.

They are relying on quantum and quantum-inspired approaches to solve routing and scheduling optimization problems.

Air

With airlines, the current focus is about optimizing aircraft fleets planning, to maximize the capacity to meet demand while optimizing the aircraft fill rate. Also, quantum computing can enable optimizing airport and aircraft gates management, the so-called flight gate assignment problem, to minimize passenger waiting time, as tested by **DLR** in Germany[4195] [4196](Figure 878). This is an NP-difficult problem that is difficult to deal with using conventional algorithms. Similar small scale tests have been run elsewhere such as with IBM [4197] [4198] and IonQ[4199].

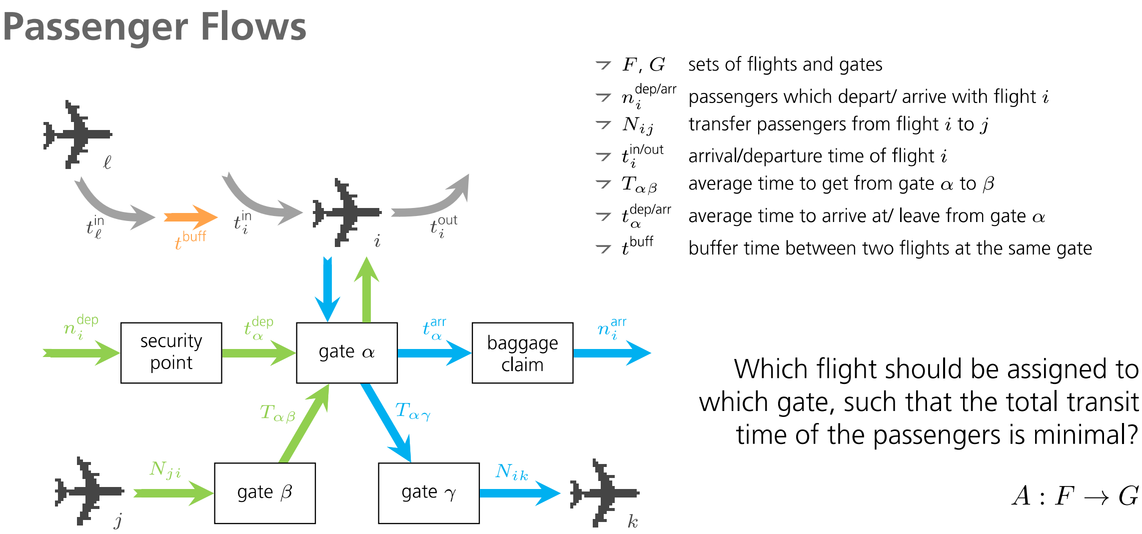

Figure 878: Flight Gate Assignment with a Quantum Annealer by Elisabeth Lobe and Tobias Stollenwerk, March 2019 (15 slides).

In Japan, **Sumitomo** launched in June 2021 a pilot experiment for optimizing flight routes for urban air mobile vehicles (air taxis and unmanned drones).

[4194] See Application of a Hybrid Algorithm Based on Quantum Annealing to Solve a Metropolitan Scale Railway Dispatching Problem by Mátyás Koniorczyk et al, September 2023 (33 pages).

[4195] See Flight Gate Assignment with a Quantum Annealer by Elisabeth Lobe and Tobias Stollenwerk of the German Aerospace Center (or DLR for Deutsches Zentrum für Luft- und Raumfahrt e.V.), March 2019 (15 slides). The case study uses a D-Wave. It shows that the solution is not obvious to develop.

[4196] The flight-gate assignment problem could also potentially be solved with a gaussian boson sampler as shown in Gaussian Boson Sampling for binary optimization by Jean Cazalis et al, December 2023 (3 pages).

[4197] See Towards Finding an Optimal Flight Gate Assignment on a Digital Quantum Computer by Yahui Chai et al, February 2023 (15 pages).

[4198] See Exploring Airline Gate-Scheduling Optimization Using Quantum Computers by Hamed Mohammadbagherpoor et al, Delta Airlines, NC State University, IBM, 2021 (20 pages).

[4199] See Simulating the flight gate assignment problem on a trapped ion quantum computer by Yahui Chai et al, September 2023 (7 pages).

They will rely on resources from Tohoku University. No precision on the problem sizing and on which quantum system they plan to pilot their solution even if D-Wave is a logic contender.

Researchers from **Chalmers University** in Sweden prototyped a promising QAOA hybrid algorithm solving the “tail Assignment problem”, which is the task of assigning individual aircraft to a given set of flights, minimizing the overall cost for the airline. They said it worked with only 2 qubits, to optimize two flights and would scale well as flights are added[4200].

These are needs that can be addressed both by machine learning algorithms to consider the past or with quantum optimization algorithms based on a description of the parameters of the problem. The former does prediction and the latter simulation. Simulations avoid the back-mirror bias that can be induced by prediction methods based on past data. A combination of the two methods is possible.

Airbus is also involved in quantum computing since 2015. They expect to use it in various fields like on quantum chemistry, quantum optimization (flight trajectories, satellite mission planning), quantum machine learning (anomaly detection, reinforcement learning) and with quantum solvers for engineering (fluid mechanics simulation, aeroacoustics)[4201].

In 2015, one of their teams based in Newport in the United Kingdom began working on the subject. In 2016, Airbus Ventures invested in the American startup QC Ware. They experimented with the use of a D-Wave for fault tree analysis (FTA), which is used to determine the origin of complex failures with a gain of a factor of 4 compared with traditional methods. This is a difficult NP-difficult combinatorial problem that is easier to solve in quantum programming.

In January 2019, Airbus launched its "Quantum Computing Challenge", a way to outsource the development of quantum solutions to help them solve their business problems, in fluid mechanics, differential equations, flight optimization, wing design, cargo bay filling, etc.[4202]. As of May 2019, 475 teams from 57 countries had competed in this challenge. They came mainly from the USA and India, followed by Europe. They announced the challenge results in October 2020 with five finalists selected out of 36 contestants.

As of 2023, Airbus had conducted some pilot proof of concepts projects:

- Improving catalytic reactions with pure platinum and platinum-capped cobalt in fuel cell stack, developed by Aerostack, a JV been Airbus and ElringKlinger. A VQE-ADAPT based hybrid algorithm was tested on Quantinuum H1-2 QPU and H1-2E classical emulator with 20 qubits. Conclusion: “*We show that ORR on Pt/Co catalyst involves strongly correlated species which are good test cases for future demonstration of quantum advantage*” and “*Practical advantage over classical methods will require improved quantum computing hardware and application of quantum phase estimation techniques.*”, meaning the need for long-term FTQC QPUs[4203].
- Optimizing aircrafts flight path, tested by Acubed and Qbraid. It is a quantum inspired algorithm that is not running on any quantum hardware. It is a classical algorithm.
- Aircraft loading optimization tested on IonQ Aria (25 qubits) and Forte (32 qubits) using a QAOA algorithm using 29 qubits, but with no quantum advantage found. As of September 2023, no preprint was available yet on this case study that doesn’t seem to be reaching production grade levels.

[4200] See Two-Bit Quantum Computer Solves Real Optimization Problem by Matt Swayne, December 2020 pointing to Applying the Quantum Approximate Optimization Algorithm to the Tail-Assignment Problem by Pontus Vikstål et al, September 2020 (11 pages).

[4201] See Bringing Aerospace into the Quantum Era by Jasper Krauser, Airbus, December 2022 (16 mn).

[4202] See Airbus gets aerodynamic with quantum computing by Michael Feldman, January 2019.

[4203] See Applicability of Quantum Computing to Oxygen Reduction Reaction Simulations by Cono Di Paola et al, Quantinuum, BMW, Airbus and Aerostack, July 2023 (35 pages).

On top of that, Airbus is a party in the EU funded project Equality with Pasqal, Inria, DLR, Da Vinci Labs, Fraunhoffer ENAS, University of Leiden working on aerodynamics, fluids, materials, energy and mission planning. The project coordinated by Capgemini Germany got publicly funded at 100% with 6M€. It started in November 2022 and is ending in 2025. Tests will be run in Europe EuroHPC hybrid centers. The project may be more promising since it will rely on Pasqal quantum simulators.

Rolls Royce partners with **Classiq** to develop quantum algorithms to model computation fluid dynamics for the development of its aircraft engines[4204].

One team from **Capgemini** devised a new hybrid matrix inversion algorithm mixing the famous HHL algorithm and a QSVM (Quantum-enhanced Support Vector Machine (QSVM). Other teams worked on plane loading optimization problems, on quantum machine learning models and on fluids dynamics applied to aircraft design.

How about designing autonomous **quantum flying drones** using a quantum computer, a project from Indian researchers[4205]? It makes no sense from a practical standpoint, at least due to real-time constraints. Since the quantum computer can't sit in the drone, a fortiori, one from IBM, there's an inevitable lag in the communication between the drone and the quantum computer which, by the way, should be entirely dedicated to a single drone to work real-time. You already have autonomous drones and robots, thanks to their embedded computing systems, and they don't yet need a quantum computer to run their symbolic and connectionist artificial to fly.

Maritime

ExxonMobil and **IBM** are working on finding algorithms to optimize maritime traffic routing. Existing solutions rely on heuristics and simplifications. They were willing to see whether quantum computing could transform these complex optimization problems and solve them more efficiently with quantum computing. Their vision is about container shipments volumes. They formatted their problem as a "vehicle routing problem with time windows" (VRPTW) which is a NP-hard problem. They compared various methods, using a QUBO algorithm that can be transformed on a lower-level VQE or QAOA hybrid algorithm and experimented it with Qiskit on the QasmSimulator IBM quantum emulator backend[4206]. As in many similar cases, the published paper does not provide any clear answers on the gain and on the quantum computer specification that would make it possible to solve a real-life problem. On top of that, it was not at all formulated as a container shipment optimization problem per se but as a simpler truck routing problem.

Maritime containers shipment is also a center of interest. **DP World**, the Dubai Port operator, is partnering with **D-Wave** to find ways to use quantum computing to optimize their port operations. At this stage, it is just exploratory work with no details at all about envisioned applications.

The ones that are easy to guess are containers loading/offloading optimizations[4207]. Another such project was also undertaken with the **Port of Los Angeles** by SavantX, also using D-Wave annealers and quantum machine learning[4208].

[4204] See Rolls-Royce Adopts Quantum Hybrid Model for Simulations by Berenice Baker, October 2022.

[4205] See Design and Simulation of an Autonomous Quantum Flying Robot Vehicle: An IBM Quantum Experience by Sudev Pradhan et al, June 2022 (7 pages).

[4206] See ExxonMobil & IBM Explore Quantum Algorithms to Solve Routing Formulations by Stuart Harwood et al, February 2021 which refers to Formulating and Solving Routing Problems on Quantum Computers by Stuart Harwood, January 2021 (17 pages).

[4207] See DP World explores quantum computing technology, April 2021.

[4208] See SavantX, D-Wave Collaborate on Quantum Algorithms to Tackle Supply Chain Problems at U.S.'s Busiest Port by Matt Swayne, The Quantum Insider, January 2022.

Supply chain

Optimizing various supply chains is an interesting field of interest to implement any sort of optimization algorithm. It is also a field of active research, particularly to enable such algorithms in NISQ platforms. I have found a couple algorithms papers in that domain.

A couple ones come from the DoE **Sandia Labs** who introduced FALQON, a Max-Cut QAOA variant performing ansatz optimizations without an expensive classical optimization loop[4209] [4210]. It converges to good approximation ratios and success probabilities with reasonable resources scaling. However, FALQON uses deeper circuits than classical QAOA which turns into a higher requirement regarding physical qubit fidelities. Sandia Labs's Alicia Magann "*has led the development of a new way to design programs on quantum computers, which she and her team think could be especially useful for solving these kinds of massive optimization problems someday in the future when quantum technology becomes more mature*"[4211].

A team from **ETSISI** in Spain developed use cases for logistics optimizations with using D-Wave quantum annealers, focused on solving the vehicle routing problem[4212]. They compared their hybrid solution to classical solutions and found some potential quantum advantage when the randomness of the data is high.

IAG Cargo (resulting from the merger of British Airways World Cargo and Iberia Cargo in April 2011) worked in 2022 with **Quantum-South** to prototype a solution to optimize cargo loading in passenger aircraft using D-Wave quantum annealers with aircraft load optimization and bin packing optimization for a single aircraft unit load device (ULD)[4213]. The solution ended up as being proposed in a quantum inspired format, running on classical computers.

In Bangladesh, a quantum machine learning based supply chain backorder prediction tool was prototyped in 2023, running on 4 qubits[4214]. It is not easy to understand how it would scale with 4 qubits.

Retail

The retail sector could benefit from quantum computing generic use cases in transportation and logistics improvements as well as in some marketing optimization techniques. Robotized warehouse operations could also benefit from some quantum-driven optimizations like what **Ocado Group** (UK) is piloting with D-Wave[4215]. Always with D-Wave, **Pattison Food Group** (Canada) is looking at ways to optimize grocery operations.

Beyond that, consultants and analysts have not much to say about it or they sometimes err in mention big data applications, which are out of scope of quantum computers[4216].

[4209] See Feedback-Based Quantum Optimization by Alicia B. Magann, Kenneth M. Rudinger, Matthew D. Grace, and Mohan Sarovar, PRL, December 2022 (7 pages). It's about implementing a Max-Cut with using QAOA.

[4210] See Lyapunov-control-inspired strategies for quantum combinatorial optimization by Alicia B. Magann, Kenneth M. Rudinger, Matthew D. Grace, and Mohan Sarovar, PRL, December 2022 (19 pages). Is using QAOA with a feedback loop using quantum measurement.

[4211] See Securing supply chains with quantum computing by Troy Rummler, Sandia National Laboratories, February 2023.

[4212] See Adiabatic Quantum Computing for Logistic Transport Optimization by Juan Francisco Ariño Sales et al, January 2023 (43 pages).

[4213] See Quantum-South Explores Quantum Algorithms for Air Cargo Optimization by James Dargan, December 2022.

[4214] See QAmplifyNet: Pushing the Boundaries of Supply Chain Backorder Prediction Using Interpretable Hybrid Quantum - Classical Neural Network by Md Abrar Jahin et al, July 2023 (26 pages).

[4215] See Towards Real Time Multi-robot Routing using Quantum Computing Technologies by James Clark et al, SFTC Hartree and Ocado, 2019 (10 pages).

[4216] See Quantum computing in the consumer and retail sectors by Linda Ellett and Ian West, KPMG, July 2021. Extract: "*quantum computing could process data sets envisioned but not feasible at the current time to provide real-time consumer insights*".

PayPal is also testing a D-Wave quantum annealer to solve some optimization problems and detect fraud using quantum machine learning[4217].

Telecommunications

At this stage, the main quantum involvement of telecom companies is mainly related to quantum telecommunications and cryptography.

In the USA, **Verizon** launched a QKD trial on three sites in the Washington, D.C., area[4218]. In 2021, they also tested a VPN using a PQC (post-quantum cryptography) solution based on the Saber NIST competition finalist, for connection between two sites in the US and UK. **AT&T** is also investing resources in QKD networks, teaming up with Caltech.

In France, **Orange** participated to a QKD trial in the Nice region with partners including the InPhyNi research lab.

In the UK, **British Telecom** has also experimented a QKD setup to demonstrate some secured backup of datacenter resources. In October 2020, they deployed a pilot 6 km long QKD infrastructure in Bristol to connect several industry sites, in partnership with **Toshiba** as part of the **AQuaSec** (Agile Quantum Safe Communications) program, cofunded by UKRI[4219]. In October 2021, BT and Toshiba announced the deployment of a commercial QKD network in London, mixed with PQC for non photonic endpoints.

Quantum computing can still play a role in solving various optimization problems in the telecom industry. The most presented are the placement, power and frequency assignment of overlapping cells in 4G/5G mobile networks[4220], the configurations of paths and wavelengths on land line fiber optics networks and similar optimization problems for satellite communications[4221] and MIMO antennas optimization[4222]. It could also be used to enhance wireless networks energy efficiency with index modulation, a task that is a NP-hard problem[4223]. It uses a Grover Adaptive Search (GAS) providing a quadratic speedup (not an exponential one) and still requires fault-tolerant large-scale quantum computers that do not exist yet.

In Italy, the telecom operator **TIM** used a D-Wave QUBO based algorithm to optimize the setup of 4G/5G radio cells frequencies.

In another realm, more in the home automation space, researchers from **Mitsubishi** developed a quantum human activity monitoring algorithm using Wi-Fi in the ultra-wideband range of 60 GHz and a variational quantum circuit (VQC) building a QNN (quantum neural network)[4224].

[4217] See PayPal: Harnessing Quantum Computing in FinTech | D-Wave Qubits 2021, December 2021 (30 mn video).

[4218] See This Executive Director Is Leading Verizon Into the Future Through Quantum Computing by Joanna Goodrich, IEEE Spectrum, November 2020.

[4219] See BT and Toshiba install UK's first quantum-secure industrial network between key UK smart production facilities, October 2020.

[4220] See High-Speed Resource Allocation Algorithm Using a Coherent Ising Machine for NOMA Systems by Teppei Otsuka et al, NTT, December 2022 (16 pages).

[4221] See Heterogeneous Quantum Computing for Satellite Constellation Optimization: Solving the Weighted K-Clique Problem by Gideon Bass et al, Booz Allen Hamilton, 2017 (17 pages).

[4222] See Evaluation of quantum annealer performance via the massive mimo problem by Zsolt I. Tabi et al, August 2021 (14 pages). A Hungarian team assessed the use of D-Wave annealers to optimize MIMO antennas configurations. They found that some interesting results were obtained with the latest D-Wave Advantage annealer but that it will require at least another generation of better-connected qubits annealers to bring some form of quantum annealing advantage for this problem's solving.

[4223] See Quantum Speedup for Index Modulation by Naoki Ishikawa , IEEE Access, July 2021 (11 pages).

[4224] See Quantum Transfer Learning for Wi-Fi Sensing by Toshiaki Koike-Akino and Pu Wang, Ye Wang Toshiaki Koike-Akino, Mitsubishi, May 2022 (6 pages).

Finance

Finance is another great playground for experimenting quantum technologies and particularly quantum computing[4225] [4226] [4227] [4228].

Both because this vertical is quite hungry with forecasting and optimization needs and also because it is a rather solvent market with many economic players having the critical mass to invest in new technologies.

As shown in both Figure 879 and Figure 880, financial applications are usually segmented in three categories: simulations, optimizations and machine learning tasks with existing classical solutions and their potential quantum equivalent or superior solutions[4229].

problem category	use cases	classical solutions	quantum solutions
simulation	• derivative pricing • risk analysis (Basel, Solvency) • financial econometrics • maximum likelihood estimation • dynamic stochastic general equilibrium modelling (DSGE) • dynamic economic models.	• Monte Carlo integration • machine learning • Black-Scholes model	• quantum amplitude estimation in quantum Monte Carlo • quantum machine learning
optimization	• portfolio optimization • trading optimization • hedging • optimal arbitrage • credit scoring • financial crash prediction	• discrete/continuous variables • branch-and-bound for non-convex cases • interior-point methods for certain convex cases	• quantum optimization • quantum annealing with QUBO or NISQ QAOA • reverse quantum annealing • VQE
machine learning	• anomaly and fraud detection • natural language modeling • risk clustering • modeling credit spread • product recommendation	• regression, classification, clustering, PCA • deep learning • unsupervised cluster analysis	• quantum SVM, PCA, … • quantum machine learning (QCNN, QGAN, QGNN, …) • quantum cluster analysis

Figure 879: source: adapted from "A Survey of Quantum Computing for Finance" by Dylan Herman et al, JP Morgan, Universities of Chicago, Delaware, DoE Argonne National Lab and Menten AI, January 2022 (56 pages). Added in 2023.

Banks have a pressing need to transform themselves to adapt to constant technological and societal changes. They manipulate valuable data dumps. They need to optimize many facets of their business, starting with investment portfolios. It is all about optimizing portfolio returns, minimizing risks, manage regulations, mainly the Basel III framework rules with their liquid coverage radio constraints, and at last detect fraud risks as efficiently as possible. Also, assets are interdependent and transaction costs are variable depending on the type of assets. Their evolution responds to varying levels of uncertainty and risk.

[4225] See overview in Quantum Computing and Finance from the Quantum World Association, August 2018, which refers to Quantum computing for finance: overview and prospects by Roman Orus et al, 2018 (13 pages).

[4226] See the review paper A Survey of Quantum Computing for Finance by Dylan Herman et al, JP Morgan, Universities of Chicago, Delaware, DoE Argonne National Lab and Menten AI, January 2022 (56 pages).

[4227] See the review paper Quantum computing for finance by Dylan Herman et al, JPMorgan, Fujitsu, Menten AI, Nature Reviews Physics, July 2023 (29 pages).

[4228] See Quantum Computing - Moving Quickly From Theory to Reality, City, July 2023 (154 pages) which is… not an excellent report.

[4229] See Quantum Computing for Finance: State of the Art and Future Prospects by Daniel Egger et al, IBM Quantum, January 2021 (24 pages).

Illustration : 14 use cases in financial institutions

	Use cases	Unlock capability / value	Typical industry	FI publicly experimenting
Optimization	1 Portfolio selection, alloc. & optimization	Increase both the scope of assets that can be taken into account and dynamic multi-period scenarios that can be considered	Asset Management, Global Markets	Natwest, BBVA, Commerzbank, Standard Chartered, CBA, Nomura
	2 Optimal execution	Design the best execution strategy for entry, exit and rebalancing	Asset Management, Global Markets	-
	3 Capital allocation	Allow dynamic capital allocation without making over-simplifications (credit risk and insurance risk in particular)	All Financial Institutions	-
	4 Asset Liability Management	Increase the number or detail of assets and run much more detailed scenarios	Universal Banks, Asset Managers	-
	5 Transaction settlement	Ensure that large volume of trades is settled in the most optimized sequence and prioritization.	Global Markets, Transaction Banking, Clearing House	Barclays, Mastercard
	6 Yield curve fitting	Can solve much complex models to improve yield curve fitting accuracy	Global Markets, HFT	-
Machine Learning	7 Credit scoring / clustering	Build more realistic models thanks to the ability to take into account more variables and speed-up the training process	Retail banking	CaixaBank, Crédit Agricole
	8 Default early warnings	Improve detection of changing customer behaviours indicative of financial stress leveraging much more complex datasets	Retail banking	-
	9 Fraud detection / AML	Identify outliers based on a growing number of variables, which will lead to better adjusted models	Retail & Transaction banking, Global Markets	-
	10 Next Best Action / Product	Rely on more consistent clusters defined with a wider range of variables to improve outputs of predictive analytics	Retail and Private Banking, Insurance	-
Simulation	11 Derivative pricing	Improve dramatically the accuracy and efficiency of complex option pricing such as path-dependent and barrier options	Global Markets	JP Morgan, Goldman Sachs, BMO, Scotiabank
	12 Valuation and regulatory ratios	Perform computation on much wider and complex scenarios (VaR, XVA, valuation of credit derivatives, Solvency 2)	Banking, Insurance	HSBC
	13 Risk assessment & tail risk simulations	Compute a wide range of intraday complex stress tests, potentially even on a real time basis	Banking, Insurance	CaixaBank
	14 Multi-factor Interest rate models	Allow to take into account more realistic assumptions	Global Markets	-

Figure 880: Source: Quantum Market & Industry Applications by Jean-François Bobier, BCG, May 2023 (10 slides).

TABLE 5: Algorithms can improve computational efficiency, accuracy, and addressability for defined use case.

	Quantum Algorithm	Description	Impact	Needs	Simulation	Optimization	ML
VQE	Variational Quantum Eigensolver	Use energy states to calculate the function of the variables to optimize	Procedure to assign compute-intensive functions to quantum and those of controls to classical	Qubit number increases significantly with problem size		x	
QAOA	Quantum Approximate Optimization	Optimize combinatorial style problems to find solutions with complex constraints	Simplify analysis clauses for constraints and provide robust optimization in complex scenarios	Uncertain ability to expand to more optimization classes		x	
AE	Quantum Amplitude Estimator	Create simulation scenarios by estimating an unknown property, Monte Carlo style	Handle random distributions directly, instead of only sampling, to solve dynamic problems quadratically speeding up simulations	High Quantum Volume required for good efficiency	x	x	x
QSVM	Quantum Support Vector Machines	Supervised machine learning for high dimensional problem sets	Map data to quantum-enhanced feature space to enable separation and better separate data points to achieve more accuracy	Runtime can be slowed by kernel computation and data structure			x
HHL	Harrow, Hassidim, and Lloyd	Estimate the resulting measurement of large linear systems	Solve high dimensional problems speeding up exponentially calculations	Hard to satisfy prerequisites and high measurement costs to recover solutions		x	x
QSDP	Quantum Semidefinite Programming	Optimize a linear objective over a set of positive semidefinite matrices	Estimate quantum system states with less measurements to exponentially speedup in terms of dimension and constraints	High Quantum Volume required for good efficiency		x	

TABLE 6: Financial services focus areas and algorithms.

Financial Services	Example Problems	Solution Approach	Quantum Algorithm
Asset Management	Option Pricing Portfolio risk	Simulation	AE
Investment Banking	Portfolio Optimization Portfolio Diversification Issuance: Auctions	Optimization	Combinatorial: VQE, QAOA Continuous: QSDP AE
Retail & Corporate Banking	Financial Forecasting Credit Scoring (e.g. SME Banking) Financial Crimes: Fraud + AML	Machine Learning	QSVM HHL AE

Figure 881: Source: Quantum Computing for Finance: State of the Art and Future Prospects by Daniel Egger et al, IBM Quantum, January 2021 (24 pages).

There are also some interesting mathematical relationships between some key equations in finance and in quantum physics. A review paper from Isaiah Hull, Or Sattath, Eleni Diamanti and Goran Wendin from Sweden, Israel and France, describes the wealth of quantum algorithms that could be used in the economic and financial spheres[4230] (Figure 881):

- Solving **dynamic economic models** using interpolation algorithms.

[4230] See One Bit, Qubits, A Dollar: Researchers Say Economists Should Prepare for Quantum Money by Matt Swayne, January 2021, making reference to Quantum Technology for Economists by Isaiah Hull, Eleni Diamanti et al, December 2020 (120 pages). The report was published by Sveriges Riksbank, the employer of one of the contributors. They didn't fund any research related to this paper.

- **Numerical differentiation** algorithms in financial econometrics, structural micro econometrics, maximum likelihood estimation, dynamic stochastic general equilibrium (DSGE) modelling, and large-scale macroeconomic modelling conducted by central banks and government agencies.
- **Pricing of derivatives** using linear systems algorithms including matrices inversions, linear regressions, matrix powers [4231] and Markov chains time discretization[4232].
- **Macro-economic models** using finite elements methods.
- **Portfolio optimization** using most of the time quantum annealers but that could run on a gates-based quantum computer [4233].
- **Macroeconomics**, **forecasting**[4234], **modeling credit spread**, **credit scoring**[4235] and **pricing financial derivatives** based on quantum machine learning algorithms including principal component analysis, including some work done by Goldman Sachs in partnership with IBM[4236].
- **Simulation of agent choices** over time using Monte Carlo simulations.
- **Fraud detection** using various methods including quantum machine learning[4237] [4238].
- **Investment portfolio optimization** including a model published in 2015 and running on a D-Wave quantum annealer and based on a QUBO model and graph modeling [4239].
- **Model market instability**, also on a quantum annealer [4240].
- **Sentiment analysis**[4241].
- **Random number generation** used beyond cryptography, for simulations and estimation, particularly with Monte Carlo simulations.

Most of the existing pilot algorithms have been tested with a small number of assets, in the 1-500 range, particularly on D-Wave quantum annealers, while real-world scenarios are based on thousands if not hundred thousand of them.

[4231] See Quantum computational finance: martingale asset pricing for incomplete markets by Patrick Rebentrost, Alessandro Luongo, Samuel Bosch and Seth Lloyd, September 2022 (31 pages).

[4232] See Quantum Computational Algorithms for Derivative Pricing and Credit Risk in a Regime Switching Economy by Eric Ghysels, Jack Morgan and Hamed Mohammadbagherpoor, November 2023 (13 pages).

[4233] See Quantum computational finance: quantum algorithm for portfolio optimization by Patrick Rebentrost and Seth Lloyd, 2018 (18 pages), Benchmarking the performance of portfolio optimization with QAOA by Sebastian Brandhofer et al, July 2022 (28 pages) and NEASQC financial application library v1.0 available, 2022, a quantum quantitative finance library released by the NEASQC European consortium.

[4234] See Improved Financial Forecasting via Quantum Machine Learning by Sohum Thakkar, Iordanis Kerenidis et al, IRIF and QC Ware, May 2023 (26 pages).

[4235] See Quantum Machine Learning for Credit Scoring by Nikolaos Schetakis et al, August 2023 (11 pages) with tests up to 18 qubits.

[4236] See A Threshold for Quantum Advantage in Derivative Pricing by Shouvanik Chakrabarti et al, , Goldman Sachs, IBM and University of Maryland, May 2021 (41 pages). This work was improved in Towards Quantum Advantage in Financial Market Risk using Quantum Gradient Algorithms by Nikitas Stamatopoulos, William Zeng et al, Goldman Sachs and IBM, November 2021 (20 pages) which reduced the QPU clock rate requirement to 30kHz, not far from the 2.5kHz record from existing IBM QPUs and their plans to reach 10kHz in the near future.

[4237] See Mixed Quantum-Classical Method For Fraud Detection with Quantum Feature Selection by Michele Grossi et al, IBM, August 2022 (11 pages) dealing with a QSVM based method tested with only 3 qubits.

[4238] See Financial Fraud Detection: A Comparative Study of Quantum Machine Learning Models by Nouhaila Innan et al, August 2023 (30 pages).

[4239] In Solving the Optimal Trading Trajectory Problem Using a Quantum Annealer, 2015 (13 pages).

[4240] See these slides in this D-Wave presentation. See also Applications of Quantum Annealing in Computational Finance, 2016 (29 slides) and the Quantum For Quants website they created.

[4241] See Applying QNLP to sentiment analysis in finance by Jonas Stein et al, LMU and Aqarios, July 2023 (5 pages).

As we will also see, some interesting pilot projects have been launched with quantum simulators ala Pasqal, showing a potential quantum advantage in a not-so-distant future.

A **BCG** 2020 review on quantum computing financial use cases included a roadmap against the estimated progress of quantum hardware in the next 5 to 20 years, as shown in Figure 882 [4242].

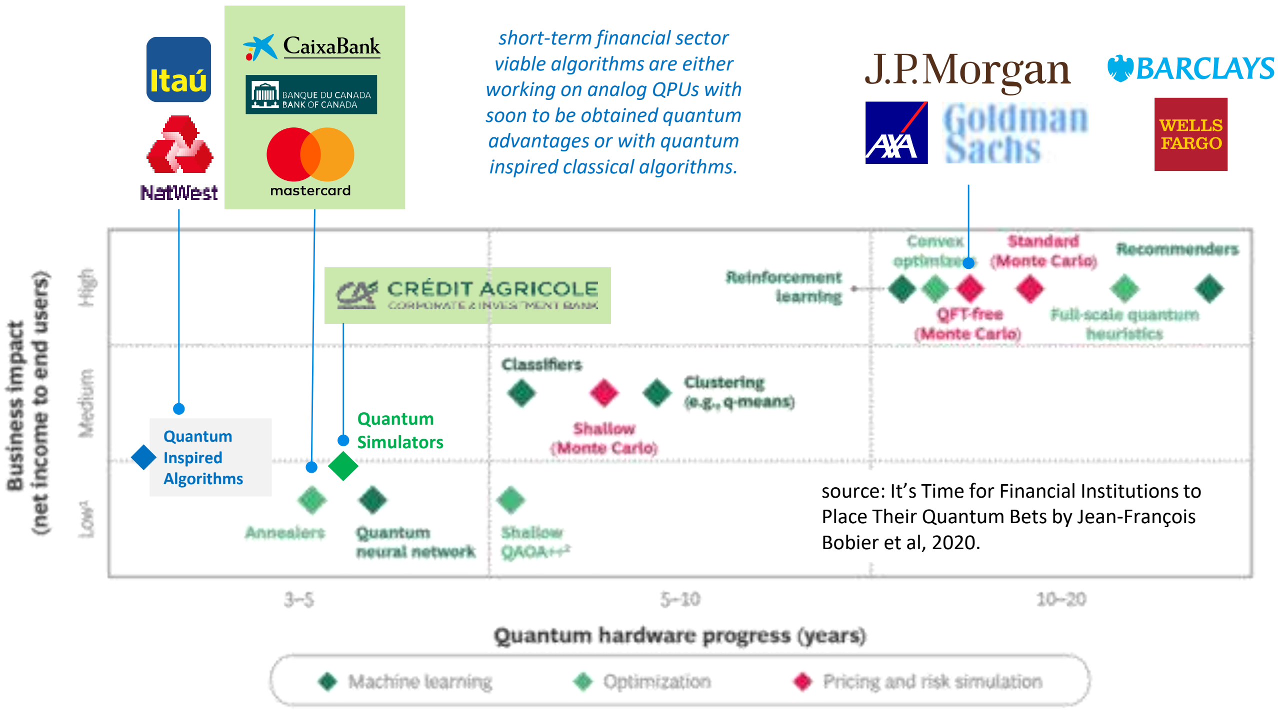

Figure 882: Source: It's Time for Financial Institutions to Place Their Quantum Bets by Jean-François Bobier et al, 2020, and logos placed by Olivier Ezratty, 2022. It shows that in the financial market, short term quantum solutions are either quantum inspired and running on classical hardware or on analog quantum computers. Most gate-based identified algorithms require fault-tolerant gate-base hardware and are positioned in the long term. Added in 2023.

McKinsey is more upbeat and is pushing banks to evaluate as fast as possible the potential use cases of quantum computing[4243]. D-Wave and IBM have been very active to push financial organizations to evaluate quantum computing benefits. So far, we have mainly proof of concepts and trials. Indeed, as the authors of another review paper published in 2020 point out: "*to date, quantum hardware is not advanced enough for solving any problem of practical relevance faster than classical computers*"[4244]. **Atos** (now Eviden) also published a white paper on quantum applications in finance[4245] .

As a result, most banks are either using quantum inspired algorithms and small scale quantum annealing and simulations algorithms or investigating highly demanding gate-based solutions that are positioned in the 10-20 year timeframe at best. Gate-based solution hardware resource requirements are indeed significant[4246].

[4242] See It's Time for Financial Institutions to Place Their Quantum Bets by Jean-François Bobier et al, 2020.

[4243] See How quantum computing could change financial services by Miklos Dietz et al, McKinsey, December 2020.

[4244] See Quantum algorithms are coming to finance, slowly by Sarah Butcher, November 2020, mentioning the review paper Prospects and challenges of quantum finance by Adam Bouland, Iordanis Kerenidis et al, 2020 (49 pages). This paper documents the quantum speedups theoretically achievable with Monte Carlo simulation and portfolio quantum algorithms.

[4245] See Quantum finance opportunities: security and computation, 2016 (20 pages). This is also the case of Everest Group with Quantum Computing in the Financial Services Industry-Infinite Possibilities or Extreme Chaos, 2018 (15 pages, $990... not really worth it).

[4246] See Credit Risk Analysis using Quantum Computers by Daniel J. Egger et al, 2019 (8 pages).

Most credit risks analysis and derivative pricing use cases mention a need for 7,500 logical qubits (as in Figure 883), which is a level above the 4,098 minimum number of logical qubits for breaking a 2048-bit RSA key with Shor's algorithm[4247] [4248]. This translates into millions of physical qubits.

	(d, T)		Error		T-count		T-depth		# Logical Qubits	
Method	**Auto**	**TARF**	**Auto**	**TARF**	**Auto**	**TARF**	**Auto**	**TARF**	**Auto**	**TARF**
Riemann Sum					$\geq 10^{43}$	$\geq 10^{18}$	$\geq 10^{43}$	$\geq 10^{18}$	-	-
Riemann Sum (no-norm)	$(3, 20)$	$(1, 26)$	2×10^{-3}		1.6×10^{11}	5.5×10^{10}	1.5×10^{8}	1.6×10^{8}	23k	17k
Re-parameterization					1.2×10^{10}	9.8×10^{9}	5.4×10^{7}	8.2×10^{7}	8k	11.5k

Table 1: Resources estimated in this work for pricing derivatives using different methods for a target error of 2×10^{-3}. As representative use cases of business interest with non-trivial complexity, we consider a basket autocallable (Auto) with 3 underlyings, 5 payment dates and a knock-in put option with 20 barrier dates, and a TARF with one underlying and 26 payment dates. Detailed definitions of these contracts and their parameters can be found in Appendix A.4. We find that Grover-Rudolph methods [10] are not applicable in practice (details in Appendix B) and that Riemann summation methods require normalization assumptions to avoid errors that grow exponentially in T. Even if those normalization issues were avoided, as detailed in the Riemann Sum (no-norm) row, the re-parameterization method still performs best. See Section 4.1 for a discussion of the Riemann summation normalization. The detailed resource estimation is discussed in Sections 4.1.2 and 4.2.3.

Figure 883: Source: A threshold for quantum advantage in derivative pricing by Shouvanik Chakrabarti et al, Goldman Sachs, IBM and University of Maryland, May 2021 (41 pages).

Large financial institutions have been studying quantum computing for a couple of years now.

D-Wave worked with a diverse set of banks including **Deutsche Bank** with the creation of the Quantumforquants website, dedicated to the uses of quantum in finance. In 2022, they announced an R&D partnership with **Mastercard** to create quantum-hybrid applications for optimizing consumer loyalty and rewards programs, cross-border settlement, and fraud management. The **Bank of Canada** and **Multiverse Computing** created a prototype quantum simulation of the cryptocurrency market using a D-Wave quantum annealer. It simulated a financial network with 8-10 players[4249].

Since 2017, IBM has been highlighting partnerships with **JPMorgan Chase**[4250], **Barclays**[4251], **HSBC**[4252] [4253] (who also works with Quantinuum) and **Wells Fargo**[4254] to study the uses of quantum in trading strategy optimization, investment portfolio optimization, pricing and risk analysis.

[4247] See again A threshold for quantum advantage in derivative pricing by Shouvanik Chakrabarti et al, May 2021 (41 pages). 7.5K logical qubits and T-depth of 54 million are needed for pricing derivatives.

[4248] See also End-to-end resource analysis for quantum interior point methods and portfolio optimization by Alexander M. Dalzell et al, Goldman Sachs, AWS, November 2022 (38 pages) which deals with a FTQC algorithm proposal with a 8 million qubits, a T-count of 8×10^{29} and some QRAM, generating a total computing time of 10,000 years.

[4249] See Bank of Canada and Multiverse Computing Complete Preliminary Quantum Simulation of Cryptocurrency Market by Multiverse, April 2022.

[4250] J.P. Morgan recruited an IBM veteran in quantum computing, Marco Pistoia, who had contributed to the development of Qiskit Aqua. See JP Morgan Chase poaches an IBM 'Master Inventor' with 26 patents for quantum computing by Hugh Son, January 2020. This quantum activity is integrated in their "Quantitative Research Group". See also Option Pricing using Quantum Computers by Nikitas Stamatopoulos, Daniel J. Egger, Yue Sun, Christa Zoufal, Raban Iten, Ning Shen and Stefan Woerner, JPMorgan Chase, ETH Zurich and IBM, May 2019-July 2020 (20 pages).

[4251] See Why banks like Barclays are testing quantum computing, de Penny Crossman, July 2018, Barclays demonstrates proof-of-concept quantum clearing algorithm by Cliff Saran, October 2019, Quantum Algorithms for Mixed Binary Optimization applied to Transaction Settlement by Lee Braine et al, October 2019 (8 pages) and Quantum Machine Learning in Finance: Time Series Forecasting by Dimitrios Emmanoulopoulos and Sofija Dimoska, Barclays, February 2022 (20 pages).

[4252] See HSBC Working with IBM to Accelerate Quantum Computing Readiness, IBM, March 2022 and Unsupervised quantum machine learning for fraud detection by Oleksandr Kyriienko and Einar B. Magnusson, University of Exeter, HSBC and Canada Square, August 2022 (7 pages).

[4253] See Approaching Collateral Optimization for NISQ and Quantum-Inspired Computing by Megan Giron et al, HSBC, May 2023 (17 pages).

[4254] See Wells Fargo prepares to take a quantum leap by Poornima Apte, CIO Magazine, June 2022.

Goldman Sachs hired Will Zeng, from Rigetti Computing, who had developed the Quil language. Will Zeng also works for the Unitary Fund which promotes open sourced quantum solutions, tools and benchmarks and is also a partner for Quantonation since 2023. With IBM, they work on different algorithms designs like the one on pricing derivatives already mentioned above. They are not yet operational but help define the hardware requirements to be able to run them. It is still at least in the mid-term.

Microsoft devised a way to make stock value predictions using topological computing, a farfetched idea given the state of the art of their Majorana fermion based qubits[4255].

Also, noteworthy is the investment by the **Royal Bank of Scotland** (RBS) in 1Qbit along with Fujitsu and Allianz.

Itaú Unibanco (South America) and **QC Ware** (USA) undertook a four-month joint project in the customer retention domain with QML algorithms improving customer churn prediction models. The main outcome of the project is a new method to do this on classical computers and improved predictions by 2% and the model precision by 6.4% (from 71%). They used 180,000 data points[4256].

Of course, you must add to this quick review all the quantum software startups that are entering this market. They are either using quantum inspired algorithms or pilot projects using gate-based or annealing-based quantum computing. Among these are **1Qbit**, **Multiverse Computing**, **ApexQubit**, **JosQuantum** and **QuantFi**.

Portfolio optimization case studies

Many prospective case studies and algorithmic research deal with optimizing portfolios. You can assess their viability with reading the fine prints on the number of portfolio assets, period and optimization constraints. When the case studies are tested on real QPUs, their sizing is always way below the usual production needs.

JPMorgan Chase tested in 2022 a mean-variance portfolio optimization algorithm with 32 trapped ions qubits, using XY mixing gates for the QAOA mixing part alternating with phase operators. This can be emulated on a classical server cluster[4257].

TCS Research India studied in 2023 how to combine quantum annealing and gate-based quantum computing to enable solving large-scale optimization problems efficiently on the available hardware. They improved the Large System Sampling Approximation (LSSA) algorithm that divides a large problem into several smaller problems and then combines the spare solutions to approximate the solution to the original problem. Tests were implemented on real-world stock data from the Indian stock market on up to 64 assets and it performed as well as classical optimization methods[4258].

CCB Fintech (China) tested another QAOA-based portfolio optimization algorithm with a quantum emulator using 18 qubits, that could fit on a simple laptop[4259].

Raiffeisen Bank International with Data Reply (Italy) tested a portfolio optimization using QUBO on D-Wave Advantage annealers. The dataset was taken from a real-world problem used in production.

[4255] As documented in Decoding Stock Market Behavior with the Topological Quantum Computer, 2014 (24 pages).

[4256] See QC Ware Applies Quantum Computing Principles to Increase Customer Retention at Itaú Unibanco by James Dargan, May 2022.

[4257] See Alignment between Initial State and Mixer Improves QAOA Performance for Constrained Portfolio Optimization by Zichang He, Marco Pistoia et al, JP Morgan Chase, May 2023 (13 pages).

[4258] See Exploring the synergistic potential of quantum annealing and gate model computing for portfolio optimization by Naman Jain and M Girish Chandra, TCS Research India, May 2023 (12 pages).

[4259] See Quantum Portfolio Optimization: Binary encoding of discrete variables for QAOA with hard constraint by Bingren Chen et al, CCB Fintech, April 2023 (18 pages).

They used two D-Wave hybrid solvers combining a quantum annealer and classical methods, and a purely classical algorithm. The portfolio was structured into three main asset classes: equity (EQ), fixed-income (FI) and money market (MM). A client portfolio typically ranges from 9 to 11 assets[4260].

In another trial, Data Reply worked on improving the value of Sharpe Ratio which evaluates the risk-adjusted returns of an investment strategy or portfolio. It calculates the excess return of an investment, relative to a risk-free rate of return, per unit of its volatility or risk. It measures how an investment compensates investors for their level of risk. The ratio is $(R_p - R_f)/\sigma_p$ with R_p being the expected return of the investment, R_f, the return of a low-risk asset such as government bonds and σ_p is the standard deviation of the investment returns, which gauge its volatility or risk. The ratio is used to compare different investment options, the higher being the better[4261].

JP Morgan Chase researchers created in 2021 the NISQ-HHL approach, a hybrid version of the HHL algorithm that works on NISQ devices small-scale portfolio-optimization problems, using 6 and 14 assets from the S&P 500. It uses various techniques like mid-circuit measurement, Quantum Conditional Logic and qubit reset and reuse in the Quantum Phase Estimation routine used for eigenvalue estimation. The technique can reduce the number of ancillary qubits to just one and qubit connectivity requirements and (usually costly) SWAP gates. The test was implemented on a Quantinuum System Model H1 with 11 qubits. In other words, we are obviously far from getting any sort of quantum advantage in such a situation[4262].

NatWest is experimenting quantum inspired algorithms running on traditional computers to optimize its investment portfolios (HQLA for High Quality Liquid Assets).

Derivatives and options pricing

Derivatives are financial contracts whose value is derived from an underlying asset, such as stocks, bonds, commodities, or market indices.

Derivative pricing is about determining the fair value of these contracts based on several factors, like the current market conditions, the price of the underlying asset, their time of expiration, interest rates, market volatility, and other specific parameters. Derivative pricing covers options, futures, swaps, and forwards.

Options pricing is a subset of derivative pricing that deals with valuing options contracts. An option gives the holder the right, but not the obligation, to buy or sell an underlying asset at a predetermined price (named strike price) within a specified period (expiration date). Options can be classified into **call options** (giving the right to buy the underlying asset) and **put options** (giving the right to sell the underlying asset). Options pricing models like the **Black-Scholes model**, estimate the fair value of options contracts. It considers the underlying asset current price, the strike price, time to expiration, interest rates, dividends (if applicable), and volatility. With all these variables, algorithms can determine the theoretical value of an option and assess whether it is overpriced, underpriced, or fairly priced. Some methods use quantum generative adversarial networks[4263].

[4260] See A real world test of Portfolio Optimization with Quantum Annealing by Wolfgang Sakuler et al, March 2023 (32 pages).

[4261] See Financial Portfolio Optimization: a QUBO Formulation for Sharpe Ratio Maximization by Mirko Mattesi et al, Data Reply, February 2023 (15 pages).

[4262] See NISQ-HHL: Portfolio Optimization for Near-Term Quantum Hardware by Romina Yalovetzky et al, January 2022 (14 pages). Presents a version of the HHL algorithms suitable for NISQ quantum computers.

[4263] See Efficient option pricing with unary-based photonic computing chip and generative adversarial learning by Hui Zhang et al, August 2023 (11 pages).

Derivatives pricing can be based on quantum Monte Carlo algorithms and simulations with the additional capability to support negative price and payoff with a new encoding method. These algorithms all require future FTQC platforms and provide a theoretical quadratic speedup[4264] [4265].

Risk analysis case studies

In both classical and quantum computing, many risk analysis are using Monte Carlo simulations. It is used to assess and analyze the potential outcomes of an investment or portfolio based on probabilistic models and random sampling. It involves creating a large number of random scenarios *aka* simulations to estimate the range of possible outcomes and their associated probabilities. It makes use of a random number generator which can be a generic QRNG or even some quantum algorithm using a quantum walk algorithm. This technique requires a large number of simulations and FTQC platforms, like in a study from Taiwan using phase kickback and amplitude estimation (QAE) plus an inverse QFT, which was tested with only 10 qubits emulated classically with Qiskit[4266].

Many other risk analysis case studies were also recently published but were tested with only a few qubits[4267] [4268] [4269] [4270]. **GE Research** with IonQ developed a risk management hybrid algorithm with 20 qubits from the IonQ Aria QPU[4271].

CACIB (France) along with Pasqal and Multiverse worked on a "fallen angel" detection for loans risk mitigation and was closer to production levels. Their dataset was covering a period of 20 years (2001-2020) with >90,000 instances characterized by about 150 features, with the historical evolution of credit ratings and financial variables. Predictors include rating, financial and equity market variables and their trends calculated on a bi-annual, quarterly and five-year basis. It was based on >2,000 companies from 10 different industrial fields and 100 sub-sectors in 70 different countries. The training set consists of around 65,000 examples from the 2001-2016 period and test set with 26,000 examples from the 2016-2020 period, with only 9% of fallen angels in the training set and 12% in the test set. A binary classification task was using QUBO and random graph sampling. It was compared with classical tensor network based algorithm. A quantum advantage was estimated to require about 150-342 neutral atoms for precision outcome with linear extrapolation and 2,800 with the subsampling method[4272]. The actual test on a Pasqal machine went up to 60 functional neutral atoms.

At last, tensor network classical solutions are also developed for derivative pricing settings[4273].

[4264] See Real Option Pricing using Quantum Computers by Alberto Manzano et al, March 2023 (20 pages).

[4265] See Quantum Monte Carlo algorithm for solving Black-Scholes PDEs for high-dimensional option pricing in finance and its proof of overcoming the curse of dimensionality by Yongming Li and Ariel Neufeld, January 2023 (46 pages).

[4266] See Preparing random state for quantum financing with quantum walks by Yen-Jui Chang et al, February 2023 (11 pages).

[4267] See Quantum Monte Carlo simulations for financial risk analytics: scenario generation for equity, rate, and credit risk factors by Titos Matsakos and Stuart Nield, March 2023 (30 pages).

[4268] See Quantum Deep Hedging by El Amine Cherrat, Iordanis Kerenidis, Marco Pistoia et al, JP Morgan Chase, QcWare and CNRS IRIF, March 2023 (43 pages) with a QML risk analysis tested with 20 qubits from Quantinuum.

[4269] See Towards practical Quantum Credit Risk Analysis by Emanuele Dri et al, December 2022 (12 pages) with 9 qubits classically emulated with Qiskit. Which explains the "towards" in the title.

[4270] See Low depth amplitude estimation on a trapped ion quantum computer by Tudor Giurgica-Tiron et al, QCWare, IonQ and Goldman Sachs, September 2021 (12 pages).

[4271] See IonQ and GE Research Demonstrate High Potential of Quantum Computing for Risk Aggregations, June 2022 and Copula-based Risk Aggregation with Trapped Ion Quantum Computers by Daiwei Zhu et al, June 2022 (10 pages). They say the results are better than full classical algorithms which always puzzles me given 20 qubits are very easily emulated on a classical computer.

[4272] See Financial Risk Management on a Neutral Atom Quantum Processor by Lucas Leclerc et al, CACIB, Multiverse, IOGS and Pasqal, December 2022 (17 pages).

[4273] See A highly efficient tensor network algorithm for multi-asset Fourier options pricing by Michael Kastoryano et al, Amazon, March 2022 (9 pages). Quantum inspired tensor networks based derivative pricing algorithm.

Currency case studies

In a Master's Thesis at RWTH Aachen University, Cameron Perot studied how to develop foreign exchange applications on D-Wave annealers[4274].

Czech National Bank compared algorithms for universal gate-based quantum computers (QAOA, VQE and Grover adaptive search), quantum annealers and classical exact algorithms for currency optimization of an investment portfolio but the results were not conclusive with any available quantum system[4275].

IBM tested a VQE algorithm to run crypto-currency arbitrage on 25 qubits in a 127 qubit QPU, running for 12 hours[4276].

Other case studies

Satisplay (Italy) announced in October 2023 its willingness to deploy in production a quantum-hybrid application which optimizes by 50% its mobile payment customer rewards initiatives with using D-Wave's CQM hybrid solver. The case study is only documented with a press release.

Quantum money

The Hull/Diamanti et al paper also reviews the broad topic of quantum money, and idea born circa 1969 and published in 1983 by **Stephen Wiesner**. The idea is to use quantum objects properties and the no-cloning theorem to avoid any counterfeiting and forging. Any bill has two unique identifying numbers: one classical serial number that is public and one secret random quantum number called a "random classical bill state" (Figure 884).

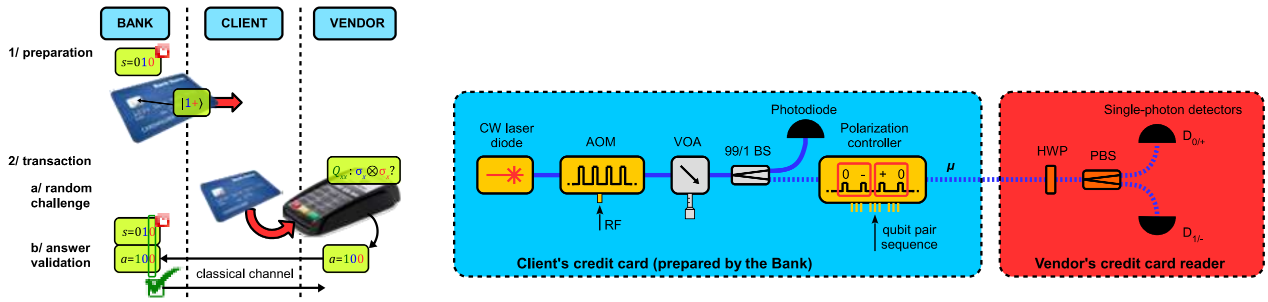

Figure 1. **Practical quantum money protocol.** The sequence of interactions between the credit card holder (client), the bank and the vendor involved in the transaction. In the preparation phase, the bank uses a secret key to prepare the quantum state loaded on the credit card, which is then given to the client. In the transaction phase, the vendor randomly selects one out of two challenge questions, measures the qubits and sends the outcome to the bank, who can then verify the validity of the credit card or detect a forgery attempt.

Figure 2. **Experimental setup of the quantum money system.** The credit card state preparation is performed using pulses carved from light emitted by a telecommunication wavelength laser diode using an acousto-optic modulator (AOM). A multi-stage polarization controller (EOSPACE) is then used to select the polarization states according to the protocol by applying suitable voltages. The average photon number of pulse μ is set by a variable optical attenuator (VOA) and is calibrated with a 99/1 beam splitter (BS) and a photodiode. The credit card reader is materialized by a standard polarization analysis setup including a half-wave plate (HWP), a polarization beam splitter (PBS) and two InGaAs single-photon avalanche photodiodes (ID201). The entire setup is synchronized using a multi-channel delay generator and is controlled by software incorporating the random state generation and data acquisition and processing.

Figure 884: Source: Experimental investigation of practical unforgeable quantum money by Mathieu Bozzio, Iordanis Kerenidis, Eleni Diamanti et al, 2017 (10 pages).

The central bank is the only one keeping the classical bill state. It is encoded using for example polarized photons on a 0° or 45° basis. Only the bank knows this sequence of encoding. There are many variations of this concept of quantum money, with different degrees of anonymity and private and public schemes.

[4274] See Quantum Boltzmann Machines: Applications in Quantitative Finance by Cameron Perot, January 2023 (62 pages).

[4275] See Finding the Optimal Currency Composition of Foreign Exchange Reserves with a Quantum Computer by Martin Vesely, Czech National Bank - Risk Management Department, March 2023 (30 pages).

[4276] See Differential Evolution VQE for Crypto-currency Arbitrage. Quantum Optimization with many local minima by Gines Carrascal et al, IBM et al, August 2023 (17 pages).

Quantum money could be physical segmented into bills, coins and lightning schemes[4277]. An untraceable quantum coin proposal was made around 2010. But there are many shortcomings with these schemes which are just non implemented ideas at this stage.

One of these being that it requires quantum memory that doesn't exist yet, and which, by the way, is therefore not miniaturizable to be embedded in devices like a credit card[4278]. The Quantum Lightning variation prevents the bank from creating multiple bills with the same serial number. You also have semi quantum money that requires no quantum communication infrastructure. All in all, it is quite difficult to assess the practicality of such quantum money ideas.

Insurance

The insurance market is a vertical that also must fix complex optimization problems, particularly related to risk modelling. The various related surveys and review papers I have found are not as rich as in the financial services vertical[4279].

Some analysts are using the usual Shor based codebreaking attacks cybersecurity red flag and explaining all the risks businesses may face in the future[4280]. The related reports are clearly misleading, stating for example that quantum-based communications could be "*quicker over long distances*" on top of being better secured[4281].

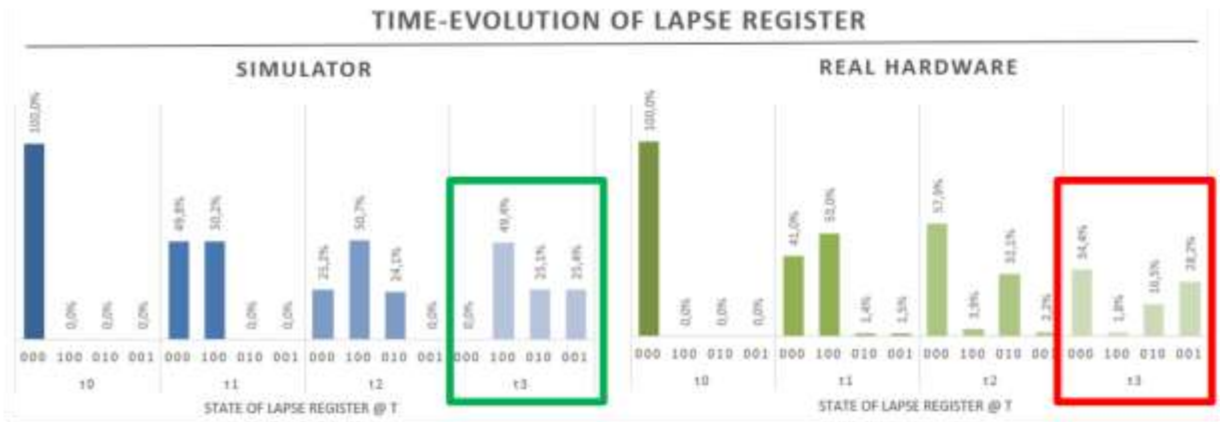

Figure 13: Comparison of simulator and real hardware results for the lapse register. We can see that the first lapse event delivers results which are close to the theoretical expectation. After a tenfold increase of the costs by the controlled linear amplitude function at step 2.2, the QPU returns more or less meaningless results.

Figure 885: AXA Konzern AG trial of a NISQ quantum amplitude estimate algorithm for insurance contract valuation showing in red the detrimental of qubit noise with 27 qubits on an IBM QPU. Source: Potential Applications of Quantum Computing for the Insurance Industry by Michael Adam, October 2022 (43 pages). Added in 2023.

A 11 pages report was published late 2019 by **Novarica**, an US insurance consulting services company[4282].

[4277] Quantum Lightning is a public key quantum money type.

[4278] See Experimental investigation of practical unforgeable quantum money by Mathieu Bozzio, Iordanis Kerenidis, Eleni Diamanti et al, 2017 (10 pages).

[4279] See The impacts of quantum computing on insurance - From theory to reality from Lloyds's, February 2021 (34 pages).

[4280] See Quantum computing a potential cyber risk for re/insurers by Charlie Wood, November 2019.

[4281] See Top 5 insurance quantum computing use cases by Danni Santana, January 2018. One speaker in Quantum Computing in Insurance - Interactive discussion, February 2021 (59 mn) estimates that the Shor threat can materialize in between 7 and 10 years, this being "conservative". Well that's kind optimistic.

[4282] See Quantum Computing to Affect Insurer Tech Strategies, December 2019.

Besides the usual generic description of the whereabouts of quantum computing, it contains only one and a half pages of insurance related use cases ideas.

They are related to risk modelling and portfolio optimization. It also mentions quantum machine learning used to better detect and mitigate fraud, risks assessment with actuarial models for enhanced pricing and risk pooling precision, investments portfolio optimization and model life expectancy algorithms for large populations (Figure 886).

Another paper was published in 2022 by Michael Adam from **AXA Konzern AG** dealt with the potential applications of quantum computing in insurance. It is an investigation of a NISQ QAE (amplitude estimate) and QFT (Fourier transform) based algorithm to implement insurance contracts valuation and compared it with Monte-Carlo classical algorithms[4283]. It was tested it with existing IBM QPUs using 27 qubits, showing the detrimental effect of qubit noise, in the chart below. It is not surprising given QAE and QFT are FTQC algorithms. The paper did not make an assessment on the FTQC required resources to obtain some quantum advantage (Figure 885).

One risk modelling algorithm created by **JoS QUANTUM** is indeed documented[4284]. Another use case was publicized by **Caixabank** with D-wave in 2022. They developed an investment portfolio hedging and portfolio optimization solution that generated a 90% decrease in time-to-solution when compared to their classical legacy system[4285].

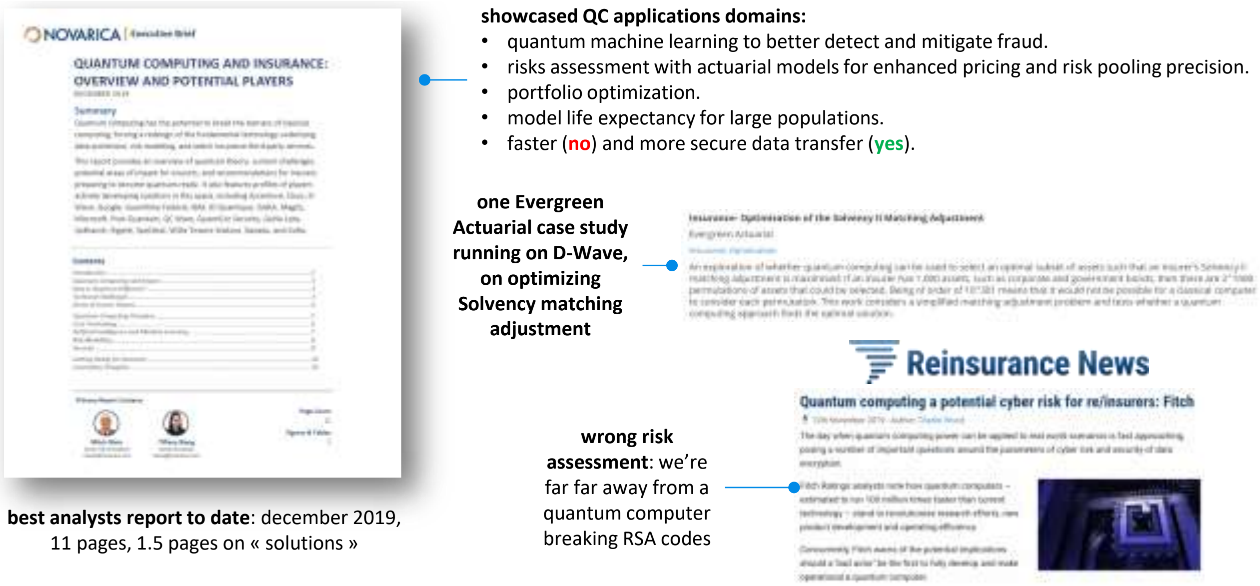

Figure 886: some use cases and constraints for quantum computing in the insurance business. (cc) Olivier Ezratty, 2021.

Marketing

Marketing is also an area where optimization algorithms for complex systems based on quantum computers could be of interest. This concerns the optimization of the marketing mix, that of media plans, or the maximization of advertising revenues, various areas that are also invested by the AI field.

Volkswagen experimented a vehicle recommendation system in online sales sites, with a D-Wave.

[4283] See Potential Applications of Quantum Computing for the Insurance Industry by Michael Adam, October 2022 (43 pages).

[4284] In A Quantum Algorithm for the Sensitivity Analysis of Business Risks by M. C. Braun et al, March 2021 (21 pages).

[4285] See CaixaBank Group, D-Wave Collaborate on Innovative New Quantum Applications for Finance Industry, March 2022.

Once again, predictive systems based on the exploitation of past data and simulation based on the knowledge of market operating rules are once again opposed to each other. However, these rules do not fall under the notion of AI expert systems, which manage logical predicates, but more complex causality models[4286].

Content and media

Wondering how we could use quantum computing to create some content and art? That's the weird idea some have, at least with regards to music creation.

Computers have played a role in music creation for a while so why not quantum computers? Quantum mechanics is about waves, like music[4287]!

The **Quantum Music** project[4288], was run by Volkmar Putz and Karl Svozil in Austria from 2015 to 2018. It led to the **QuTune** project[4289]. **Eduardo Miranda** from the Interdisciplinary Centre for Computer Music Research (ICCMR) at the University of Plymouth (UK) also works on using quantum computers to create music[4290]. He is currently preparing the release of the book "Quantum Computer Music", several of its chapters having already been published on arXiv[4291].

At this point, quantum music is about finding another source of randomness to create melodies (using quantum walks-based algorithms) and to generate credible synthetic voice. It led to the organization in November 2021 of a quantum music online event, organized, unsurprisingly, by the University of Plymouth (UK) with the sponsoring from IBM and Cambridge Quantum Computing[4292].

In 2019 **Quantum Sound** was the first music created and performed from measurements of superconducting qubits. It was not far from some form of random music generator. The project was run with the financial support of the Yale University Quantum Institute as well as from their top quantum scientists Michel Devoret and Robert Schoelkopf[4293].

At last, quantum music was generated in a more sophisticated way with a quantum annealer from D-Wave in 2021 by a very international team (India, Poland, Mexico, Estonia)[4294]. More recently, voice synthesis was studied using quantum algorithms[4295].

[4286] See for example Display Advertising optimization by quantum annealing processor by Shinichi Takayanagi, Kotaro Tanahashi and Shu Tanaka of Waseda University and A quantum-inspired classical algorithm for recommendation systems by Ewin Tang, July 2018 (36 pages). The latter classical algorithm exceeds the performance of a quantum algorithm realized for D-Wave quantum computers.

[4287] See Quantum music Physics has long looked to harmony to explain the beauty of the Universe. But what if dissonance yields better insights? by Katie McCormick, May 2021.

[4288] See Quantum music by Volkmar Putz and Karl Svozil, 2015 (5 pages).

[4289] It was linked to QuTune Project Quantum Computer Music Resources, about making music with quantum computing, and making quantum computing with music. This project started in Spring 2021.

[4290] See Quantum Computer: Hello, Music! by Eduardo R. Miranda, June 2020 (32 pages), Creative Quantum Computing: Inverse FFT Sound Synthesis, Adaptive Sequencing and Musical Composition by Eduardo R. Miranda, 2021 (32 pages) and The Arrival of Quantum Computer Music by Eduardo R. Miranda, May 2020.

[4291] See Making Music Using Two Quantum Algorithms by Euan J. Allen, Jacob F. F. Bulmer and Simon D. Small, January 2022 (13 pages), New Directions in Quantum Music: concepts for a quantum keyboard and the sound of the Ising model by Giuseppe Clemente et al, April 2022 (14 pages), QuiKo: A Quantum Beat Generation Application by Scott Oshiro, April 2022 (23 pages) and A Quantum Natural Language Processing Approach to Musical Intelligence by Eduardo Reck Miranda et al, December 2021 (41 pages).

[4292] See 1st International Symposium on Quantum Computing and Musical Creativity.

[4293] See Superconducting qubits as musical synthesizers for live performance by Spencer Topel, Kyle Serniak, Luke Burkhart and Florian Carle, March 2022 (17 pages).

[4294] See Music Composition Using Quantum Annealing by Ashish Arya et al, January 2022 (29 pages).

[4295] See Teaching Qubits to Sing: Mission Impossible? by Eduardo Reck Miranda and Brian N. Siegelwax, July 2022 (31 pages).

In 2022, a first "visual art" was created with the help of a quantum computer. The Quantum Prophet was created with the quantum artists trio Insight and **Kipu Quantum** (Figure 887). With the support of some AI, the author interfaced himself in real time with a quantum computer. Via motion capture, he manipulated qubits to modify aesthetically his animated 3D artwork which is sold with an NFT. All-in-all, the creative process was still largely in the hands of some humans!

Figure 887: The Quantum Prophet.

Another content related explored field is gaming. Proof of concepts of quantum computing based games were recently proposed for Mastermind[4296] and Go[4297] and of games using quantum principles[4298].

This is all early stuff. Don't count yet to see quantum computers having an impact on classical video games and the Metaverse, its most recent incarnation.

Another form of quantum art without pretentions is to exhibit art to explain what quantum physics and technologies are about, like was done in Switzerland in 2020[4299].

Defense and aerospace

The military-industrial complex has always been a big consumer of advanced IT. It is therefore not surprising that it is interested in quantum technologies. This is obviously the case in the USA but also in Europe, with Airbus being one of the first to take an interest in quantum applications, and also China and Russia to name a few others[4300].

The **US Air Force** has also identified various needs that can be covered by the four categories of quantum technologies with a special mention for quantum sensing in time measurement and navigation[4301]. They are also interested in quantum radars and, finally, in quantum computing applied to optimization problems (Figure 888).

In France, **DGA** funded or co-funded since 2011 about twenty theses on quantum and eight projects for €6.6M. The **Defense Innovation Agency** (reporting to the DGA) planned to launch a call for projects for quantum sensors in 2020 and in 2021 to fund a research project to support PQC in dedicated hardware. In July 2020, it became an ASTRID RFP on sensors, cryptography and quantum communications and on the creation of quantum computing algorithms[4302].

[4296] See Winning Mastermind Overwhelmingly on Quantum Computers by Lvzhou Li et al, July 2022 (27 pages).

[4297] See Quantum Go: Designing a Proof-of-Concept on Quantum Computer by Shibashankar Sahu et al, June 2022 (7 pages).

[4298] See Defining Quantum Games by Laura Piispanen et al, May 2022 (19 pages) which deals with the games using some principles of quantum physics but not games handled by quantum computers.

[4299] See "Do you speak quantum?" A quantum physics and art exhibition by Chiara Decaroli and Maciej Malinowski, June 2022.

[4300] See the review paper Quantum Technology for Military Applications by Michal Krelina, EPJ Technology, November 2021 (52 pages) that makes a pretty extensive inventory of defense use case for quantum sensing, communications and computing. On quantum radars, it showcases some adequate cautiousness. It still contains some misunderstanding like on page 27 on "*processing Big Data from surveillance and reconnaissance and identifying targets using quantum ML/AI*". Quantum computing is probably not bound to manage big data per se.

[4301] See Quantum Information Science at AFRL by Michael Hayduk, December 2019 (21 slides).

[4302] See Defense Research and Innovation: launch of a new ASTRID call for projects on quantum technologies, July 2020.

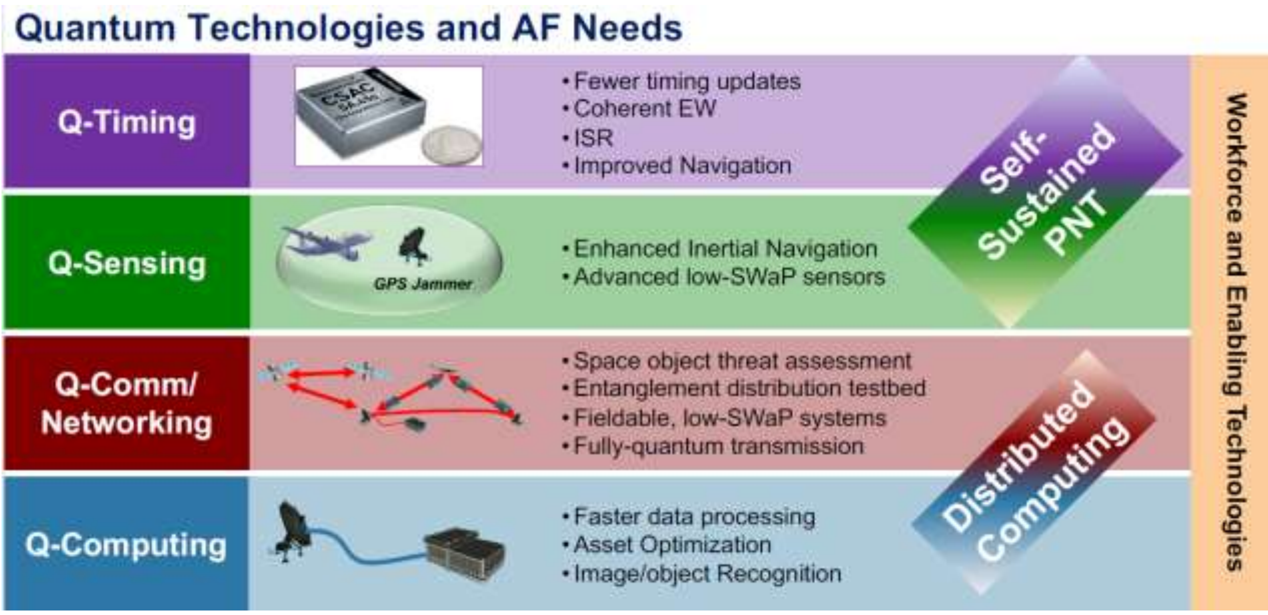

Figure 888: quantum technology and US Air Force needs. Source: Quantum Information Science at AFRL by Michael Hayduk, December 2019 (21 slides).

Quantum communications is also studies by the military around the world to secure communications[4303], particularly on the battlefield. **NATO** is experimenting it[4304]. Of course, **China** is working on it although with the visible part being only the civil use cases.

Here are some various published case studies of quantum use in this vast sector.

It starts with **Lockheed Martin** partnering with **Google** and **NASA** to test D-Wave annealers staring in 2014. They developed with it a solution for formal proof of software operation. NASA co-founded the Quantum Artificial Intelligence Laboratory (QuAIL) with Google, operating a D-Wave Two. They test quantum optimization algorithms in different directions to optimize spacecraft filling, a variant of the bin-packing problem, on quantum versions of machine learning and deep learning algorithms, on problem decomposition and embedded computing[4305] (Figure 889).

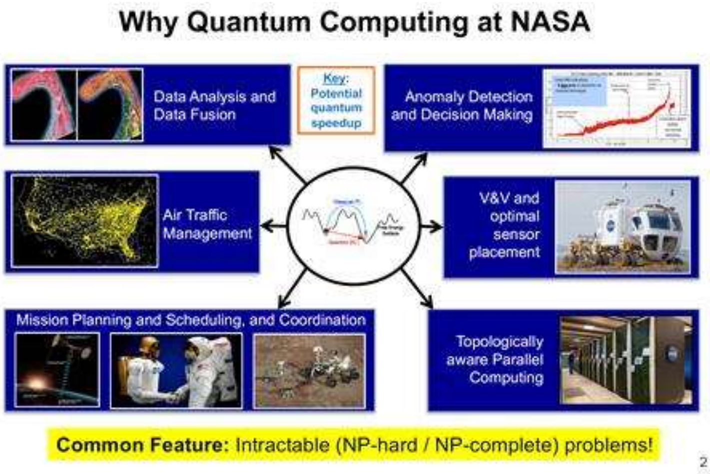

Figure 889: Source: Quantum Computing at NASA: Current Status by Rupak Biswas, September 2017 (21 slides).

In 2015, **Raytheon** and **IBM** demonstrated the efficiency of a quantum algorithm using a "black box" or "oracle" to reconstruct an unknown bit string, all running on an IBM 5 qubit general purpose quantum computer[4306]. This is obviously far from a real-world use case.

4303 See Quantum Communication for Military Applications by Niels M. P. Neumann et al, TNO, November 2020 (11 pages).

4304 See NATO cybersecurity center finishes tests of quantum-proof network by Jonathan Greig, March 2022.

4305 This is well described in Quantum Computing at NASA: Current Status by Rupak Biswas, September 2017 (21 slides).

4306 This is documented in Demonstration of quantum advantage in machine learning (12 pages).

Airbus group created a team based at their Newport site in Wales, which is tackling the uses of quantum computing, particularly in the analysis of aerial images (not obvious...) or for the design of new materials (more obvious). They also want to optimize wings air flow, a problem that is nowadays dealt with by finite element simulation. They could try to optimize the air conditioning in airplanes, the biggest source of cabin noise, above the plane's engines!

In a different field, navies are interested in quantum sensing and particularly in micro-gravimetry measurement tools used to detect submarines.

Thales prototyped in 2021 a quantum annealing solution running on D-Wave Advantage (5,000 qubits) to optimize radar configuration (the "Integrated Side Lobe Ratio" or ISLR NP-hard problem, which solution is "*the finding optimal sequences of phase shifts to minimize the mean squared cross-correlation sidelobes of a transmitted radar signal and a mismatched replica*")[4307] [4308]. Yet, it didn't reach any quantum computing advantage, but the authors think future evolutions of D-Wave annealers may be promising for this respect.

In collaboration with **Fraunhofer IAIS**, Thales also prototyped a quantum image alignment solution for satellite images with quantum-based key point extraction and feature matching. It was tested with D-Wave quantum annealers and gate-based quantum computers.

The outcome is always the same: classical systems still deliver superior results, but the proposed methods have the potential to outperform classical systems with future quantum computers[4309].

Satellite mission planning is another interesting use case, that can be addressed with variational quantum algorithms[4310]. **Thales Alenia Space** with **Terra Quantum** applied hybrid quantum computing to the optimization of earth observation satellite missions[4311]. The problem consists in maximizing the number of high-priority tasks to be realized by multiple satellites. They used a hybridized quantum-enhanced reinforcement learning agent that can achieve a completion percentage of 98.5% over high-priority tasks, an improvement over the classical baseline greedy methods having a completion rate of 63.6%. Where was that tested? On a QMWare classical emulator from TerraQuantum, thus not showing a quantum advantage by definition. Similar problems were solved at a small scale on D-Wave annealers[4312].

The use of quantum technologies in the military field also gives rise to elucubrations that hybridize the plausible and the offbeat, such as those of the American political scientist **James Der Derian**, director of Project Q at the University of Sydney[4313].

4307 See Phase-coded radar waveform AI-based augmented engineering and optimal design by Quantum Annealing by Timothé Presles, Cyrille Enderli, Rémi Bricout, Florence Aligne and Frédéric Barbaresco, Thales, 2021 (9 pages).

4308 See Phase-coded Radar Waveform Design with Quantum Annealing by Timothée Presles, Cyrille Enderli, Gilles Burel, and El Houssaïn Baghious, Thales Defense Mission Systems and University of Brest, August 2023 (11 pages).

4309 See Towards Bundle Adjustment for Satellite Imaging via Quantum Machine Learning by Nico Piatkowski et al, April 2022 (8 pages).

4310 See A Hybrid Classical Quantum Computing Approach to the Satellite Mission Planning Problem by Nils Quetschlich et al, TUM, July 2023 (6 pages).

4311 See Quantum algorithms applied to satellite mission planning for Earth observation by Serge Rainjonneau et al, February 2023 (13 pages).

4312 See Optimization of Image Acquisition for Earth Observation Satellites via Quantum Computing by Antón Makarov et al, July 2023 (12 pages).

4313 See Drones, radars, nuclear: how the quantum will change the war by Vic Castro, February 2020. Some remarks on this article: Rydberg atom-based qubits are only one of the types of qubits currently being studied. They are said to be "cold atom-based" and are moreover the specialty of Pasqal (France) and QuEra (USA). There are many other types of qubits. The text also makes a big confusion in qubits and logical gates between qubits. These gates connect qubits together. They are often systems based on the diffusion of microwaves, photons emitted by lasers or magnetic couplers. Rydberg atoms are qubits and not qubit couplers.

You also have weird plans like the UK military who seems interested to put quantum computers in tanks, which doesn't make much sense, at least for the next 20 years. The reason is they've been lured in this path by Orca which touts ambient temperature quantum computing[4314].

Intelligence services

The world of intelligence and targeted eavesdropping is obviously interested by the potential usage of quantum technologies. Shor's algorithm is the main application targeted by organizations managing electronic eavesdropping such as the **NSA** and all its colleagues. They are firefighters who are eager to decode information intercepted from various targets (embassy communications, economic intelligence, etc.) and to protect the sensitive communications of their own states against this type of decryption. They are therefore investing simultaneously in quantum computing (the "arsonist" dimension) and in quantum keys and post-quantum cryptography (the "firefighter" dimension).

On the other hand, these investments are not very public. The NSA has communicated well for almost ten years on the firefighter dimension but very little on the arson dimension.

They have surely acquired the various generations of D-Wave computers to get their hands on, in conjunction with **Lockheed Martin** which is one of their major suppliers. NSA also maintains a joint laboratory with NIST and the University of Maryland, **QuICS**, which will be launched in 2014.

One way to lift a veil on these activities is to detect laboratory and startup grants awarded by **IARPA**, the intelligence innovation agency led by the DNI (Director of National Intelligence), who oversees all American intelligence. It consolidates collaborative research funding for all intelligence agencies. It has already launched five programs around quantum technologies: in superconducting qubits (CSQ), logical qubits (LogiQ, with IBM), error correction (MQCO, also with IBM), the creation of development tools (QCS, with Raytheon and GeorgiaTech) and quantum annealing computation (QEO). But it is not clear that this has significantly advanced the state of the art.

Other Western intelligence services may also have acquired D-Wave, notably the British CGHQ. The NSA is also in contact with IBM and Google to explore the path of superconducting quantum general-purpose computers.

Industry

Industry in the broadest sense of the term is another vertical market for quantum computing. As soon as there is a complex optimization problem for scheduling, logistics or complex system design support, quantum may have its say.

The Japanese **JSR Corporation** is one of the companies working with IBM in the quantum field, mainly for the creation of new materials. **LG Electronics** announced a similar partnership with IBM in January 2022 to "*support big data, artificial intelligence, connected cars, digital transformation, IoT, and robotics applications*", you name it all.

It seems that quantum computing could be used within computer-aided design tools[4315]. But the document cited in the note comes back to the basics of quantum computing without being very elaborated on quantum computing uses in CAD, a very common phenomenon when quantum computing is pushed in various industries.

[4314] See UK Military Wants to Install Quantum Computers in Tanks for Some Reason by Lonnie Lee Hood, Future Society, June 2022.

[4315] According to Computer-Aided Design for Quantum Computation by Robert Wille, Austin Fowler and Yehuda Naveh (Google and IBM), 2018 (6 pages).

The routing of electronic circuits is also a NP-complete problem that could be partly handled by quantum algorithms, provided they have a sufficient number of qubits. This could be useful for designers of ASIC-type circuits and especially FPGAs, these circuits whose operating logic is dynamically programmable via two key parameters: the decision tables of the processing units and the links between these units.

Otherwise, various use cases have been tested, but not yet in a quantum advantage regime, since being emulated on a classical system with a small number of qubits like for the prediction of surface roughness in 3D printed specimen[4316], other artificial vision operations[4317] and with some multi-agent neural network[4318]. All this is not far from classical quantum inspired methods that are also investigated[4319].

Climate change

Climate change fixing easily comes forward when the societal benefits of quantum computing and other quantum technologies are promoted by various stakeholders in the quantum ecosystem[4320]. Some vendors like PsiQuantum, Terra Quantum, Zapata and IQM promote this use case in their communication. We'll see here what categories of quantum solutions and algorithms are being talked about and their credibility. They cover various chemical simulations enabling optimized carbon cycle processes, weather and climate forecasts, transportation, and power grid optimizations.

Most of the proposed solutions are very long term and require large FTQC QPUs[4321]. Overpromises on fixing climate change come for example from **McKinsey**[4322] or **PsiQuantum** who has no functional quantum computer yet[4323] and on weather modelling from **Pasqal** and **BASF**[4324].

[4316] See Quantum Machine Learning Approach for the Prediction of Surface Roughness in Additive Manufactured Specimens by Akshansh Mishra and Vijaykumar S. Jatti, April 2023 (18 pages) conducts a comparison of three quantum algorithms: the Quantum Neural Network (QNN), Quantum Forest (Q-Forest), and Variational Quantum Classifier (VQC) adapted for regression. They successfully experimented it with 8 qubits. You now know what to think about it.

[4317] See Quantum artificial vision for defect detection in manufacturing by Daniel Guijo et al, Multiverse, August 2022 (11 pages). It uses a quantum support vector machine (QSVM) classification algorithm for two classes separation, implemented with 16 qubits on a classical emulator, again in the "laptop emulation" zone, and on a D-Wave annealer. The results are good but it's unsure whether this is bringing a real quantum advantage.

[4318] See Quantum Multi-Agent Actor-Critic Neural Networks for Internet-Connected Multi-Robot Coordination in Smart Factory Management by Won Joon Yun et al, January 2023 (11 pages). The experimented was done on a classical emulation platform with 2 Nvidia Titan XP GPUs and 12 GB memory.

[4319] See Quantum Inspired Optimization for Industrial Scale Problems by William P. Banner, William D. Oliver et al, May 2023 (10 pages).

[4320] See for example Sustainable Development - How Quantum Technologies Can Help Drive the UN's Sustainable Development Goals by Capgemini, 2022 (15 pages).

[4321] See How Quantum Computing Can Tackle Climate and Energy Challenges by Annarita Giani and Zachary Goff-Eldredge, October 2022. This sort of document should avoid using the present tense. The great majority of the quantum algorithms that could potentially help address some climate change issues require quantum computers that do not exist yet and will not exist before a couple decades, at best.

[4322] See Quantum computing just might save the planet by Peter Cooper, Philipp Ernst, Dieter Kiewell and Dickon Pinner, McKinsey, May 2022.

[4323] See PsiQuantum Announces Qlimate Initiative Developing Breakthrough Climate Technologies Enabled by Quantum Computing, May 2022.

[4324] See Pasqal, BASF to collaborate on quantum compute-powered weather modeling, July 2022. This works seems serious from the pure mathematic standpoint, with modelizing weather models using differential equations and quantum neural network models labelled PINNs (physics-informed neural networks). What these stories don't tell is the size of the models that would be required in real use cases. Most of the time, these sizes are way beyond the capacities of the related quantum computers, even when taking into account their 5-year roadmap. On top of that, one unaddressed issue is how these quantum simulators are fed with training data. The larger the data set, the slower it is, and it requires a lot of classical pre-processing.

Quantinuum is a little more credible with working on low-emission refrigerant production which deals with more simple problems although their existing 20 functional qubits are of no real use today[4325].

Chemical and physics simulations

Chemical and material simulations are some of the most amazing potential use cases of quantum computing. It could enable us to better understand how nature works, how to imitate it and how to produce energy or various materials more efficiently[4326].

The typical futuristic quantum computing use cases are about creating:

- **New solvents** for carbon capture and adsorbents for direct-air capture[4327].
- **Fertilizers production** methods innovative catalysts, related to understanding the famous FeMoCo complex natural reaction in nitrogenase (more on this page 1056).
- **Higher density batteries**, enabling the electrification of transportation[4328].
- **More efficient solar panels** even though its impact could be minimal, given the key figure of merit is more the cost per produced Watt than solar panel yields.
- **New zero-carbon cement clinkers**, although I've not yet seen any research paper on this.
- **New materials**, replacing cement, aluminum, and steel which producing and recycling are energy intensive[4329].
- **Cleaner fuel** by replacing coal with liquified natural gas (LNG)[4330].
- **Hydrogen production techniques** with modeling fuel cell membranes, catalysts and electrical currents.

PsiQuantum is pushing this climate change agenda for a reason: it requires quantum error correction and a large number of physical and logical qubits, namely as a starter, a million physical qubits and one hundred logical qubits. The use case he is putting forward is the improvement of fertilizer production. The caveat is that even PsiQuantum's own literature positions FeMoCo complex quantum simulation in the multiple thousands of logical qubits scale and it would be only part of the solution since, after this simulation, the creation of a new catalyst and production process has to be created

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[4325] See How Quantum Computing Can Help Keep Things Cool, 2022.

[4326] See How quantum computing can help tackle global warming, Jeremie O'Brien, PsiQuantum, May 2022 which is focused on this topic.

[4327] See Carbon-capture technology could benefit from quantum computing, PhysicsWorld, April 2023. The proposed algorithms is based on a VQE to calculate how a carbon dioxide molecule reacts with an ammonia molecule. The research involved using a classical supercomputer to simulate the quantum calculation, including the noise levels expected in a NISQ. Obtaining the vibrational energy levels of NH_2COOH proved difficult and would require an FTQC QPU.

[4328] See Quantum computation of dominant products in lithium–sulfur batteries by J.E. Rice et al, Journal of Chemical Physics, 2021.

[4329] See Riverlane-led Research Shows How Quantum Computers May Simulate Materials to Reduce Humanity's Impact on The Environment by Matt Swayne, The Quantum Insider, March 2023. It is about simulating nickel oxide and palladium oxide, which are used to create chemicals and fuels. The proposed algorithm enables the quantum simulation of large solid-state systems with runtimes often associated with much smaller molecular systems. It has potential impact in fuel cells to petrochemicals and hydrogen production. It was published in Quantum computation for periodic solids in second quantization by Aleksei V. Ivanov et al, Joan Camps et al, Physical Review Research, March 2023 (22 pages). The proposed algorithm requires QEC and FTQC, between 10^{10} and 10^{15} T gates to obtain a good chemical accuracy with the small studied molecules, runtimes reaching 50 days and 100 millions of physical qubits.

[4330] See the review paper Quantum Computing and Simulations for Energy Applications: Review and Perspective by Hari P. Paudel et al, ACS Engineering, August 2022 (46 pages). This interesting paper covers fossil fuel energy optimization. It provides not many real estimations of hardware resources to process the showcased algorithms. It has a long bibliography of 455 items. With a small mistake page 155 presenting Sycamore as a "commercially accessible quantum computer". It is not.

and simulated as well. So, when he says that green ammonia could be produced and used in ships by 2030, he is very optimistic.

Weather forecasts and climate predictions

On a more operational basis and potentially shorter term, another use case for quantum computing and climate change is weather and climate forecasting[4331]. It could potentially help run simulations and predict scenarios faster and more accurately than with existing solutions, which already work well. Weather modeling can use a wide range of data like temperature, pressure, wind, moisture, and other meteorological variables[4332 4333].

This brings more operational use cases:

- For finding optimum locations for wind and solar farms.
- To make more accurate weather simulations to predict energy production and improve grid balancing and supply predictions.
- To predict CO_2 emissions peaks due to fossil fuel power plant activities in connection with renewable energy production and optimize the whole grid with better planning.

Modeling weather with quantum computers probably involves the ingestion of a lot of data which is not where they excel. Also, many weather forecasting equations are non-linear (Navier-Stokes equation). This is addressed with encoding multiple copies of a quantum state and let these copies interact with each other via an interaction Hamiltonian[4334]. Related work from researchers from Oxford University shows a need for 755 logical qubits, 10^{29} gates and 10^{20} s of computing time, so billion years! Fidelities of 10^{-29} create a huge and overhead to create logical qubits[4335]. Making weather forecast requires probably tens of billions of parameters[4336]. On the other hand, climate predictions may be more macroscopic in nature and require less data, although very diverse in type. It may be more accessible to future NISQ and FTQC QPUs.

Transportation optimizations

This topic was already dealt with in the transportation and logistics section of this part of the book (page 1065). It is about the famous TSP problem (traveling salesperson problem) and other variants but can also involve the optimization of trucks loads to avoid trucks being driven empty. This also includes route planning with routes offering better fuel efficiency.

Power grid optimizations

Power grid optimizations could potentially get the help of quantum computing when the problem complexity exceeds the capacity of classical systems and algorithms[4337].

[4331] See Will Quantum Technology Be The Silver Bullet For Climate Change? by Markus Pflitsch, Terra Quantum, Forbes, September 2022.

[4332] See Quantum and climate action - The role of quantum computing in creating a sustainable future by Daryl Pereira, January 2023.

[4333] See The ESCAPE project: Energy-efficient Scalable Algorithms for Weather Prediction at Exascale by Andreas Müller et al, 2019 (28 pages).

[4334] See Variational quantum algorithms for nonlinear problems by M. Lubasch et al, University of Oxford, PRA, 2020 (7 pages).

[4335] See Quantum Computers for Weather and Climate Prediction: The Good, the Bad and the Noisy by Felix Tennie and Tim Palmer, Oxford University, October 2022 (20 pages). It reports "*the inversion of a matrix of size $N=2^{17}$ on currently available quantum backends from IBM*" where the feat is not inverting a matrix per se but finding a matrix eigenvalues expectation values. A full matrix inversion has an exponential cost due to the number of shots necessary for reading out its results. See also the misquoted Large-scale quantum hybrid solution for linear systems of equations by Michael Perelshtein et al, 2021-2022 (8 pages).

[4336] See Strong scaling for numerical weather prediction at petascale with the atmospheric model NUMA by Andreas Müller et al, 2018.

[4337] See Qubits for climate change by Prachi Mishra, August 2022.

The laundry list of potential solutions includes:

- Optimize power generation through simulations and map demand/supply with more accuracy[4338] [4339].
- Identify leaks and gaps in power supply, impacting how it is stored, transmitted, and distributed.
- Preempt forest fires and floods.
- Predict extreme weather conditions that can impact power supply and surge in demand.

Some of these potential applications are also inventoried in a publication from the Q4Climate initiative[4340].

Science

Fundamental research is starting to test and use quantum computing, particularly in materials development and particle physics research[4341].

High-energy particles physics

High-energy particles physics is a domain that could make use of quantum computing, mostly to analyze the data coming out of particle accelerators but also for various quantum simulation tasks[4342] (Figure 890).

After having investigated quantum computing for a good number of years with a first workshop organized in November 2018, **CERN** launched a formal **Quantum Technology Initiative** (QTI) in September 2020[4343]. They want to use quantum computing to analyze the noisy data coming from their ultrasensitive particles detectors and to simulate the behavior of many-body quantum phenomena[4344]. CERN also participates to international quantum computing education and training with its online training resources. IBM worked with CERN to select/classify LHC events using QSVM[4345].

4338 See Quantum computing opportunities in renewable energy by Annarita Giani and Zachary Eldredge, SN Computer Science, September 2021 which identifies opportunities in simulation, scheduling and dispatch, and reliability analyses.

4339 See Quantum Computers Can Now Interface With Power Grid Equipment, NREL, July 2023, which deals with some partnership between NREL and Atoms Computing. They tout having put a quantum computer in the loop for grid optimization. In reality, it was a digital quantum emulator of the Atoms Computing QPU.

4340 See Quantum technologies for climate change: Preliminary assessment by Casey Berger, Agustin Di Paolo, Tracey Forrest, Stuart Hadfield, Nicolas Sawaya, Michał Stęchły and Karl Thibault, June 2021 (14 pages).

4341 See Applying quantum computing to a particle process by Glenn Roberts Jr., Lawrence Berkeley National Laboratory, February 2021, referring to Quantum Algorithm for High Energy Physics Simulations by Benjamin Nachman et al, February 2021 (6 pages). The algorithm used to detect particles using the 20 qubits IBM Q Johannesburg quantum system in the cloud is not providing any quantum advantage but would be promising with a larger number of qubits. See also the review paper Quantum Simulation for High Energy Physics by Christian W. Bauer et al, April 2022 (103 pages) and Simulating Collider Physics on Quantum Computers Using Effective Field Theories by Christian W. Baue et al, November 2021 (7 pages), Quantum computing hardware for HEP algorithms and sensing by M. Sohaib Alam et al, April 2022 (23 pages), Quantum Computing for Data Analysis in High-Energy Physics by Andrea Delgado et al, March 2022 (22 pages) and Snowmass White Paper: Quantum Computing Systems and Software for High-energy Physics Research by Travis S. Humble, March 2022 (17 pages).

4342 See Quantum Computing for High-Energy Physics: State of the Art and Challenges. Summary of the QC4HEP Working Group by Alberto Di Meglio et al, July 2023 (41 pages).

4343 It was later formalized in their CERN Quantum Technology Initiative Strategy and Roadmap by Di Meglio et al, October 2021 (46 pages).

4344 See Particle track reconstruction with noisy intermediate-scale quantum computers by Tim Schwägerl et al, March 2023 (6 pages).

4345 See Application of quantum machine learning using the quantum kernel algorithm on high energy physics analysis at the LHC by Sau Lan Wu et al, Physical Review Research, 2021 (9 pages).

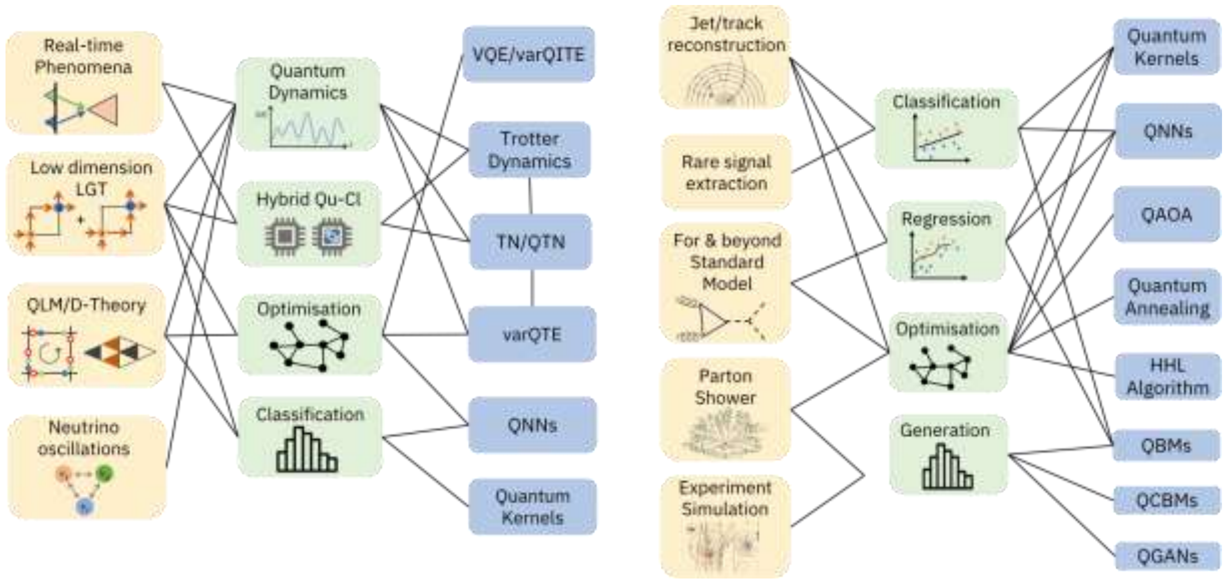

Figure 890: various use cases of quantum computing in the field of high-energy physics.
Source: Quantum Computing for High-Energy Physics: State of the Art and Challenges. Summary of the QC4HEP Working Group by Alberto Di Meglio et al, July 2023 (41 pages).

Back in 2017, **Caltech** used a D-Wave Two X quantum annealer with 1,098 qubits to "rediscover" the Higgs boson using CERN LHC data and a QAML algorithm (quantum annealing machine learning)[4346]. Later in 2020, they improved it with their QAML-Z algorithm, quantum annealing machine learning model zooming in on a region of the analyzed energy surface[4347].

Sometimes, scientists are toying with quantum computers with so few qubits that it doesn't make any sense, the underlying mathematics being much simpler to solve with a simple classical computer[4348].

Astrophysics

In astrophysics, superconducting qubits are also used to detect dark matter, in the form of axions, a dark matter candidate and hidden photons, that would interact with the photons[4349]. Other researchers are also using squeezed states to detect axions[4350]. Quantum computing is also tested to simulate exotic magnetic materials[4351].

Quantum physics

Quantum computing prospects also exist to do some quantum physics in various fields like with simulating gravitational based quantum entanglement[4352] or quantum field theory[4353].

[4346] See Solving a Higgs optimization problem with quantum annealing for machine learning by Alex Mott et al, 2017 (5 pages).

[4347] See Quantum adiabatic machine learning with zooming by Alexander Zlokapa et al, Caltech, October 2020 (9 pages).

[4348] See Simulating neutrino oscillations on a superconducting qutrit by Ha C. Nguyen et al, December 2022-February 2023 (26 pages). This is a bit absurd with one qutrit being implemented with IBM qubits, way below any quantum advantage. What's the point of doing a quantum computer simulation for a mathematical problem that is easy to solve on a simple laptop or Raspberry Pi?

[4349] See Searching for Dark Matter with a Superconducting Qubit by Akash V. Dixit et al, April 2021 (7 pages).

[4350] See A quantum enhanced search for dark matter axions by K. M. Backes et al, 2021 (8 pages).

[4351] See Quantum computing enables simulations to unravel mysteries of magnetic materials by Elizabeth Rosenthal, Oak Ridge National Laboratory, February 2021, using a 2000Q D-Wave annealer.

[4352] See Digital quantum simulation of quantum gravitational entanglement with IBM quantum computers by Carlos Sabín et al, February 2023 (11 pages).

[4353] See Towards a variational Jordan–Lee–Preskill quantum algorithm by Junyu Liu et al, December 2022 (30 pages). For simulating quantum field theory physics.

Software and tools vendors

There are already many quantum software and development tools startups, particularly with regards to what suitable hardware is available (Figure 891). Initially, many of them were developing software running on D-Wave annealers. Then, as gate-based vendors like IBM started to put their hardware in the cloud, most software vendors adopted it. Many software vendors adopt hybrid approaches that combine business knowledge, associated algorithms and their execution on classical machines and quantum computers, hybrid classical-quantum algorithms, or so-called "quantum inspired" algorithms that run on classical computers. Only a few software vendors have adopted the quantum simulation paradigm which is a pity given these systems may be the most viable in the mid-term.

Even though quantum software won't solve all business and technical problems, it is time for legacy software vendors to give a look at the value it could provide[4354].

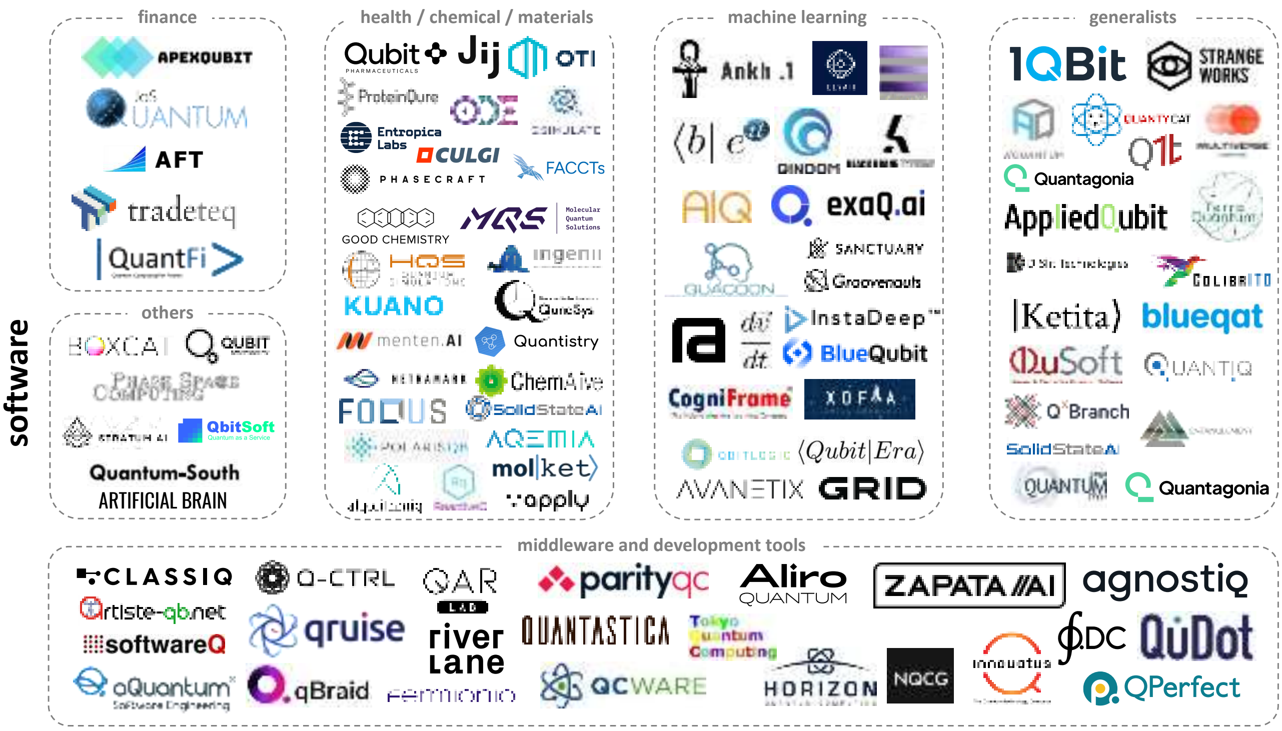

Figure 891: a nice logo map of the quantum software industry. By the way, if you want to show up properly in such market maps, please make SVG logos of your company easy to access! (cc) Olivier Ezratty, March 2024.

These approaches are essential for survival. Indeed, a startup cannot be exclusively dedicated to quantum computing at the risk of only being able to sell proofs of concepts on a very small scale that cannot generally be deployed industrially[4355].

There are real opportunities to position yourself in this emerging market! You will notice that this inventory does not include any Chinese startup. This is probably not by chance. This ecosystem is therefore still very young. It will evolve in parallel with the development of commercial quantum computers. You see already the startup scene maturing with many vendors adopting platforms approaches and developing partnerships models all over the place[4356]. China is not very well versed in software compared to hardware and seems to have put the quantum priority on cybersecurity more than on quantum computing.

[4354] See Why quantum software will be eating the world by Yuval Boger, June 2022.

[4355] This principle of reality is well described in The hard sell of quantum software by Jon Cartwright, 2019.

[4356] See a couple examples in Collaboration is Dominating Quantum Computing by Russ Fein, The quantum Leap, April 2022.

1QBit (2012, Canada, $35M) is a multi-sector quantum software company. It was funded among others by Fujitsu, as well as by Accenture and Allianz.

They have developed various low-level quantum algorithmic components that are hardware neutral. This includes, for example, the graphs processing that they apply in several markets, via their consulting activity. They cover financial markets, for the dynamic optimization of investment portfolios or to simplify the allocation of asset classes in a portfolio[4357].

They also developed QEMIST, a library for accelerating innovation in materials science and drug discovery. In addition to being a long-standing partner of D-Wave, they also work with IBM and Fujitsu (Figure 892). The startup already has over 100 employees. Their customers include Dow Chemical (chemicals), Biogen (biotechs) and Allianz. In April 2020, they launched the "Quantum Insights Network", a network of around 100 experts and content in quantum computing.

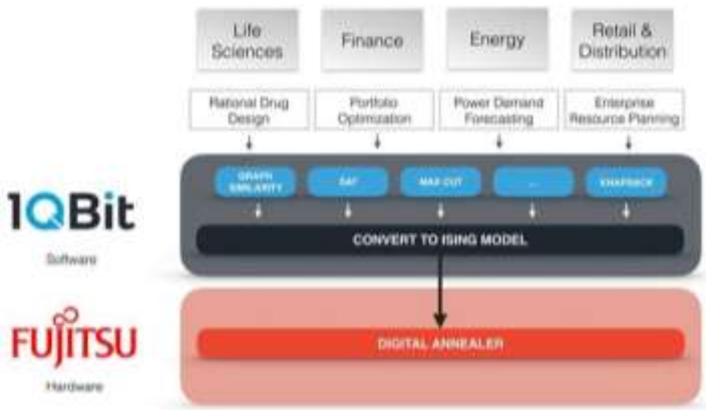

Figure 892: 1QBit Software running on Fujitsu Hardware. Source: Fujitsu.

Adaptive Finance Technologies (2020, Canada) came out of the Creative Destruction Lab. It was created by Roman Lutsiv, Vlad Anisimov and Edward Tang and develops investment and credit risk management software.

They used classical machine learning methods and are prototyping quantum machine learning solution running on D-Wave annealers.

Algorithmiq (2020, Finland, $15M) is a spin-off from the University of Turku created by Sabrina Maniscalco (CEO) which develops quantum software for life science and data science, particularly on drug discovery and genomics[4358]. They also created an online quantum science and technologies learning.

AIQTECH Inc (2018, Canada**)** is a machine learning specialist that explores the uses of QML. They are partners of the IBM Quantum Network. It is a two-person shop created by Farzad Quassemi (CEO) and Felix Motzoi.

Aliro Quantum (2018, USA, $3M) is a startup cofounded by Prineha Narang (CTO, coming from Harvard) and Jim Ricotta (CEO) with 25 employees as of December 2023. It is selling software and services for the creation of entanglement-based quantum networks.

Their software platform comprises AliroNet, released in October 2022, a solution providing emulation services of entanglement-based quantum networks, which can help design small scale pilots and universal entanglement-based quantum networks, including quantum networks connecting quantum sensors. They also have Aliro Simulator to assess the needs of quantum network and create performance estimates, Aliro Orchestrator, a configuration, management, control, and monitoring tool, and AlirOS, a quantum network operating system.

[4357] See Solving the Optimal Trading Trajectory Problem Using a Quantum Annealer, 2015 (13 pages).

[4358] See Disease Gene Prioritization With Quantum Walks by Harto Saarinen, Sabrina Maniscalco et al, Algorithmiq and University of Turku, November 2023 (12 pages).

They are partnering with HQAN (Hybrid Quantum Architectures and Networks, funded by the NSF, and belonging to the University of Illinois) to develop the seeds of a distributed quantum computing network in the USA.

Ankh.1 (2018, USA) has developed Anubis Cloud, a virtual machine in the cloud for data scientists that integrates with the open source solution Jupyter as well as with the TensorFlow and Keras learning machine frameworks.

Apply Science (2019, Italy) is an applied mathematics services company who is experimenting quantum computing in the IBM Quantum Network. They are testing QML algorithms for virtual drug development.

AppliedQubit (2019, UK) presented itself as a publisher of quantum software for businesses.

In particular, they targeted the two main markets: finance and chemical simulation, in addition to generic optimization problems and predictive analysis.

They were developing both classical/quantum hybrid computing and quantum machine learning solutions. The company stopped operating in March 2021.

ApexQubit (2018, Belarus and USA) is a drugs discovery company that develops quantum software solutions for the pharmaceutical sector targeting rare diseases. They operate in project mode and publish some research papers on their web site.

Aqemia (2019, France, 31.6M€) is a software company developing drug discovery and retargeting algorithms using statistical physics, AI and quantum inspired algorithms.

The startup is a spin-off from Ecole Normale Supérieure run by Maximilien Levesque and Emmanuelle Martiano. In December 2020, they announced a partnership with Sanofi to discover new treatments for covid-19. They raised 30M€ in October 2022 to fund their AI-enabled drug discovery pipeline, but not part of their future quantum-based algorithms efforts.

aQuantum (2018, Spain) is quantum software engineering service company doing contract research, development and consulting. They develop hybrid classical-quantum computing software and provide quantum software project management expertise, particularly in quantum machine learning.

They also developed |QuantumPath⟩ (*aka* Q|Path⟩), a quantum software development and lifecycle application platform. It contains all the tools to handle the whole software design and execution lifecycle covering both gate-based, quantum simulators and quantum annealing based computing, supporting Qiskit (IBM), Forest (Rigetti), Ocean (D Wave), ProjectQ and Quantinuum's |tKet⟩. They embed the open sourced Quirk graphical tool in their development environment. Since April 2022, QuantumPath is integrated with AWS and Amazon Braket making it possible for developers to access the quantum emulation and quantum hardware platforms supported by Amazon Braket.

Launched in 2023, QAgnostic QAOA extends the software portability from |QuantumPath⟩ to run QAOA algorithms to solve optimization problems. It is possible to launch the same optimization algorithms on QPUs from different providers as well as various quantum computing emulators while supporting gate-based and annealing paradigms.

A*Quantum (2018, Japan, $3M) specializes in the development of quantum software solutions for both annealers (including digital annealers from Fujitsu) and gate-based quantum computers (from IBM). Their ambition is to create high-level software libraries for users.

Aqarios (2022, Germany) is specialized in quantum machine learning applied to the financial sector. Its AqOPT platform is used to solve complex optimization problems, mainly with quantum annealing, with a tool to transform problems into Ising model/QUBO form, a solver recommendation tool, and a tool to analyze a potential quantum advantage.

Arclight Quantum (2020, China, $3M) is a quantum software company spun out of the Institute of Software Research of the Chinese Academy of Sciences. With CAS, they developed isQ-Core, a quantum cloud software development platform that can execute gate-based quantum code on both classical emulators and quantum processors. They also created an EDA (electronic design automation) tool for the creation of quantum processors.

Arline (2020, Germany) is developing a compiler optimizing QML algorithms execution, reducing the number of quantum gates used and taking into account all qubit characteristics such as their connectivity.

They also propose Arline Benchmarks, an automated benchmarking platform for quantum compilers. It compares gate count, circuit depth and compiler runtime.

It can also be used to combine compiled circuits and optimization routines coming from different compilers in a custom pipeline to optimize algorithms performance.

Artificial Brain (2022, India/USA) is developing hybrid quantum-classical algorithms targeting energy, aviation, finance and climate change use cases.

Its first achievement was an algorithm identifying optimal locations for electric vehicle charging. It is running on D-Wave quantum annealers (probably from the Pegasus generation) using a mix of quantum annealing and genetic algorithm developed on Qbsolv[4359]. It produces some result in 3 seconds for $8.5*10^{15}$ combinations. Its scalability depends on how D-Wave will expand the capacity of its annealers in the future. The company was founded by Jitesh Lalwani, who worked beforehand in the software industry.

Artiste-qb.net (2018, Canada) has a business model similar to that of 1Qbit: they develop algorithmic patterns of intermediate levels that they then assemble according to the needs of their customers.

They have even filed patents for certain methods. The startup was created by an international team including German researchers. They develop a Python based set of libraries in open source, available on Github.

Automatski (2014, USA) is a software company established in London, India in Bangalore and in California. They do applied contract research to develop quantum algorithms on any form of computer and quantum simulator. They have developed a software solution for emulating up to 300 qubits on conventional computers in full vector state. They focus on creating biochemistry algorithms and claim to have solved protein folding and to cure diabetes, cancers and 4,000 other diseases.

They also claim to have created a quantum annealer with a "quintillion qubit" based on some "new physics", probably some photonic coherent Ising machine. They combine huge overselling and trade secrets. No visible case studies, scientific publications or descriptions are substantiating these wild claims. It smells fishy at best.

[4359] See Towards an Optimal Hybrid Algorithm for EV Charging Stations Placement using Quantum Annealing and Genetic Algorithms by Aman Chandra, Jitesh Lalwani Babita Jajodia, November 2021 (6 pages).

Avanetix (2019, Germany) develops hybrid algorithms dedicated to solving supply chain problems. They combine classical optimization methods, machine learning and quantum computing. They target the automotive and logistics markets. The startup is founded and managed by serial entrepreneur Naimah Schütter.

Beit.tech (2016, Poland, $1.4M) is specialized in quantum machine learning. It is mainly a research project funded by the European Union, covering the period 2017-2010. The founder Wojtek Burkot is a former Google employee who even tries to make D-Wave useless by creating algorithms for optimizing complex graphs that can run on traditional computers.

Beyond Limit (2014, USA) is an AI software company focused on reasoning applications and autonomous decision-making with a first product targeting refineries in the oil industry. I discovered them when they announced some partnership with IQM to work on quantum AI technology in the APAC region around Singapore. I hope for them that for some time, they are ready to work on quantum inspired algorithms.

Blueqat (2008, Japan, $2.3M), formerly MDR for Machine learning and Dynamics Research, is creating algorithms integrating AI and chemistry, working among other with customers from the cosmetics industry like KOSÉ[4360].

They are working with D-Wave annealers. The startup was founded by Yuichiro Minato and various other alumni of the University of Tokyo.

BlueQubit (2022, USA) released in 2023 a software platform to access both classical emulators of quantum code with up to 34 qubits and quantum processing units, particularly focused on quantum machine learning applications.

It supports the Qiskit and Cirq frameworks as well as Nvidia classical infrastructures. The startup was created by alumni from Google, QcWare, Twitter, Caltech, the MIT and Stanford University. They collaborate with QuEra and won a project with DARPA in October 2023 as part of the IMPAQT program.

Boltz.ai (2020, Canada) is specialized in the development of AI and quantum software for the agriculture business. They create crop field allocation optimization tools.

BosonQ Psi (2020, India) created the cloud-based BQPhy software suite, a computer-aided engineering (CAE) solution performing complex simulations with using hybrid quantum-classical algorithms. It covers structural mechanics, thermal analysis and design optimization. The startup was created by Abhishek Chopra, Rut Lineswala and Jash Minocha.

They are partnering with Artificial Brain, another startup from India.

[4360] Blueqat developed an algorithm that analyzes the distribution of cosmetics product features in a multidimensional space. It visualizes existing areas and reveals unknown product areas they were able to open up, to create possibilities for new cosmetic designs that humans never thought of. The solution was patented and thus, is not yet publicly documented.

Boxcat (2017, Canada) is a startup created by Ystallonne Alves that develops image and video processing solutions based on hybrid quantum algorithms (Figure 893). They target the media and medical imaging markets. Their algorithms are based on currently available hardware architectures (D-Wave, IBM, Rigetti, Xanadu). The process they present on their site is an image realized on a D-Wave, which could have been realized with Nvidia's latest GPUs.

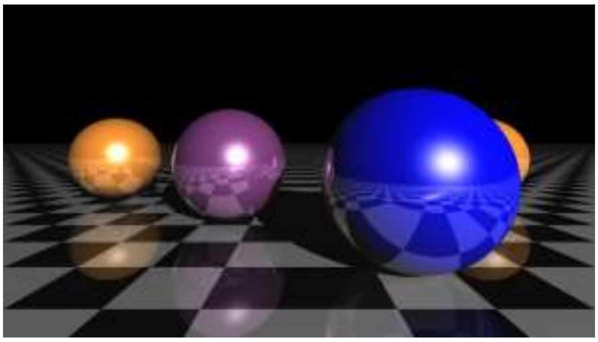

Figure 893: an artificial image generated on a D-Wave 2000Q by Boxcat. Source: Boxcat.

Cambridge Quantum Computing Limited (2015, UK, $72.8M) develops the $t|ket\rangle$ quantum operating system and various quantum algorithms including Arrow for machine learning[4361]. They are partnering with Oxford Quantum Circuits and with IBM which is one of their investors. CQC is also active in post-quantum cryptography.

$t|ket\rangle$ is available broadly and for free to everyone since February 2021 and also open source. It covers many quantum computing platforms (IonQ, Honeywell, AQT, IBM Qiskit, Rigetti, Amazon Braket and Azure Quantum) and incorporates circuit optimization and routing. It is interfaced with Python with Pytket. $t|ket\rangle$ is also used by EUMEN, CQC's quantum computational chemistry platform, and the company's QML framework. QQC is also partnering with Roche to use quantum algorithms for drug discovery targeting Alzheimer's Disease, as announced in January 2021.

In 2021, CQC launched a cloud software random quantum number generator. It is using a classical random generator, a quantum random number generator amplifying the randomness of the first and a Bell test used to check the resulting randomness, all running on an IBM quantum system[4362].

CQC has also been demonstrating how NLP (Natural Language Processing) could be implemented on current NISQ IBM quantum computers. Their researchers explain that the structure of natural language is natively quantum, which could lead to efficient translating, and better understanding of complete sentences and texts. The paper, however, doesn't provide much indication on the way data was encoded in the qubits. It lacks supporting data on actual system performance[4363]. In October 2021, this was packaged in an open source Quantum Natural Language Processing (QNLP) toolkit and library, lambeq.

The company announced a merger with Honeywell Quantum Systems in 2021. In December 2021, it became Quantinuum with a staff of over 350 employees.

ChemAlive (2014, Switzerland) is a quantum computational chemistry startup and contract research company. It provides simulation tools for getting molecular properties and synthetic reactions using basic 2D chemical syntax.

They deal with reaction mechanism elucidation and optimization, kinetic observed rate modeling, molecular design, virtual screening and drug discovery, molecular and spectroscopic property prediction, materials modeling and design, data and computation driven synthetic planning, experimental execution of chemical synthesis and experimental research on spectroscopic and electronic molecular

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[4361] See t|ket> : A Retargetable Compiler for NISQ Devices, April 2020 (43 pages).

[4362] See Quantum-Proof Cryptography with IronBridge, TKET and Amazon Braket by Duncan Jones, March 2021 and Practical randomness and privacy amplification by Cameron Foreman et al, 2020 (26 pages). It was branded as (origin)^{cQ} in December 2021.

[4363] See Foundations for Near-Term Quantum Natural Language Processing by Bob Coecke et al, December 2020 (43 pages). By the way, they are using ZX calculus in this work.

properties. They developed ConstruQt, a software tool transforming molecular drawings into 3D structures and energies.

All of this is quantum... but seems to be computed classically. Quantum chemistry doesn't necessarily mean quantum-computed quantum chemistry. Once quantum computing hardware will scale, they'll naturally switch to it.

ClassiQ Technologies (2020, Israel, $51.8M) develops a quantum programming tool providing a higher level of abstraction than classical quantum gate programming[4364].

Their platform is on Amazon Bracket as announced in June 2022. The company was created by Nir Minerbi (CEO), Amir Naveh (VP-R&D) and Yehuda Naveh (CTO, who spent 20 years at IBM Research in Haifa, including quantum, condensed matter specialist).

CogniFrame (2016, Canada) is a software publisher of data analysis platform software exploiting machine learning. They also develop hybrid algorithms for the financial sector based on D-Wave annealers.

One of their first customers is the Canadian investment bank Alterna Savings. The proposed applications are classic in the financial field: credit risk assessment and investment portfolio optimization. Late 2021, they launched **FirstQ Store**, an aggregation platform of quantum computing applications, provided as an application running on Windows, Mac and Linux desktops. It supports QUBO applications for Toshiba's Simulated Bifurcation Machine (a sort of digital annealer) and gate-based algorithms[4365]. In April 2023, the company announced a service partnership with KPMG's Global Quantum Hub.

ColibrITD (2019, France, 1M€) is a quantum software R&D company created by Hacène Goudjil and Laurent Guiraud that develops vertical use case software components with a team of a dozen skilled PhDs/post-docs, focused on gate-based computing models.

They created a set of framework tools branded "QUICK" for "Quantum Innovative Computing Kit" for solving problems in the financial, pharmaceutical, material design and logistic fields. They also contribute to the advancement of quantum software engineering tools like in functional testing[4366].

CreativeQuantum (2010, Germany) is specialized in quantum physics-based R&D in the chemical and pharmaceutical industries.

They seem however to run these many algorithms with classical computers. Which makes sense given quantum computers are not yet powerful enough to run these physics simulation tasks efficiently.

Culgi (2004, the Netherlands) is yet another computational chemistry company that will someday adopt quantum computing or simulation for its software. It was founded in 1999, changed its name in 2004 and was acquired by Siemens in 2020.

[4364] See A revolutionary approach to building quantum circuits by Amir Naveh and Yuval Boger, Classiq, November 2021 (32 minutes).

[4365] See Benchmark of quantum-inspired heuristic solvers for quadratic unconstrained binary optimization by Hiroki Oshiyama and Masayuki Ohzeki, December 2021 (11 pages).

[4366] See Principles of quantum functional testing by Nadia Milazzo, Olivier Giraud, Giovanni Gramegna and Daniel Braun, ColibrITD, Universität Tübingen and Université Paris Saclay CNRS LPTMS, September 2022 (13 pages).

Dihuni (USA) is a service company created by Indian developers working in the AI, cloud and Internet of Things domains. It created Qubrid, a QML platform that is supposed to support multiple frameworks and classical plus quantum backends hardware. It supports Qiskit. The company looks like a single person shop run by its CEO Pranay Prakash.

Dirac (2021, USA) is developing quantum software and algorithms for robotic applications. That New York City startup was created by Filip Aronshtein. I'd advise them to revisit their branding since the search engine optimization of Dirac is not obvious. On top of these nasty white logos on dark backgrounds!

dividiti (2014, UK) develops quantum algorithms, particularly in machine learning and using hybrid methods. Their solutions are open source. It is a service model, which is rather the standard in this market at the moment.

D Slit Technologies (2018, Japan) develops custom quantum software solutions for creating proofs of concept. Their website is not very talkative about their achievements.

Elyah (2018, Japan/Dubai) is developing quantum software to "*improve people's lives*". The company is made up of two people, a certain Salman Al Jimeely based in Dubai and an American, Sydney Andrew, based in Tokyo. I'm still looking for those developing software worsening people's lives, besides pirates.

Entanglement (2017, USA) is a quantum software development service company. One of their achievements was to create a quantum inspired software for vaccine distribution optimization in the USA in 2021.

Entropica Labs (2018, Singapore, $6.5M) is a startup dedicated to the creation of quantum (and non-quantum) algorithms for life sciences and in particular for genomics, based on quantum machine learning.

The result is faster development of therapies, in partnership with pharmaceutical companies. The company was founded by Tommaso Demarie (CEO), Ewan Munro (CTO), joined in 2018 by Joaquin Keller, a former Orange researcher based in France, who left them to later create **exaQ.ai**. It offers its Entropy Development Framework that manages the workflow of quantum software. They are also working with Honeywell/Quantinuum and BMW, and the Singapore Defence Science and Technology Agency to create proof of concepts for supply chain optimization[4367].

exaQ.ai (2020, Singapore) is a quantum machine learning created by Joaquín Keller, who formerly co-founded Entropica Labs and was a teacher in France and a R&D lead at Orange.

Their offering is based on the "polyadic QML Library" that implements supervised quantum machine learning for multi-class classification on NISQ architectures[4368].

It was tested on IBM Quantum hardware with accuracy levels similar to classical machine learning, doing a ternary classification of the Iris flower dataset that contains only 150 objects to classify in

[4367] See Multi-Objective Optimization and Network Routing with Near-Term Quantum Computers by Shao-Hen Chiew et al, August 2023 (19 pages) which concludes that "*that the fraction of feasible to infeasible configurations shrinks exponentially in the problem size*"

[4368] See Polyadic Quantum Classifier by William Cappelletti, Rebecca Erbanni and Joaquın Keller, Entropica Labs, July 2020 (8 pages) and https://dev.exaq.ai/.

three classes. They also provide their ManyQ quantum computer emulator that is optimized for quantum machine learning and supports CPUs and GPUs.

FAccTs (2016, Germany) is a spin-off from Max Planck Institute for Chemical Energy Conversion that develops ORCA, a quantum-chemical software package.

FAR Biotech (2016, USA) does drug discovery based on quantum representation of molecular structures done, so far, on classical computing.

FermionIQ (2021, The Netherlands, 300K€) is a quantum software company created by Harry Buhrman, Ido Niessen, Chris Cade and Jörgen Sandig.

Their first product is a quantum circuit emulator provided as a cloud platform. The company is spin-off of University of Amsterdam, Centrum Wiskunde & Informatica (CWI) and QuSoft.

First Quantum (2023, South Korea) is a company created by Suk Whan Chang (CEO) and Doyeol Ahn (CTO) that is developing quantum circuits and using algorithms optimization based on its proprietary and patented quantum Karnaugh Map technique, supporting Qiskit. The technique simplifies circuits, particularly based on Toffoli gates[4369]. They cover QFT, fluid dynamics, quantum chemistry and financial services.

Foqus (2021, Canada) The company is a spin-out from University of Waterloo, led by Michele Mosca and Sadegh Raeisi which creates a software solution to improve the performance of MRI and NMR systems with machine learning and quantum computing. Quantonation is one of its investors.

GenMat (USA-Canada, $15M) aka Quantum Generative Materials is a stealth quantum computing software startup. They got a funding of $15M (plus $35M following development milestones) from **Comstock Mining** (USA), a gold and silver mining Company also invested in lithium-ion battery manufacturing.

In March 2023, they announced ZENO, a generative artificial intelligence that simulates known and new materials more efficiently and computes electrical and thermal conductivity, heat capacity, local and charged density of states, band gap, formation energies, magnetic properties, and more. For the time being, this is still classical computation.

Good Chemistry Company (2022, Canada) is a spin-off from 1Qbit which developed QEMIST Cloud, a cloud developers platform associating classical machine learning and quantum computing to undertake computational chemistry simulations. Accenture is an investor in the company.

Grid (2009, Japan) is a specialist in deep learning and learning by reinforcement with their ReNom platform.

They have adapted this library in a quantum version called ReNomQ. And they have been IBM Quantum partners since September 2019. On the other hand, their AI was probably not very efficient to help them find a company name that is easy to be found via search engines.

Groovenauts (2012, Japan, $12M) developed in 2016 a D-Wave based cloud service called Magellan Blocks to solve complex optimization problems.

[4369] See Quantum circuit optimization using quantum Karnaugh map by J.-H. Bae, Doyeol Ahn et al, Nature Scientific Reports, September 2020 (8 pages).

They exploit hybrid algorithms combining machine learning and quantum algorithms. Their first customers include a Japanese retailer who optimizes its planning and Mitsubishi Estate who optimizes household waste collection[4370].

Hafnium Labs (2016, Denmark) develops software that provides physical property data for molecules and mixtures by combining quantum chemistry and AI. So, not yet a quantum software vendor.

Haiqu (2022, USA, $4M) was cofounded by Mykola Maksymenko (CTO) and Richard Givan (CEO). It develops a platform-agnostic software to extend NISQ quantum hardware capability into applications in finance, chemistry, life sciences, mobility and other domains.

Horizon Quantum Computing (2018, Singapore, $33.33M) creates quantum development tools. Their ambition is to compile code from classical development tools such as Matlab, C, C++ and then optimize it and run it on quantum computers, in order to make quantum computing accessible to traditional developers. In short, they want to democratize quantum software development.

They also work on quantum Internet software. As such, they announced in 2022 that they will establish a quantum communication on Singapore's National Quantum-Safe Network. The startup was launched by Joe Fitzsimons and Si-Hui Tan, both coming from Singapore's CQT research center[4371]. In November 2022, it opened an office in Dublin, Ireland, the first outside Singapore. Not a big surprise given Joe Fitzsimons is Irish.

HQS Quantum Simulations (2017, Germany, €15.3M) is a Karlsruhe Institute of Technology spin-out startup led by Michael Marthaler. They are developing quantum algorithms in the field of organic and inorganic molecular simulation of simple molecules (methane, light emission in OLEDs, diffusion of molecules in liquids). It mostly targets the chemistry industry[4372].

They announced in July 2018 an open source porting tool for ProjectQ code (IBM platform) to Cirq (Google platform). They released their Quantum Assisted Design toolbox in 2020 and qoqo, a quantum circuit representation library in 2021. They already have BASF and Bosch as customers. In practice, they also develop classical versions of their algorithms, running on datacenters or supercomputers[4373] . Their latest 12M€ funding round in February 2022 was led by Quantonation, extended by an investment of 1M€ by Trumpf in June 2023.

Ingenii (2021, USA) is a startup created by Christine Johnson (CEO) and Marko Djukic (CTO, coming from Purdue University) that wants to simplify the work of scientists using quantum computers.

They started with creating a Python package to submit data science jobs like quantum chemical simulations to quantum hardware like IonQ through Microsoft Azure for a starter. These algorithms are of course limited by the current capacities of existing quantum computers. The startup focuses on life and environmental science use cases, up to working on benchmarking and error mitigation, using machine learning. The startup was created out of the Duality accelerator in Chicago.

[4370] See Groovenauts and D-Wave collaborate on hybrid Quantum Computing, December 2019.

[4371] See the QSI Seminar presentation: Dr. Joe Fitzsimons, Horizon Quantum Computing, Abstracting Quantum Computation, April 2020 (1h26). Joe Fitzsimons created the blind quantum computing protocol with Anne Broadbent and Elham Kashefi in 2008.

[4372] See Post-processing noisy quantum computations utilizing N-representability constraints by Tomislav Piskor et al, HQS, April 2023 (13 pages) which is about optimizing a NISQ quantum simulation algorithm to estimate the ground state energy of the usual suspect molecules H_2, LiH and BeH_2.

[4373] See HQS Quantum Simulations: How to survive a Quantum winter by Richard Wordsworth, 2020.

Innovatus Q (2018, Singapore) is a spin-off from the Centre for Quantum Technologies in Singapore. They work on hybrid quantum algorithms based on trapped ions and superconductors.

Jaynes Computing (2019, Canada) is creating a cloud-based solution based on some quantum machine learning (QML) in the supply chain market. They are supposed to use some NISQ hardware, without any details. The startup was created by German Alfaro and was spun out of the Creative Destruction Labs.

InstaDeep (2015, UK, $107M[4374]) is a machine learning software vendor who started to investigate the field of quantum machine learning in a team led by Tom Barrett.

Their first announcement dealt with classical quantum simulations using machine learning[4375]. Still, they hired Jules Tilly in that team, who knows well the inner workings of both VQE and QPE quantum algorithms. The company was acquired by BioNTech in January 2023, the pharmaceutical company at the origin of the Pfizer RNA-based Covid vaccine.

Inspiration-Q (2020, Spain) is providing quantum-inspired software solutions in the areas of optimization, simulation, and machine learning, mostly in the financial and transportation sectors.

Jij (2018, Japan, $1.9M) was created by researchers from the Tokyo Tech Institute of Technology. It develops software for quantum annealing, including OpenJij, an open source framework for implementing Ising models to model particle interactions, built on D-Wave's QUBO APIs. They are also partners of Microsoft Azure.

JoS Quantum (2018, Germany) develops quantum software solutions for the financial services industry, particularly in risk management and fraud detection. They also do contract research.

Ketita Labs (2018, Estonia) develops unspecified quantum software for NISQ computers, and for good reason. It is a university spin-off.

Kipu Quantum (2021, Germany, 13.5M€) was cofounded by Enrique Solano, a prolific and outspoken researcher working in Spain (Bilbao) and Germany (Chief Visionary Officer), Daniel Volz (CEO) and Tobias Grab (Chief Strategist Officer), with Sébastien Perseguers being their CTO. As of October 2023, they had a team of 30 people. The startup's goals are to "*design and manufacture of modular and co-designed Quantum computers*" tailored to solve specific tasks with NISQ, without waiting for FTQC generations.

It is about reducing the footprint of NISQ algorithms, to enable them to deliver better value to customers in various fields. In 2021, they announced their NISQA paradigm merging digitized-counterdiabatic quantum computing (DCQC) and digital-analog QC (DAQC) for NISQ computers without error correction overhead. It uses digital and analog compression techniques to reduce the physical qubits required to solve specific industry problems (combinatorial and optimization problems, chemistry, QML, ...).

[4374] I don't account for this amount in my tracking of quantum startups funding since it was not related to their recent investments in quantum computing.

[4375] See InstaDeep launches dedicated Quantum Machine Learning team, as first QML research paper is published in Nature Machine Intelligence, April 2022 referring to Autoregressive neural-network wavefunctions for ab initio quantum chemistry by Thomas D. Barret et al, Nature Machine Intelligence, 2020 (8 pages).

Kuano (2020, UK, $3.6M) creates quantum software solutions for the design of molecules and for the inhibition of enzymes, which is used both in pharmaceuticals and to create protective agents in agriculture.

They use quantum emulation and quantum algorithms as well as machine learning. The company was founded by defectors of GTN, including their CEO Vid Stojevic who was the CTO of GTN.

Kunfeng Quantum Technology (China) is a quantum software company based in Shanghai focused on machine learning applications. It developed HQconv and FQconv, two quantum neural network algorithms for image recognition[4376]. It also proposes an EDA software to design quantum chipsets, the Quantum-chip Design Automation Platform (QDAP) that is distributed as a cloud service. It was founded by Xu Hua, who has a PhD from the University of Maryland[4377].

Menten.ai (2018, USA, $4M) develops hybrid algorithms combining machine learning and quantum programming to simulate organic chemistry and design enzymes, peptides and proteins, working with D-Waves annealers.

MolKet (2022, the Netherlands) is a company created by Taha Selim and Alain Chancé that provides cloud-based software with AI services and solutions for molecular modeling and design using quantum and high performance classical computing.

Molecular Quantum Solutions (2019, Denmark) or MQS, provides computational tools for pharma, biotech and chemical industries. It is using HPC and quantum computers.

Multiverse Computing (2019, Spain, 49M€) develops quantum and quantum-inspired software for various markets, starting with financial services, with portfolio optimization, risk analysis, market simulation and fraud detection.

They announced in August 2021 their Singularity Spreadsheet app solution which gives access to some portfolio optimization algorithm running on D-Wave annealers in the cloud, directly from within Microsoft Excel (video). It is now part of their Singularity SDK for Portfolio Optimization. They have then expanded their reach to mobility, energy, and manufacturing verticals. They also use more traditional techniques based on machine learning, digital annealing (with Fujitsu) and quantum inspired solutions using tensor networks.

They published several papers in 2022 on combinatorial problem solving using quantum annealing with ZF in Germany[4378], on artificial vision using both gate-based and quantum annealers with the research institute Ikerlan Technology Center in Basque country, Spain, showing qualitative improvements vs classical machine learning method[4379], on a quantum-inspired tensor neural network to solve partial differential equations, in partnership with CACIB (French investment bank)[4380], and on

[4376] See RGB Image Classification with Quantum Convolutional Ansaetze by Yu Jing et al, July 2021-February 2022 (17 pages).

[4377] See Release of the world's first "cloud native" quantum chip design service cloud platform, iMedia, undated.

[4378] See Multi-disk clutch optimization using quantum annealing by John D. Malcolm et al, August 2022 (11 pages).

[4379] See Quantum artificial vision for defect detection in manufacturing by Daniel Guijo et al, August 2022 (11 pages). They make a comparison with classical machine learning methods instead of deep learning convolutional neural networks, which may be misleading.

[4380] See Quantum-Inspired Tensor Neural Networks for Partial Differential Equations by Raj Patel et al, August 2022 (14 pages).

portfolio optimization with Ally and Protiviti on D-Wave annealers[4381], as well as with Bankia[4382]. This is a good practice and track record for a quantum software vendor.

Their Fair Price quantum solution computes accurate fair prices for financial institutions and seems to run on IonQ quantum processors[4383]. The solution is leased as a cloud service for 100K€ to 200K€ a year.

In November 2023, the company launched CompactifAI, a software compressor of LLM (large language models) using tensor networks with the aim of reducing energy and compute costs.

CompactifAI is designed to reduce energy requirements at multiple points in the lifecycle of an LLM, including the training phase, general operations and retraining. Multiverse's new software also reduces the overall footprint of the models, making them more portable and easier to run at the edge in applications such as autonomous vehicles and remote production facilities.

The company is partnering with Xanadu, Microsoft, Fujitsu, IBM, Rigetti, DWave, NTT, Strangeworks, Pasqal (France), IQM (Finland), Objectivity IT (UK, an IT services company), among others. They have several offices outside Spain in Toronto (Canada, with one of their cofounders), Munich (Germany) and Paris (France) with about 75 people as of February 2023. The EIC (European Innovation Council) provided them with 12.5M€ comprising a mix of grant and venture funding. They have 22 patents pending in quantum computing algorithms. Multiverse made 4.3M€ of revenue in Q1 2023 vs 2.9M€ in Q1 2022.

NETRAMARK

NetraMark (2016, Canada) develops quantum software solutions for the pharmaceutical industry to define therapeutic targets. They are part of the Quantum Machine Learning program at the Creative Destruction Lab in Toronto. It was acquired by the brain biotech **Nurosene Health** (2019, Canada) in October 2021.

novarion

Novarion (2016, Austria) is a server storage and GPU server vendor who wants to create the first hybrid quantum computer by 2025, without mentioning what sort of quantum processor it will integrate.

In October 2020, they started a partnership with **Terra Quantum AG** (Switzerland) to create a Joint Venture to create '<Qa|aS> by QMware'. It supports machine learning and big data analytics. QMware's customers will be able to develop and effectively run completely Hybrid Quantum Applications. Applications built on QMware's Hybrid Quantum Cloud are supposed to run on upcoming native quantum processors when they show up. Meanwhile, it runs on some classical emulators, seemingly a QLM from Atos/Eviden. And it is Gaia-X and GDPR compatible.

NQCG

Nordic Quantum Computing Group (NQCG) (2000, Norway) does R&D in areas at the crossroads between AI and quantum computing. They are creating a platform agnostic quantum software using superconducting and photonic qubits.

ODE L3C (2018, USA) is an American NGO involved in the creation of chemical simulation algorithms. Its ambition is to solve "difficult NP" problems with quantum computation, which is far from obvious.

This sounds more like a service provider than a software publisher. The company was created by Keeper Layne Sharkey, coauthor of a book on quantum chemistry with Alain Chancé.

[4381] See Financial Index Tracking via Quantum Computing with Cardinality Constraints by Samuel Palmer et al, August 2022 (8 pages).

[4382] See Hybrid Quantum Investment Optimization with Minimal Holding Period by Samuel Mugel et al, December 2021 (6 pages).

[4383] See Quantum portfolio value forecasting by Cristina Sanz-Fernandez et al, November 2021 (9 pages).

Opacity (2020, Australia) offers Quiver, a quantum code optimization software compatible with IBM's Qiskit.

Their hardware-agnostic solution maps processor errors at the global and individual qubit level, including parasitic interactions between qubits.

It then allows the code to be optimized to take into account this duly mapped noise. The tool seems to be dedicated to developers as well as to quantum computers designers.

OTI Lumionics (2011, Canada, $5.7M) is specialized in the design of new materials and in particular LEDs and OLEDs. They have developed quantum and quantum-inspired molecular simulation algorithms for this purpose.

In particular, this allows them to predict the properties of the created materials like their color when being excited[4384], model chemical relationships and determine geometric structures. They are partners of Microsoft Azure (video).

ParityQC (2020, Austria) is a spin-off of the University of Innsbruck created by Wolfgang Lechner and Magdalena Hauser, the first being the scientist and the second, handling business aspects of the venture[4385].

By September 2022, the company had about 25 employees plus 15 researchers working at the University of Innsbruck. As architects, they connect the dots between hardware and software.

They develop software solutions to solve optimization problems (CADD, N-body problems, constraint problems) adapted to digital and analog quantum computers (qubits with universal gates or quantum simulators[4386]). Their ParityOS software suite optimizes the software parameters of the solution as well as those of the hardware control[4387].

They support an architecture called LHZ, created by Wolfgang Lechner and Austrian colleagues Philipp Hauke and Peter Zoller, which is compatible with different hardware quantum platforms with 2D qubit connectivity architectures[4388]. Its principle consists in encoding a problem requiring n-to-n relations between qubits (all to all) to run it on a physical architecture where qubits are only connected to their nearest neighbors as is the case in most quantum computers, except for some that rely on

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4384 See Estimating Phosphorescent Emission Energies in Ir(III) Complexes using Large-Scale Quantum Computing Simulations by Scott N. Genin et al, October 2021 (32 pages). It is based on using a 72 logical qubits classical emulator.

4385 She is a family relative of Hermann Hauser, the co-founder of Arm, now a serial entrepreneur and investor in deep techs, including Graphcore (UK). He is also a member of the EIC Council.

4386 Like in Rydberg blockade based parity quantum optimization by Martin Lanthaler, Clemens Dlaska, Kilian Ender, Wolfgang Lechner, October 2022 (10 pages) which describes a solution to quantum optimization problems using MWIS and UDG on quantum simulators using neutral atoms.

4387 See Applications of Universal Parity Quantum Computation by Michael Fellner, [Submitted on 19 May 2022 (v1), last revised 2 Nov 2022 (this version, v2)]. ParityQC

4388 This LHZ architecture is documented in A quantum annealing architecture with all-to-all connectivity from local interactions by Wolfgang Lechner, Philipp Hauke and Peter Zoller, October 2015 (5 pages) for universal gated qubit platforms which they perfected in Universal Parity Quantum Computing by Michael Fellner, Wolfgang Lechner et al, PRL, October 2022 (7 pages), Applications of universal parity quantum computation by Michael Fellner, Wolfgang Lechner et al, PRA, October 2022 (8 pages), Parity Quantum Optimization: Compiler by Kilian Ender, Wolfgang Lechner et al, May 2021-March 2023 (9 pages) and Parity Quantum Optimization: Encoding Constraints by Maike Drieb-Schön, Wolfgang Lechner et al, May 2021-March 2023 (24 pages). They propose to enable more efficient Fourier transforms and Shor integer factoring implementation in NISQ regime. See also Quantum Approximate Optimization with Parallelizable Gates by Wolfgang Lechner, 2018 (5 pages) which describes the implementation of a QAOA optimization algorithm with CNOT and single qubit gates, and follow-up work in Low-depth Circuit Implementation of Parity Constraints for Quantum Optimization by Josua Unger, Anette Messinger, Wolfgang Lechner et al, November 2022 (9 pages) and Parity Quantum Optimization: Benchmarks by Michael Fellner, May 2021-March 2023 (12 pages). At last, see also Rapid counter-diabatic sweeps in lattice gauge adiabatic quantum computing by Andreas Hartmann and Wolfgang Lechner, September 2019 (11 pages) for quantum annealing computing. At last, A Josephson Parametric Oscillator-Based Ising Machine by Sasan Razmkhah et al, PRB, September-December 2023 (13 pages) is proposing an implementation of the LZH model with classical superconducting electronics.

trapped ions who enable many to many connectivity (Figure 894). Their solution also includes an in-house error correction system[4389]. They are partnering with **Pasqal** whose cold atom-based architecture seems adapted to their model.

They announced in 2021 a partnership with **NEC** to help them with their superconducting qubits.

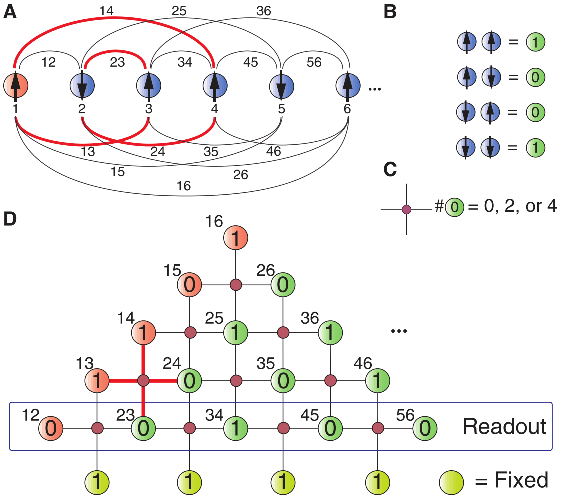


Figure 894: LHZ architecture. Source: A quantum annealing architecture with all-to-all connectivity from local interactions by Wolfgang Lechner, Philipp Hauke and Peter Zoller, October 2015 (5 pages).

Phase Space Computing (2017, Sweden) is a spin-off from the University of Linköping that develops training solutions on quantum computing for secondary and higher education.

PhaseCraft (2018, UK, £22.1M) is a quantum software company spun out of University College London and the University of Bristol by Toby Cubitt, Ashley Montanaro and John Morton (also a cofounder of Quantum Motion).

They plan to exploit quantum computing to create better energy collection and storage systems (batteries, solar PV, ...). The company had a staff of 20 as of December 2023. They launched in 2022 a partnership with Oxford PV, a perovskite photovoltaic panel company, to improve PV efficiencies thanks to quantum computer-based simulations.

Their researchers developed an optimized version of an algorithm solving the Fermi-Hubbard model with fewer resources which could be helpful to create high-temperature superconducting materials[4390]. In 2021, they also published a proposal for how to simulate a "kagome magnet". While it did not show any quantum advantage on existing quantum hardware and worked only with 20 qubits, it extrapolated that with 50 qubits, the problem could be solved by 200 two-qubit gates, something high-fidelities NISQ qubit system could achieve in the near future[4391].

PiDust (2019, Greece) is a startup launched by Vasilis Armaos, Paraskevas Deligiannis and Dimitris Badounas, who are alumni of University of Cambridge, Stanford and the University of Patras. They develop quantum algorithms in chemistry.

[4389] See Error correction for encoded quantum annealing by Fernando Pastawski and John Preskill, 2015 (4 pages).

[4390] See Strategies for solving the Fermi-Hubbard model on near-term quantum computers by Chris Cade, November 2020 (27 pages) which turned into Observing ground-state properties of the Fermi-Hubbard model using a scalable algorithm on a quantum computer by Stasja Stanisic, Jan Lukas Bosse, Filippo Maria Gambetta, Raul A. Santos, Wojciech Mruczkiewicz, Thomas E. O'Brien, Eric Ostby and Ashley Montanaro, Phasecraft and University of Bristol, Nature Communications, October 2022 (11 pages).

[4391] See Probing ground state properties of the kagome antiferromagnetic Heisenberg model using the Variational Quantum Eigensolver by Jan Lukas Bosse and Ashley Montanaro, October 2021 (12 pages).

Pine.ly (2019-2022, Canada) is positioned on software to assist in the creation of innovative materials with quantum computing. They aim in particular at the recycling of CO^2 emissions. The startup was created by three women, Nayer Hatefi, Shabnam Safaei and Rachelle Choueiri, all three scientists. The company was seemingly closed in 2022.

POLARISqb (2020, USA, $2M) is a startup that wants to use quantum computing to create new therapies. One more! It built the Tachyon platform.

But what kind of quantum computer and tools are used in Tachyon is a mystery. The startup was founded by Shahar Keinan (CEO) and Bill Shipman (CTO). They are partnering with Fujitsu, probably to use their conventional supercomputers and their digital annealing computer. Their idea is to use personalized medicine techniques to create ad-hoc therapies. Shahar Keinan received a PhD in chemistry from the Hebrew University of Jerusalem. She specializes in computational chemistry. In August 2022, they worked with **Auransa** to identify interesting target proteins for curing specific breast cancers using Tachyon and Auransa's SMarTR Engine based on classical AI.

Allosteric Bioscience (2021, USA, $920K) is a company integrating quantum computing and AI with biomedical sciences to create improved treatments for aging and longevity. In February 2022, the company announced it was teaming up with and investing in Polaris Quantum Biotech (Polarisdb). They first work on creating an inhibitor of a key protein involved in aging.

ProteinQure (2017, Canada) is a Toronto-based startup that uses various technologies including quantum computing to create and simulate new "*in silico*" therapies. They use quantum algorithms to simulate protein folding.

They are also developing hybrid algorithms using GPUs. They support different hardware architectures including D-Wave computers. In their experiments, they manage to simulate molecules with 6 atoms in universal quantum computers and reach 11 atoms with D-Wave. In practice, however, it would seem that they have put quantum computing on the backburner and are now focused on classical learning machines in the meantime.

Q1t (2018, the Netherlands) creates mathematical models and software for classical and quantum computers in the field of quantum chemistry, quantum optics and financial analysis. They have tried these algorithms on quantum simulators and quantum optics.

They developed the q1tsim quantum simulation library published on Github which implements new quantum gates types for creating simpler circuits, the ability to simulate measurements without affecting qubit quantum states and the option to re-run a circuit starting with the previous quantum state for debugging purpose.

QAR-lab (2021, Germany) aka "Quantum Applications and Research Laboratory" created the UQO middleware platform for architecture-independent programming and platform evaluations, focused on solving optimization and machine learning problems.

They support D-Wave quantum annealers, Fujitsu digital annealers, and gate-based quantum computers from Rigetti and IBM. The company was created by Claudia Linnhoff-Popien (CEO) and Thomas Gabor who both come from the Institut für Informatik at Ludwig-Maximilians-Universität München (LMU) and had a team of 20 as of April 2023.

QbitLogic (2014, USA, $1.5M) is another startup that develops quantum machine learning applications, without more precision in their communication. They also develop an AI based system, CodeAI, to debug software.

Qbraid (2020, USA) provides a quantum online coding platform. Based in Chicago, the company was created by Kanav Setia (CEO) who has a PhD from Dartmouth College in quantum simulation and worked with IBM research.

The platform supports the most popular programming frameworks (Qiskit, TensorFlow Quantum, Q#, Xanadu PennyLane, Braket, IonQ, Rigetti pyQuil, D-Wave, Intel's C++ framework...) and contains a transpiler that optimized the quantum code for the target hardware platforms. Access to IBM quantum computers seems embedded in the software solution which targets students. The company also developed quantum programming courseware.

Qbrain (Italy, 2022) created Qortex, a machine learning powered framework accelerating and optimizing quantum algorithms.

Qcentroid (2023, Spain) is a company created by Carlos Kuchkovsky and Antonio Peris that provides a quantum as a service platform with an algorithms marketplace.

It covers various verticals (financial services, transportation, energy, pharmaceuticals, retail) and does hardware benchmarking. Its early partners are Artificial Brain, Quintessence Labs, Kipu Quantum and Qulabz. It targets customers launching proof-of-concept projects but does not provide many details on their scope and supported hardware either natively quantum or in emulation mode, or even for designing quantum inspired classical algorithms.

Quantum Computing Inc aka QCi (2018, USA, $7.5M) is a quantum software, hardware consulting company that started with the creation of Qatalyst, a high-level software development and cloud provisioning tool.

It is mainly enabling developers to solve constrained optimization problems (QAOA, QUBO, graph optimization) with either gate-based accelerators, quantum annealers or classical computers (Figure 895). It is based on using six simple high-level APIs. It is commercially available. They support their home QikStart Program, a marketing initiative to accelerate real-world use cases with their customers. Qatalyst supports D-Wave, IonQ and Rigetti accelerators through Amazon's Braket cloud services. In March 2021, the company hired a "Chief Revenue Officer" (Dave Morris) and a Marketing VP (Rebel Brown) after having announced in December 2020 that they were filing for a Nasdaq IPO.

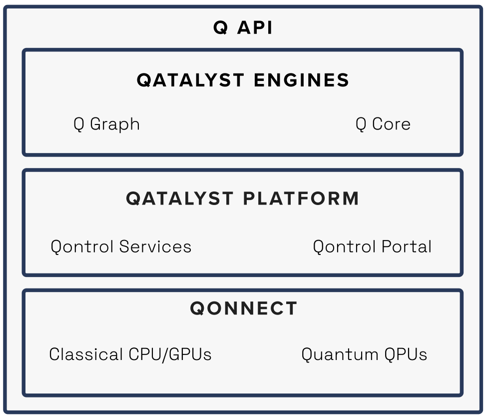


Figure 895: QCi software platform.

In July 2021, it became the first publicly traded pure player quantum computing company. QCi also created QUBT University in August 2021, an online initiative to train students on their Qatalyst tool. It is also a tactic to partner with academic institutions, like Notre Dame University in Indiana. They also have QCi Qonsulting practice, implementing their Path2Quantum (P2Q) methodology, a four-phase framework to planning quantum computing deployments.

In March 2022, QCi announced a partnership looking like an acquisition of a stealth company, QPhoton (2020, USA) from New Jersey and created by Yuping Huang (also assistant professor at the Center for Quantum Science and Engineering at Stevens Institute of Technology), and which develops some sort of photonic quantum computer of unspecified nature.

QCI with hook this system with its Qatalyst software. QPhoton was funded by various US federal agencies (DARPA, NSF, NASA, DoD). QCI launched in September 2022 its Dirac 1 Entropy Quantum Computer (EQC) cloud-based subscription based on Qphoton's technology.

In June 2022, QCi announced QAMplify, a solution supposedly increasing the processing power of quantum computers by x20. Of course, it deserves some scrutiny. Their patented process is supposed to increase the number of variables that can be handled in quantum hardware by a factor x5 for gate-based and x20 for quantum annealing. How they are doing this is not yet documented.

Q-CTRL (2017, Australia, $70.8M) is a startup created by Michael Biercuk of the University of Sydney. They develop a set of enabling software tools to improve the operations and programming of quantum computers.

Their Boulder Opal is quantum control infrastructure software working at the firmware level. It leverages machine learning to improve qubits control pulses and optimize quantum error correction codes. It is a Python toolkit used by quantum computers designers that works with IBM Qiskit, Rigetti and with Quantum Machines pulse generators. They implement error-suppression techniques that increases the likelihood of quantum computing algorithm success between 1,000x and 9,000x on quantum hardware, as measured using the QED-C algorithmic benchmarks, but probably with a very low number of logical qubits (Figure 896). These impressive numbers relate to the impact of optimized quantum error correction. In November 2023, this solution was proposed to IBM Quantum pay-as-you-go customers.

They are relying on Google Cloud and TensorFlow to run the classical machine learning algorithms of their solution[4392]. Fire Opal is a set developer tools for quantum algorithm designers while Black Opal is an educational tool for students new to quantum programming. In May 2022, Q-CTLR combined a 5-qubit QEC code (using 9 qubits, due to the additional helper qubits) with Fire Opal and could improve the code's ability to correctly identify errors by 70% on IBM QPUs (meaning: one qubit error detection rate was improved by 70%)[4393]. Fire Opal error suppression techniques can help improve the successful execution of QPE[4394] and QAOA algorithms at small scale[4395].

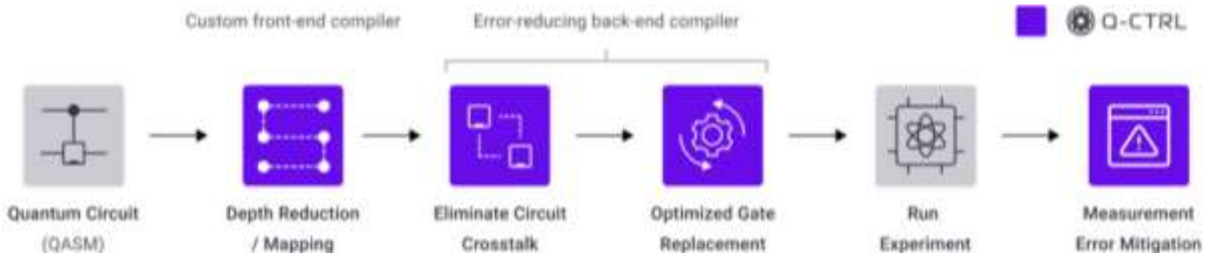


Figure 896: Q-CTRL error reduction techniques. Source: Q-CTRL.

They also work on optimizing quantum sensing solutions and are creating quantum sensors of their own for acceleration, gravity and magnetic fields measurement in space applications. They are partnering with the Australian company **Advanced Navigation**, which specializes in geopositioning. At last, they are also creating prototype algorithms for buses dynamic scheduling of buses in Sidney, Australia (*disclaimer: it can't be operational given the power of existing quantum computers*).

[4392] See Boosting quantum computer hardware performance with TensorFlow by Michael J. Biercuk, Harry Slatyer, and Michael Hush, October 2020.

[4393] Fire Opal's process is documented in Experimental benchmarking of an automated deterministic error suppression workflow for quantum algorithms by Pranav S. Mundada, Michael J. Biercuk, Yuval Baum et al, September 2022 (16 pages).

[4394] See Quantum phase estimation with Fire Opal by Rowen Wu, Q-CTRL, May 2023.

[4395] See Optimize hybrid quantum-classical algorithms directly with Fire Opal by Rowen Wu, Q-CTRL, April 2023.

QC Ware (2014, USA, $39.7M) develops a platform for cloud-based quantum software development. They create quantum algorithms and software for large companies with two layers: their proprietary Forge platform and open source libraries for optimization, chemical simulation and machine learning (Figure 897).

They provide tools to load training data from learning machine models into memory more quickly. They have also developed an algorithm for calculating the distance between objects, which can be used to train both supervised (classification) and unsupervised (clustering) machine learning models. Their first customers include **Equinor** for oil exploration optimization, Japan's **AISIN** for certification testing of automatic gearbox software, **Airbus** for aircraft flight envelope optimization and **BMW** for autonomous vehicle route optimization. They are also targeting financial markets as well. It supports universal gate quantum computers (IBM, Rigetti), D-Wave quantum annealing computers and software emulators (IBM, Google, Microsoft, Rigetti).

Airbus Ventures is one of their investors on top of the Koch group. They have received a $1M US public funding via the NSF in 2017. The startup, which already includes over 20 people, was created by Matt Johnson, who has a financial background, and Kin-Joe Sham and Randy Correll, who seem to have gotten late into quantum computing. The team also includes Iordanis Kerenidis, who is based in France and is a leading specialist in quantum machine learning. He oversees international algorithms development. Scott Aaronson is their Chief Scientific Advisor.

The startup organizes a reference annual conference on quantum computing, the **Q2B**, in Santa Clara (California), Tokyo (starting in 2022) and in Paris (starting in 2023)[4396].

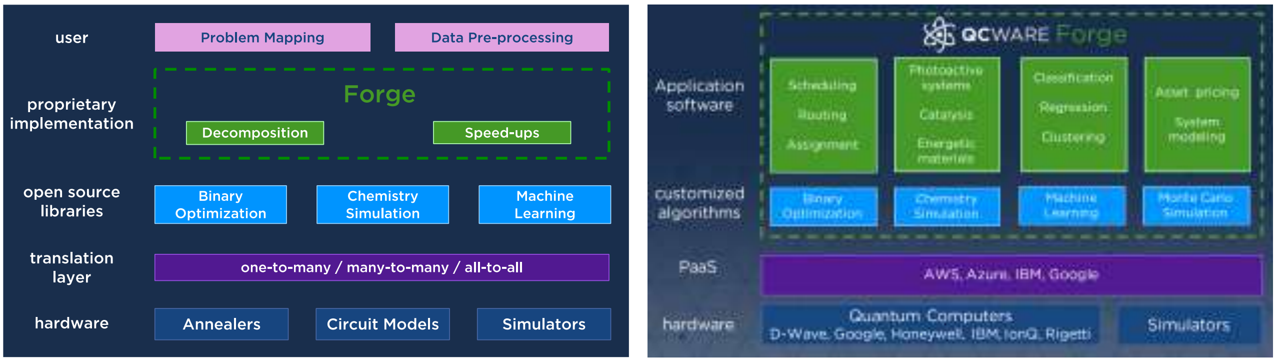


Figure 897: QC-ware software platform. Source: Enterprise Solutions for Quantum Computing by Yianni Gamvros, December 2019 (25 slides).

In April 2023, QCWare introduced its Promethium classical software-as-a-service (SaaS) quantum chemistry platform designed to help develop pharmaceutical, chemical, and materials discovery solutions. It runs on NVIDIA H100 and A100 GPGPUs. It can compute various chemical systems with about 100 atoms and up to 2,000 atoms with one to two order of magnitudes time savings compared to other classical solutions according to them. The SaaS platform runs on AWS.

QDC.ai (2022, USA/Poland) is a company developing machine learning, HPC, combinatorial optimization, scientific programming, and quantum computing algorithms, mostly using optimized classical hardware platforms, but with an eye on quantum hardware. Their software platform VeloxQ is an HPC engine that automatically selects the appropriate solver to solve various problems (QUBO, MAX-SAT, MAX-CUT, CP-SAT, Simulated Bifurcation and Heuristic algorithms.

[4396] View presentation materials and videos from the 2019 Q2B conference, in December 2019.

QEDma Quantum Computing (2020, Israel) is a quantum software company created by Dorit Aharonov of the Hebrew University, Nathaniel Lindner of the Technion and Asif Sinai. It creates quantum algorithms and software tools positioned as a quantum operating system. It includes a multi-qubit quantum gates characterization tool, a gate optimizer and a fidelity enhancing compiler implementing error suppression and error mitigation techniques.

Qindom Inc. (2018, Canada, $2M) is a startup developing quantum machine learning (QML) software running on D-Wave quantum annealers.

Qoherent (2019, Canada, $173K) is developing machine-learning based RF signals analysis tools that helps create adaptive RF communications and sensing systems in the Software-Defined Radio (SDR) realm. They analyze signals in the telecom ranges (4G, 5G, V2X, Satellite communications...). The company is investigating the usage of quantum machine learning techniques to improve its solutions.

Qrithm (2018, USA) develops quantum algorithms in diverse and rather disparate fields: machine learning, materials science, cryptography and finance.

Qruise (2021, Germany) is providing the Qruise Toolkit, positioned as a collection of algorithms (optimizers, simulators, machine learning tools and other software components) designed to accelerate the development of quantum technology and to improve its applications.

The company was cofounded by Shai Machnes, Simone Montangero, Frank Wilhem-Mauch and Tommaso Calarco and had a team of 20 people as of April 2023. Their scientists contributed to over 280 publications at the same date in both quantum physics and quantum computing, with interesting work on machine learning based quantum mitigation error techniques, parametrized quantum machine learning circuits[4397] and on the energetics of quantum computing[4398]. In 2022, they were awarded a 2.5M€ grant from the EIC Council Transition program.

Qu&co (2017, the Netherlands) was created by Benno Broer and Vincent Elfving, both quantum physicists who worked at TU Delft.

They develop tailor-made quantum software solutions for large companies, accompanied by benchmark tools, particularly for chemical simulation applications based on DFT. They develop solutions for simulating fluid mechanics with nonlinear differential equations solved with VQE algorithms (hybrid variational quantum eigensolvers). They even solve Navier-Stokes 1D equations. They partner with IBM, Microsoft and Schrodinger (USA).

In March 2021, they launched a beta release of QUBEC, a chemistry and materials science toolkit. It comprises Q-time, a tool estimating when quantum advantage can be expected for solving chemistry or materials problems. QUBEC workflow manager supports quantum systems from IBM, IonQ and Rigetti. It is available through the IBM Quantum Experience and Amazon Braket platforms. LG Electronics is one of their customers.

In August 2021, they announced a new funding round with Quantonation, Runa Capital and SPInvest, with an undisclosed amount and in January 2022, the company merged with Pasqal (France). Benno Broer is now the CCO of Pasqal (Chief Customer Officer).

[4397] See Pulse-efficient quantum machine learning by André Melo et al, November 2022 (8 pages).

[4398] See Is quantum computing green? An estimate for an energy-efficiency quantum advantage by Daniel Jaschke and Simone Montangero, May-November 2022 (12 pages).

Quanscient (2021, Finland) was created by Alexandre Halbach, Asser Lähdemäki, Valtteri Lahtinen and Juha Riippi.

It provides Quantum Simulation-as-a-Service (QSaaS) software. Its multiphysics simulation algorithms use finite element method and partial differential equations solvers. They hint about using hybrid classical/quantum computing but given the state of the art of quantum computers, it is probably relying on classical computing. They help simulate physics in the electromagnetism, mechanics and fluid dynamics fields.

QuantFi (2019, France/USA) is a young startup specializing in the creation of quantum software solutions for finance.

It was created by Paul Hiriart (French, ex of Lehman Securities, who left the startup early in 2021), Kevin Callaghan (coming from the New York financial sector) and Gabrielle Celani (on sales and marketing). They are creating goals-based investment optimization algorithms, and also handle trends detection, derivatives pricing and risk management. In 2020, they joined the IBM Quantum network.

They also have a US subsidiary named Quantistry, that develops chemical simulations solutions supporting density functional theory (DFT), post-Hartree-Fock, classical force fields, and machine learning prediction models.

Quanterro Labs (2019, Abu Dhabi) is an association of researchers and entrepreneurs created by Kaisar Parvez and Ram Soorat and AiFi Technologies, an AI software startup, working in quantum information and security. They work on middleware and software development for D-Wave, Google, IBM and others. It is mostly a consulting services company.

Quantica Computacao (2019, India) is the first Indian quantum startup and a software company working on creating a cloud development environment an QML algorithms. It is incubated in the Indian Institute of Technology Madras from Chennai.

Quantopticon (2017, UK) develops the Quantillion modeling and simulation software for the design of photonic components for quantum communication and computing applications. It is a tool for the design of new materials. The company was created by a mother-daughter duo: Mirella Koleva (CEO) and Gaby Slavcheva (CSO). Their first customers are Quandela, QuiX and Toshiba.

Quantum Benchmark Inc (2017, Canada) provides an error-correcting code software solution for general-purpose quantum computers and error evaluation. It was thus apparently a competitor to Q-Ctrl.

They also offer a quantum computer performance validation system. The package is integrated into the True-Q suite, launched in 2018, with True-Q Design, which is used to evaluate the error rate of a quantum computer and to optimize its architecture, and True-Q OS, which helps optimize the accuracy of software solutions.

The target market is initially the manufacturers of quantum computers and those who evaluate them. Eventually, it will be that of user customers. Note that they have already tested Google's Cirq framework, having been part of Google's beta test program for this language, and that Google is using their solution, as it did in its 2019 supremacy experiment. They are also partnering with IBM (since 2018, on error characterization, mitigation, correction, and performance validation) as well as with Fujitsu Labs (since 2020, to develop quantum algorithms). The company was acquired by **Keysight Technologies** in May 2021, on top of Labber Quantum in 2020. Among its key people are Joseph Emerson (CEO), Joel Wallman (CTO) and Daniel Gottesman (Senior Scientist, specialized in error correction, MIT), Stefanie Beale and Kristine Boone (both researchers).

Quantum Flytrap (2020, Poland) is a quantum computing and cryptography software company. They created a real time browser-based emulator using photonic elements instead of quantum gates, the "Virtual Quantum Lab" that is being used at Stanford University and the University of Oxford[4399]. It enables the simulation of an optical table and to run various quantum optical physics experiments. They developed the Pulser Studio IDE for Pasqal and have seemingly ceased activity since then.

Quantum-South (2019, Uruguay) is a spin-off from the University of Montevideo who is specialized in developing quantum optimization software, first targeting the cargo shipments in ships and airlines. Their software was released in 2022. The Automatic Package Selection and Ordering (APSO) is proposed as a quantum inspired solution as of 2023.

They also target the financial sector which may be more dynamic although more crowded with many existing quantum software startups. In cargo shipments, they are partnering with Quantum Brilliance (Australia) with their prototype (5) NV centers qubits which, of course, is experimental.

QBaltic (2019, Estonia) develops algorithms for quantum computing, quantum cryptography and artificial intelligence. QBaltic is a contract research spin-off from the University of Latvia, University of Tartu in Estonia and QuBalt, Germany and Latvia.

QbitSoft (2022, France) is a new software and service vendor targeting in priority the retail and logistics market. It was created by Olivier Pegeon, a former business executive from IBM France.

QPerfect (2023, France) is a spin-off from UNISTRA (University of Strasbourg) which first product is MIMIQ-circuits, a quantum code emulator supporting large circuits with hundreds of qubits and using optimized tensor networks. The company was created by Sébastien Buffechoux (CEO), Guido Masella (CTO), Guido Pupillo (Qubit control lead), Johannes Schachenmayer (Numerics lead) and Shannon Whitlock (Hardware lead).

Qsimulate (2018, USA, $9.3M) develops quantum solutions for molecular simulation for healthcare and chemistry. They are partners of Amazon Braket and Google and are already working with Amgen. The company was co-founded by Toru Shiozaki and Garnet Chan, both specialized in chemistry.

Quacoon (2020, USA) is a small startup created by Tina Sebastian and Barbara Dunn that develops software solutions for the food supply chains combining AI and quantum annealing.

Quantum Thought (2019, USA) develops quantum or quantum inspired algorithms for chemical, AI and security markets. According to their website, it seems to be mainly a service company operating in project mode and doing consulting services. Their CEO is Rebecca Krauthamer.

Quantumz.io (2019, Poland) develops the Quantum Simulator Platform (QSP), a quantum program emulation solution running with GPUs. They are also developing a PQC (post-quantum cryptography) solution called banax, including some dedicated hardware to implement it.

[4399] See Visualizing quantum mechanics in an interactive simulation -- Virtual Lab by Quantum Flytrap by Piotr Migdał et al, May 2022 (29 pages).

Quantastica (2019, headquarter in Finland with offices in Estonia and Serbia) develops hybrid quantum algorithm software tools including Quantum Programming Studio, a graphical web development environment for creating quantum algorithms executable on quantum computers or simulators, including a classical simulator they have developed themselves.

Quantistry (2018, Germany) created a cloud-based simulation platform for material research and development, using quantum simulations, molecular dynamics and machine learning.

Quantopo LLC (2017, USA) is a company specialized in machine learning algorithms. They focus on biotechs, supply chain and logistics. They are part of the Creative Destruction Lab in Canada. But as they don't have an active website, it is not certain that they still exist.

Quantum AI (2021, USA) is a company providing AI based financial trading software. It is supposed to use some form of quantum computing that is not disclosed. I suspect it is some for of (classical) quantum inspired software.

Quantum Mads (2020, Spain) was created in Bilbao by Eriz Zárate and Alain Mateo Armas and is positioned in the financial market. It offers four software tools with a mix of quantitative/classical/hybrid/quantum-inspired software.

With Q-MADS, an investment strategy analysis framework for traders, Q-RETAIL, a framework for retail banks, Q-ALLOCATE, for asset allocation optimization and Q-CRYPTO, a framework for optimal path finding in graphs. The whole is based on the HHL linear algebra algorithm.

Qausal.ai (2020, USA) is providing quantum and classical model insights and governance. Their platform uses large language models as well as causal methods to provide human understandable insights for health care, life sciences and finance industries. It provides classical and quantum machine learning model governance, monitoring, and explainability, identifies when to re-train models, meta insights combining multiple models, model discrimination analysis, population and characteristic stability and explainability.

Quantum Open Source Foundation got a $4K grant from Unitary Fund. It publishes a list of quantum open source projects on Github, with various software development tools and libraries for gate-based quantum as well as and quantum annealing computing.

It is a repository of existing open source projects including IBM Qiskit and PennyLane from Xanadu. NISQ. Provides financial funding for quantum open source software projects. Organize events. More a community than a foundation like the Apache or Mozilla foundations.

QSIMPLUS (2023, Korea) is selling QSIMpro, a software quantum code emulator used for quantum communication and optical network design.

Quantagonia (2021, Germany, $4.3M) is a company based in Frankfurt and Berlin who created HybridSolver, a hybrid quantum platform allowing users to run existing code and decision models on both quantum hardware and high performance computers.

When you look at the details, this is mostly a way to implement more efficiently mixed integer programming, linear programming and QUBO problems on classical computers, before quantum computers are able to support it with some computing advantage. I have doubts on their claim to support existing code, but it can make sense for decision models which are code-independent and are implemented as optimization problems which have their specific implementation in classical and quantum computing. The company was created by Dirk Zechiel (CEO), Sabina Jeschke, Sebastian Pokutta (CTO) and Philipp Hannemann. They have a partnership with Kvantify (Denmark) as well as with Strangeworks (USA).

QuantyCat (2020, USA) creates cloud-based APIs for quantum software development, supporting D-Wave, IonQ and Rigetti through Amazon Braket.

Qubit Engineering (2018, USA) was founded by University of Tennessee alumni. They develop classical and quantum optimization algorithms suitable for wind turbine design and location optimization. Another very niche market. They are partners of Microsoft Azure.

Qubitera (2018, USA) develops solutions combining AI and quantum.

Qubitl Quantum Technologies (2018, India) is a contract research laboratory developing quantum machine learning software. They were initially specialized in cybersecurity, having created a QRNG solution (Q-RandCon), a PQC protected healthcare solution (HealthCetra) and a Quantum Differential Phase Shift technology for QKD (Q-Shift).

Qubit Pharmaceuticals (2020, France/USA, 23M€) is a startup co-founded by Jean-Philip Piquemal, a CNRS professor-researcher at Sorbonne University, a long-time specialist in molecular dynamics simulation that mathematically models the quantum mechanics of organic molecules.

Its algorithms have been in use for a long time. Jean-Philip Piquemal is the co-author of the Tinker molecular simulation library and its Tinker-HP version adapted to supercomputers. It exploits massively parallel CPU-based systems and Nvidia GPU tensors, all with high-precision computation to compute molecular polarizable force field. Their Amoeba software computes molecular interactions and water polarization and Sirius implements machine learning. Other solutions are dedicated to variational quantum eigensolvers (VQE). At this point in time, they are using quantum inspired and quantum emulation solutions running on classical systems like their own HPC-QC Gaia HPC of 50 PFLOPS installed in the Paris region. They develop drugs in three sectors: oncology, immunology and antivirals. What about quantum computing in all this? It could be used to define optimized parameters for classical simulation, in short, within the framework of hybrid algorithms. They will also be able to exploit quantum simulators in the future, such as those being developed with cold atoms at Pasqal[4400].

Jean-Philip Piquemal's laboratory at Sorbonne University has received a €9M ERC for the development of simulation solutions for organic systems of several million atoms[4401]. They finally welcomed the investment fund Quantonation in their capital in June 2020[4402]. As of mid-2023, they had a staff of 50 spread across Paris, Boston and Sherbrooke. They also hired Alberto Peruzzo, as quantum information R&D lead, one of the creators of the VQE algorithm.

QuDot (2018, USA) develops software for the simulation of quantum circuits on traditional computers, the QuDot Net. They use techniques based on Bayesian networks to optimize the in-memory representation of qubits.

[4400] See the presentation Computational Drug Design & Molecular Dynamics: an HPC perspective by Jean-Philip Piquemal, April 2020 (28 slides).

[4401] See Extreme-scale Mathematically-based Computational Chemistry (EMC2), 2020.

[4402] See Qubit Pharmaceuticals closes a pre-seed round with Quantonation, Quantonation, June 2020.

QunaSys (2018, Japan, $12.8M) also develops quantum algorithms in chemistry and healthcare. From the universities of Tokyo, Osaka and Kyoto, they also maintain the Qulacs simulator developed at Kyoto University.

It was followed in 2021 by Qamuy, a quantum cloud platform. They also created in 2020 the Japanese consortium QPARC with 50 participating companies investigating various use cases of quantum computing. The company is also participating in the **Pistoia Alliance**. In May 2023, IBM Venture made an investment in the company of an undisclosed amount. It also organizes the Quantaggle challenges.

QuSoft (2014, the Netherlands) is a spin-off from TU Delft University specializing in quantum algorithms and software. Like its sister company QuTech, it is more a private applied research laboratory than a startup.

QxBranch (2014, USA, $8.5M) was created by former Lockheed Martin employees. It offers solutions, probably tailor-made, for the financial, insurance, aerospace and cyber security markets.

Based in Washington DC, they already have offices in Hong Kong, London and Adelaide, Australia. They are partners of D-Wave and IBM. The startup was acquired by Rigetti (USA) in July 2019.

Rahko (2018, UK, £1.3M) is a quantum machine learning and chemical simulation software development company based in London. It was founded by Leonard Wossnig. They are among the first Amazon AWS partners for the use of quantum resources in the cloud, and the first in Europe.

In May 2020, they announced that they would work with Merck on "quantum inspired" algorithms, i.e. on conventional computers. In 2021, they announced that they were collaborating with Honeywell and achieved excellent accuracy executing a Discriminative Variational Quantum Eigensolver algorithm on Honeywell's H0 6-qubit trapped ion system. The company was acquired by **Odyssey Therapeutics** in January 2022, a company specialized in drug discovery in the fields of inflammatory diseases and cancers.

ReactiveQ (2018, Canada) develops quantum simulation algorithms for the design of innovative materials such as high-temperature superconductors, all on a NISQ quantum computer.

Riverlane (2016, UK, $42.8M) is a spin-off from the University of Cambridge whose main business is to provide quantum error correction solutions to QPU vendors.

They develop Deltaflow.Decode, a software quantum decoder. It was complemented by a dedicated quantum error correction chip announced in September 2023, which implements a surface code at the error syndrome decoder level with a Xilinx Ultrascale+ FPGA and a 12 nm ASIC, the DD0A[1]. They also have Deltaflow.Control, a high speed pulse sequences to control qubits using off-the-shelf hardware. At last, Aqueduct is an automation and administration environment to run quantum circuit experiments. It seems to be an evolution of Deltaflow.OS which is dedicated to NISQ systems and optimizes access to hardware resources for qubit control. It was deployed in mid-2020 at several sites in the UK and in partnership with SEEQC and, later, CQC (Quantinuum).

Meanwhile, the company also provides services in quantum computing and develops new algorithms combining machine learning and quantum in chemistry. They created with dividiti Ltd, a one man shop created by a certain Grigori Fursin, the Quantum Collective Knowledge, a benchmark SDK for quantum hardware and software. In July 2021, Riverlane created a consortium with **Astex Pharmaceuticals** and **Rigetti UK** to develop quantum drug discovery algorithms running on Rigetti platforms in the cloud. This was part of a 18-month feasibility study funded by a grant from UKRI as part of the UK quantum plan[2].

RQuanTech (2018, Switzerland) develops RTranscender, a quantum machine learning tool for finance, healthcare, automotive, seismology and cyber security. It supports Fourier transforms, qubit-based arithmetic operations which can help craft oracles (additions, multiplications, divisions, exponentials), factorization, discrete logs, etc.

Sanctuary (2021, Canada) is a startup created by Geordie Rose, the cofounder and first CTO of D-Wave.

It wants to implement reinforcement learning algorithms with quantum computing. At this point, the company is hiring a bunch of scientists and developers, with no offering yet looming.

SavantX (2015, USA) was created by Ed Heinbockel and David Ostby who in the past worked for the FBI, the Department of Defense and the US Intelligence Community including the DIA where they developed OSINT collection tools (open source intelligence, using unstructured data).

They core skills are data search, discovery, analysis and visualization. They plan to leverage quantum computing to identify hidden patterns in data and help optimize complex systems. One of their first realization in that domain was implemented for the Port of Los Angeles in 2020, to optimize cargo flow and container handling using D-Wave quantum annealers and classical machine learning[4403]. Their target markets are the nuclear, healthcare, utilities and defense industries.

Schrodinger (1990, USA, $193) is a digital drug design company, mainly using molecules screening and doing drugs retargeting.

It is an established competitor of Qubit Pharma (France). They work with Sanofi. The company is listed on the NASDAQ. They inevitably became interested in quantum computing and have started a partnership with Qu&Co to ramp-up their skills in quantum computing.

Semicyber (2018, USA) develops algorithms in various fields: data analysis (non-quantum), quantum and others for critical applications for the USA defense sector, particularly the US Air Force.

So they are probably closer to the service company than to the product-oriented startup. The startup is co-founded and managed by Kayla Farrow, an engineer specializing in algorithm creation and signal processing.

Sigma-i Labs (2019, Japan, $3.7M) is a consulting company and private laboratory that grew out of the Tohoku University Quantum Annealing Computing Research Laboratory, based in Sendai and led by their CEO Masayuki Ozeki. They started by doing consulting around the creation of software for D-Wave's annealers, using their cloud Leap platform since 2019[4404].

SolidState.AI (2017, Canada) develops machine learning solutions for the industry covering yield improvement, production calibration and predictive maintenance.

All of this is based on hybrid classical/quantum algorithms. They work with Bosch, Applied Materials, Mercedes-Benz as well as with D-Wave, Rigetti, Microsoft and IBM Q, among others.

[4403] See SavantX: Logistics Optimization at the Port of Los Angeles, D-Wave (2 pages).

[4404] See Sigma-i and D-Wave Announce Largest-Ever Quantum Cloud-Access Contract | D-Wave Systems, July 2019.

Spin Quantum Tech (2018, Colombia) develops quantum algorithms in the field of cybersecurity that combine AI and quantum. They seem to create PQC (post-quantum cryptography) that exploits new encryption algorithms. They are also working on chemical simulation, which has nothing to do with it.

SHYN (2016, Bulgaria) develops solutions for the visualization of data coming from quantum calculations. So, some quantum dataviz! With a use case consisting in detecting quantitative fake news. It was co-funded by Google's Digital News Information Fund dedicated to the press. This €150M fund distributed funding of a few 100K€ to more than 400 projects in Europe.

SoftwareQ (2017, Canada) offers development software for quantum computing including a compiler, simulator and optimizer. The company was co-founded by Michele Mosca (on top of evolution) and Vlad Gheorghiu of the Canadian Institute of Quantum Computing.

It is a startup from the Quantum Machine Learning program at Creative Destruction Lab in Toronto.

Strangeworks (2018, USA, $28M) develops quantum software. Like many colleagues, they target the aerospace, energy, finance and healthcare markets. They are at the origin of the creation of a Q&A site on quantum computing, Quantum Computing Stack Exchange. The company was founded by William Hurley aka whurley, and is based in Austin, Texas, with a staff of about 20 as of April 2023.

In 2019, they launched a beta of their multi-platform development environment for quantum applications supporting quantum computers or emulators from Rigetti Computing (Forest), DWave (Leap), Microsoft (Q#), Google (Cirq), IBM (Qiskit) and Infleqtion (ColdQuanta) Hilbert gate-based cold atom QPU (as of late 2021).

This environment seems to facilitate collaborative work and sharing of results. In February 2021, it turned into an initiative to "humanize quantum computing".

It is based on Strangeworks QS (Quantum Syndicate) which consolidates quantum hardware vendors solutions and software tools, Strangeworks QC (Quantum Computing), a free quantum computing ecosystem to learn quantum code using common quantum programming languages and Strangeworks EQ (Enterprise Quantum), an enterprise infrastructure solution consolidating QC and QS with better security, IP protection, quantum machine access, resource aggregation, custom integrations, private deployments, project management and the likes. In January 2022, Strangeworks announced a partnership with **Entangled Networks** (Canada) to support their future Multi-QPU Computers Entangled Networks and the associated MultiQopt code compiler. This scale-out approach to quantum computers is of course to be first validated at the hardware level and with physical (optical) connections between several QPUs before becoming operational.

Stratum.ai (2018, Canada) develops a quantum software dedicated to a very specialized market, the optimization of mineral prospecting, particularly in gold.

Super.tech (2020, USA, $150K) is a startup launched by Pranav Gokhale, Fred Chong and Teague Tomesh that develops a software stack dedicated to the control of quantum computing systems ranging from a hundred to a thousand qubits.

The solution is the result of the NSF-funded Practical-Scale Quantum Computation (EPiQC) research project involving five Chicago-area and MIT universities and stars such as Peter Shor and Aram Harrow. It is creating a software infrastructure targeting the development of NISQ solutions. The company was acquired by ColdQuanta in May 2022.

Terra Quantum (2019, Switzerland, $85.8M) was cofounded by Markus Pflitsch (CEO) with, as recent hires, Valerii Vinokur (co-CTO) and Dr Florian Neukart (CPO). Their $75M funding helps them expand their R&D effort and also, their international expansion, with the opening of new offices in Munich and in the Silicon Valley with over 70 permanent staff in total (2023).

The company develops quantum software solutions in all possible quantum fields: quantum computing, quantum cryptography and quantum sensing. They modestly position themselves as being in a position to build the "*European quantum ecosystem*"[4405] and working on the "*development of revolutionary quantum computing applications*", with supporting the development of more efficient fertilizers, batteries and power grids. Their "Quantum Algorithms as a Service" is an algorithms library with the usual optimizations and QML pieces. The company then customizes it according to the needs of their customers making them a mixed product/services company. They also develop classical and hybrid chemical process algorithms[4406].

In January 2022, Terra Quantum launched an alpha release of its QMware hybrid quantum cloud data center (QMware), in partnership with Novarion Systems (Austria, covered earlier in this section). Their HPC and quantum emulation workloads are powered by Intel Xeon Platinum CPUs and Nvidia A100 GPGPUs and DGX servers. Their CPU-GPU-QPU interconnect is based on CXL (Compute Express Link), a unified in memory communication open standard protocol created in 2019 by Intel for high-speed CPU-to-device and CPU-to-memory connections, designed for HPCs and built on the PCI Express (PCIe) interface. Their hybrid quantum computing approach is described in a white paper where they claim it outperforms classical approaches[4407]. They also propose to implement the platform with some flexible virtual classical setups[4408].

Their architecture is based on a memory-centric compute architecture that supports processing with QPUs, CPUs and GPUs as well as hybrid quantum computing with 12 TB of NVRAM shared by all systems. They also developed a unified information theoretical model for classical and quantum information that allows for efficient QPU emulation via a hardware agnostic intermediate representation of the quantum circuits. In 2022, they launched QUANTON-HGX2, a new generation of servers with Nvidia GPGPU and AMD EPYC CPUs. In 2023, they benchmarked their QMware emulator against AWS's emulators on a QML use case[4409].

Their last offering is a "Quantum Security as a Service" using a QKD solution, given the physical architecture and related hardware are not specified. They are also studying with Aalto University in Finland the development of more efficient superconducting qubits based QPUs.

[4405] See Terra Quantum secures EUR10m to build the European Quantum Ecosystem by James Dargan, April 2020. On 1st April, their CTO declared: "*We plan to implement a useful quantum algorithm on the IBM machine with 20 qubits in order to test quantum supremacy*". Well, 20 qubits for a quantum supremacy? It's fine if it's an April's fool.

[4406] See Optimization of chemical mixers design via tensor trains and quantum computing by Nikita Belokonev, Artem Melnikov, Maninadh Podapaka, Karan Pinto, Markus Pflitsch and Michael Perelshtein, April 2023 (8 pages) that digitally simulates efficiently a chemical mixing process with the prospect to improve it in the future with quantum computing.

[4407] See Practical Application-Specific Advantage through Hybrid Quantum Computing by Michael Perelshtein et al, 2021 (14 pages).

[4408] See A Uniform Quantum Computing Model Based on Virtual Quantum Processors by George Gesek, QMWare, February 2023 (9 pages).

[4409] See Benchmarking simulated and physical quantum processing units using quantum and hybrid algorithms by Mohammad Kordzanganeh et al, Terra Quantum and QM Ware, November 2022 (17 pages).

In May 2023, Terra Quantum acquired **divis intelligent solutions** (Germany) which brings some knowledge in classical algorithms.

Tinubu Software (France) is a credit insurance software company that is investigating the usage of quantum computing to extend its offering to improve prediction on time series. They had participated to the ClassiQ 2022 quantum computing coding challenge and were the bronze winners with Thomas Frossard, Ayoub El Qadi, Quoc Viet Nguyen and Marcelin Gallezot.

Tokyo Quantum Computing (2017, Tokyo) wants to develop quantum annealing computer simulation software like many of the software startups from Japan.

Tradeteq (2016, UK, $6.3M) is a financial trading platform that uses AI for risk assessment and portfolio optimization. Their ambition is to use quantum computing to develop their own quantum tools.

In April 2020, they announced that they would work in this direction with the Singapore Management University (SMU) and with quantum neural network algorithms.

Turing (2016, USA) is a company created by Seth Llyod and Michele Reilly that wants to solve key societal problems with hybrid classical-NISQ software solutions using AI and quantum machine learning techniques. Seth Lloyd is a famous prolific quantum scientist in quantum computing and Michele Reilly has been working on qRAM.

Unitary Fund (international) is a kind of equivalent of the Mozilla Foundation for quantum technologies. It is a non-profit organization creating open source quantum libraries, tools, hardware and content[4410].

They fund through a microgrant program ($4K) developers. They sponsored about 20 projects including the error mitigation framework mitiq[4411], Qrack (emulator accelerator on GPUs), OLSQ (Optimal Layout Synthesizer for Quantum Computing, a pre-compiler optimizer reducing the SWAP gates count), a quantum machine training textbook and Pulser, developed by Pasqal. They partner with Rigetti and IBM.

Xofia (2019, USA) develops software solutions based on quantum machine learning for classification. They want to distribute their software in open source. They exploit Atos/Eviden' 40 qubit quantum emulator, a QLM server, sitting in the cloud.

Zapata AI (2017, USA, $67.4M) is a quantum software and services company founded by Harvard researchers including Christopher Savoie and Alán Aspuru-Guzik from the University of Toronto who has developed many founding algorithms in chemistry quantum applications.

Their partners include Google, Rigetti, IonQ and IBM. Also, Bosch (Germany) is one of their corporate investors. Honeywell invested in the company in March 2020. The company established an office in the UK in June 2021. They initially developed a complete quantum operating system serving as a hub between application algorithms and quantum accelerators of all types. In April 2020, this took the form of Orquestra, a platform for managing quantum application workflows with (Figure 898):

[4410] See their 2021 Annual Report, Unitary Fund (23 pages).

[4411] See Mitiq: A software package for error mitigation on noisy quantum computers by Ryan LaRose, William J. Zeng et al, September 2020-August 2022 (33 pages).

Building code using their Orquestra Studio. It offers a set of code libraries supporting Cirq (Google), Qiskit (IBM), PennyLane (Xanadu), PyQuil (Rigetti), Q# (Microsoft) as well as pyAQASM (Atos/Eviden).

Orchestrate its deployment across multiple QPU and classical emulation platforms using Zapata Computing Quantum Workflow Language (ZQWL), which is YAML-compatible and supports various quantum hardware architectures (NISQ, quantum annealing) and classical computing (quantum emulators such as those from Atos/Eviden, supercomputers, cloud servers) [4412]. Orquestra includes tools for managing batch computing. The Orquestra Data Correlation Service (ODCS) collects treatment data in a MongoDB database which is then exported as Excel tables, a Jupyter notebook or for the Table software.

Deploying the Orquestra runtime locally or on the cloud.

Orquestra was in beta in April 2020 as part of an Early Access Program and is released since 2021.

Zapata Computing became Zapata AI in 2023 to pivot into developing classical generative AI solutions using tensor networks. It is a path many quantum software vendors will have to follow if not already done. In September 2023, the company announced its IPO via a SPAC for a pre-money valuation of $200M.

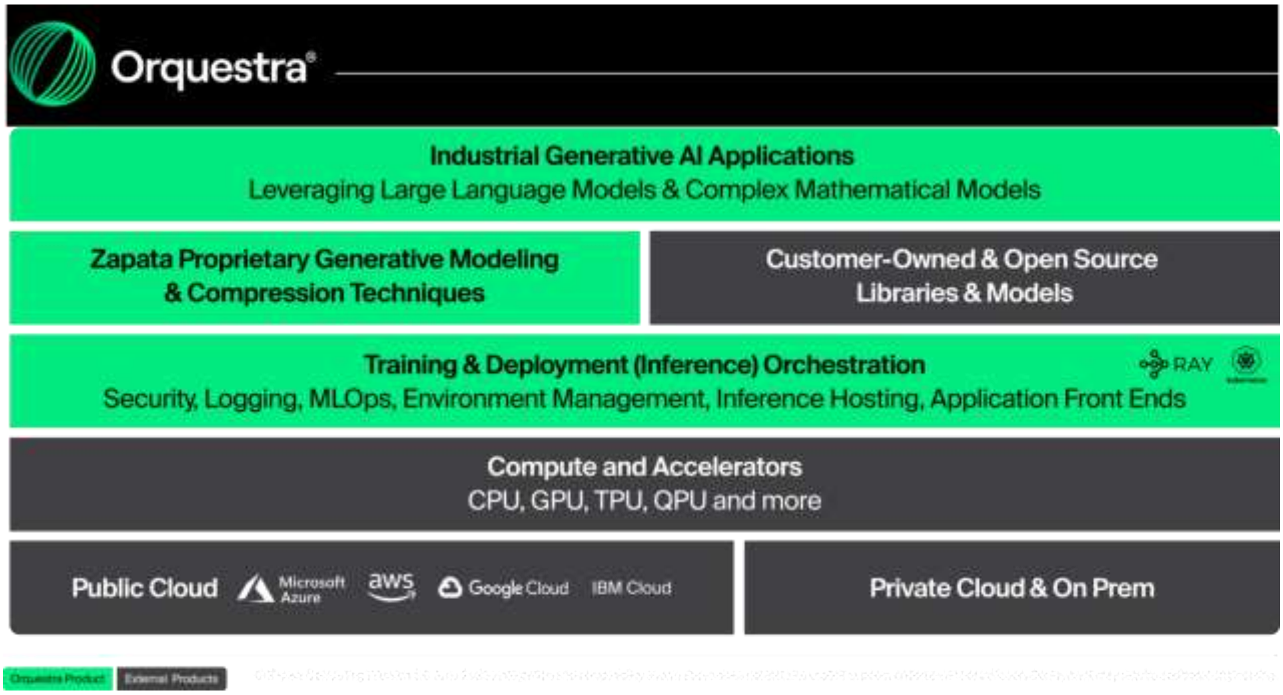


Figure 898: Zapata Computing Orquestra platform. Source: Zapata Computing.

ZebraKet (2022, Canada) is a supply chain startup created by Alex Khan (CEO) which was incubated at the Creative Destruction Lab (CDL) and University of Maryland Quantum Startup Foundry Pre-Traqtion program. It develops various algorithms supporting among other platforms D-Wave annealers.

Service vendors

On top of software vendors, the service vendors industry is getting structure to work with end-user customers and help them adopt quantum technologies, mostly focused on quantum computing.

[4412] YAML is a language that dates back to 2001. It is used to create configuration files. It is used in conjunction with Python.

Large IT service vendors like **Accenture**, **Capgemini** and **KPMG** have launched quantum technology practices, often partnering with hardware and software vendors. Accenture teams up with IonQ, Quantinuum, D-Wave and 1Qbit. KMPG is with Microsoft, selling "quantum inspired" projects. Capgemini created its Quantum Lab (Q-Lab, led out of Paris by Julian Van Velzen) early in 2022, partnering with IBM and positioned in Germany, Portugal, India and France. It covers all branches of quantum technologies (computing, telecommunications and cryptography, sensing) and focuses on life sciences, financial services, automotive and aerospace verticals[4413]. **McKinsey** has a quantum practice that is part of its QuantumBlack.AI "excellence center". **EY Global Services Limited** partners with IBM. **T-Systems** (Germany, the service branch of Deutsche Telekom) also has a quantum computing practice, a T-Lab in Berlin dedicated to quantum technologies and particularly quantum telecommunications and is also partnering with IBM.

Here are a bunch of more specialized quantum services and consulting companies throughout the world: **Safe Quantum Inc** (2020, USA) sells PQC services, **D-fine** (2002, Germany), **Q&I** (UK) also called Qandi, **Q4B** (UK), **QuantGates** (UK), **Caledonian Photonics Limited** (UK, consulting service in lasers and quantum photonics), **Cystel** (UK, security risks assessments), **Orange Quantum Systems** (2019, The Netherlands, 1.5M€), **Kvantify** (2022, Denmark), **Quantum Phi** (2018, Czech Republic). **RayCal** (UK) which is an analyst firm in quantum technologies, **Inside Quantum Technology** (2018, UK) created by Lawrence Gasman with the help of 3DR Holdings, **Quantum Technologies AB** (Sweden) which is specialized in quantum computing use cases in material design, **Qureca (**Spain) whose name means Quantum Resources & Careers and which does training, recruitment, business development and events, **Max Kelsen (**Australia), **AngelQ Quantum Computing** (Singapore), **h-bar Quantum Consultants** (Australia), **Quantum Quants** (the Netherlands). **SoftServe** (1993, Ukraine), **StrategicQC** (USA) is also specialized in recruiting talent in quantum technologies. **Quantum Computing Engineering** (USA) or QCE, provides generic consulting services on quantum computing, **Qdeeptech** (Austria), created by Angie Qarry, **Aspen Quantum Consulting** (USA) does due diligence investigation of startups for investors. **AmberFlux** (2013, India) provides quantum computing consulting services on top of existing machine learning and data science services. **QRDLab** does about the same, also in India. **Quemix** (2019, Japan) is an IT company specialized in providing quantum computing solutions. **QuRISK** (2021, France) does consulting on quantum computing originated risks on cybersecurity and also develops quantum algorithms, **Q. BPO Consulting** (2020, France) seems to be a generalist quantum computing consulting shop and **QuantumBCS** (2019, France) is doing unspecified quantum software development on top of Blockchain solutions development. At last, **Reply Data IT** (Italy) is a quantum computing service company that deployed its custom MegaQUBO solver in the cloud, first on classical computers, and supposedly on quantum computers.

Specialized head hunting HR companies also exist in the quantum market like **Quantum Futures** (UK), and **Psirch** (USA), which advocates for the creation of Chief Quantum Officers at the board level in Corporations[4414]. It is a bit extreme to say the least. At least should Corporations have a CTO in the board.

Applied Quantum Computing (2022, UK) is a service company created by Tim Thomas and Robert Cumming, who have a past in the financial industry but have respectively as past as physicist and developer. Their mission is to help customers get a good understanding of the impact of quantum

[4413] See Capgemini, IBM Launch Quantum Lab to Promote Quantum Use Cases by Matt Swayne, The Quantum Insider, January 2022.

[4414] See The Rise Of The Chief Quantum Officer And The Prospect Of Quantum At The Governance Level—An Interview With Shai Phillips, President Of Psirch by James Dargan, The Quantum Insider, October 2022.

computing on their business, plan their quantum readiness, explore and experiment practical use cases[4415].

Sigma Reply (2022, France-Italy) is the Reply company offering services in quantum computing. They cover the whole stuff: chemistry, optimizations, cybersecurity, and strategic consulting.

In November 2022, they became the first certified partner of Quandela. This branch of Reply (Italy) was created by Olivier Debliquy.

Plantagenet Systems (2014, UK) is a consulting service shop run by Roberto Desimone who runs projects for various UK organizations including their Ministry of Defense, InnovateUK, the UK National QT program, the National Quantum Computing Centre (NQCC) and the UK Quantum Hubs.

Protiviti (USA) is a large 7,000 people consulting company with a quantum practice focused like many other on the potential applications from quantum computing and on securing IT infrastructures against the potential related threats (so, PQC et al). They are partnering with Multiverse Computing.

Psi-Ontic (USA) is a quantum consulting created in Florida by Alan Martin, a management consultant coming from the automotive industry and with a physicist PhD. background. He works with large end-user customers as well as with investment funds and startups. He also creates market studies and quantum strategy assessments for various government organizations.

Quantum Computing Engineering Inc. (QCE) (2019, USA) is a company created by Gonzalo Florez Giraldo that does consulting for putting in place quantum computing solutions in most addressable markets (optimization, chemistry, healthcare, finance, machine learning). It also addresses cybersecurity and cryptography. The company is based in Houston, Texas. The company has no visible web site.

DN-Quantum Computing (2019, India) aka DishaNitish Technologies | Quantum Computing is a service company helping corporations develop quantum computing solutions, noticeably in chemistry and with transmon qubits.

Qkrishi (2021, India) develops quantum models, algorithms and kernels for applications in automotive, finance, agriculture, seismology, signal processing and other areas. The Birla Institute of Management Technology has also partnered with them to create a Quantum Computing course that integrates business and technical elements.

Q-iSIM (2016, Germany) aka "Quantum Interdisciplinary Simulations" develops quantum annealing software in physics simulation and fluids dynamics.

Silicofeller (2021, India) develops quantum solutions and advertises doing it in simulation, optimization and quantum machine learning which is fine, and then, crosses the line with mentioning the metaverse and Blockchain as application domains which are a bit farfetched.

Quanta-ly (2020, Libya) is preparing Libyan industries for the adoption of quantum products and services with training, consulting and advanced secure communications. No country, even in war, can escape quantum technologies!

Quanvia (2022, Spain) is a quantum consulting, service, research and training company targeting the usual suspect industries (retail, finance, logistics, energy, transport, automotive, pharmaceuticals, chemistry).

The company spun out of the University of the Basque Country, Bilbao, Spain. It is run by Enrique Solano (President, also cofounder of Kipu Quantum), Javier Mancilla Montero (Managing Partners),

[4415] See Using a quantum computer to solve a real-world problem -- what can be achieved today? by Robert Cumming and R. Thomas, November 2022 (62 pages).

Dawoon Choi (Quantum Machine Learning Director) and Ariela Strimling (Quantum Computing Tech Lead). The team is spread in Spain, Chile and the USA. The Latin America expansion came from the acquisition of Nimoy Cognitive Computing, a data-science service shop.

Qubitech (2021, Greece) is a quantum consulting company cofounded by Alexis Askitopoulos (CSO). Not to be confused with QubitTech, a scam crypto trading company that "*uses quantum technology to generate a monthly ROI of 25% for its investors and a total of 250% in 10 months*".

QuGeeks (2022, Switzerland) is a jobs board and search company specialized in quantum technologies. Their outreach includes "The Quantum Hubs" newsletter and its related social network activity.

Quant-X Security & Coding (2019, Germany) is a quantum service company created by Xenia Bogomolec (CEO) and Peter Nonnenmann (Scientific Advisor and Analyst). They develop quantum algorithms for quantum annealing and gate based systems. They also integrate PQC for dynamic partner authentication in QKD networks.

Unitary Zero Space (2020, Finland) quantum consulting and training services company. Their services portfolio includes helping organization commercialize their own quantum technologies, educating customers on quantum technologies and cybersecurity expertise.

They packaged their training offering in a half-day quantum computing workshop format (that's quite short...). The company was created by Topias Uotila (quantum programming), Risto Hakala (cybersecurity) and Juha Muhonen (quantum physics).

Quantum computing business applications key takeaways

- Most quantum computing market forecasts are highly optimistic and plan for an early advent of scalable quantum computers. They also sometimes tweak forecasts by pushing business value numbers instead of an actual market for quantum technologies.
- There are interesting potential use cases of quantum computing in nearly every vertical market, particularly in energy, chemistry, healthcare, transportation and then finance. Most of them are theoretical or have been evaluated at a very low scale given the capacity of existing quantum computers. Some may be useful with advanced noisy computers (NISQ) while most of them will require highly scalable fault-tolerant quantum computing systems (LSQ/FTQC). Others may find their way on quantum simulators.
- This book contains a framework proposal to assess the interest and relevance of published use cases. It helps separate the marketing from the real practicality of these use cases given that, as of 2023, no quantum application is running live in production in any corporation.
- In some cases, the potential use cases are in the overpromising twilight zone like simulating very complex molecules, fixing global warming, curing cancers or optimizing large fleets of autonomous vehicles. All these are dubious long-term promises.
- The main purveyor of case studies is D-Wave with its quantum annealer although it has not demonstrated yet a real quantum advantage. IBM is second there, having evangelized a broad number of customers and developers since 2016. IonQ and Quantinuum are also publicizing case studies, but they are usually was below a quantum advantage level, using only fewer than 25 qubits.
- Beyond computing time, a quantum advantage can also come from the system energetic footprint and/or the precision of the outcome.
- There are already many software vendors in the quantum computing space. How do they strive as there are no real functional quantum computers around yet? They sell pilot projects, develop software frameworks, build quantum hybrid algorithms and create quantum inspired algorithms running on classical hardware. On top of being funded by venture capital! We also cover in this book the burgeoning IT and consulting services in quantum technologies.

Quantum technologies around the world

Quantum computing in the broadest sense is a strategic technology domain for various reasons. In cryptography, states sovereignty is at stake with the protection of sensitive communications. Quantum computing has critical applications that may extend the scope of digital computing beyond what is feasible today, particularly in the fields of healthcare, the environment and artificial intelligence.

In terms of maturity, quantum and post-quantum cryptography represent more established fields with economic players and commercial solutions, even if the standardization of post-quantum cryptography is not yet complete. However, it has fewer scientific and engineering unknowns compared to scalable quantum computing.

Quantum computing is much less mature. The feasibility of commercially and useful quantum computers remains an open question both in the NISQ and FTQC regimes. There are significant technological challenges to overcome, including the thorny issue of qubits noise, quantum error correction, and how to scale the number of physical qubits by several orders of magnitude while preserving the quality of the qubits. So, quantum computing is full of scientific and technological uncertainties even before being economical and market ones. How countries deal with that is a good revelator of their innovation and forward-looking culture.

For the time being, fundamental research is mainly funded by governments in most countries, then from very large IT players who entertain many technology bets in parallel (IBM, Google, Microsoft, Intel, Alibaba), and a more or less well-funded startups, mainly in North America (D-Wave, IonQ, Rigetti, PsiQuantum, Xanadu), in Europe (IQM, OQC, Pasqal, Quandela) and other regions (SQC in Australia, Quantum Machines in Israel, Origin Quantum in China, etc).

The quantum computers software industry is in its infancy. The major players and startups creating quantum computers have all invested in the software arena, starting with the low-level supporting tools and sometimes, for developing quantum or quantum inspired applications.

Some QPUs are already available in the cloud, directly or through cloud services provides like AWS, Microsoft, Google and OVHcloud. Most of these players also sell cloud access to classical quantum emulators. In 2019, **Yuri Alexeev** from the DoE Argonne National Laboratory drew a parallel between the history of AI and quantum computing. He anticipated two quantum winters, the first which occurred in the late 1990s, the second in 2020 and the next around 2030[4416] (Figure 899). Well, if has not happened yet although we're in 2023 and practical applications are not there!

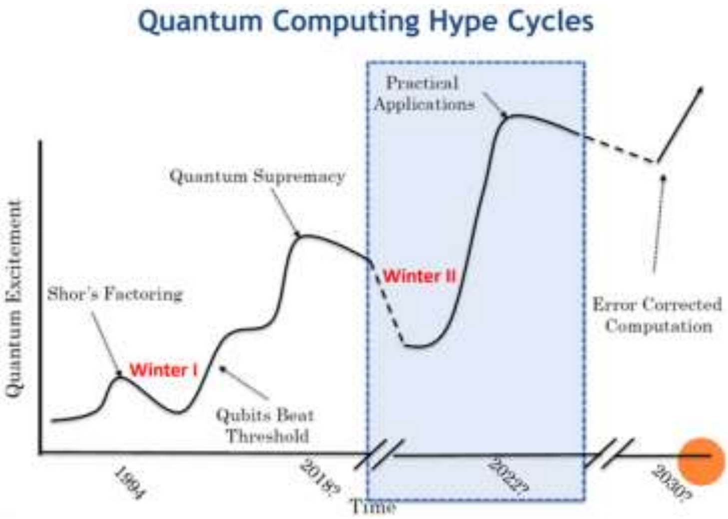


Figure 899: one view of the quantum computing hype cycle. Source: Quantum Computing Trends by Yuri Alexeev, August 2019 (42 slides).

Since this excitement is a somewhat fuzzy wave function and difficult to evaluate, it doesn't mean much. One quantum winter could still show up if NISQ system don't deliver business value.

[4416] See Quantum Computing Trends by Yuri Alexeev, August 2019 (42 slides). You can also find an historical comparison with many other technology hypes in Mitigating the quantum hype by Olivier Ezratty, February 2022 (26 pages).

It could slow down investments in various parts of the ecosystem, from public funding of academic research to investments in startups and with end-user corporations. Hardware startups will have a hard time delivering useful machines, while software startups will not have a large enough addressable market due to the lack of hardware. But this will not prevent public research laboratories and large tech companies from doing fundamental and applied research.

Quantum computing startups and SMEs

Mapping these vendors is a bit easier than in other deep techs like in artificial intelligence because there are not so many. There are many methods to inventory worldwide quantum startups and small businesses. My own database contains about 580 such companies worldwide. There are many such inconsistent inventories available all over the place. The discrepancy frequency from the way enabling technologies companies are accounted for. I put them in my database at the condition they enable "second quantum revolution" solutions. As a consequence, metal cutting lasers and classical telecommunications photonics are out!

This ecosystem began to take shape even before quantum computers were working on a small scale. It is fascinating to discover startups that make long-term bets, particularly with hardware. Software startups rely on a still limited hardware infrastructure but often reduce their risks by also supporting traditional computing architectures like Nvidia GPGPUs in machine learning and promoting so-called quantum inspired classical solutions. Their customers are large companies who test algorithms on a small scale to get their hands on quantum programming. To date, no serious application seems to be deployed in production. We are in the field of applied research and small scale prototyping within client companies. The software ecosystem is to be monitored closely. It will probably expand once hardware works on a larger scale, particularly with NISQ computers and quantum simulators[4417].

For their part, quantum cryptography systems are operational and correspond to a very separate market, just like the quantum sensing market that is more mature technologically but still in its infancy.

The stakes for many startups in this field are common with those of deep techs: how to develop real products with economies of scale, how to expand internationally rapidly, how to avoid falling into models that are too "service-oriented" and at last, how to resist to potential quantum winter or simpler investment winters not specifically related to quantum. Enabling technologies niche companies (photon sources, cryostats, ultra-vacuum, various sensors, electronics) can do well by reaching out diversified markets, notably targeting several different branches of research, telecommunications, defense and aerospace applications.

Investors

Quantum technologies investments may be impressive for the large rounds like those from PsiQuantum and IonQ and the associated "FOMO" factor ("*fear of missing out*"). But it is still small in volume in the technology sphere. The first investment funds more or less specialized in quantum technologies have already emerged with the following ones, a few of them being shown in Figure 900:

- **Quantonation**, a French seed fund created by Charles Beigbeder and managed by Christophe Jurczak, a physicist who got his PhD with Alain Aspect[4418]. They have already invested in 26

[4417] See Some Teams Go For NISQ-y Business Some are NISQ-Averse by Doug Finke, February 2020.

[4418] See Investing in the Quantum Future : State of Play and Way Forward for Quantum Venture Capital by Christophe Jurczak, November 2023 (11 pages) which describes the interplay between public and private investments, academic research and startups technology developments and the need for open research practices.

startups, making it the largest quantum dedicated fund in the world[4419]. They organize quantum meetups and hackathons in Paris (with **QuantX**, the quantum alumni association from Ecole Polytechnique), the first edition dating from October 2018. They were behind the launch **Le Lab Quantique**, which was jointly created with **Bpifrance** and many industry vendors and users.

- **Quantum Valley Investments** (QVI), a $100M Canadian investment fund, raised in 2013, dedicated to quantum technologies. Their founders invested in 1984 in Blackberry / RIM. They do not disclose their investments, except in ISARA Corporation, part of which are spin-offs from the Canadian research laboratory Institute for Quantum Computing at the University of Waterloo in Ontario.
- **Quantum Exponential** (UK) invested £450K in May 2022 in Universal Quantum who plans to build a million qubit quantum computer. It also invested in Arqit, Siloton, Aegiq and Delta G (UK) which develops gravity sensors.
- **Quantum Ventures** is a quantum investment company launched in 2016. Its "Quantum Revolution Fund" is managed from London and Switzerland. It aims to raise €100M.
- **Quantum 1 Group** is an American investment fund specializing in quantum technologies since 2015.
- **Summer Capital** is a Dutch investment fund specialized in quantum technologies, data and finance. Their investments include Horizon Quantum Computing, Rigetti and Turing.
- **Parkwalk Advisors** is a British deep tech fund. As of 2021, they have invested in Phasecraft, Quantum Motion Technologies, Riverlane, nu quantum, nu nano and Oxford Quantum Circuits. This fund is part of IP Group Plc since December 2016.
- **Runa Capital**, created by Serguei Beloussov (Russian with a Singapore nationality, founder of Acronis) is an investor in many deep tech startups, including IDQ, Qnami, Qu&Co, Pasqal and Welinq.
- **Phystech Ventures**, previously Quantum Wave Fund, created by Russians in the Silicon Valley, including Serguei Beloussov, and having already invested in the IDQ and Nano-Meta Technologies. Their fund is not 100% specialized in quantum. They also invest in robotics, drones, sensors and connected objects.
- **Machine Capital**, a UK fund focused on quantum and AI, which has so far invested in **QuantumX Incubator,** an incubator for quantum software projects launched jointly with Cambridge **Quantum** Computing (now in Quantinuum), with a 20-week incubation period.
- **SpeedInvest** is an Austrian investment fund specializing in deep tech start-ups, which invests among others in quantum technologies. They invest in seed stage with up to 1M€. They have invested in QPhoX and Kets.
- **QAI Ventures** is a quantum-focused investment fund and a startup accelerator which is part of QuantumBasel in Switzerland. Their first accelerated startups include Anaqor, Kipu Quantum, Miraex, Moonlight AI and QDC.ai. They are partnering with Redstone VC.

[4419] Quantonation invested in LightOn (France), Spark Lasers (France), which offers laser sources not specifically dedicated to quantum computers, Pasqal (France, cold atoms), Quantum Benchmark (Canada, software), Kets Quantum Security (UK, QKD component), Orca Computing (UK, hardware, photonics-based computing), CryptoNext Security (France, PQC), Qunnect (USA, repeaters for QKD), Quandela (France, photon source), Qubit Pharma (molecular simulation), Qnami (Switzerland, NV center-based metrology), Orca Computing (photon qubits, UK), Foqus (Canada, quantum sensing software), Qu&Co (the Netherlands, software), QPhoX (Netherlands, communications between quantum computers), HQS (Germany, software), Qubit Pharmaceuticals (France, software), evolutionQ (Canada, software), Inspek (France, chemical sensors), Multiverse (Spain, software), Pixel Photonics (Germany, single photons detector).

- **Redstone VC** launched in September 2023 a 52M€ dedicated seed fund to quantum technologies.
- **IQ Capital** is a deep tech UK fund who raised $400M in 2023. Out of its 48 investments, its only investment in the quantum space is Nu Quantum (entangled based QKD).
- **Qbit Capital** is another early stage investment fund based in Switzerland and dedicated to Swiss startups.
- **Quantacet** is a small early stage $20M investment fund from Québec in Canada.
- **Airbus Ventures** is an investment fund, headquartered in Silicon Valley, that operates independently from the Airbus Group which is one of its limited partners among several others. They invested in QCWare, IonQ, Q-CTRL, C12 and Qunnect.
- **2xN** (UK) launched in August 2022 is a dedicated $120M fund focused on quantum computing startups with $3M to $5M pre-seed to Series A tickets. It was created by Lars Fjeldsoe-Nielsen and Niels Nielsen. Limited partners include the Danish Growth Fund and some family offices. They invested in QuantrolOx and Sparrow Quantum.
- **Quantum Italia** is an investment fund created in 2023 by Scientifica Venture Capital, with some partnering with the Unitary Fund

You then have generalist funds who invested in one or two startups, like the Canadian pension fund **PSP Investments** that invested in D-Wave with seats in the board, **UVC Partners** who invested in HQS in Germany, **Supernova Invest**, **Elaia Partners** and **Breega Capital** in Alice&Bob in France, **Omnes** and **Serena Capital** in Quandela, again in France. And of course, corporate venture funds like **TotalEnergies Ventures, Airbus Ventures** and **Tencent Investments**.

Figure 900: a few of the key investors in quantum technologies. (cc) Olivier Ezratty, 2023.

We must also describe the **SPAC** funding mechanism that was used by IonQ, Rigetti, D-Wave and Arqit. A SPAC is first an investment fund that is created before it finds where to invest the money[4420]. It plans to invest the money in one company. When the company is found, the SPAC buys shares of the company, usually with other limited partner investors like Corporate venture funds. At last, the SPAC puts the fund on the stock market and is traded with using the acquired company name.

The process is a bit complicated. The SPAC business model is fairly unbalanced. The SPAC fund takes a significant cut of the deal and has a significant upside when the company IPO takes place, whatever the subsequent outcome in the stock market. There were hundreds of SPACs until 2022 but the phenomenon then dried up, particularly with assets reallocations in investors as inflation popped up after the start of the Ukraine war[4421].

[4420] See What is a SPAC: the step by step process going public, May 2021.

[4421] See SPACs out, the quantum community react by Karina Robinson, The Quantum Insider, June 2022.

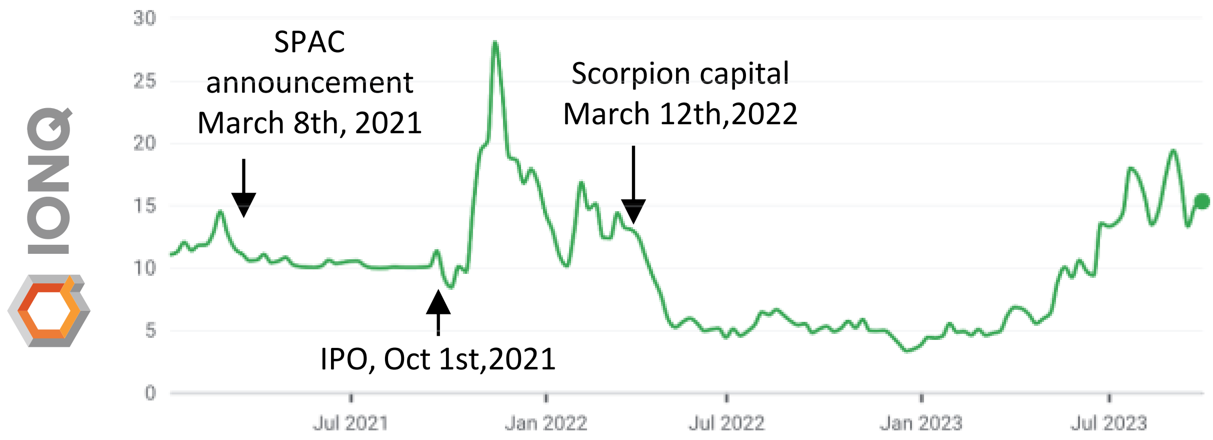


Figure 901: evolution of the value of IonQ . Source: Google Finance. October 2023 ([source](#)).

Figure 902: evolution of the value of Rigetti . Source: Google Finance. October 2023 ([source](#)).

Figure 903: evolution of the value of D-Wave. Source: Google Finance. October 2023 ([source](#)).

IonQ and Rigetti's stock value trended down after their IPO as shown in Figure 901, Figure 902 and Figure 903 due to significant overpromises in their investor pitch decks. D-Wave's IPO in August went well but things went awry staring in November 2022 and the company face delisting for a while until June 2023[4422]. First, we had a sluggish start for 2022 with large fund rounds[4423]. Then, the current post-Covid/Ukraine war recession may drive assets reallocations unfavorable for long-term risky

[4422] See Is quantum computing headed for a financial reckoning? by Dan O'Shea, May 2022.

[4423] See Shifting Quantum Investment Dynamics by Russ Fein, The Quantum Leap, July 2022.

investments[4424]. This explains indirectly why governments and their aggressive national quantum plans find workarounds to fund their local startups, like with creating an artificial market with public orders of non-existent or low performance experimental quantum hardware. As shown in Figure 904, the investment trends in amount slowly decreased in 2022 compared to 2021 which is itself a performance compared to other sectors. In 2022, the global VC investment was $432B for $618B in 2021[4425]! What is going to happen next? I don't know. Things will probably be tougher for some startups in their various rounds of investment, particularly in series B and C. The winning startups who'll escape the crisis will have good fundamentals: excellent teams, respected roadmap milestones, some IP and preferably some orders and/or revenue.

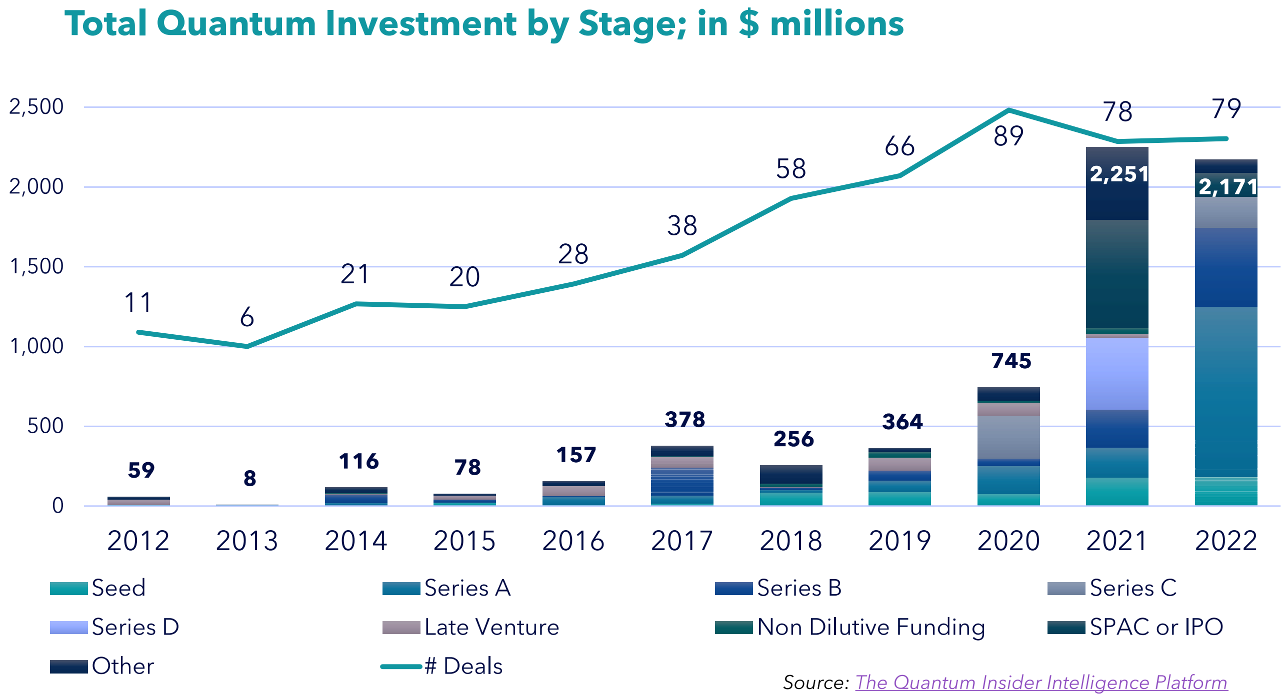


Figure 904: the trend in quantum startups funding assembled by The Quantum Insider. As a warning, they sometimes account for startups who are not pure players in the quantum field like pharmaceutical startups who are just starting to investigate the quantum computing field. Source: Quantum Technology Investment Update 2022 Review, February 2023, The Quantum Insider (33 slides).

Now, looking at the current revenue streams of these three companies, you find out that IonQ, Rigetti, and D-Wave made respectively $5.5M, $3.3M and $1.7M in Q3 2022, probably with selling computing time in the cloud either directly or through cloud vendors like AWS, Microsoft and Google with some growth YoY (Figure 905). Their yearly cash burn rate is in the tens of million $ per year which corresponds to between 3 and 9 years of "air supply" for Rigetti and IonQ. It is unclear for D-Wave when looking at their accounting[4426].

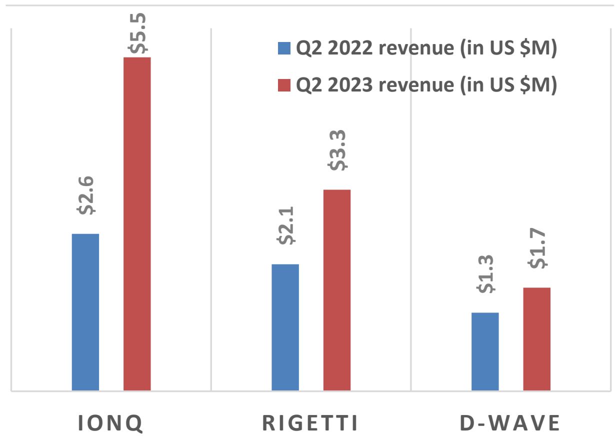


Figure 905: IonQ, Rigetti and D-Wave Q2 2023 and 2022 quarterly revenue. Source: their quarterly reports.

4424 See How the recession will affect quantum tech vendors by André M. König, June 2022.

4425 Source: Dealroom.

4426 Data source: Rigetti Q2 2022 quarterly report, IonQ Q2 2022 quarterly report and D-Wave Q2 2022 quarterly report.

Industry vendors maps

Since 2018, I have been consolidating many resources to build my own database of quantum industry vendors, starting with startups and then expanding it to small businesses and large established companies having a foothold in quantum technologies. Many other such databases are published or sold by various consulting or analyst firms and their content is very variable. They sometimes integrate companies that are not really in the "second quantum revolution" value chain, like the vendors selling metal cutting lasers. As of September 2023, my database had 666 companies including 21 established companies (IBM, Google, Microsoft, Intel, NEC, Hitachi, Toshiba, Fujitsu, Baidu, Huawei, Alibaba, …), 313 companies created between 2018 and 2023 and 140 between 2014 and 2017.

Many of the startups inventoried here are not yet in the "pure" form of the startupian model, i.e., they are far from having a scalable model or even just a product. They are often fundamental or applied research labs on wheels funded by VC and government subsidies, with a very high scientific and technological uncertainty. They sometimes fund themselves with contract research, various consulting services for large companies or public institutions, or with businesses not related to quantum technologies, like in the artificial intelligence field for software vendors.

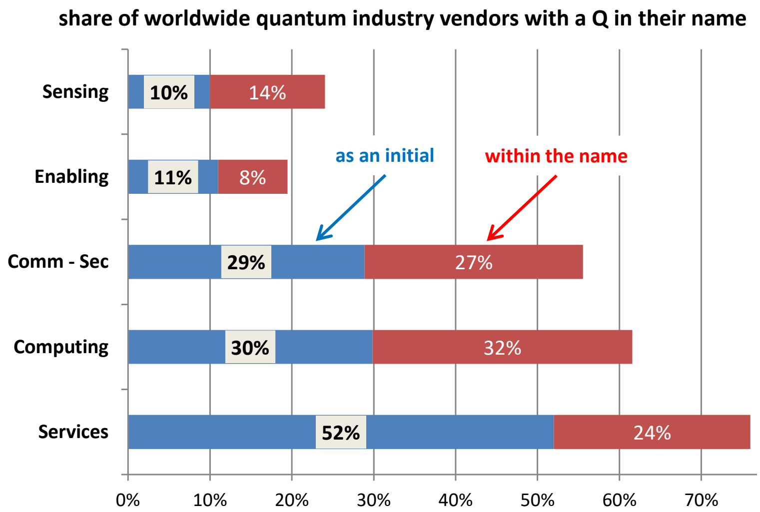


Figure 906: fun fact: in some fields like computing, telecommunications, cryptography and services, quantum startups branding shows a lack of creativity with many names containing a Q. (cc) Olivier Ezratty, 2023.

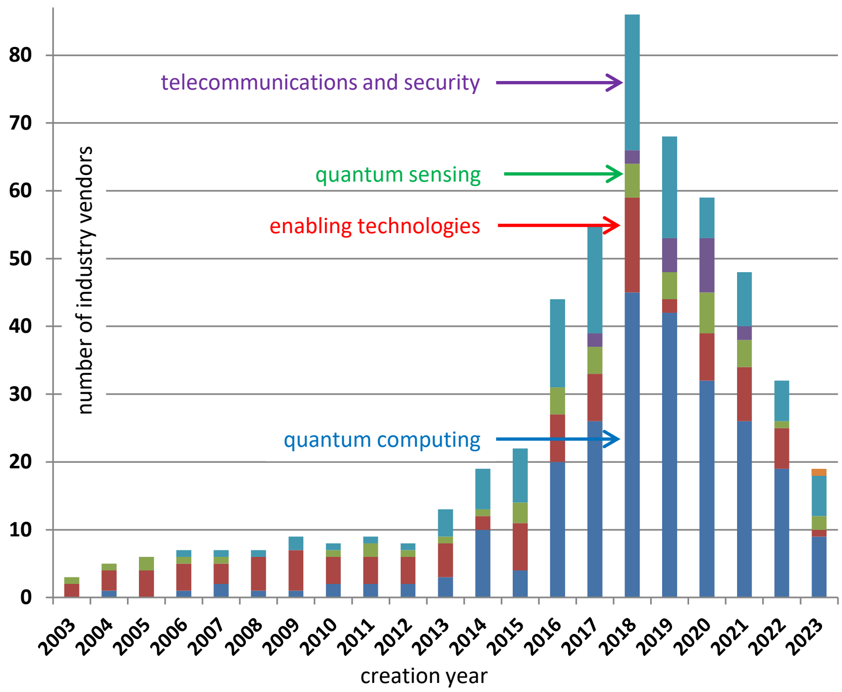
Some startups broadcast so little information about them that one may wonder if they are not scams. This lack of communication may be simply due to many creators being uncommunicative researchers, them being poorly funded, and their projects having too remote business prospects. Also, many times, they are so early-stage that they can't talk about anything that would drive interest, such as "*I have two operational qubits*".

I rely a lot on public information available on the Internet to describe what these startups do in the various parts of this book. One way to find out what they do is to identify their founders, if they are researchers, and find their original laboratories, their past scientific publications, and their PhD thesis if it is available. Finally, search for possible patents filed by the startups. This is technology intelligence using open-sourced information (OSINT). Of course, you can also meet with entrepreneurs, live or distantly.

Here are some charts extracted from my database of startups and SMBs. Figure 907 shows an indication of the number of startup creations per year and a breakdown of companies per by country. Figure 908 shows a different breakdown by country that highlights the largest funding. As usual, we see a significant funding gap between North America and Europe. One of the reasons is that European startups were created later or are more traditional small business which don't rely on venture capital for their development[4427].

[4427] See New record looms in VC funding of quantum startups by Michel Kurek, September 2020 and The European Quantum Computing Startup Landscape by Alex Kiltz, October 2020.

The other is of course different access to capital and markets. The US market size has always been an advantage for US-born companies although it doesn't prevent some worldwide leaders to emerge elsewhere in the world like ASML or SAP.



Country	Computing	Enabling	Sensing	Services	Comm - Sec	Total
USA	70	43	19	4	32	168
UK	23	17	8	5	20	73
France	14	29	10	4	6	63
Canada	34	8	5		14	61
Germany	25	22	4	1	7	59
Japan	13	7		1	4	25
China	11		2		10	23
India	13			3	5	21
Switzerland	4	9	1		5	19
Netherlands	8	4	1	1	3	17
Spain	9	4			2	15
Finland	5	3		2	3	13
Israel	6	2	2		3	13
Australia	6	1	3	1	2	13
Italy	3	3	1	1	3	11
Denmark	3	6			2	11
Singapore	6	1	1	1	1	10
Sweden	3	5				8
Poland	4	1			2	7
Russia		2			4	6
South-Korea	2				3	5
Austria	3				2	5
Taiwan	1	1			2	4
Estonia	2				1	3
Tchekia				1	1	2
Turkey		1	1			2
Hong-Kong	2					2
Belgium		2				2
UAE	2					2
Bulgaria	1				1	2
Greece	2					2
Chile					1	1
Bielorussia	1					1
Luxembourg		1				1
Columbia	1					1
Lybia	1					1
Uruguay	1					1
Slovenia	1					1
Norway	1					1
Ukraine	1					1
Total	**282**	**172**	**58**	**25**	**139**	**676**

Figure 907: chart of the creation year of small businesses and startups in "second revolution" quantum technologies. (cc) Olivier Ezratty, November 2023.

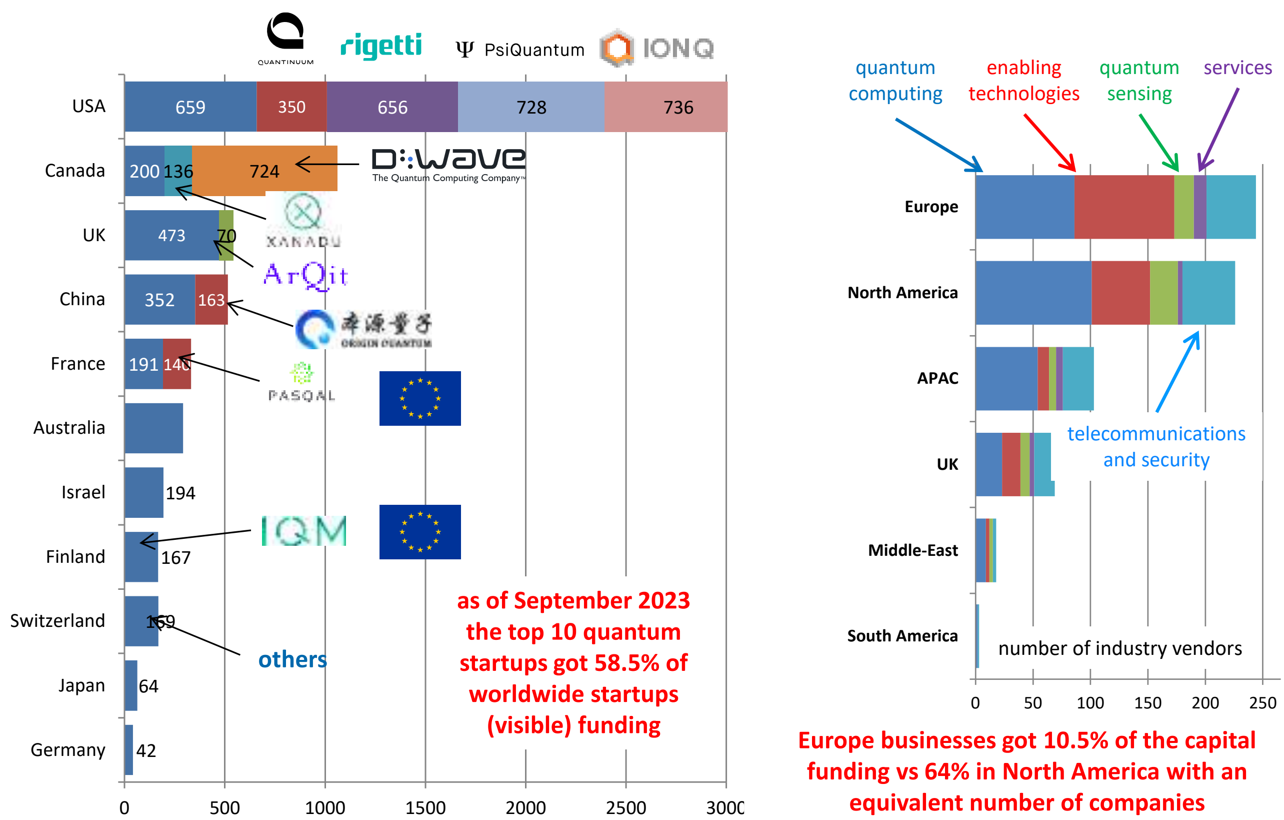


Figure 908: chart showing where the investment money went by country and its concentration. (cc) Olivier Ezratty, September 2023.

As of August 2023, 58.5% of worldwide startups funding went to the top 10 startups: IonQ, PsiQuantum, Rigetti, D-Wave, Xanadu, Quantinuum all from the North America, Arqit from the UK, IQM from Finland, Origin Quantum from China and Pasqal from France, while Europe has an equivalent number of companies compared to North America. These charts miss the undisclosed amounts raised by many startups, particularly from China.

If there are as many quantum startups and small businesses in mainland Europe than in North America, their visible equity funding represents only 10.5% of worldwide funding while North American companies got a hefty 64% as of August 2023, but it was 5% in September 2021. This doesn't include the investments from large IT corporations like IBM, Amazon, Microsoft, Google and Intel.

Quantum Startup incubation and acceleration

There are already a few incubation and acceleration programs for deep tech startups which host quantum startups in the world. One of the most famous is the **Creative Destruction Lab**, based in Toronto and other cities in Canada. **Xanadu** and **Nord Quantique** came out of it. Similarly, **Unit DX** is a deep tech incubator based in Bristol, UK, which started in the biotech industry and also helped some quantum startups[4428]. **Duality** is a quantum dedicated startup accelerator in the USA with a sponsorship from Amazon. **Quantum Startup Foundry** is the University of Maryland accelerator which was completed in 2022 by **Q-Cat** (Quantum Catalyzer), a startup studio.

In France, the **HEC Challenge+** program and the **Deeptech Founder** program created by the team behind the deep tech Hello Tomorrow event, accelerated a big share of France's quantum startups like Quandela, Pasqal and Alice&Bob. To create such acceleration programs, you need to be close to a critical mass pool of talents, such as a dynamic academic and research zone.

Disappeared startups

How about startups who disappeared? You would guess that there are many given the technological immaturity of the sector. Well, there aren't so many out of the 300 to 400 hundreds and so created so far.

Quantum Factory (Germany) was closed in January 2021. They were building trapped ions computers. **NextGenQ** (France) was pursuing the same goal and was also closed in 2021. The company had just a founder with no quantum physics skills nor any funding.

SeQureNet (2008-2017, France) was a spin-off of Telecom ParisTech specialized in the distribution of long distance CV-QKDs. It had been funded within the framework of the European research project SECOQC (secure communication based on quantum cryptography). The startup was based on work done by Philippe Grangier's team at the Institute of Optics and the Thales TRT laboratory in Palaiseau. The company closed down in 2017. It had been launched a little too early compared to the maturity of the market.

Black Brane Systems (2016, Canada) was a startup focused on the development of quantum machine learning solutions. They startup as a very "stealthy" company, got some undisclosed funding in 2018 and then closed their business.

We also have acquired startups like **CQC** (Quantinuum in 2021), **Labber Technology** and **Quantum Benchmark** (Keysight, 2020 and 2021), **Muquans** (ixBlue, 2021), **QxBranch** (Rigetti, 2019), **Qu&Co** (merged with Pasqal in 2022), **Super.tech** (ColdQuanta, 2022) and **QDevil** (Quantum Machines, 2022). So far, no quantum startup was acquired by a large IT vendor like IBM, Google and

[4428] See Incubators & Accelerators: Launchpads For Quantum Success? by James Dargan, 2020.

Microsoft. But to some extent, the investment of SandboxAQ, a spin-off from Alphabet, in August 2022, is a first[4429].

Safran (France) acquired **Orolia** (2005, France) and **Syrlinks** (2011, France) in 2022. It also seems that **Atomtronics** (2015, Italy) disappeared.

Global investments

What about global investments in quantum computing? A 2015 McKinsey study provided an overview of investments that likely compiled public research budgets[4430]. At that time there were 1,500 researchers worldwide with a total budget of $1.5B. This number has since then increased dramatically. The USA and China are obviously leading that space. But the distribution of these investments, which probably includes both quantum cryptography and quantum computers, is intriguing for other countries. As usual, Europe was fragmented with Germany, France, the Netherlands, Finland, Italy, Spain, the United Kingdom (then, in the European Union) and Switzerland (geographically in Europe). And we have strong quantum countries in other regions like Canada, Japan, Singapore and Australia.

Quantum technologies became a geopolitical issue, almost like nuclear deterrence[4431]. Governments are motivated to invest in quantum for strategic reasons: both in the idea of being able to decrypt existing or past telecommunications in the context of the activities of their intelligence services and to protect their own via quantum or post-quantum cryptography. More than almost any other digital technology, quantum is therefore becoming a strategic sovereignty tool for developed countries[4432]. The public authorities in these different countries have mobilized in very different ways on quantum. Most developed country governments coordinate efforts in the quantum field. Plans with up to $4B over 5 or 10 years periods have been announced here and there (Figure 909).

It is still quite difficult to compare these investments between countries and for a couple reasons:

- What is the **existing run-rate investment**? It is sometimes not easy to capture this data, particularly with highly decentralized research like in the USA and most European countries.
- What are the **undisclosed investments** in military and intelligence? It may be high in the USA and Russia. But lower in Europe, given these countries don't allocate a great share of their GDP to military expenses.
- Is the publicized funding **incremental** or does it include existing investments? You can easily embellish things with the latter accounting method or create misleading rankings of country investments like when McKinsey did showcase a chart with Germany investing more than the USA which was not true at all[4433].
- Are there any **double bookings** in the showcased investments? This can easily generate misleading information.
- Are some countries **overinflating** their investment? This is a hypothesis for China's investments which have been highly confusing. We provide as accurate data as possible here.

[4429] See SandboxAQ Announces a Partnership with evolutionQ as part of its New Strategic Investment Program, August 2022.

[4430] The McKinsey investment data is shown in the long piece Quantum technology is beginning to come into its own, The Economist, March 2017.

[4431] See Quantum, AI, and Geopolitics (3): Mapping The Race for Quantum Computing by Hélène Lavoix, December 2018.

[4432] See the forum Europe: Keys to Sovereignty by Thierry Breton, August 2020. He cites three pillars of this sovereignty: computing power, data control and secure connectivity. Quantum technologies have a key role to play in the first and third! However, the means cited to obtain this sovereignty are classic and relate to public funding for R&D. We know that this is clearly insufficient.

[4433] See A quantum wake-up call for European CEOs, McKinsey, December 2021.

* You can also use various metrics like **investment per GDP**, which has to be normalized over a given period of time[4434].

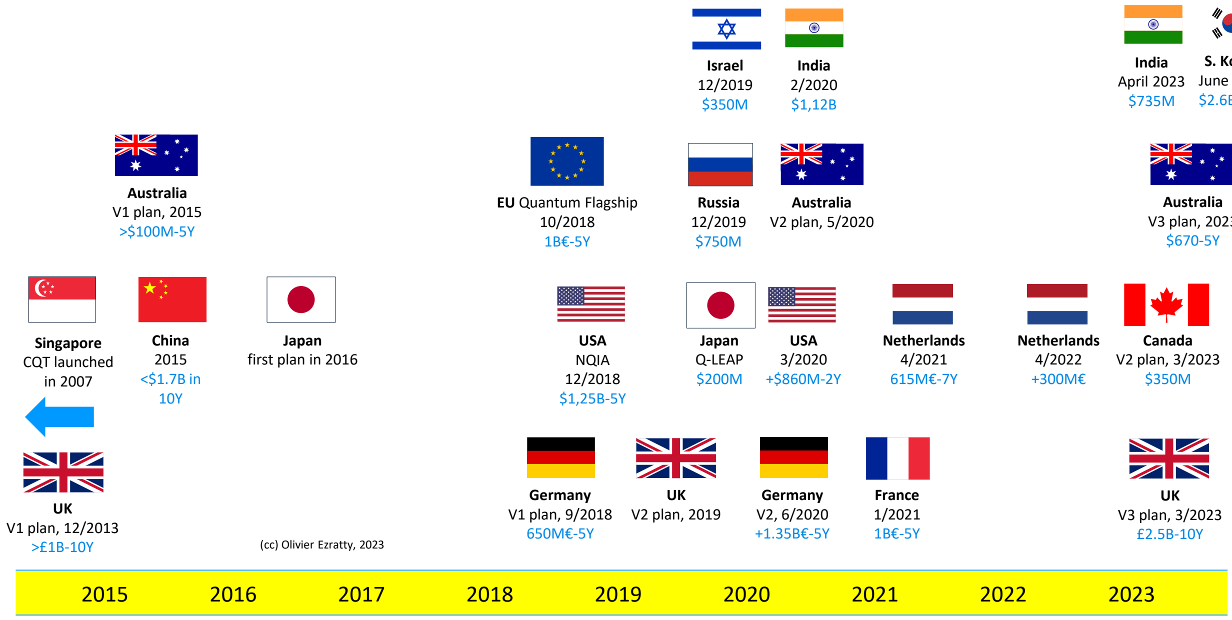


Figure 909: national quantum initiative plans across the years including only public funding. (cc) Olivier Ezratty, 2023.

Where's government money (your taxes) going? It usually funds the following in these various national quantum initiatives:

* Incremental **public research** funding efforts. This comes from both the country governments and sometimes, from local governments, like in Germany or Canada.
* Growing the **education effort** and addressing the skills shortage. This is particularly important as it deals with very highly skills roles, with a strong scientific background.
* Creating **quantum hubs** that consolidate quantum research and sometimes startup creation. These are handled within universities or public labs (like the DoE in the USA) and regions (like in Munich in Germany) or across several of them in a thematic way (UK). This may involve some real estate and building construction. Every country wants to consolidate their quantum ecosystems. What is a quantum ecosystem? Well, like with other technology ecosystems, It contains a mix of education, academia, public research labs, industry vendors of all sizes (large, small businesses and startups; hardware, software, telecommunications and cloud, services, including on enabling technologies), policy makers and users[4435].
* Create **hybrid quantum-classical supercomputing centers**, based on existing supercomputing capacities, which is being done nearly everywhere, in the USA, Europe, etc.
* Create a **quantum technologies industry** with both encouraging startups creation and existing industries to adopt quantum technologies, like in the sensing domain or just, in software development. This goes with putting in place public/private funding mechanisms for long-term investments.

[4434] See Top Quantum Spenders Based on GDP — List Offers Surprising Changes in Leadership Status by Matt Swayne, The Quantum Insider, February 2023, which compares multi-year plans with annual GDP without normalizing the plans duration, like with the (fake) $15B investment tag for China. In most other countries, however, plans are generally covering 5 years.

[4435] See Building a Quantum-ready Ecosystem by Abhishek Purohit et al, Apr 2023 (45 pages).

- Handle **procurement** with local startups with countries indirectly funding their local quantum computing startups champions through local procurement like in Finland with IQM, Australia with Quantum Brilliance, the USA with IonQ[4436], the UK with Orca and OQC or France with Pasqal. This is often related to the hybrid quantum-classical centers already mentioned. It can also come from universities procurement like in China.

- Foster **global partnerships** that are different in nature, between research laboratories within countries (as in the UK hubs), between particular labs across various countries, and between public and private research within the same country (CEA and Atos) or between different countries (Intel with Qutech). The raison d'être of all these partnerships can be identified: quantum computing is a complex scientific subject that cannot be mastered by a single laboratory or company. Collaboration is necessary to bring together talent from different specialties, between condensed matter physics, sensors, and control technologies, optronics, cryogenics, semiconductor production, algorithmics and software development.

- Launch bilateral **countries partnerships** like the USA with Nordic countries, Switzerland, the UK and Australia, or France with the Netherlands and Singapore with Finland.

I tried to consolidate in Figure 910 a more global view with public and industry investments per country or region. This is based on some guestimates for large industry vendors in the USA (IBM, Microsoft, Google, Intel) but the public investment data is rather safe. What this shows is counter-intuitive: the first region in public investment is the European Union! It is investing more than the USA and even China. Still, the USA leads the international pack thanks to their industry investments, both from large IT vendors and from VCs in startups.

Europe's Achille's heel is not having these large IT vendors on one hand and investing less in startups in proportion to GDP. The result is some pressure put on European startups who have to find ways to get bigger fundings in Series B, C and beyond with investment sources outside the EU like the USA, Asia or even the Middle East oil countries.

I did this chart after seeing so many analysts providing wrong or outdated numbers on countries and region public investments in quantum technologies. The last one comes from a **BCG** report which tried in 2022 to compare of investments between the EU, the USA and China.

Their data is wrong. Like many, in page 14, they overestimate China's investment (at $10B when it is probably way under $4B) and underestimate USA's which is about $4.7B as of August 2022 for 5 years instead of the mentioned $2.9B[4437]. They also showcase a $700M plan from France in 2019 that never existed. This is puzzling. But they showcase the harsh reality of a much smaller investment in startups in the EU compared with the USA, Canada and the UK. A similar report from the **World Economic Forum** contains the same wrong data on the USA, China, and also Japan[4438].

An evaluation of scientific publications in quantum computing done by **Insead students** in 2018 did show with no surprise to discover that the USA, Canada and China are the first countries to publish[4439].

A more detailed analysis was produced by **Michel Kurek** in September 2020 (sources of the illustrations in Figure 912) which did help relativize the influence of Chinese publications[4440]. Indeed, the Citations Per Publications is very low in China and also India, compared all Western countries.

[4436] See IonQ Secures Contract to Provide Quantum Solutions to United States Air Force Research Lab, IonQ, September 2022.

[4437] See Can Europe Catch Up with the US (and China) in Quantum Computing? by François Candelon, Jean-François Bobier, Maxime Courtaux, and Gabriel Nahas, BCG, August 2022 (45 pages). You find the same China investment overstating in The Quantum Computing Arms Race is not Just About Breaking Encryption Keys by Adm. Mike Rogers and Nir Minerbi, Classiq, June 2022.

[4438] See State of Quantum Computing: Building a Quantum Economy, September 2022 (48 pages).

[4439] See VC investment analysis Quantum Computing, April 2018 (18 slides).

[4440] See Quantum Technologies : Patents, Publications & Investissements Landscape by Michel Kurek, September 2020 (52 pages).

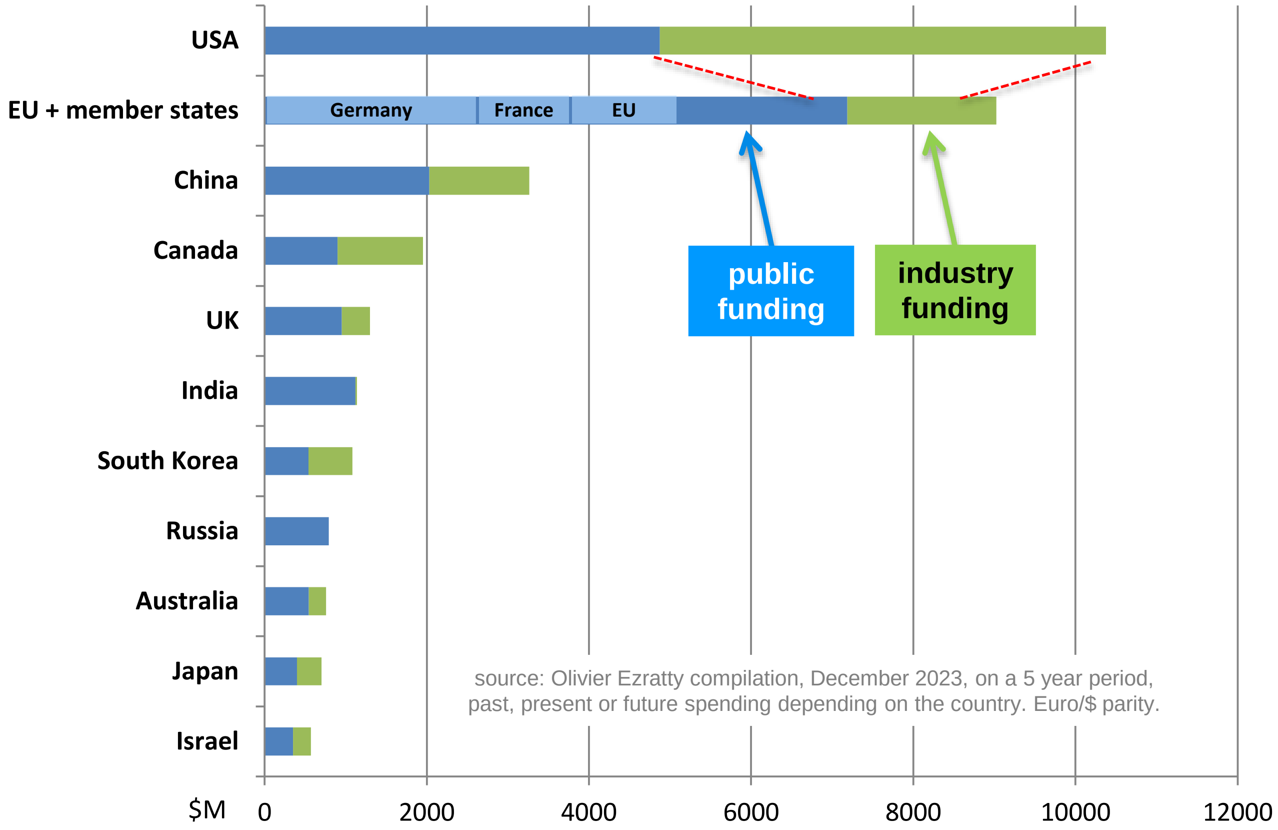


Figure 910: a consolidation of quantum technologies public and private investments with some raw estimated for large IT vendors. It creates a very different picture than what is commonly thought about the place of China and Europe. (cc) Olivier Ezratty, December 2023.

The significant investments made by developed countries in quantum technologies raise fears that computing power could end up being concentrated in the hands of a few or even a single country or company. I don't believe this, at least not in the initial phase of development of these technologies. Knowledge on the subject is highly distributed, as are enabling technologies and strategic materials. I would rather situate the risk of concentration in a second phase of the maturation of this market, one that will see a market that was initially fragmented with many players concentrating through consolidation. It will probably do so for reasons that are more macro-economic than scientific or technological, through economies of scale and the platforming of offers. This explains why it is necessary to simultaneously keep an eye on the hardware, development tools and software applications of quantum computing.

Once the main scientific and technological uncertainties are lifted, the success of each company and country will depend on the classic key success factors of technology ecosystems: execution speed, team quality, funding levels, communication, marketing, sales, the ability to promote technology platforms to a maximum number of players and on a global scale.

This is where sovereigntist approaches combining protectionism of key players while ensuring maximum trade openness to the world to enable them to achieve economies of scale will have to be carefully adopted.

We'll go through the details, country by country, continent by continent. With one exception, Africa, which is little invested in the subject, at least as a producer of quantum technologies, maybe besides South Africa which seems to have started to get involved in the academic side[4441].

Figure 911 shows which country best masters quantum computing technology per qubit type. All in all, we have a good balance between the USA and the European Union, although the USA have the

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[4441] See Will Africa miss the next computational revolution? by Amira Abbas, April 2020.

benefit from having large IT vendors invested in the field in superconducting (Google, IBM), silicon (Intel), trapped ions (Honeywell, IonQ) and topological qubits (Microsoft). China and Japan are less represented here, despite some interesting research, mostly since they only have a few quantum computing hardware startups.

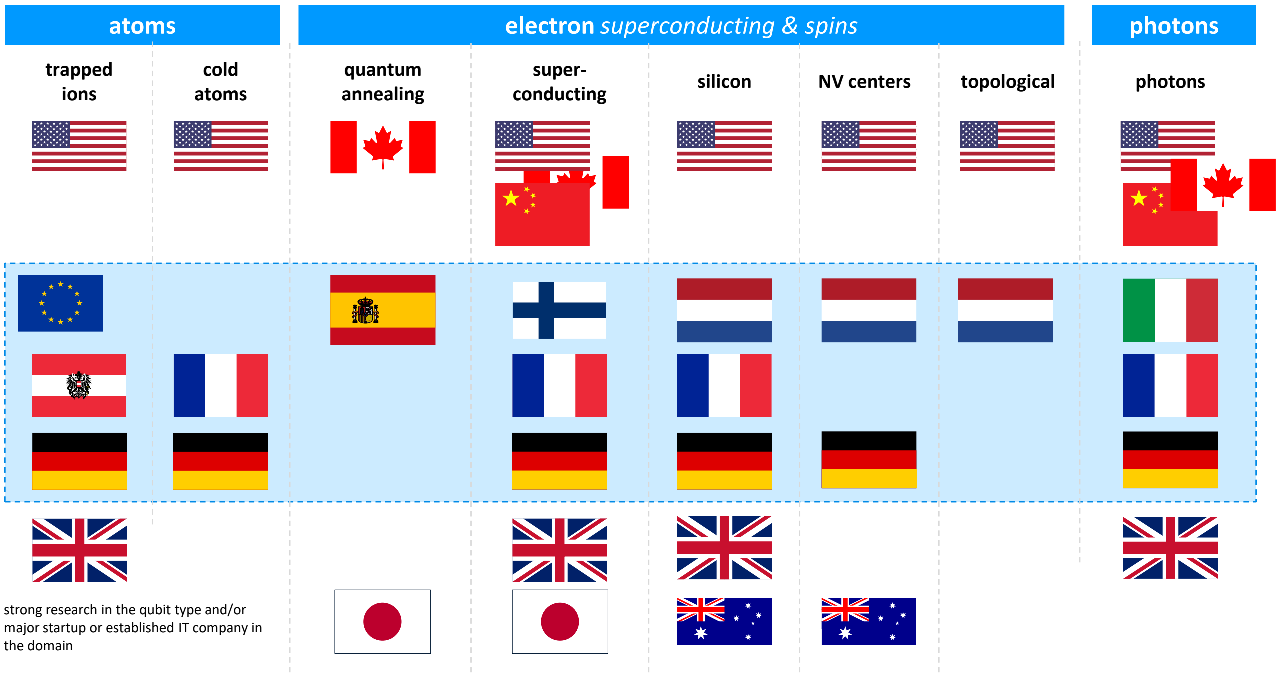


Figure 911: key quantum computing technologies per qubit type and country of origin. (cc) Olivier Ezratty, 2022-2023.

What are the key success indicators of success for countries investing in the quantum race? We'll probably have some analyst shops create their own quantum sort-of Shanghai ranking using composite metrics: public funding, scientific publications, patents and the likes, entrepreneurship spirit, number of startups, startups funding, large companies' investments, corporate adoption, skilled workforce and else. Guess what? US and China will probably rank first there. And smaller countries behind in some variable order. But what if Europe was consolidated?

Patents

As quantum technologies are the focus of hundreds of startups and many large information technology vendors like IBM, Google, Microsoft, and others, patents are becoming increasingly important to safeguard businesses. Patents always play an offensive role and a defensive one. The defensive one enables research and industry vendors to extract commercial value from their work without being disrupted by competitors. The offensive one usually plays a role later, when the market really grows up and when royalties can be obtained from licensing your technology and contribute to growing your revenue base and to fund your own R&D efforts.

On a worldwide scale, many sorts of know-how can be patented: architectures, protocols, processes, materials, tools, algorithms and even business processes (particularly in the USA). Patent applications must demonstrate novelty, non-obviousness, usefulness, and applicability, and should be clearly described to enable replication. The trick of the trade consists in saying and describing the minimum to obtain a patent grant, with some obfuscating preventing competitors to implement the invention. In some cases, organizations can also protect their IP with industry secret, when it can't be easily reverse engineered by their competitors. All these phenomena are amplified in the complicated field of quantum technologies as we've seen elsewhere in this book about some Huawei and AMD patents which were borderline patent trolls.

As they are created, quantum startups start to understand the need to get a maximum number of patents. It creates a business safety belt. It can help get good pre-money value for the company while getting term sheets from investors. Still, this work usually starts earlier, as technologies usually see the light of day in public research laboratories.

	COUNTRY	TP	%TP	TC	%TC	CPP	RCI	%ICPEI
1	USA	4,295	26.4%	108,128	44.8%	25.2	1.7	70%
2	China	3,706	22.8%	38,611	16.0%	10.4	0.7	44%
3	UK	1,428	8.8%	32,435	13.4%	22.7	1.5	120%
4	Germany	1,400	8.6%	38,339	15.9%	27.4	1.9	123%
5	Japan	1,106	6.8%	20,996	8.7%	19.0	1.3	99%
6	Canada	1,056	6.5%	23,104	9.6%	21.9	1.5	124%
7	India	991	6.1%	5,847	2.4%	5.9	0.4	33%
8	Australia	777	4.8%	20,777	8.6%	26.7	1.8	130%
9	France	699	4.3%	14,016	5.8%	20.1	1.4	117%
10	Italy	635	3.9%	10,522	4.4%	16.6	1.1	116%
	Total 10 countries	16,093	98.9%	312,775	129,5%	19.4	1.3	83.1%
	Total world	16,279		241,536		14.8		

*TP= Total Publication ; TC = Total Citation ; CPP = Citation par Publication = TC/TP ;
RCI = Relative Citation Index ; ICPEI = International Collaboration Publication Extended Index

Figure 912: publications and patents on quantum tech per country. Source: Quantum Technologies : Patents, Publications & Investissements Landscape by Michel Kurek, September 2020 (52 pages).

They are also incentivized to patent their own innovation. First, to get some economical reward and second, to best protect the startups that spin out of their labs.

Patents data is being used to rank countries and companies and quantum technologies are not an exception. Published rankings usually showcase the usual dominance of the USA and China. They have their many shortcomings given it is difficult to build well-crafted database requests in the various USPTO and other patents database due to the complex lingua of quantum technologies, and sometimes, to various abuse by patent applicants.

For example, in some patent reports, China's large number of patents granted in the quantum space seems due to the abundance of patents related to spectroscopy, something that doesn't really belong to the "second quantum revolution", which should be a beacon to identify relevant patents. It may explain why China accounts for above half of all the applications in various patent jurisdictions for quantum technologies[4442]. But the explanation may be elsewhere. China is investing a lot in patenting its R&D efforts in quantum technologies, and particularly in quantum sensing and quantum communication. In that later case, they are very active in their research labs like in Hefei, and with their first startups in the field.

[4442] See Quantum Technologies : Patents, Publications & Investissements Landscape by Michel Kurek September 2020 (52 pages).

In the quantum computing realm, the USA seems to dominate the field. IBM, Google, D-Wave, Northrop Grumman, Microsoft and Intel are the first applicants[4443].

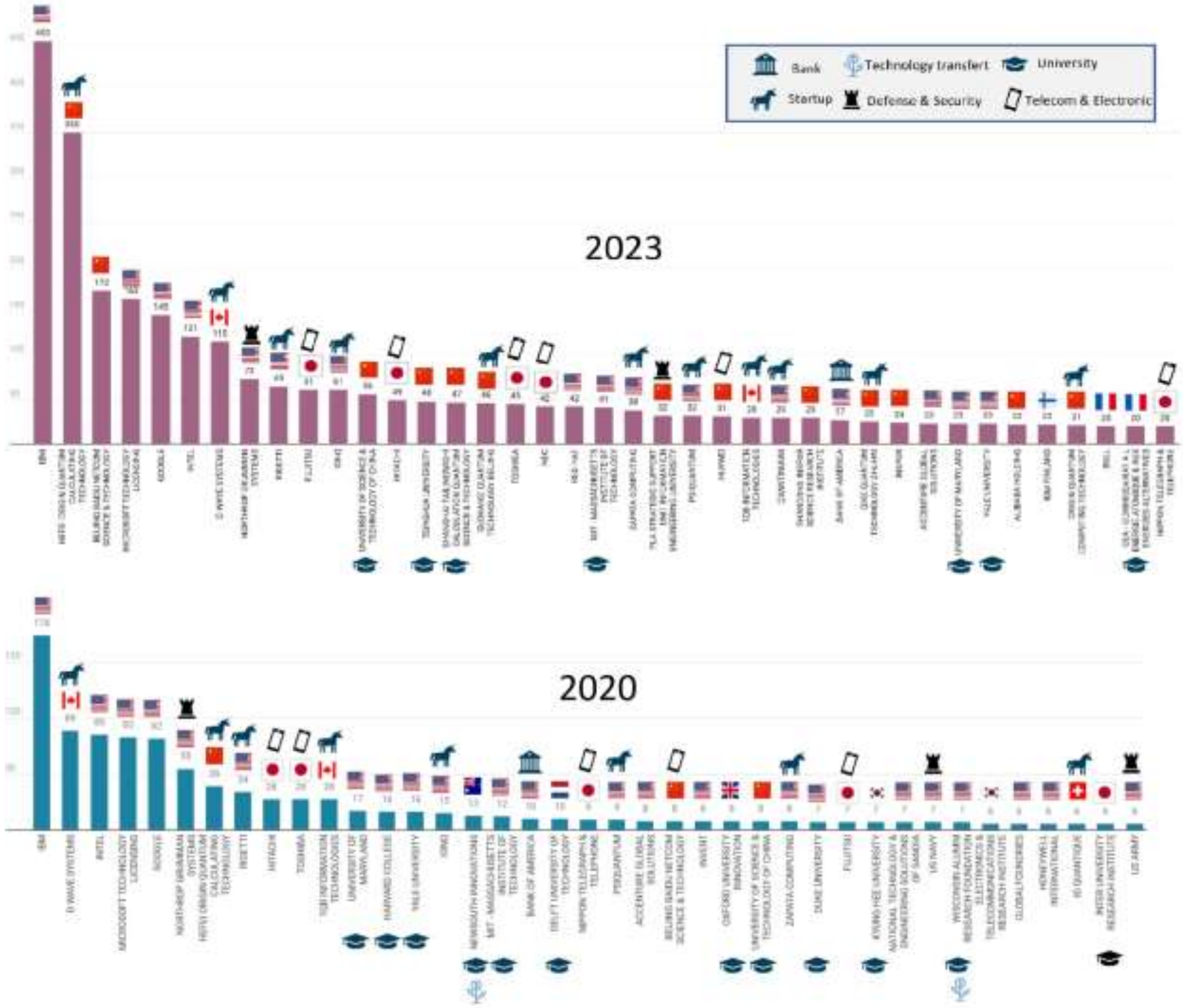


Figure 913: a quantum computing patents view per holder, comparing the situation between 2020 and 2023. Source: Michel Kurek. March 2023.

Recent charts like the one in Figure 913 from Michel Kurek shows an uptick with China companies like Origin Quantum and Baidu who, together, have more patents than IBM. It is quite surprising since both companies are not as advanced as IBM in their superconducting qubits R&D. Another report, from the EPO (European Patents Office) shows that patents related to physical realizations of quantum computers come mainly from US vendors[4444]. All this is consistent with another report coming from China's IPRdaily[4445] [4446]. While it shows that quantum computing patent applications submitted by companies in China went from 137 in September 2020 to 804 in October 2022, US companies still largely leading the field.

[4443] See Quantum patent trends update: 2022 by Elliott Mason, QED-C, February 2023. It lists Ruban Quantum Technology (2017, China) as the 8th applicant in quantum computing. This totally unknown company has patented many cybersecurity methods against quantum computing threats, showing again the issue with database requests (source).

[4444] See Quantum computing technologies on the rise, EPO, January 2023 (42 pages).

[4445] See Quantum computing patent filings surge in China by Zhu Lixin, Chaina Daily, November 2022.

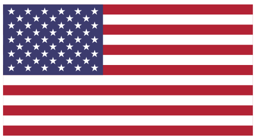
[4446] See Top 100 Global Quantum Computing Patents Announced Huawei and Tencent in the list, TechGoing, October 2022.

Europe is relatively absent from these charts. It contrasts with its significant activity in scientific publications, showing the unpreparedness of its startup ecosystem and the relative absence of large IT players in the field[4447].

The lack of sufficient government incentives may be at play here, but the lack of large players may be a more important explanation since these are the most active patent holders, whatever the business[4448].

North America

USA



Whatever the metric you use, the USA is leading the world in quantum technologies. They mix three components no other country or region has: a powerful Federal government investing significant amounts in fundamental and applied research, large IT companies investing a lot as well in research and industrialization and a healthy well-funded dense startup ecosystem.

The coordination of research in the different branches of quantum physics started in October 2014, when the Obama White House produced two reports[4449]. It was an inventory of what existed. Like almost all countries, quantum technologies were segmented in four with quantum communication, sensing, computing and simulation. In 2017, industry and research lobbying started to push the federal government to launch a national quantum plan. It started with U.S. House of Representatives organizing a hearing in October 2017 (video). For three hours, elected officials questioned a panel of scientists including James Kurose of the NSF and John Stephen Binkley of the Department of Energy, who explained the basics of qubits and the associated sovereignty issues.

The Democrats were concerned about the Trump administration's proposed cuts in funding for civilian research in favor of defense budget increases and tax cuts. But the US Congress increased federal research budgets for fiscal year 2018 and beyond[4450], given that these budgets are then traditionally mainly channeled to American universities research laboratories. This is one of the few cases where the Republican-controlled Congress opposed the Trump administration. This happened consistently throughout all fiscal years of the Trump administration.

National Quantum Initiative Act

The National Quantum Initiative Act was first proposed on June 26, 2018 by the House of Representatives Science Committee (H.R. 6227, 25 pages). An equivalent proposal came from the Senate on the same day. This project was the result of a proposal, the National Quantum Initiative-Action Plan, prepared by public and private research stakeholders (IBM, Google, Rigetti).

[4447] See Quantum technologies: patent applications vs. scientific publications across the world by Sébastien Ragot and Michel Kurek, November 2021.

[4448] See Mapping the Patent Landscape of Quantum Technologies: Patenting Trends, Innovation and Policy Implications by Mateo Aboy, Timo Minssen and Mauritz Kop, May-July 2022 (30 pages).

[4449] The report Advancing Quantum Information Science: National Challenges and Opportunities, July 2016 (23 pages) was followed by a working meeting in October 2016.

[4450] See Trump, Congress approve largest U.S. research spending increase in a decade, Science, March 2018.

An intense lobbying campaign was carried out by several professional associations[4451], with the **National Photonics Initiative** (NPI), a professional association bringing together photonics physicists and industrialists in the sector, accompanied by the lobbying firm **BGR Group**.

This association, which wanted to make photonics a priority, was launched in 2012. It is sponsored by other entities: The Optical Society (now named Optica), SPIE (The International Society for Optics and Photonics), American Physical Society (APS), IEEE Photonics Society, ALIA Laser Institute of America and many other professional associations. The lobbying was also pushed by **Jonathan Dowling** (1955-2020, American), professor of physics at Louisiana State University and a photonics specialist[4452]. Various quantum industry consortiums also played a role in lobbying the US government. The **Quantum Industry Coalition** brings together generalist vendors such as IBM, D-Wave, Intel, as well as some startups[4453]. The founder of the Quantum Industry Coalition is Paul Stimers[4454], then, a partner of KL Gate, an US law and lobbying firm ranked as the 41st largest law firm in the world making $1B annually, and since 2023, at Holland & Knight LLP, another large US law form. The **Quantum Alliance Initiative** launched in 2018 by the Hudson Institute, a conservative think tank, creates proposed standards for QKD and QRNG and of course advocates for the development of this industrial sector in the USA. **NIST** had also created with **SRI International** the **Quantum Economic Development Consortium** (QED-C) to develop the American quantum industry[4455]. It is run by Celia Merzbacher, a semiconductor industry lobbyist who worked in the White House during the Bush 43 administration.

In 2018, the Quantum National Initiative Act proposed allocating $1.275B over five years to fund civil quantum R&D, divided among the Department of Energy ($625M), NSF ($250M) and NIST which is focused on cryptography issues ($400M).

The Act also proposed the creation of a National Quantum Coordination Office within the White House Office of Science and Technology Policy. It asked the President of the United States to create a 10-year quantum plan, the first step being a five-year plan to be delivered one year after the passage of the law. This bill was pushed by elected officials fearing that China will take over the quantum space, especially in computer security[4456]. The bill was voted by the House in September and then by the Senate in December 2018[4457].

4451 See Quantum computing finds its lobbying voice by Aaron Gregg, Washington Post, June 2018.

4452 See Schrödinger's Killer App - Race to Build the World's First Quantum Computer by Jonathan P. Dowling, 2013 (445 pages) where the author was sending a warning about the risk to see China lead the quantum technology race. If the USA were not investing more: "*The future of the quantum Internet is in photons and the short circuiting of the development of optical quantum information processors in the United States means that the future quantum Internet will have 'Made in China' stamped all over it.* ", page 173.

4453 See their Quantum Industry Coalition website. Microsoft and Intel joined the Coalition when it launched but they seem to have withdrawn from it since then.

4454 See The US National Quantum Initiative by Paul Stimers, K&L Gates, October 2019 (6 pages).

4455 See NIST Launches Consortium to Support Development of Quantum Industry, September 2018. And more details in U.S. Consortium Pulls Ecosystem Into Quantum by Susan Rambo, August 2019. As of July 2020, the association has 130 members from the private sector - large corporations and startups - and about 40 laboratories from universities and the U.S. public sector.

4456 See How suspicions of spying threaten cross-border science by Patrick Howell O'Neill, December 2019, which discusses the direct and indirect methods used by China to plunder European and American quantum research and exploit it both civil and military, such as quantum radars, quantum sonars and QKD. Here is the link to retrieve the Quantum Dragon Strider study mentioned in the article, November 2019 (22 pages). You can indicate a bogus email to get it, the download does not go through an email. It evokes various partnerships in research that help the Chinese to exploit Western research. It is based on a few examples including the very detailed one from the University of Heidelberg in Germany. On the same subject, see also China's top quantum scientist has ties to the country's defense companies, December 2019, Quantum USA Vs.Quantum China: The World's Most Important Technology Race by Moor Insights and Strategy, October 2019 and New Warnings Over China's Efforts in Quantum Computing by Sintia Radu, January 2020.

4457 See SIA Welcomes House Passage of Quantum Computing Legislation, September 2018.

In September 2018, the White House published the National Strategic Overview for Quantum Information Science that included the terms of the congressional proposal[4458]. They emphasized research, training of scientists and international collaboration. At last, Donald Trump signed this law on December 21, 2018.

In December 2019, the **Quantum Information Edge** alliance was created, bringing together Lawrence Berkeley National Laboratory and Sandia Labs of the Department of Energy, the University of Maryland, Duke University (North Carolina), the University of Colorado at Boulder, Harvard, Caltech, MIT and the University of New Mexico[4459]. For the most part, the usual suspects of basic research in quantum computing, thus creating their "virtual hub" for coordinating research in this field. With a focus on error reduction at the qubit level, techniques for interconnecting qubits and the development of new quantum algorithms. For its part, the **NPI** embarked on a new lobbying campaign at the end of 2019 and early 2020 to increase once again the federal funds allocated to research in quantum technologies[4460]. In 2022, the campaign was still going on with pushes for more Federal investments in quantum technologies procurement[4461].

In February 2020, the White House published a memo from the National Quantum Coordination Office recommending the development of quantum networks[4462]. In March 2020 at the beginning of the Covid pandemic, it proposed a new increase in quantum research budgets for the years 2020/2021[4463]. It included $450M for the Department of Energy, $330M for the NSF and $80M for the NIST. This was matched by a $1B increase for artificial intelligence research programs[4464]. In August 2020, the White House announced a 30% increase in the quantum and AI budgets for fiscal year 2021. In their yearly report since then, the NQI publishing interesting data on actual spendings related to these plans on top of legacy investments, as show in Figure 914.

In December 2021, a memo was published by the NQI team showing for the first time the total Federal budget spent in quantum technologies by year with the legacy spendings and the NQI related spendings. It did show that the 5-year trend was a $4B plan[4465].

The US NQI (National Quantum Initiative) is run by the **National Quantum Coordination Office** (NQCO), hosted by the White House Office of Science and Technology Policy (OSTP).

It has a web site since October 2020[4466]. Its director has been Charles Tahan since its creation[4467]. If you wonder about the bureaucracy in your own country, here you are also with several related committees: the **National Science and Technology Council** (NSTC) Subcommittee on Quantum Information Science (SCQIS) that coordinates Federal R&D in quantum technologies, the **National Science and Technology Council** (NSTC) Subcommittee on the Economic and Security Implications

[4458] See National Strategic Overview for Quantum Information Science, White House, September 2018.

[4459] See US alliance for quantum computing by David Manners, 2019.

[4460] See NPI Brings Quantum Experts to Capitol Hill to Advocate for Additional NQI Funding by Jo Maney, March 2020.

[4461] See The US government needs a commercialization strategy for quantum by Laura E. Thomas, senior director of National Security Solutions at ColdQuanta, December 2021. Pushing for federal procurement of US quantum computers through DARPA and the NSF.

[4462] See A Strategic Vision for America's Quantum Networks, White House, February 2020 (4 pages).

[4463] See Why is Trump funding quantum computing research but cutting other science budgets? The national security implications of this technology may be exaggerated by John Lindsay, March 2020.

[4464] See White House reportedly aims to double AI research budget to $2B by Devin Coldewey in TechCrunch, February 2020.

[4465] See National Quantum Initiative supplement to the President's FY 2022 budget, December 2021 (46 pages).

[4466] The NQCO published the quick status report Quantum frontiers report on community input to the nation's strategy for quantum information science in October 2020 (32 pages).

[4467] Charles Tahan is a physicist specialized in condensed matter physics and quantum information science. He continues to publish some scientific papers from time to time, trying to not losing ground in his field.

of Quantum Science (ESIX) that handles economic and security implications across federal agencies[4468] and the **National Quantum Initiative Advisory Committee** (NQIAC) that advises the President, the Secretary of Energy and the NSTC Subcommittee on quantum information science[4469].

In April 2021, Congress made the **Quantum for Universal Advancement in Nationwide Technology Use and Modernization** (QUANTUM) for National Security Act of 2021 proposal, which didn't directly turn into law[4470]. Two Senators introduced two bills to "*better position the United States to be globally competitive in quantum information science*". It was focused on developing Department of Defense quantum networking and telecommunications use cases and workforce developments. This bill appeared as a direct response to China's massive investments in quantum telecommunications infrastructures. It was kind of a military grade version of the National Quantum Initiative Act launched in 2018 that had mostly a civilian face despite the significant DoE funding it did incorporate[4471].

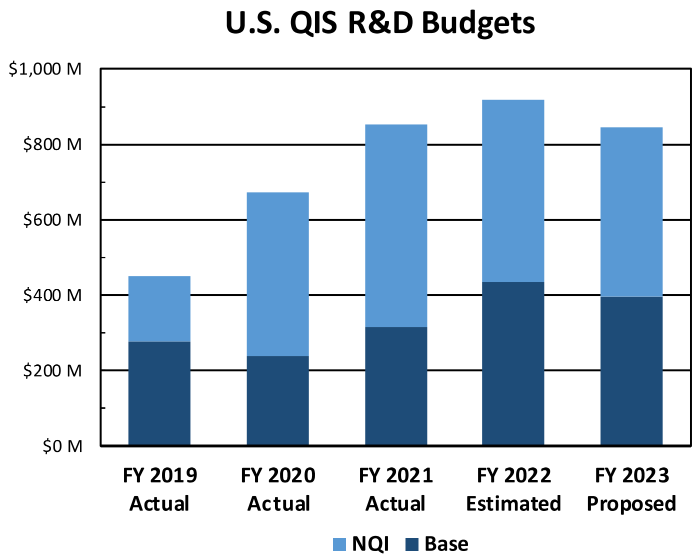
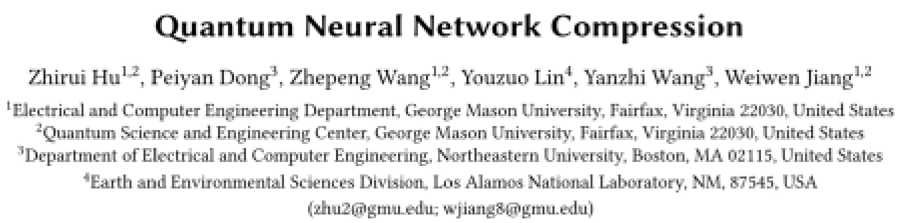


Figure 914: the most accurate Federal investment report on quantum technologies. Source: National Quantum Initiative supplement to the President's FY 2023 budget, January 2023 (47 pages).

In February 2022, the NQI released a report on the development of quantum technologies workforce in the USA[4472]. It includes exposing the public to educational content about quantum technologies and to ensure quantum technologies are as inclusive as possible.

The USA is concerned about managing its talent pool efficiently with, simultaneously caring about its national security. It is a concern fueled by a simple fact: a significant share of the quantum research talent pool in the USA are first generation immigrants. So, they need to attract more of them.

The paper shown in Figure 915 from the Georges Mason University in Virginia and the DoE Los Alamos lab has only Chinese names as authors. Are they Americans, first generation immigrants, or PhD students or post-docs who'll soon return to China?



Quantum Neural Network Compression

Zhirui Hu[1,2], Peiyan Dong[3], Zhepeng Wang[1,2], Youzuo Lin[4], Yanzhi Wang[3], Weiwen Jiang[1,2]
[1]Electrical and Computer Engineering Department, George Mason University, Fairfax, Virginia 22030, United States
[2]Quantum Science and Engineering Center, George Mason University, Fairfax, Virginia 22030, United States
[3]Department of Electrical and Computer Engineering, Northeastern University, Boston, MA 02115, United States
[4]Earth and Environmental Sciences Division, Los Alamos National Laboratory, NM, 87545, USA
(zhu2@gmu.edu; wjiang8@gmu.edu)

Figure 915: an example of a paper published in the USA with authors all having a Chinese name. See Quantum Neural Network Compression by Zhirui Hu et al, July 2022 (11 pages).

[4468] In The role of international talent in quantum information science by National Science and Technology Council of the White House, October 2021 (20 pages), the NSTC worries about the quantum talent shortage in the USA and advocates a balanced approach between hiring international talent and protecting national security. The global hunt for talent is launched!

[4469] See President Biden Announces Key Appointments to Boards and Commissions, White House, December 2022.

[4470] See Thune, Hassan Introduce Bills to Bolster the United States' Leadership in Quantum Information Science, April 2021.

[4471] And it's never enough! See for example America is Losing the Quantum Race with China by Theresa Payton, a former White House CIO, May 2022. Unfortunately, it contains only poorly documented anecdotal evidences and no significant hard data to prove that China is indeed ahead of the USA for any respect. Maybe the only real case is about QKD deployments but it's not even mentioned.

[4472] See Quantum Information Science and Technology workforce development national strategic plan, February 2022 (34 pages.

In May 2022, President Biden signed two Presidential directives[4473]. The first was an Executive order enhancing the governance of the NQI with the creation of an Advisory Committee. The second was a National Security Memorandum describing how the Federal government will prepare for the adoption of PQC cybersecurity to protect it from future quantum computing cryptology threats, in line with the first 4 NIST approved PQC standards announced in July 2022. The NIST is tasked with running a PQC migration project for the Federal government at the National Cybersecurity Center of Excellence and for the industry. The NSM also mandates new protections against IP theft and abuse. It led to the **Quantum Computing Cybersecurity Preparedness Act** that was signed into law in December 2022[4474].

In August 2022, the CHIPS Act was signed by POTUS with additional Federal funding of $152M per year for quantum technologies for the 2023-2027 period, going to NIST, the DoE and the NSF as usual. Consequently, it adds about $760M to the 5-year $4B run-rate of quantum federal research expenditures[4475].

In a Defense bill signed by Joe Biden in 2023, Congress pushed for the prototyping of useful quantum computing applications in the NISQ realm in less than 24 months, showing a sense of impatience[4476] [4477]! In a similar fashion, and a bit naively, an Iowa congressman is advocating for the inclusion of agriculture use cases in future funding appropriations for the NQIA, like for creating fertilizers[4478]. More realistic work comes from a US quantum technology roadmap proposal, the **Quantum Technology Demonstration Projects** (QTDPs)[4479].

Meanwhile, discussions started in 2023 to renew the NQIA with a second 5-year round plan[4480]. Interestingly, the NQIAC is advocating for removing a lot of red tape in the federal administration to accelerate projects funding[4481].

At the international scale, the USA has signed various bilateral partnership agreement framework on quantum technologies with the UK, Australia, Switzerland, Finland, Sweden, Denmark, France, Japan, India, South Korea and still counting.

In November 2021, the USA decided to restrict exportation of quantum technology goods to China related. The US Commerce Department added three entities in China: the Hefei National Laboratory for Physical Sciences at Microscale, QuantumCTek to the Entity List for acquiring and attempting to acquire US made items in support of military applications.

[4473] See Fact sheet: President Biden Announces Two Presidential Directives Advancing Quantum Technologies, White House, May 2022.

[4474] See H.R.7535 - Quantum Computing Cybersecurity Preparedness Act, 117th Congress (2021-2022), December 2022. Like many Congress laws, it is very succinct and leaves room for implementation to the Federal administrations.

[4475] See Quantum in the CHIPS and Science Act of 2022, QuantumGov, August 2022.

[4476] See Lawmakers aim to establish DOD pilot focused on near-term quantum computing applications by John Harper, DefenseScoop, June 2023. "*The NDAA and Appropriations bills explicitly call for U.S. policies that include all viable quantum computing systems, including quantum annealing, gate-model, and quantum-hybrid technologies (quantum plus classical computing applications). The legislation also includes an aggressive timeframe of 24-months or less for developing and deploying demos, proofs of concepts, and pilots*".

[4477] See H. R. 2670 National Defense Authorization Act (NDAA), 118th Congress, June 2023 (1,236 pages) in Section 222. Pilot program on near-term quantum computing applications, page 72.

[4478] See Quantum Fields: Iowa Congressman Pushes For Quantum Tech in Agriculture by Matt Swayne, The Quantum Insider, December 2022. All the related use cases are very long term to say the least.

[4479] See Accelerating Progress Towards Practical Quantum Advantage: The Quantum Technology Demonstration Project Roadmap by Paul Alsing, Johannes Pollanen, Mikhail Lukin, Christopher Monroe, Jun Ye et al, October 2022-March 2023 (31 pages).

[4480] See Renewing the National Quantum Initiative: Recommendations for Sustaining American Leadership in Quantum Information Science, National Quantum Initiative Advisory Committee, June 2023 (19 pages).

[4481] See National Quantum Initiative Advisory Committee, March 2023 (52 pages).

The goal is to prevent China from developing counter-stealth technology like advanced radars and counter-submarine sensors. It also blocks US technology for breaking encryption (quantum computing) or develop unbreakable encryption (PQC)[4482]. Indirectly, this will impact exports from other countries related to the US, mainly from NATO. There are few dissent voices on this exacerbated fight, that slows down international collaboration for the democratization of quantum technologies[4483]. In 2023, the Biden administration enacted an executive order to forbid American investments in China in sensitive sectors, including quantum technologies[4484].

Military and intelligence federal agencies

Public laboratories investing in quantum computing cut across much of the federal military-industrial complex with internal research or external research funded through calls for proposals or joint laboratories with universities:

DARPA funds several programs in quantum technologies. It started with long-distance quantum communications, quantum metrology applied to imaging, and neurological trauma diagnosis and PTSD. Funding goes to projects led by universities, startups and established companies[4485]. In 2020, they launched a NISQ computation challenge which led to the selection of 7 research teams as part of the ONISQ program[4486] and QAFS, a program on quantum annealing involving among others the Lincoln Lab from the MIT. In 2020, DARPA awarded ColdQuanta with a $7.5M project to build a neutral atom based quantum computer. In 2023, it selected Microsoft, Atoms Computing and PsiQuantum for the Underexplored Systems for Utility-Scale Quantum Computing (US2QC) program which focused on underexplored approached to quantum computing which could enable utility-scale operations[4487] [4488]. It also seeks to get $75M funding from Congress for 2024 to work on quantum sensing[4489].

Air Force Research Laboratory (AFRL) is a key player at the DoD, both in quantum computing with its Quantum Information and Sciences Laboratory and quantum communications with its Quantum Communications lab. The AFRL announced in December 2020 that it planned to work with the Office of Naval Research to test quantum technologies with the “Five Eyes” countries (USA, Canada, Australia, New Zealand and UK) for a Naval exercise. The Quantum Information and Sciences Laboratory does applied research in superconducting qubits, photonic qubits, trapped ions qubits, quantum algorithms and quantum sensing. They even deployed their own superconducting qubits system prototype, created with the MIT.

AFRL awarded PsiQuantum with a $22.5M contract to build a photonic quantum computer in October 2022, then, in 2023, another contract of the like, of $1.25M to Atlantic Quantum, a 5-year fab contract with Rigetti and then a $25.5M contract with IonQ in September 2023.

[4482] See Addition of Entities and Revision of Entries on the Entity List; and Addition of Entity to the Military End-User (MEU) List, November 2021.

[4483] See Democratization of quantum technologies by Zeki C Seskir, Steven Umbrello, Christopher Coenen and Pieter E Vermaas, IOPScience, February 2023 (20 pages).

[4484] See Executive Order on Addressing United States Investments in Certain National Security Technologies and Products in Countries of Concern, The White House, August 2023.

[4485] See The DARPA Model for Transformative Technologies, 2019 (511 pages) which tells the story of how the agency works. It has about one hundred program managers in total with a total budget of about $3.5B. It explains how it connects fundamental research to difficult technology challenges.

[4486] See DARPA Challenge May Boost Quantum Value of NISQ Devices by Matt Swayne, June 2020. One of the selected teams includes a certain Davide Venturelli who studied at the University of Grenoble.

[4487] See DARPA Collaborates with Commercial Partners to Accelerate Quantum Computing, DARPA, January 2023.

[4488] See Underexplored Systems for Utility-Scale Quantum Computing (US2QC), Joe Altepeter, DARPA.

[4489] See Pentagon seeks $75M for new program to accelerate quantum tech transition by Brandi Vincent, DefensesScoop, April 2023.

In April 2023, AFRL also announced a 5-year $500M funding in quantum information science covering quantum computing and quantum interconnect. It then seems that AFRL became the quantum computing "funding by procurement" arm of the DoD, a bit like DLR in Germany.

Army Research Office also has its own quantum research program covering the entire spectrum from sensing to quantum computing, cryptography and quantum communications.

Office of Naval Research (ONR) is working on the use cases of QKDs for the Navy and on using of quantum algorithms related to the Navy operational needs.

IARPA (Intelligence Advanced Research Projects Agency) funds third-party projects on quantum computing and quantum algorithms. They run several quantum programs that happen to involve academics outside the USA. **LogiQ**'s goal is to improve the quality of qubits and involves TU Delft (the Netherlands), the University of Innsbruck, Duke University and IBM. As part ot fhe **ELQ** (Entangled Logical Qubits) project that seeks to demonstrate high-fidelity entanglement between two error-corrected logical qubits in a fault tolerant manner, SuperMOOSE is led by ETH Zurich (Andreas Wallraff) with superconducting qubits. and **MODULARIS** led by the University of Innsbruck along with Cornelius Hempel from PSI and Jonathan Home from ETH Zurich is based on trapped ions. IARPA also funds programs conducted by third parties. It is a small agency that employs fewer than a hundred people.

NSA is investing heavily in quantum technologies, both in the race to implement Shor's algorithm for breaking RSA-based public-key protected communications and for protecting sensitive communications with quantum keys and cryptography. This work is obviously not public. The NSA subcontracts some of its research to private companies such as Lockheed-Martin. It is also part of a joint laboratory with NIST and the University of Maryland, QuICS, which was launched in 2014.

Federal civil agencies

In quantum science and technologies, the key Federal civil agencies are the Department of Energy, the NIST, the NSF and NASA.

Department of Energy (DoE) has many research laboratories that are big consumers of supercomputing capacities like in Oak Ridge and Argonne. It also operates the Los Alamos National Laboratory (LANL) and its Quantum Institute (QI) launched in 2002 that also invests in quantum computing and cryptography. In particular, they fund research at UNSW in Australia as well as in Maryland. The DoE also runs the Sandia National Laboratories, which also conducts applied research in all areas of quantum physics. The DoE launched a call for proposals to award 158 grants totaling $32M to 118 SMEs through the SBIR program. The grants are delivered in two phases, a first phase of $200K followed by a second phase of $1.1M for the best projects, spread over a period of two and a half years.

The DoE also announced in August 2020 the funding of five research centers in quantum technologies, all led by DoE laboratories, with $300M coming from the DoE and the rest from relevant institutions and the industry (IBM, Microsoft, Intel, Lockheed Martin)[4490]. These new research centers are **Q-NEXT** (Next Generation Quantum Science and Engineering Center, David Awschalom) led by Argonne National Laboratory which focuses on the industrialization of quantum hardware, **C²QA** (Co-design Center for Quantum Advantage, Steve Girvin) led by the DoE Brookhaven National Laboratory which will focus on ways to achieve quantum advantage in scientific applications, the **SQMS** (Superconducting Quantum Materials and Systems Center, Anna Grassellino) led by the Fermi National Accelerator Laboratory which will focus on superconducting qubits, the **QSA** (Quantum Systems Accelerator Center, Irfan Siddiqi) led by the Lawrence Berkeley National Laboratory which works on quantum computing hardware and software along with 250 experts from 14 other US and

[4490] See National Quantum Information Science Research Centers by Ceren Susut, December 2020 (17 slides). Unstable link.

Canada research institutions[4491] and the **QSC** (Quantum Science Center, David Dean) led by the Oak Ridge National Laboratory which will focus on quantum computing scalability issues.

The DoE then launched a $30M program in March 2021 on nanoscale matter and their use case in energy applications. It will fund the five existing DoE Nanoscale Science Research Centers and their research partners over 3 years. The awards size is between $1M and $2.5M[4492]. It also launched a $25M program in April 2021 on Quantum Internet including quantum repeaters, quantum memory and quantum communication protocols, opened to the 17 DoE labs.

NSF funds various research projects[4493]. In 2019, it launched a call for **Quantum Leap Challenge Institutes**, to fund research institutes conducting interdisciplinary research projects advancing the state of the art in quantum technologies[4494]. Their format is reminiscent of the UK quantum program hubs. Three hubs were selected in July 2020 for a total of $75M spread over five years: a first dedicated to quantum sensing led by the University of Colorado, a second dedicated to quantum computing led by the University of Illinois - Urbana-Champaign and a third also on quantum computing and rather software side led by the University of Berkeley[4495]. These three hubs bring together 16 academic institutions, 8 national laboratories and 22 industrial partners.

On top of that, the NSF is also funding the consolidation of other initiatives like the one around Purdue University in Indiana[4496]. The NSF also launched a **Quantum Algorithm Challenge** in March 2020[4497].

NIST is a federal research institute with the Department of Commerce. Its historical role is sensing and the definition of weights and measures. Its work on atomic clocks naturally led it to look after quantum technologies. It has an annual budget of $1.2B and employs 3,400 people in two campuses, one in Boulder, Colorado and another one in Maryland, next door to the University of Maryland and north of Washington DC. Several of its research groups are dedicated to quantum technologies with the Quantum Processing Group for quantum computing, another for spintronics, one for quantum sensing and another for superconducting electronics. On top of this, the Computer Security Division of the Information Technology Laboratory (ITL) manages the call for tenders on the standardization of PQC (Post-Quantum Cryptography)[4498]. NIST's PQC standardization strategy has wide implications. It will sediment the market around a dozen standards that will be royalty-free. This may favor large cybersecurity vendors instead of enabling new players to disrupt the market.

NIST is also a stakeholder in and a cofounder of three joint laboratories with major universities, each located near its own campuses in the states of Colorado and Maryland, the JQI, QuICS with the NSA and JILA with the University of Colorado.

The University of Maryland's **Joint Quantum Institute** (JQI)**,** established in 2006 is a fundamental quantum physics laboratory. It is the home of David Wineland, a long-time specialist in ion control

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[4491] See Impact report – Quantum System Accelerator, LBLN, Sandia National Laboratories, April 2023 (32 pages). The QSA was awarded a funding of $115M for 5 years in August 2020.

[4492] See DOE Announces $30 Million for Quantum Information Science to Tackle Emerging 21st Century Challenges, March 2021.

[4493] See for example NSF Awards $2M For Research on Quantum Machine Learning With Photonics, September 2019 for the University of Maryland.

[4494] See Quantum Leap Challenge Institutes (QLCI), NSF, 2019.

[4495] See NSF establishes 3 new institutes to address critical challenges in quantum information science, NSF, June 2021.

[4496] Purdue University launched in July 2021 a new Center for Quantum Technologies funded by the NSF with an established team of 50 quantum scientists and engineers coming from various research institutions in Indiana working on many quantum fields (atomic and molecular optics, solid state quantum systems, quantum nanophotonics, quantum information and communication).

[4497] See Dear Colleague Letter: Quantum Algorithm Challenge, Anne Kinney and Margaret Martonosi, NSF, March 2020.

[4498] See this overview of NIST's scientific activities: Quantum Information Science & NIST - Advancing QIS Technologies for Economic Impact, 2019 (39 slides).

by laser cooling, who won the Nobel prize in Physics in 2012 along with Serge Haroche. It is in this laboratory that the IonQ startup by Christopher Monroe was launched in 2015. Many alumni from this lab also joined Honeywell's quantum team in Denver, Colorado. This laboratory employs 35 permanent researchers, 55 post-docs and 85 PhD students with an annual budget of $6M supplemented by various external funding.

The **Joint Center for Quantum Information and Computer Science** (QuICS) at the University of Maryland (UMD) launched in 2014 in partnership with NSA's research directorate that focuses on quantum computing architectures, algorithms and complexity theories to complement the JQI.

The **JILA** at the University of Colorado at Boulder which is dedicated to sensing technologies[4499]. It is home to two Nobel Prize winners: Eric Cornell (in 2001, for his work on Bose-Einstein condensates) and John Hall (in 2005, for his work on laser frequency combs).

NIST employs a fourth Nobel Prize winner in physics, William D. Phillips for his work on atoms laser cooling using the Zeeman effect in 1997, shared with Claude Cohen-Tannoudji from France.

NASA created in 2013 the Quantum Artificial Intelligence Laboratory (QuAIL) jointly with Google, located at the Ames Research Center near the Google's headquarters in Mountain Views to explore the field of quantum algorithms, in particular on a D-Wave quantum annealer they acquired and installed there.

USA local quantum ecosystems

The main geographical quantum hubs in the USA combine a mix of national labs like those from the DoE, Universities and commercial companies. So here we are:

- **California** with the Silicon Valley and in the Los Angeles area, with Stanford, UCLA[4500], Caltech[4501], UCSB, plus the labs from Google, Microsoft and Amazon, and also Rigetti. The Los Angeles area seems on par with the Silicon Valley which is not at all the case in classical computing.
- **Massachusetts** with the MIT, Harvard, UMass Amherst, Atlantic Quantum and QuEra[4502].
- **Colorado** at Boulder, with Quantinuum, NIST and the University of Boulder Colorado[4503].
- **New Haven** with its very influential Yale University, particularly in the superconducting qubit domain, and Qci which is a spin-out startup from Yale. Yale and the University of Connecticut want to establish a quantum hub in the state with the help of NSF funding, starting with an initial grant of $1M.
- **Illinois/Chicago** with two DoE labs (Fermi and Argonne), several universities and the Chicago Quantum Exchange ecosystem which regroups these labs, the University of Chicago, the University of Illinois, the University of Wisconsin and Northwestern University[4504]. The University of Chicago Polsky Center and the Chicago Quantum Exchange launched the first national quantum

[4499] JILA was created in 1962 as the Joint Institute for Laboratory Astrophysics but they now use only the acronym without this meaning given its extended activities beyond astrophysics.

[4500] UCLA got a $5M grant from Boeing to support the Center for Quantum Science and Engineering, as announced in May 2022.

[4501] In January 2022 was announced the creation of the "Dr. Allen and Charlotte Ginsburg Center for Quantum Precision Measurement" thanks to a donation from the couple.

[4502] See Massachusetts Quantum Computing Ecosystem Study, The Innovation Institute at the MassTech Collaborative, November 2023 (51 pages).

[4503] The University of Boulder created the Qubit Quantum Initiative to foster interdisciplinary quantum research in a 4-floor building.

[4504] See Chicago Quantum Exchange welcomes new partners focused on manufacturing, computing, and the Chicago region, March 2022, and University of Chicago forges new bonds with European partners through Quantum and Sustainability conference in Paris, May 2022. It established a UChicago Center in Paris to foster collaboration between the Chicago and European quantum ecosystems.

startups accelerator program in April 2021. In September 2022, the NRF (National Research Foundation of South Korea) awarded $1M (over 5 years) to David Awschalom and Liang Jiang from the University of Chicago to create a joint lab, “The Center for Quantum Error Correction”. The University is also partnering with IBM and Google as part of a partnership announced in May 2023 with the University of Tokyo with the goal to create a superconducting QPU with 100,000 qubits. In May 2023, the University of Michigan announced an investment of $55M in the new Quantum Research Institute which will start with 8 new principal investigators.

- **New York State** with Princeton, the Flatiron Institute, IBM, GlobalFoundries and SEEQC.
- **Maryland** with the University of Maryland, IonQ, NIST, NSA, the Quantum Catalyzer quantum startups accelerator (Q-CAT) launched by Ronald Walsworth that creates quantum startups from scratch and the National Quantum Laboratory Qlab from the University of Maryland, established in partnership with IonQ.
- **Tennessee** and **New Mexico** host three DoE labs and their quantum research centers.
- **Washington State** with the University of Washington and the Pacific NorthWestern DoE lab, given Microsoft and Amazon HQ are there but their quantum teams mostly sit in part in Santa Barbara, in Southern California.

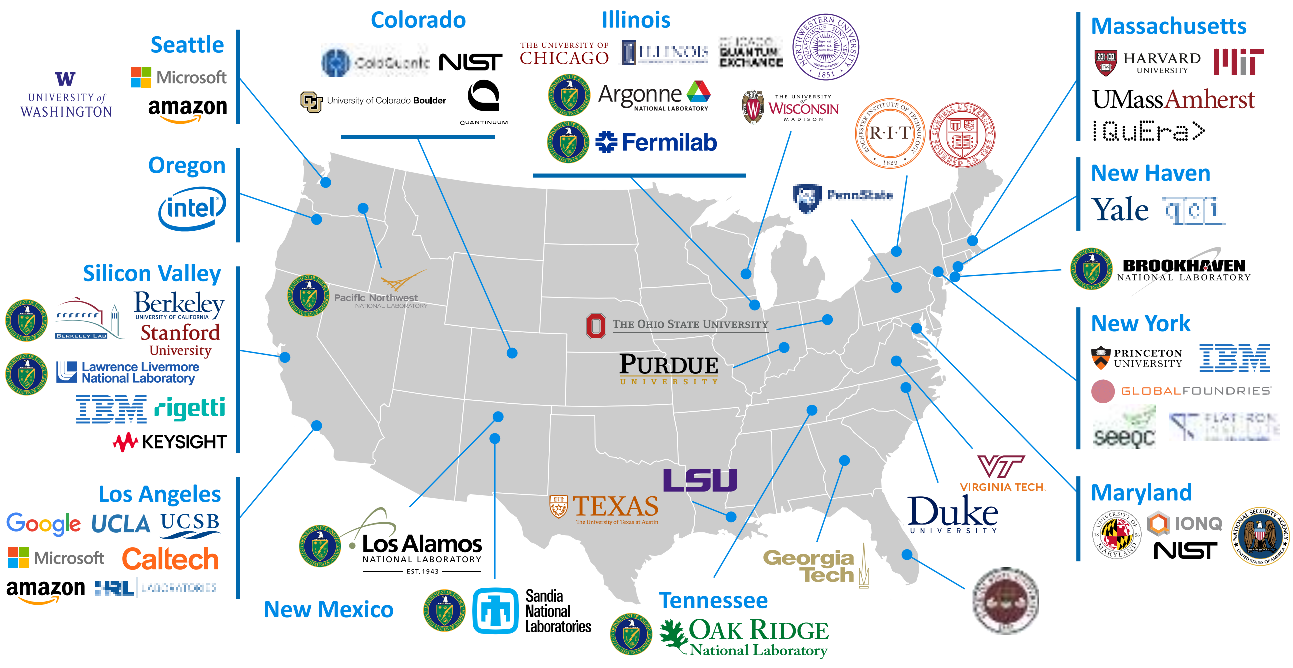


Figure 916: a map of the USA quantum ecosystem. (cc) Olivier Ezratty, 2022-2023.

- Lesser developed ecosystems in **Pennsylvania** (PennState, Pittsburgh University with its Western Pennsylvania Quantum Information Core – WPQIC, launched in May 2023[4505]), (**Indiana** (Purdue University, Midwest Quantum Collaboratory, a joint lab between Purdue University, Michigan State University and University of Michigan), **Virginia** (Virginia Tech), **Georgia** (GeorgiaTech) and **Florida** (Florida State University).

The above map in Figure 916 showcases this geographical distribution of USA’s quantum technology R&D areas. The distribution is more even than in classical digital technologies, which are more concentrated on the country’s West Coast and particularly in the Silicon Valley.

[4505] See Western Pa. is set to 'level up' its quantum capabilities with an $11.6 million investment from Pitt, May 2023.

Finally, let us recall a market reality that echoes economist **Maria Mazzucato**'s thesis on the public origin of technological innovations: the major American players are sourcing at different levels and throughout the USA and the world to advance their quantum technologies. Figure 917 is a good illustration of this phenomenon, showing how large IT players like IBM, Google, Intel, Microsoft, Amazon, Honeywell and even IonQ, surf on the work of publicly funded research labs not only in the USA but throughout the world. The last example being the creation of a Google AI lab in Australia in partnership with UNSW, the University of Sidney, Macquarie University and UTS, for the development of quantum applications.

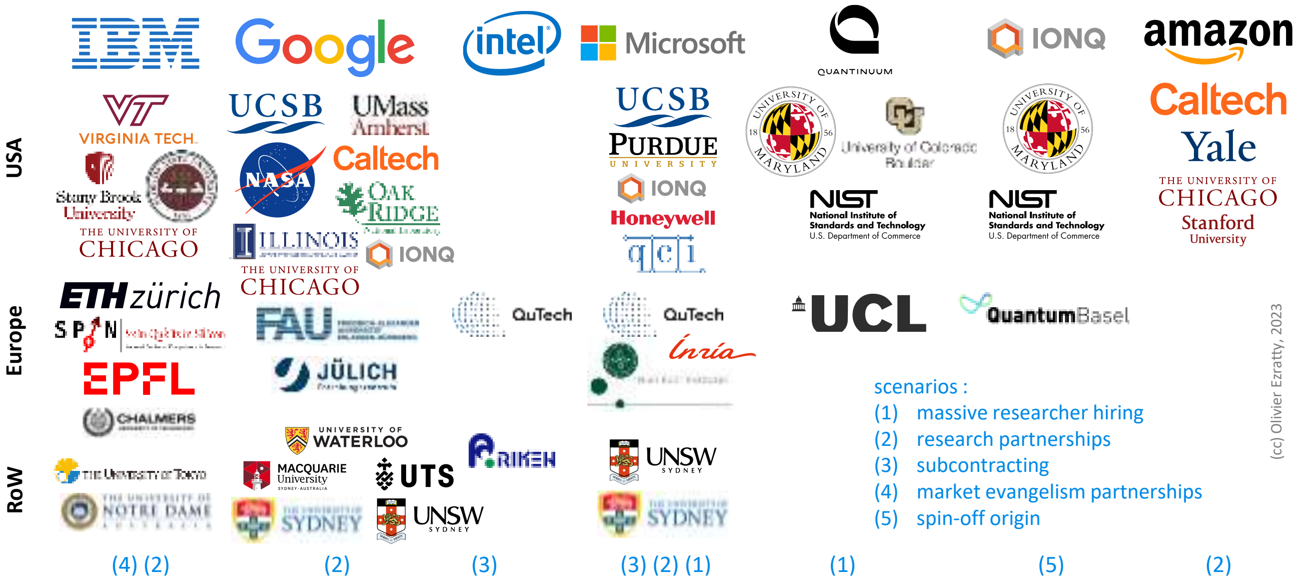


Figure 917: large IT vendors are reusing a lot of research and talent from universities both in the USA and in the rest of the world. Here's a map of who works with whom. (cc) Olivier Ezratty, 2021-2023.

Canada

In Canada, a parallel can be drawn between artificial intelligence and quantum technologies. In both cases, the country's influence is far greater than its economic weight, at the basic research level, with a healthy startup ecosystem and best-in-class investment per capita.

This is due to constant and early-stage investments in research by government and universities and to a certain entrepreneurial dynamism.

Canada has two great quantum stars in research with **Gilles Brassard of the** University of Montreal who is with **Charles Bennett of** IBM Research the co-inventor of QKD's BB84 protocol.

Research

Canada is distinguished by a strong investment in basic research in quantum computing, including more than $1B of public investment over a decade, mainly in three institutions[4506]. Let's look at the Canadian ecosystem from East to West (Figure 919).

[4506] See Quantum Canada by Ben Sussman, Paul Corkum, Alexandre Blais, David Cory and Andrea Damascelli, February 2019 (6 pages) for an overview on Canada's quantum investments.

In **Québec**, the quantum ecosystem is spread on at least four places, Montreal, Bromont, Sherbrooke and the city of Québec. Polytechnique Montréal is home to the quantum INTRIQ laboratory, specialized in quantum photonics. Bromont is a skills and infrastructure center in semiconductors assembly and testing with the C2MI (Miqro Innovation Collaborative Center), which partners with an IBM assembly line that is nextdoor. IBM installed one of its quantum computers there as part of a regional investment of CAN $68M and CAN $62M from IBM. The ibm-quebec 127 qubit systems was inaugurated in September 2023. PINQ^2 (Platform for Digital and Quantum Innovation of Quebec) will facilitate the access to the 127-qubit IBM QPU to researchers and other users. In Sherbrooke, the University is the home of 3IT (Institut interdisciplinaire d'innovation technologique) which runs a small superconducting component fab.

The University of Sherbrooke Quantum Institute**,** near Montreal hosts researchers like Alexandre Blais and startups. SBTech (metrology), Nord Quantique (GKP bosonic qubits based quantum computing) and Quantic (sensing) came out of there (shown in Figure 918). In February 2022, the Québec Region announced an impressive plan of public/private investment of CAN $435M (US $350M) in the Sherbrooke region. It includes CAN $131M to create "Sherbrooke Quantique" coming from the region and the rest from industry investors including IBM. As part of this program, the DistriQ Espace quantique 1 is to open in fall 2023 with 5,000 m^2 of facilities (offices, conference rooms, coworking spaces, a quantum studio built with Quantonation, assembly zones, hosting quantum computer dilution fridges, datacenters). It will host teams from 1Qbit, Multiverse and Pasqal. Pasqal has a subsidiary there which includes a QPU assembly line and a future cloud data center covering the North American market. The rest of the region's investments (about $62M) covers various research funding and real-estate investments, including the innovation platform PINQ^2. 1QBit, Pasqal and Eidos-Sherbrooke installment in Sherbrooke will represent an investment of CAN $205M over 5 years. Eidos-Sherbrooke is a video games studio that is planning to use quantum computing.

Figure 918: Institut Quantique in Sherbrooke, which hosts among other things the startup Nord Quantique. Photo: Olivier Ezratty, May 2023.

A region funding of CAN $3.6M out of a total of CAN $8.1M is allocated to setting up a collaborative platform on quantum software design with CMC Microsystems who also manufactures some electronic components (CMOS down to 22 nm density, III-V, Si Photonics, superconducting).

In **Ontario**, the quantum ecosystem is centered around Toronto and Waterloo. The University of Waterloo Institute for Quantum Computing, near Toronto obtained $120M in 2017 to fund its various quantum research institutes, complemented by $53M in Australian funding from UNSW, the operator Telstra and the Commonwealth Bank of Australia. The IQC does both research and teaching. They offer short courses of one to two weeks in the summer on quantum cryptography and quantum computing. The IQC is directed by Raymond Laflamme, one of the fathers of QEC. They cover all aspects of quantum technologies with about thirty teams of theorists and experimentalists, about fifty post-docs and 125 PhD students. A dozen startups have been created since 2002. The IQC is leading the Transformative Quantum Technologies (TQT), a seven-year research commercialization program funded by the Canadian government and its First Research Excellence Funds to the tune of $76M.

In January 2020, TQT launched the Quantum Alliance, an umbrella for IQC and TQT to link them to their Canadian and international ecosystem, including the fabric of quantum startups. The University of Waterloo also runs its Quantum-Nano Fabrication and Characterization Facility (QNFCF) with cleanrooms located in the Lazaridis Quantum-Nano Centre. Among others, it builds the superconducting qubit chips for Anyon Systems (Montréal).

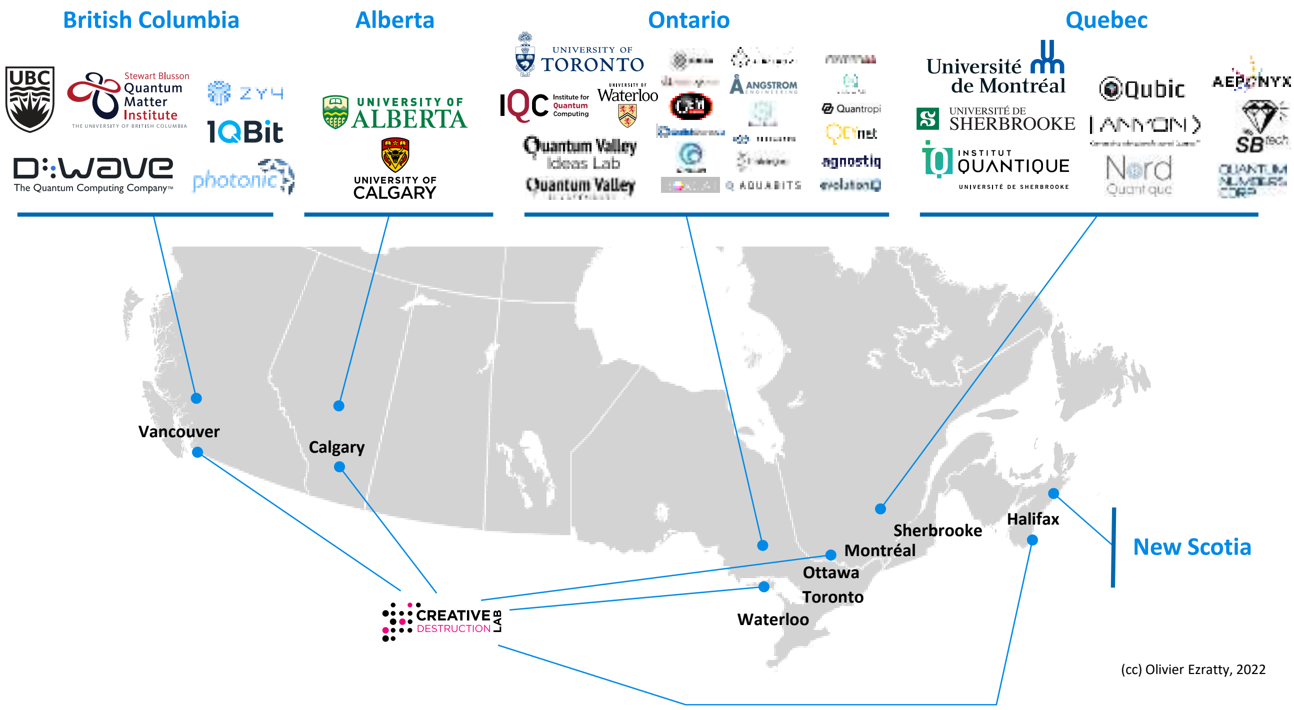


Figure 919: a map for Canada's quantum ecosystem from East to West where you see a startup concentration in Ontario. (cc) Olivier Ezratty, 2022.

The Waterloo quantum ecosystem also hosts the Research Accelerator Center of Quantum Technologies.

In **Alberta**, the University of Calgary is working on quantum communications and has deployed an experimental QKD network of a few tens of kilometers. The University of Alberta in Edmonton, north of Calgary, is also involved in this work[4507].

In 2020, the government of Alberta dedicated $11.8M to the creation of an international hub for quantum computing, $3M of which will fund quantum research.

In **British Columbia**, the region launched its own quantum initiative in December 2022 with CAN $14M funding over six years, focused on business and on the commercialization of quantum products.

[4507] See Quantum Communication Network Activities Across Canada by Barry Sanders and Daniel Oblak, June 2019 (10 slides).

The region ecosystem key research player is the UBC (University of British Columbia) with its Quantum Matter Institute (QMI), located primarily in Vancouver. Then, you have of course D-Wave and 1Qbit.

Government funding

As in most countries, the government is funding quantum research and the industry.

The Canadian government invested over CAN $1B in quantum research and education between 2009 and 2020. In March 2021, it announced a public funding of $40M for D-Wave[4508]. In April 2021, it launched its National Quantum Strategy with a CAN $360M (US $300M) plan spread over 7 years[4509]. All this was packaged with highly optimistic business forecasts. The National Research Council of Canada (NRC) estimated that quantum technologies would generate CAN $139B turnaround and create 209,000 employments in Canada by 2045[4510]. It is even larger than the most bullish worldwide forecasts! The forecast in 2022 was of 1,100 jobs creation by 2024. In January 2023, the Canada federal government announced, again, its National Quantum Strategy with more details on the CAN $360M funding allocation spread on research ($141M), education and talent development ($45M) and applied solutions development ($169M). They target both applied quantum computing and a quantum Internet of thing dimension (with CAN $50M funding)[4511]. It contains also a specific investment in defense quantum technologies (quantum-enhanced radar, quantum-enhanced light detection and ranging (LiDAR), quantum algorithms for defense and security and quantum networking)[4512].

In October 2022, the EU and Canada announced three joint quantum technology fundamental research project funded as part of the EU Quantum Flagship: MIRAQLS for image quantum sensing, FoQaCiA for quantum computing algorithms and HYPERSPACE for quantum communications with CAN $5M each[4513]. Various other international partnership exists like the CAFQA (Canada-France Quantum Alliance) with France led there by CNRS[4514].

Quantum industry

In the industry side, you can’t escape **D-Wave** and the quantum software specialist **1QBit**. With about 60 quantum startups and SMEs, they are the third largest ecosystem in the world in this respect after the USA and UK, mapped in Figure 920.

Private funding includes donations from Michael Lazaridis, one of the RIM BlackBerry co-founders, with $75M to the **Institute for Quantum Computing** at the University of Waterloo and $128M in 1999 to the **Perimeter Institute for Theoretical Physics** also located in Waterloo. Together with Doug Fregin, also co-founder of RIM, they also created the **Quantum Valley Investment Fund** with a total of $100M in funding and the **Quantum Valley Ideas Lab**.

Let us also note the existence of the **Creative Destruction Lab**, a deep techs startup acceleration structure with a specialty on quantum technologies. They are located in Canada (Toronto, Montreal, Vancouver, Calgary, Halifax), in the USA (Atlanta) as well as in Oxford and Paris.

[4508] See Government of Canada contribution strengthens Canada's position as a global leader in quantum computing, March 2021. This funding looks curious considering the company was created back in 1999. But it’s probably not yet break even and has a strong need for cash to maintain its activity and leadership in a yet unmatured market.

[4509] See National Quantum Strategy consultations, July 2022 (28 pages).

[4510] See Economic impact of quantum technologies, National research Council Canada, Government of Canada, 2019.

[4511] See Government of Canada launches National Quantum Strategy to create jobs and advance quantum technologies, January 2023.

[4512] See Overview of Quantum 2030, Government of Canada, March 2023 (22 pages).

[4513] See Joint EU/Canada quantum research projects, October 2022.

[4514] See Quantum Technology: A New France-Canada Network, CNRS, November 2022.

In 2020, of group of industry vendors created **Quantum Industry Canada** (QIC), an association promoting the Canadian quantum industry. It includes D-Wave, 1Qbit, Xanadu, Zapata Computing (now Zapata AI) and ISARA.

In April 2023, **Ericsson** announced that it would invest in quantum technologies R&D in Canada as part of a multi-domain investment of CAN $470M[4515]. Quantum R&D will probably represent a small share of this investment. Indeed, Ericsson already employs 3,100 people in Canada and invested CAN $7.6B in telecommunications R&D. This quantum investment will probably in the $1M-$2M range per year.

Figure 920: the Canadian industry ecosystem by product category. (cc) Olivier Ezratty, 2023.

South America

We add in this edition a first country for South America with Brazil.

Brazil

Brazil has many academic institutions doing research in various fields of quantum physics and technologies. It was consolidated in December 2022 in a national quantum initiative funded through EMBRAPII (a sort of governmental innovation agency). $11M will fund the creation of a Competence Center in Quantum Technologies driving the creation of startups.

The main academic groups in quantum are **ISP-SC** (University of Sao Paolo São Carlos Institute of Physics) with its Group of Quantum Information (GIQ) and Group of Theoretical Physics (FT-FCI), **UFABC** (Federal University of ABC) where the Quantum Information Science & Technology group works in spin qubits (NMR), photonic qubit and atomic (trapped ions) qubit labs, **UFSC** (Federal University of Santa Catarina) with its Southern Quantum Information Group and **Unicamp** (State University of Campinas) with it Quantum Information Theory group. The **Brazil Quantum** community launched in 2020.

[4515] See Ericsson establishes Quantum Research hub in Canada, Ericsson, April 2023.

In 2021, Atos (now Eviden) and **SENAI CIMATEC**, which hosts a supercomputer center, launched the creation of a Center of Excellence in Quantum Computing. It hosts an Atos QLM simulation appliance, nicked-named CIMATEC KUATOMU.

Europe

Just making things clear, we're dealing here with geographical Europe, including European Union member states, the UK and Switzerland!

United Kingdom

As with many continental European countries, the United Kingdom has contributed to many advances in quantum physics since the 18th century with precursors and founders, followed by a new generation of scientists in the second half of the 20th century.

Let's mention **Thomas Young** (1773-1829), **Ernest Rutherford** (1871-1937), **Joseph John Thomson** (1856-1940), **James Chadwick** (1891-1974), **Paul Dirac** (1902-1984), **Brian Josephson** (1940), **David Deutsch** (1953), **Andrew Steane** (1965) and even more recently the creators of the QML language, **Thorsten Altenkirch** and **Jonathan Grattage**.

Research

In the UK, the main quantum research laboratories are located in the Universities throughout the country. All of these have one or several quantum physics laboratories. Their main specialties are found later in the UK quantum plan rollout with a lot of advanced photonics (Bristol, Oxford), electron spin (UCL), telecommunication and cryptography (nearly all of them), sensing (same) and the likes. See the list of UK Universities in the UK map in a forthcoming page.

Government funding

At the instigation of the physicist Sir **Peter Knight** (1947), **UK** was the first large country to launch a quantum technology structured plan, the **UK National Quantum Technologies Programme**, announced in November 2013. It had an initial funding of £270M over five years[4516].

This represented a much larger amount of funding than for previous initiatives in innovative materials or robotics. The plan did not start from scratch. It was built on an existing ecosystem of university research laboratories in quantum physics.

The plan was and remains coordinated by the **EPSRC** (Engineering and Physical Sciences Research Council), a non-governmental organization funded and supervised by the government. The plan involves **Innovate UK** (basic research funding), the **Department for Business, Energy and Industrial Strategy**, the **NPL** (National Physical laboratory, where Peter Knight had been Chief Science Advisor, it is UK's metrology lab), the **CGHQ** (their NSA) and **dstl** (army research) (Figure 922).

Figure 921: the key public stakeholders of the UK quantum plan. (cc) Olivier Ezratty, 2021.

[4516] See The UK National Quantum Technologies Programme Current and Future Opportunities by Derek Gillespie, November 2014 (29 slides) and Delivering the National Strategy for Quantum Technologies (5 pages).

In a conventional way, the UK plan targeted all the usual quantum fields: computing, security, and sensing with a strong focus on medical imaging. Funding was based on thematic hubs bringing together universities and selected by call for projects (£124M), training, technology transfer and industrialization[4517] (Figure 922).

From the outset, the plan showed a strong commitment to creating business and attracting private capital. The original plan was to move research into startups as quickly as possible.

Four quantum hubs cover the major fields of quantum technologies and bring together teams spread over the territory in some thirty universities. All the hubs' managers are scientists, supplemented by a business development director and a board of 8 people including industry vendors CTOs.

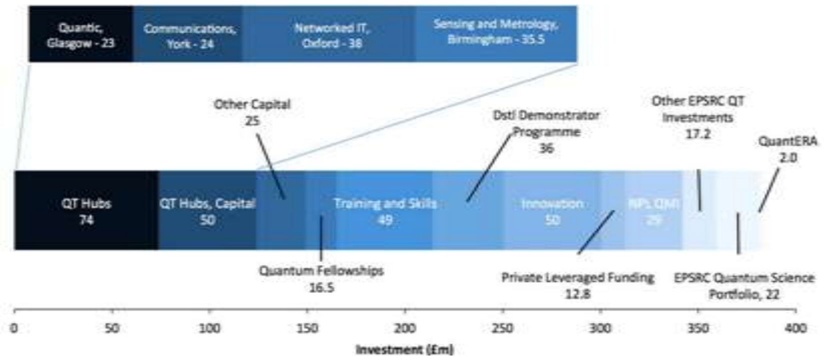


Figure 922: UK's investments in quantum technologies in the first phase of their plan from 2014 to 2019.

The **UK Quantum Technology Hub Sensors and Timing** covers sensing, including time measurement and involves the universities of Birmingham, Glasgow, Nottingham, Southampton, Strathclyde and Sussex.

The **Quantic** hub brings together the Universities of Glasgow, Bristol, Edinburgh, Heriot-Watt, Oxford and Strathclyde and focuses on quantum imaging. This gives us two hubs in the field of quantum sensing.

The **Quantum Computing & Simulation Hub** brings together 17 universities and is led by Oxford University. It took over from the NQIT (Networked Quantum Information Technologies) hub in 2019. It focuses on computing and security issues[4518]. They are working on creating a network of trapped ions quantum computers.

The **Quantum Communications Hub** consolidates a dozen universities: Bristol, Cambridge, Glasgow, Heriot Watt, Kent, Oxford, Queen's Belfast, Sheffield, Strathclyde under the leadership of York University, companies such as Airbus, Toshiba, ID Quantique, Kets, and public agencies.

[4517] Diagram source: UK national quantum technology programme by Peter Knight and Ian Walmsley, October 2019 (10 pages).

[4518] This includes the QuOpaL (Quantum Optimization and Machine Learning) initiative funded by Nokia and Lockheed Martin.

They are developing a quantum communication network between Bristol, Cambridge and Ipswich via the **UK National Dark Fibre Infrastructure Service** launched by the EPSRC (also linking Southampton and UCL in London)[4519] (Figure 923).

This did not prevent the state security agency from expressing skepticism about the suitability of QKD in a four-page white paper published in April 2020[4520].

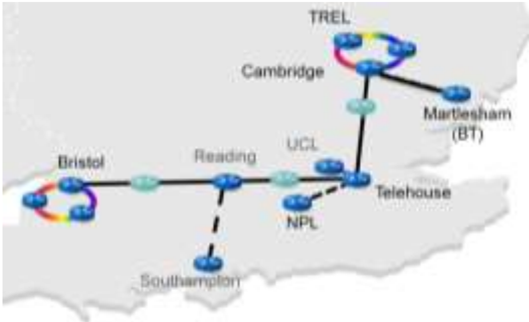


Figure 923: the UK National Dark Fibre Infrastructure Service.

These hubs are finally very multipolar, bringing together universities that are involved in several different hubs, according to the map in Figure 924[4521]. The United Kingdom has had a lot of ideas in managing this plan over the long-term.

In August 2022, as part of the NQTP, the EPSRC launched a new “Materials for Quantum Network” program led by Peter Haynes (Imperial College London) and Richard Curry (University of Manchester) to focus on quantum matter research. It seems that this topic will soon complement the usual computing/sensing/communications trio of all national quantum plans.

A progress report was published in 2015 by the EPSRC and Innovate UK followed by another interim report, the Quantum Age-Blackett review in 2016 investment launched in 2014 and extending the effort to the algorithmic and software part, in particular in liaison with the **Alan Turing Institute** and the **Heilbronn Institute for Mathematical Research** to propose case studies of computational problems to be solved[4522].

This was followed by a parliamentary report published in November 2018 which supported the continuation of the plan, the launch of a second phase of £350M over the period 2019-2024 and some fine tuning on the coordination between the different stakeholders (hubs, innovation centers, companies) [4523].

This led to the official announcement of Phase 2 in June 2019, following the recommendations of the House of Commons[4524]. With the expected private sector investments, the total of the two phases of the UK Quantum Plan was estimated at $1,227B. Phase 2 funding renewed funding for hubs (£94M over 5 years), industrialization projects (£153M from the Industrial Strategy Challenge Fund, over 6 years[4525]), training (£25M over 5 years[4526]).

4519 Diagram source: The Quantum Communications Hub, 2016 (11 slides).

4520 See Quantum Security Technologies, NCSC, March 2020 (4 pages).

4521 Map source: UK National Quantum Technologies Plan Strategic Intent, 2020 (38 pages). I added some of the large universities logos. See also UK national quantum technology programme by Peter Knight and Ian Walmsley, October 2019 (10 pages).

4522 See The Quantum Age Technological Opportunities, 2016 (64 pages) and A roadmap for quantum technologies in the UK, 2015 (28 pages).

4523 See Quantum technologies, House of Commons Science and Technology Committee, November 2018 (75 pages).

4524 See UK government invests $194M to commercialize quantum computing by Frederic Lardinois.

4525 The Industrial Strategy Challenge Fund (ISCF) was a multi-domains initiative of £2.6B backed by £3B of private investments, created to invest in challenges having a strong economical and societal impact. A dedicated Commercialising Quantum Technologies Challenge was then launched in two stages, first in 2018 with £20M and second with £153M completed by £205M from the private sector, in July 2019. To date, in 2021, over 40 such projects were funded.

4526 Doctoral training in quantum technologies is not managed in the hubs but in doctoral centers such as the Quantum Engineering Centre for Doctoral Training in Bristol.

It added the launch of the **NQCC** (National Quantum Computing Centre**)** for the development of quantum computing solutions, with £93M over 5 years[4527].

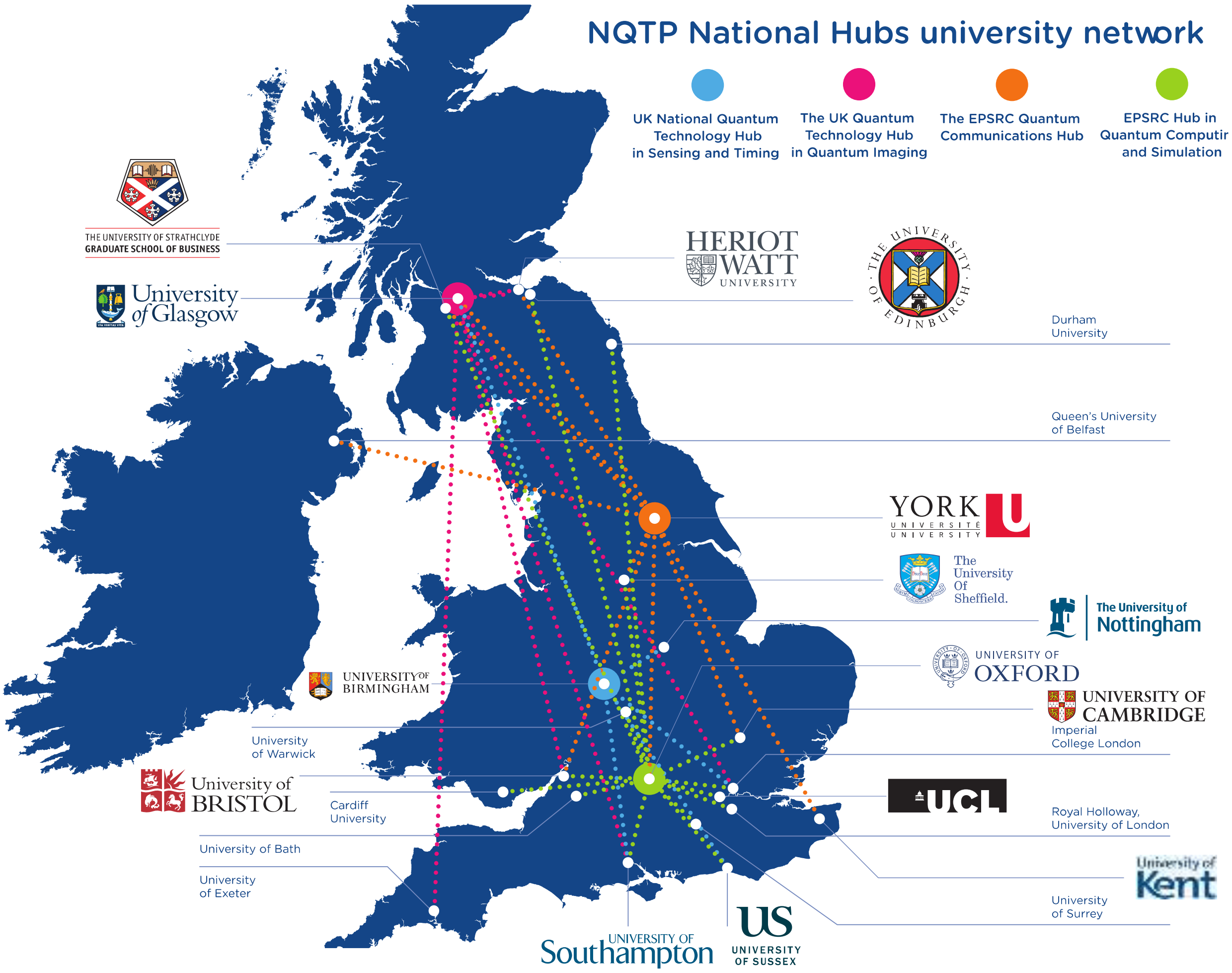


Figure 924: the UK universities map. Source: UKRI and logos added by Olivier Ezratty, 2021.

The first "UK" quantum computer was to be built by Rigetti (US). How can this be? It is linked to Rigetti having acquired a local startup, QxBranch and to its various connections with the local ecosystem and universities. But Oxford Quantum Circuits announced the launch of its cloud based superconducting qubits based computer in July 2021. All in all, the UK government has invested £100M per year in quantum technologies between 2014 and 2022.

The bulk of Phase 2 was the creation of the NQCC, which is led by **UKRI**, the **EPSRC** and the **STFC** (Science and Technologies Facilities Council), a government agency that conducts research in physics and astronomy and manages the country's major scientific instruments (particle accelerators, lasers, space engineering, etc.) [4528]. This center is tasked to create NISQ and then LSQ computing demonstrators, develop quantum algorithms and software and their uses, and build a community of users around them. The center became fully operational in 2022. It is to set up a NISQ machine that should be operational by 2025. In 2020, the preferred technologies were superconducting and ion-trapped qubits.

[4527] See [Establishing the National Quantum Computing Centre (NQCC)](), August 2019 (64 slides). The construction started in September 2021.

[4528] UKRI (UK Research and Innovation) is an autonomous non-governmental organization created in April 2018 with an annual budget of £7B and consolidates seven former research councils including the EPSRC and STFC, Innovate UK and Research England.

In May 2022, NQCC launched its SparQ Applications Discovery Programme and a collaboration with OQC. SparQ is a sort of directory aimed at UK-based companies and researchers who are looking for case studies of quantum computing[4529] (Figure 925). In November 2023, NQCC announced a partnership with IBM which will provide UK researchers with an access to its online QPUs.

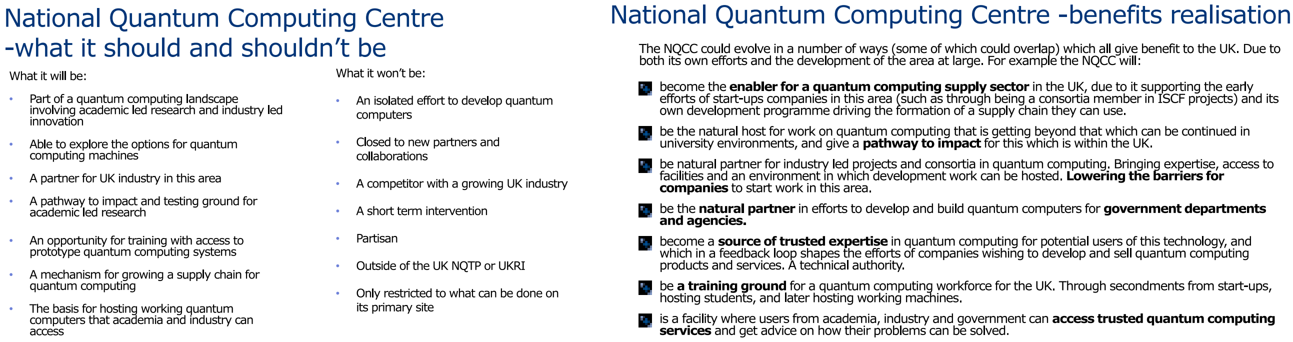


Figure 925: NQCC positioning. Source: NQCC. 2021.

The UK had recovered approximately 14% of the budgets for the first wave of European Quantum Flagship projects by October 2018. Despite the Brexit, the country will continue to benefit from it, as the collaboration with Europe on research survives the Brexit. For example, John Morton's UCL laboratory is part of the flagship project QLSI on silicon qubits driven by CEA-Leti and awarded in March 2020.

In November 2021, the UK signed a partnership agreement with the USA[4530], the first of a long list of USA bilateral partnerships later signed with Australia, all Nordic countries and Switzerland.

In March 2023, the UK government announced its Phase 3 wave of quantum investments with a new round of investment of £2.5B UK over the 2024-2033 period, doubling current public investments[4531]. The plan was motivated by the increase investments from countries like Germany and France, but also, the perceived threat coming from China. The plan KPIs (key performance indicators) are to generate an additional £1B of private investment, train 1,000 postgraduate research students, launch bilateral arrangements with 5 countries, increase the share of worldwide investments in startups from 12% to 15%, and grow quantum technologies market share from 9% to 15%, plus drive 75% large accounts adoption of quantum technologies, influence international standards developments, launch new research hubs, accelerators and establish the Office for Quantum in the Department for Science, Innovation and Technology (DSIT). This new investment is part of the UK Science and Technology Framework that focuses the UK on five technologies: artificial intelligence, quantum, telecommunications, semiconductors and biology[4532].

In May 2023 was launched the University of Edinburgh Quantum Software Lab, led by Elham Kashefi who was also named as the chief scientist of the National Quantum Computing Centre (NQCC) in November 2022[4533] [4534]. It strengthens UK's investments in quantum algorithms and software engineering[4535].

4529 See Industry engagement prepares UK for quantum transformation, PhysicsWorld, November 2021 and Early adopters position themselves for quantum advantage, June 2022.

4530 See Cooperation in Quantum Information Sciences and Technologies Joint Statement, November 2021.

4531 See National Quantum Strategy, March 2023 (61 pages).

4532 See Policy paper - UK Science and Technology Framework, March 2023 (19 pages).

4533 See New horizons beckon for UK quantum computing, PhysicsWorld, May 2023.

4534 See The National Quantum Computing Centre 2022 Annual Report, 2023 (23 pages).

4535 See Collaboration provides catalyst for quantum acceleration, PhysicsWorld, January 2023.

Quantum industry

On the entrepreneurial side, I have identified 71 quantum technologies startups and small businesses in the UK with a good balance by category (Figure 926). It is the second country in the world in terms of the number of startups, behind the USA. In October 2022 was created **UKQuantum**, a quantum industry association with Kets, Orca Computing, Arqit, Nu Quantum, Riverlane and Oxford Instruments among its funding members.

The intellectual property management company **IpGroup,** launched in August 2020 a £12M fund to fund startups, these being selected by the independent agency **UKRI**. Projects funding range from £125K to £2M. Let's also mention the **Quantum Technology Enterprise Centre** from the University of Bristol which was a sort of startups incubator and training program for quantum startup founders. The QTEC incubation program offered a 12-months salaried fellowship to quantum scientists during the build-up of their startup and business skills training.

Since 2016, QTEC helped the creation of 31 startups including KETS, QLM, Nu Quantum, Quantum Dice and Vector Photonics. The program funding ended in 2021 and QTEC is looking for funding to launch a new "cohort" of quantum entrepreneurs.

Figure 926: the UK startup scene is the most active in Europe (the old Europe, with them, before Brexit...). (cc) Olivier Ezratty, 2022.

Notable players are **Oxford Instruments** (cryogenics), **Oxford Quantum Circuits (**superconducting qubits[4536]), **Quantum Motion Technologies (**silicon qubits), **Cambridge Quantum Computing (**operating system, software, services, which merged with Honeywell Quantum Systems in 2021), **TundraSystems (**photonic qubits), **Orca Computing (**photon qubits) and **River Lane Research** (software). On the other hand, no major company in the country seems to be particularly invested in quantum computing, except perhaps in telecommunications.

[4536] Oxford Quantum Circuit obtained funding from Innovate UK in April 2020 in a consortium of four companies and two universities. See Oxford Quantum Circuits-led consortium wins Grant to Boost Quantum Technologies in the UK by Quantum Analyst, April 2020.

Germany

Germany is a land of dense basic research in quantum technologies. It builds on a strong history of the many German founders of quantum physics with **Max Planck**, **Albert Einstein**, **Werner Heisenberg** and many others[4537]. It also has a strong ecosystem of industry vendors, particularly in quantum enabling technologies.

Research

The main research organizations and laboratories involved in quantum technologies are shown in Figure 927 and described below:

Max Planck Institute for Quantum Optics (MPQ), based in Munich, is one of the 84 MPIs and their 24,000 employees. It was the home of Klaus von Klitzing who discovered the quantum Hall effect in 1980 and got the Nobel prize in physics in 1985.

They specialize in cold atom-based qubits in particular. This MPI is associated with the International Max Planck Research School also based in Munich. Two other MPIs are dedicated to information technology, but do not seem to be invested in quantum.

Munich Center for Quantum Science and Technology (MCQST) in Munich was launched in 2019 and brings together Munich's quantum research centers: the MPQ, the Walther-Meißner-Institute for Low Temperature Research (WMI) and the city's two leading scientific universities: Ludwig-Maximilians-Universität München and Technical University of Munich (TUM). It covers all quantum technologies (simulation, computing, communication, sensors). The whole with a budget of 31M€ over five years and about 55 permanent researchers. In February 2022, the Bavaria region announced an additional funding of 300M€ for the Munich quantum valley supplemented by 80M€ of federal funding. Among other things, the funding will help build the upcoming Center for Quantum Computing and Quantum Technologies (ZQQ), a center that will provide access to superconducting, ionic and atomic qubits quantum computers. Meanwhile, in January a D-Wave Advantage was installed at Julich in JUNIQ (Jülich UNified Infrastructure for Quantum computing) that was created in 2019.

Fraunhofer Institutes for Applied and Partnership Research with its 72 institutes and 26,600 people. They comprise several institutes specialized in quantum physics: the **IAF** (Institute for Applied Solid State Physics) in Freiburg, the **IOF** (Institute for Applied Optics and Precision Engineering) in Jena, the **ILT** (Institut for Laser Technology) in Aachen, **FOKUS** (Open Communication Systems) in Berlin, **SCAI** and **IAIS** (quantum machine learning) in Sankt Augustin and **ITWM** (quantum HPC) in Kaiserslautern. On top of that must be accounted cleanrooms from **IPM** in Freiburg and **IPMS** near Dresden. In March 2022 was launched EIN Quantum NRW, a competence network for photonics-based quantum technologies in the North Rhine-Westphalia (NRW) lander, involving the Fraunhofer Institutes **FHR** (High Frequency Physics and Radar) and **IAIS** (Intelligent Analysis & Information).

Helmholtz Association groups 18 Research Centers that conduct basic research in response to major societal challenges, with a total of 40,000 people. It includes the Quantum Laboratory of the **Jülich Forschungszentrum** (*aka* Jülich FZ or Jülich Research Center) located between Aachen and Cologne and headed by Kristel Michielsen[4538], where Tommaso Calarco, who coordinates the European Quantum Flagship, also works. He is associated with the University of Aachen in the **JARA Institute Quantum Information** (IQI). The Helmholtz Network also includes the **Institute of Photonics and Quantum Electronics at the** Karlsruhe Institute of Technology (KIT).

[4537] See The Innovation Potential of Second-generation Quantum Technologies by the National Academy of Science and Technology, July 2020 (96 pages) which contains a list of key people in German quantum research at the beginning.

[4538] Jülich Forschungszentrum started in 1956 in nuclear research. It also houses a number of supercomputers, such as the CEA's DAM at Bruyères-le-Châtel in France or the various US DoE research centers across the USA.

Leibniz Association with its community of 96 centers conducting basic research includes the Institute for Solid State and Materials Research (IFW) in Dresden, Germany, which focuses on superconductivity and magnetism, the Institute of Photonic Technology (IPHT) in Jena, Germany, the Max-Born-Institute for Nonlinear Optics and Short Pulse Spectroscopy (MBI) in Berlin, Germany, and the Paul Drude Institute for Solid State Electronics (PDI) in Berlin, Germany.

Institute for Complex Quantum Systems at the University of Ulm between Stuttgart and Munich.

PTB is the federal office of sensing, which is obviously investing on quantum sensing like the NIST.

BSI is the federal office for information technology security[4539].

At last, let's mention here the **Quantum Alliance** which regroups the German clusters of excellence and research centers working in quantum science and technology.

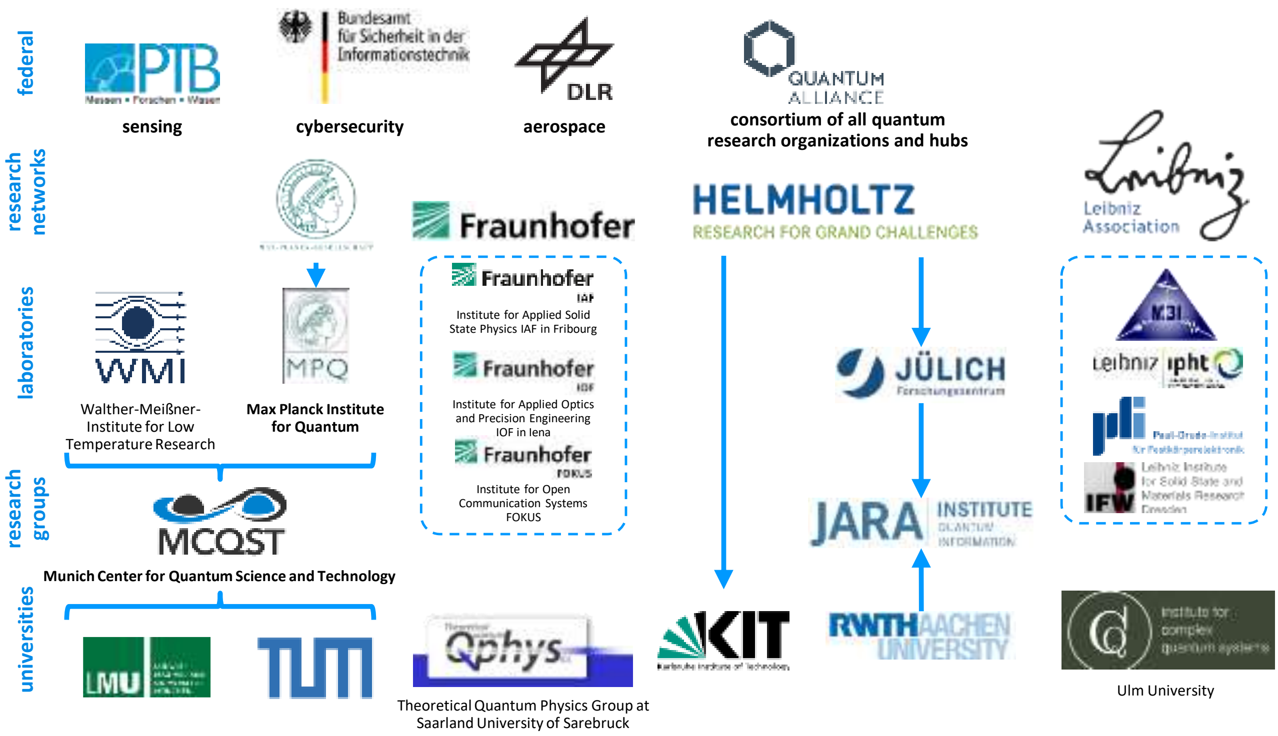


Figure 927: understanding the research ecosystem in Germany. (cc) Olivier Ezratty, 2022.

Government funding

In September 2018, the German Federal Research Ministry announced €650M in funding for quantum technologies over four years (2018 to 2022)[4540]. Like all such plans, it funds projects in quantum computing, quantum communication and quantum metrology. In September 2019, IBM announced that it would join this plan and install a quantum computer in Germany addressing researchers and cloud usages. It is not certain that this is the best approach to develop a German and European quantum industry, at least on the hardware side.

[4539] In Germany, the federal agency that protects information systems, which is the counterpart of the French ANSSI, published in May 2018 the report Entwicklungsstand Quantencomputer (*State of the art of quantum computing),* which provided an update on quantum computing, focusing in particular on cybersecurity issues (231 pages, in English). This was a very good overview of global quantum computing research. It provided a surprisingly accurate inventory of efforts in the field, particularly in US public research. But things have changed a bit since 2018.

[4540] See German Government Allocates €650M for quantum technologies, the German government's announcement (in German) and the plan itself (51 pages).

The computer was actually launched early in 2021 in the Stuttgart region in an IBM facility[4541].

In June 2020, the German government more than doubled its efforts by announcing a seemingly incremental €2B in funding for its quantum plan, including investment in two quantum computers[4542]. The $2B included the initial 650M€ of the 2018 plan. The German government put in place a scientific and industry experts board of 16 members to propose a roadmap and funding allocation with a Joint presidency of a scientist (Stefan Filipp from TU Munich) and an industry member (Peter Leibinger from Trumpf). It made some proposals in December 2020 including creating an independent coordination body, Deutschen Quantengemeinschaft (DQG). In January 2021, the BMWi disagreed with some of these proposals, estimating that too much funding was directed to fundamental research at the expense of startups.

In May 2021, the plan was split in two parts with 1.1B€ managed by the Federal Ministry of Education and Research (BMBF) and 878M€ by the Federal Ministry of Economic Affairs and Energy (BMWi), focused on applications developments.

One key showcased goal for this plan is to build two national quantum computers with 24, then 100 and later 500 functional (physical) qubits. DLR (Germany's Aerospace Center) was to receive the bulk of this funding (740M€) to work with small, mid-sized and large companies and create two related consortia. In 2022, DLR awarded QuiX (the Netherlands) a 14M€ contract to build a photonic quantum computer and Universal Quantum (UK) a 67M€ contract to build two trapped ions-based quantum computers to be installed in Hamburg. Their investment in trapped-ions caps 208M€ also involves EleQtron (Germany) and Universal Quantum (UK). Planqc also got a 29M€ contract for a cold atom QPU. DLR deals with startups outside Germany at the condition they install a significant part of their R&D and/or manufacturing in Germany.

In May 2023, the German federal government "revealed" a 3B€ plan to build a quantum computer by 2026 with from 100 to 500 physical qubits, the same goals as before. The funding is split between 2.2B€ including 1.37B€ to the research ministry and 800M€ for the National research institutes[4543]. It seems that this amount is a repackaging of the 2B€ plan announced in 2020 with some additional funding. It also covers quantum sensing and quantum cryptography/communications. But how about the missing 800M€? Is it including the Lander existing funding which is about 750M€?

The Federal government is indeed not the only public body funding quantum research. Landers added these 750M€ including 300M€ from the Bavarian region (probably with half of it coming from the Lander and the rest from the industry). How is that possible? German Landers (regions) have a very large budget independent from the Federal government (in a 42%/58% ratio). Bavaria being a large Lander, they have the means to invest significantly on research[4544]. For example, the lander cofounded two quantum research projects, NeQuS (quantum networks between quantum computers) and IQ-Sense (quantum sensing) for a total of 3.5M€ in August 2022.

Like with each country, the German quantum plan covers quantum computing, communications and sensing. The 2B€ effort is planning many projects, particularly in quantum computing research. Let's list some identified projects.

[4541] It lead to misleading information regarding a supposed investment of the German government in IBM of $717M corresponding to these €650M in German government to invest $717M in IBM quantum computing efforts, WRAL Techwire, September 2019. At best, only a small share of these 650M€ were dedicated to cofund the IBM initiative in Germany. Looking at what was done in 2022 in Canada, you can guess that the German government participation was around 40-60M€.

[4542] See Germany: 2 Billion euros for quantum technology, June 2020.

[4543] See Handlungskonzept Quantentechnologien der Bundesregierung, April 2023 (48 pages).

[4544] A false number on German public investment circulated with a hefty 4,456M€ from Interference Advisors. This report is amazingly entirely wrong, making double bookings of the 2018 and 2020 announcements, using unsafe press reference ($717M allocated to IBM's effort...) and double-accounting subsequent quantum projects which are already part of the 2B€ 2020 announcement.

GeQCoS (German Quantum Computer based on Superconducting Qubits) with 14.3M€ BMBF funding, involving Fraunhofer Fribourg and Infineon). It was launched in February 2021 and will seemingly use German originated quantum technologies. The ambition is modest with a goal of 50 qubits.

QSolid is another superconducting qubit computer that would be tied to a HPC at the Jülich Supercomputing Centre, including optimized firmware and software. It will start with a 10-qubit system. The project budget is 76.3M€ with 89.8% being funded by the BMBF over five years starting in January 2022. On top of Julich, it involves Fraunhofer IPMS and Fraunhofer IZM-ASSID, Karlsruhe Institute of Technology (KIT), Leibniz IPHT in Jena and the PTB.

DAQC is a project launched in February 2021 which got 12.4M€ from BMBF. It is coordinated by IQM Germany and involves Jülich, Infineon and ParityQC from Austria as well as the Leibniz Computing Center and Free University of Berlin. It will create a digital-analog superconducting qubits system using IQM's architecture. This project is related to the Quantum Flagship OpenSuperQ project.

MANIQU is a project running from 2021 to 2023 related to the usage of NISQ hardware to undertake quantum simulations to simulate new materials. The participants are HQS, Bosch, BASF, the Friedrich-Alexander University in Erlangen, and the Heinrich-Heine University in Düsseldorf.

Q-Exa is yet another superconducting qubits project announced in November 2021, involving IQM. Their systems will be integrated with HPCs. The project funding is of 45.3M€. It seems to be a follow-up project to DAQC.

PhotonQ is a photonic quantum computer project launched with a funding of 16M€ and led by the University of Stuttgart with participations from the Universities of Würzburg, Mainz and Ulm, TUM Munich, the Institute of Microelectronics Stuttgart and Vanguard Automation GmbH. The four-year project first goal is to create a demonstrator of 8 qubits using MBQC and deterministic photon sources, silicon photonics circuits and new single-photon detectors.

PhoQuant is another photonic quantum computer project, led by Q.ANT, a subsidiary of Trumpf, with the notable participation of Christine Silberhorn's Paderborn University lab and many others (Universities of Münster, Jena, Ulm, Humboldt Berlin, Fraunhofer IOF and IPM, HQS, Swabian Instruments, TEM Messtechnik, ficonTEC Service and MenloSystems).

ATIQ is a trapped ions computing project. The German government is funding 81.1% of the 44.5M€ project that will last from 2021 to 2026. Participants include Toptica (lasers). The project is coordinated by the Leibniz University Hannover with participations from Johannes Gutenberg University Mainz, University of Siegen, TU Braunschweig, RWTH Aachen, Physikalisch-Technische Bundesanstalt, Fraunhofer-Gesellschaft, AMO, AKKA Industry Consulting, Black Semiconductor, eleQtron, FiberBridge Photonics, Infineon, JoS QUANTUM, LPKF Laser & Electronics, ParityQC, QUARTIQ, Qubig, AQT, Boehringer Ingelheim, Covestro, DLR-SI, Volkswagen and QUDORA Technologies.

QUASAR is a semiconductor-based project using shuttling electrons with a QuBus, a quantum bus to transport electrons and their quantum information over distances of 10 µm. The partners are Infineon, HQS, Fraunhofer (IAF, IPMS), Leibnitz Association (IHP, IKZ) and the Universities of Regensburg and Konstanz. The project will run until 2025 to create 25 coupled qubits.

The resulting computer is to be deployed at JUNIQ. Jülich is also participating to the European Flagship QLSI project driven by CEA-Leti in France. QUASAR got a 7.5M€ funding from BMBF[4545].

QuaST is an enabling technologies project (Quantum-enabling Services and Tools for Industrial Applications) that will develop high-level libraries automatically decomposing and optimizes a solution into classical and quantum parts. The project is run by the Fraunhofer Institute for Cognitive Systems IKS with other Fraunhofer Institutes (AISEC, IIS, IISB), the Leibniz Supercomputing Center, and the

[4545] See Quanten-Shuttle zum Quantenprozessor "Made in Germany" gestartet, Jülich, February 2021.

TUM, plus DATEV, Infineon, IQM and ParityQC. The project sponsor is German Aerospace Center (DLR). The project will last 4 years and got a funding if 7.7M€.

QLindA is a quantum machine learning project led by Siemens with participations from Fraunhofer IIS, IQM and others.

We also have three enabling technologies projects: **QuMIC** (Qubits Control by Microwave Integrated Circuits, 6.3M€, 2021-2024), **qBriqs** (2M€, 2021-2024, compact cryogenic connectors, qubit readouts TWPA and HEMT amplifiers, filters and attenuators, DACs and ADCs and DC flux current generators) and **HIQuP** (2021-2024, 2,2M€, superconducting and cryogenic qubit control electronic circuits).

Germany also launched the creation of two QKD-based telecommunications networks, both funded by BMBF:

QuNET (165M€) which uses a standard QKD associated with terrestrial and satellite links. The project involves several Fraunhofer Institutes including the Heinrich Hertz Institute (HHI), the Max Planck Institute for the Physics of Light and the German Aerospace Center (DLR)[4546]. The project launched in November 2019 was scheduled to last seven years and aims to create a communications protection infrastructure for the German government. This should lead to the creation of a secure European network. The private sector is also involved with Deutsche Telekom, ADVA Optical Networking and Tesat-Spacecom. Test sites will be implemented in Bavaria, Saxony and Thuringia.

Q.Link.X (14.8M€) for the creation of a terrestrial network in optical fiber and QKD based on quantum repeaters, managed by the Fraunhofer HHI[4547].

Germany leads or participates to various European Flagship programs: **MetaboliQs** (NV center based medical imaging), **UNIQORN** (photon qubits chips), **S2QUIP** (hybrid photonic chips), **QRANGE** (QRNGs).

At last, the German national plan is funding three other initiatives associating research labs and industry vendors: **BrainQSens** (medical imaging with NV centers, 2.8M€), **Opticlock** (compact optical clock to synchronize communication networks, 6M€) and **QUBE** (space QKD with Cube-Sats, 3.12M€).

Quantum industry

On the private sector side shown in Figure 928 and in the map in Figure 929, Germany has a various set of quantum startups including **Avanetix** (hybrid algorithms), **InfiniQuant** (CV-QKD cryptography), **PicoQuant (**photon counters), **Kiutra** (magnetic cryogenics), **HQS Quantum Simulations (**algorithms), **JoS Quantum** (software in finance), **QuantiCor Security, QuBalt (**both in post-quantum cryptography) and **QuTools** (sensing).

Only a few quantum computing hardware startups have been created and recently like **eleQtron** (trapped ions), **ARQUE** (silicon spins), **XeedQ** and **SaxonQ** (NV centers), **Planqc** (cold atoms).

Many of the country's major industrial companies are also interested in quantum applications, particularly in chemistry (**BASF**), health (**Merck),** telecommunications (**Deutsche Telekom)**, components and automotive (**Bosch**, **Daimler**).

4546 See Germany's QuNET Receives €165 Million To Establish Quantum Communications Infrastructure, 2019, German ministry and research sector join forces to launch major quantum communications initiative, May 2019 and German Aerospace Center In QuNET Working On Satellite-Based Quantum Communication, November 2019.

4547 See Germany splashes further €15m in quantum networks R&D project, October 2018.

PlanQK (Platform and Ecosystem for Quantum-Assisted Artificial Intelligence) is a project to build a marketplace of quantum assisted artificial intelligence components, with first, quantum inspired algorithms. It gathers scientists from various universities (Stuttgart, Berlin, Munich) on top of Accenture, HQS, Deutsche Bundesbahn, Deutsche Telekom and many other corporations and mittelstands. It is supported by BMWi with a total funding of €19M. In 2023, it turned into the company **Anaqor**.

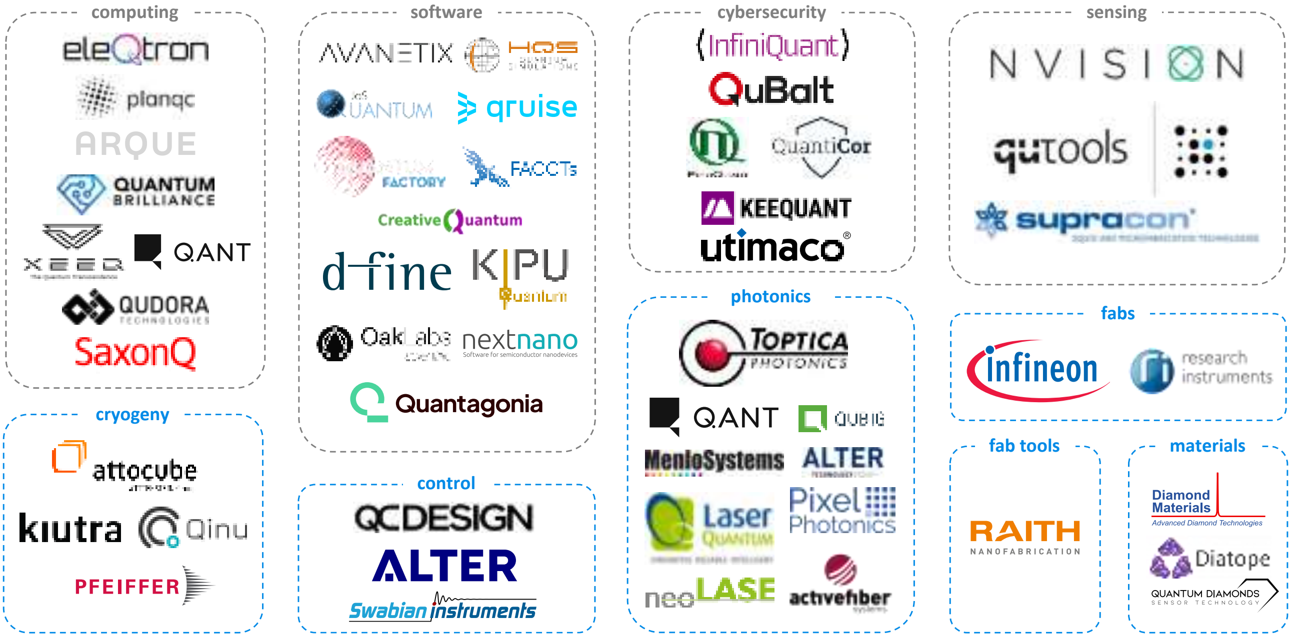


Figure 928: the German quantum industry. (cc) Olivier Ezratty, 2023.

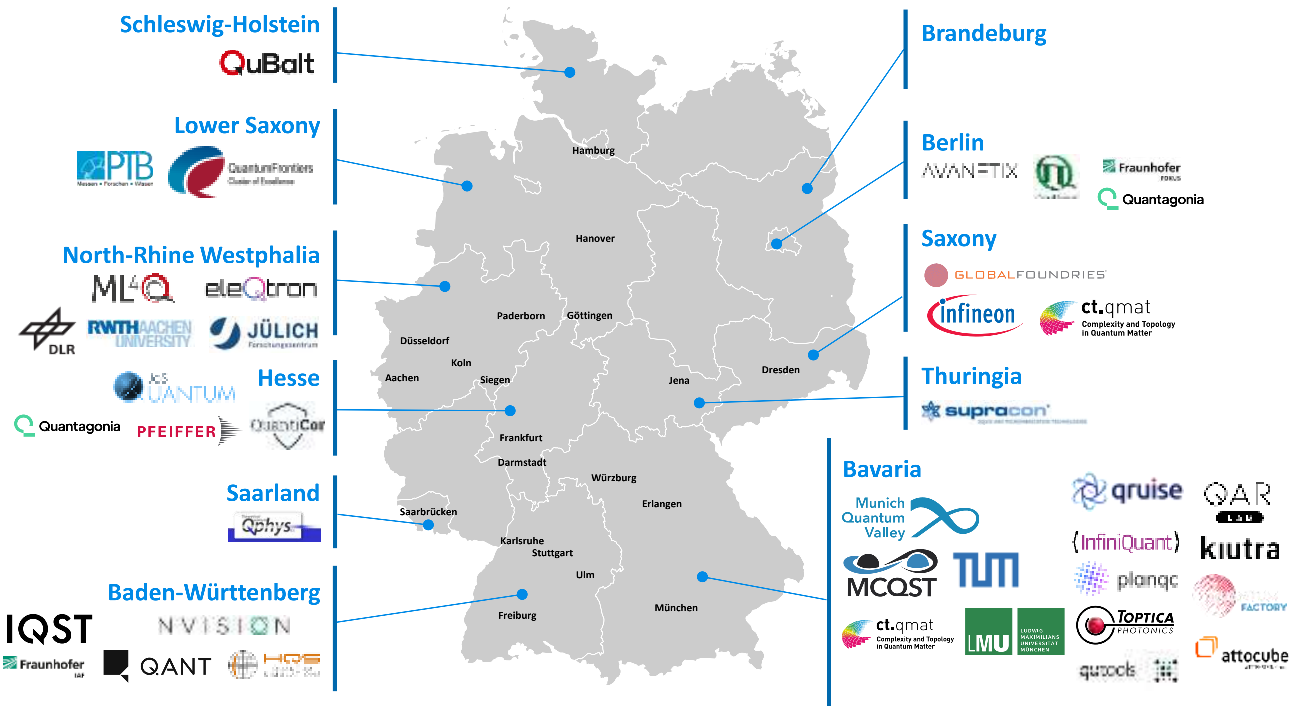


Figure 929: a map for Germany's quantum ecosystem. (cc) Olivier Ezratty, 2023.

In June 2021, ten German companies created **QUTAC** (Quantum Technology and Application Consortium) to develop quantum computing usable industrial applications in the technology, chemical and pharmaceutical, insurance and automotive industries.

The consortium was launched by BASF, BMW Group, Boehringer Ingelheim, Bosch, Infineon, Merck, Munich Re, SAP, Siemens, and Volkswagen. One of its goals is to create a cross-industry application portfolio.

There is another thing in Germany called **QuCUN** (Quantum Computing User Network) which also comprises SAP and BASF as partners. It is supported by the German Federal Ministry of Education and Research (BMBF).

The QuCUN platform launched in 2022 is "*designed to give potential quantum computing users – from SMEs to industrial giants – a central point of entry*". Does it mean that QUTAC is peripheral?

Let's also mention **PushQuantum**, a student initiative born in Munich that organizes lectures, workshops and entrepreneurship labs for wannabee quantum entrepreneurs.

Austria

Austria is one of the key European countries in quantum history and scientific development, starting with Ludwig Boltzmann, Paul Ehrenfest, Wolfgang Pauli and Erwin Schrodinger. Then, you can count on key scientists like Anton Zeilinger (2022 Nobel prize in physics), Reiner Blatt, Peter Zoller, Tracy Northup, Matthias Troyer (Microsoft) and Markus Aspelmeyer.

Austria's investment in quantum computing is concentrated in the **IQOQI**, the Institute for Quantenoptik und Quanteninformation in Innsbruck and Vienna. It focuses in particular on the design of trapped ions qubits. This led to the start-up **Alpine Quantum Technologies (AQT)**, founded by Rainer Blatt and Thomas Monz from IQOQI. It received €12.3M in public funding and competes with **IonQ** and **Quantinuum**. One other notable Austrian startup is **ParityQC** which develops ParityOS and a related architecture to codevelop quantum hardware and software platforms.

The Vienna Center for Quantum Science and Technology (**VQC)** is a partnership between the University of Vienna, Vienna University of Technology and the Austrian Academy of Sciences. It brings together a critical mass of about 20 quantum physics research laboratories.

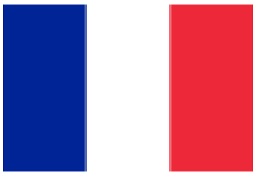
Austria is also invested in quantum cryptography and is associated with China, with whom it has conducted experiments in sending quantum keys via the Micius satellite to set up secure video communication. One key reason is that Jian-Wei Pan is a former PhD student of Anton Zeilinger.

IQOQI is also collaborating with the **Grenoble University Space Center** (CSUG) in the development of a CubeSat-type quantum key relay satellite, similar to the one in Singapore, in the **Nanobob** project[4548].

The Austrian government announced a formal quantum plan in June 2021 with 107M€ over 2022-2026 covering research and quantum technology developments. Quantum Austria is part of the Austrian 10 years Research Technology and Innovation strategy 2030 launched in 2020, in a country that spends overs 3.18% of its GDP in R&D. Part of the funding comes from the European Resilience and Recovery Facility (NextGenerationEU). The plan is managed with RFPs from the Austrian Research Promotion Agency (FFG) and the Austrian Science Fund (FWF).

[4548] See NanoBob - A Secure Quantum Communication CubeSat Concept for Quantum Key Distribution by CSUG, 2017 (13 slides).

France



France has a good breadth of research and industry activities in quantum technologies. Let's first mention its greatest scientists with **Henri Poincaré** (1854-1912), **Louis de Broglie** (1892-1987, Nobel prize in physics in 1929), **Alfred Kastler** (1902-1984, Nobel prize in physics in 1966), and **Claude Cohen-Tannoudji** (1934, Nobel prize in physics in 1997).

Serge Haroche (1944, Nobel prize in Physics in 2012) is a pioneer in cavity quantum electrodynamics and on the interaction between photons in a superconducting cavity and Rydberg atoms passing through the cavity). Of course, we must add **Alain Aspect** (1946, Nobel prize in physics in 2022), who invalidated Bell's inequalities in 1982 and verified the principle of nonlocality of entangled photons, a cornerstone of the second quantum revolution. Other domains worth mentioning are in quantum photonics with **Pascale Senellart**, neutral atoms physics with **Jean Dalibard** and **Antoine Browaeys**, silicon spin qubits with **Maud Vinet**, **Tristan Meunier** and **Silvano de Franceschi**, quantum cryptography and telecommunications with **Philippe Grangier**, **Frédéric Grosshans** and **Eleni Diamanti** and PQC with various cryptographers like **Damien Stehlé**.

Research

Public research is organized around three national research organizations: **CNRS**, **CEA** and **Inria**. The first is involved in fundamental research in physics, mathematics, and algorithms. The second also does fundamental research in physics, particularly on superconducting qubits, and applied research on electron spins qubits as well as on photonics. Finally, Inria is doing research in computer science, and for quantum technologies, on quantum error correction, cryptography, and quantum algorithms. Many laboratories are joint research units ("Unités Mixtes de Recherche" in French) between Universities, these national organizations and sometimes industry vendors like Thales.

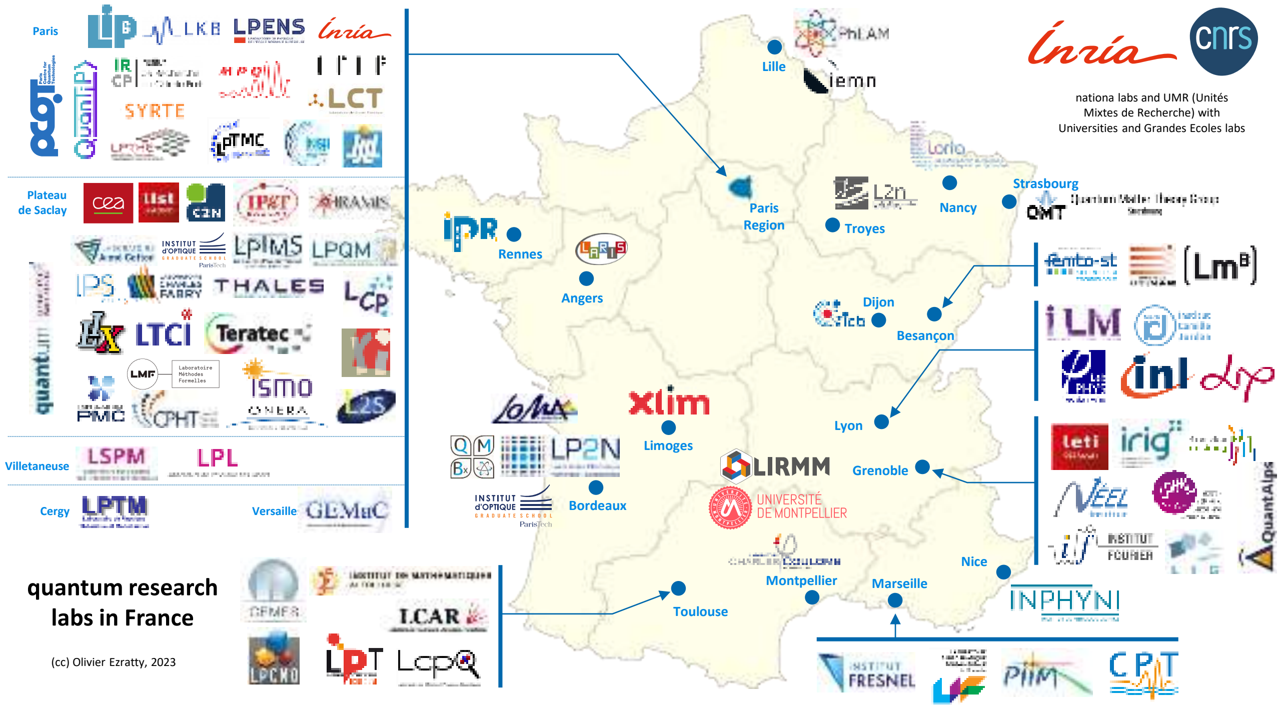


Figure 930: a map of France's quantum related research labs. (cc) Olivier Ezratty, 2021-2023.

These research laboratories are mainly located in Ile de France and in Grenoble, but other regional locations are active such as Toulouse, Montpellier, Marseille, Lyon, Bordeaux, Besançon and Lille as shown in Figure 930[4549]. Like many large countries, French laboratories are exploring many qubit tracks: superconducting, cold atoms, electron spins, photons and topological matter.

Public sector researchers get projects funding by answering various country and European RFPs[4550]. Of the more than 20 quantum startups in France 2021, 7 are from CNRS, two from Inria, two from ENS and one from CEA.

Paris region

The Paris Region (aka "Ile de France") is home to a good half of the country's research laboratories devoted to quantum technologies. Let's start with the laboratories that are located within Paris.

Inria's efforts in the Paris region are concentrated in the Quantic (Quantum Information Circuits) team of Pierre Rouchon, Mazyar Mirrahimi, Zaki Leghtas and Alain Sarlette, which is a joint venture between the CNRS, ENS and the Ecole des Mines de Paris.

They work on mathematical models of superconducting qubits, on quantum error correction (including cat-qubits), on proof of superiority of quantum algorithms and on cryptographic issues[4551]. The Cosmiq team led by Anne Canteaut, works on cryptographic algorithms, and David Pointcheval's Cascade team, works in cryptography and PQC. Inria also jointly runs many other teams with various labs from CNRS.

Other Inria teams are dedicated to quantum science and technology: IQA (LTCI, Saclay) is working on networking aspects in quantum computing, cryptography and photonics and quantum machine learning, QI with LIP6, MOCQUA with LORIA, CAPP (LIG, Grenoble) on contextuality and quantum combinatorial games, AlgoComp with IRIF, MC2 with LIP Lyon and PACAP with IRISA and Inria Rennes working on mapping quantum circuits to particular architectures and the new QUACS team on quantum algorithms in Saclay.

LIP6 (Laboratoire d'Informatique de la Sorbonne) hosts several recognized specialists in cryptography and quantum telecommunications (QKD): Eleni Diamanti was awarded a European Synergy Grand ERC for her work in the QUSCO (Quantum Superiority with Coherent State) project. Elham Kashefi is co-founder of the VeriQloud startup. She is also working on verified quantum computing, secure multiparty quantum computing, and features to achieve quantum advantages.

The **LPENS** (Laboratoire de Physique de l'Ecole Normale Supérieure) is the result of the merger in early 2019 of several physics research laboratories at ENS Paris, including the **LPA** (Laboratoire Pierre Aigrain), which specializes in nanotechnology and photonics.

They are working on numerous nanotechnologies used for the creation of qubits and the transport of quantum information: superconducting thin films, superconducting and microwave circuits for their

[4549] For this purpose, I consulted the websites of these laboratories and the fields of research they present, plus, when they were easy to find, the scientific publications of the researchers of these laboratories.

[4550] Some obtain ERC Grants (European Research Council): Synergy Grants for a few handfuls of teams (up to €14M over 6 years), and more often Starting (young researchers, up to €1.5M), Consolidators (experienced researchers, up to €2M) and Advanced (emeritus researchers, up to €2.5M spread over 5 years). Then European FET funding, funding via the European Quantum Flagship, or finally through various calls for projects at the national level (ANR).

[4551] This is specified in Inria strategic scientific plan 2018-2022, 2018 (93 pages), pages 47 and 48.

control, two-dimensional electron gases with very high mobility, semiconductor quantum boxes, qubits based on carbon nanotubes.

Taki Kontos and Audrey Cottet's teams are at the origin of the creation of carbon nanotubes used as electron traps potentially usable in electron spin qubits, which led to the creation of the **C12** startup, already mentioned. The lab is also a participant on the work on cat-qubits related to the startup Alice&Bob.

The **LKB** (Laboratoire Kastler Brossel) from ENS Paris focuses on quantum information and photonics, interactions between light and matter (Nicolas Treps and Valentina Parigi), quantum simulation and precision sensing with cold atoms (Christophe Salomon).

Thibault Jacqmin is working on microwave photon generation with NEMS (nano MEMS).

The IRIF (Institut de Recherche en Informatique Fondamentale) from CNRS and the University Paris Diderot is led by Frédéric Magniez who also teaches at Collège de France and hosts Iordanis Kerenidis, Sophie Laplante and two Inria teams. It works in quantum computing, cryptography and communications.

The **MPQ** laboratory (Materials and Quantum Physics) of the University Paris Diderot is particularly interested in trapped ions in the Quantum Physics and Devices (QUAD) and QITE (Quantum Information and Technologies) groups. But also, to the generation of entangled photon pairs (Sara Ducci).

The **LPTHE** (Laboratoire de Physique Théorique et Hautes Energies) of the University Paris Sorbonne works in condensed matter and statistical physics with applications in superconducting qubits.

The **INSP** (Institut des Nanosciences de Paris) of Paris-Sorbonne University is a generalist laboratory on nanosciences. They work in particular in different branches of photonics, on NV centers, on color centers qubits in silicon carbide, on spin and magnetism and on photonics components in III-V materials.

The **IRCP** (Institut de Recherche de Chimie Paris) associated with the Ecole Nationale Supérieure de Chimie ParisTech conducts research in innovative materials.

Philippe Goldner is working on the creation of qubits based on nanocrystals doped with rare earth ions such as europium or erbium, and is involved in the SQUARE project of the European Quantum Flagship, coordinated by the Karlsruhe Institute of Technology and also involving Thales. The laboratory is also involved in the European Quantum Flagship **ASTERIQS** project which is working on NV-based qubits in diamonds.

The **LPEM** (Laboratory of Physics and Study of Materials) of the ESPCI and the UMPC works in particular in superconductivity as well as on Majorana fermions.

The **LPTMC** (Laboratoire de Physique Théorique de la Matière Condensée) of the University Paris-Sorbonne has among other things several teams working on condensed matter physics like Jean-Noël Fuchs and Julien Vidal on topological insulators and Majorana fermions and Rémy Mosseri working on quantum information with topological qubits.

SYRTE (Laboratoire Systèmes de Référence Temps-Espace) located at Paris Observatory works in quantum sensing, in particular gravimetry, quantum gyroscopes and on time measurement with atomic and optical clocks. They are partnering with NIST. The quantum gravimeter and interferometry team is led by Franck Pereira dos Santos. SYRTE is led by Arnaud Landragin.

The **Laboratoire Jacques Louis Lions** (LJLL) is specialized in applied mathematics. It focuses on the analysis, modeling and high-performance scientific computation of phenomena represented by partial differential equations. Mario Sigalotti and Ugo Boscain, who specialize in the control of quantum systems and are also members of Inria, are among others.

The **Laboratory of Theoretical Chemistry** at Sorbonne University is directed by Jean-Philippe Piquemal (co-founder of Qubit Pharma) and is interested in computational chemistry, including quantum.

In September 2020, the **Quantum Innovation Center Sorbonne** (QICS) was inaugurated, a collaborative research structure associating LIP6, the LKB of the ENS and Inria.

The **Saclay plateau** has an even higher density of laboratories, located south-west of the Paris region. Most of these entities are consolidated in **Université Paris Saclay**.

At the **CEA**, Daniel Esteve's Quantronics team at the Iramis laboratory in Saclay has been working on superconducting qubits for nearly 20 years. Daniel Esteve's laboratory includes about fifteen people and is now managed by Hugues Pothier.

IphT (Institut de Physique Théorique de Saclay) associates CEA and CNRS. They work on the physics of condensed matter, including high-temperature superconductors, and on Majorana fermions. But their main focus seems to be mainly astrophysics.

The **LAC** (Laboratoire Aimé Cotton) is located at the ENS Saclay. It also works on cold atoms and interactions between atoms and light. In particular, they create qubits by combining an optically active erbium ion and a nuclear spin of yttrium.

The **C2N** (Centre des Nanosciences et des Nanotechnologies) of CNRS and Université Paris Saclay is a key quantum photonics laboratory. It is the home to Pascale Senellart and Jaqueline Bloch's labs. They work in particular on light-matter coupling in semiconductors. It also host quantum electronics teams (Frédéric Pierre).

The **LPS** (Laboratoire de Physique des Solides) works on magnetism, Josephson junction superconductors, thermodynamics, superconducting spintronics and quantum dynamics. They also develop codes for quantum and semi-classical dynamics and quantum control with applications in quantum information.

The **LPTMS** (Laboratoire de Physique Théorique et de Modèles Statistiques) has several strings to its bow in quantum physics without the link with quantum computing being immediately detectable.

The **LCP** (Laboratoire de Chimie Parisud) works on superconductors and on the dynamics and control of trapped ions controlled by laser pulses. They develop hybrid computational models of quantum chemistry (quantum+traditional) using MCTDH (Multi-configuration time-dependent Hartree) which allows to solve the Schrödinger equation for the simulation of interactions between atoms in molecules.

On the program: condensed matter physics, modeling of classical and quantum systems via statistical physics, quantum chaos, number theory and quantum chaos, theoretical aspects of quantum information; cold atoms, quantum integrable systems, quantum groups, etc.

TelecomParistech's **LTCI** (Laboratoire Traitement et Communication de l'Information) is an industry laboratory operating with partnerships with the private sector and via chairs. Its "Quantum Information and Applications" (QIA) team specializes in the theoretical and experimental aspects of quantum communications.

They develop hybrid CV-QKD-based quantum cryptography protocols compatible with telecom operators' fiber networks and QKD repeaters. They are contributor, founding member and reporter to the ETSI QKD-ISG on the QKD standardization processor. The team is led by Isabelle Zaquine and includes Romain Alléaume.

ISMO (Institut des Sciences Moléculaires d'Orsay) works on quantum dynamics, interactions between heavy particles and electrons at low temperature, light/matter coupling and on software for the simulation of quantum physics.

The **CPht** (Centre de Physique Théorique de Polytechnique) is specialized among other things in the physics of condensed matter. But not to the point of creating superconducting qubits! We find there Karyn Le Hur's group, who is specialized in condensed matter physics and Laurent Sanchez-Palencia's Quantum Matter Group.

The **Charles Fabry Laboratory** of the Institute of Optics Graduate school (IOGS) is specialized in lasers and quantum optics. It is home to Alain Aspect, Philippe Grangier as well as Antoine Browaeys, co-founder of the startup Pasqal and its laser-controlled cold atom qubits.

The **LIX** (Laboratoire d'Informatique de l'Ecole Polytechnique) is particularly active in post-quantum cryptography algorithms.

The **PMC** (Laboratoire de Physique de la Matière Condensée) is another laboratory of the Ecole Polytechnique. They work in particular on spin dynamics in semiconductors and magnetic thin films.

The **L2S** (Signals and Systems Laboratory) of CentraleSupelec is active in quantum systems research. In particular, the L2S is staffed by Zeno Toffano, who is focused on quantum states measurement.

The **LPQM** (Laboratory of Quantum and Molecular Photonics) associates the ENS Paris Saclay and the CentraleSupelec school. Their domains are coherence and quantum correlations.

The **LRI** (Laboratoire de Recherche en Informatique) located at CentraleSupélec is managed by Benoît Valiron, who teaches and conducts research in quantum computing, a field that is still relatively under-taught in engineering schools.

The **LMF** (Formal Methods Laboratory) was created in 2021 as a joint research center of University Paris-Saclay, CNRS, ENS Paris-Saclay, Inria, and CentraleSupelec with a focus on formal methods, combining 100 members from the former Laboratoire Spécification et Vérification (LSV) and the VALS team of Laboratoire de Recherche en Informatique (LRI). They target computational paradigms ranging from classical to emerging ones such as biological and quantum computing.

Thales **RT** (Thales Research and Technology) carries out R&D to create industrialized quantum sensing solutions. In particular, they have developed expertise in diamond NV centers.

Onera studies quantum optics at its Palaiseau site. It is in this capacity that it coordinates the ASTERIQS project of the European Quantum Flagship, "Advancing Science and Technology through diamond Quantum Sensing".

They also have teams of researchers in photonics, in III-V semiconductor materials (gallium, ...) with a prototype manufacturing unit located in their premises in Palaiseau, in metrology (gravimeter, atomic clock, accelerometer) and in QKD.

Let's move on to other parts of the Ile de France: Cergy-Pontoise, Villetaneuse and Versailles.

The **LPTM** (Laboratoire de Physique Théorique et Modélisation) of the University of Cergy-Pontoise is interested in cold atoms, in liaison with the Institut Francilien de Recherche sur les Atomes Froids (IFRAF). They also study graphene, electronic quantum transport, topological phases and entanglement.

The **LSPM** (Laboratoire des Sciences des Procédés et des Matériaux) of the University of Paris 13 in Villetaneuse is working on the manufacturing processes of NV centers, carbon nanotubes and graphene centers and associated applications.

The **LPL** (Laboratoire de Physique des Lasers) of the University Paris 13 in Villetaneuse works in photonics and cold atoms, their traps and on quantum metrology. It is the laboratory of Hélène Perrin, already mentioned, who is its Deputy Director.

The **GEMaC** (Groupe d'Etude de la Matière Condensée) of Versailles also works in the field of diamonds and graphene, on spin electronics and magnetism. It also works on QKD and photonic quantum memory.

QuanTiP (Quantum Technologies in Paris Region) is a community that groups research laboratories in the Ile de France region that are focused on quantum communications technologies. According to them, there are 650 quantum researchers in the Ile-de-France region in all (physics, algorithms, telecommunications, cryptography) spread over 100 teams in 30 research laboratories. It followed-on SIRTEQ in 2022, that was created in 2017.

Launched in 2014, the Paris Center for Quantum Computing (PCQC) which was later rebranded into Paris Center for Quantum Technologies (PCQT) brings together several dozen researchers from various laboratories in the Paris region.

We can also mention the initiative of the high-performance computing cluster **Teratec** (based in Bruyères-le-Châtel, near the CEA's Military Affairs Department) around quantum physics[4552].

It aims to develop quantum algorithms, hybrid development methods, use cases, and to inform, train and animate a community. They benefit from an Atos QLM simulator installed at the CRTT (Centre de Calcul, Recherche et Technologie) of the CEA in Bruyères-le-Châtel.

Grenoble

Grenoble's quantum ecosystem is dense, well-organized and very focused on the creation of qubits based on electron spins but also on superconductors, all with good skills in photonics. It is probably the place where coordination between research teams works best, particularly by integrating the key stages of industrialization.

Quantum research in Grenoble is led by different branches of the CEA (Leti in nanoelectronics and IRIG in fundamental physics), the CNRS with Institut Néel, LPMMC and two joint CNRS and CEA teams: NPSC (NanoPhysics and Semiconductors) focused on quantum sensing, quantum photonics, quantum thermodynamics and the quantum foundations, and Quanteca, created in 2019, which deals with all kinds of solid states qubits (electron spins, superconductors).

Institut Néel[4553], launched in 2007, is a CNRS laboratory specialized in condensed matter physics with a critical mass of researchers in quantum physics. Its researchers are exploring the possibilities of electron spin qubits (Tristan Meunier), superconducting qubits (Nicolas Roch), topological matter (Adolfo Grushin) and photonics. It also works cryogenics (Sébastien Triqueneaux) and quantum foundations (Cyril Branciard).

CEA-Leti (Electronics and Information Technology Laboratory) in Grenoble is the CEA's micro and nanoelectronics laboratory. It is notably at the origin of the SOI wafer technology that led to the creation of SOITEC. Leti is focused on CMOS electron spin qubit engineering. The project was coordinated by Maud Vinet until November 2022 when she created Quobly and Jean-Charles Barbé since then. It federates the efforts of several CEA, CNRS and UGA laboratories.

[4552] Teratec brings together several private and public HPC players including Atos, CEA, CERFACS (European Center for Advanced Research and Training in Scientific Computing), Dassault-Aviation, EDF, IFPEN, PCQC (Paris Centre for Quantum Computing), Total and the University of Reims.

[4553] The institute takes its name from Louis Néel (1904-2000, French), a physician of Lyon origin who was awarded the Nobel prize in physics in 1970 for his studies on magnetism and the discovery of antiferromagnetism. He is at the origin of the creation of the Polygone Scientifique de Grenoble, which brings together numerous research institutes and companies in the peninsula between the Isère and Drac rivers. The place hosted the first CEA site outside the Paris region in 1956, launched by Louis Néel. The CNRS established a foothold there in 1962, and in 1967 CEA-Leti was created. CEA-Leti is one of the world's largest civilian laboratories for applied research in nanoelectronics and nanotechnology. The Grenoble Science Park is also home to several international research organizations, the Institut Laue-Langevin, the European Synchrotron Radiation Facility and one of the branches of the European Molecular Biology Laboratory. In 2005 the CEA-Liten was created, a branch of the DRT specialized in new energies (photovoltaic solar, batteries, fuel cells, complete management of the carbon cycle, mixed energy management, innovative materials). In 2006, Minatec was launched, a nanotechnology commercial development center, later complemented by the Minalogic competitiveness cluster. In 2012, the Clinatec research center, founded by Alim-Louis Benabid, was launched, which is at the origin of the first complete exoskeleton for tetraplegics.

CEA IRIG (Grenoble Institute for Interdisciplinary Research) is the counterpart of Institut Néel in fundamental research at CEA. It includes the Laboratory PHotonique ELectronique et Ingénierie QuantiqueS (PHELIQS), which works on the physics of condensed matter including silicon spin qubits, topological qubits and unique photon sources.

LPMMC (Physics of Condensed Matter) of the University Grenoble Alpes is a CNRS UMR focused on the theoretical physics of condensed matter and quantum physics, N-body quantum interactions, superconductivity and superfluidity, and on the temporal evolution of quantum systems under the effect of magnetic and electric fields. It hosts researchers like Anna Minguzzi, Robert Whitney and Benoit Vermersch.

The **IJF** (Institut Joseph Fourier) of the University of Grenoble is working on quantum dynamics and in particular on issues of decoherence and thermal quantum noise.

The **LIG** (Laboratoire d'Informatique de Grenoble) is interested in quantum algorithms in general. One of its researchers is Mehdi Mhalla, who works on the quantum resolution of graph problems.

Research in quantum computing in Grenoble is currently structured around three initiatives: **QuantAlps**, **QuantECA** and **QuCube,** which are not on the same level.

QuantAlps (formerly, between 2017 and 2021, QuEnG for Quantum Engineering Grenoble) is the Grenoble ecosystem ranging from philosopher to industrialist, a trans-laboratory, trans-disciplinary and trans-sectoral umbrella initiative. Anna Minguzzi is the Director of QuantAlps.

The teams are working in physics on many other fields: in photonics, on superconducting qubits, electron spin qubits and qubits based on molecular magnets. Teams also make the link between quantum physics and philosophy. The initiative also includes training engineers in physics and quantum computing with various courses, including a project with Ensimag, Grenoble's leading computer science school.

Lyon

Research in Lyon is well balanced between the physical part and the mathematical and software part of quantum.

The **Physics Laboratory of ENS Lyon** studies condensed matter. The Quantum Circuit Group of Benjamin Huard is working on superconducting qubits and their error correction codes. He was notably joined by Audrey Bienfait in 2019, who works on electron spin resonance and its applications in quantum sensing. It was also there that Théau Peronnin finalized his thesis in 2020 while creating the startup Alice&Bob with Raphaël Lescanne.

The **INL** (Lyon Nanotechnology Institute) is located at Centrale Lyon (Ecully). They work on semiconductors and photonics. They have a technological platform for component prototyping, particularly in photonics.

The **iLM** (Institut Lumière Matière) of Lyon is specialized as its name says in photonics.

The **Camille Jordan Institute** in Lyon is a research laboratory in mathematics that works in particular on quantum probabilities. It is distributed on several sites: Villeurbanne, Saint-Étienne and on the Centrale Lyon campus in Écully.

The **LIP** (Laboratoire de l'Informatique du Parallélisme) of ENS Lyon associates CNRS, Inria and Claude Bernard Lyon 1 University. Its MC2 team works on theoretical computer science and complexity theory. It includes Omar Fawzi, CNRS 2019 bronze medalist and specialist in quantum information theory. He leads his work in the MC2 team at LIP.

Occitanie

Quantum research in Toulouse is very focused on fundamental physics and quite far from quantum computing with the exception of **LPTT**. There are also two laboratories in Montpellier, one of which is associated with IBM. Let's mention the **QuantUM Hub** initiative launched by IBM Montpellier, the University of Montpellier, and the Occitanic Region.

The **Institute for Quantum Technologies in Occitanie** was created in January 2021 to consolidate all the Occitan research and industry organizations, including the research labs below from Toulouse and Montpellier.

The **CEMES** (Centre d'Élaboration de Matériaux et d'Etudes Structurales) in Toulouse is specialized in physics and optronics. It is interested in light-matter coupling at scale and the creation of sensors oriented more towards connected objects than quantum applications.

LCAR (Laboratoire Collisions-Agrégats-Réactivité) of the Paul Sabatier University of Toulouse works on Rydberg atoms. It is in the team of Juliette Billy and David Guéry-Odelin.

The **LPCNO** (Laboratory of Physics and Chemistry of Nano-objects) of INSA Toulouse is specialized in photonics and quantum electronics. They study electron and nucleus spins, quasiparticles and quantum dots. They aim at applications in quantum computing. Their research is looking at applications in the health sector.

The **ITM** (Institut de Mathématiques de Toulouse) of the University of Toulouse studies statistical and quantum physics. It is home to Clément Pellegrini who studies quantum information theory and quantum state measurement.

The **LPTT** (Laboratoire de Physique Théorique de Toulouse) works on superconductors and SQUID Josephson effect loops. They are involved in the Quantware project which has been co-funded among others by the NSA!

The **LCPQ** (Laboratory of Quantum Chemistry and Physics) of the Paul Sabatier University of Toulouse develops generalist quantum chemistry codes, contributing to molecular simulation efforts.

The **L2C** (Charles Coulomb Laboratory) of the University of Montpellier is working on quantum metrology, spin dynamics and graphene, with applications in magnetic microscopy.

The University of Montpellier is an IBM partner in the setting up of a joint laboratory on quantum which actually aims to evangelize customers on the general principles and tools of the IBM Quantum platform.

The **LIRMM** (Montpellier Laboratory of Computer Science, Robotics and Microelectronics) focuses on the creation of quantum algorithms. It collaborates with IBM, Total and CERFACS.

Aida Todri-Sanial is one of their Research Director and works on quantum algorithms used for classical integrated circuits routing and on classical algorithms improving qubits gates mapping taking into account calibration data[4554].

Nouvelle Aquitaine

The Nouvelle Aquitaine ecosystem is specialized in sensing and enabling technologies like lasers, with its industry ecosystem comprising **Muquans**, **Azurlight Systems** and **ixBlue**. Since March 2021, this ecosystem is federated under the umbrella **Naquidis**, as part of the AlphaLRH cluster and with the support of the Region.

Besides the local branch of IOGS (Institut d'Optique Graduate School), here are two quantum research labs in the region.

The **LP2N** (Laboratoire Photonique, Numérique et Nanosciences) of the Institut d'Optique de Bordeaux does research in photonics and metrology based on cold atoms (microgravitometry). This is where the startup Muquans started.

QMBx (Quantum Matter Bordeaux) from the University of Bordeaux is a laboratory created in 2019 that gathers researchers in quantum science and quantum technologies and is specialized in quantum matter and particularly on nano-optomechanical systems, on quantum transduction, on topological matter and quantum simulators.

The **LOMA** (Laboratoire Ondes et Matières) from CNRS works on quantum matter and is investigating, among other things, nanomechanical qubits based on carbon nanotubes.

The **XLIM** (Limoges) does among other things photonics. They are notably partners with Thales TRT. They are working on applications in polariton metrology, in particular SPR (Surface Plasmon Resonance).

Sud

There are also a few quantum physics laboratories in Marseille, three of which are directly related to the needs of quantum computing. And one laboratory in Nice.

INPHYNI (Institut de Physique de Nice) of the Université Nice Côte d'Azur is interested in cold atoms, wave transport, interactions between light and atoms. It deploys a QKD test network between Nice in Sophia-Antipolis since 2019 in partnership with Orange. The quantum laboratory is directed by Sébastien Tanzilli.

The **CPT** of the Universities of Marseille and Toulon is working on quantum dynamics and wave diffusion in optical fibers and light guides. They are partners of various foreign universities: Aalborg University (Denmark), Pontificia Universidad Catolica de Chile, Karlsruhe Institute of Technology (Germany), Kyoto Institute of Technology and the Moscow Institute of Physics and Technology.

[4554] See A Hardware-Aware Heuristic for the Qubit Mapping Problem in the NISQ Era by Siyuan Niu, Aida Todri-Sanial et al, October 2020 (14 pages).

The **Fresnel Institute of** Marseille is involved in photonics, so inevitably, it can contribute to advances in photon-based qubit management and QKD-based quantum cryptography.

The **PIMM** (Physics of Ionic and Molecular Interactions) laboratory at the University of Marseille does research in plasmas, more related to the ITER nuclear fusion project than to quantum computing.

The **Laboratoire d'Informatique Fondamentale** de Marseille is particularly interested in quantum computing. Their Discrete Time Quantum Simulator project was launched in 2018. They are working on Quantum Walks and the Quantum Cellular Automata.

Burgundy Franche-Comté

Besançon is home to three quantum laboratories and Dijon to a fourth.

The **LmB** (Laboratoire de Mathématiques de Besançon) of the Université Bourgogne Franche-Comté studies quantum groups and probabilities.

The **UTINAM** Institute of the University of Besançon studies quantum decoherence, control, diagnosis, processing and transport of quantum information in the field of quantum sensing.

Femto-St is a research institute in Besançon focused on nanosciences, optics and optoelectronics. They work in particular on optical telecommunications, nonlinear optics, optics-based Ising machines and quantum imaging.

Icb (Interdisciplinary Carnot of Burgundy) of the University of Burgundy, based in Dijon, includes a team studying quantum and nonlinear dynamics (DQNL).

Great East

The region includes three quantum laboratories located in Strasbourg, Nancy and Troyes.

The **Quantum Matter Theory Group** from the University of Strasbourg is involved in condensed matter physics and also works on the interactions between light and matter, with Rydberg atoms. Run by Shannon Whitlock, the lab is developing cold atom-based quantum systems.

The **L2n** (Lumière Nanomatériaux Nanotechnologies) of the Technology University of Troyes is specialized in optoelectronics and photon sources and is run by Christophe Couteau. It has a fab of 700 m^2 that manufactures and characterize these components.

The **Loria** (Lorraine Laboratory for Research in Computer Science and its Applications) is based in Nancy. Two teacher-researchers are interested in quantum computing and algorithms: Simon Perdrix and Emmanuel Jeandel. The first is one of the main contributors of ZX-Calculus. Since 2021, Simon Perdrix is a PI at Inria Nancy.

Elsewhere in France

And finally, here are a few quantum physics laboratories located in other regions, in Rennes, Lille, Bordeaux and Limoges, but with no apparent direct link to quantum computing.

The **IPR** (Institut de Physique de Rennes) is attached to the University of Rennes. They are interested in quantum dynamics, the evolution of quantum states over time.

The **LARIS** (Laboratoire Angevin de Recherche en Ingénierie des Systèmes) based in Angers deals with various IT subjects. Within it, François Chapeau-Blondeau and Etienne Belin are interested in the impact of noise on quantum algorithms.

The **PhLAM** (Laboratoire de Physique des Lasers Atomes et Molécules) in Lille is interested in photonics and cold atoms.

The **IEMN** (Institute of Electronics, Microelectronics and Nanotechnology) is a laboratory located on four sites in Lille, Villeneuve d'Ascq and Valenciennes. They specialize in the design of quantum nanostructures.

International collaborations

International partnerships are very common in research. Many of the works of French researchers are carried out with researchers from other countries, including the USA, the UK, Austria, the Netherlands and Germany, Japan and Singapore (notably with joint international units of the CNRS IFLI and MajuLab).

CEA-Leti is a partner of **IMEC,** its counterpart in Belgium, based in Leuven, covering AI and quantum computing[4555]. Like CEA-Leti in Grenoble, they have a clean room for etching up to 28 nm on 30-cm wafers and another on 20-cm wafers for MEMS.

Since 2017, the **Grenoble University Space Center** has been collaborating with the Austrian **IQOQI** on sending quantum keys via satellite in the Nanobob project.

And there is another international collaboration on quantum involving France, the Netherlands (QuSoft) and Latvia.

Government funding

After Atos launched in 2015/2016 its venture in quantum computing emulation, the French government started to look at the opportunity to launch a quantum plan. Back then, it was involved in the European Quantum Flagship which was announced in October 2018.

Things really started with the creation of a parliamentary investigation commissioned by the Prime Minister in March/April 2019 and led by MP **Paula Forteza**, accompanied by **Iordanis Kerenidis** (CNRS researcher specialized in quantum machine learning) and **Jean-Paul Herteman** (former CEO of Safran). The parliamentary mission submitted its report on January 9, 2020, titled "Quantum: the technology disruption that France will not miss". The report made fifty proposals, 37 of which were made public. The government then created a national quantum strategy that included some but not all of the parliamentary mission's proposals. All this during the early stages of the covid pandemic. It was finally announced a bit late, in January 2021, but by President Emmanuel Macron, a premiere in the western world.

[4555] See Partners Double-Team AI & Quantum Computing by Mathew Dirjish, November 2018.

The ambitions of the strategy and its roadmap revolve around rather classical themes: NISQ quantum computing, Fault Tolerant Quantum Computing, algorithms and software, quantum telecommunications overall (including quantum cryptography and distributed quantum computing), quantum sensing, and at last, enabling technologies. This includes cryogenics, cabling, control electronics, vacuum control, lasers and photon sources.

The plan is spread over 5 years from 2021 to 2025 with 1B€ public funding and an additional 850M€ funding expected from European funds and the private sector (industry R&D and startups funding)[4556].

In 2021 and 2022, many components of the French quantum strategy were launched. There was an incremental 150M€ research program handled by CNRS, CEA and Inria announced in September 2021 and launched in 2022. Then, a hybrid classical/quantum platform announced in January 2022, which will be located at the TGCC supercomputing center handled by CEA. It will use the Joliot-Curie supercomputer in association with a QLM classical server from Atos for emulation and QPU drive, and a Pasqal quantum simulator as a starter before other QPUs are selected. This HQI (Hybrid HPC Quantum Infrastructure) project is partly funded by the EU HPC-QS program. An education program was launched with universities to expand training from license to PhD. Other programs were launched for FTQC-related enabling technologies, for quantum startups accelerations, for cryogeny and for the deployment of PQC cryptography systems.

Many of France's National Quantum Strategy's accomplishments and related projects are described in its Annual Report published in March 2023[4557].

In December 2022, France and the USA signed a joint statement to enhance cooperation on quantum technologies, like the USA are doing bilaterally with so many countries as you'll see afterwards.

Quantum industry

On the industry vendors scene mapped in Figure 931, France has a handful ventures in quantum computing hardware front with **Alice&Bob** (cat-qubits), **C12** (carbon nanotubes electron spins qubits), **Pasqal** (cold atoms qubits), **Quandela (**single photon sources and photons qubits), **Quobly** (silicon quantum dots spin qubits) and **Crystal Quantum Computing** (using trapped ions in Rydberg states).

In the software side, we have **Qubit Pharmaceuticals** (healthcare), **QuantFi** (finance), **MolKet** (chemistry), **ColibrITD** (multisector), **QbitSoft** (logistics and retail), **QPerfect** (emulation), **Aqemia** (biotech), **VeriQloud** (quantum telecommunications) plus a bunch of companies specialized in cryptography, mostly PQC with **CrytoNext**, **CryptoExperts**, **Ravel**, **QuRisk** and **Secure-IC**.

In quantum sensing, we can count on **Muquans** (microgravimeters, acquired by **iXblue** in 2021, becoming **Exail** in 2022), **Chipiron** (NV centers imaging), **Kwan-Tek** (NV centers imaging), **Mag4Health** (imaging) and **Thales** (NV centers, SQUIDs and cold atoms sensing, lightweight cryogeny).

In addition to **Bpifrance** and the investment fund **Quantonation**, the **Deep Tech Founders** trains entrepreneurs/researchers in deep techs. It is an international program created by the Hello Tomorrow team. All these organizations are behind the creation of a structure to support the quantum ecosystem in partnership, the **Lab Quantique**, launched officially in April 2020. The Lab Quantique is a think tank for the development of talent, particularly at the crossroads between science and entrepreneurship. From a practical point of view, le Lab Quantique organizes regular meetings that bring together mainly quantum technology entrepreneurs from France and abroad.

[4556] See How France Is Becoming a Quantum Computing Power by Peter Suciu, The National Interest, January 2022 and What Europe can learn from France when it comes to quantum computing by Andersen Cheng, Sifted, November 2021.

[4557] See France National Quantum Strategy Annual Report, March 2023 (51 pages).

These meetings took place in the form of videoconferences on Zoom during the covid-19 pandemic period in 2020 and 2021. Its objectives are to connect industry players, startups and researchers, to build bridges with the international community, to launch a program to accelerate quantum startups and to organize a major annual high-level conference bringing together all the stakeholders in the ecosystem, as well as an International Prize (attracting talent). It takes the form of a trade association mixing the quantum industry (large organizations, small businesses and startups) and its users (mainly, large companies like EDF, Airbus and the likes).

Figure 931: France's quantum industry ecosystem. (cc) Olivier Ezratty, 2021-2024.

One France specificity in Europe is its large corporations directly invested in quantum technologies and quantum enabling technologies :**Eviden**/**Atos** (software, emulators, quantum accelerators), **Thales** (sensors), **Air Liquide** (cryogenics), **Orano** (isotopes production like silicon 28), and small/medium businesses like **Radiall** (connectors, cabling, switches, attenuators, couplers, optical links), **ATEM** (cabling) and many in photonics (like **Exail, Azurlight Systems, Aurea Technology, Lumibird** and **Cailabs**) and even semiconductor manufacturing machines with **Plassys Bestek** and **Riber**. We can also add **OVHcloud**, a European cloud services vendor, which launched in 2022 its first quantum offerings, in partnership with Eviden (emulation), Pasqal (quantum simulation), Quandela (QRNG with a first QPU) and C12 (emulation).

The Netherlands

The Netherlands is one of the most active European countries in quantum technologies research and development, mainly around the University of Delft (**TU Delft**) and its **QuTech** branch.

It has long been a historical melting pot of quantum physics research in Europe. We have thus cited many great names at the beginning of this book: **Hendrik Antoon Lorentz** (1853-1928), **Heike Kamerlingh Onnes** (1853-1926), **George Uhlenbeck** (1900-1988), **Hendrick Casimir** (1909-2000) and **Samuel Goudsmit** (1902-1978).

In 2015, the government launched a 10-year, 135M€ plan to create a quantum computer[4558]. The investment was made in **QuTech**, TU Delft's quantum research center launched in 2014 with a 10-year budget of 145M€, half of which comes from TU Delft University and the other half from the NWO, the national funding agency[4559]. Qutech employs more than 180 people in all, of which only 37% are Dutch, with 25 permanent researchers.

The Netherlands government then announced a 7 years 615M€ plan in April 2021, complemented by an addition of 228M€ in 2022[4560]. This makes the country probably the greater investor in quantum technology in proportion to its GDP. Then, Quantum Delta « raised » an additional 60.2M€ from its government National Growth Fund as part of a joint partnership with France and Germany.

All this public funding should drive private sector investments of 3.6B€, a very ambitious goal in comparison with the similar 565M€ expected in France. It is managed by the non-profit foundation **Quantum DELTA NL** that was created in 2020[4561]. The Netherlands plans to create 30,000 high-tech jobs and create a cumulative economic impact of at least 5B€ with quantum technologies. The country plan is organized around the creation of three technology demonstrators, four generic action lines and shared cleanroom facilities.

- **Quantum Inspire**, their cloud superconducting computer service that is already available and got 90M€ of funding.
- **Quantum Network** project on quantum telecommunications and cryptography, connected to the related European projects, with a funding of 62M€. They expect to quantumly connect three quantum computers by 2023 and five by 2026.
- **LightSpeed** is a program connecting startups with investment funds. It is overselling a bit its value touting access to 13.6B€ in investment capital, representing the totality of the various funds managed by these investors. The 2022 funding extension added 15M€ for a startup seed fund.
- **House of Quantum** is a startup ecosystem facility which opened in 2023 in Delft, with a budget of 182M€. It will accelerate part of the 100 startups the country wants to consolidate by 2027. Within this house, the **Living Lab QT** will focus on ethical, legal and societal aspects of quantum technology with research collaborations between universities, the public and private sector, with a funding of 20M€. They will open two related interdisciplinary university positions, create a desk and a toolkit for responsible innovation and entrepreneurship and create a covenant to be signed by private and public stakeholders promoting sustainable and safe use of quantum technologies.
- They also plan to invest 150M€ in the 5 cleanrooms from **NanoLabNL**, have a quantum sensors plan with 23M€ funding and a training program that should create 2000 PhDs and engineers by 2027 with a funding of 41M€.

QuTech is also associated with **Intel** and **Microsoft**. QuTech has received $50M in funding in 2015 from Intel as part of a partnership on their superconducting and electron spin qubits.

Microsoft has also been a partner of QuTech since 2010, which they have also depleted by hiring **Leo Kouwenhoven** in their Microsoft Research laboratory which is on site and working on topological

[4558] See the state of play of the Dutch National Quantum Plan in National Agenda for Quantum Technologies, Quantum Delta Netherlands, September 2019 (51 pages).

[4559] See QuTech's 2018 Activity Report (80 pages) as well as an independent valuation report published in 2019 and covering the period 2015-2018.

[4560] See Quantum Delta NL awarded 228 Million Euro for second phase of its programme to Accelerate Quantum Technology, April 2022.

[4561] See the plan details in Quantum Delta NL in a nutshell, 2021 (20 pages). Look also at the excellent Economic Impact of Quantum in the Netherlands, Quantum Delta NL, May 2020 (60 slides) which contains a lot of interesting market data. DELTA stands for Delft Eindhoven, Leiden, Twente and Amsterdam, completed by Nijmegen, Maastricht and Groningen.

quantum and fermion of Majorana in liaison with a team of QuTech dedicated to the same subject. The Netherlands looks like a brain reservoir for the American quantum industry.

Collaborative research approaches are making good progress, particularly with a view to recovering European funding. In October 2017, QuTech launched a partnership with the Institute of Photonic Sciences, the University of Innsbruck in Austria and the Paris Centre for Quantum Computer. QuTech is also a partner of the University of Aachen in the CMOS qubit. The University of Delft has also obtained for the European part of the QuNET project mentioned about Germany an ERC of 1.5M€ with a launch in November 2019 and an end planned for October 2024[4562].

Figure 932: map of the Netherlands research ecosystem. Source: Venturing into Quantum Technology in the Netherlands by Deep Tech Fund and Invest-NL, October 2023.

Other initiatives with blurred contours have been launched such as **Quantum Helix,** funded under the European Quantum Flagship Program and Horizon 2020.

The **Quantum Software Consortium** runs for 10 years from 2017 and has received €18.8M in public funding from the country's Gravitation Program. It brings together various Dutch laboratories: **TU Delft**, **QuTech** (part of the latter), **QuSoft** (a research laboratory dedicated to quantum software, launched by CWI, UvA and VU in 2015), **CWI** (Centrum Wiskunde & Informatica), the **University of Leiden, UvA (**University of Amsterdam) and **VU** (Free University of Amsterdam) to conduct research in quantum software and cryptography.

Their vibrant startup ecosystem includes **QuantWare** (superconducting qubits), **Delft Circuits** (superconducting cabling), **Qblox (**electronics for controlling superconducting qubits), **Single Quantum (**single photon detectors), **QuiX** (photonic processor, a subsidiary of Lionix, a foundry capable of producing photonics wafers in nitrates on SiO2), **Qu&Co** (quantum software), **QuSoft (**quantum software), **QPhoX** (quantum computer interconnection), **ipCLock** (quantum clock) and **Leiden Cryogenics** (high-power dilution cryostats). As in most non-US countries, the local ecosystem is striving to get enough funding for its startups and calling for more private equity investments[4563].

In December 2020, the Dutch quantum industry created the **IMPAQT** consortium. The first members were Orange QS, Qblox, Delft Circuits, QuantWare and Qu&Co (now in Pasqal). Their goal was to improve the coordination of how they are creating quantum computer enabling technologies.

At last, The Netherlands and France signed in September 2021 a Memorandum of Understanding to expand collaborations in quantum technologies. The bilateral collaboration includes research partnerships in silicon qubits as part of the European flagship project QLSI, research-industry collaboration involving companies like Atos and Qu&Co, the creation of a joint portal listing job opportunities in

[4562] See A quantum network for distributed quantum computation, Cordis, 2019.

[4563] See Venturing into Quantum Technology in the Netherlands by Deep Tech Fund and Invest-NL, October 2023 (34 pages).

France and the Netherlands (www.quantumjobs.fr and quantumjobs.nl) and collaboration to increase EU venture capital in the domain (involving Quantonation).

Belgium

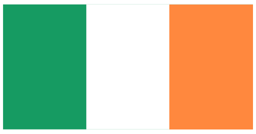
Belgium is the host of the famous Solvay conferences created in the early 20th by Ernest Solvay. Their presence in the quantum science and technology scene is exemplified by **IMEC**, the international semiconductor and nanotechnologies research center based in Leuven, an equivalent to CEA-Leti in France, with 4,000 employees.

IMEC's quantum technology activities are centered on producing superconducting and electron spin qubits on behalf of various laboratories and vendors as well as some cryoelectronics systems. Among other projects, they participate to the European Quantum Flagship QLSI project that is coordinated by CEA-Leti. They announced in August 2021 a partnership with **Xanadu**, for the development of fault-tolerant photonic qubits chips based on silicon-nitride.

Let's also mention the **Centre for Quantum Information and Communication** from the Free University of Brussels (Vrije Universiteit Brussel). It works on quantum measurement, quantum entanglement, quantum communication, quantum cryptography and quantum algorithms. It has also worked on continuous-variable quantum cryptographic protocols and developed quantum adiabatic algorithms.

In the vendor space, I have identified a company that was already mentioned, **QBee.eu**, a quantum accelerator and incubator created by Koen Bertels, who leads the Quantum Computer Architectures Lab in TU Delft and also works at Qutech.

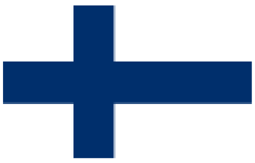
Ireland



In Ireland, a first quantum computing initiative (QCoIr) was launched in 2020 with a funding of $11M. It included global companies like IBM and Mastercard, plus the Tyndall National Institute in Cork. It established a Quantum Center of Excellence.

In May 2023, the **Trinity Quantum Alliance** (TQA) was launched to build a local quantum ecosystem, starting with establishing a custom space in Trinity East, a new zone of Trinity College in Dublin. It is associated with several foreign quantum industry vendors who have a foothold in Ireland like Microsoft, IBM, Horizon Quantum Computing, Algorithmiq and Moody's Analytics and will rely on Trinity College's quantum Master of Science. In November 2023, Ireland published its national quantum strategy dubbed Quantum 2030 organized along 5 pillars (research, education, entrepreneurship, international collaboration, public awareness), but with no announced budgeting.

Finland



Finland has a couple very interesting assets in quantum technology. In research you can count with Aalto University, the University of Helsinki, Tampere University and VTT Technical Research Centre of Finland. The Finnish Quantum Institute federates the efforts of Aalto University, the University of Helsinki and VTT.

It is an organization fostering collaborative research (ResQ) and education particularly with public broad audiences (EduQ) and business adoption of quantum technologies within Finland (BusinessQ).

On the industry side, **Bluefors** is the worldwide leader in low temperature cryogeny used with quantum computers. **IQM** is by far the largest and best funded superconducting qubits quantum computing startup. Other Finish startups are **SemiQon** (silicon qubits), **Algorithmiq**, **Ampliconyx**, **Kronus**,

Qplaylearn, **Quanscient**, **Quantastica**, **SSH Communications Security**, **Unitary Zero Space** and **Vexlum**.

And you have the **CSC** computing center which hosts the LUMI supercomputer and will put in place an hybrid quantum/classical computing architecture with IQM as part of the EuroHPC HPCQS (High Performance Computer – Quantum Simulator hybrid) program.

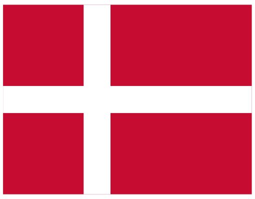
In January 2022, Finland and VTT also launched **QuTI** with 10M€ to pool the expertise and resources of four research and eight industry partners over a three year period. It is centered around enabling technologies for quantum computing (materials, cryogeny, electronics, software, systems architecture). The program industry partners are Afore, Bluefors, IQM, Quantastica, Saab, Vexlum, and for companies outside Finland, Picosun and Rockley Photonics.

In April 2022, Finland and the USA signed a Joint Statement on Cooperation in Quantum Information Science and Technology very similar to the ones signed with Australia, Sweden, Denmark, Switzerland, Australia, and the UK in 2021/2022. A similar partnership was launched with Singapore in September 2022.

In December 2022, the EU awarded a 19M€ Specific Grant Agreement (SGA) funding for the project Qu-Pilot to upgrade the existing European micro, nano and quantum technology manufacturing infrastructures, mostly in superconducting chips. It is led by VTT with 24 member organizations from 9 EU countries (Finland, the Netherlands, Germany, Belgium, France, Austria, Italy, Spain and Czech Republic). VTT got $3.8M of this funding.

In September 2022, the Finnish quantum community formed a working group which published in February 2023 a list of recommendations for the development of quantum technologies in the country. It included a better coordinated research and innovation program, long term investments, education investments and the support for national and international cooperation[4564]. It estimated the ongoing R&D effort at $45M per year including the Swedish Wallenberg Foundation investment of 130M€ for 12 years starting in 2018. In January 2024, the Research Council of Finland awarded a 5year 13€M of funding to the new Finnish Quantum Flagship project covering funding for various research organizations including VTT, Aalto University, the University of Helsinki, and Tampere University.

Denmark



Quantum research in **Denmark** is organized around the Center for Quantum Devices (**QDev)** at the **Niels Bohr Institute** (NBI) at the University of Copenhagen[4565] and **DTU** (Danish Technology University) with a QuantumDTU Center for Quantum Technologies, led by Lydia Baril. NBI is focused on various fields in condensed matter physics.

They also study quantum photonics (optical memories, photon sources) and fluxonium qubits.

The Danish government invested $12M on quantum research between 2017 and 2019. In June 2023, the Danish government announced a formal National Strategy for Quantum Technology in two parts. Part 1 corresponds to $140M of funding focused on research with an eye on use cases in healthcare, green transition and cyber security over the 2024-2027 period ($35M per year). Part 2 is focused on commercializing quantum technologies in Denmark ($7M per year) for the same period[4566].

[4564] See Finish Quantum Agenda, InstituteQ, February 2023 (31 pages).

[4565] See Quantum technology in Denmark by KPMG, November 2020 (34 slides). This report from a well-known consulting company is highly disappointing. It doesn't mention any research lab besides NBI, any scientist besides Niels Bohr or any startup from the Danish scene. It contains only generalities.

[4566] See Denmark makes decision to spend 1 billion dkk on quantum research and innovation strategy, June 2023.

This was completed in September 2022 by the launch of a $200M initiative by **Novo Nordisk** to build a generic quantum computer in 2034, most of the funding going to NBI as part of the Novo Nordisk Foundation Quantum Computing Programme (NQCP).

In April 2022, NATO announced the setup of a NATO Accelerator for Quantum Technologies in Denmark. This innovation accelerator is installed at NBI. This was completed in June 2022 by the announcement of a global quantum partnership with the USA.

Industry wise, Denmark hosts about a dozen quantum companies, mostly in enabling technologies, like **QDevil** (filters), **Sparrow Quantum** (single photon sources) and **Cryptomathic** (QRNG).

Sweden

On the **Swedish** side, there is mainly the WACQT (Wallenberg Centre for Quantum Technology) which is part of the **Chalmers University** of Gothenburg and is co-financed by the Wallenberg Foundation. The WACQT has been funded under a 12-year plan with over $100M.

As in all countries, the center targets all quantum technologies domains (computing, communications and sensing). They are invested in superconducting qubits as well as in continuous variable qubits. They plan to create a 100 qubits superconducting computer. WACQT is also working on cold atom qubits from Rydberg... named after a Swedish physicist! It has also launched a "Women in WACQT" initiative to develop gender diversity in quantum science.

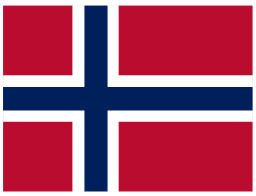
In March 2021, the Wallenberg budget nearly doubled to $9M per year, allowing the hiring of 40 more researchers. In April 2022, Sweden and the USA announced a partnership on quantum technology similar to the ones with the UK, Finland, Denmark, Australia, Switzerland and France.

Vinnova (Sweden Innovation Agency) with RISE (Research Institutes of Sweden), Swelife (life science community), The Swedish Research Council and Chalmers-WACQT published in March 2023 a position paper calling for a Swedish quantum agenda[4567]. It remains to be launched by the Swedish government.

In January 2023, WACQT announced to would deliver a 25-qubit quantum computer to the country ecosystem in academia and industry, with a funding of $9.5M coming from the Knut and Alice Wallenberg Foundation. They later plan to upgrade the system to 40 and then 100 qubits.

At last, let's remind a key influence from Sweden in quantum physics. Its Royal Academy of Science is awarding every year the Nobel prize in physics!

Norway



Norway feared in 2018-2019 to miss the second quantum revolution. In 2020, it created a Gemini-center on quantum computing with SINTEF, the University of Oslo, NTNU, and in 2021, Simula Research Laboratory, a fundamental research organization belonging to the Norwegian Ministry of Education and Research, where Shaukat Ali leads research in quantum software engineering (among other things)[4568].

The project was led by Franz Georg Fuchs from SINTEF, an independent research organization with the goal to make Norway "quantum ready".

[4567] See A Swedish quantum agenda, Vinnova, March 2023 (25 pages).

[4568] See QuSBT: Search-Based Testing of Quantum Programs by Xinyi Wang, Paolo Arcaini, Tao Yue and Shaukat Ali, April 2022 (5 pages).

Later in 2021, the Norwegian Quantum Computing Centre was created consolidating research from 13 scientists from three partner institutions (SINTEF, the Norwegian University of Science and Technology *aka* NTNU and the University of Oslo) including Jeroen Danon from the Center for Quantum Spintronics at NTNU[4569].

In June 2023, the QCNorway consortium published a white paper calling for a national quantum initiative, like in many other countries[4570].

Italy

Italy has very active research in place in various technologies in quantum computing. Photonic qubits are explored by **Fabio Sciarrino** at Università La Sapienza in Rome. He is a European pioneer of boson sampling experiments and wants to make it programmable. The **Università di Padova** launched its own Quantum Technologies Research Center working on trapped ions computing.

Francesco Tafuri from the Università Federico II in Naples works on superconducting qubits. The Italian National Institute for Nuclear Physics (INFN) is also working with the US DoE on superconducting quantum materials at the FermiLab in Chicago, which happens to be run by Anna Grassellino, an Italian. In the quantum communication realm, **Paolo Villoresi** from the Instituto Nazionale di Ricerca Metrologica in Turin pioneered photons polarization encoding with a satellite in 2015. Italy also deployed its **Italian Quantum Backbone** (IQB) with a total of 1,850 km fiber link based on commercial fibers. It connects INRIM's premises in Turin to Milan, Bologna, Firenze, Rome, Napoli, Pozzuoli and Matera. From Turin, a 150 km fiber reaches Modane in France, and connects to Grenoble, Lyon and Paris, then Europe.

The public supercomputing center **CINECA** entertains a quantum computing lab. It tests the capacities of various quantum computers, develops quantum algorithms and hybrid solutions associating classical supercomputers and quantum accelerators.

The big shortcoming of Italy is its weak private sector with not many industry vendors and startups engaged in quantum technologies. It led in 2023 to the creation of **Quantum Italia**, an investment vehicle dedicated to quantum technologies and created by Scientifica Venture Capital. They partner with the Unitary Fund to support the open source quantum ecosystem in Italy, as well as with Quantonation. As part of its recovery plan announced in April 2021, the Italian government allocated a budget of 1.6B€ to fund 7 new research organizations, one of these being focused on quantum technologies[4571].

Spain

Spain's quantum academic efforts are spread in various places like in Madrid, Barcelona and in Bizkaia. **ICFO** (Instituto de ciencias fotónicas) of Barcelona is mainly specialized in photonics. Other quantum research are carried out at the Quantum Information and Computation Laboratory (GIC-UB) of the **University of Barcelona** as well as the **Autonomous University of Barcelona**[4572].

[4569] See Protected Solid-State Qubits by Jeroen Danon et al, October 2021 (6 pages).

[4570] See Contributions Towards a Norwegian Quantum Computing Strategy by Are Magnus Bruaset, Shaukat Ali, André Brodtkorb, Gunnar Bøe, Sergiy Denysov, Hans Eide and Sølve Selstø, Simula Research Laboratory, Oslo Metropolitan University and Sigma, June 2023 (48 pages).

[4571] See Italy's quantum scientists jostle for a superposition by Francesco Suman, April 2021.

[4572] See Quantum Technologies in Catalonia, July 2019 (43 slides) which describes very well the quantum ecosystem of this key region of Spain.

The **IFAE QCT** is the Quantum Technology Group from the IFAE (Institut de Fisica d'Altes Energies) from the Autonomous University of Barcelona opened its new lab and fab in October 2020.

The Basque country led by the **Provincial Council of Bizkaia** is also fostering the adoption of quantum computing in the industry. Participating members are Technalia (again), IBM, Telefónica, Accenture, the Bilbao City Council, Gaia, Silicon Europe, the UPV/EHU (the University of the Basque Country), the University of Deusto in San Sebastien and Mondragon Unibertsitatea (east of Bilbao). The region established a partnership with IBM which will deploy one QPU in San Sebastian in Spain[4573].

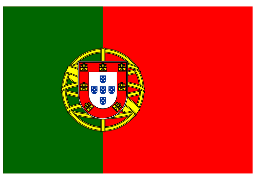
Spain is invested in testing and deploying quantum networks. The **MadQ-CM** (Madrid Quantum-CM) project is about building a quantum communications ecosystem in Spain involving various regions (Galicia, Basque Country, Castilla y León, Cataluña and Valencia). It involves the Polytechnic University of Madrid, the Autonomous University of Madrid (UAM), Complutense University of Madrid (UCM), IMDEA Software Foundation (IMDEA SW), IMDEA Networks Foundation (IMDEA NW), National Institute of Aerospace Technology (INTA), the Spanish Metrology Center (CEM), the Vithas Foundation (FV), Telefónica and RedImadrid (Madrid city infrastructure project). It will support the **MadQCI** project (Madrid Quantum Communications Infrastructure), which will link the area of Madrid with other European quantum communications network as part of the EU EuroQCI project, and within a radius of 30 km.

On the startup scene, **Multiverse** is a software player focused on quantum inspired algorithms and quantum annealing, operating in the financial and other verticals. **Qilimanjaro** develops a cloud-based quantum software platform and a superconducting quantum annealer. **Entanglement Partners** is a service provider that is clearly succeeding in selling quantum-related cybersecurity services. **aQuantum** creates development tools facilitating quantum programming, particularly for quantum annealing.

In February 2022, UMIQ aQuantum-UEx was created, a Joint Quantum Software Research Unit between the Spanish quantum software and engineering company aQuantum and the University of Extremadura (UEx) consolidating a partnership that started in March 2020. They are collaborating on software research projects including the "QHealth: Quantum Pharmacogenetics Applied to Aging" project. Seven Spanish quantum companies (Amatech, BBVA, DAS Photonics, GMV, Multiverse Computing, Qilimanjaro Quantum Tech and Repsol), five research centers (BSC, CSIC, DIPC, ICFO and Tecnalia) and the Universitat Politècnica de València launched in 2022 the **CUCO Project** to foster quantum computing research and development in Spain, particularly in the industry.

In 2023, **Ametic**, the digital trade association in Spain, published a position paper on quantum technologies in Spain[4574]. It looks like a call to action for the government to launch a strategic quantum initiative like the ones that exist in many other countries, particularly in the EU. It led to the launch of Quantum Spain initiative in 2023 which plans to build the first local quantum computer at the Barcelona Supercomputing Center in a project led by Qilimanjaro with the help from IQM.

Portugal



Portugal's key investment in quantum technologies sits with **QuantaLab**, a collaborative research center launched by **International Iberian Nanotechnology Laboratory** (INL) and the **Universidade do Minho**, both in Braga, Portugal. It focuses on quantum materials and quantum technologies. Portugal is also participating to European quantum projects including the QCI (Quantum Communication Infrastructure).

[4573] See IBM and Fundación Ikerbasque Partner to Launch Groundbreaking Quantum Computational Center, March 2023.

[4574] See Report Spain Quantum Industry, Ametic 2023 (90 pages).

Launched in 2022, **PQI** (Portuguese Quantum Institute) consolidates research in quantum science. Led by Yasser Omar, it has 13 principal investigators. The Institute is involved in various EU funded projects like IPAS (Quantum Information in Malta), DigiQ (EU masters program), EuRyQa (infrastructure for neutral atoms quantum computing) and QuantHEP (quantum computing for high-energy physics).

Poland

Poland launched its National Quantum Information Centre in Gdansk (KCIK) in 2007 with 9 research institutions. Other interesting labs are the Quantum Physics Research Center focused on quantum cryptography and the International Center of Theory of Quantum Technologies (ICTQT).

The University of Warsaw is also very involved in quantum research, particularly in photonics with the Center of Quantum Optical Technologies (QOT), led by Konrad Banaszek[4575].

The Polish National Science Centre also coordinates the international research network **QuantERA,** itself funded by the European Union's Europe 2020 budgets. It does this in coordination with the French ANR. The countries involved, in addition to those of the European Union, are Switzerland, Israel (Bar-Ilan University) and Turkey. About thirty research projects had been funded after a call for proposals in 2017, some of which were subsequently funded in the European Quantum Flagship, such as SQUARE. They are all quantum physics projects (photonics, cold atoms, ...).

Some other Polish quantum research groups worth mentioning are The Quantum Research Group from the Polish Academy of Sciences which works on quantum computing, qubit measurement, error mitigation and software engineering and the Quantum Resources Group from the Jagiellonian University in Kraków that was created in 2020 and works on quantum information science.

In the ecosystem, The Quantum AI Foundation was created in 2019 by Paweł Gora. It organizes meetings of the Warsaw Quantum Computing Group (WQCG) and hackathons.

Hungary

Hungary launched in 2018 its quantum plan with a funding of about $10M, consolidated in a new consortium named HunQuTech consolidating the countries quantum research groups from various institutions and industry vendors.

The participants are the Wigner Research Centre for Physics, the Institute of Physics the Faculty of Electrical Engineering and Informatics of the Budapest University of Technology and Economics, the Institute of Physics of the Eötvös Loránd University, completed by industry partners (Bonn Hungary Electronics, Ericsson Hungary, Nokia-Bell Labs, and Femtonics). The plan goals were to work on single photon sources for quantum telecommunications, pairs of entangled photons at telecom wavelength, free-space QKD systems, quantum memories, qubits and quantum gates and new quantum algorithms. Researchers also work on QRNG and PQC. In total, Hungarian researchers got 4 ERC from the EU in quantum physics and information science.

In 2022, the country launched the Quantum Information National Laboratory initiative that consolidates quantum research covering quantum computing and quantum telecommunications. Hungary

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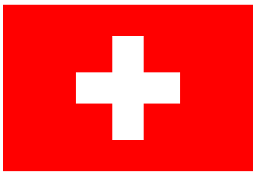
[4575] They however have not created the "first world's first quantum processor" as claimed in One-of-a-kind: Warsaw-based scientists build groundbreaking quantum processor by Jo Harper, The First News, February 2022. The related paper is Optical-domain spectral super-resolution via a quantum-memory-based time-frequency processor by Mateusz Mazelanik, Adam Leszczyński and Michał Parniak, Nature Communications, February 2022 (12 pages). It is about a (rather interesting) high-resolution spectrograph.

also participates to the EuroQCI project since 2019. The country seems to have several intra-European partnerships like with Germany and the Netherlands.

In 2022, the Faculties of Science and of Informatics of the Eötvös Loránd University selected QuiX as a vendor for a photonic circuit to build their own research photonic quantum computer.

Switzerland



Switzerland is mobilized on quantum technologies, particularly at **ETH Zurich**, which is collaborating with IBM and especially on quantum cryptography, notably with its startup **IDQ,** which is a leader in quantum random numbers generation used in quantum cryptography and elsewhere. And also with the Lausanne's **EPFL**. Other ecosystems are very active like in Geneva and Basel.

The **Swiss Quantum Hub** brings together the Swiss quantum ecosystem. It published a manifesto to promote Swiss research and industrial efforts in quantum[4576].

The **Quantum Science and Technology Initiative** (QIST), a joint initiative of ETH Zurich and the University of Basel, which also involves the University of Geneva and EPFL Lausanne, has 34 faculty members and 300 students. It was funded with $120M between 2010 and 2017.

It covers all the usual fields of quantum with, with a particular effort in quantum telecommunications. In August 2021, the EPFL launched though its own multidisciplinary Quantum Science and Engineering Center consolidating its research and academic efforts in all branches of quantum technologies.

The **Swiss Quantum Investor Club** was created in 2020 to link investors and quantum entrepreneurs and organize events in Geneva, Lausanne and Zurich, as well as the **Swiss Quantum Hub,** a think tank and accelerator for quantum startups, and the Quantum Computing Garage, a permanent hackathon. In November 2020, Martin Haefner, an alum from ETH Zurich donated $44M to the ETH Foundation to have them build a quantum research facility. We could wish more wealthy people would make such long-term investments for their community! In another similar initiative, ETH Zurich and the **Paul Scherrer Institute** (PSI, which has its own proton accelerator and an electron synchrotron, the Swiss Light Source, that is equivalent to SOLEIL in France) created the Quantum Computing Hub in 2021, a joint quantum computing research center, focused on trapped ions and superconducting qubits with the goal to host 30 researchers. ETH Zurich invested $36M there[4577].

In October 2022, Switzerland and the **USA** signed yet another bilateral quantum cooperation agreement[4578].

The Basel quantum ecosystem is structured around **QuantumBasel**, its Center of Competence for Quantum and Artificial Intelligence whose CEO is Damir Bogdan. It belongs to uptownBasel Infinity which is itself a subsidiary of the uptownBasel Group, an international competence center for Industry 4.0 and which opened a first building in use by Bouygues and Vinci (Axians and Actemium) and is planned to host 50 to 100 companies with up to 2,500 jobs (not just in quantum technologies). QuantumBasel has several quantum computing partnerships in place, with D-Wave (December 2022, which announced the opening of their European headquarter in Basel in October 2023), IBM

[4576] See Switzerland: At the Quantum Crossroads, 2019 (8 pages).

[4577] See ETH Zurich and PSI found Quantum Computing Hub, May 2021.

[4578] See Joint Statement of the United States of America and Switzerland on Cooperation in Quantum Information Science and Technology, US State Department, October 2022.

(December 2022) and IonQ (June 2023 with the upcoming installment of an IonQ QPU with 35 qubits, the first outside the USA).

European Union

The **European Union** has a lot of programs operating in the quantum technologies field (European Quantum Flagship, EuroHPC, EuroQCI, ERC, QuantEra, Chips Act, COST, …) totaling about 4.5B€ of funding over 10 years. These various programs are funded through Horizon Europe and Digital Europe.

European Quantum Flagship

The European Quantum Flagship was formally launched in October 2018. It funds collaborative research on all aspects of quantum information: sensing, communications, computing and simulation[4579]. It was endowed with 1B€ over a 10 year period with over 20 programs described below. It is one of the three European "flagships" that aim to place Europe at the forefront of major technological breakthroughs with strong community investment in research[4580]. The first phase of the Flagship included €132M spread onto 20 projects selected out of 140 applicants and for a period of three years. 130 additional projects will be later selected[4581] (Figure 933). The flagship projects involve an average of at least half a dozen countries, even partner countries like Switzerland and Israel.

Figure 933: the European Quantum flagship projects as of 2022. (cc) Olivier Ezratty, 2019-2023.

[4579] See the motivations behind the European Flagship: The Impact of quantum technologies on the EU's future policies: Part 1 Quantum Time, 2017 and Part 2

[4580] The two other flagships are the "Human Brain Project" led by the Swiss Henri Markram and the Graphene project in nanotechnologies.

[4581] See the Press Kit (28 pages), the complete list of projects and Europe Accelerating the Industrialization of Quantum Technologies, October 31, 2018, the title of which is somewhat misleading in that the majority of projects funded are research projects and not industrialization projects. And then there is The quantum technologies roadmap: a European community view, October 2017 (25 pages), which takes stock of the state of the art in Europe and around the world. See also The EU Quantum Technology Flagship by Elisabeth Giacobino, 2018 (41 slides and video).

It starts with three side projects related to quantum computing:

- **AQTION** (Austria, €9.57M) is trapped ions qubit computer project, planning to reach 50 qubits. Austria has a long history here and is quite legitimate. Atos is participating in this project.
- **MicroQC** (Bulgaria, €2.36M) plans to create another trapped ion computer.
- **OpenSuperQPlus** (Germany, €20M) is a follow-up to the OpenSuperQ project launched in 2018 with teams from Spain, Sweden, Switzerland and Finland and a total of 10 research laboratories. Its ambition was to create a 100-qubit system. The new 7-year project led by Forschungszentrum Jülich is now shooting for 1,000 qubits, with an intermediate 100 useful NISQ qubit stage (which will require very high quality qubits). It gathers new partners from the Netherlands, France, Finland, Germany, Hungary and Sweden, for a total of 28 partners from 10 countries.

Then we have four quantum simulator projects:

- **PASQuanS2** (Germany, €16.6M) aka Programmable Atomic Large-Scale Quantum Simulation is shooting for the creation of large-scale atomic quantum simulators with up to 10,000 atoms. It is coordinated by the Max Planck Institute of Quantum Optics, with 25 academic and technology partners from Austria, France, Germany, Italy, Slovenia and Spain. It is funded through the Horizon Europe Framework Programme. It is the successor of PASQuanS (€9.25M) that was launched in 2018 and aimed at creating is a cold atoms or trapped ions quantum simulator project based on with up to 1,000 qubits. A goal that has not been achieved yet.
- **PhoQuS** (France, 3M€) is a photonic based quantum simulator project led a PSL research team. It involves the use of polaritons.
- **Qombs** (Italy, €9.3M) is another photonics-based quantum simulator project.
- **SQUARE** (Germany, €2.99M) a quantum simulator project using trapped ions led by the University of Karlsruhe which involves laboratories from Denmark, Sweden, Spain and France, including Thales.

Let's continue with projects in quantum communication and telecom security.

- **Quantum Internet Alliance** (the Netherlands, 10M€) (QIA) aims at deploying an Internet network protected by quantum key distribution (QKD) in mesh network mode and not just point-to-point. The quantum nodes or relays will be made up of systems using cold atoms. They will start with a three or four-node network. The project is led by TU Delft University. The CNRS participates in it, notably Eleni Diamanti, Elham Kashefi and Iordanis Kerenidis. The Sorbonne University also participates. Other participants include Swiss, Germans, Danes and Austrians (complete list).
- **QRANGE** (Switzerland, 3.87M€) is a project to improve quantum random number generation techniques.
- **CiViQ** (Spain, 9.9M€), or Continuous Variable Quantum Communications, is another QKD-based fiber telecommunications security project. The project involves 21 stakeholders covering the academic and industrial world, including CNRS, Institut Mines-Telecoms, Nokia Bell Labs France, Inria, Orange, as well as Mellanox.
- **Uniqorn** (Austria, 9.9M€) is in the same niche and is working on a random number generator and a QKD system. It associates 17 organizations from 9 countries (Austria, the Netherlands, Italy). The Israeli Mellanox is also involved there.
- **S2QUIP** (the Netherlands, 3M€), Scalable Two-Dimensional Quantum Integrated Photonics, is another QKD-based secure communication project.

* **2D-SIPC** (Spain, 2.9M€) is a project for the development of photoelectronic components made for networks secured by QKDs.
* **QMICS** (Germany, 3M€) or "Quantum Microwave Communication and Sensing" is about creating a microwaves-based links and networks between superconducting network nodes with applications in distributed quantum computing and also in quantum sensing.
* **NEASQC** (NExt ApplicationS of Quantum Computing, France) is a collaborative project launched in September 2020, to develop practical applications of NISQ (noisy quantum computers, an intermediate step before scalable quantum computers). It is an H2020 project that brings together European players including Atos, Total, EDF, the Loria laboratory from the University of Lorraine, Astrazeneca, HQS Quantum Simulations, HSBC and the University of Leiden (the Netherlands).
* **QLSI** (France) is a Quantum Flagship project that started in March 2020 to fund four years of fundamental research in silicon qubits. It is being driven by the Grenoble team from CEA-Leti. The project funding of 14M€ is spread over 19 organizations: Atos/Eviden, STMicroelectronics, SOITEC, CNRS Institut Néel, TU Delft, University of Twente and TNO in the Netherlands, IMEC in Belgium, UCL and Quantum Motion in the UK, Infineon, RWTH Aachen, University of Konstanz, Fraunhofer and IHP Frankfurt in Germany, University of Copenhagen and University of Basel.
* **QTEdu** (Italy) is about creating the quantum education ecosystem in the European Union. It is funded by H2020[4582].
* **QUCATS** (France) is the project handling the coordination for European Quantum Flagship projects. It covers the 2022 to 2025 period and is coordinated by Philippe Grangier (CNRS, France). It is a follow-up to the QFLAG (Germany, €3.48M) project launched in 2018. The EU quantum strategy is now formalized in its **SRIA** (Strategic Research and Industry Agenda) that was published in November 2022[4583]. Is consolidated the SRA and SIR. The **SRA** (Quantum Flagship Strategic Research Agenda) defines goals for the Quantum Flagship, and details them for the next three years, with an outlook for six to ten years. It contains typical goals like helping NISQ and FTQC scale through better qubits, error correction mechanisms and various enabling technologies, developing software algorithms and tools, cloud offerings. The **SIR** (QuIC Strategic Industry Roadmap) describes the needs of the EU quantum industry, as represented by the QuIC consortium, in education and skills buildup, standards, IP and trade and governance principles.

Then we have the five quantum sensing projects already seen in the Sensing section of this book.

Large countries are present in many of these projects. As an example, France is involved in many of these projects. CNRS (France) alone was involved in 14 of the 20 projects when the Flagship was launched[4584].

EuroHPC

The EuroHPC Joint Undertaking program is a joint initiative between the EU, European countries and industry partners to develop a supercomputing ecosystem in Europe. The quantum side of the project will be funded with 1.5B€ over 10 years, over a total of 5B€ which covers the buildup of exascale HPCs.

[4582] See Expanding the European Competence Framework for Quantum Technologies, January 2022.

[4583] See Quantum Flagship publishes preliminary Strategic Research and Industry Agenda, Quantum Flagship, November 2022 (50 pages). Roadmap until 2030.

[4584] See New Strategic Research Agenda on Quantum technologies, February 2020 (114 pages) which details the state of play of the European Quantum Flagship projects.

HPCQS corresponds to the first deployments of quantum simulators is planned in Germany and France as part of the project.

In 2023, EuroHPC is funding six gate-based quantum computers to be integrated into existing supercomputers centers in Czechia, France, Germany, Italy, Poland and Spain for a total of 100M€.

HPCQC is an initiative created by Technical University of Munich (TUM), the Danish Technical University (DTU) and the Leibniz Supercomputing Centre (LRZ) which wants to consolidate a community of HPC and quantum computing specialists, focused on creating interfaces and benchmarks.

EuroQCI

The European quantum Internet research program is a joint program with ESA and is funded with 1B€ over 10 years with a terrestrial and a satellite segment.

Chips Act

The Chips Act launched in 2022 has allocated about 500M€ to quantum technologies manufacturing. It will certainly have several components and RFP (request for proposals).

Qu-Pilot and **Qu-Test** are two new projects launched in 2023 as part of the Chips Act. Coordinated by VTT, Qu-Pilot aims to federate and expand European production facilities for quantum technologies in a trusted supply chain. It gathers 21 partners from 9 different countries. It covers quantum computing, communication and sensing needs through 13 service-provider organizations. Qu-Test is complementing Qu-pilot with three test beds. One the Quantum Computing Testbed for the characterization and validation of cryogenic quantum devices, cryogenic qubits such as superconducting and semiconducting qubits, photonics qubits and ion traps circuits. A Quantum Communication Testbed will characterize devices for QKD and QRNG solutions. And the Quantum Sensing Testbed will benchmark sensing and metrology instruments.

Other programs

Figure 934 makes an inventory of the various EU programs funding quantum technologies R&D.

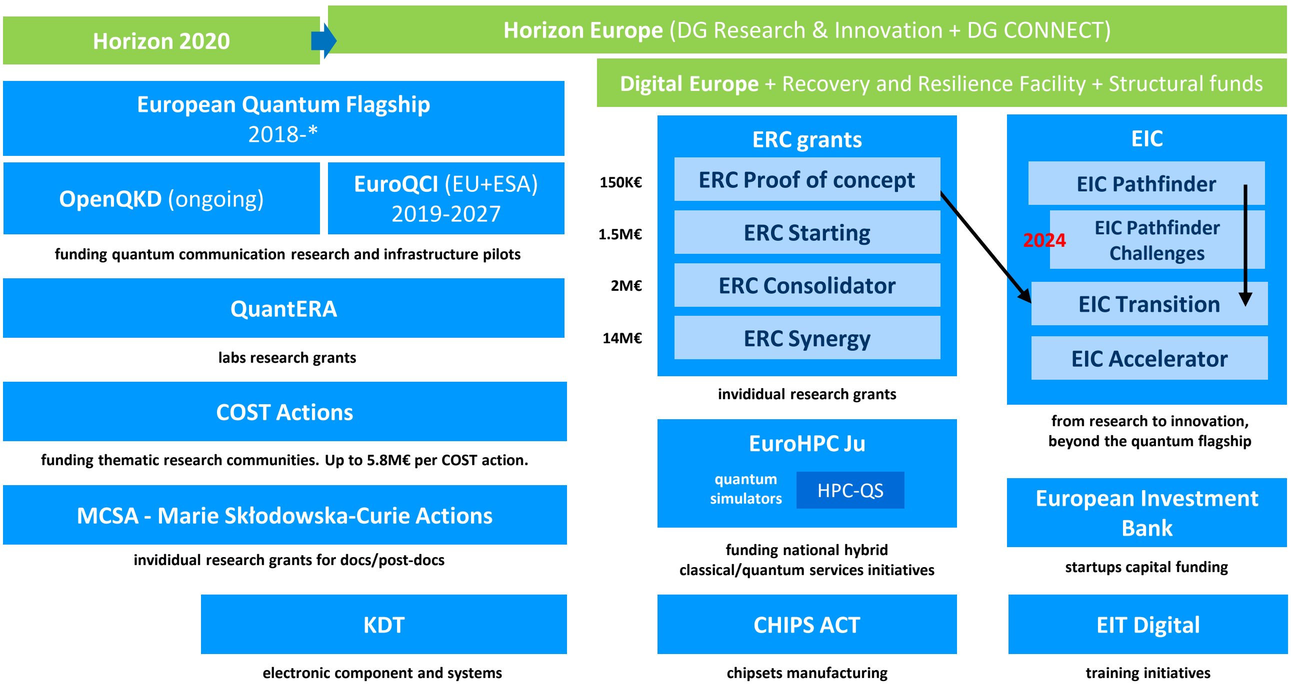


Figure 934: the web of European projects funding the quantum field both at the research and industry levels. (cc) Olivier Ezratty, 2023.

ERC grants funds for individual researchers with four levels, from 150K€ to 14 M€ (Proof of Concept, Starting, Consolidator, Synergy). A couple dozen quantum researchers are awarded with such a grant every year.

EIC programs fund transfer from research to innovation, and mostly startups, with several levels (Pathfinder, Transition, Accelerator and an upcoming Challenge). In one year, 1B€ funds were allocated in deep tech startups.

COST Actions are funding the buildup of research communities with up to 5.8M€ each. If covers the organization of scientific events, short-term scientific missions, training schools, communication activities, and virtual networking tools.

QuantERA I (2014), II (2021) and III (2023) is an alliance of research funders from member states created to reinforce transnational collaborations in inspiring multidisciplinary quantum research. The QuantERA II Consortium assembles 38 Research Funding Organizations from 30 countries, some being extra-EU. It complements the Quantum Flagship in early stages, serving as an incubator of new ideas which then can get integrated in Quantum Flagship projects but comes from participating countries (45M€ for QuantERA I and 40M€ for QuantERA II) and the EU (11.5M€ for QuantERA I and 15M€ for QuantERA II). There were two calls for projects in QuantERA I and one in QuantERA II with a selection done late 2021 and projects funding starting in 2022.

Spare projects

These various quantum projects are also funded or cofounded by the EU through various funding sources.

MATQu (Materials for Quantum Computing) is developing a European value chain to manufacture superconducting qubits. This H2020 project running from 2021 to 2024 with a total cost of 21M€ with EU funding of 6.5M€ is led by Fraunhofer Mikroelectronics, IAF and IPMS with the participation of CEA-Leti, IMEC, Soitec, BE Semiconductor Industries (semiconductor assembly equipment), IQM, VTT, Keysight, Siltronic (silicon wafers production), Kiutra, Atos, Mellanox, Beneq (atomic layer deposition equipment), Orange Quantum Systems and Technic France (engineering).

SPROUT (Scalable Platform for Quantum Technology) is a project launched in November 2021 by Delft Circuits and kiutra to provide a scalable cryogeny platform funded by the EU Eurostars program. They develop a demonstrator for a <1K cryogen platform based on cryogen-free magnetic cooling (the Kiutra specialty) and a multi-channel electrical cabling.

LSQuanT is an initiative funded by the EU that was launched in 2021 and is dedicated to promoting "large-scale quantum transport methodologies", which deals with the physics of quantum transport digital simulation, to invent new quantum materials and devices[4585].

EQUIPE (Enable Quantum Information Processing in Europe) project aims to advance the industrialization of the creation of quantum computing and telecommunications solutions for industry[4586]. It was funded by an ERC for 1.4M€.

DigiQ (Denmark, €17M) is a quantum training initiative providing master courses. It gathers contributors from Denmark, Finland, France, Germany, Italy, the Netherlands, Portugal, Spain and Czechia. It is funded by Digital Europe.

[4585] See a related review paper: Linear scaling quantum transport methodologies by Zheyong Fan et al, December 2020 (61 pages).

[4586] See Simulation on / of various types of quantum computers by Kristel Michielsen (40 slides).

QTIndu (Quantum Technology Courses for Industry, €5.6M) is a project launched in January 2023 which is creating industry-specific Quantum Technology (QT) training programs targeting SMEs and large companies from various vertical markets.

It is led by a pan-European consortium with 11 academic and industry partners that are coordinated by Qureca (Spain). It is also funded by Digital Europe.

EQSI (European Quantum Software Institute) is an initiative supported by the EU which ambitions to create a strong research and industry ecosystem in quantum software.

It was launched in November 2022 by the Paris Centre for Quantum Technologies (Iordanis Kerenidis), PQI – Portuguese Quantum Institute (Yasser Omar), QMATH / Copenhagen (Matthias Christandl), QuSoft / Amsterdam (Harry Buhrman), the Technical University of Munich (Robert Koenig), and the University of Latvia (Andris Ambainis). Funding is TBD.

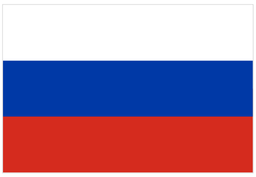
Industry associations

European research is federated under the umbrella of **QCN** (Quantum Community Network). Its industry counterpart is the **QuIC** (Quantum Industry Consortium) announced in June 2020 and formally launched in April 2021[4587]. Founding members are companies that were involved in at least two European Quantum Flagship projects. They include Bosch, SAP, Atos, Thales, Muquans, Airbus and many others. The consortium has an extensive work plan covering market needs assessment, analysis of the quantum technology value chain, development of standards and regulations, sharing of best practices in intellectual property protection and market evangelization, access to infrastructure, linking startups and investors, skills development issues and coordination with public authorities.

In January 2023, Quantum Industry Canada, QED-C (USA), Quantum Strategic Industry Alliance for Revolution (Q-STAR, Japan) and European Quantum Industry Consortium (QuIC) created the meta-consortium "International Council of Quantum Industry Associations" (ICQIA) to consolidate their joint work in developing the worldwide quantum ecosystem.

Another association, **|QBN⟩**, the Quantum Business Network, is an European Quantum Community, which connects the industry, users, vendors and research.

Russia



Russia is not very visible in the quantum scene, maybe because they have not built the same research and industry partnership that are seen in the western world. Like with AI, its government realized that quantum technologies were critical for sovereignty. In December 2019, Russia announced its own plan of attack on quantum technologies, which seemed very focused on military, intelligence and cryptanalysis applications[4588].

This plan got a five-year funding of $790M. In practice, it covers almost all fields of quantum technologies[4589] as shown in Figure 935. In January 2022, Russia created it National Quantum Laboratory

[4587] See Announcing the creation of the European Quantum Industry Consortium by Laure Le Bars (SAP), the first President of QuIC, April 2021.

[4588] See Russia joins race to make quantum dreams a reality by Quirin Schiermeier, December 2019.

[4589] Source: Quantum communication in Russia: status and perspective by Vladimir Egorov, 2019 (22 slides).

(NQL) run by Rosatom and with NRU HSE and MIPT, MISIS, the Lebedev Physical Institute of the Russian Academy of Science, the Russian Quantum Center and the Skolkovo Foundation. The center hosts a nano-fabrication center of 2,000 m^2 in Skolkovo. One might wonder how things will fare with exports restrictions to Russia after it started its war against Ukraine in February 2022. Many Western countries equipment vendors won't be in position to sell their hardware and Russia will not be able to count on China this time since it does not seem to manufacture the sophisticated equipment used in nanofabrication facilities. Still, Russia is expanding its international collaboration with eyes on India and China.

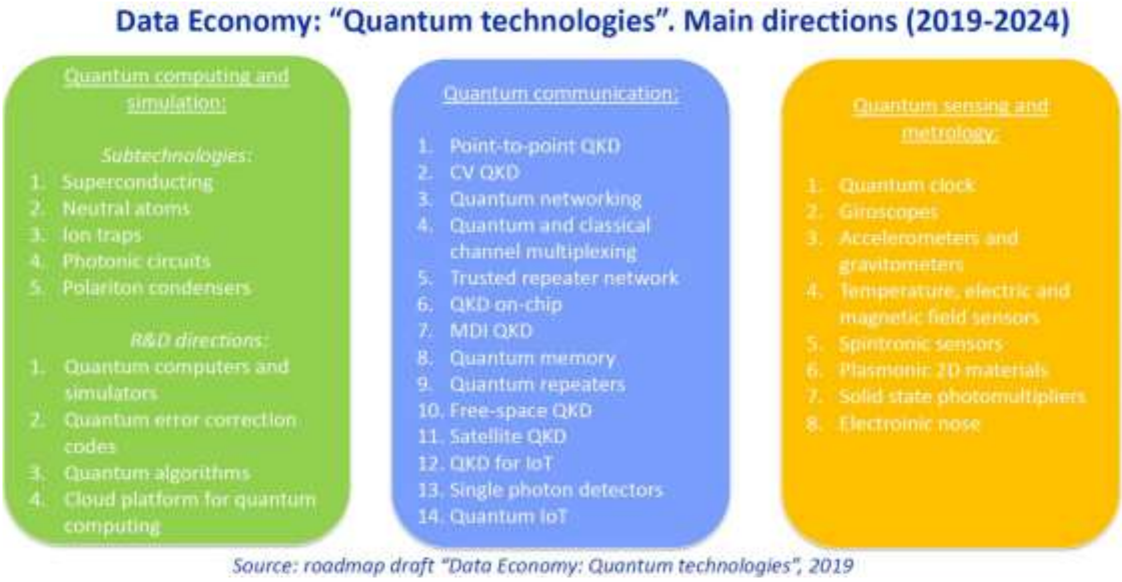


Figure 935: Russia's quantum plan priorities as of 2019. Source: Quantum communication in Russia: status and perspective by Vladimir Egorov, 2019 (22 slides).

In January 2022, Rosatom announced a plan to build a trapped ions quantum computer that would be made available on the cloud by 2024. It is being developed by the Russian Quantum Center and the Lebedev Physics Institute of the Russian Academy of Sciences. They are starting with 4 qubits and, as such, are very late compared to state of the art quantum systems coming from AQT, IonQ and Quantinuum who have about 20 to 30 qubits in-store. In July 2023, Russian scientists presented to their President Vladimir Putin a good intermediate result with a 16-qubit trapped ion quantum computer. Putin made a speech dedicated to the role of quantum technology.

As a reminder, 16 perfect qubits can be simulated on your own laptop with the open-source Quirk emulator, so this Russian computer won't bring any competitive advantage nor change the course of the aggression war in Ukraine. How about getting some details on the ion traps used in the Russian computer and on the qubit fidelities? Not so fast my friend. You'll have to wait for a while.

RQC Russian Quantum Center

Before all of that, the **Russian Quantum Center** was created in 2010, a private research center dedicated to the various application areas of quantum computing, including quantum cryptography. It employs over 200 researchers.

It covers many quantum computing branches: superconducting, trapped ions, photons and NV centers qubits, quantum sensing, QKD and a single photon detector. They collaborate with some international research organizations in the USA (MIT), Canada (University of Calgary), Germany (Max Planck Institute for Quantum Optics) and UK (University of Bath)[4590].

[4590] This information comes from Evaluation Report of Russian Quantum Center, 2017 (7 pages). See also Quantum technologies in Russia, October 2019 (9 pages).

The St. Petersburg **ITMO University** has a QKD research laboratory as well as the **Kazan Quantum Center** which has deployed a QKD on a 160 km network in Kazan. The country also plans to launch a QKD quantum key communication satellite in 2023. A few other laboratories are invested in quantum technologies such as the **NTI Center for Quantum Communications** at MISiS University and the **NTI Technologies Centre** at Moscow University.

On the industry vendors and startup scene, most of the players are in the cybersecurity and quantum cryptography domain, with only three identified startup, **Photonanobeta** (a producer of diamonds with cavities), **Qrate Quantum Communications** and **QSpace Technologies**, the others being established companies, such as **Infotecs**, **Scontel** and **Smarts QuantTelecom**[4591].

Africa, Near and Middle East

Israel

Israel was relatively quiet about quantum technologies until 2018 apart from Gil Kalai from the Hebrew University of Jerusalem who, since 2013, has shown a deep-rooted skepticism about the future of quantum computers. I was relatively surprised in 2018 to find out that the country was not very visible in the quantum research and entrepreneurship.

It was a stark contrast with other fields where this relatively small country has a significant impact worldwide: in software, artificial intelligence (where they have in excess of 400 startups), Internet, semiconductors, electronics, medtechs and biotechs to name a few.

Government funding

Then, things started to change[4592]. After a study carried out in 2017 by Uri Sivan from Technion to evaluate the country's quantum technologies efforts, a first initiative to better fund its research was launched in 2018 by the country's government and endowed with 75M€. It went mainly to Technion and the University of Haifa, which wants to design its own quantum computer and had also received a donation of $50M.

In December 2019, a panel of specialists commissioned by the government proposed a plan to invest $350M over 6 years in quantum technologies[4593]. In just a few months, the government approved this proposal which ended up with a funding of $400M, 60% of it supposed to fund academic research. The plan is fairly standard with an investment in human capital (faculty hiring and launching training courses), research and scholarships funding, attracting foreign researchers and the likes. The usual quantum technologies domains were picked with computing hardware and components, telecommunications, cryptography and sensing (with $40M), particularly with its military applications.

The Israel National Quantum Initiative (INQI) plan follows-up in naming the US plan announced in December 2018.

In March 2021, Israel announced it planned to create its own quantum computer, allocating a budget of $60M taken out of the national initiative. The goal is to create 30- to 40-qubit quantum systems. It was to take bids from both local players and international vendors, with a build or buy approach depending on the outcome. In July 2022, the government announced the creation of a quantum

[4591] Here are some elements on this ecosystem: Quantum communication in Russia: status and perspective by Vladimir Egorov, 2019 (22 slides).

[4592] But not to the extent of this title; Israel has become a powerhouse in quantum technologies by David Kramer, Physics Today, December 2021 (4 pages).

[4593] See Israel joins the quantum club by Uri Berkovitz, December 2019 and Israel joins the race to become a quantum superpower by Anna Ahronheim and Maayan Hoffman, Jerusalem Post, December 2019.

computing R&D center with a budget of $30M over a three-year period. It selected their local star vendor, Quantum Machines to create the center that is supposed to work on superconducting qubits, cold atoms, trapped ions and quantum optics computers[4594]. The company will work with Classiq and Elbit Systems. Still, the ploy became a "buy" with Israel announcing the same month the procurement of an Orca Computing (UK) photonic quantum computer. They also work with Inleqtion who already partners with Quantum Machines (like Pasqal in France) and Classiq[4595].

In January 2023, $32M were added by the Israel Innovation Authority to a new quantum computing consortium assembling defense industry companies and startups, to be focused on superconducting and trapped ion QPUs. It is probably related to the Weissman Institute (trapped ions) and Bar-Ilan University where Michael Stern's team is working on fluxonium qubits.

Research

There are about 125 "principal investigators" in quantum technologies research in Israel spread in the following research institutions:

Ariel University with its Wireless Communication & Radars Lab, located in the Israeli settlement of Ariel in the middle of the Palestine West Bank. They work on millimeter wave and Terahertz sensors.

Bar Ilan University (Ramat Gan near Tel Aviv) with the Photonics and Optics group (Avi Peers, Eli Barkai and Dror Fixler) and the Institute of Nanotechnology and Advanced Materials which works on superconducting qubits (Michael Stern). These are integrated in QUEST (Quantum Entanglement in Science and Technology), a quantum research center launched in 2017.

Ben-Gurion University of the Negev (Beer Sheva) with the Sensing Technologies Lab (Asaf Gros), Quantum Magnetometry Group (Reuben Shuker) and the Atom Chip Group (Ron Folman). The startup AccuBeat (1993) which produces rubidium quantum atomic clocks, is a product of this university.

Hebrew University of Jerusalem with the Nano-Opto Group and Time Dissemination Group, and the Center for Nanoscience and Nanotechnology that works on superconducting qubits. The University also established its Quantum Information Science Center in 2013 to focus on secure quantum communication (QKD).

Technion (Haifa) with Quantum Information Processing lab (Tal Mor) which works on photonics, NMR and silicon qubits, the Russell Berrie Nanotechnology Institute created in 2005 (Gadi Eisenstein) and the Helen Diller Quantum Center which is working on photonics, quantum dots, superconducting qubits and cold atoms (Yosi Avron). On top of this, the Technion AdQuanta Group is a sort of research accelerator to develop new concepts and proof-of-concepts (sensing, quantum optics, imaging, polaritons, etc.), led by Ido Kaminer.

Tel Aviv University with its QuanTAU, their Quantum Science and Technology Center, working among other things on superconducting qubits.

Weizmann Institute of Science in Rehovot with the Center for Quantum Science and Technology (Adi Stern), the Quantum Circuits Group (Serge Rosenblum) working on superconducting qubits[4596]

[4594] See Quantum Machines to establish Israeli quantum computing center in $30 million deal by Meir Orbach, CTECH, July 2022.

[4595] See Inside ColdQuanta's Role in the Israel National Quantum Initiative (INQI) by Brian Siegelwax, The Quantum Insider, August 2022.

[4596] See A superconducting quantum memory with tens of milliseconds coherence time by Ofir Milul et al, Weizmann Institute, February 2023 (19 pages).

and the trapped ions group (Roee Ozeri) which created the first Israeli quantum computer in 2022 with 5 trapped ions qubits[4597] and is also looking at ways to use it in simulation mode[4598].

Quantum industry

The Israeli startup scene was a little modest until 2020 but is developing strongly since then. The most visible startups in the field are **Quantum Machines** (qubit control hardware and software), **ClassiQ** (error control and compilers), **Accubeat** (atomic clocks), **Elta** (sensing), **Hub Security** (PQC in HSMs), **Mellanox Technologies** (quantum communications, part of Nvidia), **PhotonicsQ** (cold atoms interconnection), **Qedma Quantum Computing** (software tools), **QuantLR** (QKD), **Quantum Source Labs** (photonic computing), **Quantum Transistor** (NV center QPU), **Raicol Crystals** (photonics), **Tabor Electronics** (RF electronics). Google's R&D lab in Tel Aviv also hosts researchers in quantum computing.

The **Quantum Technologies Consortium** created in 2019 assembles research institutions and industry vendors.

South Africa

South Africa launched a formal Quantum Technologies Initiative in 2023 that consolidates the efforts from the University of the Witwatersrand (Wits University), Stellenbosch University, University of KwaZulu-Natal, Cape Peninsula University of Technology and the University of Zululand. Wits University is coordinating this group with an initial funding of $3M.

These five universities will expand the number of quantum research centers. They will focus on software and applications.

United Arab Emirates

Each and every country wants « its » quantum computer. Even the emirate Abu-Dhabi got the quantum virus and decided to "build" its own quantum computer, even if "build" or "buy" are interchangeable in such a situation since it is a quantum annealing superconducting system coming from Qilimanjaro.

Still, it comes after the establishment of a Quantum Centre at the Technology Innovation Institute (TII) which hosts about 20 researchers coming from the Emirates and from various countries: Italy, Spain, Brazil, Greece, UK and Germany. This lab complements other labs in robotics, cybersecurity and energy. It even has some qubits manufacturing tooling.

Jose Ignacio Latorre is the chief scientist of this quantum research laboratory. He is a professor of theoretical physics at the University of Barcelona currently on leave, cofounder of Qilimanjaro and the director of the Singapore CQT since July 2020. Their key partners are Qilimanjaro, Universitat Catania in Sicilia and INFN, an Italian research network.

The QC-TII organized a webinar conference in June 2021, Atomtronics@Abudhabi with about 500 participants. In April 2023, IonQ announced an agreement with TII to provide it with access to IonQ Aria QPU. In August 2023, Abu Dhabi University created a quantum academic lab in Abu Dhabi in partnership with Vernewell Group. The lab will use SpinQ's NMR educational quantum computers.

[4597] See A huge leap: Israeli researchers build country's first quantum computer by Ricky Ben David, Times of Israel, March 2022 and Trapped Ion Quantum Computer with Robust Entangling Gates and Quantum Coherent Feedback by Tom Manovitz, Yotam Shapira, Lior Gazit, Nitzan Akerman and Roee Ozeri, PRX Quantum, March 2022 (14 pages). These qubits use strontium. Their fidelity if 99.64% for a single qubit gate and 97.3% for two-qubit gates, which is not stellar.

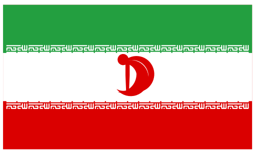
[4598] See Programmable quantum simulations on a trapped-ions quantum computer with a global drive by Yotam Shapira, Roee Ozeri et al, August 2023 (7 pages).

It will also work on cybersecurity, optimizations algorithms, quantum blockchain. Vernewell Group (Dubai) is a service and management consultant company.

Intqlabs (2022, Dubai) is a contract research company created by Ankur Srivastava, from India, formerly an independent researcher, not affiliated with any lab. The company works in various fields: quantum computing, reversible classical computing, geo-magnetism and cybersecurity. They are talking about some form of "quantum and reverse computing" that would solve all current problems with quantum computing.

It is quite difficult to figure out whether it is some form of quantum computing or of classical reversible computing. All of this is patent pending, meaning that for at least a year and a half, you will have no idea what it's all about. Wait and see. They plan to license their technology to hardware manufacturers. They have otherwise created NGNSS (New Global Navigation Satellite System), an alternative to classical GPS solutions which as its name doesn't tell, is not requiring a satellite to function and uses magnetism detection and mapping.

Iran



Israel is not the only country in the Near and Middle East that seems to be invested in quantum research. **Iran** is also involved with at least two research laboratories, **Sharif University** which is working on quantum physics in partnership with Canada and the **Quantronics Lab** of the Iranian Technological University which is dedicated to quantum communication (QKD)[4599].

The **University of Tehran** also hosts quantum researchers working on spin qubits[4600]. The country even organizes its conference on quantum computing, the **IICQI,** since 2007[4601]. It opened its first large laboratory dedicated to quantum physics, the **National Center for Quantum Technology**, also named ICQTS, in 2021[4602]. Its website is not accessible. The Iran research community is obviously not well connected to the Western world. You can see from time to time their contribution in various arXiv preprints, mostly on theoretical work[4603].

In June 2023, the country's Navy officials showcased the first Iran "quantum processor" in an event. The pictured processor was a classical Digilent ZedBoard Zynq-7000 ARM/FPGA SoC Development Board bought for $600 in the USA[4604]. At best, it is using some quantum inspired algorithm running on the FPGA board or the board is used to drive some qubits.

The announcement was ridiculed but it is not that far from many quantum computing case studies showcased here and there in the Western world which happen to be just quantum inspired or running in classical emulation mode. In the End, the Iran government withdrew its statement and said it was misunderstood by their local media[4605].

[4599] Source: Iranian research in quantum information and computation, June 2016.

[4600] See Characterization and Coherent Control of Spin Qubits with Modulated Electron Beam and Resonator by Soheil Yasini et al, March 2023 (7 pages).

[4601] See http://iicqi.sharif.edu/.

[4602] See Iran opens National Center for Quantum Technology, April 2021.

[4603] See Case Study of Decoherence Times of Transmon Qubit by H. Zarrabi et al, September 2023 (7 pages).

[4604] See Iran's Claim Of Quantum Processor Draws Ridicule, June 2023.

[4605] See Iran Admits Its Quantum Computer Had Zero Quantum in It by Francisco Pires, Tom's Hardware, July 2023.

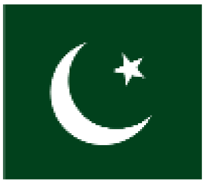
Qatar

Qatar has also some ambitions in quantum technologies. In 2022, Hamad Bin Khalifa University (HBKU) announced the creation of the Qatar Center for Quantum Computing (QC2).

It received a $10M research grant from Barzan Holdings, a local defense industry vendor.

Let's also mention **QUANTUN**, the Quantum Tunisian Network launched in October 2021 to consolidate the work of quantum researchers in the country and **South Africa**'s Quantum Initiative (Sa QuTI) launched in September 2021.

Pakistan



Pakistan started to appear in the quantum radar in October 2022 when announcing a partnership with Ireland[4606]. Two quantum research teams can be identified in the country, **QuantuC**, with five full time researchers at the University of Lahore is working on quantum chemistry quantum algorithms and another one works on quantum physics at **LUMS University**[4607].

Asia-Pacific

China

As in many technology sectors, **China** is loudly and clearly asserting its ambitions and power in quantum technologies[4608] [4609]. As in the UK, this investment was taken seriously by the government and as early as 2013 with the involvement of Chinese president Xi Jinping, during a visit to the Anhui laboratory, focusing on quantum cryptography, combined with a training session.

Government funding

In 2015, Xi Jinping integrated quantum communication into the country's scientific priorities, in 13th plan covering 2016-2020. Maybe was it a benefit from having a government comprising a majority of politicians with some scientific background and also the result of Snowden's revelations on NSA's spying capabilities in 2013.

Assessing the government and regional investments in quantum technology is not an exact science. Many amounts are hard to check, and some are clearly false. The current $15B to $25B estimates are most certainly largely exaggerated and should be used using the conditional tense[4610] [4611] [4612] [4613].

[4606] See President Arif Alvi stresses on enhancing quantum of bilateral trade with Ireland, October 2022.

[4607] See Syed Babar Ali School of Science and Engineering.

[4608] See Quantum Hegemony? China's Ambitions and the Challenge to U.S. Innovation Leadership, CNAS, 2018 (52 pages).

[4609] and Quantum information technology development in China by Yuao Chen, June 2019 (25 slides).

[4610] See The quantum tech arms race is on by Stuart Rollo, Asia Times, March 2022.

[4611] See The Quantum Insider Report Details China's Emergence as a Global Leader in Quantum Investment and Research by Matt Swayne, The Quantum Insider, March 2023.

[4612] See A quantum revolution: report on global policies for quantum technology, CIFAR, April 2021 (57 pages) is reporting on China investing $15.4B in the NPL lab.

[4613] See New Funding For Quantum Computing Accelerates Worldwide by Gil Press, Forbes, January 2023 contains a mention of $25B China investments in a chart from GQI Government Funding in Quantum Tech, 2023.

You see them in various McKinsey, BCG and World Economic Forum. It is tiring to see these unchecked and non-official numbers since China's government has never communicated in a consistent and consolidated manner about its investments in quantum contrarily to most developed countries. Why are these false numbers circulating broadly in the Western world without being checked? For many different reasons. Unchecked numbers from untrusted sources circulate easily when broadcasted by Western trusted sources.

People don't have time to check numbers or just look at their scale. There are sometimes currency errors[4614] or aggregate amounts with funding not related to quantum science[4615]. There is also some internal China politics at play with some regions playing an announcement game to compete with each other or with the country's government[4616]. Many economic indicators are known to be false in China's communication. Some, like in the USA, are also happy to amplify the strategic and business threat coming from China to drive more policy makers attention and public investments[4617].

Very few sources, like a February 2022 report from **Rand Corporation**, are pinpointing this and detailing China's real quantum investments which are probably either on par or below similar investments in the USA and Europe, in the broad range of $2B to $4B across 10 years[4618]. The report estimated that only 1,700 students earned a PhD in quantum technology in China.

In a long New Yorker 2022 piece, a senior researcher from China also did cast doubts on the $15B estimate, saying the truth was probably below $4B, in line with what I had found initially[4619].

So, let's try to find some realistic numbers here and identify their sources. Government's funding of quantum R&D was of $160M in the **11th plan** covering 2006-2010, from $490M to $800M in the **12th plan** covering 2011-2016 and $320M to $337M in the **13th plan** starting in 2016-2020,

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[4614] See China is building a massive multi-location national-level quantum laboratory by Runhua Zhao, Technode, September 2018, which mentions the 10 billion total funding for the Hefei National Laboratory for Quantum Information Sciences announced in 2017. It happens that these billions are in RMB which stands for the Chinese Yuan and has a value of $0.14. It then means that the Hefei lab announced budget was of $1.4B. But the paper also mentions a long term funding of RMB 100B, meaning $14B, by the Anhui province. Which is enormous. But the investment duration is not indicated. 5 or 30 years?

[4615] See Chinese Quantum Companies and National Strategy 2023 by Jakob P, The Quantum Insider, April 2023, mentions "*Two legislative initiatives aiming to prepare the future Chinese for a quantum future funded by about 4% of the Chinese GDP, approximately $150-200bn yearly.*". It happens to be a funding for scientific education which cover quantum and many other topics.

[4616] See Industrialization of Quantum Communication: China is planning to build a 100-billion-level national laboratory by Qian Tongxin, Yicai, translated from Chinese by Google Translate, September 2018.

[4617] See Why the US needs a 'quantum Oppenheimer' to beat China in the quantum race, by Duncan Earl, Qubitekk, May 2023 which write "*The Chinese quantum programme [...] is well resourced, with an annual budget estimated to be in the billions of dollars…*".

[4618] See An Assessment of the U.S. and Chinese Industrial Bases in Quantum Technology by Edward Parker, Rand Corporation, February 2022 (140 pages). Extracts on China: « *Even higher levels of announced investment are associated with the main Chinese quantum research facility, the Hefei National Laboratory for Physical Sciences at the Microscale (HFNL), which is led by Pan and a part of the University of Science and Technology of China in Hefei, Anhui Province. Chinese-language news media reported $1.06 billion in laboratory funding in 2017, and the Anhui Business Daily newspaper reported plans (though not confirmed funding) for $2.95 billion per year over the 2017–2022 period.* ***These announced spending goals are in stark conflict with the government-wide spending estimates*** *given in Table 4.11. The $1.06 billion start-up funding that Chinese news media announced in 2017 for Pan's quantum laboratory alone hugely exceeded Pan's own 2019 estimate for the PRC's total government spending over the same time period.* ***Our team's China experts assess that these conflicting reports of funding levels are not unusual in China; the PRC government often announces ambitious (and often highly politicized) spending goals, and it is not uncommon for these goals to go unmet****… […] In summary, official reports of the PRC's government investment in quantum R&D in recent years have varied widely, from a low of $84 million per year (Pan's estimate) to a high of at least $3 billion per year (the Anhui Business Daily's reported funding for Pan's laboratory). We are unable to assess from public information which figure is more accurate. By comparison, the U.S. government has spent $450–$710 million per year in recent years;* ***we cannot determine whether the PRC total is higher or lower than this amount***.".

[4619] See The World-Changing Race to Develop the Quantum Computer by Stephen Witt, The New Yorker, December 2022. Quote: "*In a paper published in Science, in 2020, a team led by the scientists* ***Lu Chao-Yang*** *and Pan Jian-Wei announced that their processor had solved a computational task millions of times faster than the best supercomputer. […] Lu Lu immediately began debunking claims made by his competitors, and even claims made about his own effort.* ***One widely reported figure stated that China has invested fifteen billion dollars in developing a quantum computer. "I have no idea how that was started," Lu said. "The actual money is maybe twenty-five per cent of that***.".

supplemented by $640M of region funding[4620] [4621] [4622]. We don't know if this is covering legacy or just incremental funding.

Later on, China's government communicated an amount of $34B corresponding to several scientific priorities including quantum. This represented probably less than $1B between 2016 and 2020 and a total of $1.76B over 10 years. Other estimates were lower, in the $1.5B range for the 2006-2020 period[4623]. In 2021, China announced its new **14th year plan** on research, with a 11% global funding increase but with no details regarding quantum investments[4624]. At most, it would bring their 2006-2027 total quantum investments to about $4B, but definitively not $10B to $25B.

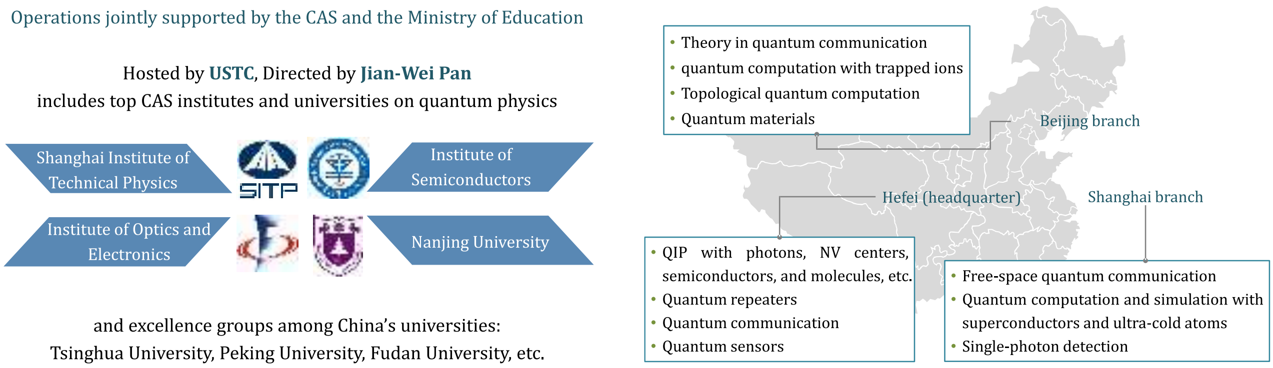


Figure 936: China's quantum ecosystem. Source: Chinese QC Funding by Xiaobo Zhu, 2017 (35 slides).

These investments are mainly spread over Beijing, Shanghai, and Hefei in the Wuhan province (500 km west of Shanghai) (Figure 936). They specialize respectively in quantum communications, trapped ion computing, topological qubit computing and quantum materials for Beijing, silicon qubit computing, NV centers and photons, quantum communications and metrology in Hefei, and communication, superconducting and cold atom qubit computing and photon detection in Shanghai. The Chinese plan is coordinated by the **USTC** (University of Science and Technology of China) of the Chinese Academy of Sciences (CAS) and under the leadership of Jian-Wei Pan[4625].

The most ambitious project is the "$10B" national quantum research lab that partially opened in 2020, the **NLQIS** (National Laboratory for Quantum Information Sciences) of Hefei. This laboratory is focused on quantum computing and metrology for military and civilian applications. The lab currently hosts between 300 and 600 researchers depending on the estimates (Figure 937).

Figure 937: Hefei's quantum lab as pictured in 2022 (source). Added in 2023.

[4620] See China's Quantum Ambitions by Elsa B. Kania and John K. Costello, Center for New American Security, 2018 (9 pages).

[4621] The 2016 quantum roadmap is available in "Quantum Leap: The Strategic Implications of Quantum Technologies by Elsa Kania" and John Costello (part 1 and part 2). See also Chinese QC Funding by Xiaobo Zhu, 2017 (35 slides).

[4622] See Quantum information research in China by Qiang Zhang, Feihu Xu, Lili, Nai-Le Liu and Jian Wei Pan, IOPScience, 2019 (8 pages) which provides other estimates for the 11th, 12th and 13th plan investment in quantum science.

[4623] See FactBasedInsight's Quantum Landscape 2020: China, March 2020.

[4624] See Translation: 14th Five-Year Plan for National Informatization, December 2021, DigiChina.

[4625] See The man turning China into a quantum superpower by Martin Giles in MIT Technology Review, December 2018.

The initial plan announcement was about 1,800 researchers, including 560 full-time researchers spread across two labs, three universities and a fab[4626 4627]. The zone has its own “Quantum Avenue” of several hundred meters in Hefei, Yunfei Road, with the NLSIS lab and 20 startups including the three largest quantum startups in China (Origin Quantum, QuantumQTesk, Ciqtek). These seem to be the largest employers here thanks to the burgeoning business of selling quantum computer to universities in China[4628].

Research

On the quantum computing side, Chinese laboratories are testing all imaginable qubit technologies and regularly announce technological progresses. They seem to be rather ahead in photon qubits as we have seen about their boson sampling experiments but not really with other qubit types. In 2017, the Hefei laboratory announced the realization of a test system of 10 superconducting qubits in aluminum and sapphire[4629] (Figure 938). The two qubit gates error rate of 0.9% was not best in class.

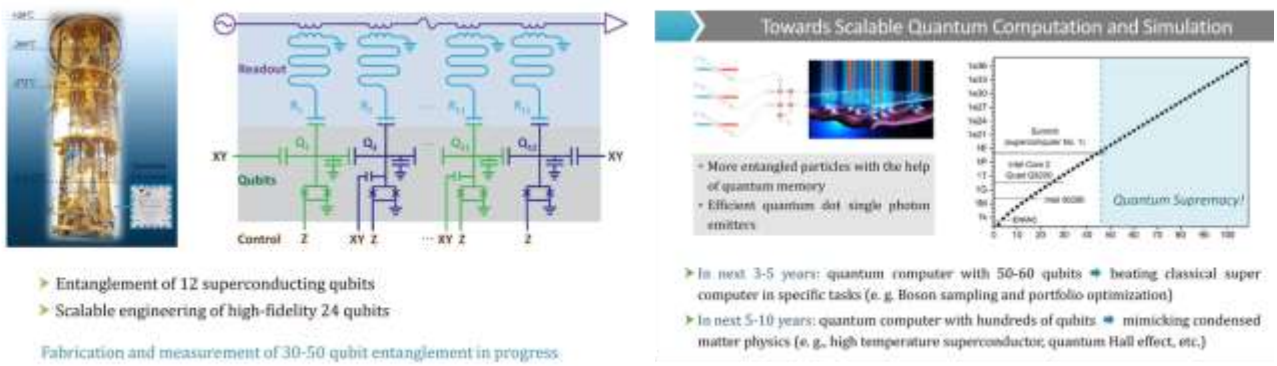


Figure 938: China superconducting state of the art in 2017. Much progress has been accomplished since then. Source: 10-qubit entanglement and parallel logic operations with a superconducting circuit by Chao Song et al, 2017 (16 pages).

They were at 24 superconducting qubits in 2019 (Figure 939). Their fidelity is 99.9% on single-qubit gates and 99.5% on two-qubit CZ gates, is much better[4630].

Their T_1 duration, which defines the coherence time of the qubits is 40 µs, equivalent to what IBM obtains with its Q System One at 20 qubits. The Jian-Wei Pan team planned to reach 50 superconducting qubits by 2023.

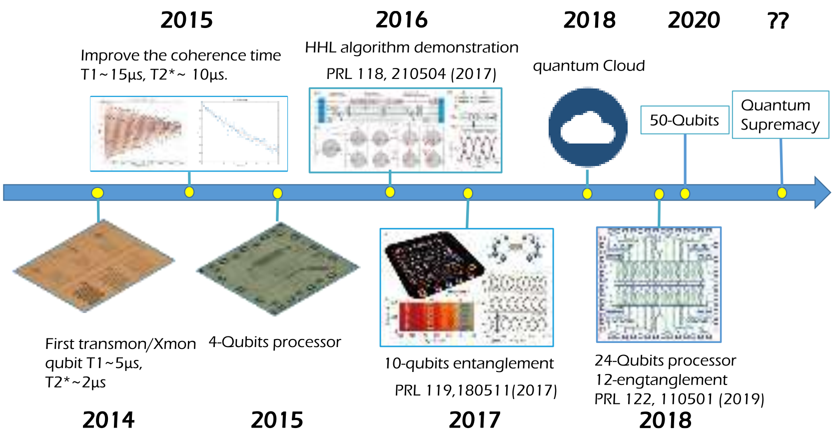


Figure 939: Source: Superconducting Quantum Computing by Xiaobo Zhu, June 2019 (53 slides).

It delivered early on this promise in May 2021 with 62 superconducting qubits, implementing quantum walks, which makes comparisons difficult, for example with IBM’s 65 superconducting qubits

[4626] See China building world’s biggest quantum research facility by Stephen Chen by September 2017.

[4627] See Hefei’s plan to create a national laboratory for quantum information science has been reported to the state council, EEworld, May 2018.

[4628] See University of Science and Technology of China, hiding a Chinese quantum sky group, iNews, September 2023.

[4629] See 10-qubit entanglement and parallel logic operations with a superconducting circuit by Chao Song et al, 2017 (16 pages).

[4630] Source: Superconducting Quantum Computing by Xiaobo Zhu, June 2019 (53 slides).

system launched online in September 2020[4631]. They followed with the announcement of a 66 superconducting qubits system quantum advantage, being seemingly a copycat of Google's Sycamore processor architecture and benchmarking and in 2023, they had 176 qubits with a new version of Zuchongzhi-2, which puts them ahead of all other players besides IBM which is currently at 433 qubits. This doesn't account for gate fidelities that were not available for Zuchongzhi-2 as of this writing.

The Chinese scientific level is good but not yet stellar. They mostly improve technologies developed in Western countries and do not generate many new ideas. On the other hand, they create experiments like boson sampling or QKD deployments at a large scale.

China does not seem to have a significant influence in the academic world on quantum algorithms and programming tools. We must never forget the strategic role of software and platforms in the digital economic battles! It looks like History is repeating itself in China for this respect.

Quantum industry

Public-private partnerships have been put in place, such as with **Alibaba,** who invested in the USTC to launch in 2015 the Shanghai Alibaba Quantum Computing Laboratory. It focuses on quantum cryptography and quantum computing. Quantum cryptography could be used to secure e-commerce transactions and data centers connections. In January 2018, Alibaba even launched a cloud-based 11 qubits system developed by USTC (Figure 940).

Figure 940: Alibaba's 11 qubit processor.

Alibaba is a serious contender in the superconducting qubit space with its fluxonium qubits variation.

Baidu launched in 2018 its **Institute for Quantum Computing,** which is being deployed in their Technology Park in Beijing with around ten people as of September 2019. It is led by Runyao Duan, a specialist in quantum information theory, with Artur Ekert as board member. They are mainly developing quantum software stacks in their QIAN full stack quantum software and hardware platform. They developed a quantum emulation solution in their cloud resources named Quantum Leaf[4632] and a bunch of other tools: Paddle Quantum (quantum machine learning), Quanlse (quantum pulse control for superconducting qubits), Qulearn (a quantum knowledge base), plus quantum error processing, measurement and control[4633], network architecture and quantum electronic design automation. They are also working on a quantum Internet. In August 2022, they announced their first superconducting qubits quantum computer with 10 qubits named Qian Shi to be expanded later to 36 qubits.

Tencent also launched a Quantum Lab in 2018, led by Shengyu Zhang and based in Shenzhen. They plan to offer quantum computing resources in the cloud. The lab publishes work in quantum simulation and machine learning algorithms. It is developing QuAPE, a quantum control microarchitecture for superconducting qubits supporting multi-core QPUs parallelism that was prototyped with a FPGA circuit.

We can also mention the involvement of **ZTE** and many telecom operators and manufacturers in the deployment of secure fiber networks by QKD (**China Telecom, China Cable, China Comservice, China Unicom**) as well as various banks that use them. **China Telecom** announced it invested $434M in the creation of a quantum technology group in May 2023.

[4631] See Quantum walks on a programmable two-dimensional 62-qubit superconducting processor by Ming Gong, Science, May 2021 (34 pages).

[4632] See Introduction to Baidu Quantum Program by Shuming Cheng, June 2019 (9 slides). They notably propose the Paddle Quantum library, released on GitHub, which supports neural network QML, quantum chemistry and optimization tools. All this in quantum emulation on classical data centers.

[4633] See for example Efficient characterization of quantum nondemolition qubit readout by He Wang and Ya Cao, August 2022 (11 pages).

They deployed “Tianyi Quantum Secret Talk”, a smartphone service seemingly supporting a QNGR security key generation within a SIM card, a solution jointly developed with ZTE and QuantumCTek which had one million users as of October 2023.

China’s quantum startups ecosystem is quite poor compared to the country GDP and its public investments in quantum science, whether you use $4B or $25B as a reference (Figure 941). The country quantum startup and small business ecosystem is the 10th largest in the world in number of entities and the 5th in capital investments, in between France and Israel[4634]. When considering only startups, China is 6th in number of entities[4635]. One of the reasons being that public research laboratories are well funded and have less incentive to create companies. One exception is **Origin Quantum Computing** which raised a record $163.4M to develop a full-stack superconducting qubits offering.

Most other startups in China are in the quantum communication field, like **QuantumCTek** and **Qasky Science,** and have joined with ID Quantique and Battelle to form the **Quantum-Safe Security Working Group,** which federates the quantum cryptography industry.

A project led by Shan Lei and Wang Shaoliang from **Anhui University** is to create a dilution refrigerator operating at 8.5 mK. They will spin out a startup from their lab in Hefei[4636]. It is mandated by the export controls from Wassenaar’s agreements which limits China’s capacity to import large dilution refrigerators like the ones coming from Bluefors.

Figure 941: a market map of China’s quantum industry vendors. (cc) Olivier Ezratty, 2023.

China’s dominance often touted is also exaggerated in the patents area. There was indeed an increase of China patents in quantum technologies from 137 in September 2020 to a total of 804 by October 2022 according to the Global Quantum Computing Technology Patent Filings Ranking List (Top 100) from IncoPat Global Patent Database[4637]. It includes 234 patents from Origin Quantum. The US accounts for 40% of the patents, IBM being first with 1,323 patents, China with 15% and Japan with

[4634] Source: my own database of quantum and small business startups used to create various charts in this book and as of July 2023.

[4635] See The landscape of the quantum start-up ecosystem by Zeki Can Seskir, Ramis Korkmaz and Arsev Umur Aydinoglu, EPJ Quantum Technology, 2022 (15 pages).

[4636] See China to make its own quantum computer fridges by Jeff Pao, AsiaTimes, June 2023.

[4637] See Quantum computing patent filings surge in China by Zhu Lixin, China Daily, November 2022.

11%[4638]. IBM alone has more patents on quantum technologies, mostly on quantum computing, than China.

This can also be found in the work from Zeki Seskir and Kelvin W. Willoughby who created a supervised machine-learning method to create a new classification to build a cleaner quantum technologies patents dataset[4639]. The hybrid human/AI tool analyzed 11,600 patents and eliminated 6,000 patents that were not describing a genuine quantum technology invention. In the end, the USA and China seem on par with regards to the number of patents in quantum technologies.

Japan

Let's move to the rest of Asia, starting with **Japan**. The country stands out for its very active and long-term oriented fundamental research initiation of two key technological waves in quantum computing.

It started with the creation of the principle of quantum annealing by **Hidetoshi Nishimori** in 1998[4640]. Then, there was the creation of the first superconducting qubits in 1999 by **Yasunobu Nakamura**, **Jaw Shen Tsai** (both then at NEC) in liaison with **Yuri Pashkin** (Lancaster University, UK). Unfortunately, this was not turned into some industry lead.

Research

Japan's public research is conducted by several independent agencies attached to various ministries that fund public laboratories, university laboratories and research partnerships with companies[4641]:

- **JST** (Japan Science and Technology Agency) funded by the Ministry of Research and which funds deep techs research projects and also promotes science to the general public and international scientific collaboration. In 2016, JST launched a project by Yasunobu Nakamura of "Macroscopic Quantum Machines" to assemble 100 superconducting qubits.
- **RIKEN** (Institute of Physical and Chemical Research) also funded by the Ministry of Research (MEXT), with a total of about 3,000 researchers including a total staff of 210 in quantum technologies and a yearly budget of $30M[4642]. It includes a laboratory in theoretical quantum physics, headed by Franco Nori, and another in photonics, headed by Katsumi Midorikawa. They work in particular on silicon and superconducting qubits. It collaborates with Fujitsu since 2020 to build a supercomputing qubits computer.
- **NICT** (National Institute of Information and Communication Technologies) includes the Quantum ICT Advanced Development Center, which specializes in quantum cryptography. In July 2017, the institute carried out a demonstration of quantum telecommunications using a microsatellite, reminiscent of the Chinese experiment with the Micius satellite carried out the same year.
- **NII** (National Institute of Informatics) includes a hundred or so researchers and focuses on research in theoretical quantum computing but also works on superconducting and silicon qubits.

[4638] See Top 100 Global Quantum Computing Patents Announced - Huawei and Tencent in the list by James Lopez, TechGoing, October 2022.

[4639] See Global innovation and competition in quantum technology, viewed through the lens of patents and artificial intelligence by Zeki Can Seskir and Kelvin W. Willoughby, February 2023 (22 pages).

[4640] See Quantum annealing in the transverse Ising model by Tadashi Kadowaki and Hidetoshi Nishimori, 1998 (9 pages).

[4641] The most active quantum laboratories are located at the universities of Tokyo, Kyoto, Tohoku, Osaka, Nagoya, Keio, Tsukuka and Hokkaido. See Activities on Quantum Information Technology in Japan by Akihisa Tomita, June 2019 (19 slides).

[4642] See RIKEN Center for Quantum Computing Activity Report, May 2023 (58 pages).

The Japanese-French Laboratory for Informatics (JFLI) created in 2009 is based in Tokyo and hosted at both the NII and the University of Tokyo. It brings together researchers from the Universities of Tokyo, Keio, NII, CNRS, Sorbonne University (LIP6), Inria and Université Paris-Sud. It covers fundamental physics, algorithms and studies the feasibility of large-scale quantum computing as well as quantum cryptography.

The laboratory is co-directed by **Kae Nemoto**, from the NII, one of the few women in this whole panorama.

- **NEDO** (New Energy and Industrial Technology Development Organization) which is attached to the Ministry of Economy and Industry, METI. It is particularly invested in quantum annealing with a project running from 2018 to 2022 with $4.5M per year.
- **AIST** (National Institute of Advanced Industrial Science and Technology) also funded by METI. It employs about 2300 researchers in all. Several laboratories appear to be dedicated to nano-material sciences. There is also a research group on precision measurement.
- **QST (**National Institutes for Quantum and Radiological Science and Technology**)** was launched in April 2016 with an annual budget of $487M. This impressive amount is not exclusively allocated to quantum technologies. It mainly covers the vast field of quantum sensing and in particular medical imaging.

The Japanese government had launched various quantum initiatives such as **PRESTO** (since 2016) or the **CREST** cross-cutting program (also since 2016) as well as the **ERATO** projects in 1981 (Exploratory Research for Advanced Technology).

The country's quantum initiatives are currently part of its Fifth Science and Technology Plan, running from 2016 to 2022. In a typical Japanese way, this plan is linked to a societal goal "Society 5.0" to bring cyberspace and physical space closer together to solve society's social problems and create a human-centered society (Figure 942). All this with AI, quantum sensors and cybersecurity.

Figure 942: Japan's classical societal angle to sell some new technology wave.

Here are a few leading researchers in Japan in addition to those mentioned above[4643]:

Akira Furusawa of the University of Tokyo has the ambition to create a large-scale quantum computing solution with photon-based qubits. He's teaming up with NTT (see below) and plans to create a scalable computer by 2030[4644].

Kohei Itoh of Keio University has been managing the Q-LEAP project since 2018, which focuses on assembling different silicon isotopes into CMOS components and on NV center based quantum magnetometry (video). He is also a partner of IBM's Q Lab in Tokyo.

[4643] Source: Q2B 2019 - International Government Panel, December 2019.

[4644] See Quantum computing: Japan takes step toward light-based technology - NTT, University of Tokyo and Riken aim for full-fledged system by 2030, Nikkei Asia, December 2021. The paper mentions a Japan $1.75B quantum plan. It is probably a mistake. See more reliable numbers in Concept of Quantum Technology Innovation hubs, 2021 (6 slides, broken link).

Yoshihisa Yamamoto (1950), a Stanford alumni and director of the NTT Physics and Computer Science Laboratory, who worked in photonics, QKD and quantum dots. He is very influential in Japan on the country's technological choices[4645]. He is the pilot of the Quantum Information Project (QIP), one of the national research program projects from FIRST selected in 2009 and which covered all branches of quantum applications[4646].

Yasuhiko Arakawa (1952) of the University of Tokyo specializes in semiconductor physics and optoelectronics, at the origin of new processes for the exploitation of quantum dots in sensing.

François Le Gall (1959) is a French researcher based at Kyoto University who specializes in quantum computing theory, mathematics, quantum algorithms and cryptography. He is also interested in distributed quantum computing (video). He has been living in Japan for more than 20 years.

Yasunobu Nakamura (1968) who specializes in superconducting qubits and serves at the RCAST (Research Center for Advanced Science and Technology) of the University of Tokyo and at the CEMS (Center for Emergent Matter Science) of RIKEN[4647].

Masahito Hayashi of Nagoya University was originally a mathematician who then became a specialist in theoretical quantum computing. He coordinated the ERATO project on theoretical quantum computing.

Masahiro Kitagawa of Osaka University specializes in atomic nucleus spin-based quantum sensing in nuclear magnetic resonance with notable applications in medical imaging.

Mio Murao who created and manages the Quantum Information Group at the University of Tokyo that bears his name (Murao Group). This group specializes in distributed quantum computing, quantum systems simulation algorithms, quantum telecommunication protocols and quantum algorithms. She is very fluent in English, which has enabled her to serve as a connecting point between Japan and research teams in the USA (video).

Nobuyuki Imoto of Osaka University is leading research in quantum cryptography and telecommunications.

Masahide Sasaki of NICT leads much of Japan's quantum cryptography efforts. In particular, he has contributed to the SOTA project for quantum key communication using satellites[4648].

Government funding

The *flagship* project **Q-LEAP** launched late 2019 by the Ministry of Research (MEXT) seems the most ambitious and aims to catch up with both China and the USA, even if an alliance with the USA also seems to be on the agenda[4649]. The roadmap extends to 2039 with $200M spread over 10 years. The program targets quantum computing, quantum sensing and next-generation lasers.

[4645] He is notably the co-author of the briefing note Quantum information science and technology in Japan, February 2019 (8 pages).

[4646] See First program overview.

[4647] See his presentation of the state of the art of quantum computing Development of quantum hardware towards fault fault-tolerant quantum computing by Yasunobu Nakamura (19 slides).

[4648] See QKD from a microsatellite: the SOTA experience, October 2018 (10 pages).

[4649] See Japan plots 20-year race to quantum computers, chasing US and China by Noriaki Koshikawa, November 2019 and Land of the Rising Qubit: Japan's Quantum Computing Landscape by James Dargan, December 2019.

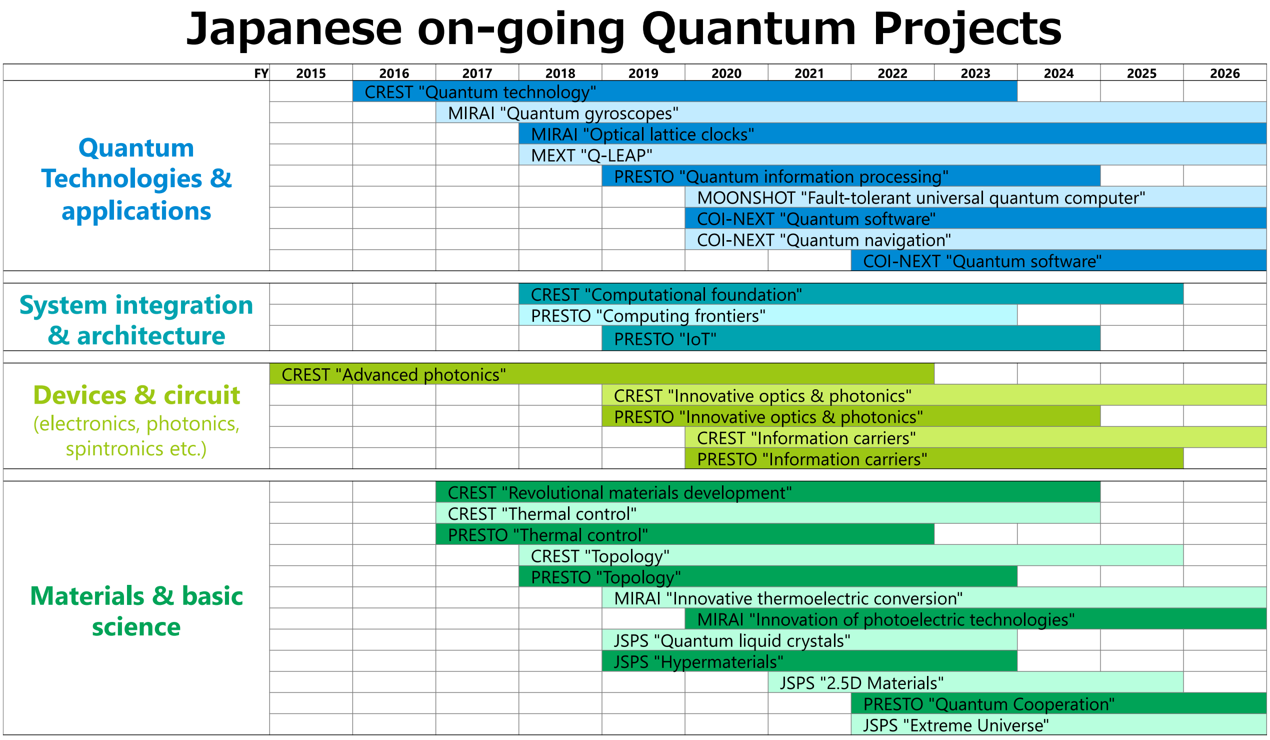


Figure 943: Japan's quantum project inventory.
Source: QC Benchmarks seen from science policy & science communication perspective by Yoshiaki Shimada, Center for Research and Development Strategy, Japan Science and Technology, May 2023 (15 pages).

Most qubits technologies are funded: superconducting, cold atoms, trapped ions and electron spin. This "Flagship" will run until 2027[4650].

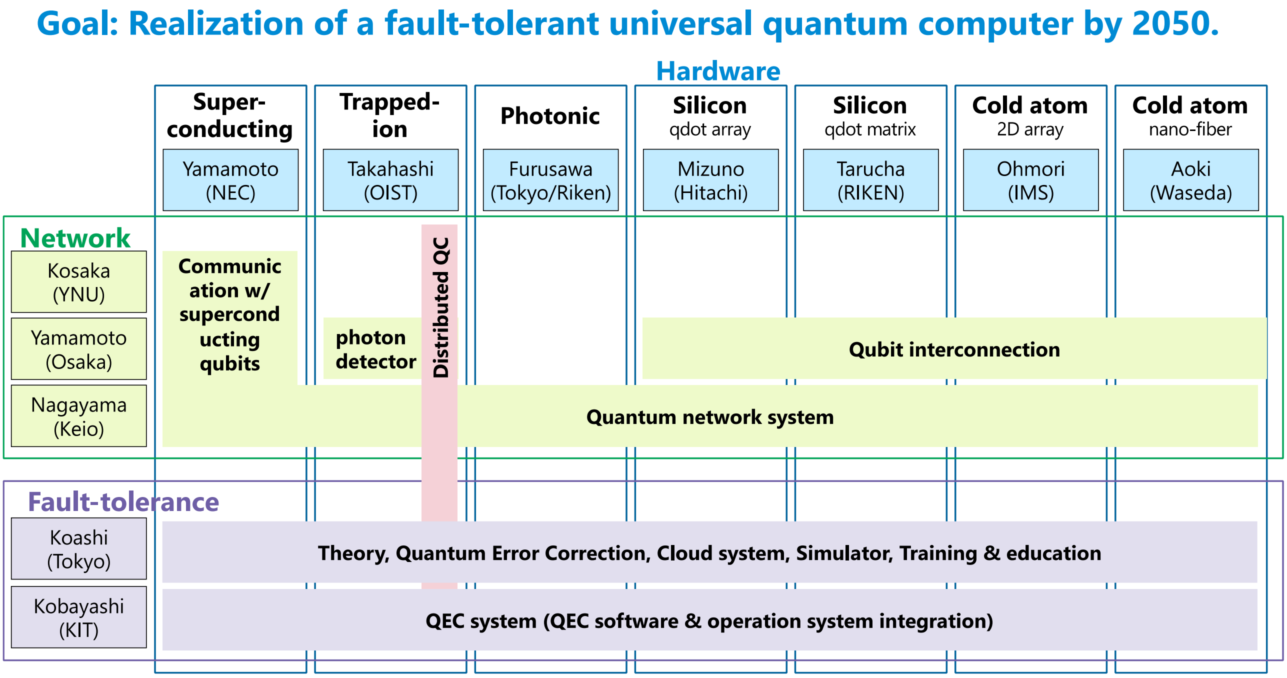


Figure 944: Japan's quantum computing project inventory per qubit type.
Source: QC Benchmarks seen from science policy & science communication perspective by Yoshi-aki Shimada, Center for Research and Development Strategy, Japan Science and Technology, May 2023 (15 pages).

[4650] See Activities on Quantum Information Technology in Japan by Akihisa Tomita, June 2019 (19 slides).

As part of its broad quantum computing plan, the Ministry of Economy, Trade and Industry is investing $31.7M to create a cloud quantum computing offering, led by the University of Tokyo. This cloud platform will enable access to both 27-qubit and 127-qubit QPUs from IBM, the first one being installed at the University[4651].

So, how much is Japan investing in quantum technologies? The reconciliation of their public and private investments is not easy. It seems that the country doubled its annual expenditure to $590M in 2022 as part of its new quantum strategy announced that year[4652] [4653]. One of this strategy's goals includes having 10 million users by 2030 which is typical from a very dubious claim made for politics.

An inventory of Japan's various quantum and quantum computing projects can be found in Figure 943 and Figure 944. In quantum computing, we then have multiple goals with 100,000 qubits by 2033.

Quantum industry

Japanese startups are rather specialized in software and in particular for quantum annealing computing running either on D-Wave quantum annealers or on Fujitsu digital annealers. We have **A*Quantum** (2018, QA software), **D Slit Technologies** (2018, software), **Groovenauts (**2012, QA software), **Jij (2018,** QA software framework), **MDR** (2008, chemical simulation), **QunaSys (2018,** healthcare), **Sigma-I (**2019, QA software) and **Tokyo Quantum Computing** (2017, QA software) (Figure 945).

Softbank's investment fund abounded with Saud family's money up to $100B was also planning to invest in quantum technologies[4654].

However, several years after its announcement, the fund does not seem to have a single stake in quantum technologies. They have first to get rid of Wework!

Figure 945: Japan's quantum industry vendors. (cc) Olivier Ezratty, 2022-2023.

In the private sector, Japan's major industry groups are mainly focused on quantum telecommunications and cryptography, as well as on quantum and non-quantum annealing-based computing.

Hitachi also has a research laboratory located at the University of Cambridge (UK) that works on quantum key distribution, quantum computing and the creation of SQUID components for superconducting qubits. They are also working on silicon spin qubits quantum computing.

Toshiba Corporation has been involved in quantum cryptography since 2003. They are working on it with the Quantum Information Group (QIG) at the University of Cambridge, UK. They performed a first demonstration of quantum communication in 2014, sending 878 Gbits/s of secure data over a 45 km fiber between two areas in the Tokyo area over a cumulative period of 34 days, at a rate of 300 kbits/s.

[4651] See Japan Invests $32 Million (US) For Shared Quantum Computing by Matt Swayne, The Quantum Insider, April 2023.

[4652] See Japan's first domestic quantum computer targets 10m users by 2030, Techwire Asia, April 2022

[4653] See Strategy of Quantum Future Industry Development Summary, Secretariat of Science, Technology and Innovation Policy Cabinet Office, April 2023 (14 slides).

[4654] See SoftBank's Vision Fund Eyes Investment in Quantum Computing by Jeremy Kahn, Bloomberg, June 2017.

They were continuing the experiments in 2019 and beyond and with British Telecom in the UK[4655].

NTT maintains four applied quantum research laboratories, focused on quantum telecommunications and quantum cryptography, all with about 40 researchers[4656].

In 2017, **NTT** launched a prototype photonics-based Quantum Neural Network (QNN) in collaboration with the **National Institute of Informatics** and the **University of Tokyo**. It was available on the cloud at qnncloud.com (video) but the service was discontinued in March 2019[4657]. This was done with Toshiba, NEC and the NICT in Tokyo with three nodes and 45 km apart[4658].They also work in the CMOS quantum dots qubits. NTT also developed LASOLV, a photonic based coherent Ising system with 2000 nodes[4659]. NTT is also involved in the development of superconducting qubit control software along with Fujitsu and RIKEN.

Finally, several non-quantum annealing optimization computation projects on CMOS components have been launched. There is the **Fujitsu** offering, and also the NEDO project led by Masanao Yamaoka and Masato Hayashi at **Hitachi** in partnership with the AIST, RIKEN and NEDO (New Energy and Industrial Technology Development Organization, the equivalent of the energy branch of the CEA) laboratories[4660].

And then the **NEC** project in quantum annealing led by **Yuichi Nakamura** in liaison with Waseda, Yokohama and Kyoto Universities, AIST and Titech (Tokyo Institute of Technology). They are optimizing the classical part of annealing processing with NEC vector processors. The quantum part seems to be managed on D-Wave machines. NEC is also versed in quantum keys (QKD).

Recruit Communications Ltd (1960), a large $16B CDN company specializing in HR, communications and marketing, distinguished itself by launching a partnership with D-Wave in 2017 to develop quantum annealing-based solutions for the operational optimization of marketing, communications and advertising. In particular, they have developed the PyQUBO open source library, which simplifies the development of quantum annealing software applications[4661].

In September 2021 **Q-STAR** (Quantum Strategic Industry Alliance for Revolution) was launched. It is an industry alliance to promote the usage of quantum technologies and particularly quantum computing and cryptography in various industries with the participation from Toshiba, Toyota, NEC, NTT, Hitachi, Fujitsu, Mitsubishi Chemical and Sumitomo among others.

IBM announced at the end of 2019 the opening of a Q Lab in Tokyo in partnership with the University of Tokyo. IBM's investment in Japan follows a model already inaugurated in France in Montpellier in 2018, in Germany in September 2019 and in Canada with the Institut Quantique in June 2020.

[4655] See Performance Limits for Quantum Key Distribution Networks by Andrew Shields, June 2019 (16 slides).

[4656] This leads to raising wages inflation for the most talented people, a bit like in Silicon Valley. See NTT offers researchers $1 million salaries in bid to lure top talent in cryptography, quantum computing, November 2019.

[4657] See Japan launches its first quantum computer by Walter Sim, November 2017.

[4658] See Tokyo QKD Network and its application to distributed storage network by Masahiro Takeoka, June 2019 (22 slides).

[4659] See LASOLV Computing System: Hybrid Platform for Efficient Combinatorial Optimization by Junya Arai et al, 2020 (6 pages).

[4660] See CMOS Annealing Machine - developed through multi-disciplinary cooperation, November 2018, Overview of CMOS Annealing Machines by Masanao Yamaoka, Hitachi, (4 pages) and A 2 x 30k-Spin Multi-Chip Scalable CMOS Annealing Processor Based on a Processing- In-Memory Approach for Solving Large-Scale Combinatorial Optimization Problems, November 2019.

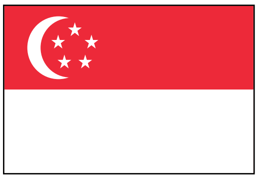
[4661] See Recruit Communications and D-Wave Collaborate to Apply Quantum Computing to Marketing, Advertising, and Communications Optimization, May 2017.

It contains a partnership with a university, investments in training and above all, a technical and marketing investment to evangelize quantum among major customers[4662].

In May 2023, it formalized two partnerships, on the road to the buildup of a 100,000 qubits quantum computer. The first is with **IBM** which will bring $100M to the table over 10 years, including funding for the University of Tokyo and also for the University of Chicago. It seems to be an extension of the partnership established in 2019 with the University.

The second is with **Google** which will do the same, with $50M[4663]. On top of that, **Intel** is partnering with RIKEN in a broad field encompassing classical and quantum computing[4664].

Singapore



The small state of **Singapore** is known for its economic and entrepreneurial dynamism. Its quantum ecosystem is built around the **National University of Singapore** (NUS), **Nanyang Technological University** (NTU), **Singapore University of Technology and Design** and the **Agency for Science, Technology and Research** (A*STAR), mapped in Figure 947.

Within NUS, quantum research was consolidated in 2008 in the **Center for Quantum Technologies** (CQT) with an annual funding of about $15M. It was created thanks to some real political leadership, coming from the then Defense Minister of Singapore[4665]. It is vested in both quantum computing (cold atoms in Berge Englert's group, photons and superconductors in Dimitris Angelakis' group, trapped ions in Dzmitry Matsukevich's group), quantum cryptography (Kwek Leong Chuan's group) and quantum metrology (notably atomic clocks in Murray Barrett's group). The CQT was led from its inception until July 2020 by Artur Ekert. Since then, it is run by José Ignacio Latorre. It brings together about twenty teams representing 22 permanent researchers, 60 research fellows and 60 PhD students, covering the four usual fields of quantum technologies. This represents a total of 300 people. Of the 22 research supervisors, about a quarter are Singaporeans who have usually done a thesis abroad. Singapore is doing well to attract talented foreigners and to ensure that they settle permanently in this country of five million people.

Several startups emerged from CQT like **Entropica Labs** (quantum algorithms), **Horizon Quantum Computing (**software), **Innovatus Q (**hybrid algorithms), **S-Fifteen Instruments (**quantum cryptography) and **SpeQtral** (satellite QKD).

Quantum communication is one specialty from Singapore. Since 2016, CQT has been associated with the telecom company **Singtel** and the NUS for the deployment of QKD on optical fibers with repeaters.

4662 See IBM Takes Its Quantum Computer to Japan to Launch Country-Wide Quantum Initiative by Anthony Annunziata, December 2019. In partnership with the University of Tokyo and IBM and the University of Tokyo Launch Quantum Computing Initiative for Japan by IBM, 2019. In August 2020, IBM embellished this partnership by announcing the creation of a consortium for the adoption of quantum technologies in Japan. See IBM Launches Global Consortium for Quantum Innovation by Chris Duckett, August 2020, which refers to an announcement that is really only about Japan: IBM and the University of Tokyo Unveil the Quantum Innovation Initiative Consortium to Accelerate Japan's Quantum Research and Development Leadership by IBM, August 2020.

4663 See A quantum computing partnership with the University of Chicago and the University of Tokyo, Google AI, May 2023.

4664 See Road to Zettascale: Intel and RIKEN Announce Strategic Partnership by Francisco Pires, Tom's Hardware, May 2023.

4665 Artur Ekert says he was persuaded to join Singapore in 2000 by Tony Tan, who was then the country's defense minister. He had met him at a conference where his visionary speech, for a politician, had impressed him. In 2005, Tony Tan took charge of the sovereign wealth fund Singapore Investment Corporation and then Singapore's National Research Foundation. He was at the origin of the strategy of targeted investment in cutting-edge research fields, which today we call deep tech. This Tony Tan then became the President of Singapore between 2011 and 2017. The CQT was launched in 2006. The story is told in the book 50 years of science in Singapore pages 362 to 387, February 2017. His personal credo: to be successful, you need to attract the right people, original ideas and then funding. Too often, this happens through funding.

At the end of 2019, a team from NTU developed a 3 mm-sided chip capable of integrating a CV-QKD, a continuous variable quantum key-based encryption system[4666].

In 2015, Singapore launched its Galassia-2U nanosatellite, created by CQT and used to experiment encrypted QKD based quantum communications. Galassia is integrated in a two-unit CubeSat format (two cubes on top of each other, see Figure 946).

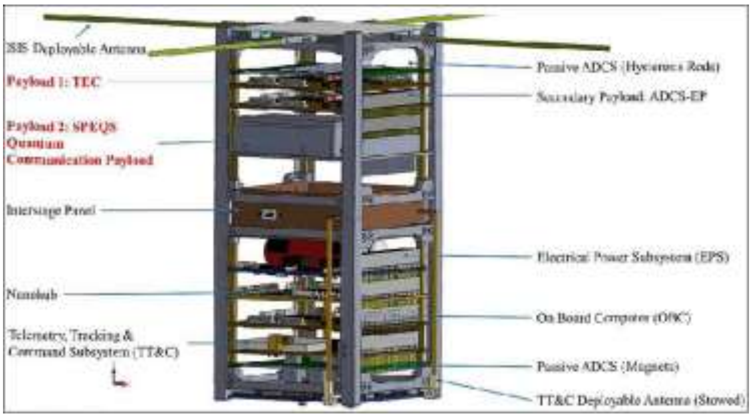


Figure 946: Source: https://directory.eoportal.org/web/eoportal/satellite-missions/g/galassia

It weighs only 3.4 Kg in total. It was sent to space with 5 other satellites including the telecommunications satellite TeLEOS-1 (400 kg) at the end of 2015 by an Indian launcher[4667].

The lifetime of this type of satellite is six months[4668]. These experiments led to the creation of the S-Fifteen Space Systems. However, solutions have yet to be found to ensure that these satellites last longer in their low orbit and do not contribute even more to low orbit pollution.

In May 2022, as part of its **Quantum Engineering Programme** (QEP) started in 2018, Singapore launched three national platforms with a total funding of $23.5M for 3.5 years with a pooling of the resources and skills from CQT at NUS and NTU Singapore, A*STAR's Institute of High Performance Computing (IHPC) and the National Supercomputing Centre (NSCC).

- The **National Quantum Computing Hub** develops quantum computing capabilities and use cases for the industry.
- The **National Quantum-Safe Network** conduct trials of quantum-safe communication technologies for critical infrastructure.
- The **National Quantum Fabless Foundry** supports microfabrication techniques for quantum devices and enabling technologies. Hosted at A*STAR's Institute of Materials Research and Engineering (IMRE), it manages the micro and nanofabrication of quantum devices and related enabling technologies. This complements the Quantum Science and Engineering Centre (QSec) launched in December 2021 by NTU to design and manufacture various quantum chips using classical semiconductor manufacturing technologies.

Several partnerships and linking the French and Singaporean quantum ecosystems. The CQT welcomes several researchers from France, including **Steven Touzard**, **Miklos Santha** and **Christian Miniatura**. Since January 2023, **Alexia Auffèves** is leading **MajuLab**, the joint CNRS-NTU/CQT/NUS research laboratory on quantum science since January 2023. NUS is also partnering with the **Thales TRT** research lab based in Singapore, in security and sensing[4669]. **Sondra Lab** is another Franco-Singapore lab with CentraleSupelec, ONERA, NUS and DSO which is working in the fields of electromagnetism and signal processing applied to radar.

[4666] See Quantum chip 1,000 times smaller than current setups, November 2019.

[4667] See India's PSLV Rocket orbits six Satellite for Singapore in year-closing Mission, SpaceFlight, December 2015

[4668] See Quantum Tech demos on CubeSat nanosatellites (41 slides).

[4669] See Singapore's NUS and Thales developing quantum technologies for commercial applications by Jamilah Lim, October 2021.

Figure 947: Singapore quantum ecosystem.
*Adapted from [Overview of Quantum Computing Efforts in Singapore - QC Benchmarks Views](), by Ye Jun, A*Star, May 2023 (30 slides).*

South Korea

In South Korea, visible quantum investments started with the telecom operator **SK Telecom** making inroads in quantum telecommunications[4670]. They are partnering with Florida Atlantic University. They also took a controlling share of the Swiss startup ID Quantique in 2016. It is also partner since 2017 with Nokia in the QKD field as well as with Deutsche Telekom with whom they have established a "Quantum Alliance" to create secure telecommunications.

SK Telecom has deployed a QKD network in the backbone of its 4K network in the city of Sejong on two links of 38 and 50 km respectively[4671]. They also announced a memorandum of understanding with the government of Luxembourg in October 2023, in relation to their LuxQCI satellite QKD project led by SES Astra.

Samsung is also investing in QKD and cryptography. They integrated a quantum random number generator in a dedicated version of a Galaxy smartphone for the Korean market in April 2020, with a component from ID Quantique. A new version was launched in April 2021.

Hyundai has invested in IonQ and is teaming up with them to build quantum algorithms for the simulation of batteries chemistry. These algorithms require fault-tolerant quantum computing resources that will be available only in the distant future. As a result, IonQ signed a partnership with the country to develop its regional quantum computing ecosystem[4672].

In June 2023, the South-Korea government announced an ambitious quantum investment plan funded with $2.6B over 12 years including industry investments by Samsung, SK Telekom and Hyundai[4673].

[4670] See SK Telecom Continues to Protect its 5G Network with Quantum Cryptography Technologies, March 2019.

[4671] See Quantum Safe Communication - Preparing for the Next Era by Dong-Hi Sim, June 2019 (21 slides).

[4672] See IonQ Signs Agreement With South Korea's Ministry of Science and ICT to Cultivate Regional Quantum Computing Ecosystem by Matt Swayne, July 2023.

[4673] See S.Korea to invest $2.6 bn in quantum technology by 2035 by Jin-Won Kim, The Korea Economic Daily, June 2023.

Like most developed countries in the world, it wants a greater piece of the upcoming quantum technologies market, from 1.8% to 7.3%. It also plans to extend PhD training from 300 to 2,000 and workforce extension from 953 to 10,000, and to grow the number of startups from 9 to 100 as well as the number of technology companies making use of quantum technologies. Korea will start to invest about $300M in quantum computing with focusing on cold atoms and superconducting qubits. Their goal is to reach 1,000 physical qubits. They plan to create their own dedicated fab.

Of course, the USA is part of the game with their generic country level partnership template signed with so many developed countries, particularly in Europe[4674].

South Korea startup ecosystem is rather small with **First Quantum** (quantum circuits optimization software), **HEaaN CryptoLab** (PQC cryptography), **QSIMPLUS** (software emulator for quantum communication and optical network design) and **Qunova Computing** (quantum software). It leads the country to establish various partnerships with foreign quantum vendors like IBM with the Yonsei Institute of Quantum Information Technology in 2022, and between KAIST and QuEra in September 2023.

Taiwan

Taiwan is a land of digital technology with its leadership semiconductors manufacturing (TSMC, UMC), consumer and professional electronics manufacturing (Foxconn, Quanta), but also all sorts of technology products (personal computers with Asus and Acer, PC boards with Gigabyte and MSI, Internet of things, etc).

It was logical in these conditions that the country became interested by quantum technologies. Public research is organized through various organizations: the NTU and NTHU Universities with various principal investigators in all typical quantum technologies domains, National Cheng Kung University (NCKU), Academia Sinica, and ITRI (Industrial Technology Research Institute) which is an equivalent to KAIST in South-Korea and AIST in Japan. I used to see some of their research output in the consumer electronics domain when I was visiting the CES in Las Vegas from 2006 until 2020.

In December 2020, Taiwan launched a quantum plan with a funding of $282M over 5 years. It consolidated its investments in the Southern Campus of Academia Sinica, the national academy of Taiwan, in Tainan. They planned to create a Quantum Technology R&D center between 2022 and 2024. In April 2022, the government announced as part of this plan the selection of 17 research teams in universities and the hiring of 72 project directors and 24 IT companies. They are looking at ways to improve quantum hardware and software and to leverage, if possible, their semiconductor industry. The country will probably play a key role in the future in the manufacturing and integration of large scale quantum computers, a bit like what they do today in the chips and PC industry.

The selected projects cover silicon, cold atom and superconducting qubits and their manufacturing, cryo-CMOS (particularly at ITRI and NTU), QKD light sources, QRNG sources, multinode quantum networks, VQE algorithms and error mitigation, GKP error correction codes, financial applications, and at last, code verification and compilation[4675]. On top of that, Hon Hai (FoxConn) created a Quantum Computing Research Center in January 2021 that is focused on quantum information science (algorithms for their own use, mostly for solving manufacturing optimization problems), on trapped ion qubits, on enabling technologies (which they could manufacture) and on cybersecurity.

[4674] See [Joint Statement of the United States of America and Republic of Korea on Cooperation in Quantum Information Science and Technologies](), April 2023.

[4675] Source: https://site.etop.org.tw/qt/index.php?c=pub&m=loadpage&d=pub&mid=1008 (to translate from Chinese).

I visited Taiwan in September 2023 and met various researchers at NTU, NTHU and ITRI and from their national quantum initiative team, on top of Foxconn (Min-Hsiu Hsieh). I was welcomed by Ching-Ray Chang who is a professor at NTU and the president of TAQCIT, the Taiwan Association of Quantum Computing and Information Technology. Also, let's mention that IBM established a foothold in the country to help it adopt quantum technologies, with an IBM Quantum Lab at NTU, who is accessing online QPU from IBM's Poughkeepsie datacenter in the USA.

Australia

Australia is a key APAC region player in quantum technologies. Its academic research stars are Michelle Simmons, Andrew S. Dzurak and Andrea Morello (UNSW), Andrew White, Gerard Milburn and Jacqui Romero (University of Queensland) and Gavin Brennen (Macquarie University) to name a few.

Government funding

The Australian National Innovation and Science Agenda announced in 2015 included 24 initiatives and $820M in funding over 4 years, of which $19M were allocated to the Center for Quantum Computation and Communication Technology (**CQC2T**) over 5 years in quantum computing. **CQC2T** was created in February 2019 at UNSW, headed by Michelle Simmons. The goal was to create an electron spin quantum computer. With federal funding of $33.7M, it brought together a community of 200 researchers[4676]. An investment fund of the Ministry of Defence, the **Australian Next Generation Technologies Fund** allocated $730M in 2016 to 9 areas including one on quantum technologies over 10 years[4677]. Assuming that these funds were distributed evenly among the 9 initiatives, this gives us $8M of additional funds per year on quantum technologies for military uses, including sensing.

In May 2020, CSIRO published a quantum opportunity white paper[4678]. Their (fairly optimistic) ambition is to turn it into a $4B industry creating 16,000 jobs by 2040 out of a projected global total revenue of $86B. The projected breakdown was $2.5B and 10,000 jobs for computing, $900M and 3000 jobs for sensing and $800M and 3000 jobs for telecommunications. The goals? To define a coordinated strategy, to finance research and business creation, to train talents and to create a coherent industrial value chain. A relatively new point in such a plan, is to explore the ethical, social and environmental issues that could be raised by quantum technologies. The subject has been growing in importance since 2020. They also address the question of the supply chain of key components and materials for quantum technologies.

In November 2021, the Australian government allocated an additional US $80M to its quantum efforts, particularly for supporting the commercialization, adoption and use of quantum technologies and create new jobs. It includes US $51M for the creation of a "quantum commercialization hub" with the task to build strategic partnerships with "like-minded countries" to sell Australia's quantum technologies, starting with the usual Commonwealth country partners and the USA.

As a follow-up from the famous nuclear submarine deal with the USA (at the expense of an existing classical submarine deal with France) announced in September 2021, Australia cemented a global quantum partnership with the USA in November 2021, covering in a fuzzy way the exchange of quantum knowledge and skills. The University of Sydney is already part of an international consortium integrated in the US IARPA **LogiQ** program.

[4676] In early 2019, UNSW's CQC secured an additional $33M in funding at its official launch. See Federal govt funnels $33.7 million towards UNSW's quantum research by Matt Johnston, February 2019.

[4677] See Next Generation Technologies Fund, 2016.

[4678] See Growing Australia's Quantum Technology Industry by CSIRO, May 2020 (56 pages) and Australia could lose its quantum computing lead, CSIRO warns by John Davidson, May 2020.

In terms of international partnerships, the country is also associated with the University of Singapore for the creation of quantum telecommunication satellites and also with Japan.

In June 2023, Australia formally launched its national quantum strategy. Like every country, the goal is to become a global and leading player in quantum technologies.

The plan funding is $670M spread over 7 years. It contains investments in research, workforce development, standards, quantum infrastructure and materials, and a trusted and inclusive ecosystem[4679].

On a regional basis, the **Queensland Quantum and Advanced Technologies Strategy** announced in 2023 is a multipronged plan of $62M[4680]. It covers various typical aspects (education, research, fabs), with some original items like funding application research in quantum biology through the ARC Centre of Excellence in Quantum Biotechnology and on decarbonation.

Research

Key universities are University of Sidney, University of Technology of Sidney, UNSW and Macquarie University (all in South Wales), University of Queensland, and the University of Melbourne (Figure 949).

In December 2020, Australia launched the **Sydney Quantum Academy,** a joint effort from Macquarie University, UNSW Sydney, the University of Sydney and UTS. It consolidates training offerings implemented by the partner Universities for undergraduates, PhDs plus some fellowship's programs.

Quantum Industry

On the entrepreneurial side, there are three startups in the field of quantum technologies with **QuintessenceLabs (**QKD optical keys), **QxBranch** (software and consulting, an American startup with an office in Australia, acquired by Rigetti in July 2019), **Silicon Quantum Computing (**silicon qubits), **Quantum Brilliance** (2019) on top of which should be added **Archer** and their carbon electron spins qubits. These are shown in in Figure 949. They highlight Microsoft and IBM. So be it. Rigetti is there thanks to the acquisition of the local startup QxBranch.

In September 2022, the Tech Council formed the **Australian Quantum Alliance** to consolidate its quantum tech industry.

The country is also prolific in public-private partnership projects associating Australia with other countries[4681].

In 2017, the University of New Wales (UNSW), the Commonwealth Bank of Australia and telecom operator Telstra provided $52M in funding for the creation of a silicon quantum bit processor. One could hope that Orange will do the same in France with the CEA and/or a startup!

Also, **EQUS** (Arc Center of Excellence for Engineered Quantum Systems) is a national quantum sensing research center. It partners with Microsoft, Moglabs and Lockheed Martin, among others.

In July 2022, Google announced new partnerships with Australian universities including UNSW and the University of Sydney. It extends what they already do with US universities, mostly for the development of quantum algorithms on Sycamore QPUs.

[4679] See National Quantum Strategy - Building a thriving future with Australia's quantum advantage, 2023 (51 pages).

[4680] See Queensland Quantum and Advanced Technologies Strategy, 2023 (34 pages).

[4681] See Charting the Australian quantum landscape, February 2019 (5 pages).

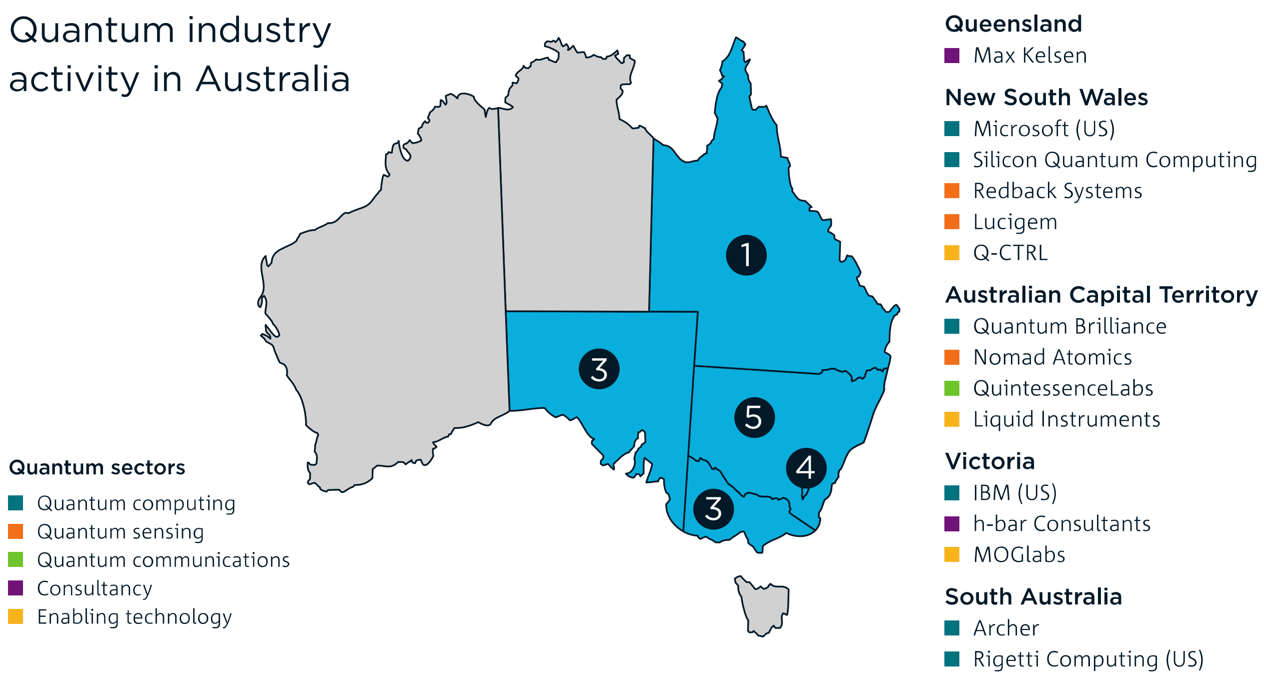


Figure 948: Australia quantum industry map.
Source: [Growing Australia's Quantum Technology Industry](#) by CSIRO, May 2020 (56 pages).

In December 2022, the **Australian Quantum Software Network** (AQSN) was created to consolidate the quantum software R&D ecosystem, with 110 participants from nine universities and several local startups (SQC, Quantum Brilliance and Diraq in hardware, and two in software). It also has extended partnerships with other countries ecosystems in the USA (Google Quantum AI), Japan (Okinawa Institute of Science and Technology), Finland (Aalto University). A little later, the government signed a 7-year procurement agreement with IBM related to various digital needs, which was beautified with some quantum computing nuggets[4682].

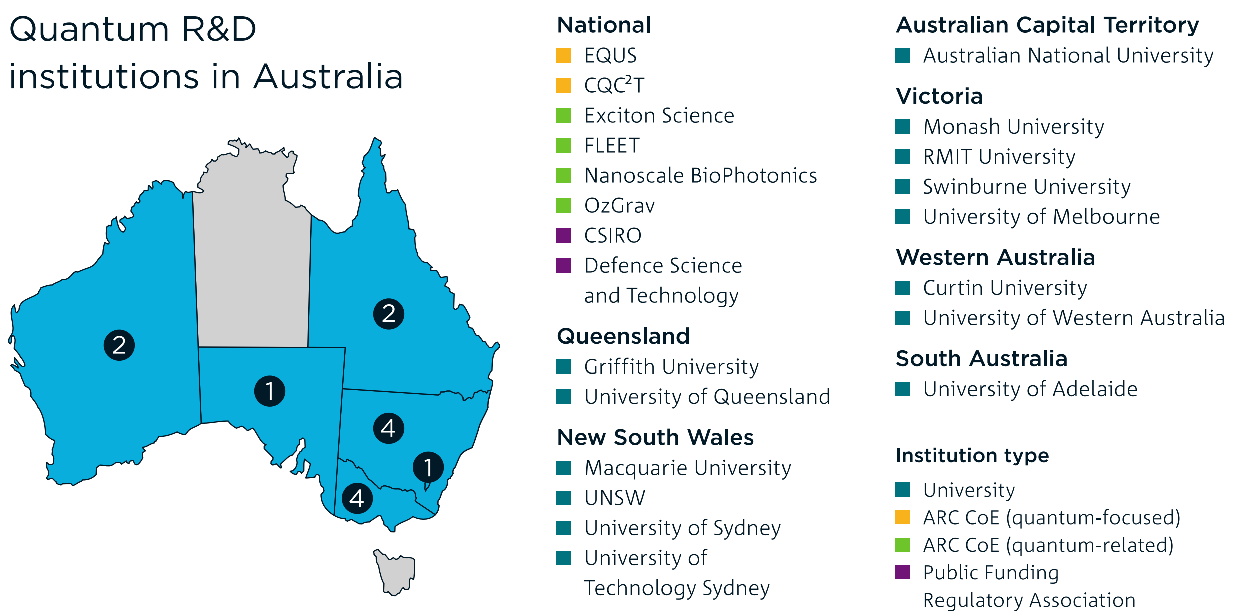


Figure 949: Australia's research ecosystem.
Source: [Growing Australia's Quantum Technology Industry](#) by CSIRO, May 2020 (56 pages).

[4682] See Part of IBM's $725 Million Deal Will Help Australian Government Connect to Quantum Computers by Matt Swayne, January 2023.

India

At the beginning of 2020, India launched an investment plan in quantum technologies, the **NM-QTA** (National Mission on Quantum Technologies & Applications). This plan is well funded as a proportion of the country's GDP, with $1.12B over 5 years, at the same level as the American Quantum Initiative Act of 2018 or the European Flagship launched the same year[4683].

The plan covers the usual suspects: quantum computing, quantum telecommunications and quantum sensing. Ironically, the CEOs of IBM, Google and Microsoft who are strong investors in quantum computing are all of Indian origins (Arvind Krishna, Sundar Pichai and Satya Nadella)! The Indian 'National Quantum Mission' plan has an eye on China and wants to turn the country into a quantum leader, particularly in computing, telecommunications, and cryptography. The quantum plan received an additional funding of $732M covering the 2023-2030 period with one goal being to create 50 to 1,000 qubits QPUs and 2,000 km of quantum communication networks. The plan also deals with satellite QKD and quantum sensing[4684].

Academic research is centered around a couple public labs:

- **MeitY** (their Ministry of Electronics and Information Technology) which launched in August 2021 the Quantum Computer Simulator (QSim) toolkit that was created by IISC Bangalore, IIT Roorkee and C-DAC. This software emulator of gate-based quantum code must not be confused with Qsim from Qsimulate and Google.
- The **Quantum Computing Applications Lab** (QCAL) launched by the Ministry of Electronics and Information Technology (MeitY) with AWS provides access to quantum computing resources (CPU, software tools, training) to researchers and developers.
- The **Quantum Measurement and Control Laboratory** (QuMaC) works on superconducting qubits.
- The **Indian Institute of Science** (IISc) in Bangalore works on quantum information sciences.
- The **Indian Institute of Technology** (IIT) Madras and the **Harish-Chandra Research Institute** (HRI) in Allahabad conduct research in quantum computing. It also partners with IBM[4685].
- **IIIT-Delhi** (Indraprastha Institute of Information Technology Delhi) created its Center for Quantum Technologies (CQT) in 2022, consolidating R&D in all quantum technologies, a bit like Singapore's CQT. It is starting with 9 principal investigators[4686].
- The **Indian Army** launched its Quantum Research Laboratory in January 2022. So be it.

The **Chatterjee Group** (TCG) is a private equity firm with assets petrochemicals, pharmaceuticals, biotech, financial services, real estate and various other industry sectors. Its TGC CREST (Centres for Research and Education in Science and Technology) contain the Centre for Quantum Engineering, Research and Education (CQuERE) which is planning to build a quantum computer with, as a starter, 4 superconducting qubits by 2027 with a first investment of $1.2M to be extended to $12M[4687]. They started with important a Bluefors dilution refrigerator and to work on the qubit designs with the **Tata** Institute for Fundamental Research in Mumbai. They also have some partnership with Qilimanjaro.

[4683] See India finally commits to quantum computing, promises $1.12B investment by Ivan Mehta, February 2020.

[4684] See National Quantum Mission gets Cabinet approval to scale up R&D for quantum technologies by Srinivas G. Roopi, ETGovernment, April 2023.

[4685] See IIT Madras joins IBM Quantum Network, IBM, September 2022.

[4686] See IIIT Delhi Launches Center on Quantum Technology by Sukanya Nandy, November 2022.

[4687] See TCG Crest to make 4-qubit quantum computer in 3 yrs, India Times, April 2023.

In February 2022, **Avasant** (an US consulting firm with a branch in India) and **NASSCOM** (the Indian software and IT service trade association) published a report on the opportunities of quantum technologies for India[4688]. It contained several nuggets like a quantum tech potential providing between $152B and $310B cumulative value to the Indian economy by 2030 with a maturity inflection point positioned in 2027. It reuses similar forecasts at the worldwide scale from McKinsey et BCG.

They expect that 10K logical qubits will be available by 2030 (100 would be so nice…). But they forecast a moderate workforce impact of 25K to 30K people in 2030. Also, India plans to develop a quantum computer with about 50 qubits by 2026. Also, they position the quantum Internet to be related to FTQC in their roadmap for 2027 and beyond. The document also shows the key role of the large Indian IT services companies in the adoption of quantum computing. As an example, in September 2021, Infosys Cobalt was partnering with AWS Braket to explore the business potential of quantum computing.

Among other things, the Indian plan has also accelerated the creation of startups in India, some being king in overselling their technology advances. This is the case of **QPI** and their projects of a one million silicon qubits and hybrid processor. Many of their new startups are multi-domains, such as:

- **QRLAB** (2020, India) who is a contract research, education and consulting company focused on quantum computing. They develop quantum inspired software, QML and also work on quantum Internet and cryptography.
- **Qulabs.ai** (2017, India) which builds quantum networks and has some expertise in QML in finance and for new drug discovery. That's quite broad in scope! Their QuAcademy facilitates students training.
- **Fractal Analytics** (2000, India, $685M) is an AI/data analytics company and a unicorn. It is creating an in-house quantum computing lab.

Indian IT services company have also started to launch various ventures in quantum technologies:

- **Infosys** has a Quantum Living Labs that helps customers develop quantum computing proofs of concepts in various verticals.
- **Tata Institute of Fundamental Research** (TIFR) created a superconducting 5-qubit QPU with a 7-qubit version in the making.
- **Wipro** partners with Tel Aviv University.
- **Tech Mahindra** partners with IQM since December 2022 to deploy various use cases of quantum computing[4689].
- **HCL Technologies** works with Sydney Quantum Academy in various student programs.
- **Mphasis** partners with IIT Madras to fund startups and train the workforce.

[4688] See The quantum revolution in India: betting big on quantum supremacy, Avasant, February 2022 (48 pages).

[4689] See Tech Mahindra Chases Quantum Dreams With IQM by Anirudh VK, December 2022.

Quantum technologies around the world key takeaways

- The quantum startup scene has seen its peak company creation in 2018. A small number of startups like D-Wave, IonQ, Rigetti, PsiQuantum and Xanadu collected about 70% of the worldwide quantum startups funding. The investors FOMO (fear of missing out) and the "winner takes all" syndrome explain this situation.
- Most developed countries now have their "national quantum plans" and want to lead that space, particularly with quantum computing. The first ones were Singapore in 2007 and the UK in 2013. Investment comparisons are not obvious since these plans accounting are not the same from country to country (incremental funding vs legacy plus incremental, private sector included or not, European Union investments included or not). All these plans invest a lot in fundamental research and on developing a startup and industry ecosystem, including workforce training.
- China's quantum investments have been overestimated for a while, both because of the ambiguity of China's communication and since various lobbies in the USA were pushing for increased federal investments to counter China's perceived threat. This worked particularly well during the Trump administration and seems to persist with the Biden administration.
- Europe and the USA are the greatest investors in quantum science so far. The European Union as a whole is the largest region for public investments in quantum research. The USA has a larger industry investment than Europe due to its large IT vendors investments (IBM, Google, Microsoft, Intel) and a traditional lead in startups funding, and, certainly, with its domestic market size and dynamics.
- Many countries did put quantum technologies in the critical field of "digital sovereignty" like if it was some sort of nuclear weapon equivalent.
- Each country has its own strengths and specialty although most of them invest in all the fields of quantum technologies (computing, telecoms/cryptography and sensing).
- Some analysts are wondering whether we'll get soon into a quantum winter, like the ones that affected artificial intelligence in the 1970s and the 1990s. One way to avoid it is to limit overpromises.

Corporate adoption

Out of the many audiences of this book interested in quantum technologies are companies that may wonder what to do about it. They are the target of a deluge of hype, information, complexity and uncertainty. This comes in addition to other technological waves to assimilate such as artificial intelligence and their disrupting large language models (LLM ala ChatGPT), cybersecurity with or without quantum computing threats, cryptocurrencies and other Blockchain, the metaverse (in decline after the 2021 hype), not to mention cloud deployments and their perpetual classical business applications backlog.

What corporations should do with quantum technologies as they are encouraged by influential analysts to jump now on the bandwagon or to lose against their competitors? The pressure is indeed significant. BCG and McKinsey are the most vocal analyst shops on this.

Understand the imperative

Large customers and their IT, digital transformation, innovation and R&D departments are exposed to a continuous stream of industry analyst and vendors pitches creating a sense of urgency for the adoption of quantum computing. It comes for example from **Capgemini**[4690 4691], **McKinsey**[4692], **Deloitte**[4693], **Harvard Business Review**[4694], **Arthur D. Little**[4695], from **Pathstone**, a financial advisor company[4696]. It can take the form of a survey commissioned by a vendor, like **Zapata Computing**[4697]. Some even are completely off-the-mark on the role of quantum computing, such as in cybersecurity or are largely overselling the real practicality of existing quantum computers[4698].

BCG started to promote in 2021 the "*it's not an if but a when*" about the advent of practical quantum computing when it should be said with much more caution given the many scientific and technological uncertainties that are still present[4699].

4690 See Capgemini: Organizations need to get moving on quantum by Dan O'Shea, Fierce Electronics, April 2022.

4691 See Quantum computing: the hype is real—how to get going? by Christian Knopf, Capgemini, April 2023.

4692 See Quantum computing use cases are getting real—what you need to know, McKinsey, December 2021.

4693 See Quantum computing in 2022: Newsful, but how useful? by Duncan Stewart et al, Deloitte, December 2021. Their assessment is the most honest of all, with cautious tales like: "*Many of the tasks that they currently do can be replicated on a standard laptop computer at a fraction of the cost. The problem with QCs' usefulness is not a lack of use cases, money, effort, or even progress. It's that current QCs are not yet powerful enough to tackle problems that can't be performed by traditional computers*".

4694 See Quantum Computing for Business Leaders by Jonathan Ruane, Andrew McAfee, and William D. Oliver, HBR, January–February 2022. With an interesting statement: "*Quantum computing will enable businesses to better optimize investment strategies, **improve encryption**…*". How can you trust this sort of report with such misunderstanding of how quantum computers will impact cryptography (it potentially endangers it and you won't be saved by a quantum computer)?

4695 See Quantum Computing - The state of play and what it means for business by Albert Meige, Rick Eagar, Lucas Könnecke and Olivier Ezratty. I indeed participated (pro-bono) to the fact-checking of this work.

4696 See "Quantum Impact" - The Potential for Quantum Computing to Transform Everything by Pathstone, December 2021 (35 pages).

4697 In Report: 74% of Executives Warn Either Adopt Quantum Soon, or Risk Falling Behind Forever by Matt Swayne, The Quantum Daily, January 2022, reporting on the first annual report on enterprise quantum computing adoption commissioned by Zapata Computing: The First Annual Report on Enterprise Quantum Computing Adoption, Zapata, January 2022 (42 pages).

4698 See Quantum Computing: 5 Potential Applications, January 2022 where this nugget can be found: "*Quantum computing could also help in the development of new encryption techniques, known as quantum cryptography*". Somebody should tell them that quantum cryptography does not run on a quantum computer!

4699 See What Happens When 'If' Turns to 'When' in Quantum Computing? by Jean-François Bobier, Matt Langione, Edward Tao, and Antoine Gourévitch, BCG, July 2021.

Late 2022, according to BCG, 50% of corporate customers spend $1M to create a quantum computing proof of concept, mainly in the finance and energy verticals[4700].

They tout an imperative to build an IP strategy coupled to a discourse of a "*winner takes all the value*" for end-user customers, due to scarce resources, both in talent pools and in quantum computers availability. This doesn't make a lot of sense given the maturity of the sector and what happened in other digital domains[4701].

They upped the ante in 2023 in writing that "*Quantum computing is becoming business ready*"[4702]. The given business use case examples? As shown in Figure 950, three applications in the financial sector, including two that were tested with only 3 and 4 qubits. This would fit in a $50 Raspberry Pi if not in your smartwatch, definitively not showing any computational or economic quantum advantage! The third, related to a CACIB fallen angel detection application running on a Pasqal quantum simulator looks promising but did not yet deliver a quantum advantage in any dimension, given the adapted hardware won't be available for a couple years.

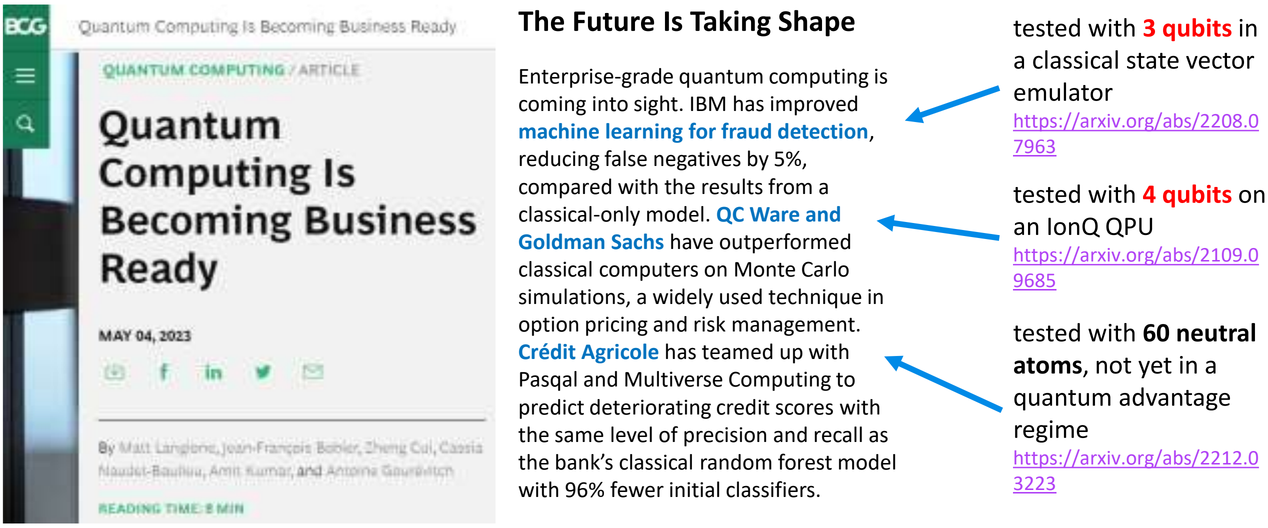


Figure 950: when the BCG is overpromising on the business readiness of quantum computing. Source: Quantum Computing Is Becoming Business Ready by Matt Langione, Jean-François Bobier, Zheng Cui, Cassia Naudet-Baulieu, Amit Kumar, and Antoine Gourévitch, BCG, May 2023 and arXiv.

But it takes some time and effort to check all of that and cut through the hype. The wave of quantum technologies is unique in that it is even more unpredictable and difficult to grasp than the other digital technologies waves. This large book is the largest I have ever published, which demonstrates this indirectly. It is nearly twice as large as my largest book on artificial intelligence[4703]. Also, the discrepancy between the real technology readiness and what industry vendors and analysts say about it is huge.

And yet, the topic is worth the attention, particularly in certain key verticals such as finance, healthcare, utilities and transportation.

Despite all the required prudence that is mandated here, there are several reasons why, still, corporations should evaluate quantum technologies:

[4700] It was not 50% of corporate customers, but 50% of the corporate customers who started to evaluate quantum computing.

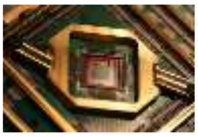
[4701] See Building Toward Q Advantage Today: Enterprise Activity & Investment by Matt Langione, BCG, Q2B 2022, December 2022 (19mn47s).

[4702] See Quantum Computing Is Becoming Business Ready by Matt Langione, Jean-François Bobier, Zheng Cui, Cassia Naudet-Baulieu, Amit Kumar, and Antoine Gourévitch, BCG, May 2023.

[4703] See Les usages de l'intelligence artificielle 2021 by Olivier Ezratty, February 2021 (742 pages), in French.

- They need to build some understanding of the trend, the buzz and the technology.
- Building a small workforce around quantum technologies is an excellent to attract high-level and skilled talent with a lot of potential. Corporations are competing with startups and IT vendors and need to find ways to attract PhD level algorithms developers, mathematicians and even physicists, when they are in the chemical and drug businesses.
- Investing in quantum technologies is not just about quantum computing but deals with other technologies like quantum sensing and quantum communications which are more mature than quantum computing.
- As the quantum threat on current public key cryptography is also part of the buzz, it has to be factored in. Even though this threat is largely exaggerated, many cryptography legacy infrastructures will need to be upgraded, pushed by standardization and, sometimes, government regulations.
- Working on quantum computing is an excellent way to revisit classical computing solutions. All the fuss on quantum computing potential benefits is creating a strong emulation with classical computing specialists, whether it works or not. As a matter of fact, classical algorithms are making a lot of progress in optimization, operational research, chemical simulations and machine learning.

To undertake this, here is a relatively simple and, all in all, fairly classic approach for corporations, which is laid out in a dozen points, some of which come from the experience of major large companies (Figure 951).



technology screening
- understand quantum technologies
- concepts and wording
- decipher vendor's messages and hype
- understand the news
- what can quantum algorithms do?
- case studies applicability and range

needs analysis
- existing unsolved problems or problems that are too lengthy or costly to solve?
- create an internal communauty
- involved security specialists
- security protocols mapping

evaluation
- test some quantum algorithms at small scale
- on universal gates qubits as well as on quantum annealing or quantum simulators

education and training
- some developers, IT architects and line of businesses R&D scientists.
- study the link between quantum computing and R&D unsolved problems.
- online training
- initial training

resources
- «Understanding Quantum Technologies» ebook (free, >1380 pages).
- ecosystem events (Q2B, QCB, Lab Quantique, QIP, APS MM, SQA, …)
- vendors quantum offerings (IBM, Amazon, Microsoft, D-Wave, Pasqal, Quantinuum, IonQ, …)
- independant software vendors offerings (QC-Ware, Multiverse, …).

Figure 951: a simple method to adopt quantum technologies. (cc) Olivier Ezratty, 2022-2023.

Technology screening

Of course, you don't adopt any new technology after reading a news clip. But your management may push you to look at the trend and understand it. This top-bottom approach is amplified by the strategy consultants and analysts who are ringing the alarm bell in the direction of "business decision makers", up to, if they can, to CEO and executive boards.

The task is hardened with quantum computing because we are still in an intermediate exploratory phase where quantum computers are not yet functional.

- Of course, you start with looking at the various **use cases** of quantum technologies in a "top-to-bottom" approach and what added value it could potentially bring to a business. This book can help you with its inventory of business applications, starting page 855, where I propose a methodology to classify these case studies in a reasoned way, sorting out the future and the present. They are organized by vertical market. If your market is not there, it doesn't necessarily mean that you shouldn't care. If you have a developer and/or mathematical background, you can have a look at what can be done at a lower level with quantum algorithms by looking at the algorithms part of this book, starting page 855.

- Then, you still need to understand the **technological dimensions** of quantum computing and related telecommunications and cryptographic matters. One thing is to understand what the state of the art is, how it is changing over time, what are the scientific and technology challenges. This technology screening must be done on a continuous basis. Things are changing fast in this domain.
- Don't miss the potential of **quantum sensing**. It may be enormous in various industries where precision is mandated. Quantum sensing helps measure with greater precision nearly any physical dimension: time, gravity/acceleration, magnetism, electro-magnetic waves and the likes.
- Learn how to **decode** analysts, research labs and vendors lingua, particularly in the field of over-promises. I provide a few examples in this book, about the fact that quantum computing is not a miracle solution that can speed up all computer processing. Learn some tricks to assess the real technology readiness level (TRL) of advertised quantum innovations. Also, understand that quantum computing is not adapted for big data applications. One important aspect here is the timing of innovations given the analysis timeframe is quite large, sometimes accounted in decades.
- Attend **ecosystem events** such as the QC Ware Q2B conference, Lab Quantique meet-ups, or quantum business conferences that are now organized all around the world. Also prefer those events where at least some scientists are delivering keynotes.

Needs analysis

Many vendors will push you to look at your needs even before describing what is really possible to do today. This is one reasonable approach, but it should not be implemented independently from a real technology assessment.

- Identify **intractable problems** in the company's applications and business needs portfolio. This is a question that developers and data scientists can sometimes answer. For example, these are complex optimization problems involving the orchestration of many resources. You have also to look at your current existing or potential usage of high-performance classical computing. What if scenarios can also be built on the power quantum computing can bring. For example, what if you could solve such or such complex business problem that was never addressed, particularly related to some optimization process?
- Then, back to technology, look at the related case studies, existing algorithms that are supposed to solve these problems. Understand the **scope of existing case studies**: are they small scale pilot projects or deployable applications? Most of the time, they are in the first category with no quantum advantage at all. Then, ask vendors and independent specialists on the size and characteristics of the quantum computers and/or hybrid systems associating a quantum computer and classical computers that would solve the business problem with a real-world size.
- Create an **internal community** of engineers and business specialists interested in quantum technologies, as Goldman Sachs, Morgan Stanley, BMW, Volkswagen, Airbus, EDF and Total have done, for example. It can be fed with presentations from research labs and vendors and sharing the understanding engineers have about quantum technologies, identify key questions to ask, brainstorm about business needs where quantum technologies could help.

- Launch a mapping of **security protocols** threatened by quantum computers and the infamous Shor integer factoring algorithm. What data in the present that could be intercepted now could have some value in the future for an attacker? If present data has some value more than 5 to 10 years from now, you may need to start worrying and looking at QKD and PQC solutions or even revisit the way you implement applications in the cloud. Many service vendors are proposing methodologies and services to run security audits[4704].
- Look at what your **peer companies** and those from your own ecosystem are doing with quantum technologies. Some may be vocal, like in the financial sector, some less. But there's now no lack of industry events where this topic is discussed.

Training

Training a core team of people will be necessary to launch the two previous steps.

- Train a **few developers** in quantum programming. This can be done by letting people interested in the matter spend time on it on their own. The information and tools are available online with IBM, Microsoft, Eviden, D-Wave and many other places. Open-source cloud-based tools are already there. The youngest and most curious developers will probably be the ones who will best adapt to quantum computing programming paradigms, which are difficult to assimilate when being trained for classical programming. These must also have a stronger mathematical background than the average developer. Analog electronics engineers can also be interested with quantum programming giving the analog nature of the underlying processes like interferences between qubits.
- Understand the links between **quantum computing and artificial intelligence**. Quantum machine learning is a new sub-discipline of quantum algorithms that deserves to be explored and understood. You may discover that classical machine learning and deep learning will fare well for a long time.
- The small hidden advertising in this book is here: I propose a **one to three days customized training** for corporate engineers, IT people, R&D and innovation specialists who are curious to discover the whereabouts of quantum computing and other quantum technologies.

Evaluation

- Talk to the many quantum computing **independent software vendors**, particularly with those who are specialized in your vertical.
- Test **some algorithms** in the cloud with universal quantum computers (IBM, AWS and Microsoft cloud, OVHcloud, etc) or quantum annealer (D-Wave) or with emulators (Quirk, Eviden, IBM, Microsoft, AWS, Google). The available case studies are discussed in this book in the section on algorithms and applications by market, starting page 1037.
- Do not hesitate to test algorithms on D-Wave **quantum annealers** despite their relatively poor image among universal quantum computer purists. Quantum algorithms for these computers are suitable for solving complex optimization problems and represent a large part of what quantum computing can bring, whether in biology or finance, to take just two examples.

[4704] See Quantum Readiness Toolkit: Building a Quantum-Secure Economy White paper, World Economic Forum and Deloitte, June 2023 (17 pages).

- Also keep an eye on **quantum simulations** which are useful for solving two main classes of problems: materials and chemical simulations, and complexity problems. Pasqal (France) and QuEra (USA) are not far from delivering very interesting hardware here. The first Pasqal system with 100 cold atoms qubits is one of the quantum computers that is the closer to enabling some potential quantum advantage.
- Avoid the **do nothing approach**. Since quantum technologies adoption takes a while, you would be left behind against your competition. This may look contradictory with the need to avoid falling into the current quantum hype. Well no. Sort the hype and find what is useful! You'll find stuff!

Congratulations, you have saved yourself an overpriced McKinsey or BCG study!

Quantum technologies and society

We will leave quantum physics, hardware, mathematics and algorithms to focus on the links between quantum technologies and society. We are still at the very beginning of this technological revolution. What will follow is a mixture of observations and interpolations. Like with any digital technology wave, the quantum wave will affect society and industries at several levels, some of which can be anticipated, others less easily.

I am interested in connecting the potential impact of quantum computing with regards to mankind ambitions, the role of science fiction in the buildup of quantum imaginary, the philosophy of quantum physics, the way in which religions and spiritual movements may embed quantum whatever in their thinking, quantum technologies ethics, education and training in quantum computing, the role of gender balance in the sector and, at last, quantum vendors marketing side effects.

Human ambition

Quantum computing is easily presented to the general public, or understood, as bringing a computational power defying imagination, going beyond anything that has been done so far. Quantum computing would thus be a way to circumvent the current sluggishness of Moore's law. It would make it possible to maintain some sort of eternal technology growth exponentiality. This may give the impression that, with quantum computing, mankind will have a tool providing him with infinite power and total control of information, in the line of many myths built around artificial intelligence and its ultimate mythical destiny, Artificial General Intelligence (AGI). In 2018, the futuristic American physicist and author **Michio Kaku** predicted that quantum computers will be the ultimate computers capable of surpassing human intelligence[4705]. Here we go again with the Singularity!

Artificial intelligence and quantum computing seem to have no boundaries. They illustrate mankind's desire for power and omniscience, to shape matter if not minds, and to have the capacity to predict the future, making it almost deterministic. So much that it would be the abandonment of free will[4706] (Figure 952). Of course, not!

Figure 952: no, quantum computers won't end free will!

Quantum physics has generated its share of questions about the nature of the world. The indeterminism of quantum state measurement has become that of life. Quantum entanglement has given rise to pseudo-scientific explanations of telekinesis and the transmission of thought. We will see in the following section how quantum medicine mixes nano and macro worlds in a fancy way.

[4705] See The World's Most Disruptive Technology (That No One Is Talking About), Part II by Ian Connett, 2018.

[4706] As suggested by this article in The Atlantic of June 2018, the title of which has little to do with the content!

The mechanical nature or not of consciousness is at stake. For **epiphenomenalism** (definition), our consciousness is the result of physical phenomena in our body and brain but without direct external physical effects. Behavior is the result of the brain's action on the muscles.

For **mysterianism**, the understanding of consciousness is beyond the reach of Man. As consciousness depends at a low-level on quantum phenomena which govern a-minima the relations between atoms of the molecules of our brain, some people deduce a little quickly that quantum computing would allow AI to become general as in this debate! But these are at this stage fancy elucubrations.

Ambitious projects such as the European **Human Brain Project** led by Henri Markram aim to simulate the brain's behavior in a computer and thus to understand how it functions from start to finish, even if it is not possible to do so on even a molecular scale. In another fashion, the ability of quantum computers to simulate quantum phenomena has also sustained the idea that we are objects of a great simulation. An idea that ignores the constraints of dimensionality.

An exploration of the mysteries of quantum computing and complexity theories allows us to put our feet back on the ground. Complexity theories describe various limits to the nature of problems that can be solved with quantum computing. Computational omnipotence does not exist. We will always be obliged to use various forms of reductionism to simulate the world, i.e. we will only be able to do it correctly at "macro" scales and not at "micro" or "nano" scales for matters related to computational magnitude[4707]. A bit like predicting the weather thanks to the finite element method applicable to large portions of sky and not at the level of each water molecule.

The limits of the possible will be constantly pushed back, but they will remain. Just like those of understanding the world which are confronted with the temporal and spatial limits of the Universe. We will probably not be able to know what was happening before the big bang nor to evaluate the existence of multiverse. Being unverifiable, these interpretations of the world can only remain speculations and not become real science. In the same way, our physical means will probably never make it possible to simulate our world in-extenso.

Quantum physics also introduces a lot of chaos and randomness into biology that no computer will ever be able to fully simulate and control.

Finally, this quote from Scott Aaronson sums up the quest for quantum computing. This would be justified by the desire to counter those who say it is impossible. The rest is the icing on the cake[4708]. This is obviously some humor, not to be taken at face value!

Figure 953: Source: A tale of quantum computers by Alexandru Gheorghiu (131 slides).

Science fiction

Science fiction and particularly movies and TV series have been great sources of inspiration and also of delirium about the potential of quantum technologies (Figure 954).

4707 See Three principles of quantum computing by Yuri I. Ozhigov, Moscow State University of M.V. Lomonosov, June 2022 (10 pages) which tries to address this topic.

4708 The quote comes from A tale of quantum computers by Alexandru Gheorghiu (131 slides, slide 31). The Combination Problem for Panpsychism by David Chalmers (37 pages) and Why Philosophers Should Care About Computational Complexity by Scott Aaronson (59 pages) ?

They have created an imaginary world made of teleportation (**Star Trek)**, supraluminal speed transportation (**Star Trek**, **Star Wars)**, various entanglements and miniaturization (**Ant Man**[4709]), states superposition (**Coherence)**, parallel or multiverse worlds (**Fringe**, **Spiderman**, **Counterpart**, **Dark**, **Doctor Strange in the Multiverse of Madness**) or time travel (**Interstellar**, **Umbrella Academy**).

Figure 954: quantum in science fiction movie and TV series.

In some cases, the quantum term is used without any scientific connection to quantum physics, as in the 2013 James Bond **Quantum of Solace,** which means approximately "an ounce of regret".

Or it plays the role of the "MacGuffin", popularized by **Alfred Hitchcock,** the gizmo that the protagonists will look after from the beginning to the end of a movie without us really understanding what's inside or all about. This is the case of the **Ronin** movie. We find another one in the movie **Hard Kill** with Bruce Willis released in February 2020. Bad guys are trying to get the "code" that will activate a "quantum AI", but its contours are quite blurred. All we know is that it could eventually do some "good" things just like hacking an airliner to crash it. In short, it is a banal "dual" civil-military solution. The bullets will rain down until the bad guy is dead without us really knowing what it's all about. However, a small 11-page guide tries to explain quantum physics to screenwriters[4710]! It contains some language basics that can be used to create scripts. Another 2022 paper portrays the role of quantum computing in movies[4711]. The usual scriptwriters do not hesitate to twist things, like **Christopher Nolan** with his elastic vision of time arrows in Interstellar or Tenet. Although not being about science fiction, Nolan's **Oppenheimer** released in 2023 showcases famous quantum scientists like Niels Bohr, Albert Einstein, Isidor Isaac Rabi and even Richard Feynman.

In March 2020, the eight-episode TV mini-series **Devs** was the first to be built around the prowess of a quantum computer capable of reconstructing the past up to Christ's crucifixion and predicting the future anywhere on Earth. With a video! Of course, this doesn't make any sense with today's technologies, but also with those of tomorrow[4712].

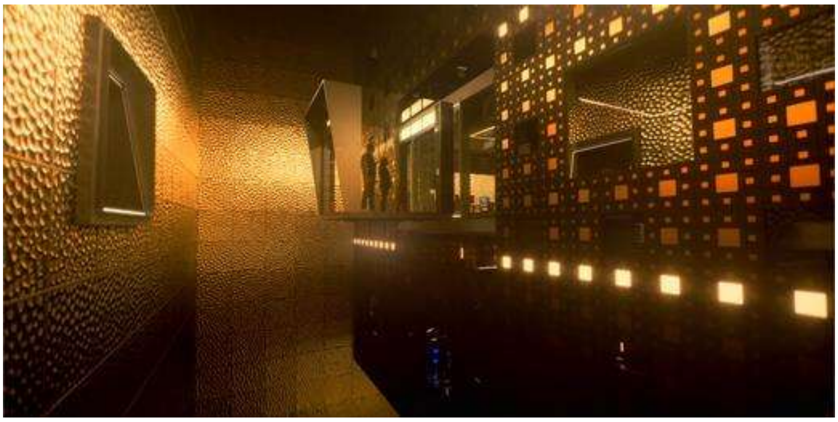


Figure 955: Dev's series quantum computer is sitting in a suspended huge cage.

[4709] See 'Ant-Man' science adviser explains the real-life physics behind the film by Denise Chow, July 2018, which explains the links between quantum physics and the scenario of the last Ant Man. Well, knowing that there is none to enlarge or miniaturize a character.

[4710] See The Sci-Fi Writer's Guide to Quantum Physics by Radha Pyari Sandhir, 2019 (11 pages).

[4711] See Quantum Cinema and Quantum Computing by Renate C.-Z. Quehenberger, May 2022.

[4712] The stylized quantum computer features an elongated candlestick that resembles those of IBM and Google quantum computers. It is not connected to anything at all from the top, but that's okay! The whole thing is enclosed in a huge cube that is suspended and magnetically isolated. See this beautiful analysis of the series: "Devs" by Alex Garland: a quantum thriller in Silicon Valley by Romane Mugnier in Usbek&Rica, May 2020.

It is a level of complexity problem and also about getting the training data. Even assuming that the Universe is totally deterministic, it is impossible to capture the precise position of all particles in space to determine their past and future. And this comes up against one of the key principles of quantum physics, Heisenberg's indeterminacy.

Figure 956: Dev's quantum computer is not well isolated…!

Its derivative states that one cannot accurately capture the position and velocity of an elementary particle. From this point on, everything falls into place to model and simulate the world with precision (Figure 955 and Figure 956)!

In 2015, episode 11 of season 2 of **Scorpions** featured a quantum computer made of lasers and a large plexiglass cube capable of injecting ransomware into the US Federal Bank with just 4 qubits! Quite a feat! The heroes hack the computer dressed as cosmonauts and by redirecting the beam of one of the lasers towards the luminous cube (Figure 957). We are far off from any realistic quantum computer here!

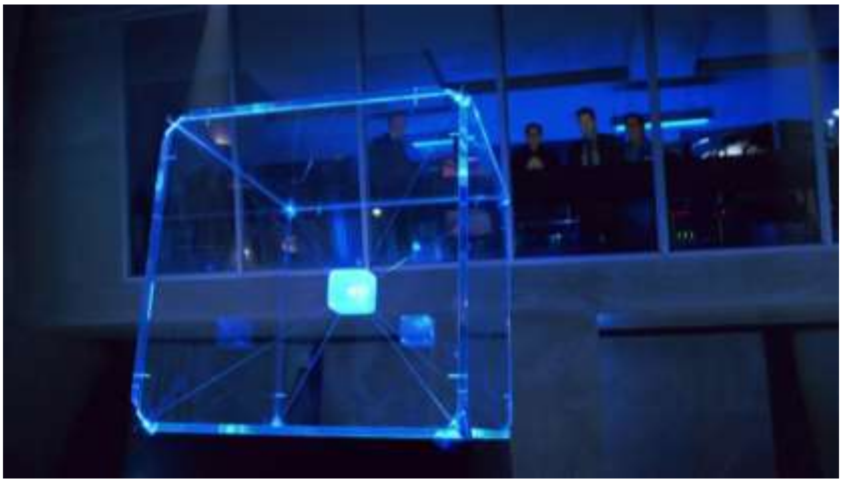


Figure 957: Scorpion's quantum computer could endanger banks with… 4 qubits!

Quantum science being trendy, it is showcased in more TV series. In 2023, the **Bird Box** movie and Netflix series is playing with the nerves of the audience with unseen monsters which are quantum beings due to these being in an undefined state. **Hijack** episode 3 deals with quantum, a Schrödinger's cat and the nerves of the audience, with blank (or not blank) cartridges in a gun. **Black Mirror** season 6's first episode "Joan is awful" showcases a mysterious quantum computer that runs a virtual world. Antman's **Quantumania** has a good name, way too many special effects and a bad script[4713]. In the **1899** series that takes place in a Titanic like ocean liner, one episode showcases a quantum computer driving a giant simulation. **Heart of Stone** (Netflix) also showcases some omniscient quantum computer. Finally, the **Quantum Leap** series from NBC is about a time machine.

Science-fiction is fine when it stays in the science-fiction realm. The problem starts when pseudo-researchers present science-fiction as if it was actual science instead of classifying it in a rough "hard science-fiction" category that is looking for some form of scientific credibility although being most of the time heavily farfetched. So, when some singularists tell you science and quantum physics could help resuscitate the dead using some fancy Dyson sphere and the likes, just forget it or just have some

[4713] See Argonne Researchers Demystify Marvel's Quantumania, by Laura Thomson, AZOquantum, July 2023.

fun[4714]. The same can be said about this work by two researchers from Germany and Georgia (the country) who published an amazing pre-print explaining that some quite advanced extra-terrestrial could have built a very powerful quantum computer out of a black hole and propose to detect it through SETI. It seems like an updated version of Dyson's sphere, a hypothetical spherical structure envisioned by Freeman Dyson in 1960 that would capture all energy from a star of galaxy and would be detectable in the infrared spectrum. Needless to say, the paper is weak on quantum engineering on how an ETI (extra-terrestrial intelligence) would control qubits within a black hole from a practical standpoint[4715 4716]. Science fiction can even come from quantum startups, like this paper from a team at Terra Quantum dealing with quantum simulation worlds[4717].

These science-fiction dreams are far removed from the science of today and probably tomorrow. Their benefit is to create vocational aspirations. Dreaming drives innovation. Even when a young person discovers that science does not allow them to realize the scenarios of these fictions, they can discover the infinite field of applications of quantum physics and still be creative. If real-world quantum technologies are less impressive than Star Trek magic, it still can do wonders and bring new generations of researchers and innovators.

The use of quantum physics in Hollywood movies can also be used to convey other messages. As is often the case, they can agitate the potential of an external threat against which the USA should respond with strength. It would not be surprising to see fictions emerge in which the quantum threat comes from China. These movies often illustrate the myth of the hero who can get through adversity, also illustrating an alternative to the centralized powers of governments[4718].

In novels, fiction can also have pedagogical virtues. This is to some extent the case of **The Key of Solomon,** a novel by Portuguese author José Rodrigues Dos Santos published in 2015. In an affair mixing espionage and quantum computing, the hero spends his time teaching quantum physics to the other protagonists of the story. This gets the message across in a didactic way and without overly taxing science.

[4714] See A Dyson Sphere Could Bring Humans Back From the Dead, Researchers Say by Stav Dimitropoulos in Popular Mechanics, March 2021. Which refers to Classification of Approaches to Technological Resurrection by Alexey Turchin (Digital Immortality Now, Foundation Science for Life Extension) and Maxim Chernyakov (Russian Transhumanist Movement), not dated (39 pages). It suggests a Dyson sphere, some quantum algorithm based on a QRNG and some weird magic with an Everetian parallel universe could help resuscitate the dead. The science-fiction, not science at all. It also suggests quantum physics could help read data from the past, a bit a la Devs.

[4715] See Black holes as tools for quantum computing by advanced extraterrestrial civilizations by Gia Dvali and Zaza N. Osmanoval, January 2023 (18 pages).

[4716] See Black holes destroy nearby quantum superpositions, thought experiment reveals by Vlatko Vedral, PhysOrg, March 2023.

[4717] See Do we live in a [quantum] simulation? Constraints, observations, and experiments on the simulation hypothesis by Florian Neukart, Anders Indset, Markus Pflitsch and Michael Perelshtein, Terra Quantum, December 2022 (27 pages).

[4718] See Quantum Computing, Hollywood and geopolitics by Jean-Michel Valentin, March 2019. The author is a French specialist in strategic studies, sociology of defense and American strategy. The article relies heavily on the film Mortal Engines (2018), whose scenario only indirectly emphasizes quantum, with a past quantum war that ravaged the planet.

Quantum foundations

Philosophy is a process of critical reflection and questioning about the world, knowledge and human existence. It creates a connection between all these dimensions. The discovery of quantum physics at the beginning of the 20th century created a real philosophical shock wave through the upheavals it brought to our understanding of the world at the microscopic level[4719]. It called into question key notions such as the links between reality and observations, or between ontology and epistemology. And the debates are still raging about it. If you meet a group of quantum physicists and want to have some fun, ask them simple questions like “what is a quantum state?”, “what does the wavefunction mean?” or “what is contextuality?”! They may not agree on a (simple) answer.

Quantum physics and its missing ontology

Science has always been closely linked to philosophy. It is not by chance that a doctorate is a "PhD", or Doctor of Philosophy, whether in humanities or in so-called hard or exact sciences. The great physicists and mathematicians of the 19th and early 20th centuries were also philosophers, which is less common now, due to a process of accelerating specialization.

The creators of quantum physics were constantly questioning the impact and meaning of their discoveries. **Niels Bohr** was also both a physicist and a philosopher, influenced in particular by **Søren Kierkegaard** (1813-1855, Danish). **Erwin Schrödinger** was even more of a philosopher than a physicist (Figure 958). He had studied Western and Indian philosophy before creating the famous wave function that bears his name[4720]. An assistant to Niels Bohr, **Werner Heisenberg** had also invested a lot of his time in philosophy and it related well with his work around the mathematical modeling of quantum physics and the famous indeterminacy principle.

The debates that agitated the physicists of quantum mechanics often took as much the form of philosophical jousting as of physical or mathematical debates, all the more so since the founders of quantum physics were not experimenters and were rather theoreticians[4721]. History has, moreover, forgot the names of the experimentalists[4722].

Quantum physics has generated endless debates since its beginnings because its formalism is difficult to associate with the principles of reality usually applicable in classical physics. Intuitive classical physics understanding has historically been associated with an ontology (Figure 959).

[4719] In practice, these upheavals occur mainly at the nanoscopic scale, that of atoms and their constituents, the nuclei and electrons. However, quantum effects can also be observed at the scale of large groups of particles that can be microscopic, as is the case with large molecules and their wave-particle duality, in Bose-Einstein condensates or superconducting currents. Knowing in all this that the frontier between quantum physics and classical physics has regularly evolved over the last century.

[4720] Michel Bitbol indicates that in the epilogue of "What is life? Mind and matter", Erwin Schrödinger wondered whether consciousness was singular or plural. If consciousness is only experienced in the singular, its extension to a global consciousness such as that of the Universe is only a risky extrapolation and difficult to prove experimentally. The thesis of a consciousness of the Universe is defended by some scientists. See for example Is the universe conscious? It seems impossible until you do the math by Michael Brooks, April 2020, which refers to the work of German mathematicians who try to define mathematically the notion of consciousness, allowing them to apply it then to the universe as a whole. Details are in The mathematical structure of integrated information theory by Johannes Kleiner and Sean Tull, 2020 (22 pages). It's cold and abstract!

[4721] The book Fantaisies quantiques - dans les coulisses des grandes découvertes du xx^{e} siècle by Catherine d'Oultremont and Marina Solvay, 464 pages (2020) tells the story of the famous 1927 Solvay conference. It is a very beautiful history of quantum physics that tells touching stories of its various protagonists in the first half of the 20th century. It seems the book is not yet available in English.

[4722] We mentioned many of them at the beginning of this book, such as Johann Balmer, Theodore Lyman, Friedrich Paschen, James Chadwick, Arthur Holly Compton, George Paget Thomson, Clinton Davisson and Lester Germer. The names of these experimental physicists generally do not ring a bell to the general public and scientists, whereas the general public has heard much more about Max Planck (with his constant more than for the black body radiation explanation), Albert Einstein (for the theory of relativity more than for the photoelectric effect explanation), Niels Bohr (for his model of the atom), Erwin Schrödinger (more for his cat analogy than for his wave equation) and Werner Heisenberg (for his indeterminacy principle, commonly called uncertainty, but not much for this huge work on quantum physics mathematical foundations). Among the founding fathers, Paul Dirac, Wolfgang Pauli and John Von Neumann were geniuses but are way behind in notoriety.

In Newtonian physics, the notion of state with position and motion of an object and the laws of evolution of these properties allow the prediction of phenomena such as the motion of planets. These evolutions are perfectly observable and deterministic.

Figure 958: some books on quantum physics and philosophy.

Quantum physics was founded without such an ontology although it served to explain some known physical phenomenon like the blackbody radiation, the photoelectric effect or hydrogen's absorption and emission spectrum[4723]. It was created as a set of mathematical postulates that could help predict experimental results. You have mainly the Schrödinger wavefunction for non-relativist massive particles and others like Dirac and Klein-Gordon equations for relativist particles. In quantum physics, the prediction instrument is a probabilistic wave function that is difficult to apprehend. It is coupled with a whole host of new notions that have no equivalent in the macroscopic and classical world: energy quantification, wave-particle duality which applies to matter (electrons, atoms) and photons (all have a momentum p related to a wavevector k using Planck's constant $\hbar$, as $p = \hbar k$), the influence of measurement on the quantities to be measured[4724], measurement indeterminacy and the notion of chance.

Quantum physics is a predictive, not descriptive theory. It doesn't describe physically the electrons and other particles when they behave quantumly. It does not physically explain entanglement nor wave-particle duality.

Einstein's position was that quantum physics was an incomplete theory when creating his famous EPR paradox in 1935. Werner Heisenberg asserted in 1927 that quantum physics established the final failure of causality! The knowledge of the present no longer made it possible to predict the future from the application of the laws of physics, all the more so as the knowledge of the present with precision is also impossible[4725].

Some like Niels Bohr concluded that it was useless and even counterproductive to create some quantum physics ontology. Many attempts were contradicting each other or weren't even realist per se. Some like Hugh Everett believed that reality was a universal $|\Psi\rangle$ function, David Bohm devised some pilot waves explanations, Christopher Fuchs et al are focused on the role of agents actions and experiences in quantum Bayesianism and its derivative QBism, CSM's ontology argues that states pertain to systems and contexts, and so on. We end up having competing postulated ontologies frequently enabling the same predictions. These are hard to sort out.

[4723] An ontology deals with what is, types and structures of objects, properties, events, processes and relationships in all areas of reality. It is usually opposed to epistemology, which covers how to obtain valid knowledge.

[4724] This is not valid only in quantum physics and the infinitely small. It works regularly at the macro scale, as in any survey with biased questions for example.

[4725] "*In the strong formulation of the causal law, 'If we know the present with exactitude, we can predict the future,' it is not the conclusion, but rather the premise that is false. We cannot know, as a matter of principle, the present in all its details.* " vu dans One Thing Is Certain: Heisenberg's Uncertainty Principle Is Not Dead by Ava Furuta in Scientific American, 2012.

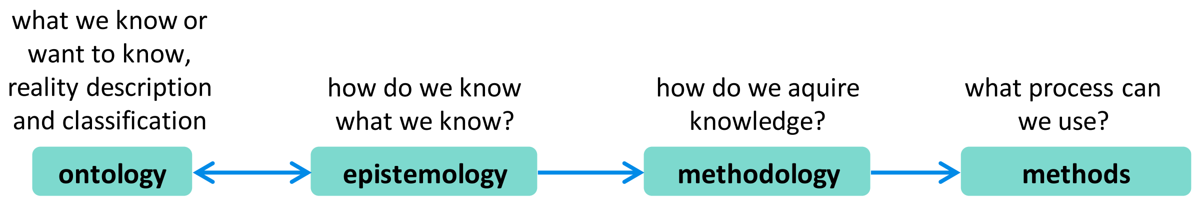


Figure 959: ontology, epistemology, methodology and methods defined.

The relationship between measured values, measurement and the observer is also debated. Would a true measurement be one that does not alter the quantity to be measured at all, a feat hard to attain in the infinitely small? In fact, quantum mechanics is contextual, the measurement depends on its context, which does not detract from its objectivity[4726].

The mathematical formalism of quantum physics from 1900 to 1935 was not at all disconnected from the observable physical world. It made it possible to explain experimentally studied phenomena such as black body radiation, interference from Young's slits with light and matter waves, or spectral excitation lines of atoms under a wide range of conditions. We have seen how important they are in photonics, with trapped ions, cold atoms or NV centers. Electron spin explained the hyperfine energy levels of atoms observed in 1922 in the Stern-Gerlach experiment[4727].

Relativistic quantum chemistry derived from Paul Dirac's equations explained spectral shifts of transitions involving low layer electrons of heavy atoms moving at relativistic velocities. The list is long.

If quantum physics explained experimental measurements, linking the observed reality and the theory, it was however insufficient to produce an unanimously accepted representation of reality. It is part of a history of science that described matter step by step, with nested Russian dolls. Atoms were initially abstract, theoretical entities before being embodied and accurately described and then directly observed as we do now with electron microscopes or cryogenic microscopy (Cryo-EM).

The very existence of atoms was debated at the end of the 19th century between Ludwig Boltzmann who believed in them and Wilhelm Ostwald and Ernst Mach who opposed them. Protons and neutrons were then discovered. These were split into quarks and gluons with particle accelerators, turning the physical world into a maybe endless fractal. Obstacles to understanding it could simply be related to the enormous amount of energy that needs to be injected into particle accelerators, which is increasing the more elementary the particles are.

Quantum Physics interpretations

Quantum physics philosophy belongs to the broad field of **quantum foundations**. It focuses essentially on the multiple possible interpretations of the same theory and their mathematical formalism. They all ask related questions such as: does reality exist independently of the observer? What is the physical meaning of the wave in the wave-particle duality? Is it a real wave of an indeterminate nature or is it a simple statistical and probabilistic mathematical model incomplete in its ability to describe

[4726] This approach is challenged by the Bayesian quantum interpretation (QBism for Quantum Bayesianism) promoted from 2002 onwards by Carlton Caves, Christopher Fuchs, Rüdiger Schack and then David Mermin. See in particular QBism The Future of Quantum Physics by Hans Christian von Baeyer, 2016 (268 pages).

[4727] This one made a beam of heated silver atoms pass through a non-homogeneous magnetic field, which generated two distinct spots on a screen.

physical reality[4728]? Other quantum foundation fields include work on the measurement problem and its indeterminism, the associated notions of causality, local realism and the completeness of quantum mechanics[4729].

Many of these theories deal with the complicated notion of **contextuality** in quantum physics according to which quantum measurements is not revealing pre-existing values but depends on the measurement context like the angle of a spin measurement in the famous Stern-Gerlach experiment or of a photon polarization angle[4730]. Contextuality is dealt with in the **Kochen-Specker** theorem *aka* the Bell–Kochen–Specker theorem demonstrated by John S. Bell in 1966 and by Simon B. Kochen and Ernst Specker in 1967. It is a "no go theorem" that creates constraints on the types of putative hidden-variable theories trying to explain the predictions of quantum mechanics in a context-independent way. It was complemented by Spekkens's theorem in 2004 that shows that entanglement can't be fully explained using classical-like models with hidden variables[4731].

Several interpretations of quantum physics have thus emerged to try to provide answers to these many questions (Figure 960)[4732]. The debates are still open[4733]!

Copenhagen interpretation is the canonical version of quantum foundations[4734]. It is essentially probabilistic. Quantum physics postulates and the wave function describe all that we can know about reality but not reality itself, which is neither accessible nor meaningful. It adopts the positivist approach according to which one sticks to observations, laws and phenomena, without seeking to know their intrinsic nature. It was the "Bohrian" side of the historical debate between **Niels Bohr** and Albert Einstein, mainly between 1927 (in the famous Solvay Conference in Brussels) and 1935 (the EPR paradox paper and subsequent discussions). Adopted by Werner Heisenberg, Max Born, Wolfgang Pauli, Paul Dirac, it is the classical and dominant interpretation of quantum physics that is still mostly

4728 These different interpretations can be evaluated according to the criteria of scientificity of Karl Popper (1902-1994, Austrian/English), according to which a theory is scientific if it can be refuted by crucial experiments giving precise results. The theory cannot be shown to be irrefutable. A proven scientific theory is therefore always between two waters, in the state of a theory corroborated by facts, until proven otherwise. The history of physics has shown, however, that the "serious" theories of the past were mainly challenged by the broadening of their perspective and context: with large masses and high velocities (for relativity) and in the microscopic (for quantum physics). In their initial contexts, they remained perfectly valid. I like the very current example of the search for dark matter, which would represent 85% of that of the Universe. Its existence is not yet experimentally demonstrated but is assumed by the application of the laws of gravity and relativity applied to the cohesion of galaxies. It can be refuted or partially verified at present in at least three ways: by discovering elementary particles associated with dark matter (quantum detectors are built in this sense, and have so far given nothing), by modifying the laws of general relativity as the Israeli Morchedai Milgrom is trying to do, or by discovering hidden matter such as the dust of galaxies that could explain all or part of their cohesion without using dark matter. Belief in God and many areas of metaphysics are not part of science because they are neither demonstrable nor refutable.

4729 See Argument for the incompleteness of quantum mechanics based on macroscopic and contextual realism: EPR and GHZ paradoxes with cat states by Jesse Fulton et al, August 2022 (20 pages). The hidden-variables debate is not closed yet!

4730 Works on contextuality frequently deal with discrete variable quantum systems. It can be extended to continuous variable systems. See The Interplay between Quantum Contextuality and Wigner Negativity by Pierre-Emmanuel Emeriau, April 2022 (221 pages), a thesis under the direction of Elham Kashefi and Shane Mansfield and the related Continuous-variable nonlocality and contextuality by Rui Soares Barbosa, Tom Douce, Pierre-Emmanuel Emeriau, Elham Kashefi and Shane Mansfield, May 2019 - April 2022 (44 pages).

4731 See Contextuality for preparations, transformations, and unsharp measurements by R. W. Spekkens, June 2004-March 2005 (16 pages).

4732 See Taxonomy for Physics Beyond Quantum Mechanics by Emily Adlam, Jonte R. Hance, Sabine Hossenfelder and Tim N. Palmer, September 2023 (33 pages).

4733 See Contra multos verbos: On scandals of quantum mechanics by Theodorus Maria Nieuwenhuizen, University of Amsterdam, July 2023 (22 pages) which contains this interesting quote: "*Hence a cat can not be in a pure state. The statement "the cat is in a superposition of a state dead and a state alive" is a poetic construct with no bearing on reality. The conclusions drawn from it already convinced Schrödinger that this line of reasoning leads to absurdity (to scandals). We side with him*".

4734 The "Copenhague interpretation" naming was created by Werner Heisenberg in 1955. It consolidated Heisenberg's and others contemporary views on the realism of quantum physics and concepts that were not promoted by Niels Bohr in the 1920s like the wave packet collapse and the views on the subjective observer in quantum measurement. It's a post-mortem consolidation on the quantum physics foundations. See Who Invented the "Copenhagen Interpretation"? A Study in Mythology by Don Howard, Department of Philosophy of University of Notre Dame, Indiana, 2004 (15 pages).

taught like in the Cohen-Tannoudji/Laloe/Diu bible of quantum physics. It is satisfied with an essentially mathematical and probabilistic model that does not seek to physically describe the entire real world. There are, moreover, sub-branches in the Copenhagen interpretation, notably around the open and closed theories that had opposed Heisenberg and Dirac from 1929 onwards.

	Copenhagen	**Bohm**	**Everett / DeWitt**
world entities	macroscopic quantum objects	wave function and particles position	wave function with quasi-classical world
determinism	indeterminism	determinist	deterministic
probabilities interpretation	objective	epistemic	objective
theories predictions goal	measurement results	particles position	agents bet
locality	non-locality	non-locality	locale
theory mathematical formalism	Schrödinger equation, projections, probabilities	Schrödinger equation and pilot waves	Schrödinger equation

Figure 960: the top three interpretations of quantum physics. Source: the excellent thesis The plurality of interpretations of a scientific theory: the case of quantum mechanics by Thomas Boyer-Kassem, 2011 (289 pages).

Bohm interpretation came from **David Bohm** (1917-1992, American then Brazilian and British). He proposed in 1952 a so-called deterministic version of quantum mechanics, called "De Broglie-Bohm theory". It was inspired by ideas initially promoted - but partly abandoned - by the French physicist and took up the idea of the existence of hidden variables, insinuated by Albert Einstein in the 1930s, and by Louis de Broglie, in the form of pilot waves[4735]. The existence of local hidden variables was disproved in 1982 with Alain Aspect's famous experiment.

Many Worlds Interpretation (MWI) and its Universe wave function that was proposed in 1957 by **Hugh Everett** (1930-1982, American) and after being almost forgotten, revived by **Bryce DeWitt** in 1970. It then became the multiple worlds or multiverse interpretation in an article published in Physics Today[4736]. It is said to be realistic in the sense that the Universe is a huge wave function with a (immensely) large number of parameters. It never collapses and the world is deterministic, but split in parallel branches. DeWitt's interpretation transforms quantum probabilities within this universal wave function into parallel worlds that exist simultaneously. Since it is impossible to verify that parallel worlds exist, the theory is not refutable, but counter arguments abound[4737]. We are therefore far from an experimentally supportable interpretation[4738].

[4735] The Bohmian approach is well popularized in Quantum Physics Without Quantum Philosophy by Detlef Dürr, Sheldon Goldstein and Nino Zanghì, 2013 (304 pages). It is completed by Quantum solitodynamics: Non-linear wave mechanics and pilot-wave theory by Aurélien Drezet, July 2022 (29 pages).

[4736] See Quantum mechanics and reality by Bryce S. DeWitt, 1970 (6 pages) as well as The Many-Worlds Interpretation of Quantum Mechanics by Bryce DeWitt and Neill Graham, 1973 (146 pages) which contains "The theory of the universal wave function" by Hugh Everett, 1957. DeWitt's interpretation is also called EWG for Everett-Wheeler-Graham. John Wheeler was supervisor of High Everett's thesis and Neill Graham, a student of DeWitt. Seen in Everett's pure wave mechanics and the notion of worlds by Jeffrey A. Barrett, 2011 (27 pages).

[4737] See On playing gods: The fallacy of the many-worlds interpretation by Luis C. Barbado and Flavio Del Santo, November 2023 (11 pages).

[4738] See Making Sense of the Many Worlds Interpretation by Stephen Boughn, 2018 (36 pages) which dismantles a bit the model of parallel universes, especially in terms of dimensioning. By calculating the number of bifurcations of the Universe since its birth, and taking Planck's time as a basis, we end up with a number of parallel worlds that is beyond comprehension and all imaginable analogies. As for Schrödinger's cat, the dead cat and the living cat cohabit in two parallel worlds and the matter is settled!

This theory has also been promoted by David Deutsch, also known for his quantum algorithms and still keeps quantum foundation physicists busy[4739]. It has a "Everything is a Quantum Wave" Interpretation (EQWI) variation[4740]. It feeds many science fiction dreams and mad mysticism, everything being linked to everything and vice versa, especially souls and consciences. Howard Wiseman extended this theory with embedding interactions between these many worlds[4741].

GRW theory published in 1986 by the Ghirardi-Rimini-Weber trio proposes a different formulation of Schrödinger's equation with a spontaneous reduction of the wave function that is not simply related to the notion of measurement. This is a rare formalism that could experimentally validated.

QBism is a derivative from quantum Bayesianism, starting with some ideas by **Edwin Jaynes** (1922-1998, American, yes, the Jaynes from the Jaynes-Cummings Hamiltonian) and pushed by **Christopher Fuchs**, **David Mermin**[4742] et al, based on **Frank Ramsey**'s anti-realist interpretation of probability (1903-1930, British philosopher) and **Ludwig Wittgenstein**'s work (1889-1951, Austrian philosopher)[4743]. It interprets quantum physics through the eye of the observer agent actions and experiences.

Relational Quantum Mechanics (RQM) was crafted by **Carlo Rovelli** in 1994. This relational ontology considers that a quantum state is defined by the relation between any pairs of systems. One of them can be an observer. It is inspired by special relativity principles and its observer reference model.

CSM ontology was proposed in 2015 by **Alexia Auffèves** and **Philippe Grangier** in order to reconcile the Copenhagen interpretation with realistic models[4744]. CSM is a minimalist ontology designed to pacify somewhat these old debates. In this model, the properties that are measured, called **modalities,** are attributed to a **system** (studied system, as isolated as possible) within a **context** (completely specified measurement device and settings like a photon polarizer or Stern-Gerlach experiment angle) (Figure 961).

[4739] See The Many-Worlds Interpretation of Quantum Mechanics: Current Status and Relation to Other Interpretations - Research workshop of the Israeli Science Foundation, Tel-Aviv University, October 2022, 6 days and 41 hours of videos!

[4740] See The Everything-is-a-Quantum-Wave Interpretation of Quantum Physics by Vlatko Vedral, University of Oxford, March 2023 (4 pages).

[4741] See Quantum Phenomena Modeled by Interactions between Many Classical Worlds by Michael J. W. Hall, Dirk-André Deckert and Howard M. Wiseman, PRX, 2014 (17 pages).

[4742] See Is the moon there when nobody looks? Reality and the quantum theory by David Mermin, April 1985 (14 pages).

[4743] See Quantum Wittgenstein - Metaphysical debates in quantum physics don't get at 'truth' – they're nothing but a form of ritual, activity and culture by Timothy Andersen, Aeon, May 2022.

[4744] CSM results from the creation with Nayla Farouki of the CEA of a group dedicated to the foundations of quantum mechanics in Grenoble. In 2013, they form a group with Philippe Grangier, who has long defended contextual objectivity. CSM is documented in several papers: Contexts, Systems and Modalities: a new ontology for quantum mechanics, January 2015 (9 pages) lays out the key principles of CSM ontology, tying physical properties to the system, and to the context in which it is embedded. Violation of Bell's inequalities in a quantum realistic framework, International Journal of Quantum Information, February 2016 (5 pages) reuses a lot of content from the first paper, commenting on observed "loophole free" violation of Bell's inequalities. Recovering the quantum formalism from physically realist axioms, Nature, December 2016 (8 pages) derives Born's probabilistic rule and unitary transforms from CSM. Then What is quantum in quantum randomness?, Philosophical Transactions of the Royal Society A, April 2018 (9 pages), Extracontextuality and Extravalence in Quantum Mechanics, Philosophical Transactions of the Royal Society A, May 2018 (7 pages), A generic model for quantum measurements, July 2019 (8 pages) and Deriving Born's rule from an Inference to the Best Explanation, October 2019 (6 pages). See one critic of CSM in Comments on New Ontology of Quantum Mechanics called CSM by Marian Kupczynski, 2016 (8 pages). And the older Contextual objectivity : a realistic interpretation of quantum mechanics by Philippe Grangier, 2001 (5 pages). Philippe Grangier et al continued the brainstorming on CSM with working on contextuality in Contextual inferences, nonlocality, and the incompleteness of quantum mechanics by Philippe Grangier, December 2020-January 2022 (7 pages), A contextually objective approach to the extended Wigner's friend thought experiment by Maxime Federico and Philippe Grangier, January 2023 (8 pages) and Revisiting Quantum Contextuality in an Algebraic Framework by Mathias Van Den Bossche and Philippe Grangier, April 2023 (8 pages).

Modalities are jointly associated to the system and its context, not just the system, building a contextual objectivity[4745] (Figure 962). In CSM, randomness doesn't just come from Heisenberg's indeterminacy principle but is a direct consequence of the quantization postulate and the contextual nature of reality.

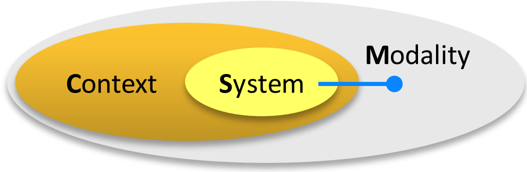


Figure 961: CSM's simple view.

CSM also helps explain the origin of probabilities, nonlocality and quantum-classical boundary. Nonlocality, aka the EPR paradox, has nothing to do with an action at a distance, but appears because a modality belongs to both a system and a context. It also solves the Wigner's friend thought experiment paradox based on a recursive observer of a measurement agent[4746].

There are many more interpretations of quantum physics than qubit types around! Like **superdeterminism** (which deterministically the observed violations of Bell's inequalities in entanglement experiments with some yet unknown hidden variable[4747] [4748]), **consistent histories** (which avoid the use of a wavefunction collapse to describe physical processes and tries to get rid of the measurement theory[4749]), **modal interpretation** (class of interpretations created starting in 1972 by Bas van Fraassen which introduced a distinction between a dynamical state over time and a value state at a given time[4750]), **quantum darwinism** (which explains how the classical world emerges from the quantum world [4751]), **dynamic histories** (which reinterprets quantum mechanics as deterministically evolving dynamical world lines in a 5D universe not far from a many-worlds interpretation[4752]), **Quantum Coherentism** from Claudio Calosi and Matteo Morganti, **Foundationalism** (there must be a source of being), the **Geometrodynamic Model of Reality** from Shlomo Barak and the **Quantum Conceptual Turn** from Diederik Aerts and Massimiliano Sassoli de Bianchi[4753] [4754]and the **category theory** of Gennaro Auletta, a sort of generic unifying meta-logic theory applicable to quantum physics[4755].

4745 Within the usual quantum formalism, a modality is a pure quantum state and a context is a complete set of commuting observables (CSCO). For a given context, CSM defines N distinguishable modalities that are mutually exclusive. If one modality is "true", or "realized", the others are "wrong" (or "false"), or "not realized". The value of N, called the dimension, is a characteristic property of a given quantum system, and is the same regardless of the context.

4746 The Wigner's friend paradox is driving hot debates among physicists in quantum foundations. See for example A general framework for consistent logical reasoning in Wigner's friend scenarios: subjective perspectives of agents within a single quantum circuit by V. Vilasini and Misha P. Woods, ETH Zurich, September 2022 (47 pages).

4747 See Rethinking Superdeterminism by Sabine Hossenfelder and Tim Palmer, May 2020 (13 pages) and What does it take to solve the measurement problem? by Jonte R. Hance and Sabine Hossenfelder, June 2022 (11 pages).

4748 See What does the world look like according to superdeterminism by Augustin Baas and Baptiste Le Bihan, British Journal for the Philosophy of Science, 2020 (21 pages).

4749 See Consistent histories and the interpretation of quantum mechanics by Robert B. Griffiths, Journal of Statistical Physics, 1984 (55 pages).

4750 See The scientific image by Bas Van Fraassen, 1980 (248 pages) and Modal Interpretations of Quantum Mechanics, Stanford Encyclopedia of Philosophy.

4751 Also see Quantum Darwinism, Decoherence, and the Randomness of Quantum Jumps by Wojciech Zurek, 2014 (8 pages).

4752 See A Dynamic Histories Interpretation of Quantum Theory by Timothy D. Andersen, August 2020 (13 pages).

4753 See Diederik Aerts and Massimiliano Sassoli de Bianchi - The quantum conceptual turn, May 2021 (48 mn) from the International Workshop on Quantum Mechanics and Quantum Information, Quantum Ontology and Metaphysics, April 2021.

4754 See Are Words the Quanta of Human Language? Extending the Domain of Quantum Cognition by Diederik Aerts and Lester Beltran, December 2021 (27 pages) which makes a symbolic projection from quantum physics phenomena to the way human language works

4755 The category theory is described at the end of The Quantum Mechanics Conundrum by Gennaro Auletta, 2019 (879 pages) which contains a good primer on quantum physics in its first 150 pages.

Other ontologies abound like **Structuralism**, **Perspectival Objectivity**[4756], **Pluralism** (atomism), **Monism** and **Infinitism**, but their scope goes beyond quantum physics.

Where are the schools of quantum foundations? In Europe, there is a quantum foundations epicenter in Italy but you find contributors in most countries. There are also two foundations who provide research grants on quantum foundations: the **Templeton Foundation** and **FQXi** (Foundational Questions Institute) which covers quantum foundations and cosmology and was created in 2005 by cosmologists Max Tegmark and Anthony Aguirre.

Other interpretations

Among the physicists who have contributed to the field of quantum physics philosophy. **Pascual Jordan** (1902-1980, German) built a theory of free will according to which one is not freer by acting randomly or in a determined way, breaking the idea that quantum non-determinism would be a proof of human free will. **Henri Stapp** (1928, American) worked on consciousness and believes that it governs the world and reality and that it can only be explained by quantum physics[4757].

Roger Penrose (1931, English) considers that consciousness results from the reduction of the wave packet and **Elizabeth Rauscher** (1931-2019, American) was a physicist who first became interested in philosophy and then moved on to parapsychology[4758].

On the other hand, **Steven Weinberg** (1933-2021, American), Nobel prize in Physics in 1979 for his work on the unification of the weak and electromagnetic forces, thought that philosophy is of little use in quantum physics other than to protect us against the errors of other philosophers[4759]. This view was shared by **Stephen Hawking** (1942-2018, English) .

In France, in addition to the CSM ontology creators, **Michel Bitbol**, a biophysicist and philosopher of science, is interested in particular in the question of consciousness, **Etienne Klein,** originally an engineer and physicist, is specialized in the philosophy of science at the CEA, as well as **Alexei Grinbaum** and **Vincent Bontemps** who are both part of Etienne Klein's LARSIM laboratory.

[4756] See Perspectival Objectivity or: How I Learned to Stop Worrying and Love Observer-Dependent Reality by Peter W. Evans, University of Queensland, 2020 (16 pages).

[4757] See Mind, Matter and Quantum Mechanics by Henry P. Stapp, 2009 (303 pages). This is the kind of book that makes non-testable hypotheses that then become gospel for the quantum medicine quacks we are talking about in the dense section dedicated to quantum fumbling. And yet, the basic idea is nothing extraordinary: brain chemistry, like all chemistry, is based on many facets of quantum physics. This becomes complicated when the hypothesis is put forward of an implementation of entanglement in consciousness. Quantum medicine goes out of the scientific game when it claims that these mechanisms can be controlled from the simple will, without counting the action on the other organs (preferably sick ones) of the human body.

[4758] Elizabeth Rauscher was the cofounder of the Fundamental Fysiks Group in 1975 with George Weissmann to work on quantum foundations. She is co-author with Richard Amoroso of The Holographic Anthropic Universe, 2009 (510 pages). They discuss a model for creating a scalable quantum computer called "Universal Quantum Computing" that is difficult to grasp between real science and crackpot science and is based on a theory called "Unified Field Mechanics" that is difficult to evaluate. The subject is detailed in Brief Primer on the Fundaments of Quantum Computing by Richard L Amoroso, 2017 (140 pages). Richard Amoroso is Director of the Noetic Advanced Studies Institute in Oakland, California. Noetics is interested in the links between quantum states and consciousness. And this goes well beyond the realm of science with Pragmatic Proof of God (Part I & Part II, 2017 by Richard L. Amoroso (34 and 13 pages).

[4759] See the chapter "Against Philosophy" in "Dreams of a Final Theory", 1994, Steven Weinberg, which is contradicted in Physics Needs Philosophy / Philosophy Needs Physics by Carlo Rovelli, 2018. Carlo asserts that saying that science does not need philosophy is to be doing some sort of philosophy of science! See also The Trouble with Quantum Mechanics, 2016.

The story goes on and on with new interpretations popping up regularly, at least on arXiv (and excluding viXra)[4760] [4761] [4762] [4763] [4764]! Some scientists are even trying to explain quantum mechanics with the Bible[4765].

Quantum physics raises other physical-philosophical questions such as does a total vacuum exist? Indeed, quantum physics describes the energy of the vacuum, which would always be crossed by various real and virtual particles. From a practical point of view, it is therefore difficult to create an empty space that is not crossed at all by electromagnetic waves or particles of all kinds. If therefore nothing exists, what was there before the big bang? And let's not talk about the nature of time, which is still a matter of debate.

Beyond Quantum Foundations

The current philosophical approach to quantum physics baffled me a bit when I discovered it around 2019. Most of the writings in this discipline are full of mathematics and physics. They must break records in this respect compared to any other subject covered by the field of philosophy. Above all, they do not deal much with human sciences per se.

What are the human consequences of these different interpretations of quantum physics? Are there philosophical questions other than these related to the interpretation of realism at low-level? There is much to be done in this area. The notions of uncertainty and indeterminism inevitably lead to the notion of free will and destiny (as seen by Pascual Jordan). The quantum philosophical focus on the microscopic and nanoscopic scale of physics could also be a form of reductionism preventing a wide-angle view of its societal impact.

Also, is the extension of the scientific field infinite? What are the limits of human knowledge that seeks to explain and interpret everything about the way the Universe works? What do we miss and why? What links can be made with our humility? What are the structural limits to our insatiable curiosity? This is only reformulating the very notion of Kantian metaphysics, the "*science of the limits of human reason*".

The philosophical question thus concerns the notion of the feasible and the unfeasible and its evolution over time, a perspective provided by the history and philosophy of science. What are the limits of human ingenuity? What is superhuman? Will we be able to create ultra-reliable and *scalable* quantum computers? Complexity classes, that are described in this book starting page 938, should also serve as tools for this kind of thinking.

How to extend the interpretation of quantum physics to the metaphor of quantum computation: highly rich and complex inside but simple after measurement is done? Could it be used to simulate the living and create it in silico? This will then raise questions about man's power over nature and the associated responsibilities. We will also see the resurgence of debates on scientism, the "*science-led society*", as well as on technology solutionism, a concept described by **Evgeny Morozov**, which could provide answers to all problems, especially environmental and health problems, which cannot be treated properly with the required urgency.

[4760] See An alternative foundation of quantum mechanics by Inge S. Helland, May-June 2023 (45 pages).

[4761] See A Theory of Inaccessible Information by Jacopo Surace, ICFO, May 2023 (31 pages).

[4762] See Is the Statistical Interpretation of Quantum Mechanics ψ-Ontic or ψ-Epistemic? by Mario Hubert, November 2022 (26 pages).

[4763] See The solution to the Einstein-Podolsky-Rosen paradox by Roman Schnabel, August 2022-February 2023 (6 pages).

[4764] See Epistemic-Pragmatist Interpretations of Quantum Mechanics: A Comparative Assessment by Ali Barzegar, October 2022 (40 pages).

[4765] See A biblical view restores reality to quantum mechanics by D. Russell Humphreys, Journal of Creation, 2020 (6 pages).

These questions arise more and more at a time when precaution prevails over everything, when there are fears of technological blunders in almost every field (nuclear, GMOs, fertilizers, vaccines, artificial intelligence and 5G), when the very notion of scientific progress is no longer accepted and when cognitive relativism no longer allows us to distinguish between the serious and the farfetched, leading to a collective mistrust in science. In the following section, we will precisely study a question that belongs to the field of philosophy, the question of the ethics of quantum computing.

Are these questions really specific to quantum physics and quantum computing? Aren't they recurrent as soon as a major new technology shows up? Perhaps, but these questions deserve to be asked, like those raised by the commoditization of artificial intelligence since 2012, and more recently, with large language models ala chatGPT. The interpretations of quantum physics are in any case there to remind us that in all matters, we must multiply the angles of view of problems to better analyze them. This is obviously full of lessons from a metaphorical point of view.

I wonder about all these questions by observing that, if they are not dealt with, they tend to become the field of esotericism and charlatanism as we will see in a following section dedicated to quantum fake sciences. It is a bit as if the philosophy of quantum physics had remained at the stage of fundamental research without moving on to the stage of applied research. In a way, it is in line with the level of market maturity of the technologies of the second quantum revolution. Let's bet that, as quantum technologies mature, the more this applied philosophy will develop and allow us to write a new chapter in this exciting history of science.

Responsible quantum innovation

We'll cover here the broad topic of responsible innovation and ethics pertaining to quantum technologies. We'll draw some lessons from the current quantum hype and also from what happened with artificial intelligence. We'll look at the various initiatives around the world.

Quantum hype side effects

The "quantum hype" has been perceived as being problematic for a while, mostly by many scientists who fell that the field was oversold, particularly by industry vendors of all sizes[4766] [4767]. It is hard to position the peak oil of the quantum hype ("Quipe" for some authors[4768]) but at least, with following the money invested in startups, 2021-2022 were defining years with the large funding rounds of startups like PsiQuantum and the initial public offerings (IPOs) through SPACs (special purpose acquisition companies) of IonQ, Arqit, Rigetti and D-Wave. Governments have been fueling this hype with their large national(istic) quantum initiatives and their aspirations for some technological sovereignty.

While technology hypes have always existed and contributed to drive emulation, innovations, and a field attractiveness, it works well when scientists and vendors deliver progress and innovation on a continuous basis after a so-called peak of expectations. It fails with exaggerated overpromises and under deliveries that last too long. In that case, it could cut short research and innovation funding, creating some sort of quantum winter, although government investments are reducing the risk[4769] [4770].

[4766] See Quantum computer researcher warns that the industry is full of ridiculous hype by John Christian, Futurism, March 2022, Quantum Computing Hype is Bad for Science by Victor Galitski, Maryland University, July 2021 and the quite exaggerated view from Nikita Gourianov from Oxford University, as described in Oxford scientist says greedy physicists have overhyped quantum computing by Tristan Greene, TheNextWeb, August 2022 and with a response in Separating quantum hype from quantum reality - Are the sceptics too sceptical? By Simon Benjamin, Financial Times, September 2022.

[4767] See also Disentangling the Facts From the Hype of Quantum Computing by James Clarke, Intel, IEEE Spectrum, September 2022

[4768] See Hope and Hype in Quantum Computing by Philip Nikolayev and Susmit Panda, Quantum Poet, June 2022.

[4769] See What if Quantum Computing Is a Bust? by Chris Jay Hoofnagle and Simson Garfinkel, January 2022.

[4770] See Quantum Computing's Hard, Cold Reality Check by Edd Gent, IEEE Spectrum, December 2023.

In the "Mitigating the quantum hype" paper published in January 2022[4771], I drove some lessons from past technology hypes and investigated the current quantum hype and its specifics. I laid out the structural changes happening like the vendors hype profound and disruptive impact on the organization of fundamental research. I then made some proposals to mitigate the potential negative effects of the current quantum hype including recommendations on scientific communication to strengthen the trust in quantum science[4772], vendor behavior improvements, benchmarking methodologies, public education and putting in place a responsible research and innovation approach. But the hype did not stop. In 12 months alone (October 2022-September 2023), it was fueled again and again, showing a huge discrepancy on the promoted and real capacities of quantum computers. The messages usually come from visible people who have a very poor knowledge of the real whereabouts of quantum computers.

Figure 962: Source: Quantum Computers Could Solve Countless Problems—And Create a Lot of New Ones, Time Magazine, January 2023.

They are themselves exposed to the overselling from different players, both from industry vendors and some researchers. You've had this infamous 2023 **Time Magazine** piece and misleading cover touting "*This machine can solve problems that used to take years*", in your dreams only (Figure 962). The numerous paper mistakes were hopefully debunked by a UK Twitter account[4773]. Even John Preskill stated on Twitter as well: "*Quantum science and technology is exciting when discussed accurately and responsibly. It's disappointing that @TIME would publish such a misleading article*".

It was followed by the promotional tour of **Michio Kaku's** Quantum Supremacy book that is full of hyperboles and exaggerations on the potential impact of quantum computers. Up to saying that these computers will replace digital computers. Scott Aaronson published a harsh review of the book[4774] but he didn't have the visibility of Michio Kaku who was invited in a TV show of 46 mn by Neil deGrasse Tyson[4775]. That's the very best example of Brandolini's law at play according to which "*The amount of energy needed to refute bullshit is an order of magnitude bigger than that needed to produce it*". It looks that we could turn this into an exponential law for the case of quantum computing[4776]. Debunking is done in specialized media[4777] [4778] while the nonsense is broadcasted more widely.

[4771] See Mitigating the quantum hype by Olivier Ezratty, February 2022 (26 pages).

[4772] See Hype in Science Communication: Exploring Scientists' Attitudes and Practices in Quantum Physics by María T. Soto-Sanfiel, Chin-Wen Chong, and José I. Latorre, November 2023 (23 pages) which shows the ambiguous attitude from the scientific community.

[4773] See Here is our takedown of the recent @TIME magazine article on quantum computing, January 2023 with this very nice nugget: *" "In November, IBM unveiled its new 433-qubit Osprey chip—the world's most powerful quantum processor, the speed of which, if represented in traditional bits, would far exceed the total number of atoms in the known universe.". Since when are we measuring speed in bits?"*. Also, IBM Osprey has qubits that are so noisy that they are useless and IBM unplugged its QPU late 2023.

[4774] See Book Review: "Quantum Supremacy" by Michio Kaku (tl;dr DO NOT BUY) by Scott Aaronson, May 2022.

[4775] See How the Quantum Computer Revolution Will Change Everything with Michio Kaku and Neil deGrasse Tyson, June 2023 (46 mn).

[4776] See How quantum computing could transform everything everywhere, but not all at once by Alan Boyle, GeekWire, April 2023, provides a good example with a webzine promoting all the debunkable messages from Misho Kaku. Without doing it.

[4777] See Guest Post: Michio Kaku's Book "Quantum Supremacy" Neither Supreme Nor Particularly Quantum by from James Sanders, The Quantum Insider, June 2023.

[4778] See Dulwich Quantum Computing and its Twitter/X account which rightfully "*hopes to encourage quantum computing experts to be more outspoken. If quantum hype and bullshit are left unchecked, they will eventually damage the reputation of the whole field, thus hurting everyone, including those who are trying to make legitimate progress*".

You can then wonder why Michio Kaku was invited to talk by Google in July 2023[4779]! But they are also hype contributors like with their paper on a traversable wormhole simulation[4780]. Whatever the limits of a simplistic mathematical simulation[4781], it was run on 9 noisy qubits on a machine with a power of 25 kW while it could be emulated with better precision on your own smartphone!

When you know the field a little, you can easily spot the mistakes. But how about regular corporate people? Let me showcase some typical mistakes fueling the hype.

Quantum and cryptography confusion like in this paper from Inquirer.net which started with "*The NIST announced four quantum computing algorithms*" which is totally wrong[4782]. The NIST standardized four post-quantum cryptography protocols which are classical, not quantum, and designed to resist attacks of (far in the) future quantum computers.

Quantum computing and big data, when the idea is conveyed that quantum computers will be adapted to big data applications, and why not, for training and running large language models *ala* ChatGPT to save time and energy[4783]. It will probably not, as we show in this book on the part dedicated to quantum machine learning.

Corporate adoption. Another one consists in making economical predictions based on the assumption that the problem is the speed of adoption by (sluggish) corporate customers when it is about delivering the technology itself. On top of thinking that quantum computers will solve complicated problems in milliseconds[4784]. It will not[4785].

Classical and quantum computing confusion. When some algorithms are promoted to be quantum ones when they are classical ones[4786].

Interestingly, this hype and the debunking imperative have two consequences on scientific education. First, the hype can attract students in the field which is good news. Then, it drives professors and teachers to adapt their course to correct major misconceptions coming from the hype without discouraging the students[4787].

Learnings from AI

Artificial intelligence ethical concerns became a real political issue in 2018. This was very apparent in France in the **Villani Mission Report on Artificial Intelligence published in** March 2018 as well as in a **Report of the House of Lords** published the same month and on the same subject in the UK[4788].

[4779] See Michio Kaku | Quantum Supremacy | Talks at Google, July 2023.

[4780] See Making a Dual of a Traversable Wormhole with a Quantum Computer by Alexander Zlokapa and Hartmut Neven, Google AI, November 2022.

[4781] See some Twitter explanations here and there by Nirmalya Kajuri and Sabine Hossenfelder, and by Peter Woit in This Week's Hype, November 2022.

[4782] See Why 2023 Is The Year Of Quantum Computing by Dale Arasa, Inquirer.net, January 2023.

[4783] See Why Quantum Computing Is Even More Dangerous Than Artificial Intelligence by Vivek Wadhwa and Mauritz Kop, Foreign Policy.

[4784] See How to introduce quantum computers without slowing economic growth by Chander Velu and Fathiro H. R. Putra, Nature, July 2023.

[4785] See Disentangling Hype from Practicality: On Realistically Achieving Quantum Advantage by Torsten Hoefler, Thomas Häner and Matthias Troyer, Communications of the ACM, May 2023 (6 pages).

[4786] See 3 Most Important Advantages Of Quantum Computing by James Dargan, The Quantum Insider, June 2023, which in the case of quantum based chemistry is using examples which are all classical methods for quantum simulations.

[4787] See How media hype affects our physics teaching: A case study on quantum computing by Josephine C. Meyer et al, January 2023 (10 pages).

[4788] See AI in the UK: ready, willing and able?, March 2018 (183 pages).

It highlighted the need to ensure, at least morally, but, if possible, practically, that AI-based solutions respect society and avoid generating or perpetuating training data-originated discriminations. Hence two salient topics such as the explicability of algorithms and the limits of the manipulation of our emotions, particularly via more or less humanoid robots and voice agents.

The difficulty of explaining how deep learning algorithms work has been exaggerated. If it is true that multilayer neural networks are somewhat abstract. But it is equally abstract for almost any software, with or without AI, that can affect our daily lives. But we've forgotten that a little. When a software from the Visa group rejects your credit card payment abroad, we almost never get an explanation of the why and how it was rejected and how to avoid it. Bayesian fraud and machine learning based detection techniques are not explained to consumers.

Ethical quantum

Quantum computation is likely to amplify this quest for explicability. It is even less obvious to satisfy with quantum algorithms, which follow a logic that few developers can grasp. Quantum algorithms are likely to be even more complicated and less understandable than those of today's AI. This is amplified since we cannot observe their inner working and intermediate quantum states. Only the "classical" result is measured at the end of the operations. Moreover, from about fifty qubits, it becomes impossible to emulate a quantum algorithm on a classical computer.

Their possible biases will not necessarily come from the data that feeds them because, for a certain period, quantum computers will probably not exploit large volumes of data. We can therefore speak literally of the term algorithm bias, whereas when we talk about AI, we are dealing more with training data bias rather than algorithms bias.

But this will be judged on a case-by-case basis. Depending on whether the applications of quantum computing optimize automobile traffic, manage energy distribution, optimize financial portfolios of the wealthy, create new molecules in chemistry or biology or help the NSA break the codes of private communications, the stakes will not be the same[4789].

Ethical questions related to quantum technologies will undoubtedly emerge. They will be associated with a whole range of applications of quantum computing: the simulation of the dynamics of organic molecules. It will probably be limited at the beginning to the simulation of relatively simple molecules. The simulation of complex proteins folding is a hypothesis that has not yet been validated. In a distant hypothetical future, we may be able to simulate larger biological ensembles.

When this is simulated and then altered, for example to create new therapies, the rejection of GMOs or vaccines will seem like distant memory. New fears will show up and scientists will have to get involved to prevent them from spreading. These irrational fears will emerge because of exaggerations about the capabilities of quantum computers.

Each time, we will have to decode and take a step back. In 2022 emerged a proposal to quantumly link the brain with a quantum computer[4790]. Not only was it totally farfetched but it is probably not a good idea to create trust with quantum computing to elaborate such crazy scenarios. We have enough of it with Neuralink, the Elon Musk startup that works on neural implants that would supposedly be connected to some AI to augment the brain capacities, when it is more relevant for treating some neurodiseases like Parkinson's.

A good approach for the quantum scientific community would be to pre-empt these fears by analyzing them as early as possible and defusing them if possible, so as not to be in a situation that would block

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[4789] This is the point raised by Emma McKay in Should We Build Quantum Computers at All? A Q&A with Emma McKay, quantum physicist turned quantum skeptic by Sophia Chen, APS News, August 2022.

[4790] See An approach to interfacing the brain with quantum computers: practical steps and caveats by Eduardo Reck Miranda, Enrique Solano et al, January 2022 (6 pages).

scientific progress and innovation useful to society because of these irrational fears[4791] [4792]. Also, good examples can be given with researchers who admit having published scientific erroneous papers[4793].

Various initiatives started to pop-up in 2021 around quantum ethics in Australia, the UK, the Netherlands, Canada and the USA. It is following a similar pattern than with artificial intelligence but earlier given the maturity of the sector. So far, contrarily to what's happening in the AI field, it has not yet been hijacked by large industry vendors or even regulators. Most initiatives were born out of the research community.

Still, there are already some similarities and overlaps between the AI and quantum ethics frameworks that are showing up. On the AI side, many AI charters have been published since 2018. One of these comes from the **GPAI** (Global Partnership on AI[4794]) launched by 15 countries in June 2020 including France and Canada. Its goal is to foster the development of responsible and inclusive IAs based on human rights, favoring diversity, while driving innovation and economic growth. GPAI did organize experts working groups on responsible AI, data governance, the future of work and at last, on innovation. OECD launched its **AI Policy Observatory** (OECD.AI) in February 2020, an online platform consolidating information to help states craft their public AI policies.

OECD defined its own AI principles (OECD AI Principles) that were adopted by 42 countries in May 2019. Also in 2020, the **AI Rome Call for AI ethics** gathered the Vatican, Microsoft, IBM and others to whitewash about the same goals as GPAI. And these are just a couple initiatives among many others, frequently driven by industry vendors who are lobbying for self-regulation instead of tight government-based regulations.

In the quantum space, **Australia** was the first country that launched a quantum ethics initiative. Early on, in 2019, **CSIRO**, the Australian scientific research agency, mentioned the need to explore and address any unknown ethical, social, or environmental risks that may arise with the next generation of quantum technologies[4795]. It was followed in 2021 by a white paper published by Elija Perrier from the Centre for Quantum Software and Information at the Sydney University of Technology[4796]. It was spun out of the **Association for the Advancement of Artificial Intelligence** (www.aaai.org). The paper starts with defining the quantum physics postulates[4797], then cover ethical quantum computation and asks many ethical related questions that could be asked for any kind of classical computing. They mention the complicated question of quantum algorithms auditing. Quantum algorithms indeed may become a black box like with deep learning, leading to some explainability issues. So, on top of the various XAI (explainable AI) initiatives like the one launched by DARPA in the USA, will we see emerging the field of XQC for Explainable Quantum Computing?

4791 See Ethics education in the quantum information science classroom: Exploring attitudes, barriers, and opportunities by Josephine Meyer, Noah Finkelstein and Bethany Wilcox, University of Boulder, Colorado, February 2022 (15 pages) where the authors argue that quantum ethics and social responsibility should be incorporated in quantum information science education from the beginning.

4792 See Introducing a Research Program for Quantum Humanities: Applications by Astrid Bötticher, Zeki C. Seskir and Johannes Ruhland, February 2023 (36 pages) which still, lists use cases that are far from being realistic.

4793 See one good example with On a gap in the proof of the generalised quantum Stein's lemma and its consequences for the reversibility of quantum resources by Mario Berta, Fernando G. S. L. Brandão et al, May 2022 (22 pages) which shows that an initial proof from one of the authors was incorrect. An author who works at Amazon!

4794 With Canada, Germany, Australia, South Korea, USA, Italy, India, Japan, Mexico, New-Zealand, UK, Singapore, Slovenia and the European Union.

4795 See Growing Australia's Quantum Technology Industry, CSIRO, 2019 and The 'second quantum revolution' is almost here. We need to make sure it benefits the many, not the few by Tara Robertson, June 2021.

4796 See Ethical Quantum Computing: A Roadmap by Elija Perrier, Centre for Quantum Software and Information, Sydney University of Technology, February 2021-April 2022 (40 pages).

4797 They define only the first four quantum physics postulates and not the whole 6, and their fourth postulate doesn't correspond to the canonical Born rule related principle.

It also mentions the need for some Quantum Fair Machine Learning (QFML). It may not be such a problem since QML may not be used to process huge volumes of personal data due to quantum computing limitations in data loading techniques, which may last for a long time. They even go as far as asking whether quantum interferences implemented in quantum algorithms are ethical in nature. They also cover privacy and cryptography matters. Is Shor going to kill our private life? How could some differential privacy be implemented with quantum computing[4798]? Other topics involve distributional ethics and fair distribution which are classical economical questions arising with any new technology. Finally, they wonder about the impact of quantum simulations and whether it could be implemented to simulate people's personal behavior. The paper seems highly influenced by the works on ethical AI and sometimes mixes science-fiction with real state of the art understanding of what can and will be done with quantum computing[4799]. But it asks good questions. Another Australian paper focused more recently on the need to involve all stakeholders, beyond the classical market awareness creation[4800].

In the **UK**, ethical quantum computing became a topic promoted by the media The Quantum Insider starting in December 2020. They released a short video documentary trying to explain what quantum computing is and the related ethical issues involved with researchers like John Martinis and entrepreneurs like Ilana Wisby[4801]. They are highlighting the need for democratizing quantum technology skills, mentioning the risks on privacy and security and the need to address quantum AI bias. They also pinpoint the "Hype-Fear-Disappointment Cycle" and recommend setting realistic expectations to avoid triggering fears and biases. Researchers from Oxford University are also studying responsible innovation in quantum technologies[4802]. It spun the ResQCCom project (Responsible Quantum Computing Communications) out of the university's Responsible Technology Institute (RTI).

In **the Netherlands**, the government 615M€ initiative launched in April 2021 includes a 20M€ plan on quantum ethics and societal impact research run out of the Living Lab Quantum and Society spun out of Quantum Delta NL, the foundation established to run the Netherlands quantum program. They also create ethical, legal and societal standards for quantum technologies and their applications. There was even a quantum hype PhD proposal, as of October 2023 at the University of Leiden[4803].

The **World Economic Forum** launched its Quantum Computing Governance initiative in February 2021[4804]. It wants to standardize an ethical framework enabling the responsible design and adoption of quantum computing. They ask the ever-lasting question: will the public trust technologies which they cannot understand and whose results they cannot verify as if they could do it with existing digital technologies. They advocate the use of preemptive involvement in technology design to make sure ethical issues are addressed as early as possible. With that, they are assembling a "*global multistakeholder community of experts from across public sector, private sector, academia and civil society to formulate principles and create a broader ethical framework for responsible and purpose-driven design and adoption of quantum computing technologies to drive positive outcomes for society*". They

4798 See Quantum Differential Privacy: An Information Theory Perspective by Christoph Hirche et al, February 2022 (26 pages).

4799 See Ethics of Quantum Computing: an Outline by Luca M. Possati, Philosophy and Technology, July 2023 (21 pages).

4800 See Talking about responsible quantum: Awareness is the absolute minimum... that we need to do by Tara Roberson, December 2021-January 2023 (15 pages).

4801 See Quantum Ethics documentary, December 2020 (13 mn) published by TheQuantumInsider as part of a series of "conversations". It was followed by several posts like Quantum Ethics Series: Understanding the Issues and Expanding the Conversation by Matt Swayne, 2021.

4802 See Asleep at the wheel Responsible Innovation in quantum computing by Philip Inglesant, Carolyn Ten Holter, Marina Jirotka & Robin Williams, October 2021 (14 pages).

4803 See PhD candidate - hype and public engagement with quantum science and technology, University of Leiden, October 2023.

4804 See Quantum Computing Governance Principles, Insight Report, World Economic Forum, January 2022 (35 pages).

will frame the conversation, drive quantum ethical issues awareness, study quantum related risks, design quantum computing ethics principles and framework and test it with some case studies.

Other responsible quantum frameworks are proposed, with similar focus on cybersecurity and privacy issues, workforce diversity and societal dialogue[4805].

In **Canada,** The Perimeter Institute launched the **Quantum Ethics Project** that is creating full-length courses on ethics and social impacts of quantum technology[4806]. Also, **Q4Climate** is an initiative for using quantum technologies in climate research, an initiative coming from the Institut Quantique, the University of Waterloo and Zapata Computing. It looks like a small think tank. It explains how some quantum chemistry algorithms could potentially solve some environmental problems[4807].

In the **USA**, some spare initiatives are launched by academics like Chris Hoofnagle from Berkeley, or a while ago, by Scott Aaronson[4808].

In **Switzerland**, the GESDA (Geneva Science and Diplomacy Anticipator) is working on the creation of an Open Quantum Institute that is working on the development of quantum use cases designed towards accelerating the achievement of the UN Sustainable Development Goals (SDGs). It was inaugurated at CERN fwainwamin Geneva in October 2023.

Other ethical issues to be addressed cover the potential harmful manipulation of the human genome fears, the (positive) quantum use cases to find environmental solutions and the energetic footprint of quantum computing.

Interestingly, none of these initiatives mention the field of quantum sensing, which could also have some underlying ethical issues to be addressed, particularly when used in the military. Quantum radars, quantum imaging, precision gravity measurement and its impact on underground resources exploitations are a couple examples.

Religions and mysticism

In recent millennia, the human race has developed the habit of devoting a cult to one or more higher divine powers of an imprecise nature but explaining everything and everything else.

Mankind probably began to attribute this power to natural phenomena that he could not explain like the Sun or the stars. Mankind then went from multiple systems of gods to a single all-powerful God. In a way, the monotheistic religions realized before time the theory of unification so much sought after by physicists. This story is told with hindsight by **Yuval Harari** in Sapiens and with cynicism by **Richard Dawkins** in "The God Delusion".

For some scientists or believers in an afterlife, quantum physics renews the desire to explain the inner works of the Universe by some divine power. It gives the impression of providing an ultimate scientific explanation for everything, of God, and of his ability to control and supervise everything[4809]. The quantum function most often emphasized in these explanations is entanglement.

[4805] See Towards responsible quantum technology, safeguarding, engaging and advancing Quantum R&D by Mauritz Kop, Raymond Laflamme et al, March 2023 (22 pages).

[4806] See A Holistic Approach to Quantum Ethics Education by Joan Étude Arrow et al, The Quantum Ethics Project, May-July 2023 (10 pages).

[4807] See Quantum technologies for climate change: Preliminary assessment by Casey Berger, Agustin Di Paolo, Tracey Forrest, Stuart Hadfield, Nicolas Sawaya, Michał Stęchły and Karl Thibault, June 2021 (14 pages).

[4808] See Law & policy for the quantum age : a presentation by Chris Hoofnagle, February 2021 (58 mn), Law and policy in the quantum age, by Chris Jay Hoofnagle and Simson L. Garfinkel, January 2020, free download (602 pages), and Why Philosophers Should Care About Computational Complexity by Scott Aaronson, 2011 (53 pages).

[4809] On this subject, see the Wikipedia fact sheet that briefly describes quantum mysticism.

It makes it possible to envision a Supreme Being who, thanks to this physical phenomenon, can control all the particles of the Universe and at a distance. It would also explain strange synchronicity phenomena.

The wave-particle duality also makes it possible to imagine or explain many magical scenarios such as remote healing, telekinesis or telepathy[4810].

Some of the protagonists of these theories are themselves quantum physics scientists. One of the best-known is **David Bohm** (1917-1992), already mentioned in the quantum foundations section, who came closer to Indian spiritualism in the 1960s, simultaneously with the Beatles! He was convinced that the laws of the Universe were governed by some spirit[4811]. He is one of the initiators of the theories of **quantum cognition**, a field of cognitive theories based on the mathematical formalism of quantum mechanics and relying on analogies.

Public education

Quantum computing will amplify a situation observed with artificial intelligence: a huge gap between those who understand it and those who use it, coupled with a shortage of skills. It is right now definitely a world of specialists, and it is even harder to grasp than most other digital-related disciplines. Today, this world is balanced between specialists in condensed matter physics and quantum algorithms and software[4812].

By extrapolating a little and drawing inspiration from the history of computer science, we can anticipate that software will gradually take over when quantum computing becomes commonplace, especially if it leads to applications in all sectors of industry.

In today's digital economy, there are many more software specialists than there are with semiconductors. The economies of scale are actually much greater with the latter between producers and users. Quantum will probably not escape this, even if initially the market for quantum computers will not be a volume market.

In the short term, there is a great need to popularize the field and avoid its technical jargon. You must proceed step by step, broadening the audience in a progressive way from the techie to the non techie[4813]. In parallel, training decision-makers in industry and institutions must also be done. It is becoming even more important as the quantum technologies hype is peaking with the flurry of vendors and research labs announcements that are regularly showing up[4814].

Many initiatives around the world are launched to train the public on quantum physics and technologies. Let's mention the **National Q-12 Education Partnership** launched as part of the National Quantum Initiative and targets middle-school and high-school students. Industry participants include

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[4810] A good inventory of these different debates can be found in The Quantum God An Investigation of the Image of God from Quantum Science, 2015 (81 pages) which evokes the notion of consciousness of the Universe. See also the almost parodic Nothing is solid "All is energy".

[4811] See Lifework of David Bohm - River of Truth by Will Keepin, 2016 (22 pages).

[4812] See Eleven risks of marrying a quantum information scientist by Nicole Yunger Halpern, 2020. A second degree but realistic inventory of the life of a quantum scientist in the USA.

[4813] See for example The Quantum Prisoner, a free scientific and technological video game is now available online, CEA, October 2020.

[4814] In Democratization of Quantum Technologies by Zeki C. Seskir, Steven Umbrello, Christopher Coenen and Pieter E. Vermaas, August 2022 (22 pages), the authors define the various aspects of a democracy and include educational efforts, like IBM's continuous evangelization efforts that started in 2016. They also pinpoint the asymmetry between the quantum ecosystem stakeholders and the general public which "has to be educated" but do not participate to a democratic decision making process. It also describes the counter-narrative on the democratization of quantum technologies like cybersecurity threats, its geopolitical dimension, and the habit to position quantum physics as impossible to understand, abusively quoting Richard Feynman. The authors do not describe the process followed by governments when launching their quantum plans, which embedded some participatory process with quantum ecosystem stakeholders but had their share of discretionary decision making, sometimes even escaping the minds of top policy makers.

IBM, Google, Microsoft, AWS, Rigetti, Intel, Lockheed Martin, Boeing, Zapata Computing, APS Physics, Optica, IEEE USA and QubitbyQubit (a training organization). They organized the event QuanTime in spring 2022 with hundreds of quantum activity classrooms for K-12.

Other initiatives are gamifying the learning process like with **Quantum Odyssey** from Quarks Interactive (2020, Romania, 230K€) that were launched in 2020. They replace Dirac's bra-ket notation and linear algebra used by scientists by a visual puzzle-building approach. It is proposed to discover and learn gate-based quantum algorithms. **Quantum Chess** from Quantum Realm Games (2019, USA). The game embeds quantum phenomenon in the game play with pawns able to make multiple moves simultaneously in sort of superposition.

Likewise, the very serious CEA in France launched **The Quantum Prisoner** in English in 2020, a free online adventure game inspired by quantum logic and targeting kids over 12 (meaning, adults are welcomed…). It has 10-12 hours of gameplay with a journey across the globe to find out what happened to a physician who mysteriously disappeared in the 1960s. Playing as Zoe, a young woman, gamers must solve over 30 technology, science, and engineering-based puzzles.

Many other quantum games were created for educational purpose, like **Alice Challenge**, **Hello Quantum** and **Hello Qiskit** by IBM and the University of Basel, **Particle in a Box**, **Psi and Delta**, **QPlayLearn** (covered later), **Quantum Cats**, **Quantum Flytrap**, the **Virtual Lab** by Quantum Flytrap, **ScienceAtHome**[4815].

Some educational tools are specialized in quantum optics such **Quantum Games with Photons** from the MIT which is an open source puzzle game with 34 levels and a sandbox, and **The Virtual Quantum Optics Laboratory**, an optical lab running in a browser which enables you to build quantum optics experiments with all sorts of optical devices (lasers, PBS, depolarizer, etc, in Figure 963 *on the right*).

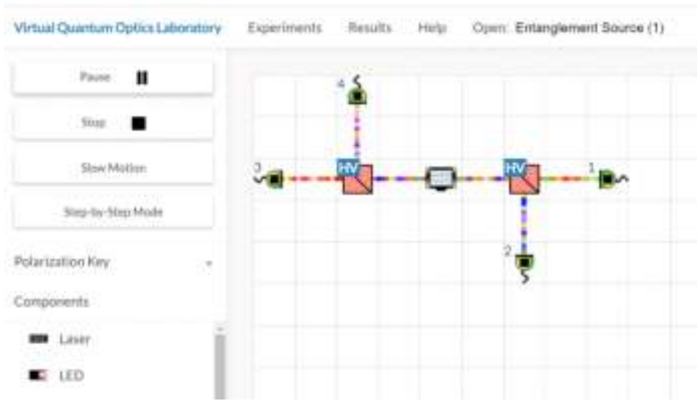


Figure 963: The Virtual Quantum Optics Laboratory.

CoSpaces is a similar simulation tool created in Italy and aimed at teaching quantum computing[4816].

Professional education

All these countries launching well-funded quantum plans create a significant challenge with professional education along the whole cycle from bachelor to doctorate. Before being an industry competition, quantum technologies are a talents one. We can expect that there will be more money to spend than talent to hire with it for a while.

Existing quantum professional training is significant in quantum physics. Most universities in the world have special programs from bachelors/licenses and master's to PhDs. Some groups are organizing summer schools for PhD level students like the famous **Ecole de Physique des Houches** doctoral and summer school in the French alps with various sessions on quantum physics. Look for example at the 2019 summer school agenda's speakers!

4815 See an inventory of quantum games in Quantum Games and Interactive Tools for Quantum Technologies Outreach and Education by Zeki C. Seskir et al, July 2022 (48 pages).

4816 See Quantum computing teaching with CoSpaces by Francesco Sisini, Igor Ciminelli and Fabio Antonio Bovino, September 2022 (8 pages).

There are a couple new disciplines where more and more people will need to build knowledge and skills on. Quantum systems engineering is about creating real machines that work from start to finish. This requires decompartmentalizing disciplines and bringing together physicists and engineers.

The technologies involved are varied and include photonics and lasers, analog and digital electronics, thermodynamics, fluid mechanics, various components manufacturing techniques, and the design of complete systems. Quantum engineering involves many complementary disciplines (Figure 964). With AI, it is a new challenge for higher education that is being prepared.

In the purely mathematical and software fields, very important disciplines come into play for creating end-to-end quantum solutions: algorithms design, software tools design and applications software development. Added to this is the field of post-quantum cryptography.

The creation of business applications also requires skills at the crossroads between the above and vertical markets, which are often themselves scientific as in life sciences (organic chemistry, protein folding, photosynthesis, ...), materials sciences (battery chemistry, superconducting materials) or other branches such as portfolio management and risk assessment in finance or optimization problems in logistics, transportation and marketing.

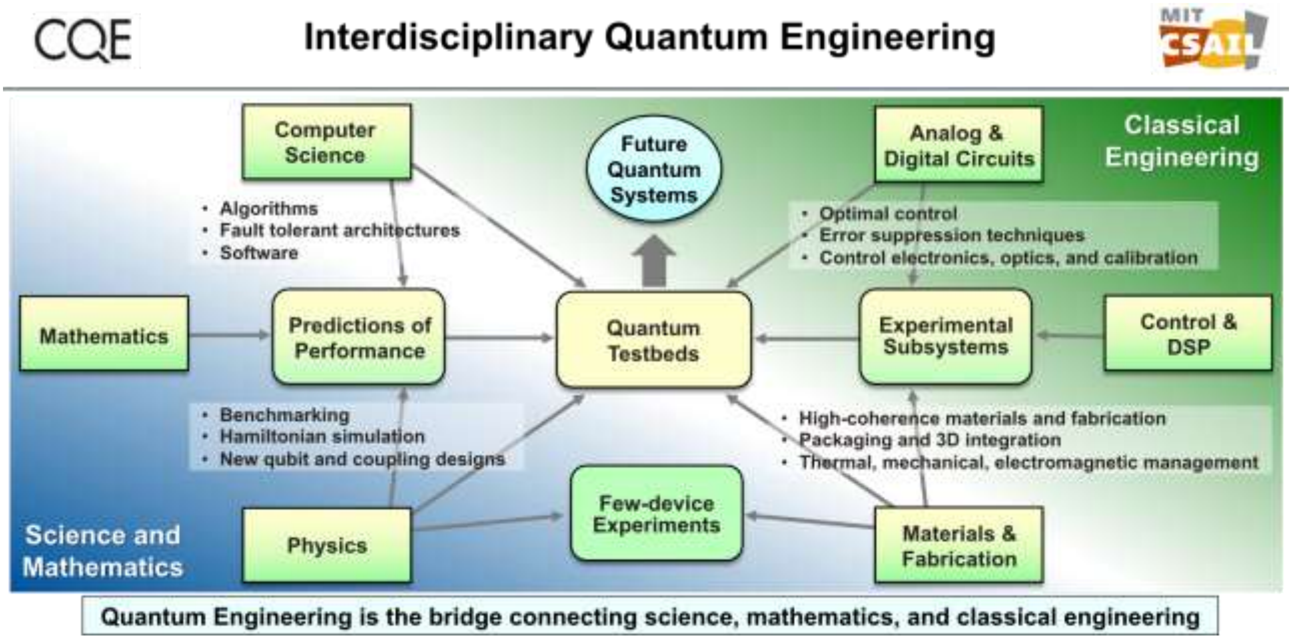


Figure 964: a definition of quantum engineering.
Source: Introduction to Quantum Computing by William D. Oliver, MIT, November 2022 (35 slides).

Quantum technologies will be found with many different professions:

- **Fundamental physicists** (solid-state physics, condensed matter physics, light-matter interaction, quantum optics) who combine theoretical and experimental approaches to understand low-level phenomena.
- **Quantum technologies researchers** who turn fundamental discoveries into first proofs of concept in the laboratory. These research teams combine physics, technology and engineering researchers.
- **Design engineers** who create technical subsets of quantum computers to complete finished products. They essentially do the "D" of "R&D" by relying on the R of physicists.
- **Research engineers**, who participate in the development of new materials and new technologies in semiconductor fabs, or process engineers who design the manufacturing processes for these integrated circuit systems supporting qubits.

- **Technicians** for certain components manufacturing and/or for the deployment of technologies such as quantum cryptography in the telecom space. But only once this technology is deployed on an industrial scale, probably by generalist or specialized telecom operators.
- **Software tools developers** who must be associated with previous researchers and engineers. Indeed, for the time being, the design of these tools still has to take into account the physical characteristics of quantum calculators/accelerators.
- **Application developers**, whose numbers will increase as the computing power of quantum computers grows. Most of the time, they will need to have three key sets of technical skills: one is being able to program quantum computers, the second will be the ability to turn business problems into quantum programs (including the related mathematical/physics related know-how) and at last, they will have to know about classical programming since most quantum algorithms are hybrid.
- **Project managers** who manage projects and teams that combine these different professions.
- **Business strategists**. Brian Lenahan goes as far as defining the job of "quantum business strategist" which looks like an equivalent of the chief digital officer for quantum technologies related projects, creating the link between IT and business managers. This role is about crafting a quantum plan with mission, vision, goals, strategies, KPI's and tactics. In other words, it is an old-fashioned consultant[4817]!
- **Support activities** like in HR, marketing, business development (managing industry partners) and sales (to end-customers), communication, public relations, legal, finance, startup creation and acceleration.

As in many disciplines, researchers and engineers are increasingly required to be versatile. Teams must be structured around a strong interdisciplinarity and transversality. They need "technological polyglot" teams that link all these professions and skills. In particular, physicists will have to be increasingly interested in engineering and engineers in physics[4818].

Finally, when you turn to the business side with actual products that can be marketed and sold, you need the whole mix of skills usually found in technology marketing and sales: product marketing, operational marketing, business development and partnerships, creating ecosystems and, above all, pure and simple B2B sales for a starter. This is completed by the generic skills associated with deep techs startups creation (organization, business planning, recruitment, funding, etc.) and with intellectual property attorneys who must grasp the specificities of the quantum vocabulary (Figure 965).

Quantum sensing products are beginning to be marketed, and in a market that is currently niche.

Quantum cryptography systems are in the experimental field phase and could be deployed on a larger scale in the coming decade.

Quantum communications with the objective of leading to quantum communications networks will develop in a second phase, combining fiber and satellite networks with quantum ground relays. This is a complementary field to the development of quantum computers.

[4817] See What is a Quantum Business Strategist? by Brian Lenahan, April 2021 and his related book Quantum Boost: Using Quantum Computing to Supercharge Your Business by Brian Lenahan, May 2021. Brian Lenahan also created in September 2021 the Quantum Strategy Institute with various people from Spain, UK, France and the USA with the goal to bridge the gap between quantum science and businesses.

[4818] See Defining the quantum workforce landscape: a review of global quantum education initiatives by Maninder Kaur and Araceli Venegas-Gomez, Qureca, February 2022 (35 pages) which makes an inventory of the various quantum educational resources across the world and Building a Quantum Engineering Undergraduate Program by Abraham Asfaw, Alexandre Blais et al, 2021 (25 pages). See also the QTEdu, the European Quantum Flagship program on education which launched 11 pilot programs on education in quantum technologies and the associated report The Future Quantum Workforce: Competences, Requirements and Forecasts by Franziska Greinert et al, August 2022 (16 pages) which is based on a survey on quantum skills needs in Europe.

Finally, quantum computing will progressively evolve and see their field of application widen as the qubits number and quality in quantum computers grows. It will be a process of continuous innovation. As in the case of classical computing, the weight of software is bound to become dominant in skills requirements (Figure 967). This explains why many publications insist on the need for quantum application developers. This is what the major players such as IBM, Google and Microsoft, not to mention D-Wave, Rigetti and IonQ, are "evangelizing" about[4819]. Nevertheless, in parallel with the software market development, an intermediate phase will require a lot of skills in engineering and in the different branches of quantum technologies.

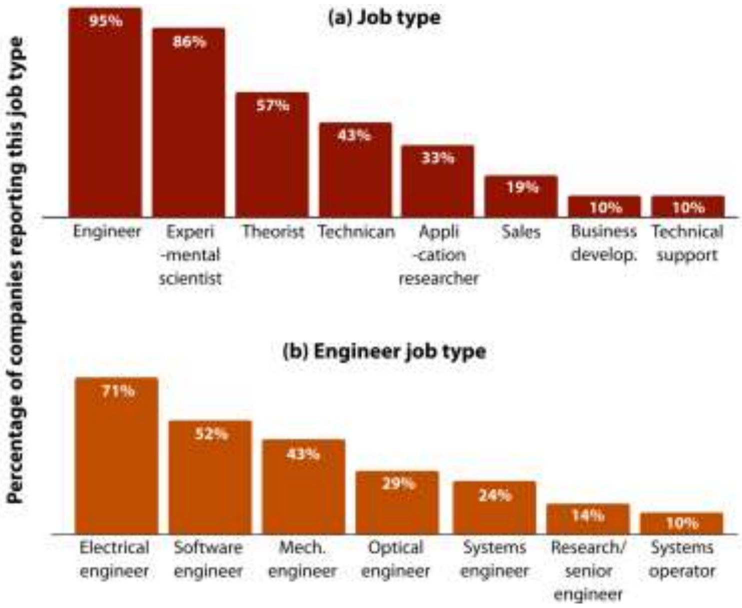


Figure 965: an American inventory of engineering jobs and skills in quantum technologies. Source: [Preparing for the quantum revolution -- what is the role of higher education?](#) by Michael F. J. Fox, Benjamin M. Zwickl et H. J. Lewandowski, 2020 (23 pages).

In some cases, training can be shared between universities, particularly when teachers are scarce. That's what is implemented in the Université Paris-Saclay with the ARTEQ interdisciplinary year positioned before masters M1 and M2, to feed M2 masters in quantum physics and quantum information science (Figure 966).

Figure 966: ARTEQ training in Saclay.

Training in public higher education should introduce quantum science and technology as early as possible in the bachelor's and master's degree programs. It will also be necessary to create master's degrees in quantum engineering, bringing the world of research and engineering closer together.

The training offer will depend on several parameters: funding for teacher-researchers or teaching positions, the creation of vocations, the ability to attract teachers and students from wherever they come.

4819 Look at it this way: Quantum Computing Demands a Whole New Kind of Programmer by Edd Gent, May 2017 (slightly ahead of schedule), The Hitchhiking Cat's Guide to Getting a Job in Quantum Computing by Jay Gambetta, October 2019, Building Quantum Skills With Tools For Developers, Researchers and Educators, IBM Research, September 2019 and Some useful skills for quantum computing by Chris Granada, January 2020, which also emphasizes mathematical and software skills.

Continuing education may include both scientific and technological courses (quantum physics, quantum communications, quantum algorithms and software) and strategic courses (understanding of the issues, knowledge of the players, economics of the sector, good practices). This is probably the less well address market need, so far. It can or could be delivered by private organizations, by higher education organizations as well as via online courses offered by Coursera and the likes.

Self-training allows enthusiasts to discover these sciences and technologies by themselves, but it is not self-sufficient as it is sometimes the case in artificial intelligence.

It must be complemented by quality pedagogical support, if only to do and correct exercises. As far as the software part is concerned, this will perhaps change the day when development tools will be possible with higher levels of abstraction than today.

Scientific events organized by quantum hubs, research laboratories and companies serve to facilitate transdisciplinarity among researchers and engineers. They can be interdisciplinary symposia, thematic conferences or workshops.

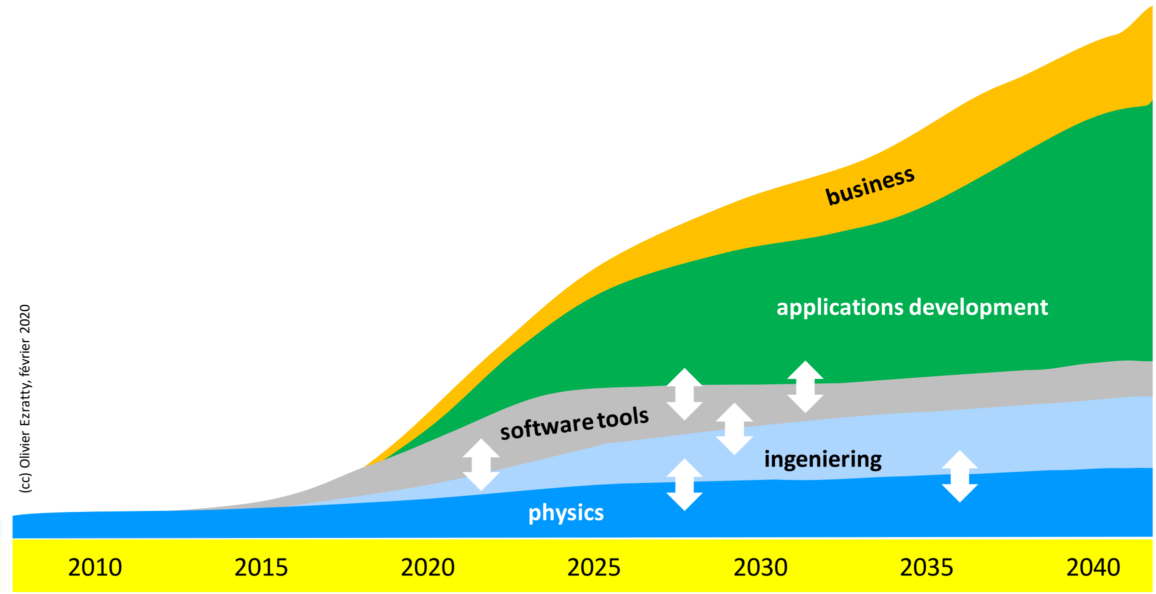


Figure 967: how quantum tech skills need will evolve over time. More engineering and then more software and more business skills. (cc) Olivier Ezratty, 2020.

It will also be necessary to attract as many women as men in these courses, otherwise there is a risk that a whole sector will develop, as in AI, which is far too masculine. Not to mention the increase in the diversity of students' social backgrounds, which remains a key means of republican promotion, despite its current decline.

Upstream of all these courses, the creation of vocations among young people is indispensable. Science fairs can also contribute to this. It is a long-term task, as is the creation of vocations in science in general and in the scientific and technical professions of the digital world in particular.

There are some pure players around in the quantum computing educational market, many of them offering open sourced eLearning contents:

Q-munity

Q-munity (2019) is a training organization and community connecting young individuals in quantum computing. With its 1,000 members, it organizes summer camps (well, outside pandemics), conferences and workshops.

It was created by Anisha Musti, a quantum computer scientist who worked on Shor’s algorithm, quantum teleportation and quantum machine learning.

QubitbyQubit (USA) is a quantum programming online learning initiative from The Coding School, created by a Brown University undergraduate in 2014. It was created by Kiera Peltz and is sponsored by IBM and Google.

Qplaylearn (2020, Finland) develops an online visual quantum programming training tool targeting a broad audience including high school students. They collaborate with various universities in Finland as well as with IBM.

Quantum Country (USA) is a tutorial web site on quantum programming created by Andy Matuschak and Michael Nielsen. It contains "mnemonic medium" that makes it easy to remember what you read. These are long reads including some good story telling and some exercising. It starts with the basics of quantum programming, then covers key algorithms like Grover search.

QuTech Academy is offering free online courseware on quantum technologies for engineers[4820].

CERN has a series of introductory conferences on quantum computing from **CERN** (7 x two hours tracks), broadcasted in November and December 2020, also targeting engineers[4821].

Qureca (2019, UK) sells online "Quantum for Everyone" courses for business people (at £400). These courses are delivered by Araceli Venegas-Gomez, the founder of Qureca, Bruno Fedrici, a French consultant and lecturer on quantum technologies and QuantFi, a French Startup specialized in Finance applications. The company also offers hiring services and other professional services for startups and businesses (community management, events organization, business development and strategy).

EFEQT (Empowering the Future Experts in Quantum Science and Technology for Europe) provides a learning experience between academic research and free interdisciplinary exploration for 25 students and young researchers. It takes the form a scientific hackathon, the first one was organized in October 2021 and ended with a graduation in September 2022. The best participants will receive a fast-track access to do a PhD or post-doc at EFEQT partner universities in Germany and Strasbourg in France. The program is supported by the Quantum Flagship's Quantum Technology Education Coordination and Support Action (QTEdu CSA).

Jobs impact

Finally, what about the future of quantum-related employment, a question that Sophia Chen asked herself in Wired in June 2018[4822]? It is difficult to assess because we're thinking over several decades and about use cases that are still uncertain. There will be, as with AI, those who know and those who don't, those who code and those who use stuff, those who create wealth and those whose jobs are threatened.

For the moment, quantum computing does not generate any specific jobs threats, because it will enable us to do things that mankind can't do today. There is no logic of replacement, at most optimization as for applications based on graph optimization like those of the traveling salesperson problem.

Gender balance

Gender unbalanced in all STEM jobs and particularly in computer science is a known fact and it has been so for a long time. You can look at all the statistics and they are not good. It started to go awry in computer science in the early 1980s when computing became mainstream.

[4820] See QuTech Academy Online Learning.

[4821] See Online introductory lectures on quantum computing from 6 November, 2020.

[4822] See Quantum Computing Will Create Jobs. But Which Ones? by Sophia Chen, Wired, June 2018.

Many initiatives have been launched worldwide to rebalance gender in all these domains. They have mostly failed, or maybe did they just made things better than if nothing was done. Are quantum technologies different for gender balance?

Figure 968: some women role models around the world, from research to the industry. (cc) Olivier Ezratty, 2021-2023.

Problems

This domain is already highly male-dominated, in the lineage of computer science and artificial intelligence. The specialty is still too masculine as it stands. Quantum physics founder in History books are mostly men, particularly in the seven first quantum wave theoreticians narrow club with **Planck-Einstein-De Broglie-Schrödinger-Heisenberg-Dirac-Born**.

You have to really dig into the History of science to recognize the role of **Emmy Noether** and **Chien-Shiung Wu**, the few female scientists of this era. Also, only three women were awarded a Nobel prize in Physics with **Marie Curie** (1901), **Maria Goeppert** (1963, for her work on nuclear physics) and **Donna Strickland** (2018 for her work on pulse lasers). But besides Marie Curie, they don't yet have the recognition status of **Linda Lovelace**, **Grace Hopper** and **Margaret Hamilton** in computer science.

The statistics are depressing with only 20% women in STEM (in the USA) and it doesn't seem to be better in quantum science[4823] [4824] [4825] [4826]. Women's representations in culture, media and toys still play a leading role in crafting this unbalanced world. And this is just about gender balance. In the USA quantum scientific community, many groups are fighting against discrimination beyond gender balance issues.

[4823] See The Quantum Computer Revolution Must Include Women by Chandralekha Singh, Scientific American, January 2021.

[4824] See The Upcoming Women In Quantum Summit III And Its Secret 70 Year-Old Legacy par Paul Smith-Goodson, December 2020.

[4825] See also Women in Quantum Technologies - What are the challenges, February 2020.

[4826] See Fledgling quantum industry is heavily male dominated, finds report, Physicsworld, April 2023.

Only a few countries are faring better, like in Asia. Is the condition of women in universities and research labs different than in business organizations? It probably depends on their values, leadership and culture. The scientific world seems as competitive and tough than the private sector even if its rules are different, based on h-indexes, conference talks and the likes. Still, in most places, research is a longer term activity which may create better conditions for women.

On top of that, the language used in quantum science is very masculine[4827]. It evokes the notions of superiority (supremacy) and auxiliaries (ancilla), the former echoing a higher authority, and the current "white supremacy" resulting from South African Apartheid and which is still stirring the US political scene. The second notion takes up the notion of "female servant" in Latin, slavery and racial segregation, whereas the technical term was coined in 1995. These are symbolic items but they deserve to be corrected. One solution is to talk about quantum advantage even if the meaning is slightly different from quantum supremacy (doing impossible things in classical computing vs doing things better). Some are advocating the usage of "useful advantage" in reference to use cases that provides some value with data inputs and outputs, but it doesn't embed the distinction between supremacy and advantage. Another solution already mentioned consists in using the term **primacy**[4828].

A mix of other authors ask for creating a language void of competition connotations that should be comprehensible, specific and practical, open, accessible, responsible, culturally embedded and meaningful[4829].

Hope

There's still hope. It seems easier to identify dozens of women who are real inspiring role models and play a key role in quantum science and technologies and anywhere in the world (Figure 968). Many of these were at the origin of key scientific advancements in quantum technologies.

You may know the famous threshold theorem co-demonstrated by **Dorit Aharonov**! There are a few startups created by women like **Silicon Quantum Computing** (SQC, Australia), created by **Michelle Simmons, Oxford Quantum Circuits** that is led by **Ilana Wisby**, Quobly (France) which was co-founded and is run by **Maud Vinet**, **Quandela,** co-founded by **Pascale Senellart, VeriQloud** co-founded by **Elham Kashefi** and Qureca, created by **Araceli Venegas-Gomez**. In the Corporate world, **Krysta Svore, Patty Lee** and **Anne Matsuura** play leading roles at respectively Microsoft, Quantinuum and Intel. In Europe, **Laure Le Bars** leads SAP's quantum research efforts on top of being the first President of the QuIC industry consortium.

The Quantum Insider launched in 2021 a series of interviews of key women working in the quantum industry. It included for example **Mercedes Gimeno-Segovia**, Systems Architecture VP for PsiQuantum, **Olivia Lanes** North-America lead for Qiskit at IBM, **Helena Liebelt** from Deggendorf Institute of Technology, Intel and SheQuantum, **Christine Johnson**, CEO of Ingenii, **Lior Gazit** quantum software engineering team lead at Classiq, **Ying Lia Li**, CEO from Zero Point Motion (UK), **Katerine Londergan**, CMO at Zapata AI, **Anindita Banerjee** Security VP at QNU Labs, and **Rojalin Mishra** senior hardware verification engineer at Riverlane.

[4827] As Karoline Wiesner of the University of Bristol points out very well in her succinct The careless use of language in quantum information, 2017 (2 pages).

[4828] See Quantum Computing 2022 by James D. Whitfield et al, January 2022 (13 pages).

[4829] See Quantum Technologies and Society: Towards a Different Spin by Christopher Coenen, Alexei Grinbaum, Armin Grunwald, Colin Milburn and Pieter Vermaas, November 2021 (8 pages).

Also, quantum tech is still a green field, and it is not too late to attract young women in this emerging and promising discipline. There are already women playing leading technical and business roles in quantum startups on top of the cofounders mentioned above[4830].

Initiatives

Some initiatives have been launched around the world to promote and help women in quantum technologies. They are matching what has been done for a while in the computer science and information technology fields. Gender oriented actions are a mix of associations, events and media visibility initiatives. Too many of these are seasonal, and centered around the Woman's Rights day, on March 8th, each and every year.

Let's mention a few of these:

- **Women in Quantum** by **OneQuantum** is a think tank gathering quantum leaders worldwide in dedicated chapters, with the goal to influence government action, vendor relationships and the quantum ecosystem. It offers a resources, services and events platform for quantum startups to collaborate. Women in Quantum is one of the "chapters" of this organization, run by Denise Ruffner (also, Chief Business Officer of Atom Computing), organizing quarterly Women in Quantum events, the last one being held online in June 2021[4831].
- **Women in Quantum Development** is a professional network of quantum tech enthusiasts in the Netherlands, with events and mentoring programs. It belongs to a new trend, with national quantum plans containing specific initiatives around the ethics and social impact of quantum technologies. Netherlands is a good best practice for that respect. There's also a gender equality workgroup in the **EU Quantum Flagship**.
- The **University of Bristol** organized a two-day Women in a Quantum Engineering event in December 2019.
- Some research labs and organizations showcasing their women quantum scientists and engineers like at the **Lawrence Berkeley National Lab** from the DoE in the USA[4832], at the **Harvard** Center for Integrated Quantum Materials[4833], at **Yale University**[4834], with **IBM**[4835] and **Microsoft**[4836].
- **SheQuantum** (2020, India) is an eLearning provider offering quantum computing education content targeting women.
- In France, the association **Quelques Femmes du Numérique!** promotes women in tech, particularly engineers and scientists using quality photography portraits with over 800 women in various fields (artificial intelligence, Blockchain, cybersecurity, IT, etc) and over 20 in quantum techs[4837]. It launched many initiatives including promoting quantum science to girls in schools.

[4830] See 52 Wonder Women Working In Industry As Quantum Scientists & Engineers by James Dargan, The Quantum Daily, August 2021.

[4831] See the casting of the Fall 2020 edition.

[4832] See Women of Quantum Computing Go Tiny in Big Ways by Elizabeth Ball, June 2021.

[4833] See Ask a Scientist: Women in Quantum Science and Technology, November 2020.

[4834] See WIQI (Women in Quantum Information) Group.

[4835] See Encouraging more women in quantum: four insights from four women, IBM UK, March 2021.

[4836] See Women of Microsoft Quantum Part 1 and Part 2, March 2020.

[4837] See A la découverte des femmes des technologies quantiques, 2021-2022. Disclaimer: I'm a cofounder and was the photographer of this association from 2011 to 2023.

Solutions

Like in any domain, particularly in social science, there's not yet a common agreement on what should be done to create a better gender balance in STEMs and in quantum science.

Should we encourage some affirmative actions or not? Some are worth the effort like the European Union ERC Grant program which extends since 2010 the age limit by 18 months per child plus other anti-bias measures. Paternity leaves are also taken into account[4838].

In the way women scientists and entrepreneurs are promoted, I believe we should be more engaged but with subtlety. For example, it is more efficient to value scientists and entrepreneurs for their achievement and who happen to be women instead of doing this explicitly because they are women. Implicit communication is sometimes more efficient than an explicit one. Finding women talents should be a sort of backstage work. It requires some discipline. When organizing training and events, and with any media speaking opportunity, make sure gender balance is respected. It involves having some knowledge of the field ecosystem and of its female leaders. Don't say "there are only a few of them", but "where are they?" and look for them. Also, let them talk about their science.

We should also promote a broad range of role models in different fields and jobs to inspire young talents. It is also about building inclusive and welcoming work environments in universities, research labs and commercial vendors.

Of course, on a broader scale, media and fiction play a key role. The geek in TV series and movies is too frequently an introverted male. We need more Felicity Smoak, the geek from the TV Series Arrow! At last, do that all year long and not just on March 8th.

Quantum technologies marketing

The last point to be mentioned here is the role of marketing and propaganda. Quantum technologies are the perfect spot to broadcast extraordinary and impressive claims that few specialists can fact-check. It is a world of superlatives and exaggerations. It started in 2019 with Google's supremacy claim.

We are going to be drowned in innovation propaganda that will blur things. Scientists in the field will no longer recognize their creations. Popular news related to quantum computing will continue to start explaining qubits with their superposed states 0 and 1 and... stop there! Consulting and analysts firms will also strive in simplifications.

Marketing and communication are all about making fancy claims and simplifying facts with wild exaggerations. One can wonder, how is the bs created in marketing when there's so much science behind most projects? It starts with the "businessification" of quantum technologies. The rules of the game for a staring looking for some VC funding is to talk about customer use cases and market size, creating an echo chamber to the crazy numbers published by industry analyst firms. You will therefore have plenty of quantum computing hardware and software companies web site presenting the same story about the beauties of quantum computing in pharma, financial services, transportation and the industry, if not to fix climate change, but nearly nothing on their actual technologies and products.

A key form of bs shows-up when quantum hardware startups are hiding simple information like the number of qubits of their QPU (a practice from Anyon Systems in Canada as of 2022 and OQC in the UK in 2021). It usually means that they are too shy to say that they have fewer than 5 operating qubits and therefore are not competitive against companies like IBM (433 qubits as of August 2023) and Rigetti (80 qubits as of the same date). How about qubit fidelities? They are usually hidden as well.

[4838] See ERC Gender Actions, 2021 (14 slides). It provides some data on the share of women applicants vs men who get ERCs and H2020 grants based on the discipline. Across the board, women have about 20% less chance to get a funding.

Another doubtful practice from quantum hardware vendors connects the dots between customer orientation and unmature hardware platforms is to say: we create application-specific quantum hardware. While it may make sense in some cases, it is economic and technological nonsense. Successful hardware companies create economies of scale, like IBM in the 1960s with its family of IBM 360 mainframes. Hardware must be generic for a large range of applications. Like Nvidia GPGPUs that are used for both machine learning and scientific applications thanks to a broad software support. On top of that, if one hardware platform has so many limitations that it is bound to be used for only one category of application, you as a customer will be locked in. Then, you can listen to the technical rationale behind custom-hardware platforms. But specialists will tell you it doesn't make much sense in general.

At last, some quantum hardware vendors will sell you fancy customer-oriented application benefits, without presenting any real quantum advantage whether in results precision, execution time, solution price or spent energy for the environmentally conscious as compared to best-in-class classical solutions.

Quantum technologies and society key takeaways

- Quantum technologies can become one of the artefacts of Mankind's technology ambitions, pushing the limits of what can be achieved in the line of some works done in artificial intelligence. It may give the impression that mankind's power has no limit. A sound scientific mind will however understand that quantum computing has its own limits. The world can't be simulated, the future can't be predicted, and apparent free will can persist.
- Science fiction has built an imaginary of what quantum technologies could achieve, with teleportation, supraluminal traveling speeds, various entanglement and miniaturization feats, parallel or multiverse worlds and time travel. While none of these things are possible given our current scientific knowledge, it can create scientific vocations and drive new generations to solve actual problems.
- Quantum foundations is the branch of science philosophy that aims to build some understanding of the real world. Quantum physics' formalism is difficult to associate with the principles of reality usually applicable in classical physics. While classical physics understanding has historically been associated with an ontology with objects position and motion enabling the prediction of phenomena such as the motion of planets. Quantum physics lacks such an ontology describing the physical world. Beyond the canonical Copenhagen interpretation (psi and the wave equation), many scientists tried to create such ontologies and the debate is still raging.
- The quantum scientific community is starting to investigate the ethics of quantum technologies. Like with artificial intelligence, it will be questioned on algorithms explainability and auditability, on what it will do to simulate if not tweak matter and life and on how to handle public education. Some related initiatives have already been launched by scientists in Australia, The Netherlands, Canada and the UK.
- The education challenge around quantum sciences and technologies is enormous, both for the general public and with specialists. There's a need for better pedagogy, accessible educational content and also for sound fact-checking information.
- Gender balance is already an issue in quantum technologies with a low share of women in the field, particularly with vendors. Hopefully, there are many top women scientists and entrepreneur role models around who can inspire a new generation of women teenagers. Many initiatives around the world have been launched for that respect.
- At last, quantum technologies vendors marketing must be watched carefully. It is and will be full of exaggerations and approximations. The worse will happen with vendors outside the quantum technology sphere.

Quantum fake sciences

One of the most fascinating topics in the mainstream impact of quantum physics is the way some people integrate it into alternative dubious scientific approaches. The vast framework of "quantum medicine" is a coherent stream of thought and practice from this point of view. It has given rise to the proliferation of gurus of all kinds and to voluntary or involuntary scams based on miracle machines for detecting electromagnetic waves or vague energies and restoring your body balance. It is at best a subset of the vast placebo effect industry!

Other fields took over quantum physics and long before quantum computing became a visible subject: management and marketing, not to mention politics[4839]. Quantum physics is essentially used there as a source of inspiration by analogy. But the "gurutisation" of these sectors is also quite common, linking together currents of thought that revolve a lot around magical thinking.

This chapter of this book on quantum fake science was created in its first edition, back in 2018, and updated since then. In 2023, **Chris Ferrie** published an entire book on this matter that drew some attention[4840] [4841]. It gives some useful quantum physics insights and background to separate real quantum science from fake science. This part is pinpointing with more details various examples of fake sciences and their associated commercial products.

Quantum biology

The starting point of quantum medicine is, however, scientifically highly relevant and interesting. Many low-level biological phenomena can be well explained at a low-level by quantum physics. Of course, since everything is quantum on the atomic scale! All the field of chemistry depends on quantum physics.

To mention just a few examples, this is obviously the case of **photosynthesis** in chlorophyll and plants, which uses the photoelectric effect transforming a photon into electron displacement, leading after the Calvin cycle to the production of glucose that is used to store energy. All the **nitrogen fixation** pathway in cyanobacteria in soil and legume roots extensions relate to quantum physics and quantum chemistry as we've seen when covering the famous FeMoCo simulation case study. The same applies to **retina cones and rods** which capture light. **UV-B rays** participate in the synthesis of Vitamin D3 precursors in the skin again using the photoelectric effect but with a different wavelength[4842]. Quantum physics also explains the **capture of terrestrial magnetism** in the brains of many birds via a special protein called cryptochrome.

[4839] The concept of quantum politics is still in its infancy. Here is some literature from economic and social researchers on the subject. For example, Quantum like modelling of the non-separability of voters' preferences in the US political system by Polina Khrennikova, University of Leicester, 2014 (13 pages) seeks to model the choices of US voters and the entanglement or not of the choice of presidential candidate and congressional candidates showing that it can decouple under certain conditions. And Quantum Politics: New Methodological Perspective by Asghar Kazemi, 2011 (15 pages) creates a link with chaos theory and the butterfly effect. The paper was written just after the 2011 Arab revolutions. See also Schrodinger's Cat and World History: The Many Worlds Interpretation of Alternative Facts by Tom Banks, who uses Bryce DeWitt's Multiple Worlds Thesis to explain the election of Donald Trump in 2016 by a giant tunnel effect. That maaaayyyy be a little exaggerated! In 2022, some writer tried to explain the Russia invasion of Ukraine with quantum physics. See Quantizing the Invasion of Ukraine by Nicholas Harrington, 2022 (not precisely dated…) and another paper described a quantum parliament, in A two-party quantum parliament by Theodore Andronikos and Michael Stefanidakis, January 2022 (23 pages).

[4840] See Beyond the quantum woo-niverse: getting to grips with the fundamentals of quantum mechanics by Philip Moriarty, Physics World, April 2023.

[4841] See Quantum Bullsh*t: How to Ruin Your Life with Advice from Quantum Physics by Chris Ferrie, 2023 (224 pages).

[4842] See The Relationship between Ultraviolet Radiation Exposure and Vitamin D Status by Ola Engelsen, 2010.

This mechanism relies on the protein's ability to detect magnetic variations through some electron quantum entanglement[4843]. Likewise, some phenomena in DNA replication and mutation can be explained by some **proton tunneling** between the two DNA strands[4844]. Understanding quantum biology can lead to biomimetism innovations such as the development of artificial nose sensors[4845].

So, quantum biology is a serious scientific domain, and it deserves its proper attention while it stays in the scientific realm[4846] [4847] [4848] [4849] [4850]. Some would obviously say, "*in the official scientific realm*". We won't argue about it since it is an endless debate.

So far so good.

Then, some renowned scientists want to explain the origin of consciousness with quantum physics. Several major schools of thought are related to each other like the **Orch-OR theory** (Figure 969), the **holographic** dimension of DNA and **biophotons**. And then there are all the works around **structure of water** and **water memory**.

None of these works obtained the agreement of most scientists, but it still deserves a little review. If only to understand how they are quickly being misused by the quantum medicine charlatans over the world.

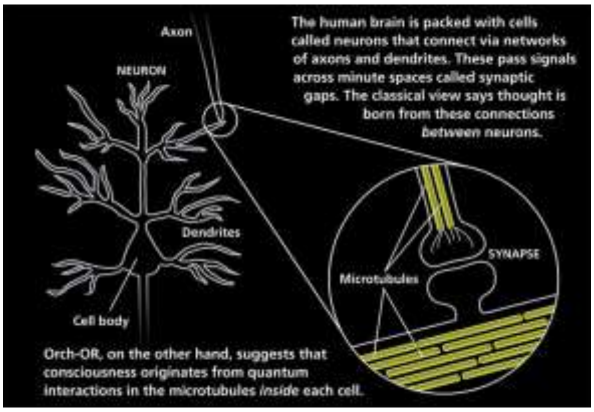


Figure 969: Orch-OR top-level view.

Orch-OR Theory

According to **Roger Penrose** (English, 1931[4851]) and **Stuart Hameroff** (American, 1947), consciousness is housed and managed by microtubules, the complex fibrous structures that, together with actin filaments and intermediate filaments, constitute the structure of neuron cells, called the cytoskeleton, and in the case of neurons, the dendrites, synapses and axons[4852].

In 1996, they proposed the Orch-OR (Orchestrated Objective Reduction) model according to which these microtubules were coherent quantum systems explaining consciousness. For them, consciousness is managed in the neurons within these microtubules and not by their interconnections via dendrites/synapses pairs.

[4843] See Resonance effects indicate a radical-pair mechanism for avian magnetic compass by Thorsten Ritz et al, 2004 (4 pages), Cellular autofluorescence is magnetic field sensitive by Noboru Ikeya and Jonathan R. Woodward, January 2021 (6 pages) and Magnetic sensitivity of cryptochrome 4 from a migratory songbird by Jingjing Xu et al, June 2021.

[4844] See An open quantum systems approach to proton tunnelling in DNA by Louie Slocombe et al, Nature Communications Physics, May 2022 (9 pages).

[4845] See A Quantum Biomimetic Electronic Nose Sensor by Ashlesha Patil, Nature Scientific Reports, 2018 (8 pages).

[4846] See Coherent Excitations in Biological Systems, 1983 (233 pages).

[4847] See What is Quantum Biology? by Filippo Caruso, 2016.

[4848] See Quantum Biology: An Update and Perspective by Youngchan Kim et al, January 2021 (48 pages).

[4849] See Vibrations, quanta and biology by S.F. Huelga &M.B. Plenio, Contemporary Physics, 2013 (26 pages).

[4850] See The Future of Biology is Quantum - A proposal for a new scientific research organization by Arye and Clarice D. Aiello, May 2022 which calls for the creation of a dedicated research lab on quantum biology, that would be directed by one of its authors.

[4851] He was awarded the Nobel prize in physics in 2020 for his seminal work on black holes.

[4852] Illustration source: Is our brain a quantum computer? by Laurent Sacco, April 2018.

In 2011, Roger Penrose and Stuart Hameroff even suggested that these microtubules, shown in Figure 970 would be quantum nano computers capable of managing qubits and associated calculations[4853]. If this were true, the power of this computer in number of qubits would be immeasurable because a single neuron comprises about 100 million tubules, the brain 86 billion neurons and more than 600 trillion connections between neurons! These theories obviously do not specify how the entanglement between these qubits would work on this scale.

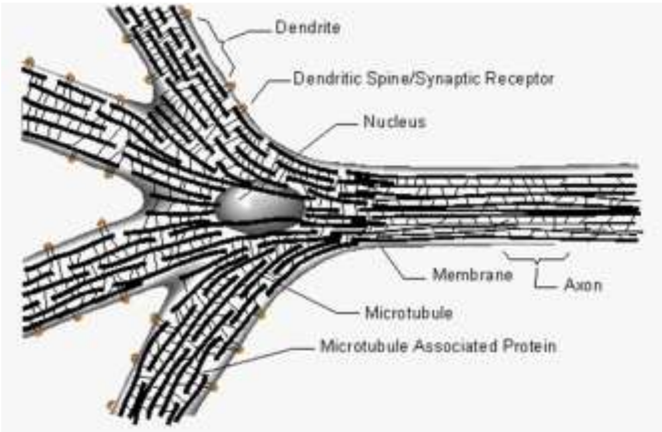


Figure 970: Orch-OR low level view with neurons and their microtubules.

Ironically, the indirect impact of this gargantuan sizing would be to push back even further in time a possible singularity, the moment when a computer would reach the computing capacity of a human brain in raw computing power[4854]. We are dealing here with another current of thought, promoted by **Ray Kurzweil**.

The Orch-OR theory was revived in 2014 with the discovery of quantum vibrations in microtubules by **Anirban Bandyopadhyay** from the National Institute for Materials Science in Japan[4855]. But that doesn't explain anything. Roger Penrose and Stuart Hameroff also asserted that this behavior is influenced by some type of gravity-related wavefunction collapse[4856]. Consciousness is a "macro" phenomenon. Trying to explain a "macro" phenomenon by a single "nanoscopic" process is meaningless because it completely gets rid of the entire biological hierarchy between the two and the other nanoscopic mechanisms at stake in the nervous system: neurons themselves, neurotransmitters, synapses and dendrites, neurons nucleus, brain regulatory glial cells, and on a larger scale, senses and brain macro-organization[4857].

For example, we can explain a good part of living things via the weak hydrogen-hydrogen bonds (which are of quantum nature, of course) that are linking together the two DNA strands, or with the oxygen and phosphorus bonds, in DNA and RNA, which are strong and can thus explain the cohesion of these fundamental molecules of living things, as shown in Figure 971.

[4853] Other theories think that quantum entanglement also works elsewhere in the brain, at the level of phosphorus atoms associated with calcium. This would allow the creation of quantum bonds between neurons. See Quantum Cognition: The possibility of processing with nuclear spins in the brain by Matthew Fisher, 2015 (8 pages). As the article indicates, this raises questions but does not provide answers! Therefore, any rather rapid interpretation of the "quantum brain" is to be taken with a grain of salt.

[4854] See Consciousness in the Universe Neuroscience, Quantum Space-Time Geometry and Orch OR Theory by Roger Penrose, 2011, 50 pages). All this is documented in Orchestrated Objective Reduction of Quantum Coherence in Brain Microtubules: The "Orch OR" Model for Consciousness, 1996 (28 pages) as well as in Consciousness, Microtubules, & 'Orch OR' A 'Space-time Odyssey' by Stuart Hameroff, 2013 (28 pages), Are Microtubules the Brain of the Neuron by Jon Lieff, 2015 and popularized in The strange link between the human mind and quantum by Philipp Ball, 2017. Roger Penrose has collaborated with Stephen Hawking on gravitational singularities and radiation emission from black holes. Hawking had developed a cosmological theory combining the theory of relativity and quantum physics.

[4855] The discovery is disputed by Matti Pitkanen in New Results about Microtubules as Quantum Systems, 2014 (18 pages).

[4856] An physics experiment did invalidate most of this theory. See Quantum theory of consciousness put in doubt by underground experiment, PhysicsWorld, July 2022 referring to At the crossroad of the search for spontaneous radiation and the Orch OR consciousness theory by Maaneli Derakhshani et al, Science Direct, September 2022.

[4857] See this interesting discussions on Orch-OR in Why is Orch-OR ignored by the mainstream scientific community?, Quora, and also Falsifications of Hameroff-Penrose Orch OR Model of Consciousness and Novel Avenues for Development of Quantum Mind Theory by Danko Dimchev Georgiev, 2006 (32 pages) which debunks many of Stuart Hameroff and Roger Penrose assertions in the Orch-OR model with an in-depth neurobiology analysis.

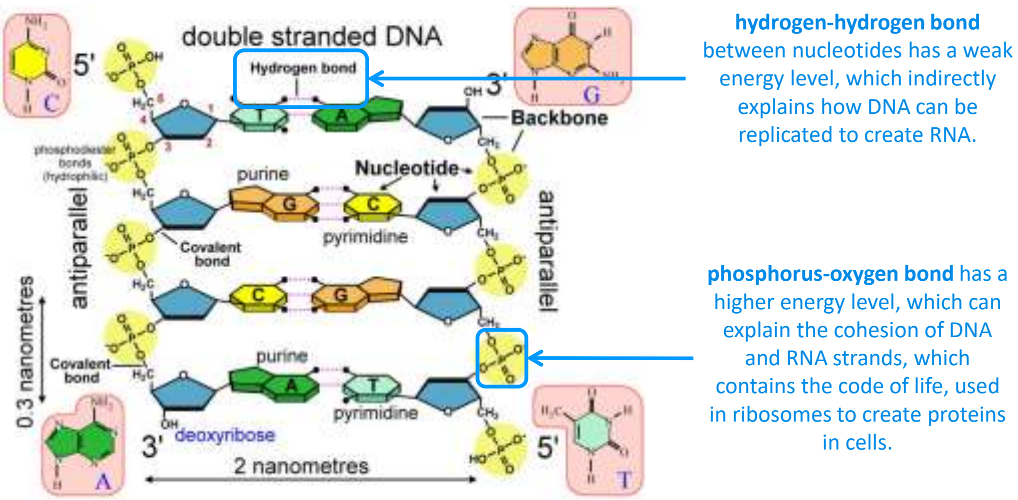


Figure 971: you can build a whole explanatory theory on life with just two chemical liaisons (hydrogen-hydrogen and oxygen-phosphorus). Source: http://universe-review.ca/F11-monocell08.htm.

However, this is obviously not enough to explain consciousness or how your heart, eyes and kidneys work. One could also easily build a bozo theory associating consciousness with electrons if not with quarks and gluons. Indeed, without electrons, there's no chemistry and no consciousness! It explains the chemical bonds between atoms.

Fortunately, nobody has yet ventured into this kind of explanation. In short, explaining consciousness by the possibly quantum nature of a particular structure of neurons is the most simplistic reductionism possible, ignoring all the other knowledge available... or yet unavailable[4858]. This comes from the fact that some are playing with words and confusion between biological processes[4859] [4860] and philosophical concepts. Conscience relates to both.

DNA would also have a quantum function. A curious paper of Russian, German and English origin describes quantum and non-localized phenomena in DNA, verified in a famous experiment based on laser light diffraction, as shown in Figure 972[4861].

Through quantum entanglement, the chromosomes of several cells would interact with each other via these radiations.

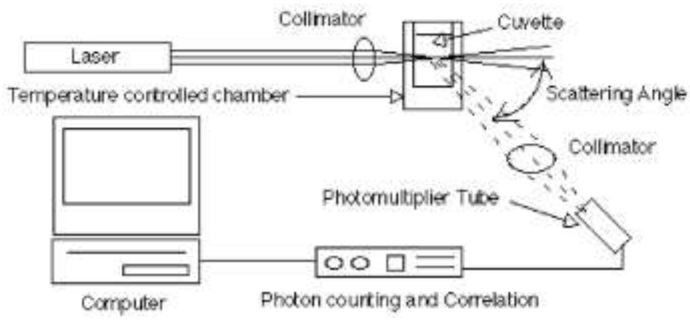


Figure 972: Source: DNA as Basis for Quantum Biocomputer, 2011 (22 pages),

The leader of this work is a certain **Peter Gariaev**, creator of the concept of BioHolograms within his **Wave Genetics Institute** in Moscow[4862].

4858 A similar reductionism process shows up in Scientists think quantum tunneling in space led to life on Earth by Tristan Greene, TheNextWeb, March 2022, that refers to A pathway to peptides in space through the condensation of atomic carbon by S. A. Krasnokutski et al, Nature Astronomy, February 2022 (13 pages). They explain the appearance of life on Earth for some low-level chemical reaction that could happen on an asteroid. But this reaction is even more likely on Earth given the conditions on the planet!

4859 See Experimental evidence of non-classical brain functions by Christian Kerskens and David López Pérez, Journal of Physics Communications, June 2018-June by 2022 (9 pages).

4860 See Are Qualia Reducible, Physical Entities? by Christian Kerskens, viXra, March 2020 (5 pages). Note that this was published on viXra, not even on arXiv.

4861 See DNA as Basis for Quantum Biocomputer, 2011 (22 pages).

4862 The history of the theme is explored in Quantum BioHolography A Review of the Field from 1973-2002 by Richard Alan Miller, Iona Miller and Burt Webb (23 pages), but these texts do not give any idea of its scientific validity.

Other attempts to explain consciousness by quantum physics have been created. **Matthew Fisher** from UCSB wanted to investigate the brain's potential for quantum computation, based on phosphorus ions spin entanglement[4863]. He launched his **Quantum Brain Project** (QuBrain) with a 1M€ funding in 2018 from the Heising-Simons foundation. Since then, the Project was discontinued. Others like **Johnjoe McFadden** from the University of Surrey in the UK try to explain consciousness with electromagnetic waves circulating in the brain[4864]. At last, some journalists invent some quantum influence where it doesn't exist[4865].

Biophotons

Another alternative school of thought is related to **biophotons**. These are the low light emissions in the visible generated by living beings. They were discovered in 1922 by the **Alexander Gurwitsch** (Russia). The theory of biophotons was perfected by the **Fritz Albert Popp** (Germany). It complements at a low-level the hologram DNA thesis.

It describes the emission of photons from molecules such as DNA, but also the emission of photons related to the energy metabolism of cells such as the transformation of ADP molecules into ATP in the mitochondria of cells.

The biophotons are ultraviolet and visible light emissions, at levels that are much lower than the mid-infrared emission occurring at around 12 microns wavelength. Up to a few hundred photons per square centimeter of organ analyzed could be detected, often at the skin level.

These biophotons are also made of coherent light - photons with the same frequency. They would constitute a form of inter-cellular communication[4866].

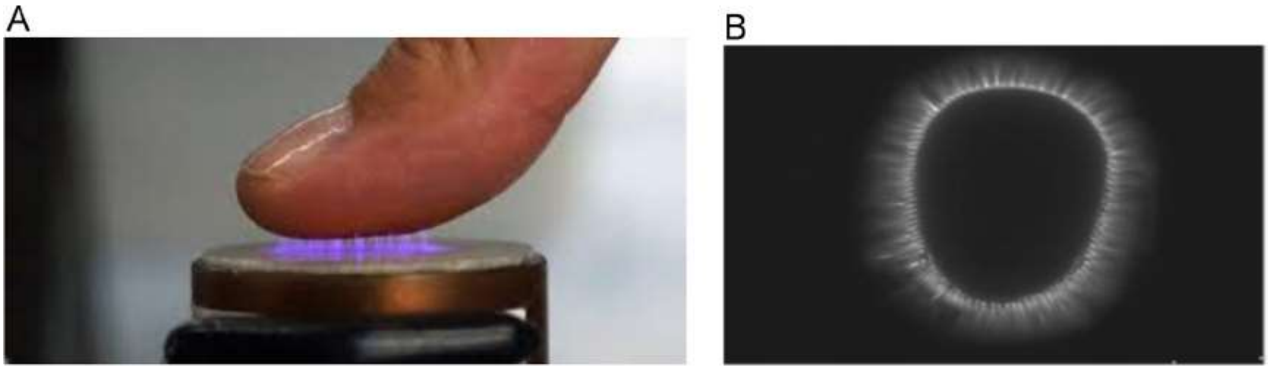


Figure 973: Biophotons. Source TBD.

I wonder how this communication works: at what range, due to the obvious attenuation of photons scattering, and with what precision targeting (direction, orientation).

According to Fritz Albert Popp, raw foods emit more biophotons than cooked foods, and organic raw plants emit five times more biophotons than traditionally grown plants. Conclusion: eat raw and organic! This is also a reason to have prehistoric men regret having discovered fire!

4863 See Quantum Cognition: The possibility of processing with nuclear spins in the brain by Matthew P. A. Fisher, 2015 (8 pages).

4864 See Integrating information in the brain's EM field: the cemi field theory of consciousness by Johnjoe McFadden, September 2020 (13 pages) covered in New research claims that consciousness itself is an energy field - a professor says this could be the key to building conscious machines by Victor Tangermann, in Futurism, October 2020.

4865 See Your brain might be a quantum computer that hallucinates math by Tristan Greene February 2022 referring to Neuronal codes for arithmetic rule processing in the human brain by Esther F. Kutter et al, Science Direct, March 2022 (15 pages).The words quantum and entanglement do not appear in the scientific paper. Ergo the first title is pure clickbait.

4866 As described in Photonic Communications and Information Encoding in Biological Systems by S.N. Mayburov, 2012 (10 pages) and popularized in Biophoton Communication: Can Cells Talk Using Light?, 2012 in the MIT Technology Review.

In any case, the detection of biophotons on the 10 fingers of the hand would make it possible to detect cardiac pathologies[4867]. The **ClearView** scanner used exploits a curious process: it sends a high-voltage pulse that creates an electromagnetic field around the finger that amplifies the biophotons that are emitted. This excites molecules in the air, creating a plasma between the sensor and the finger (Figure 973, *left*) that ionizes the air, generating the emission of UV and visible light. This is the **Kirlian effect**, discovered by the Russian Semyon Kirlian in 1939.

The ionization that is captured by the camera (Figure 973, *right*). The software analyzes the generated shape and compares it to a pathology database. I have a hard time figuring out the exact link between bioluminescence and this process! And what about the receptors of these biophotons?

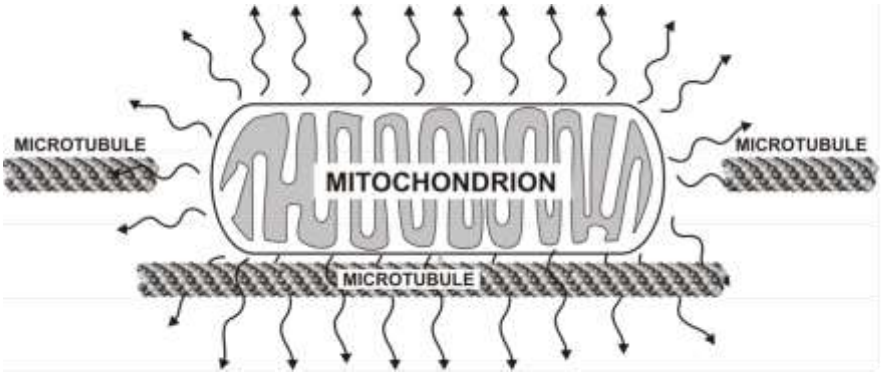


Figure 974: Source: Emission of Mitochondrial Biophotons and their Effect on Electrical Activity of Membrane via Microtubules by Majid Rahnama et al, 2010 (22 pages).

Well, it comes from the neuron microtubules, of course, closing the loop[4868] (Figure 974)! According to Popp: "*matter would only be condensed light*"[4869]. By the way, biophotons would be a way to explain chi. Why not.

David Muehsam mentions many biological effects of biophotons, which would be involved in the regulation of neurotransmitters secretion (for rats) but without the distinction between correlation and causality being visibly made in the associated publications[4870].

If all that was just science and research! But hell no. It helps snake oil vendors to sell miracle healings through the control of the body by conscience. Practitioners of quantum medicine are very often psychosomaticians exploiting mysticism and autosuggestion to generate, in the best of cases, a good placebo effect that can work with certain mild pathologies. Even so, they justify their methods on the contested work of researchers such as Roger Penrose and Stuart Hameroff, already mentioned, but also Karl Pribram and Henry Stapp, who want to explain human consciousness by quantum phenomena intervening at a low-level in the brain that would also explain a so-called immortality.

Wikipedia's Quantum Mind fact sheet reports on the evolution of this branch and the associated criticisms. It underlines the fact that there is no way to apply possible quantum phenomena such as entanglement at the scale of macroscopic brain molecular or cellular structures.

Entanglement is even less justifiable to connect the brain at long distance to the "*holographic global consciousness of the Universe*" promoted by **Karl Pribram** and **Paola Zizzi**[4871].

[4867] According to Detecting presence of cardiovascular disease through mitochondria respiration as depicted through biophotonic emission by Nancy Rizzo, 2015 (11 pages).

[4868] This is what comes out of Emission of Mitochondrial Biophotons and their Effect on Electrical Activity of Membrane via Microtubules, 2010 (22 pages).

[4869] See Introduction of Consciousness in Matter from Quantum Physics to Biology (18 pages) by Jacqueline Bousquet, a former CNRS researcher who died in 2013.

[4870] See The Energy That Heals Part II: Biophoton Emissions and The Body of Light by David Muehsam, April 2018.

[4871] See Consciousness and Logic in a Quantum-Computing Universe, 2006 (25 pages).

Similarly, it does not necessarily make sense to link mind and matter as waves and particles and their famous duality. This otherwise leads to absurdities that explain psychic phenomena of synchronicity by the consciousness wave function collapse, an explanation that is as absurd as is the Schrödinger's cat thought experiment. Even if Penrose and Hameroff's theories were verified, the shortcut would be a little hasty, connecting way too fast a nanoscopic phenomenon to a macroscopic phenomenon!

The other commonly proposed method involves the use of various electromagnetic waves, including the famous and smokey **scalar waves**. The idea is to exploit them to restore the balance of unbalanced organs, exploiting the wave-particle duality and the ability to restore the basic energy level of... we don't know. Particularly given the proposed waves are not really targeted.

It is notable, however, that few scientific specialists in quantum medicine mention the capabilities of future quantum computers to simulate the operations of organic molecules and create new therapies. Maybe because known applications of quantum computing in health care are part of traditional allopathic medicine, that they usually avoid or at least complement.

However, I found a vague trace of with **Matti Pitkanen** (Finland) who, in the framework of his work on TGD (Topological Geometrodynamics), proposes a unified theory of physics, and puts forward the idea of creating DNA-based quantum computers[4872]. He believes that DNA communicates "with the Universe". It is also based on **Luc Montagnier**'s (France, 1932-2022) experiments on DNA. Matti Pitkanen provides the basis for highly speculative theories on the supposed consciousness of the Universe[4873]. His theories of the unification of physics are so complex that they are impossible to understand, and eventually to validate by experience or to refute.

In the field of light-based therapy, one puzzling solution being sold comes from **Bioptron AG** (1988, Switzerland), part of **Zepter Group** (1986, Switzerland), since 1996. Its "Bioptron Quantum Hyperlight" uses "hyperpolarized light" generated with fullerene (C_{60}), a molecule also used by Archer for trap its electron spin qubits. Among other benefits, it treats injuries pain, avoiding pain killer drugs. So far, so good. The system generates some vertically linearly polarized light which passes through a filter containing these fullerene molecules which happen to rotate at a $1.8x10^{10}$ frequency per second. It creates "*perfectly ordered hyperpolarized light*" that is supposed to have some quantum properties similar to those of the biomolecules inside our bodies. Practically, this light is made of both vertically and horizontally polarized photons that "*without exaggeration, [...] reestablish the balance and harmony of energetic processes in biostructures and to harmonize cells, bringing them to back to their initial state of natural equilibrium*". Contrarily to many of the pseudo-quantum scams we'll cover later in this section, this offer is fairly well documented, even scientifically[4874]. You're flooded by tons of scientific information, historical references, links to Nobel prize inventions and scientific publications. But many indices generate serious doubts[4875]. Among others, it mentions these dubious Emoto's research on water structure and the way it can be changed with music and good mood.

[4872] See Quantum Mind, Magnetic Body, and Biological Body by Matti Pitkanen, August 2018 (186 pages).

[4873] See TGD Universe as a conscious hologram, February 2018 (612 pages).

[4874] See the Bioptron Quantum Hyperlight brochure (60 pages) and Hyperpolarized light 2018 (318 pages) by Djuro Koruga.

[4875] Some are well documented in an extensive analysis, although a bit dated, in Cancer and the magic lamp, February 2009. It shows that most scientific surveys were of small scale and non audited and with no control group trials. It was done only on wounds healing. But the vendor web site touts many medical indications that their device is supposed to treat, without any scientific evidence, beyond wounds healing: osteoarthritis, arthroses, lowered motivation and the inability to feel happy. All are good indications, in the best case, of some placebo effect. On top of that, the Zepter also sells blue and red LED light therapy devices, for 500€. The Bioptron is priced at about 1,000€.

Water memory

The last area on the fine line between science and charlatanism is that of water. It features a model of thought close to Roger Penrose's **Orch-OR** theory, which consists in explaining everything about life based on a few isolated physical phenomena at the microscopic level. The phenomenon of the **memory of water**, its explanation by **electromagnetism**, and parallel theories on the **water structure** are all mixed together.

One of the starting points around the role of water is **Jacques Benveniste**'s work on water memory. This immunology and allergies specialist was a director of an Inserm research laboratory in Clamart, France. He conducted experiments that led to the conclusion that "*water could preserve a memory, a print, of substances that have passed through it*". With Israeli, Italian and Canadian researchers, he published a landmark article in Nature in 1988, which was soon contested[4876]. He described a series of experiments that showed the effectiveness of anti-IgE (anti-immunoglobin E) causing the loss of histamine-containing granules by a type of white blood cell, basophilic cells, even when this anti-IgE is repeatedly diluted to the point where no anti-IgE molecule can be found in solution. For this to work, solutions must be shaken vigorously after each dilution, using the “dynamization” principle!

In the article, Benveniste hypothesized that the phenomenon could be explained by the creation of structured networks in water or by persistent electric or magnetic fields. They would constitute some sort of “water memory” which would “record” the allergen characteristics and reproduce its effects on basophilic cells. This was supposed to explain high dilutions used in homeopathy (Figure 975)!

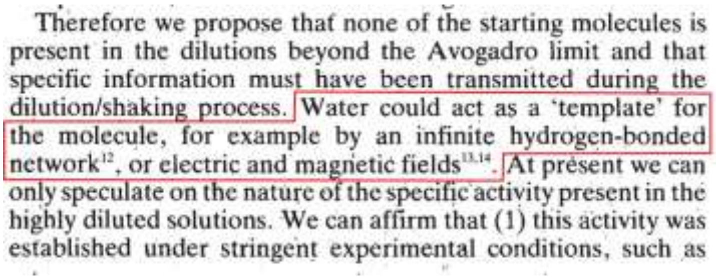

Therefore we propose that none of the starting molecules is present in the dilutions beyond the Avogadro limit and that specific information must have been transmitted during the dilution/shaking process. Water could act as a 'template' for the molecule, for example by an infinite hydrogen-bonded network[12], or electric and magnetic fields[13,14]. At present we can only speculate on the nature of the specific activity present in the highly diluted solutions. We can affirm that (1) this activity was established under stringent experimental conditions, such as

Figure 975: water memory key description in Benveniste's Nature paper. Source: Human basophil degranulation triggered by very dilute antiserum against IgE, Jacques Benveniste et al, June 1988 (3 pages).

The promoters of this empirical medicine devised by **Samuel Hahnemann** around 1810 and explained in the book "The Organon " thought they had finally found their scientific support.

Testing and evaluation protocols were flawed in many ways. Solutions were not analyzed by spectrographic analysis to deduce their molecular composition[4877]. Only electrophoresis was used to detect the presence of ions[4878]. The presence of histamine resulting from the release of granules from the basophiles had not been assessed.

It was realized in other experiments that there was none! Moreover, the phenomenon presented a cyclic character of a period of 8 dilutions (in Figure 977Figure 976), according to the successive dilutions, but being out of phase by four dilutions from one experiment to another. No explanation is given for this cyclic phenomenon[4879]. The electromagnetic theory that would explain the phenomenon is his other Achilles' heel.

[4876] See Human basophil degranulation triggered by very dilute antiserum against IgE, Jacques Benveniste et al, June 1988 (3 pages) and Ma vérité sur la mémoire de l'eau by Jacques Benveniste, 2005 (122 pages). The book contains a preface by the Nobel Prize winner Brian Josephson. In this book, published after his death in 2004, Jacques Benveniste recounts his experiences, his tumultuous relations with the medical mandarins over several decades, the story of the publication of his famous article in Nature in 1988 and other experiments conducted during the 1990s and early 2000s.

[4877] Raman spectrometry will be used in other experiments, much later from 2007, on various homeopathic strains.

[4878] At high dilutions, electrophoresis showed that there was no anti-IgG molecule left in the active ingredient.

[4879] Ironically, the process used does not prevent allergic reactions as is expected in homeopathy, which wants to treat evil with evil, but in low doses. Here the anti-IgE causes the production of histamine and does not prevent it. Some debunking came with “Memory of Water” Experiments Explained with No Role Assigned to Water: Pattern Expectation after Classical Conditioning of the Experimenter by Francis Beauvais, 2018 (20 pages).

It is weakly substantiated. These waves are not characterized, measured nor their source explained. The story of Jacques Benveniste is the story of a curious experimenter who lacks, however, the bases in adjacent disciplines around electromagnetism. However, he did investigate long-range electromagnetic fields, inspired by the work of Italian physicists specialized in quantum electrodynamics, **Giuliano Preparata** (1942-2000) and **Emilio Del Giudice** (1940-2014). In 1990, he set up an experiment with the CNRS Central Laboratory of Magnetism in Meudon, France, which showed that the activity of the diluted solution is modified by prolonged exposure to a magnetic field. The experiment used animal hearts with an electrical apparatus invented by **Oskar Langendorff** (1853-1908).

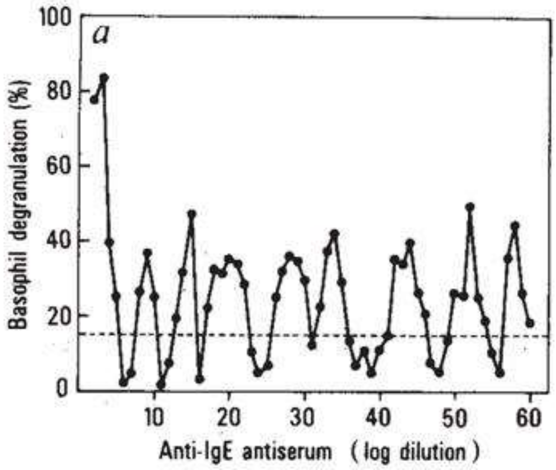


Figure 976: Source: Human basophil degranulation triggered by very dilute antiserum against IgE, Jacques Benveniste et al, June 1988 (3 pages).

In another experiment carried out over several years, he also uses an amplifier using a sound card from a microcomputer to transmit the properties of a solution to another neutral liquid. This leads to the concept of "digital biology"[4880].

After the death of Jacques Benveniste in 2004, his work was taken over by **Luc Montagnier** (1932-2022, French), who created the first AIDS treatment and got the Nobel Prize in medicine in 2008. He described low frequency waves (7 Hz) that would be emitted by DNA strands. He set up an experiment in which the waves of DNA molecules are transmitted through a coil fed at 7 Hz to pure water in another test tube. A PCR is then used to regenerate the DNA in this test tube (DNA multiplication process, "polymerase chain reaction"). And gel electrophoresis is used to decode the replicated DNA! In the experiment, this DNA corresponds exactly to the original DNA.

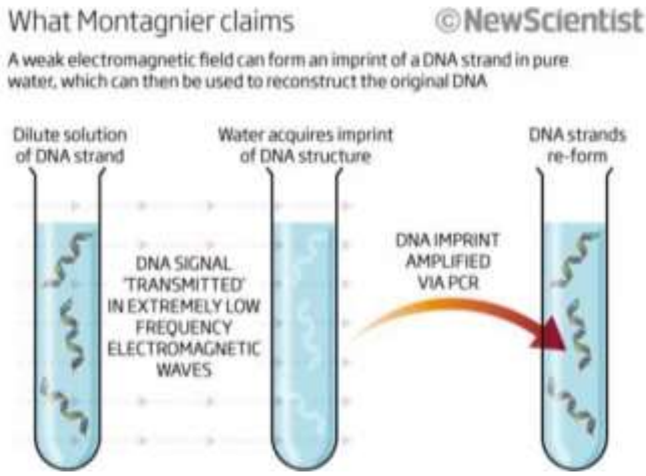


Figure 977: Montagnier claimed DNA could be created in water containing just… water molecules. How did carbon, phosphorus and nitrogen atoms appear? Source: NewScientist.

His code would have been transmitted by electromagnetic wave[4881]. But the documentation does not specify which DNA was used as a primer for PCR! Indeed, a PCR does not start from zero and a bunch of nucleotides but uses DNA strands to replicate them[4882]. The work of Luc Montagnier is related to that of the Italian **Emilio Del Giudice**, again, on the structure of liquid water[4883]. It will not surprise you to learn that this kind of discovery is rather controversial among specialists[4884].

4880 This story is well told in L'âme des molécules, une histoire de la mémoire de l'eau by Francis Beauvais, 2007 (626 pages). The author was one of Jacques Benveniste's experimenters.

4881 See explanations in Luc Montagnier's article DNA waves and water, January 2010 (10 pages). Montagnier and the quantum teleportation of DNA by Vincent Verschoore, January 2011, is the source of the illustration.

4882 This PCR problem is noted in The Nobel disease meets DNA teleportation and homeopathy, January 2011.

4883 See Mae-Wan Ho's Illuminating Water and Life, 2014 (18 pages) which describes the theories of Emilio Del Giudice, who died that same year.

4884 See Luc Montagnier and the Nobel Disease by David Gorski, June 2012.

Luc Montagnier's paper was not published in a peer-reviewed journal, but he continued to publish, with international teams, on interesting research explaining with quantum field theory how DNA polymerase works[4885]. The relationship between water and quantum physics is being emulated by others and drove the creation of many scams selling structured water and the likes[4886].

Konstantin Korotkov (Russia) did some experiments supposed to show that projecting negative emotions on water reduced its energy level and vice versa[4887]. This guy created IUMAB (International Union of Medical and Applied Bioelectrography), an organization that promotes the use of bioelectrography devices[4888]. He is promoting DGV Bio Well cameras, aura detection systems around patients that would materialize the chakras, via the analysis of the "gas discharge" (Figure 978).

Figure 978: Bio-Well measure human energy using bio-electrography, inspired by Konstantin Korotkov. These are clearly scams.

We then have **Mazaru Emoto**'s MRA (Magnetic Resonance Analyzer) (1943-2014). He conducted experiments analyzing the impact of emotions on the structure of water. Experiments that were never reproduced independently[4889]. You probably guessed it!

OK, emotions can generate infrared waves and gases that can be exhaled, producing in turn a minute reaction on exposed water[4890]. This makes it possible to sell a concentrated structured water that can be used to prepare distilled water, **Indigo Water** (*in* Figure 979). Here is the description: "*A geometrically perfect water with the "Message" your body is waiting to receive. Dr. Emoto's Indigo Water contains eight ounces of highly charged hexagonally structured concentrate. By mixing one ounce of concentrate with one gallon of distilled water, you are creating eight gallons of structured water from*

[4885] See Water Bridging Dynamics of Polymerase Chain Reaction in the Gauge Theory Paradigm of Quantum Fields by Luc Montagnier et al, 2018 (18 pages).

[4886] See Hypotheses quantum of mechanism of action of high homeopathic dilutions, is a doctoral thesis by Mathieu Palluel, 2017 (252 pages). Its first part is a fairly well-supplied history of homeopathy. It also covers the experiences of Jacques Benveniste and Luc Montagnier. The quantum part starts on page 181 and is quite weak. This PhD student was definitely not a physicist. He makes a countersense on Schrödinger's equation on page 189. He uses quantum field theory and quantum electrodynamics in a weird context, water at room temperature. On page 201, the paper states that water molecules have a diameter of approximately 3 nm while it is 0.27 nm. It also talks on page 221 about the Nobel Prize of "Serge Laroche" instead of Serge Haroche. In short, this thesis document was poorly reviewed by the people who validated it, and who were not at all up to date in quantum physics.

[4887] See The First Korotkov Intention Experiment by Konstantin Korotkov, January 2018 as well as The Intention Experiment on H2O, 2007 (18 pages) which reproduced his experiments in the USA.

[4888] He is also the author of The Emerging Science of Water: Water Science in the XXIst Century by Vladimir Voeikov and Konstantin Korotkov, 2018 (253 pages), a work or current of thought that certainly influenced Marc Henry's work, unless the opposite is true.

[4889] This is well explained in The pseudoscience of creating beautiful (or ugly) water by William Reville, 2011. See also the site Structure-altered water nonsense which makes a good inventory of commercial offers of structure water in the USA. The 1995 style layout serves the site but the inventory of solutions is edifying. Masaru Emoto also certified an effect of exposing zam zam water that is produced at Mecca to Quran. The water is supposed to have similar miraculous effects, a bit like Lourdes' water in France.

[4890] See The experiments of Masaru Emoto with emotional imprinting of water, 2018 (11 pages).

this 8 ounce Indigo water. This is about a one month supply of structured water". For $35. By the way, it doesn't mention if it's drinking water or shower water!

The delirium continues with the structured water of **Rustum Roy** (American). Structured water is said to be an antibiotic: "*One molecule of structured water in 100 million molecules of drinking water can destroy all germs present in a wound. The American army has used this water in Iraq and Afghanistan. Obama uses structured water to wash his hands*". Verification made, the only example that can be found is the healing of a foot wound and it's water associated with money[4891]. And how can we restructure water, so to speak? Simple: by heating it, with vortexes, magnetic fields, music, the force of thought, "frequencies" or minerals!

Figure 979: a structured water scam source..

The concept of wormholes comes from the astronomer **Nicolaï Kosyrev** (1905-1983) who discovered lunar volcanism and the biologist **Rupert Sheldrake** (1942), who became an expert in telepathy. This led to **Vodaflor**'s Voda vortexors which generate vortexes in water to structure it with models ranging from 936€ to 3,300€ depending on the desired water structuring rate.

More recently, the discourse around the benefits of water in homeopathy was renewed with the integration of quantum electrodynamics as an explanatory feature. Why not, since almost nobody can understand anything about it, except the few physicists in this domain[4892]. Not to mention the lack of experimental protocols to verify anything. Again, we are confronting a fake science because it cannot be refuted[4893]!

The structured water business has evolved a little. Instead of selling structured water, some companies are now selling bottles that create this structured water with regular water. It is just a bottle, or sometimes contains a blender. Gullibles can buy it for about $60 on Amazon (Figure 980).

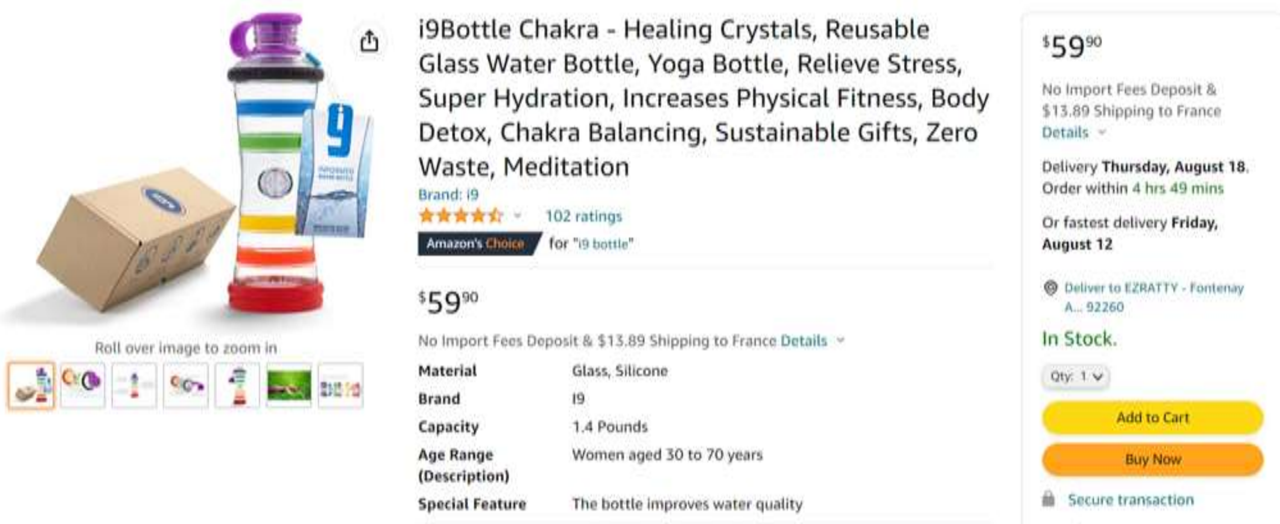


Figure 980: you can buy plastic bottles that will structure your drinking water. It is even not a thermos!

To conclude this part before moving on to the most beautiful scams of pseudo-quantum medicine, let us recall that there is a fine line between low-level science and its high-level interpretation, especially when it is then exploited by unscrupulous entrepreneurs.

[4891] In Ultradilute Ag-Aquasols with extraordinary bactericidal properties : the role of the system Ag-O-H2O, 2006 (13 pages). Rustum Roy is also the author of The Structure Of Liquid Water; Novel Insights From Materials Research; Potential Relevance To Homeopathy by Rustum Roy, 2009 (33 pages).

[4892] See Explaining Homeopathy With Quantum Electrodynamics by Antonio Manzalini and Bruno Galeazzi, 2018.

[4893] Fortunately, some scientists address this nonsense, such as L'homéopathie confrontée à la physique by Alain Bonnier, 2014 (34 pages), which dismantles homeopathy in a very didactic way, relying in particular on Planck's constant.

And we are not done finding more of the same such as some weird quantum behavior of water in carbon nanotubes[4894], superconductivity in the brain[4895] or other elucubrations on quantum cognition[4896]. This will undoubtedly fuel new waves of quantum mysticism!

Quantum medicine

As Wikipedia's quantum healing page on quantum medicine points out, this discipline misuses the jargon of quantum physics to make people believe in magical cures for certain pathologies that traditional medicine, well or badly practiced, cannot treat properly[4897]. The phenomenon is already over a decade old.

Method for detecting false sciences

The methods used to promote false quantum science in health (and in general for that matter) are easily detectable to an educated person, or just with some common sense:

- It starts with some **scientific statement** associating very quickly humanities and biology and making approximate shortcuts on quantum physics.
- The solutions are being promoted with some **esoteric jargon** using unprecise terms like wave, matter, vibration, vortex and energy[4898].
- When they exist, **tests are performed with small samples** that are not statistically representative. The arguments are often based on non-verifiable anecdotes. The miraculous healings observed in Lourdes, France, are even better documented and, moreover, as probable as those occurring in the hospital environment[4899], i.e., between 1/350,000 and 1/100,000 cases.
- Many specialists sell various, rather expensive, **healing materials or devices**, not considered as medical devices, and whose effectiveness is clearly related to the placebo effect.
- These solution’s marketing target **vulnerable people** (sick, elderly, etc.). It can be seen in the media used for advertising it.
- The **vague side of the pathologies covered**. Some are related to pain management or to what can be treated by placebo effect, such as psychonomy [4900]. Others target all the major pathologies of the moment: chronic diseases, cancers and in some cases even neurodegenerative diseases.
- **Extended resumes** with impressive diplomas and scientific guarantees to be taken with a grain of salt for many quantum medicine specialists. There are even "diploma mills" in the USA, where you can buy a doctorate in medicine or another junk discipline at a reasonable price. A bit like in the late Trump University.

[4894] See Evidence of a new quantum state of nano-confined water by G. F. Reiter et al, 2011 (5 pages).

[4895] See Possible superconductivity in brain by P. Mikheenko, 2018 (10 pages).

[4896] See What is quantum cognition? Physics theory could predict human behavior by Nicoletta Lanese, January 2020.

[4897] These methods are also well described in Richard Monvoisin's Quantox - Ideological Misuses of Quantum Mechanics, published in 2013 (in French).

[4898] You find a marvelous example with the Quantum Field Medicine web site that consolidates all these fancy alternative quantum medicines, mostly all based on placebo effect. You have consciousness awareness techniques, acupuncture, homeopathy, electro-magnetic resonance, Timewaver (another electrical product scam), color and light therapy and sound/music therapy.

[4899] See Miracles de Lourdes, Charlatans.info, March 2022.

[4900] Which is yet another false science associating mind and body.

- Rare **scientific publications** and when they exist, rarely published in peer-reviewed journals, knowing that this validation is already not enough to be a guarantee of seriousness. These therefore become "private" publications. Or it can't be falsified, like with this paper on quantum immortality that is based on a mathematical approach the many-worlds interpretation[4901].
- Some **conspiracy theories** about the pharmaceutical companies lobbying and other healthcare professionals who will do anything to prevent alternative solutions from emerging.

Nonetheless, there are positive comments from readers of these books that show that the market for gogos is a thriving one. It takes place in a context of loss of confidence in politics, media and science and the development of many conspiracy theories, fueled by the fluidity of the Internet and social networks.

Quantum medicine marketing

Let's review some of the reference books that promote this curious quantum medicine.

Quantum Healing by Deepak Chopra (1988) seems to be foundational. It comes from a former endocrinologist. He became an Ayurvedic practitioner, coming from traditional Indian medicine. According to him, quantum thinking explains some cases of psychosomatic healings that resemble self-healing. The author is a star in the field, especially in India and the USA, with a total book sale of over 10 million copies and a personal fortune estimated at over $80M[4902] (Figure 981). The content of his works is of course quite weak scientific speaking, especially when he deals with quantum physics, mostly in metaphorical terms[4903]. Of course, all of this is plain nonsense and has been widely debunked[4904]. The author himself admitted that his work was “parabolic”.

Amit Goswami's **The Quantum Doctor** (2004) is along the lines of Deepak Chopra's theories. The author is an Indo-American physics teacher who practiced in Oregon between 1968 and 1997, but in nuclear physics. He defines himself as a quantum activist who even has his own Quantum University which seems to be to healthcare what Trump University was to business schools.

According to him, quantum activism through consciousness can save civilization. He also demonstrates scientifically (!) the existence of God by building upon Deepak Chopra's consciousness of the Universe thesis. In his work, he explains the therapeutic effectiveness of “integral medicine” which combines allopathic medicine and more or less soft, alternative and traditional medicines, particularly Indian and Chinese. But god's existence can also be proven with some laser beams[4905]!

[4901] See Theoretical Quantum Immortality and its Mathematical Authority by Ce Han, February 2021 (8 pages).

[4902] See Alternative medicine is not medicine by Joel Gottsegen, Stanford Daily, October 2014.

[4903] On this subject, I watched the enlightening debate between Deepak Chopra and Richard Dawkins (Mexico, 2013, 1h13) which highlights the difficulty of reconciling Chopra's emotional and symbolic approach with Dawkins' rationalist and scientific approach. At one point, the debate focuses on the supposed Universe intelligence that exists according to Chopra and at all levels, from elementary particles to the entire Universe. While this makes no sense to Dawkins beyond biological beings with brains, or computers imitating them. It is a homothetic debate with the link between consciousness and the pathologies that consciousness would or would not necessarily control. The other interesting part of this debate concerns the notion of quantum leap on the appearance of language or certain biological evolutions that are a view of the mind for Richard Dawkins. The latter even denounces Chopra's “deliberate obscurantism”. For Richard Dawkins, consciousness is explained or will be explained by neuroscience and certainly not by Deepak Chopra's meta-consciousness galimatias.

[4904] Chopra's discourse has been thoroughly debunked in Problems of Deepak Chopra's discourse: a metalinguistic analysis of "Quantum Healing" by Caderno Brasileiro de Ensino de Física, December 2021 (29 pages). He was also awarded the “Ig Nobel prize” in 1998 “*for his unique interpretation of quantum physics as it applies to life, liberty, and the pursuit of economic happiness*”.

[4905] See Interference of Two Independent Laser Beams – Scientific Evidence of God by Henok Tadesse, December 2021 (5 pages). Published on viXra!

The scientific content of the book fits on a tiny postage stamp. It looks even like a giant quantum joke. The idea is the following: your organs are born in good health. A time passes, like a qubit would become after a Hadamard gate, it becomes superposed in good and bad health. Then, with the strength of your consciousness, you could provoke a quantum wave function collapse of your organs into the health version. That simple! It’s a scam version of this poor Schrödinger’s cat.

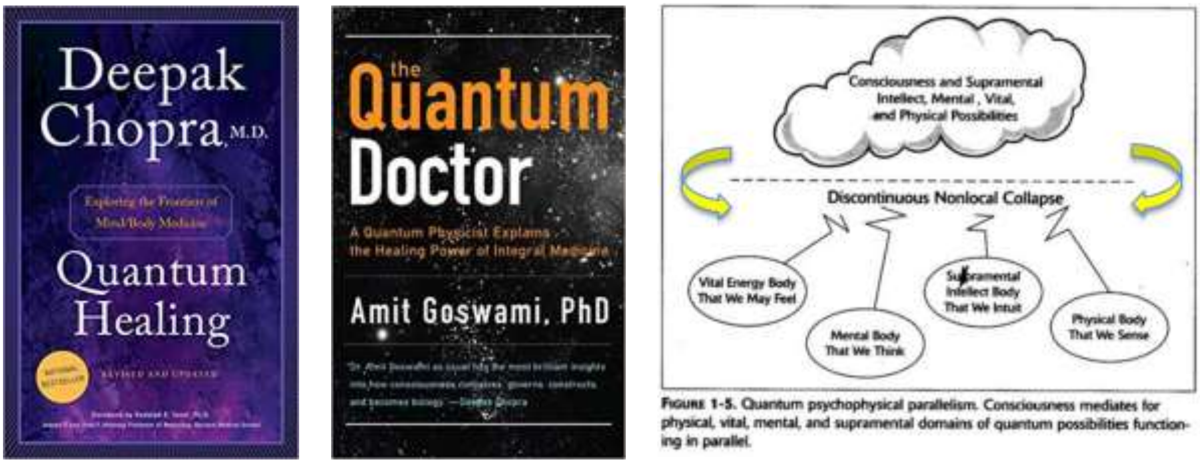


Figure 981: Deepak Chopra and Amit Goswami are promoting a quantum medicine with no scientific content. At best, it is a placebo.

The work also seeks to explain the effects and precepts of oriental medicines (chakras, reincarnation, ayurvedic medicine, acupuncture)[4906]. Here are a few selected excepts with the "*morphogenetic fields of the vital body*", "*when the mind creates the disease, sometimes healing is impossible to achieve on the mind level. One must then make a quantum leap to the supramental to heal*" or "*quantum collapse is also fundamentally non-local. Therefore, the nonlocality of healing, as in healing through prayer, finds a clear explanation within the framework of quantum thinking*”. With quantum entanglement, one can relate everything to everything and explain everything.

Amit Goswami mentions distant healings through prayer by referring to an experiment by physicist **Randolph Byrd** in 1988. The statistical representation was very weak with 6 healings out of 26 patients of not well specified cardiac pathologies. It may not surprise you to find out that it was demonstrated that prayers did not have any large-scale effects[4907].

He also quotes the telepathy experiment of **Jacobo Grinberg-Zylberbaum** (Mexico)[4908]. It involved measuring EEG waves on a participant to assess the impact on him of a flash of light arriving on one of the participants, both of whom were in Faraday cages. The experiment was repeated later between 2000 and 2004 using MRI[4909].

A small technical detail: there cannot be any radio waves transmission between the participants who are in Faraday cages, no photon either, nor particles with a common history in the brain of the participants.

Others have a slightly more scientific view of the quantum nature of consciousness, such as **Ervin Laszlo**, even if the latter relies a bit too much on quantum entanglement in his explanations[4910].

Other pseudo-scientists promote fancy theories related to the so-called quantum medicine.

[4906] Illustration source: Messengers and Messages-then, now, and yet to come (15 pages).

[4907] See Studies on intercessory prayer, Wikipedia.

[4908] Documented in The Einstein-Podolsky-Rosen Paradox in the Brain: The Transferred Potential, 1994 (7 pages).

[4909] See details and results.

[4910] In Why Your Brain Is A Quantum Computer, 2010. This thesis is partly deconstructed in The Myth of Quantum Consciousness, 2002 (19 pages), although it is an earlier work.

James Oschman (USA) promotes a concept of life energy, based on electric currents and water related quantum phenomenon. He invented the concept of perineural brain cells, which are obviously only the glial cells that surround neurons, but with a different name and which generate energy that goes to the hands[4911].

Kiran Schmidt is a German who does "information medicine". He also promotes strange machines that are supposed to cure everything, especially under the brand **Inergetix CoRe**.

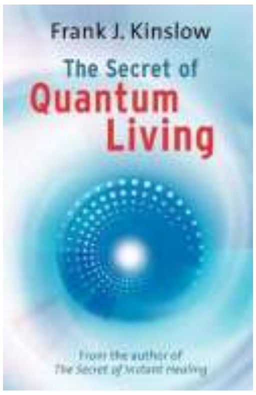
Nassim Haramein deals with the energy of creation and also water memory. He is selling fancy products through his Resonance Science Foundation. The starting point? Some work on his unified field theory, an old Holy Grail of fundamental physics[4912]. This scientist thinks he has discovered an infinite source of energy. Of course, none of the work of this "scientist" was validated by his peers.

This guru markets ARK crystals, which are magical crystals that heal or improve the performance of athletes (Figure 982). They even published a study on how to improve athlete performances. It used a double-blind method with a placebo effect for half of the test subjects. Given the study involved only 10 athletes, 5 men and 5 women, with progress of about 10%, thus within the margin of error of the sample. The study was done by the Energy Medicine Research Institute laboratory, versed in studies of fancy products such as LifeWave placebos marketed in a Tupperware-style pyramidal model.

Figure 982: there's no science in ARK crystals, it is a scam.

These crystals would also help accelerate plant growth! Prices range from 277€ to 1,850€. This is part of a trend in the sale of magic crystals that dates from a few years ago and where the offer is plethoric[4913].

Frank J. Kinslow's **The Secret of Quantum Healing** (2011) introduces the notion of "Quantum Training", a "*scientific, fast and effective* method that *reduces pain and promotes healing*". In a few words, it is about having your consciousness send vibratory waves to your organs to heal them. By the play of interferences, they will cancel the evil.



Another Schrödinger's cat trick with the application of the quantum mechanics of the pico (elementary particles) to the macro (the organs). It is mainly aimed at physical and emotional pain. It is a variant of meditation. It should be avoided for the treatment of hypothyroidism or about anything else by the way!

This kind of work has the particularity of always being very vague on the notion of pathology treated, especially if a pseudo-medical apparatus is involved, as is the case here. Even if the "Quantum Training" is supposed to work remotely.

[4911] He is the author of Energy Medicine, James L. Oschman, 2000.

[4912] His list of scientific publications deals with neutrons and protons. A part of the articles have been published in the journal Neuro Quantology which is not considered as being serious and whose review committee does not include any scientist in quantum physics or neuroscience. This publication process is known and exists in other fields such as medicine.

[4913] See Dark crystals: the brutal reality behind a booming wellness craze by Tess McClure dans The Guardian, September 2019, A Cynic's Search for the Truth About Healing Crystals by Katherine Gillespie in Vice, September 2017 and The Sickening Business of Wellness by Yvette d'Entremont, December 2016. Those guys also promote the water memory theory. See Does Water Have Memory?, ArkCrystals, July 2021.

Quantum Intronic Medicine from Christian Daniel Assoun deals with quantum biology. It is a form of epigenetic treatise describing DNA memory by quantum mechanics. According to him, "*WATER is the first quantum liquid: its current state is liquid* whereas *its state should be gaseous"*. This book describes the presence of a third DNA catenary in the form of physical plasma (hydrogen)[4914]. He states that his "*work is currently focused on the INTRONIC parts which represent 95% of our DNA and which are unfairly classified as silent or even useless"*. Intronic is used in the sense of DNA "introns", the part of DNA genes that is transcribed into RNA when the genes are expressed.

These are eliminated during splicing which generates mature RNA that will then be used in the ribosomes to make proteins. In fact, introns represent only 25% of human DNA. The rest, about 73%, corresponds to sequences that are effectively non-coding in the DNA of our chromosomes, but whose role in the regulation of genes is progressively revealed with research. Exons, the coding part of genes, represent 1.5% of human DNA (source).

Christian Daniel Assoun believes that DNA could be strengthened with "the *help of new tetravalent elements such as Germanium or Silicon (reverse optoquantic properties)*". Why germanium and silicon? Because they are in the same column of Mendeleev's table as carbon with four free electrons. This is a good idea for creating extraterrestrial life. So why didn't life on Earth use silicon, which is as abundant as carbon? One of the reasons is that silicon oxide (SiO_2) is inert and solid while carbon oxides (CO, CO_2) are gaseous and therefore more easily recombinable with other atoms and molecules. Also, carbon is more abundant than silicon on the surface of the Earth.

Christian Daniel Assoun is also the founder of the **Glycan Group**, in 1996, a company selling organic silicon for various uses and notably as a food supplement. Their subsidiary Glycan Pharma was struck off the commercial register in 2012. The company is in competition with Silicium Espana, a company linked to Loic Le Ribault, who died in 2007, who was also passionate about organic silicon. The two companies had a legal dispute in 2011 over the use of the G5 trademark.

At last, you also can count on the many books on Transurfing by **Vadim Zeland** who introduces himself as a physicist. This quantum model of personal development is based on the idea that "*When the parameters of mental energy change, the organism moves to another lifeline. When the parameters of mental energy change, the organism moves towards another life line*". So be it!

Scalar wave generators

The best of the quantum medicine scams are the **scalar wave generators**. These are electromagnetic waves associating a supposedly horizontally polarized electro-magnetic wave and another vertically polarized wave of the same frequency but 90° or a quarter wavelength out of phase.

Scalar waves were initially promoted by a certain **Thomas Bearden** in the USA as well as by the Russian **Sergei Koltsov** with his Functional State Correctors (CEF[4915]). Bearden explains this in a 1991 interview. He had also invented a **MEG** (Motionless Electromagnetic Generator) capable of extracting free energy from the vacuum and thus, of generating more energy than it consumed. A product that has of course never been commercialized.

[4914] Ebook downloadable here. It is also documented in The 3rd Strand (or 3rd Catenary) of DNA by the same author and which dates from 2011/2012.

[4915] Watch this video Functional State Correctors (FCS) - Koltsov Plates, 2014 (55 minutes) which is a good digest of any scientific theory.

Scalar waves were also promoted by a German scientist, **Konstantin Meyl**, with a paper that had to later be retracted[4916]. The general public propaganda on scalar waves is a big fantasy and always linked to alternative medicine literature. These waves would come from the Sun with neutrinos and have no energy loss over distance. The brain is supposed to produce and sense scalar waves with its own interferometer. It would explain telepathy and other paranormal effects. Well well.

Scalar waves would also make it possible to treat diabetes (I or II? Who cares...), kidney stones, Parkinson's disease, heart attacks, osteoarthritis, cancer and also aging. As for type I diabetes, which is linked to the autoimmune destruction of beta cells of the islets of Langherans in the pancreas, it is not clear how waves of any kind would bring dead cells back to life. The proposed solution?

Figure 983: scalar waves cost a lot and do nothing.

Scalar wave generators such as the **INDEL** at 8,820€ (Figure 983). Given its price, it targets professionals in a kind of Ponzi model. This generator produces a scalar wave field with a voltage of 2V. It also includes a music modulation accessory for therapy practices and wellness centers. It is also available at **QuWave**.

You can also (not safely) rely on the **ETHX-SCIO Biofeedback** from William Nelson, which combines global therapies and advanced quantum physics (in Figure 984). The device scans the body on 10,400 different frequencies to detect many pathologies. It then rebalances the body's energy with quantum biofeedback. The toy also runs 200 biofeedback therapies with the world's largest health software that integrates Western and Eastern philosophies[4917]. The EPFX-SCIO includes a wave diffuser box, connected to the patient with sensors attached to his ankles, wrists and skull. One could almost do both an EEG and an ECG with it! All this for getting some placebo!

Figure 984: SCIO Biofeedback is not better.

In the scam devices category, you also find the **Healy** and its bioresonance features using some electrodes and supposed to cure many illnesses[4918]. At best, it can be a temporary pain killer. Another device, the **TimeWaver**, is based on "quantum field theory" from a certain Burkhard Heim (1925 - 2001, German) on the 12-dimensional composition of the universe where "*the light quantum effect communicates mainly with the Global Information field (GIF) i.e. at a nonenergetic, non-phenomenal and therefore more causative level*". It looks like a biofeedback device similar to the one above. Burkhard Heim did try to unify all quantum theories but he was neither a Dirac or a Feynman[4919]! The TimeWaver site also mentions of **Kozyrev mirrors** using cylindrical aluminum sheets that were used for extrasensory perception experiments in Russia. But it doesn't seem to be involved in the TimeWaver device.

[4916] See "Way out there" paper claiming to merge physics and biology retracted, RetractedWatch 2013 and Scalar Wave Transponder device by Konstantin Meyl, 2005 (70 pages).

[4917] See How one man's invention is part of a growing worldwide scam that snares the desperate ill.

[4918] See A Skeptical Look at the Healy "Bioresonance" Device by Stephen Barrett, July 2020.

[4919] See TimeWaver System website. They hopefully have a: « *Disclaimer: Science and conventional medicine does not acknowledge the existence of information fields their medical and other important TimeWaver systems and their applications due to lack of scientific evidence. The said application is based on, treatment options, experiences and anecdotal reports from the practice*".

Other various Russian 'quantum' scientists, dead or alive, are frequently used in support of these scam devices like Nikolai Kozyrev, Vlail Kaznacheev or Alexander Trofimov. When you look at their biographies online, you quickly find that they were not at all mainstream quantum scientists. This is all full of esoterism, not science.

Quantum medallions

Quantum medallions for smartphones have become commonplace for several years and target another phobia, electromagnetic waves and 5G. This is the case of **Quantum Science**'s Quantum Shield medallions (on [Amazon]() and [Alibaba]()). One also finds some in the form of USB keys **5G BioShield** which contain a "*quantum holographic catalyst*" (Figure 985).

Figure 985: quantum medallions and 5G quantum keys are fancy gadgets for the gullible. That's a huge market!

It is obviously a huge quantum bullshit of the first kind. It is accompanied by a scientific justification that is not worth a lot of money[4920]. The American FTC has flagged these products as vulgar scams[4921]. It was even later discovered that some of these medallions were radioactive due to their component metals. And worn on a long period of time, they could actually be dangerous[4922].

In the field of wacky quantum devices, let's finish with the **Quantum 5 Ozone Generator** using Neos Technology from the **Longevity Resources** ([sources]()). It uses a quartz electrode (Figure 986). It is supposed to help purify indoor air. There is one major drawback: ozone can also be toxic to the human body and cause respiratory problems. It can also affect plants health. In short, quantum medicine may one day emerge in the wake of scientific discoveries, but the ones proposed today is for the time being full blown charlatanism.

Figure 986: a quantum ozone generator to purify indoor air. It may work but it is not quantum.

They have the advantage of generating at least a placebo effect for users and filling the wallets of their promoters. Except that this can be dangerous if the placebo effect is used instead of a traditional treatment that is essential to stay alive.

I will not, however, trash all the techniques and approaches mentioned here. Some may make sense, even though there is still a lack of both a scientific corpus and more solid evidence to support them. But most are fake sciences and are quite easily detectable.

[4920] See ["Aton" True Cell, Atom and Particle Concept]() by Ilija Lakicevic, 2019 (8 pages).

[4921] See [Cell Phone Radiation Scams](), 2011.

[4922] See [Anti-5G "quantum pendants" are radioactive]() by Jennifer Ouellette, ArsTechnica, December 2021.

Quantum skin care

I discovered the scam category of **quantum cosmetics products**, coming mainly from China that was mentioned in a 2022 Rand Corporation on China and the USA investments in quantum technologies. In it, Jian-Wei Pan is said to have criticized companies claiming to sell "quantum skin care" products in China[4923].

Let's make a roundup of these scams. We have for example a Quantum Health Super Lysine+ Cold-Stick sold on Amazon, Energecia Quantum Beauty sells quantacosmetics bullshit stuff, BioEqua sells energized nanospray skin care, probably with stirred magic water, Age Well Fundamentals sells Phyto5 quantum energy facial skin care that balances the energy vitals and is made in Switzerland (but not at ETH Zurich), Quantum Botanika, Halcyon Skin Care, Ratzilla Cosme, Quantum Aestetics, Quantum Life and Igesto. In the same vein, Boiron is selling X-ray homeopathy products for curing skin rash. You read it well!

Quantum management

Quantum management is a new and fashionable practice that seeks to draw inspiration from the general principles of quantum mechanics. Its practitioners are frequently followers of more or less occult sciences who have converted to target corporate markets that are more financially attractive than consumer markets. The vulnerability of educated executives and managers to the most outlandish proposals is always amazing.

However, we can indeed identify many analogies between quantum physics and management in the broadest sense of the term. For this purpose, I have pushed the envelope and reused advanced quantum physics fundamentals and applied it to your usual business life. Any resemblance with a real-life situation would be totally fortuitous or entirely intentional, as you will guess[4924]. As a warning, I must precise that all of this is not serious at all. It is a way to make fun of many things, both the various gurus using quantum physics in scams and, also, life in the enterprise (Figure 987).

Quantization means that certain physical values can only be very precise, discontinuous and not arbitrary, like the energy levels of a hydrogen atom. After all, an employee is just a cell in a spreadsheet. He's there one day and gone the next. Workforce management is indeed quantum. A company's workforce at a given time is a discrete integer number.

But if we average it over a period of time, taking into account departures during the period, part-time employees, fixed-term contracts, apprenticeships, subcontractors and people whose real activity we are not sure of, it is no longer an integer but a number of FTEs (full time employees) or FTEs (full time equivalents) that is at least a sum of fractions. Fortunately, it is never a complex number and one escapes Hilbert's spaces to represent them.

Quantization also manifests itself with sand bagging, when a sales manager is distributing his own sales goals to his team by adding a quantum margin of safety. The last link in the chain, the unfortunate poor salesperson, will be assigned a goal that is greater than that of all the layers of management above. Only certain layers of sales management have this flexibility.

[4923] See An Assessment of the U.S. and Chinese Industrial Bases in Quantum Technology by Edward Parker, Rand Corporation, February 2022 (140 pages).

[4924] I didn't rely on the proposal in Toward a Quantum Theory of Humor by Liane Gabora and Kirsty Kitto, January 2017 (10 pages) that is quite poor in its scientific and mathematical content.

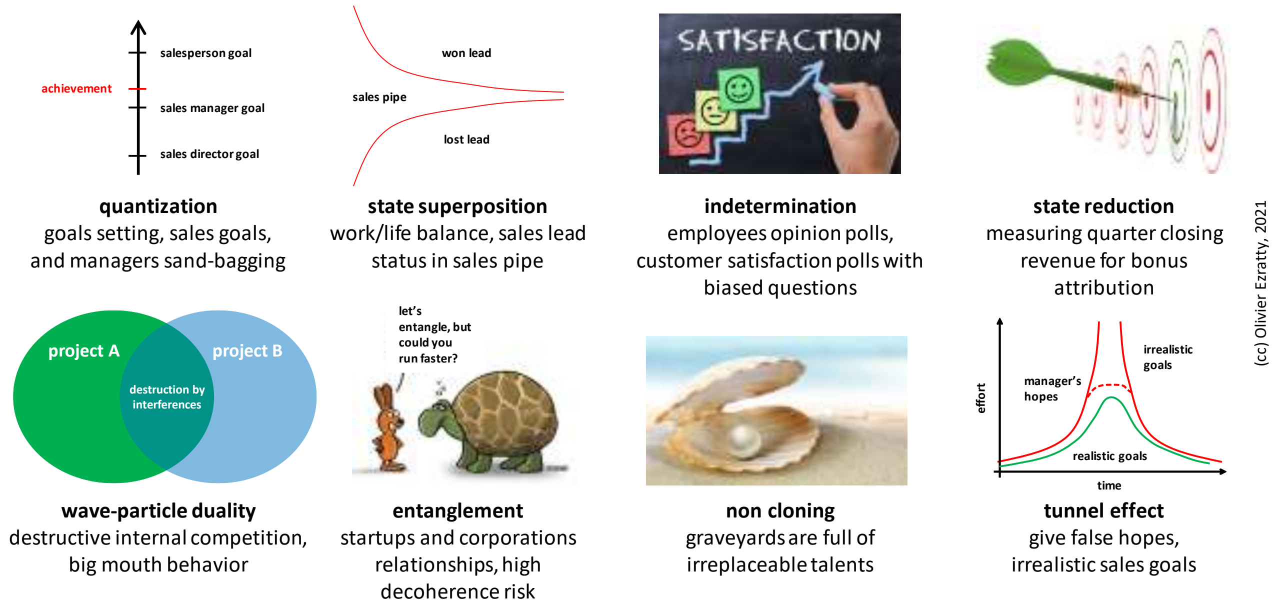


Figure 987: my useless framework for quantum management. (cc) Olivier Ezratty, 2022.

The end result is that salespersons become Rydberg state atoms: they are excited with a very high level of energy and they sometimes burn out. This system is designed so that the base salesperson does not reach his or her objective and is penalized on the bonus side, unlike managers above him or her. Particularly if you want to fire him or her. Judgment about individuals is also subject to quantization. A person is often smart or nice, or a total moron. Personal judgments are rarely nuanced in grey. Yet, in a purist application of quantum physics, this kind of judgment should be a more vague and subtle wave function, until you measure it during a stressful experience.

The top of quantization? Those nasty Internet popups where the given choice is "OK" or "Later". For the quantum measurement guru, it is a real-life example of an exaggerated POVM (see Glossary).

Superposition is very common in business. For example, thanks to smartphones and other laptops, employees are kept both at work and in their personal lives all day long. It can also manifest itself in regulatory compliance, which is variable geometry in many companies. And then, of course, in the application of the company values defined in Powerpoint slides and rehashed by managers or the HR department. States superposition also manifests itself in the evaluation of leads that are closed or not in a sales pipeline. They are usually assigned a closing rate which is an amplitude and phase $|\psi\rangle$ until it is known whether the deal is lost or won, which is like the wave-packet collapse happening with quantum measurement, on a basis state $|0\rangle$ or $|1\rangle$. This collapse also occurs if an external event creates a lead quantum state decoherence. For example, a competitor who wins the deal under the nose of the salesman. This quantum analogy, however, will not help you improve your sales pipe closing rate.

Indeterminacy works with the measurement of employee satisfaction, where the measuring tool always influences the quantity to be measured. This is true as well in the questions asked in opinion polls, which are often oriented. More generally, the measurement of any parameter in a company by a consulting firm like McKinsey, particularly during an audit, will probably lead to changes in the measured quantities (e.g., downsizing, management change, reorganization and the likes). You just hope that your enterprise won't become a planar wave afterwards.

One variation of Heisenberg's principle of indeterminacy is that one cannot accurately measure both the position and velocity of a particle. The analogy in business would be the observation of a growing startup: by the time one understands where it is at a given moment, it has already changed its situation (headquarters, staff, CEO, turnover, M&A, company name, product, done a pivot). This is why it takes an infinite amount of energy to create an up-to-date startup base in one country or worldwide, even with only quantum technologies startups. So, thank you Crunchbase for the effort!

Measurement is in line with the history of quantization when measuring revenue at the end of a fiscal quarter. In this case, one is obliged to provide numbers and not to rely on some closing rates fuzzy logic. If only to determine the bonuses of sales representatives. Otherwise, Bill Gates said loud and clear in 1997 that "*bad news should travel fast in efficient companies*". But not too fast my dear, otherwise you'll get fired. That's what is called a non self-destructive measurement.

Wave-particle duality manifests itself with real people in companies who work on competing projects and happily annihilate each other. It is the phenomenon of interference linked to the waveform aspect of each and every projects! You also have the loudmouthed managers facing their teams (thus, in the state of a solid particles) who turn into wipes in front of their own management (thus, in the state of very low-energy waves).

This behavioral duality is also often observed with irascible managers who become docile sheep once at home, or who fail to properly educate their children. Can a trendy startup Chief Happiness Officer be quantum? In any case, this person must fight on a daily basis against a universal phenomenon: a good number of passions quickly fade with time, such as the amplitude of a Rabi oscillation, which is commonly observed in quantum physics and is related to superconducting qubit decoherence (Figure 988).

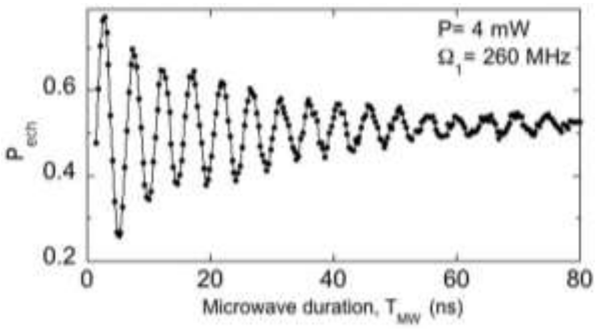


Figure 988: the Rabi oscillation of your motivation over time.

The Doppler effect also allows to indirectly put an end to a messed-up project with light, for example, via a well-managed leak in the media. Remember Theranos?

Entanglement applies to startups that are integrated into the open innovation programs of large companies. Everything goes well until the appearance of decoherence between the startup and the corporation! I create a product and you want a customized solution, I need speed and you're too slow, etc!

Entanglement also occurs with the teleportation of rumors faster than light. It is also known that the coherence time of qubits is linked to their good physical, magnetic and vacuum insulation, often at very low temperature, in order to avoid any external disturbance. This is the opposite of the open spaces in companies where employees are crammed together! It has long been claimed that open spaces improve teamwork, whereas their main purpose is to compress real estate costs!

No-Cloning Theorem says that it is impossible to identically clone the state of a qubit or quantum, has an application in business life with all those people who are believed to be irreplaceable until the day they leave or die. The theorem applies in particular when the departing manager is not replaced and whose role is then distributed among several existing managers, a bit like a quantum error correction with ancilla qubits and projective measurements. The theorem also works with successful entrepreneurs who find it difficult to replicate a success from one area to another.

Tunnel effect makes it possible to implement change management. It consists in presenting a wonderful future situation and making people forget the difficulties to get there. The principle could also be adopted by the Gartner Group with its famous innovation adoption cycle curves ("hype cycle"), as some technologies do not necessarily pass through the valley of death, as was the case for smartphones (Figure 989). It had benefited from the reality distortion field of a certain Steve Jobs, a great adept of quantum management principles. By the way, the trajectory Apple-Next-Apple was a great application of the tunnel effect, Next being a relative failure while both Apple were successes.

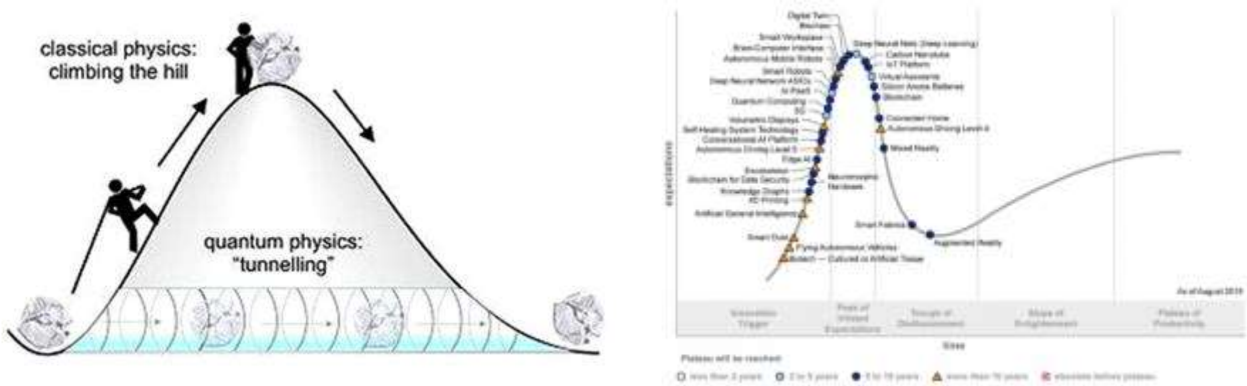


Figure 989: the quantum tunnel effect and hype effects. Sometimes, hype is so strong that it creates a pass-through from hype to success without a valley of death.

Superconductivity is linked to meeting rooms. Employees and managers are conditioned to be even-spin bozos, who can be assembled in meeting rooms or covid-zooms. Organizational superconductivity also avoids resistance to change. You freeze employees and their resistance to change disappears. Which is a bit paradoxical because once frozen, you are as solid as a rock, and defrosting is not obvious. If we take the principles of Deepak Chopra's quantum pseudo-medicine, a company is in a superposed state between a healthy leader and a declining star. The strength of leadership should theoretically allow the wave packet function of the company's quantum state to collapse in the healthy leader state. In real life, this collapse is tricky to achieve and companies simply collapse. The processes that lead the company to find itself in a declining situation are most often irreversible and linked to a slow decoherence with the environment, competitors and customers who have not waited to adapt. Corporate life is not a reversible quantum gate nor any sort of linear algebra. It is mostly nonlinear. Try, for example, to turn Nokia into the leader of Android smartphones!

Universal Gates Quantum Computing has a beautiful analogy in the life of companies with the management of calls for tenders such as those for communication agencies. The responses of candidate agencies are superposed states of a quantum register.

They undergo a simultaneous evaluation process, as in an oracle-based quantum algorithm. In the end, only one offer emerges: the winner. But during this process, there may be some quantum entanglement affecting the winner final proposal. Translation: the elements of certain answers will magically appear in the winner's answer. Again, perhaps via the enterprise quantum tunnel effect.

Finally, let us mention this other universal principle, the very famous **quantum teleportation** of human stupidity to large sections of the company or in the population. It uses superdense falsehoods encoding. And contrarily to actual teleportation, it travels faster than light. It is so fast that it is the only plausible explanation.

All of these analogies are amusing but not very useful for improving management. Even if its scientific dimension is more than questionable, parody is finally an interesting form of pedagogy!

Other exaggerations

There are many startups or ventures surfing on the quantum technology wave with various intentions. Some are just quantum startups with fluffy claims and others have only quantum in their name but nothing else.

One classical exaggeration comes with making cross-predictions between one digital trend and quantum computing. This one predicting that we'll have **quantum digital twins** is based on the usual wrong premise that quantum computing is made for compiling huge swaths of data[4925].

Dark Star Quantum Lab (2020, USA) introduces itself as a contract defense and space research company covering applied quantum physics and quantum information science (QIS). They develop tons of quantum stuff: quantum software and quantum emulation solutions, unspecified hardware, a 'Sentinel' mobile phone including a QRNG.

They also claim to have developed a Qloud high-frequency trading, a Qoin (quantum-secured cryptocurrency), a BloQchain (quantum-secured blockchain working with Qoin), a use-case of Nash embedding to create error-free qubits and, also, some Star Trek Tricorder fancy stuff.

This laundry list of things is not credible. And they don't seem to have any real defense customer. Looks like it's not really serious[4926].

Also quite weird is this **Quantum_AI Group Of Companies** with its 15 branches dealing with aerospace, artificial intelligence, naval, finance, energy, automotive, electronics and... quantum computing. They develop, take it or leave it: Nano-Flux, a range of flux-qubits superconducting computers, Q-optic, the most powerful optical qubits quantum computer, BEC, the fastest Bose-Einstein condensate based trapped ions computer (seems they mixed some things here) and also SSL, a solid-state quantum computer and Infinity-Q a high-performance heavy load quantum computer. Interestingly, these 4 ranges of systems have respectively 40, 200, a 1.6 and 128 billion qubits and they look the same in their 3D rendered pictures.

They are supposed to be based in Stanford, Boston, India, Abu Dhabi, Dublin and Tel Aviv. They still have a CEO, a certain Ranobijoy Bhattacharya. If it is not an April's fool, what is it? Some new form of mythomania?

Quantum physics abuse can be found in various other product categories. In China, for example, a so-called **quantum satellite camera** was used to produce high-resolution panoramas. The view presented was that of Shanghai with 195 billion pixels. Practically, the pictures were captured from the top of a skyscraper - there is no shortage of them in Shanghai - and not by any satellite. It used conventional high-resolution cameras that have nothing more quantum than the very classic photoelectric effect used in CMOS sensors to transform photons into electric current. The information is totally bogus and was only used to generate buzz.

Unfortunately, many media outlets around the world have taken the bait without any doubt [4927].

For its part, a French SME **What-Innove** from the East of France, specialized in renewable energies, claims it is creating an engine that captures energy from vacuum. How does it work? An unlikely mix combining a quantum field generator, the creation of photons from vacuum energy exploiting the Casimir effect, the combination of magnetodynamics and space-time, ambient temperature and pressure superconductors (which would win them a Nobel Prize if it worked), and negative entropy. They just need €2.7M of funding to move ahead!

[4925] See Why we need Quantum Digital Twins by Ian Gordon, Head of Data at Houses of Parliament Restoration & Renewal, January 2021.

[4926] See one scientific publication of their own, which is quite short: How many physical qubits are needed exactly for fault-tolerant quantum computing? by Faisal Shah Khan, Dark Star Quantum Lab, December 2021 (4 pages).

[4927] See 60 seconds over sinoland: quantum satellite camera used to do movable, panoramic photos of Shanghai, December 2018 (video) and Truth Behind Viral 24.9 Billion Pixel Image Taken By Chinese "Quantum Satellite" by Anmol Sachdeva, December 2018 and the Bigpixel website to view the view.

You are also entitled to a beautiful **quantum cooler** from **Chillout Systems** that has only quantum in its name. It uses a compact classic compressor[4928] (Figure 990).

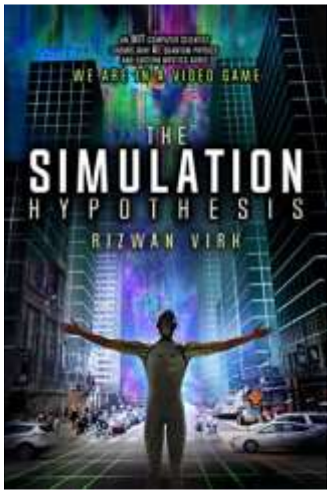
Figure 990: the non-quantum cooler from Chillout.

Other cases extrapolate to the macro scale of quantum phenomena observed at a nano scale. This is the case of **time inversion** with quantum computing, a view of the mind that is linked to the reversible nature of quantum gates but does not mean that one can go back in time scale in macroscopic practice[4929].

We also have equally wild theories willing to **predict the future** with quantum computing. If it is true that quantum computing allows us to evaluate all the solutions of a complex problem, it is reduced to simple problems in view of the complexity of macroscopic life, even if it could be deterministic[4930].

The next step is to consider that we are actually living in a **simulation**.

This is the theory presented in Rizwan Virh's The Simulation Hypothesis. The author presents himself as an MIT Computer Scientist, whereas he is more of an entrepreneur in video games, more accustomed to books on entrepreneurship than on science. This kind of simulation scenario is roughly equivalent to believing in a kind of omnipotent God who controls everything or who created the simulation tool. The question can moreover be recursively implemented: if this creator has developed a simulation tool, who created his universe and isn't this one also a simulation?



Another case that should inspire the utmost caution is that of this curious company **Precog Technologies**, which claims to offer solutions for teleportation, time travel and anti-gravity systems. Miracles one stop shopping!

The company was created to valorize the intellectual property of a certain Anisse Zerouta that is covered in a dubious scientific paper[4931].

Another guy, from **Quanta QB** (South Africa) thinks he has also found a qubit architecture that showcases a miraculous 0% error rate[4932]. Good luck with that!

[4928] See Chillout Systems Quantum Cooler. It is sold for $2199.

[4929] See Arrow of time and its reversal on the IBM quantum computer by G. B. Lesovik et al, 2018 (14 pages) and Does the IBM quantum computer violate the second principle of thermodynamics?, 2019.

[4930] See Interfering trajectories in experimental quantum-enhanced stochastic simulation by Farzad Ghafari et al, 2019 (7 pages).

[4931] See The Seed Theory: Unifying and replacing quantum physics and general relativity with "state physics" by Anisse Zerouta, 2017 (28 pages) which develops a theory of parallel worlds that does not seem to meet the criteria of a scientific publication worthy of the name. Anisse Zerouta is a company manager in Paris born in 1973, first with Elysee Communication (2008-2011) then with Avenir Optique, an optician (2011-*), companies with only one employee, its founder (source). Created in September 2018, Precogtec would have a CTO, a certain François Bissege, who has a PhD in sociology (source) and another employee, Julien Darivel, who has a DUT and worked at PSA. It's bizarre!

[4932] See I made the Quantum Breakthrough, June 2019.

We also saw the first quantum financial scam appear in 2018 with this fake article in the Guardian reporting a quantum computer project for Elon Musk's finance[4933]. The trained eye quickly detects that it is a montage, like this series of **QuantumAI quantum** computers that are nothing more than D-Waves annealers whose logo has just been photoshopped. Second, the article is supposed to come from the Guardian, but not the url (Figure 991)!

Figure 991: do you spot the scam? If not, you need some training.

It quotes a number of scientists from research laboratories around the world, all with Russian names. The article points to **QuantumAI**'s online service which would be able to go around robbing the rich and redistributing the money to the poor. And the site indicates that the startup has Jeff Bezos and Bill Gates as advisors and IBM, Microsoft and OpenAI as partners (Figure 992).

Figure 992: QuantumAI and its financial scam.

There is another, called **Quantum Code**. Obviously, run away! It is in fact a scam designed to rob users of their savings, but in an indirect way. The site offers to create an account by providing its coordinates. These are then resold to unscrupulous companies of shady financial products that exploit leads of prospects easy to fool.

[4933] See Elon Musk to Step Back From Tesla And SpaceX, Jumps on Quantum Computing Financial Tech (not dated).

Other financial scams promising skyrocketing return on investment on mysterious quantum investments in the stock market are now current[4934].

We can also quote **Qubole** which launched its Quantum SQL server, which has nothing quantum[4935]. The **Samsung** Quantum 8K processor launched in 2018 was not particularly quantum either, except via its classical CMOS transistors.

In consumer products goods, we have this washing powder **Quantum Max** of the brand **Finish** from the Reckitt Benckiser group (Figure 993). Then **Quantum American** PQ rolls and Quantum red wine from **Beringer,** a brand from Napa Valley, California. You can even buy a **Qbit troller** for your babies[4936].

Figure 993: anything can be quantum, your washing machine powder, your toilet paper and your wine or beer.

Otherwise, **Quantum Corp** (USA) does nothing quantum and just manages tape storage. The same goes for **Quantum Entanglement Entertainment** (Canada) which, as its name indicates, is in the content market. **Quantum Surgical** (France) makes surgery robots for liver cancer, which have nothing quantum. **QuantumScape** is a solid-state lithium battery manufacturer in the USA. **QuantumSi** created a silicon-based DNA advanced sequencing machine that doesn't seem to use any second-generation quantum technology, **Quantic** Executive MBA has only quantum in name. **Quantum Metric** provides cloud based digital content design software tools. There is even a Chinese company created in 2016 named **Quantum Technologies** (officially "Guandong Technology") that sells some augmented reality product with nothing quantum at all. **Quantum Switch** is not a quantum switch from the physical standpoint. It states that it creates "*A New Era In Quantum Defined Data Centres*" but does just service classical colocation data centers (Figure 994).

At last, **QuantumLeaf** is a cannabis software company servicing the cannabis industry in the USA. **Quantum of the Seas** is a famous cruise ship covering all oceans. Even **McKinsey** got into this game, labelling its data-science consulting and service offering QuantumBlack! And don't be confused between **Quantum Health** which provides healthcare services to company's employees and **Quantum Health** which sells among other things an insect repellant. Hopefully, despite the company name, they don't claim the repellant use a specific quantum effect. It is just branding washing, not more. We're safe!

[4934] That one is a concentrated feat of bullshit: https://secure2.wealthdaily.com/o/web/362894 using the one line sentences tactic to strengthen its messaging. Each and every line would deserve some debunking.

[4935] See Qubole launches Quantum, its serverless database engine by Frederic Lardinois, June 2019.

[4936] See Qbit + All Terrain Velvet Black Stroller, viewed in August 2023.

Figure 994: all those companies chose to use quantum in their name, but they have nothing quantum.
Consolidation: Olivier Ezratty, 2021-2023.

Quantum fake sciences key takeaways

- Quantum physics has been for a while integrated in highly dubious offerings, particularly in the healthcare and energy domains.
- There is a proliferation of gurus and scams-based miracle cures machines for detecting electromagnetic waves or vague energies and restoring your body balance. It is at best a subset of the lucrative placebo effect industry targeting the gullible!
- The shift from some low-level physics studies on water and matter led some scientists to explain consciousness with quantum physics. This form of reductionism is unproven. It's the same with scalar-waves detectors or generators, miraculous healing crystals, structured water and other quantum medallions.
- This part proposes a simple methodology to detect these healthcare related scams, with using some common sense.
- We uncover some other scams in the free energy generation category. These systems are supposed to extract some energy from vacuum when their only actual effect is to pump money out of your wallet.
- Quantum physics is sometimes used in management and marketing. This book offers you a nice in-depth parody of these methodologies.
- At last, we showcase a few companies using quantum in their branding when they have nothing quantum at all to offer.

Conclusion

Updating this book is definitively a crazy task, trying to cover such a broad spectrum of topics, with physics, mathematics, technology, computer science, human science and the rest. It is extreme in complexity and diversity. I try to be a "quantum engineer", keeping a holistic view of the domain. That is what makes this field so interesting. It is full of uncertainties. Too many business-biased people think the uncertainty is about "the market" when it is above all about the underlying science and technology. The advent of useful quantum computers is tied to our ability to control large numbers of entangled quantum objects and it is quite hard to determine the limits there.

I also discovered how understanding classical technologies is so important here. It plays a role in the various enabling technologies needed to build quantum products, particularly quantum computers. But also, in knowing the playing field of classical computing, both hardware and software that runs along quantum computers or is compared to it.

After reading parts or the totality of this book, you may find out that its premise was maybe misleading. "Understanding quantum technologies" is indeed a quest and a journey, but you never reach the destination. There's always something you didn't understand well and need to review again and again. It is true for me as a writer. It takes me a long time to understand how things work, like MBQC, and later, when reviewing my text, I don't get it at all. As an end-user company, you are probably still wondering about the use cases that will make sense for your business and when they will become implementable with some real business benefit. And you have to manage conflicting information.

To write this book over the course of the last six years, I downloaded and compiled thousands of documents, viewed hundreds of hours of conferences and courses on YouTube, and met several hundred researchers and entrepreneurs. The only reassuring thing is that this book pagination is growing linearly over time, not exponentially. Whatever the many productivity tricks that I use in that production, I am still a human!

Like in a thesis, I have here to thank many people and friends who helped me craft this book over my journey in quantum science and technologies. First of all, **Fanny Bouton**, with whom I started this quantum adventure in 2018 and who now runs quantum operations at OVHcloud (Figure 995).

Figure 995: the first quantum scientists we met in 2018, Alain Aspect, now a Nobel prize awardee, Cyril Allouche and Philippe Duluc from Atos, now Eviden, Daniel Esteve and Maud Vinet from CEA.

In chronological order, the first scientists and other people we met back in 2018 were **Alain Aspect** (IOGS), **Daniel Esteve (**CEA Quantronics), **Christian Gamrat (**CEA LIST), **Maud Vinet (**CEA Leti in Grenoble, now at Quobly), **Tristan Meunier (**CNRS Grenoble), **Alexei Tchelnokov** (CEA Grenoble), **Laurent Fulbert (**CEA-Leti Grenoble), **Cyril Allouche** and **Philippe Duluc (**Atos), **Bernard Ourghanlian** and **David Rousset (**Microsoft), **Pat Gumann (**IBM), **Etienne Klein (**CEA), **Christophe Jurczak** and **Zoé Amblard (**Quantonation), **Nicolas Gaude** (Prevision.io) and **Françoise Gruson (**Société Générale).

Then, in 2019, with **Philippe Grangier** (IOGS), **Elham Kashefi (**LIP6 and VeriQloud), **Marc Kaplan (**VeriQloud), **Pascale Senellart (**C2N and Quandela), **Franck Balestro** (UGA, Institut Néel), **Alexia Auffèves (**CNRS), **Matthieu Desjardins** (C12), **Jacqueline Bloch (**C2N), **Iordanis Kerenidis (**CNRS), **Heike Riel** (IBM Zurich) and **Vern Brownell** (then D-Wave CEO).

In 2020, **Artur Ekert** (CQT Singapore), **Patrice Bertet (**CEA SPEC), **Xavier Waintal (**CEA IRIG), **Yvain Thonnart** (CEA LIST), **Rob Whitney** (LPMMC Grenoble), **Damian Markham** (CNRS LIP6 and JFLI in Tokyo), **Robert Whitney** (CNRS LPMMC, also part of the QEI), **Bruno Desruelle (**Muquans), **Georges-Olivier Raymond** and **Antoine Browaeys (**Pasqal), **Théau Peronnin** and **Raphaël Lescanne (**Alice&Bob) as well as **Jeremy O'Brien (**PsiQuantum), **Magdalena Hauser** and **Wolfgang Lechner (**ParityQC), **Roger McKinley** and **Peter Knight** (UK) and the IBM Zurich research teams. I also had discussions with the teams from **Qblox**, **Qilimanjaro**, **Quantum Machines**, **Seeqc**, **Quantum Motion**, **Strangeworks**, **IQM**, **EeroQ**, and **Quantum Brilliance**.

There were these countless discussions with **Jean-Christophe Gougeon** of Bpifrance, **Neil Abroug**, who is since 2021 the coordinator of the quantum strategy in France, as well as **Charles Beigbeder** and **Christophe Jurczak** from Quantonation and Le Lab Quantique, who wrote the foreword of this book, page vii. I should also mention the numerous exchanges related to quantum investments with **Cédric O** and his team, in the French government from 2019 to 2022. He was onboard early on and became its driving force within the government.

The fourth edition in 2021 and fifth edition from 2022 benefited from many contributions and reviewers [4937] [4938]. In 2022 and onwards, I had a chance to discuss about the quantum ecosystem with **Rainer Blatt** (AQT and Munich ecosystem), **Jonathan Home** (ETZ Zurich), **Tommaso Calarco** (Jülich), **Jay Gambetta** and **Jerry Chow** (IBM) as well as with **Simone Severini** (AWS) and **John Preskill** (Caltech).

I also met with countless other quantum physicists and entrepreneurs.

Help for crafting this 6th edition came from **Vincent Pinte-Deregnaucourt** (research consultant), **Jeremy Stevens** and **Cécile M. Perrault** (Alice&Bob), **Neil Abroug** (SGPI), **Philippe Grangier** (IOGS), **ChatGPT** (always available, but trust is optional), **Ekaterina Alekseeva** (Eurodecision), **Jonas Landman** (LIP6, QC Ware), **Nicolas Delfosse** and **Vivien Londe** (Microsoft), **Marie-Elisabeth Campo** (Air & Space Force, France Minister of the Armed Forces), **Yves Colombe** (Infineon), **Cornelius Hempel** (PSI/ETH Zurich), **Davide Boscheto** (ENSTA), **Alex Challans** (The Quantum Insider) and **Jean-Philippe Nominé** (CEA).

[4937] 2021 edition reviewers: Alexia Auffèves (measurement, energetics of quantum computing, quantum foundations, photon qubits), Antoine Browaeys (IOGS and Pasqal, cold atoms), Christophe Chareton (CEA LIST, linear algebra, quantum algorithms and development tools), Cyril Allouche (Atos, supercomputing, emulators, European projects), Daniel Esteve (CEA DRF, superconducting qubits), Eleni Diamanti (CNRS LIP6, quantum telecommunications and cryptography), Elvira Shishenina (BMW, proof-read all the document), Frédéric Nguyen Van Dau (Thales, quantum sensing), Georges Uzbelger (IBM, quantum algorithms and software tools), Jonas Landman (CNRS IRIF, quantum algorithms), Léa Bresque (CNRS Institut Néel, quantum physics 101, quantum postulates and measurement), Marc Kaplan (Veriqloud, quantum telecommunications and cryptography), Michel Kurek (who patiently proof-read several times all the book and checked all hyperlinks), Peter Eid (Arm, classical and unconventional computing, telecommunications/cryptography), Philippe Grangier (Institut d'Optique, quantum foundations), Pol Forn-Díaz (Qilimanjaro, superconducting qubits), Théau Peronnin and Jérémie Guillaud (Alice&Bob, cat- qubits) and Valérian Giesz (Quandela, photon qubits and photonics).

[4938] 2022 edition reviewers: Jean-Philippe Fauvarque (Plassys-Bestek, for the fabs part), Antoine Gras (Alice&Bob, also for the fabs part), Frédéric Wyczisk (for the quantum matter part, formerly at Thales), Michel Kurek (the proof-reading master who does an incredible work spotting all the details), Antoine Browaeys (Pasqal/IOGQ, neutral atoms computing), Bruno Desruelle (ixBlue, quantum sensing), Christophe Jurczak (updated foreword), Marco Fellous-Asiani (energetics, control electronics, superconducting qubits), Clément Barraud (MPQ, quantum matter), Georges-Olivier Raymond and Nicolas Proust (Pasqal, cold atoms qubits), Léa Bresque (Institut Néel, Quantum technologies energetics), Luc Gaffet (Air Liquide, cryogeny), Olivier Hess (Atos, software), Jérémie Guillaud, Blaise Vignon (Alice&Bob, superconducting qubits), Thomas Ayral (Atos, various pats), Loïc Chauvet (CACIB, software tools), Daniel Vert (CEA, quantum annealing), Xavier Vasques and Jean-Michel Torres (IBM, on IBM quantum software stacks), Stéphane Louise and Christophe Chareton (CEA LIST, on algorithms), Marc Kaplan (VeriQloud, quantum telecommunication and cryptography), Maud Vinet (history, silicon qubits) and André M. Konig (Global Quantum Intelligence, various places).

And a special thanks to **Michel Kurek** (Multiverse) who carefully reviewed several times the whole book as with many previous editions and **Christophe Jurczak** (Quantonation) for his foreword. This book is also supported by **le Lab Quantique** (not financially, but for its visibility).

And maybe you, next time :)!

Cheers,

Olivier Ezratty, September-October 2023

Resources

Here are a few books and other sources of information on quantum technologies that I consulted or discovered to prepare and update this book.

Events

There are numerous conferences on the different scientific branches of quantum technologies and a growing number of quantum "business" conferences associating some scientific content, industry vendors talks (and sponsoring) and customer use cases testimonials.

I found online various inventories of quantum related scientific events on quantum.info (which also inventories some quantum physics predatory journals), Conference service, Conference Index and Quantum Computing Report.

Many of these events are fee-based for both participants and speakers. It costs up to $1,000 to participate as an attendee, plus extra fees for speakers and poster sessions. It is a business!

Figure 996: a yearly timeline of some notable quantum events, from science to business. (cc) Olivier Ezratty, 2022-2023.

Quantum Scientific events

Let's quickly cover the main quantum related events where the audience is mainly made of scientists and the content is likewise highly scientific[4939].

APS March Meeting is the largest physicists conference in the world with over 12K attendees, hundreds of sessions, thousands of presented papers, and a significant part of them are in the quantum physics disciplines, including enabling technologies like cryogeny. Other talks cover different parts of physics like high-energy particles physics or astronomy. They have over 100 exhibitors. Their industry tracks showcase scientific advancements from the main quantum computing players like

[4939] See Q-Turn: Changing Paradigms In Quantum Science by Ana Belén Sainz, February 2022 (9 pages) about how to organize scientific quantum events.

IBM, Google, IonQ and others. The 2022 edition was in Chicago and will be in Las Vegas in 2023. This four-day event is organized by the American Physical Society which also publish the reference journals PRX Quantum, PRX, PRL (Physical Review Letters) and PRA (Physical Review A), that are frequently mentioned in this book bibliographical references.

Photonics West in January/February is the largest photonics related conference and vendors exhibition, including quantum photonics, their related enabling technologies (lasers, photon counters, …) and also covering medtechs, organized at the Moscone Center in San Francisco. It also hosts industry related events.

Laser World of Photonics and the **World of Photonics Congress** is another photonic major congress, happening in Germany. The 2023 edition takes place in June in Munich with over 30K visitors and 6,000 congress participants.

QIP (Quantum Information Processing) is an important quantum information conference with prestigious scientific speakers. The 2023 edition was organized in Ghent, Belgium (videos) and the 2024 edition is planned in Taipei in January.

QUANTUMatter gathers various communities in quantum information and quantum matter involved in all branches of quantum technologies (computing, sensing, telecommunications). Its second edition was organized in Barcelona in June 2022. The next one is in San Sebastian, Spain, in May 2024.

QPL (Quantum Physics and Logic) is about the mathematical foundations of quantum computation, quantum physics and related areas with a focus on the use of mathematics, formal languages, semantic methods and other mathematical and computer scientific techniques to the study of physical systems and processes. The 2023 edition was organized in Paris, France.

IEEE organizes many quantum related scientific conferences including QSW (International Conference on Quantum Software), in Barcelona, Spain in July 2022, the IEEE Quantum Week, in September, 2022 in Broomfield, Colorado and IEDM (International Electron Devices Meeting) in September in San Francisco which covers silicon spin qubits and qubit control electronics among other topics.

ASC (Applied Superconductivity Conference) is a superconducting related event. The 2022 edition happens in Honolulu, Hawaii in October. It covers quantum systems, computation, sensing and networking, control and readout electronics, fabrication, packaging, and scalable infrastructure, and hybrid or novel quantum systems.

SQA (Superconducting Qubits and Algorithms Conference) covers science, technology, and algorithms related to superconducting quantum computing organized by IQM. The summer 2023 edition took place in Munich, Germany.

Spin Qubit 5 is a conference on spin qubits. The last edition was organized in Pontresina, Switzerland in September 2022. It covered NV and SiC centers qubits, quantum dots spin qubits and the likes. The conference chair is none other than Daniel Loss.

ICDCM (International Conference on Diamond and Carbon Materials) is the conference were NV center and SiC cavities are talked about. The 2023 edition happened in Mallorca, Spain, in September.

ICQTQMS is the International Conference on Quantum Technologies, Quantum Metrology and Sensors. The 2022 edition is planned in September in Rome, Italy.

QTML (Quantum Techniques in Machine Learning) focuses not surprisingly on quantum machine learning, gathering researchers and industry players. The 2023 edition takes place at CERN in Geneva in November.

QRE is a yearly workshop on Quantum Resource Estimation also dealing with benchmarking and performance analytics. The 4th edition took place in June 2023 in Orlando, as part of the IEEE ISCA (International Symposium on Computer Architecture).

QTD2023 (Quantum Thermodynamics Conference) took place in Vienna in July 2023.

FQMD (Frontiers in Quantum Materials and Devices).

QEC'23 is the 6th International Conference on Quantum Error Correction held between October 30th and November 3rd, 2023 in Sydney, Australia, with 230 attendees.

QEI 2023 is the first Quantum Energy Initiative workshop takes place in Singapore, on November 20-24th, 2023. It covers the energetics of quantum technologies, mostly computing and communications.

ICQE (International Conference on Quantum Energy) is organized in Sidney, Australia, in December 2023. It covers the topic of energy in quantum systems, like energy harvesting, conversion, storage, and transport processes.

Quantum Computing Scalability Conference was organized in March 2023 in Oxford, UK.

GDR TEQ gathers international quantum scientists covering all fields, with tutorial sessions, docs/post-docs sessions and poster sessions. It has been organized by the research group on quantum technologies from CNRS in France every November since 2011, formerly as GDR IQFA. The 2023 edition takes place in Montpellier in November.

Quantum Business conferences

And now, onto the quantum business events which usually provide a mix of scientific and business related content.

Q2B is a conference organized by QC-Ware since 2017. Initially happening in California each December, it will also happen in Tokyo in July 2022 and later in Europe. It is a good opportunity to learn about the scientific progress from major industry players in quantum computing.

Inside Quantum Technology is a series of quantum business conferences organized in several like The Hague (February 2022), San Diego (May 2022), New York (October 2022) and San Jose, California (April 2023).

Quantum.Tech is another conference focusing on industry use cases of quantum technologies. The last edition was in Boston in June 2022. A London edition takes place in September 2022.

Commercialising Quantum was organized in May 2022 and 2023 in London by the Economist, London with a mix of in-person and virtual events. The 3rd edition will take place in May 2024.

QBN (Quantum Industry Summit) is organized in Munich in October 2023. Sessions cover politics, business and Ecosystem, quantum technologies and supply chains, industry applications and user access, and business and investment opportunities:

World in Quantum Munich is organized as part of Laser Worlds and Photonics. The June 2023 edition attracted about 9K visitors and 75 exhibitors. The next edition is planned in June 2025.

Quantum World Congress had its first edition in December 2022 in Washington DC with 700 attendees and delegations from 18 countries with 120 speakers (videos). It mainly covers innovation policy. The 2023 edition is planning in Virginia in September.

QHack is a mix of a fan expo, hackathon and scientific conference for quantum software developers. The next edition is organized online in February 2024.

Quantum Business Europe is an international conference launched in 2021 that has a third edition in September 2023 in Paris with a mix of vendors, scientists, and user talks.

World Quantum Day is a virtual global event, consolidating events happening one day in April all over the world, targeting broad audiences.

Quantum World Congress gathers experts from the industry with plenary sessions and breakout tracks on market acceleration, science and engineering, government and security, complemented by an exhibition. The 2022 edition takes place in Washington DC in November/ December.

France Quantum is a conference that was launched by Startup Inside and OVHcloud in June 2022 at the Eiffel Tower to promote the French quantum ecosystem. The 2023 edition was organized at Station F in Paris in June.

Quantum Eastern Europe is a 2-day online event gathering quantum stakeholders from Eastern Europe. The 2022 edition happened in May.

Investment Summit for Quantum Startups was organized in October 2021 at Maryland University as a gathering of investors and startups.

Metaverse Quantum Computing Summit is not a joke. Or still, yes, it's really a gigantic joke. You even have the opportunity to "*Learn best practices, strategies and ideas you can implement today*". This is an extreme case of mixing two trendy B.S. into one compound B.S (source). By the way, none of the speakers talk about the metaverse, but only about crypto-financial stuff.

Websites and content sources

The Quantum Insider (USA, formerly The Quantum Daily) is a media and market intelligence company dedicated to the quantum technology industry. They provide data on companies, investors, academic groups, and government programs related to quantum technologies through their subscription data platform and their advisory services. The company also provides daily news updates, interviews, and content related to quantum technology. They are a portfolio company of Resonance. The company is run by Alex Chahans out of the USA.

AzoQuantum (UK/Australia) an information site on quantum science news with a supplier's directory, focused on manufacturing equipment and some interviews, mostly written by young researchers.

Quantum Zeitgeist (USA) is another quantum news media that also maintains an online database of startups in the field and a quantum jobs board. Their about talks about "we" but they provide no name of who's behind the site. The site belongs to a company named Hadamard LLC based in Wyoming, and created in July 2021.

Global Quantum Intelligence is an analyst shop created by André M. Konig, David Shaw and Doug Finke that consolidated in 2022 the activities from The Quantum Report (Doug Finke), Fact Based Insight (David Shaw) and André M. Konig's consulting activities. They provide data and insights on an annual subscription basis.

The Quantum Leap is a blog of news on the quantum ecosystem published by Russ Fein, a venture investor passionate of quantum computing and based in the USA.

Quantum Journal - the open journal for quantum science, a site of scientific news on quantum physics, mostly showcasing preprints published on arXiv.

Quantiki is an information site on quantum computing. It looks like they became mainly a jobs board.

The Qubit Report (USA) is a news media focused on quantum vendors launched in 2017.

Qosf, a site that inventories guides and training for developers of quantum applications.

Enter Quantum is a news webzine on quantum technologies.

Podcasts

The Qubit Guy's podcast series is run by Yuval Boger, with short-format interviews.

Quantum Tech Pod are podcasts by Christopher Bishop on quantum technology news, published on Inside Quantum Technology's web site. These are mostly half-an-hour interviews of quantum startup founders. It started mid-2021.

Consulting firm Protivity also launched its own series of podcasts, in May 2021. Like Chris Bishop's podcasts, it's about interviewing quantum startup founders. They also last half an hour.

Quantum Computing Now by Ethan Hansen covers quantum computing news, basic concepts, and what people in the field are doing. The first episode was aired in July 2019. They are biweekly and cover news as well as science and learning tutorials.

The Quantum Analysts Roundtable is a podcast launched in January 2022 and run by Doug Finke, David Shaw, Shahin Khan, James Sanders and André M. König.

The Quantum podcasts that I have been recording regularly (in French) since September 2019 with Fanny Bouton (OVHcloud). They are available on all audio platforms (Spotify, iTunes, Deezer, ...) as well as on YouTube in video version. It covers quantum news including what's happening in the ecosystem, with startups and in research. We decipher many lead scientific announcements. Our first episode was on Google's supremacy! A of September 2023, we had recorded 50 episodes.

They are complemented by the Decode Quantum interviews that we have been publishing since March 2020 with a great variety of personalities (lead researchers, startup founders, investors, user companies, public servants, etc.) in partnership with Frenchweb. The first episodes featured Pascale Senellart, Alexia Auffèves, Maud Vinet, Eleni Diamanti, Elham Kashefi, Théau Peronnin and Raphaël Lescanne from Alice&Bob, Alain Aspect, Philippe Grangier, Michel Devoret, Daniel Esteve and many others since we had 63 episodes in-store as of October 2023! They last about one hour.

insideQuantum is an equivalent series of podcasts in English which started in 2022 out of Spain with slightly shorter formats. As of September 2023, 15 episodes had been produced.

QViews is a news podcast from Anastasia Marchenkova launched in May 2022. It's a pity since it just reads the titles and headlines from press releases. It seems the series was short lived.

Books and ebooks

If you wander in Amazon or your other preferred real-life of virtual scientific bookstore or University library, you'll find an abundant literature on quantum physics and quantum information. Many people willing to learn in these domains have a hard time finding the "right" book that is adapted to their existing knowledge and particularly, to their fluency in mathematics. Here's a not-too long list of books for this purpose. It's mostly adapted to students since, if you work in the industry, you probably won't have much time to read many of these thick books. Many of these ebooks are open source and/or free to download[4940].

Quantum physics

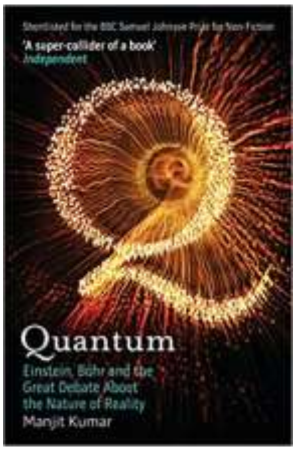
Quantum Mechanics, Volume 1: Basic Concepts, Tools, and Applications, Second Edition, 2017, by Claude Cohen-Tannoudji, Bernard Diu and Franck Laloë (879 pages) is an undergraduate reference series of books to learn quantum physics. It is considered to be the reference or the bible by many students and teachers of quantum physics.

[4940] See Publicly available quantum computing books (WIP) for a list of open access book on quantum computing, a bit lazily done with urls without titles.

Quantum Mechanics, Volume 2: Angular Momentum, Spin, and Approximation Methods, Second Edition, 2017, by Claude Cohen-Tannoudji, Bernard Diu and Franck Laloë (688 pages).

Quantum Mechanics, Volume 3: Fermions, Bosons, Photons, Correlations, and Entanglement, Second Edition, 2017, by Claude Cohen-Tannoudji, Bernard Diu and Franck Laloë (747 pages).

Do we understand quantum mechanics? Second Edition by Franck Laloë, 2019 (550 pages) is an interesting piece that documents the debates on quantum foundations and how to interpret quantum physics. Do we really understand quantum mechanics? by Franck Laloë, 2004 (118 pages) is a shorter and older version of this book, in public access.



Quantum: Einstein, Bohr, and the Great Debate about the Nature of Reality by Manjit Kumar, 2009 (480 pages) is an excellent history book about the creation of quantum mechanics. It centers a lot on the works from Max Planck, Niels Bohr, Albert Einstein, Max Born, Werner Heisenberg and Erwin Schrodinger. It's a good account of the history of ideas and how quantum physics saw the day of light. It also showcases a lot of lesser known scientists who played key roles around the most famous ones and the balance between theoreticians and experimentalists. On top of that, the book scientific content is quite good and easy to understand, without any mathematics! Other history books and papers, mostly available in open access, are also mentioned throughout this book, particularly in the History and Scientists section, starting page 26.

Lecture notes on Quantum Mechanics by Frédéric Faure, 2015 (397 pages) which provided me with some leads to link quantum mechanics to its mathematical formalism and notably to explain the Born equation.

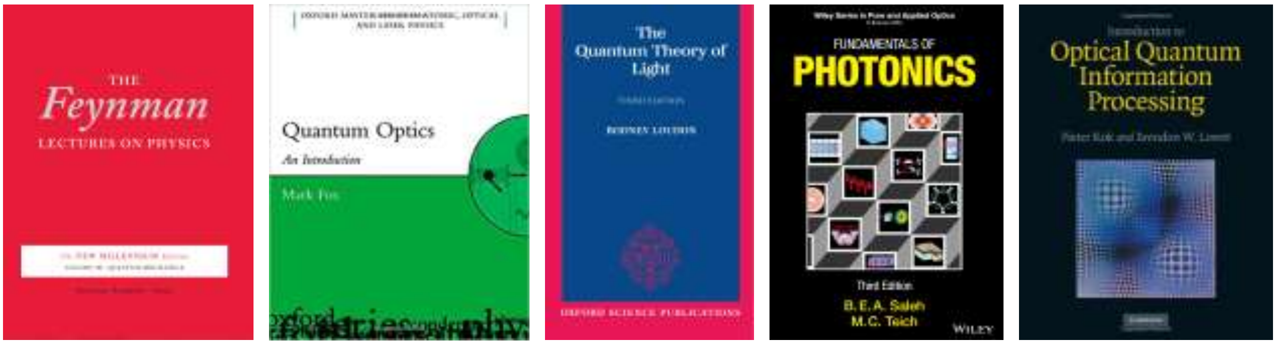
The Feynman Lectures on Physics - Volume III on Quantum Mechanics by Richard Feynman, Robert Leighton and Matthew Sands, (688 pages). It contains lecture notes of Feynman legendary courses from the early 60s. These are treasures of pedagogy with a content that is still up to date to grasp the fundamental of quantum physics. One advantage is it doesn't make any abuse of mathematics.

Quantum Optics An Introduction by Mark Fox, 2015 (397 pages) an excellent coverage of the broad field of quantum optics and the second quantization.

The Quantum Theory of Light by Rodney Loudon, 1973-2001 (450 pages) is a classic book on quantum light, that is useful to later better understand the physics of photon qubits used in quantum computing, telecommunications and cryptography. It classically starts with Planck's radiation law, then covers lasers, light-matter interactions, Mach-Zehnder interferometry, light quantization, single mode, multi-mode and continuous-mode optics and nonlinear optics.

[Fundamentals of Photonics](#) by Bahaa Saleh and Malvin Teich, 2019 (1,401 pages, a little above this book) is a comprehensive quantum optics books that also covers instrumentation, which means it's good stuff for experimentalists.

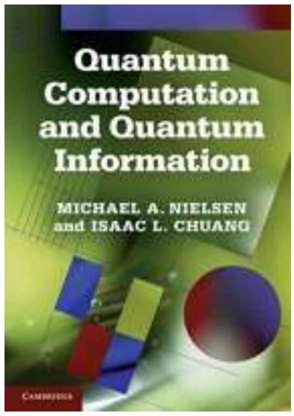
[Introduction to Optical Quantum Information Processing](#) by Pieter Kok and Brendon W. Lovett, 2010 (506 pages) is another classic quantum photonics books covering quantum information systems.



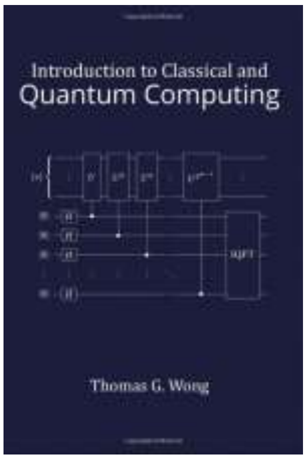
[I Don't Understand Quantum Physics](#) by Douglas Ross, 2018 (104 pages) is a nice primer to conceptualize and visualize many quantum phenomena. It describes the founding experiments of quantum physics (blackbody radiation, photoelectric effect, Compton scattering, etc), the wave-particle duality, matter wave, indeterminacy, Schrödinger's equation, superposition and the EPR paradox.

Quantum information

[Quantum Computation and Quantum Information](#) by Michael Nielsen and Isaac Chuang, 2010 (10th edition, 704 pages, public access) is the definitive reference on the basics of quantum computing. It answers many key questions, in particular on the mathematical models of linear algebra used in quantum computing. It also covers the basics of quantum physics, quantum postulates, problems complexity classes, quantum measurement, quantum algorithms, how qubits are realized (harmonic oscillators, trapped ions, photons, NMR), the impact of quantum noise and decoherence, how quantum error corrections work, what is fault-tolerant quantum computing, how about Shannon and Von Neumann information entropy and the likes. It also covers quantum key distribution and cryptography.



[Introduction to Classical and Quantum Computing](#) by Thomas Wong (2022, 400 pages), a book that is free to download in PDF and available in paperback on Amazon (😊). It makes a good comparison between classical and quantum computing. Like most quantum books for developers, it covers only classical gate-based algorithms with nothing on quantum annealing and quantum simulation. And it says nothing about the hardware and what it can do. It also makes some confusions between phase flip errors and decoherence related errors, which create mixed state when a simple phase error preserves a pure state.



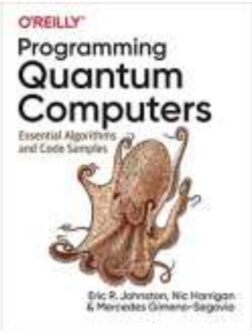
[Elements of Quantum Computing by Seiki Akama](#) (133 pages), which is at the same time concise, precise and quite complete on the nuts and bolts of quantum physics and quantum computing, with a good historical overview.

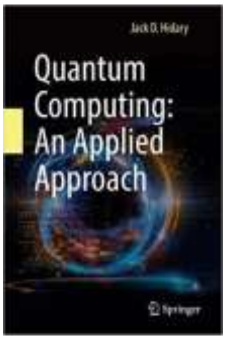
[Quantum Information Meets Quantum Matter](#) by Bei Zeng, Xie Chen, Duan-Lu Zhou and Xiao-Gang Wen. It is available in a February 2018 version [on arXiv as a](#)free download (373 pages).

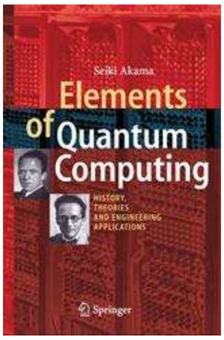
[Programming Quantum Computers - Essential Algorithms and Code Samples] by Eric R. Johnston, Mercedes Gimeno-Segovia and Nic Harrigan (2019, 336 pages) is an excellent and detailed description of key quantum algorithms like the QFT, phase estimation and the likes.

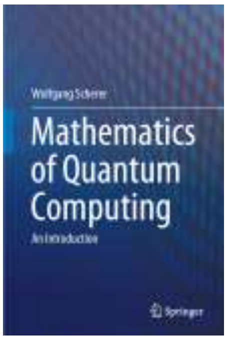
[Quantum Computing: An Applied Approach] (2021, second edition, 445 pages) by Jack Hidary, a fairly comprehensive book covering quantum algorithms and their mathematical foundations. It briefly describes the different architectures of quantum computers.

[Quantum Machine Learning - What quantum computing means to data mining], by Peter Wittek, 2014 (176 pages) is a good introduction to machine learning and quantum machine learning although many progresses were made since this book's publication.





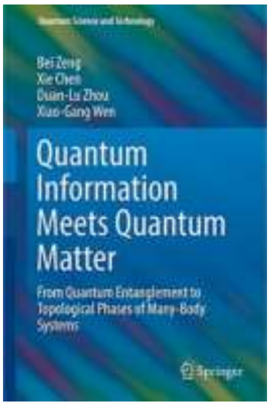


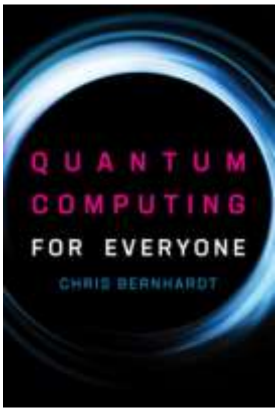


[Quantum computing- from quantum physics to quantum programming in Q#] by Benoit Prieur, 2019 (244 pages). It starts with the general principles of quantum physics. The section on quantum computers themselves is rather thin and explores only a few technologies (superconductors and NMR, which is little used). The rest is dedicated to learning programming in Q#, Microsoft's quantum programming language.

[Principles of Quantum Computation and Information, A Comprehensive Textbook] by Giuliano Benenti, Giulio Casati, Davide Rossini and Giuliano Strini, December 2018 (598 pages).

[Quantum Computing for Everyone by] Chris Bernhardt, 2019 (216 pages) which describes the basics of quantum computing starting with the inevitable qubit, quantum gates, accelerations brought by quantum algorithms and the main components of a quantum computer.



[Quantum computing] by Joseph Gruska (1999, 390 pages), another fairly comprehensive base covering all aspects of quantum computing and communication.

[An Introduction to Quantum Computing] by Phillip Kaye, Raymond Laflamme and Michele Mosca, 2007 (284 pages) which starts with some mathematical foundations of quantum physics and quantum computing. By reference authors such as Raymond Laflamme (Canada) who is one of the fathers of error correction codes.

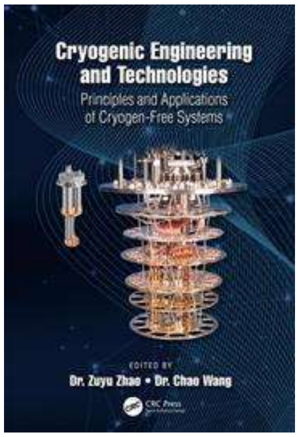
[Introduction to quantum computing algorithms] by Arthur Pittenberger, 2001 (152 pages) which describes classical quantum algorithms with a good part dedicated to error correction codes.

[Quantum Software Engineering] is a book edited by Manuel A. Serrano, Ricardo Pérez-Castillo and Mario Piattini from University of Castilla-La Mancha in Spain, 2022 (321 pages). It provides an overview of this emerging discipline, describing key concepts, the related vocabulary and formal methods. It also presents Q-UML, a quantum modeling language.

[Quantum Internet], a 60-page magazine presenting the different sides of quantum computing, published by TU Delft (2019).

[Quantum computing for dummies] by William Hurley, April 2023, has yet to deliver its nuggets!

[Cryogenic Engineering and Technologies] by Zuyu Zhao and Chao Wang, October 2019 (386 pages) is a reference book on cryogenic issues with a very extensive and well-documented history. There is an excellent chapter on dry dilution cryostats used in quantum computers. This helped me to prepare the part of this book on [cryogenics] (starting page 562) in addition to an interview with the team of the French startup CryoConcept and with researchers from CNRS Institut Néel in Grenoble.

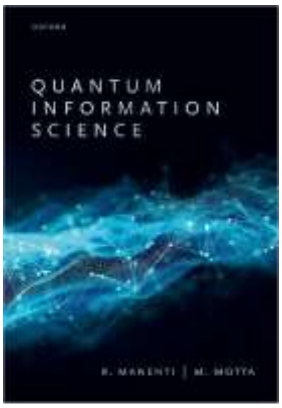


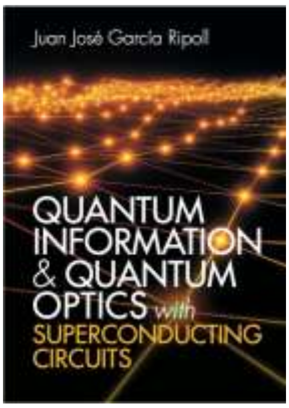
[Unconventional Computation] by Bruce MacLennan, University of Tennessee, October 2019 (304 pages) which discusses the energy issues of computation (reversible, non-reversible) and various alternative methods of computation including quantum computing and molecular computing.

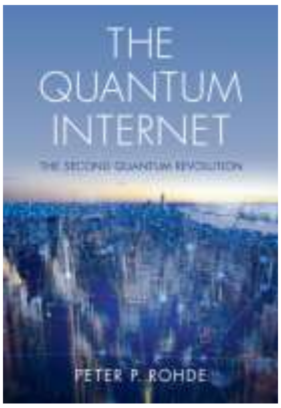
[Quantum Information Science] by Riccardo Manenti and Mario Motta, August 2023 (768 pages) covers quantum computing with going through mathematics, quantum physics, quantum algorithms including quantum chemistry and quantum hardware, focused on superconducting qubits.

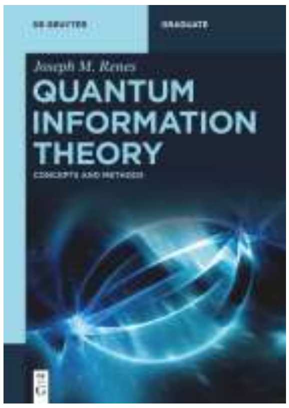
[Quantum Information & Quantum Optics with superconducting qubits] by Juan José García Ripoll, August 2022 (300 pages) covers superconducting quantum circuits and qubits, up to their applications in quantum computing and quantum simulation.

[Quantum Information Theory] by Joseph M. Renes, August 2022 (250 pages) is a graduate textbook introducing the mathematical theory for quantum communication, computation, and cryptography with focusing on the concept of quantum channels, understanding fidelities and entropies in the quantum world, and the interrelationship of various quantum information processing protocols.







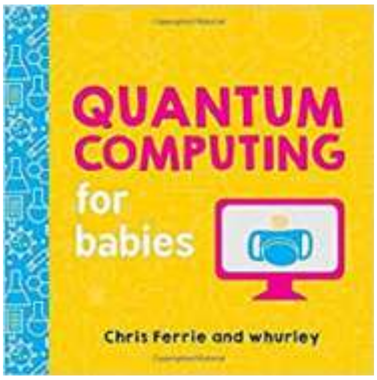


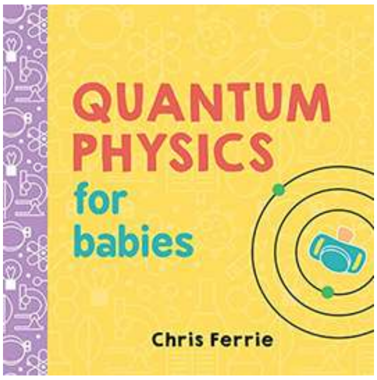
[The Quantum Internet] by Peter P. Rohde, December 2021 (300 pages) examines in detail how the quantum internet will take share, focusing on the technology and on its economical and political implications.

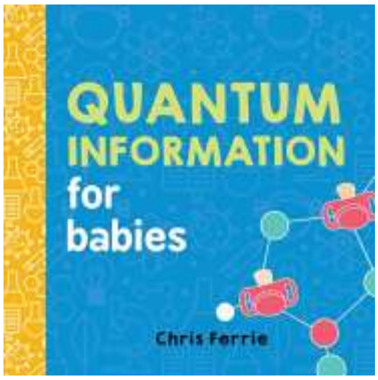
[Introduction to Quantum Cryptography] by Thomas Vidick (Caltech) and Stephanie Wehner (TU Delft), September 2023, provides background theory and mathematical techniques on the design of quantum cryptographic protocols, including QKD, quantum money, and delegated quantum computation.

Comics

Quantum Computing for babies by Chris Ferrie and William Hurley, April 2018, is aimed more at children or even older children. The book popularizes the major concepts of physics and quantum computing in a very colorful way. It comes from a professor of quantum computing at Sydney University of Technology and the founder of the American startup Strangeworks. William Hurley is the founder of **Strangeworks**. Two other books in the same vein from Chris Ferrie were also released: Quantum Physics for babies and Quantum Information for babies, all for less than $10. I "read" the last one (24 pages) and I'm not sure even an adult would really understand how quantum computing after looking at it. This is the danger of oversimplification and information dilution.







Presentations

Here are a few conferences and presentation materials rather well done to popularize quantum computing.

Quantum computing for the determined by Michael Nielsen, a series of 22 videos on quantum computing, 2011, accompanied by a long text of explanations.

CERN's Quantum Computing for High Energy Physics workshop in November 2018, with presentation materials and videos and interesting talks by various players in quantum computing, including Intel, who are not often seen. The specific content may be overtaken by the basic principles remain valid.

Quantum computing Overview by Sunil Dixit, September 2018 (94 slides) is a presentation by Northrop Grumman that takes a fairly broad look at quantum computing and the underlying mathematical models.

A Practical Introduction to Quantum Computing From Qubits to Quantum Machine Learning and Beyond, by Elias Combaro, CERN course, 2020 (251 slides) is a good course on quantum algorithms, debugging, validation, verification and benchmarking.

Quantum computing, a four-part course by Hélène Perrin at Université Paris 13, February 2020 (lecture 1 of 77 slides on trapped ions, lecture 2 of 36 slides on superconducting qubits, lecture 3 of 39 slides on silicon, molecular and NV Centers qubits, lecture 4 of 75 slides on cold atoms). This requires a good background in physics to be understood from start to finish. The references provided allow to deepen the topics covered.

Training

Berkeley courses for 2013: Quantum Mechanics and Quantum Computation on YouTube.

Videos from the Stanford Quantum Computing course.

The Quantum Computing Fundamentals course offered by MIT.

An online training course on quantum programming offered by Brillant in partnership with Microsoft.

The QSIT course (FS 2016) at ETH Zurich with its slides and lecture notes.

Quantum Computing as a High School Module, a curriculum with exercises on the basics of quantum physics for students at the BAC level.

Reports

Inside Quantum Technology, an analyst company dedicated to quantum technologies, which sells industry reports.

You can also find analysts reports on quantum technologies with McKinsey, BCG, Yole Development and other analyst companies.

Miscellaneous

Designing and Presenting a Science Poster, Jonathan Carter, Berkeley (20 slides) which is intended to help researchers design a good research project presentation poster.

Glossary

What is the purpose of a glossary? It allows you to find your way around in a new terminology and to step back to understand new concepts. For the author, it was also a good checkpoint of his own understanding and ability to popularize scientific and technological concepts. Some of these descriptions are simplified versions derived from Wikipedia definitions. Welcome to the lingua franca of quantum sciences and technologies!

137: constant used to compare different equivalent quantities in quantum physics. It turns out that 1/137 is a value that corresponds approximately to the fine-structure constant, a ratio that is found in several places in quantum physics and compares data of the same dimension. It is for example the ratio between the speed of an electron in the lower layer of a hydrogen atom and the speed of light or the probability of emission on the absorption of a photon for an electron. 137 is a bit like 42 in quantum physics. Wolfgang Pauli died after an operation for pancreatic cancer, while his hospital room was number 137.

ADC: analog-digital converter. Converts an analog signal into a digitized signal. In quantum technologies, one example of an ADC use case is the conversion of the reflected microwave pulse signals used in superconducting and electron spin qubits readout.

Adiabatic: quantum computation method used with D-Wave quantum annealing computers and also more classical gate-based quantum computers. A complex Hamiltonian describing a complex system is first determined whose fundamental state describes a solution to the problem under study. A system with a simpler Hamiltonian is then prepared and initialized in its fundamental state. This Hamiltonian then adiabatically (meaning, with no energy or mass exchange with the outside environment) evolves into the complex Hamiltonian. According to the adiabatic theorem, the system remains in its fundamental state, and its final state describes a solution to the problem under consideration.

Adiabatic theorem: quantum mechanics concept created by Max Born and Vladimir Fock in 1928. It states that a quantum mechanical system subjected to gradually changing external conditions adapts its form, but when subjected to rapidly varying conditions there is insufficient time for the functional form to adapt, so the spatial probability density remains unchanged. This can be used to find Hamiltonian energy minimums with quantum algorithms running on various architectures: gate-based, annealing and quantum simulation.

Advantage: see quantum advantage.

aka: shortened version of “also known as”.

Algorithm: a method of problem solving that is made up of a finite sequence of operations or instructions. The word comes from the name of the 9th century Persian mathematician, Al-Khwârizmî.

Algorithmic qubit: benchmark metric proposed by IonQ which corresponds to the number of qubits usable with an equivalent computing depth with a randomized benchmark producing a good result in $2/3^{rd}$ of the runs. It’s actually $\log_2$ of IBM’s quantum volume.

Amplitude: this term has various meanings depending on the context. It can be the classical amplitude of a wave, i.e. half of its maximal variation, as opposed to its phase. For a quantum object, it can be the complex amplitudes of its basis states or eigenvectors. With a qubit in its Bloch sphere representation, the amplitude is related to the projection of the qubit vector on the z axis. But the $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ describing the qubit vector are also amplitudes, although, precisely, complex amplitudes. These complex amplitudes define the qubit amplitude (1-cos $(\theta/2)$) and its relative phase (angle φ).

Anharmonic oscillator: contrarily to harmonic quantum oscillator that have the same energy difference between ach each consecutive energy levels, an anharmonic quantum oscillator has different energy differences between consecutive energy levels. This is the case of superconducting qubits, in order to create two manageable energy levels that are controlled with microwaves with the highest energy transition level of the oscillator.

Angular momentum: generally speaking, speed of rotation of a rotating object. In quantum physics, angular momentum is quantized and can have only discrete values.

Ansatz: another name for the parametrized quantum circuits or parametric quantum circuits that contain rotation gates and the parameters of the problem to encode in a variational quantum optimizer defined by a classical optimizer. Typically used is most variational algorithms (QAOA, VQE, QML).

Anyons: type of elementary quasiparticle found in two-dimensional systems. It is a generalization of the concept of bosons and fermions. Anyons have intermediate statistical behaviors between the two types of elementary particles. They are in fact virtual particles that live in two spatial dimensions and are generally based on electrons or electron gaps moving in superconducting metallic 2D structures. Anyons are a particular type of quasiparticle. They are used in topological quantum computers and would be used in computers based on the hypothetical Majorana fermions studied at Microsoft.

arXiv: Cornell University's site that allows researchers to publish scientific papers prior to publication in peer-reviewed journals such as Nature, Science or Physical Review. It can take up to 9 months between publication of an article on arXiv and publication in a peer-reviewed journal. In the latter case, the article will have eventually evolved. The interest of arXiv in literature search is that publications are open and free of charge whereas most of the peer-reviewed journals are not free. The disadvantage is that the articles are not necessarily validated and that one has to make his own evaluation. It should be noted that in a researcher's publication, there are often several authors, up to several dozen. The first author is generally the PhD student who has carried out a large part of the work, particularly its experimental part. Others are contributors who helped him/her. The last author is the thesis director or principal investigator (PI), the group leader or the laboratory director who has closely or remotely supervised the project. He/she probably contributed significantly to the article writing and cleanup.

Antisymmetry: is related to the behavior of particles with half-integer spin like electrons. They obey the Pauli exclusion principle which states that two identical fermions cannot occupy the same quantum state simultaneously leading to the way electron shells are filled in atoms. Antisymmetry also shows up mathematically with the wave function of a multiple fermions system to change sign when the location of two fermions are exchanged.

Atoms: the smallest constituent element of matter that manifests chemical properties. It consists of a nucleus, with one or more positively charged protons and zero or more neutrally charged neutrons, around which negatively charged electrons gravitate. In a neutral atom, the number of electrons is equal to the number of protons. Otherwise, the atom is negatively or positively charged and forms an ion. The number of protons determines the nature of the atom in Mendeleev's Elements Periodic Table. An atom with one proton is hydrogen, with two protons it is helium, etc. Uranium has 92 protons. The nucleus represents the bulk of the atom mass. The isotopes of an element correspond to variations in the number of neutrons. In general, the number of neutrons of an element is equivalent to that of protons. Electrons are distributed in layers whose number depends on the atomic number. They are numbered from 1 to 7. Each layer can contain a maximum of $2n^2$ electrons, n being the number of the layer (thus 2, 8, 18, 32, 50, 72 and 98). This model was developed by Niels Bohr between 1909 and 1913. The chemical properties of the element depend on the number of electrons of the last layer which is called the valence layer. If this number is $2n^2$, the atom will be inert and will not combine chemically with other atoms. Carbon has three layers of electrons, the last one having 4 which allows it to combine with other atoms such as hydrogen (1 layer, 1 electron) or oxygen (6 electrons in the last layer).

Autonomous quantum error correction (AQEC): quantum error correction codes and architectures that does not require error syndrome measurement. It replaces real-time feedback by analog feedback circuits using engineered dissipation with the reservoir engineering technique. It couples the system with a dissipative reservoir to transfer the entropy created by errors to an ancillary system, the reservoir. This entropy is then evacuated via the strong dissipation of the ancilla. This technique is used in cat-qubits and GKP qubits.

Back action: in quantum measurement, this is the physical impact of the measurement device on measured quantum objects. Quantum measurement usually modifies the state of the measured quantum object unless it is already in a basis state (mathematically, one of the eigenvectors of the measurement observable operator...). After measuring a quantum object state, performing the same measurement on the already measured system will not provide any additional information. In order to increase our knowledge on the final state of some quantum computation, the only solution is to start again the computation from the beginning and measure again the final state. The subtlety being that this new final state has not yet been measured and thus corresponds exactly to the one we are trying to infer. Then, we compute an average of the obtained results across several experiments. Some measurement techniques like gentle measurement or weak measurement are designed to minimize this back action and are sometimes used in quantum error correction codes.

Balmer series: set of four spectral emission or absorption lines with the hydrogen atom, generated by electron transitions between the second and higher energy levels of the atom.

Band gap: forbidden energy level in semiconductors and transistors. It refers to the energy difference between the top of the valence band and the bottom of the conduction band and corresponds to the energy required to excite an electron from the valence band to the conduction band. The notion is related to the electrical conductivity of solid materials. Wider band gap materials are generally more insulating and also are transparent while narrower band gap materials are generally more conductive and are opaque. Semiconductors have an intermediate band gap. Band gap size is determined by the arrangement of atoms in the material crystal lattice. The closer they are, the smaller the band gap is.

Beam splitter: optical device that splits a beam of light in two. It's usually made with two glued triangular glass prisms. Polarizing beam splitters are a particular class of beam splitters that use birefringent materials to split light into two beams of orthogonal polarization states.

Balanced beam splitter: beam splitter where the light is equally divided in two streams.

Baryon: class of elementary particles of the first level of the nuclei of atoms. It contains protons and neutrons.

Bell inequalities: Bell's 1964 are the relations that measurements on quantum entangled states must respect under the hypothesis of a local deterministic hidden variable theory. Experiments show that Bell's inequalities and related statistics are systematically violated, forcing scientists to give up one of the three following hypotheses on which Bell's inequalities are based. The first is the locality principle according to which two distant objects cannot have an instantaneous influence on each other, which means that a signal cannot propagate at a speed greater than the speed of light in a vacuum. The second is causality, according to which the state of quantum particles is determined solely by their experience, i.e., their initial state and all influences received in the past. The third is realism, which means that individual particles are entities that have properties of their own, carried with them (source).

Bell states: or EPR states are maximally entangled states of two qubits. There are four of them: $(|00\rangle + |11\rangle)/\sqrt{2}$, $(|01\rangle + |01\rangle)/\sqrt{2}$, $(|00\rangle - 11\rangle)/\sqrt{2}$ and $(|01\rangle - |01\rangle)/\sqrt{2}$. The first of the Bell state is generalized in GHZ states with n qubits and the second is generalized as W states.

Bell test statistic: it is a test of correlation of quantum state detection with two entangled quantum objects which can have values A and A', and B and B'. Quantum entanglement showing a correlation of the values of these two objects will yield and average value of: $|S| = \langle AB\rangle_{lim} + \langle AB'\rangle_{lim} + \langle A'B\rangle_{lim} - \langle A'B'\rangle_{lim} = 2\sqrt{2}$ (about 2.828). In this formula, $\langle A'B\rangle_{lim}$ is the probability to have outcome A with the first quantum object and outcome B' for the second. It's usually a photon polarization. This test is also a way to evaluate the entanglement of two qubits in quantum computers.

Black body: a body that is in thermal equilibrium with the radiation it emits. It can be the inside of a furnace or a star. It is by studying the radiation of the black body and its frequency distribution as a function of the body temperature that Max Planck uncovered the existence of energy quanta in 1900. Also written blackbody or black body depending on the source.

Blind Quantum Computing: technique for distributing quantum processing in remote quantum processors and securing the confidentiality of the processing.

Bloch sphere: geometric representation of a qubit state with a vector in a sphere of radius 1. The qubit ground state is an upwardly directed vector $|\mathbf{0}\rangle$ and the excited state is a downwardly directed vector $|\mathbf{1}\rangle$. An intermediate state vector is defined by its amplitude and phase, in line with the wave-particle duality of the qubits. It models the state of a qubit using polar coordinates with two angles, one indicating the amplitude of the quantum and the other its relative phase.

Born rule: postulate of quantum mechanics created by Max Born in 1926 giving the probability that a measurement of a quantum system will yield a given result. It states that the probability density of finding a particle at a given point, when measured, is proportional to the square of the magnitude of the particle's wavefunction at that point.

Born-Oppenheimer approximation: mathematical approximation used in molecular dynamics that treats separately the wave functions of atomic nuclei and electrons in molecules. The nucleus being much heavier than the electrons, its coordinates are approximated as being fixed, while the coordinates of the electrons are dynamic. This approximation was proposed in 1927 by Max Born and his 23-year-old graduate student J. Robert Oppenheimer.

Bose-Einstein Condensate, or BEC, state of very low density boson gas cooled to a temperature close to absolute zero (-273.15°C) where a large part of the bosons are in the lowest possible quantum energy state and exhibit particular properties such as interferences. A special case of BEC is superfluid helium, discovered in 1938, which, at very low temperatures, has no viscosity, i.e., it can move without dissipating energy. These condensates were imagined and theorized by Satyendra Nath Bose and then Albert Einstein in 1924. Their existence was demonstrated experimentally in 1995 by Wolfgang Ketterle, Eric Cornell and Carl Wieman who were awarded the Nobel prize in Physics in 2001. In quantum computing, this field is related to the field cold atom-based qubits.

Boson sampling: typical experiment with photons qubits that mixes photons in an interferometer. It's hard to emulate on a classical computer and is used to show a specific quantum advantage. The only caveat is these experiments are not programmable and are therefore entirely useless and irrelevant to compare any calculation capacity between systems.

Boson: particles with gregarious behavior, which can accumulate in arbitrarily large numbers and in the same state. Bosons comprise photons and composite objects with whole integer spin such as hydrogen, lithium-7, rubidium-87, carbon and silicon atoms in crystalline structures. These particles escape Pauli's exclusion principle. They have a symmetrical wave function.

Bosonic codes: hardware system that implement quantum error corrections with bosonic modes, using a quantum harmonic oscillator with continuous energy levels. It includes cat-qubits, GKP codes (Gottesman-Kitaev-Preskill) and binomial codes.

BQP (problem class): complexity class of problems that can be handled by quantum algorithms. Means a bounded-error quantum polynomial time. It is the class of problems that can be solved in polynomial time relative to the size of the problem with a probability of obtaining an error not exceeding one third of the results. This class is positioned between class P (problems that can be solved in polynomial time on a classical machine) and NP (problems for which a solution can be verified in polynomial time on a classical machine).

Bra-ket (notation): A notation model describing the state of a quantum and a qubit in the form $|\boldsymbol{\psi}\rangle$ and $\langle\boldsymbol{\psi}|$. It was created by Paul Dirac in 1939. A bra psi vector is a quantum state described as a column vector. A ket is its transpose, a row vector. It facilitates the writing of operations with quantum states, like inner products $\langle\boldsymbol{\phi}|\boldsymbol{\psi}\rangle$, outer product $|\boldsymbol{\phi}\rangle\langle\boldsymbol{\psi}|$ and projection $\langle\boldsymbol{\psi}|\boldsymbol{A}|\boldsymbol{\psi}\rangle$.

Break-even: said of a quantum computing hardware and associated quantum error correction code enabling the creation of logical qubits having an error rate below the error rate of its physical qubits, usually assessed as two-qubit gate errors.

Cap table: list of equity investors in a company.

Cavity: enclosed space used in quantum technologies that reflect light or other electromagnetic waves like microwaves. It creates standing wave patterns. They are characterized by their quality factor (Q factors), the higher it is meaning it can trap light for a longer time. Cavities are often placed in controlled environments, such as cryogenic or vacuum chambers, to minimize thermal and electromagnetic noise coming from the outside world. The two most used cavities in quantum technologies are superconducting cavities which are made of superconducting materials with very low resistance and trap light in the microwave spectrum and optical cavities that use mirrors or other reflective surfaces trapping light in the visible or infrared spectrum. Cavities are used in quantum computing, quantum communications and quantum sensing. In superconducting qubits, the cavity component is the resonator. Superconducting 2D cavities are made of parallel waveguides using aluminum or niobium nitride thin film materials deposited on silicon or sapphire. Superconducting 3D cavities are made of niobium or copper and shaped into cylindrical or spherical structures. It confines microwave photons more efficiently than 2D cavities with longer coherence times but is more bulky and less scalable. Optical cavities are frequently used to trap optical fields that interact with cold atoms, often, in combination with a magneto-optical trap.

Cavity quantum electrodynamics (CQED): field of quantum physics coupling trapped atoms in physical cavities and microwaves. It is about the interactions between photons and electrons and atoms.

Chandelier: nickname of the quantum computing system located inside the cryostat of a superconducting or electron spins quantum computer. It contains several stages made of copper disks covered with gold. These disks are crossed by numerous coaxial cables that are used to drive the qubits and read their state with microwaves. It is completed by filters, attenuators and amplifiers for the microwaves that circulate in these wires, various sensors, and heat exchangers that cool the copper disks, which in turn cool the elements that are placed on them.

Chi: Greek letter used to define the level of nonlinearity of an optical medium. A $\chi^{(i)}$ medium had a nonlinearity of level i. When i=2, the medium has a second order nonlinearity. It is used for example for the frequency doubling of laser light. With i=3, it is a third order nonlinearity, which is used for example in four-wave mixing. Chi is a coefficient of the polynomial relation between the phase P change of a light beam traversing the medium and the beam energy E, with the formula $P = \epsilon_0(\chi^{(i)}E^i)$.

Chip: integrated circuit built on semiconductor material, usually silicon, but also III/V materials like gallium or indium.

Chipset: refers to specialized integrated circuit containing various features that complement the features from a CPU.

Chiplet: packaging containing several chips or integrated circuits usually built with different technologies.

Circuit quantum electrodynamics (cQED): architecture used in solid state qubits systems using superconducting qubits and microwave photons. Science behind the interactions between microwaves and electromagnetic circuits.

Clifford group: group of unitary quantum gates that can be easily simulated in polynomial time on classical computers according to Gottesman-Knill's theorem. A Clifford gate is a quantum gate that can be decomposed into gates of the Clifford group. It is sufficient to have one unitary gate rotating on the X axis and another on the Z axis to create a complete set of Clifford gates. They must be completed with at least one two-qubit gate as a CNOT. These gates make quarter turns or half turns in the Bloch sphere. They are not sufficient to create a universal gates set. You need non-Clifford gates like the T gate.

Circuit: describes a set of quantum gates applied in an orderly fashion to a register of qubits. In other word, it is a quantum program for a gate-based quantum computing system. A graphical representation of a quantum circuit shows qubits in stacked horizontal lines, quantum gates as boxes labelled X, Y, Z, H and others applied to single qubits and two or three quantum gates with vertical lines connecting qubits. The X axis represents time and gates are executed from left to right.

Cluster state: the starting point for an MBQC (Measurement Based Quantum Computing) calculation with a grid of embedded qubits that are usually initialized in an entangled state. Used mostly with photon qubits.

CMOS: a common semiconductor fabrication technique used to produce processors and memory, and which is reused to create qubits that manipulate electron spins.

Code distance: notion used in quantum error correction codes and with the stabilizer codes formalism which is linked to the smallest number of simultaneous qubit errors that can be fixed with a given quantum error code. A code distance d means that the error correction code can correct errors for up to (d − 1)/2 qubits. These are usually even numbers (3, 5, 7, …). So a code distance of 7 can correct at most 3 qubits.

Coherence: quantum coherence is the ability of a quantum system to demonstrate interference. The coherence between different parts of a wave function allows for the famous double-slit interference and the formation of short quantum wave packets propagating in space. Two wave sources are coherent when their frequency and waveform (or phase for an electromagnetic signal) are identical. There are temporal coherence (same waveforms with some time delay), spatial coherence (in 2D or 3D such as with plane waves) or spectral coherence (waves of different wavelengths but with a fixed relative phase form a wave packet). In quantum physics, coherence comes with linear superposition of various states of a quantum system containing one or several quantum objects (represented by a wave due to the wave-particle duality). Quantum coherence progressively degrades naturally due to the interactions with the environment and ends after a certain time for qubits (the coherence time) and also when measuring the state of a qubit.

Cold atoms: atoms cooled at very low temperatures, generally with techniques using lasers and the Doppler effect. They are used in certain types of quantum computers called cold atom quantum computers. The atoms used are neutral atoms (not ionized) and quite often rubidium, an alkali metal.

Compatible properties: physical properties of a quantum system that can be measured in any order or simultaneously.

Commutativity: mathematically, two variables A and B commute when A×B=B×A. They do with integers but not with non-square matrices. Even square matrices don't necessarily commute. They are then "noncommutative".

Commutator or commutation operator: Characterize the level of non-commutativity between two variables, usually matrices. For two matrices A and B, their commutator is [A,B]=AB-BA.

Complementarity: principle of quantum physics introduced by Niels Bohr in 1927 according to which quantum objects have certain pairs of complementary properties which cannot all be observed or measured simultaneously. These are incompatible properties. Another version of this principle is that it's not possible to simultaneously observe a quantum object as a particle and as a wave, like in the Young slit experiments.

Complementary variables: pairs or complementary variables or properties according to the Bohr complementary principle. The common example are the position and momentum of a particle which are continuous. A more "quantum" example is the spin of an electron. You cannot evaluate the spin in a direction (X) and another one (Y or Z) which are both discrete quantities yielding +1/2 or -1/2.

Complex number: set of complex numbers created as an extension of the set of real numbers, containing in particular an imaginary number noted i such that $\boldsymbol{i^2 = -1}$. Any complex number can be written in the form a + ib where a and b are real numbers. These numbers are used in particular to describe the state of a qubit and to represent the phase of a quantum object with its complex component.

Complexity (theory): branch of theoretical computer science and mathematics that plays an important role in quantum computing to evaluate its performance compared to traditional Turing/Neumann machine computing. It defines classes of problems by levels of complexity, in terms of computing time or even the memory space required, with, in particular, problems that are solved in polynomial time in relation to their complexity (class P) and whose results are verifiable in polynomial time (class NP). The methods used to solve these problems are most often based on the brute force of navigating through an increasingly large space of combinations to be evaluated according to the size of the problem to be solved.

Compton effect: effect which demonstrates that photons can have some momentum and behave as particles, that was demonstrated by Arthur Holly Compton in 1922 with scattering of X rays and gamma rays photons by atomic electrons.

Computational basis: naming of the basic states of a qubits register. For a single qubit, this corresponds to the $|\mathbf{0}\rangle$ and $|\mathbf{1}\rangle$ states. For a register of N qubits, the computational basis is made of the 2^N combinations of series of N 0s and 1s, named in Dirac's notation $|\mathbf{000}\ldots\mathbf{000}\rangle$ to $|\mathbf{111}\ldots\mathbf{111}\rangle$. All these states are mathematically orthogonal with each other. A N qubits register in a pure state mode is a linear superposition of all these states using complex amplitudes.

Concatenated codes: describes the recursive application of error correction codes where in an error correction code, a physical qubit is replaced by a logical qubit, and so on.

Condensed matter physics: branch of physics that studies the macroscopic properties of matter (solids, liquids, glasses, polymers) and in systems where the number of constituents is large and the interactions between them are strong. Condensed matter physicists seek to understand the behavior of these phases using the laws of physics (quantum mechanics, electromagnetism and statistical physics). In practice, it mainly covers low temperature superconducting, ferromagnetic, antiferromagnetic and ferrimagnetic phases of spins in crystalline lattices of atoms, spin glasses, spin liquid, and Bose-Einstein condensates. Physicists working on superconducting qubits are part of this discipline.

Conjugate variables: pairs of dynamic variables describing the state of a quantum object, like position and momentum, that are related to the other with the Heisenberg indeterminacy principle which prevents a precise measurement of both variables.

Contextuality: fundamental concept in quantum physics where the outcome of a measurement on a quantum system depends not only on the system intrinsic properties but also on the context of other measurements being performed concurrently. As a result, measurements of quantum observables do not reveal pre-existing values. In classical physics, the value of a property of an object is intrinsic and independent of how or when it is measured. See Kochen–Specker theorem. The study of contextuality is a major topic of discussion in quantum foundations. Quantum contextuality is also said to be one of the origins of quantum computational speedups and quantum advantage in quantum computing.

Continuous variables quantum computing (CV): a type of quantum computer that uses qubits whose values are continuous and not binary. Used in two types of quantum computers: analog quantum simulators (particularly based on cold atoms) and CV photon-based systems.

Cooper pair: pairs of tightly coupled electrons creating electric current flow in superconducting materials, usually at very low temperatures and without resistance. Cooper pairs have an integer spin because they accumulate two electrons with a spin of ½. They become bosons and can accumulate and form macro quantum objects.

Copenhagen interpretation: interpretation of quantum physics elaborated by Niels Bohr in Copenhagen and by Werner Heisenberg, although it was never clearly formalized. Applied to individual quantum objects, it is mostly based on Bohr's correspondence and complementarity principles, Heisenberg's indeterminacy principle, Born's probability interpretation of the Schrodinger wave function and on the wave function collapse and its fundamental indeterminism. It avoids describing any reality beyond what can be measured like an exact position of an electron. The completeness of this theory was challenged by Albert Einstein. Physicists are still debating about this interpretation, as part of the quantum foundation field.

Correspondence principle: principle formulated by Niels Bohr in 1920 which states that the behavior of systems described by quantum physics matches classical physics in the limit of large quantum numbers (large orbits and large energies or electron quantum numbers).

Coulomb blockade: decrease of electrical conductance at small bias voltages of small electronic devices containing at least a low-capacitance tunnel junction. As a result, the conductance of the devices may not be constant at low bias voltages, but disappears for biases under a certain threshold, i.e. there is no current flows.

Coulomb force: electrostatic force between electrically charged particles like electrons and protons. Its strength is inversely proportional to the square of their distance and proportional to the product of their respective charge.

Counterfactual models: concepts extending from the counterfactual phenomena observed in quantum mechanics. It deals with the ability to obtain information or produce an effect in the absence of a direct physical interaction. It includes quantum entanglement where quantum systems can be interconnected so that the state of one seems to instantaneously affect the state of the other, regardless of the distance between these systems and with no persistent physical connection. Another such model is the quantum Zeno effect where the frequent observation of a quantum system inhibits its evolution and freezes it. It is used in some communication protocols where information can be passed between two locations without any physical particles traveling between them. It uses specific settings of quantum channels and particles, such that the presence or absence of an object like a message encoded in a quantum state is inferred through quantum measurements without the object directly traversing the space be-tween sender and receiver.

CPTP map: completely positive and trace preserving map or operator also referenced as a quantum channel or superoperator. It is a linear operator that turns a density matrix describing a mixed state system into another density matrix. Its size is then the square of the density matrix size, so 2^{4N} for a system of N qubits. It can describe any operation on a mixed state system: some quantum gates, any sort of measurement, quantum filters, as well as feedback networks in quantum control theory.

Crosstalk: in the qubit field, is a phenomenon where an action on a given qubit or set of qubits has a side effect on other qubits. Of course, various techniques are employed to minimize it like, in the case of superconducting qubits, with using tunable couplers between qubits. Then, there are several crosstalk types like IX, IY, IZ, ZX, ZY, ZZ which depend on the type of interactions between qubits.

Cryogenics: cooling technology. Very low temperature cryogeny is used with superconducting and electron spin qubits computers. The temperatures required to stabilize qubits and reduce their error rate are very close to absolute zero: around 15 mK. The most commonly used systems are dilution refrigerators that use helium-3 and helium-4. Cryogenics is also used for photon generators and photon detection systems, but at a higher temperature situated between 2K and 10K.

CSCO: complete set of commuting observables, the most complete measurement of a quantum system comprised of compatible properties that can be measured in any order.

CVD: chemical vapor deposition, an additive manufacturing technique for semiconductors, where the target surface is exposed to one or more volatile precursors, which chemically react and/or decompose on the target surface to leave a thin film deposit on the target, e.g. using silane SiH_4 to deposit Si on the wafer, generating 2 H_2 molecules.

DAC: digital-analog converter. Classical electronic device converting a digital signal into an analog signal. Is used in the microwave generation systems implemented to control superconducting and electron spin qubits.

Dark count: photons detected by photon detectors that come from the environment and thermal or tunneling effect. This explains why most single photon detectors must be cooled at a temperature usually below 10K.

Dark state: aka "coherent population trapping state" or "dark resonance" is a specific quantum superposition of states in a quantum system that does not couple to a particular external field or interaction. The dark state remains unaffected by certain interactions that would normally cause transitions between other states.

Dark state protocols: methods used to create, manipulate, or utilize "dark states" that are special quantum superpositions insensitive to certain interactions. These are used in quantum optics, atomic physics, and quantum information processing to enhance precision and reduce sensitivity to noise. Examples include techniques like electromagnetically induced transparency (EIT, which can be used in quantum memories) and coherent population trapping (CPT, used in quantum clocks and magnetometers).

De Broglie wavelength: wavelength of a particle calculated with its momentum p with h/p, with h being Planck's constant.

Decoherence: marks the end of the coherence of a quantum object or a qubit. It is notably caused by the interactions between quantum objects and their environment. One often uses indifferently the expression coherence time (time during which the qubits are in a state of superposition and entanglement with other qubits) or decoherence (time at the end of which this superposition and entanglement end), which is the same.

Degenerate: a quantum system energy level is degenerate if it corresponds to two or more different measurable states with different quantum numbers. Mathematically, a quantum state is degenerate when several linearly independent eigenvectors may have the same eigenvalue. A normalized linear combination of these eigenvectors is also an eigenvector with the same eigenvalue. The number of linearly independent eigenvectors having the same eigenvalue corresponds to the degree of degeneracy of the quantum system. The number of different eigenstates corresponding to a particular energy level is the degree of degeneracy of the level. This happens for example when the energy level alone is not sufficient to characterize the state of a quantum system. That's where we need other quantum numbers to characterize the state. This is the case with the hydrogen atom electron. Its energy level depends only on its principal quantum number n (the electron layer), and not the three other electron quantum numbers (orbital angular momentum, magnetic moment and spin, although this degeneracy can be broken with using relativistic quantum mechanics and hyperfine structure splitting of electron energy levels). But you also have degenerate quantum error correction codes, which are supposed to correct more errors than they actually detect, particularly with noisy quantum channels (meaning practically qubits gates). Another example is an atom's nucleus energy level that is only dependent on its orbital angular momentum in the absence of magnetic field. Different energy levels arise with a magnetic field due to the nucleus magnetic quantum number.

Density matrix: matrix of complex numbers used to describe the statistical state of a physical system that is more precise than the computational state vector used in quantum computing. Density matrices are useful to describe so-called mixed states versus pure states that are sufficiently described with state vectors. They are used to describe what happens to subsystems of entangled systems, when decoherence happens and also, during measurement.

Dequantization: said about some quantum algorithm where an efficient classical equivalent is found. Term initiated with Ewin Tang's work on recommendation systems in 2018, when she found a dequantized classical equivalent to a quantum recommendation algorithm devised in 2016 by Iordanis Kerenidis and Anupam Prakash. Interestingly, quantization is a term used in artificial intelligence and deep learning when the numbers used in these models are integers (or even binary numbers) instead of floating point numbers.

Determinism: philosophical view that events are completely determined by previously existing causes. Applied to classical mechanics, this relates to the ability to predict an object's position and momentum based on its initial conditions. Contrarily, in quantum physics, it is not possible to determine simultaneously the position and momentum of any particle with precision at any instant. This indeterminism is also observed with quantum measurement of a quantum object in a superposed state. These concepts are also applicable to computing. Classical computing is usually deterministic. Same entry data, same output data. In quantum computing, one circuit execution doesn't yield the same result when implemented in the exact same way. The result is usually probabilistic, at least, when the result is not a single computational basis of the state vector.

Deutsch-Jozsa (algorithm): quantum algorithm created in 1992 by David Deutsch and Richard Jozsa. It can check whether a given function is balanced or not, i.e. whether it always returns 0 or 1, or 0 and 1 in equal proportions. The alternative between equilibrium (as many 0's as 1's) or not (as many 0's or 1's in output) is a starting postulate. The gain in performance compared to classical algorithms is exponential. In the case of N qubits, the function should be classically evaluated on at least half of the possible input values, i.e. $2^{N-1}+1$. Unfortunately, this algorithm is not very useful.

DFT (Density Functional Theory): mathematical model used to describe the structure of molecules at rest as a function of inter-atomic interactions. Used in high-performance computing as well as in quantum computing for chemical simulation.

Diffraction: phenomenon created when a wave encounters an obstacle or opening, like a small hole or slit. It is generated by the bending of photon waves around the corners of the obstacle. It creates interferences between the passing waves as they are detected in a plane further down the waves path. The phenomenon can be described classically with the Huygens–Fresnel principle that considers points in the hole or slit as a collection of individual spherical wavelets. The interference pattern shows up with laser light and can also be explained by the photon wave functions and their probability distribution. All-in-all, you can consider that a Young single-slit experiment also create quantum interferences!

Dilution refrigerator: name given to the very low temperature cryostats used to cool quantum computers below 1K. They cool superconducting or electron spin chips to respectively 15 mK and 100 mK. Dilution is related to the use a mixture of two helium isotopes (3 and 4), which are diluted in a mixing chamber, the two isotopes having slightly different physical properties. A helium 4 cryostat only goes down to about 2.8K, a helium 3 cryostat goes down to 300mK while a cryostat using both goes down to 10mK. The most common variant is the "dry" as opposed to "wet" dilution refrigerator. This version uses less helium and leaves more space in the chandelier to house electronic and quantum devices.

Dirac's notation: see bra-ket.

Dirac constant: Planck constant divided by 2π, also called reduced Planck constant and denoted $\hbar$ (h-bar). Some physicists called sometimes, abusively, this constant "Planck constant".

Discrete log problem: mathematical problem consisting in finding a log of a number that happens to be an integer. It is used in finding the solution of cryptographic problems with quantum algorithms. Shor's dlog algorithm is a quantum algorithm solving discrete log problems.

Distillation: technique used in quantum error correction codes based on magic states. It consists in combining several magic state qubits to feed others with a lower error rate. Distillation has the effect of purifying the state of qubits, meaning turning mixed states into pure states.

Doppler effect: shift in the electromagnetic spectrum due to the speed at which the source moves away from or closer to the observer. If the source moves away from the observer, the light wavelength is shifted towards the red (redshift), otherwise towards the blue. This effect is used in particular in the technique of atoms laser cooling. It consists in illuminating atoms that are in thermal motion with a wavelength that is just below the absorption level of the atoms. Those atoms moving towards the laser beam will absorb the photon, which will reduce their kinetic energy and movement. Those atoms moving in the other direction will not absorb it because the apparent frequency of the photon will be too low to change the energy state of the atoms. As atoms get cooled, the cooling laser wavelength has to be adjusted. This technique allows atoms to be cooled to below mK (milli-Kelvin).

D-Wave: Canadian company designing quantum annealing computers. They do not have the same power as universal gate quantum computers with equal numbers of qubits. The current generation of D-Wave "Advantage" using Pegasus chips includes 5,000 qubits.

Dressed state: resulting quantum state of an atomic or molecular system when interacting with an electromagnetic field. For example, when exposed to a continuous magnetic field or an electronic field, an atom can showcase additional energy level splitting, known as respectively Zeeman splitting and Stark splitting. One other case is when the external electromagnetic field is applied with a frequency close to the energy difference between the atom's original energy levels. The atom then undergoes Rabi oscillations between the dressed states.

EBL: electron beam lithography, a lithography technique that is focusing a beam of electrons on an electron-sensitive resist film to remove matter in specified areas, without requiring a mask like with photolithography. It is used to create 1nm precision nanostructures like with photon-generating quantum dots and also superconducting qubit chips. It's a rather slow process compared with photolithography that is adapted to low volume and custom productions.

Eigenstate: for a quantum object, these are the elementary wave functions in which it is possible to decompose it. They are represented by eigenvectors.

Eigenvalue: see eigenvector.

Eigenvector: for a square matrix A, an eigenvector x of A is a vector that verifies the equation $Ax = \lambda x$, λ being a real number called eigenvalue. Their direction do not change once multiplied with matrix A.

Electromagnetic spectrum: all electromagnetic radiation from the largest radio waves to X-rays and gamma rays. Visible light is only a very small part in the middle of this spectrum. An EM wave is decomposable in a number of photons, the smallest elementary unit of an EM wave.

Electron: elementary particle found in atoms, orbiting the nucleus, but also in freeform traveling between atoms and creating something we know as being electric current. According to Bohr's model developed in 1913, there is a finite number of electron orbits around the nucleus of atoms. The movement of electrons from one orbit to another corresponds to the absorption or emission of a photon. Electrons are elementary particles in the standard model because it is not composed of sub-particles, unlike neutrons and protons which are composed of quarks. According to quantum physics, electrons, as many other particles, behave as a particle and as a wave. Electrons are often used in qubits, in the form of electrons circulating in semiconductors loops or who are trapped in quantum dots or electromagnetic cavities and whose spin is controlled.

Electron gas: describes the behavior of free valence electrons in metals and semiconductors when they move around free of the atom nucleus. Their behavior is governed by the Pauli exclusion principle (1925), Dirac-Fermi statistics (1926) and Arnold Sommerfeld's quantum theory of metal (1927). Electron gas enables the modeling of electric conductivity, electron heat capacity and electric thermal conductivity as well as the Hall and quantum Hall effects. There are 1D, 2D and 3D electron gases. 1D electron gas are observed in semiconductor nanowires and carbon nanotubes. 2D electron gas show up in semiconductor quantum wells and in graphene sheets.

Elliptic Curves Cryptography (ECC): a type of public key cryptography that is potentially broken by Shor's quantum algorithm. One of its advantages is that it requires small keys, about three times smaller in number of bits than RSA public keys.

Energetics of quantum technology: cross-disciplinary research field and sector studying the energetics of quantum computing but also quantum telecommunications, cryptography and sensing. It's about making sure quantum technologies are not power hungry and also dealing with the energetic constraints related to quantum computing scalability. It's about balancing the act between cooling requirements, cabling, control electronics to ensure quantum computers can scale in number of physical and logical qubits.

Entanglement: quantum phenomenon where two quantum objects are related with each other in a way that a measurement done on these two objects generates a correlated (but random) value. Mathematically speaking, two quantum objects are entangled when their quantum state (psi, vector state) cannot be expressed as the tensor product of individual quantum states. This process is used to link qubits together through two or three qubit quantum gates in quantum computers. It is also used in quantum cryptography and telecommunication systems based on entangled photons in QKDs.

Entropy: measures the degree of disorder and randomness of a physical system. Key concept related to the second law of thermodynamics that states that the entropy of an isolated system cannot decrease spontaneously. In quantum mechanics, the (Von Neumann) entropy of a system is $-tr(\rho log\rho)$ where ρ is the density matrix describing the system state.

Erasure error: type of qubit error which can be easily detected and located. Qubits generating these errors can reduce the overhead tolerance implementation overhead. An erasure error usually com-pletely destroys a qubit like resetting it or maximally mixing it. The qubit superposition is gone but it is easy to detect it is gone. Various quantum error correction codes are designed to maximize the detection of erasure qubits which make it lighter particularly for syndrome detection classical computation. A qubit leakage is a form of erasure.

ERC Grants : European Research Council grants. Funding of European research projects with several levels, the top of which is the Synergy Grant which funds "moonshots" in European research associating at least two principal investigators (PIs) from public or private research laboratories. 14M€ is the maximum funding for such a project with 10M€ of core funding and 4M€ which can notably finance heavy investments or access to large infrastructures. Other levels include the Starting grants with up to 1.5M€ for 5 years and the Consolidator grants with 2M€ also for 5 years.

Ergodicity: capacity of a moving system to explore all parts of the space in which it can move in, in a uniform and random manner. The phenomenon occurs with many physical systems like electrons. Quantum ergodicity states that in the high-energy limit, quantum objects tend to uniformly distribute in the classical phase space.

Ergotropy: maximum amount of work that can be obtained from a quantum system.

Error Correction Codes: describes both logical methods and physical architectures to correct physical errors happening in both classical and quantum computing and telecommunication technologies.

Errors: a major concern in the operation of quantum computers. Operations on qubits: one and two qubit gates and qubit readouts generate errors that must be minimized. Error rates are in 2023 between 0.1% and 2% for two-qubit quantum gates. When several quantum gates are chained together, the rates of correct results (1 - error rate) multiplies quickly to the point of distorting everything. This is avoided either by reducing the physical level error rate like with cat-qubits, using shallow algorithms (low number of gates) or with error correction code systems. Errors also happen at qubit initialization (aka preparation) and readout.

Eta letter: η, used to describe error rates or efficiency, Carnot engine efficiency, and also Landau symmetry breaking.

Exclusion principle: see Pauli exclusion principle.

Expectation value: average or mean value of an observable. With an observable operator A on a quantum state ψ, the expectation value is $\langle a \rangle = \langle \psi | A | \psi \rangle$. In other words, it's a scalar product of the ψ vector and the vector resulting from the projective measurement of ψ using the observable A. In layman's term for a qubit, it is either the average value that would be obtained when doing an experiment, a large number of times and measure the value of the qubit yielding 0 or 1, and making an average, or the result of its mathematical evaluation if you have a clear idea of the qubit quantum state description. For example, after an Hadamard gate is applied to a $|0\rangle$ qubit, the expectation value of its measurement in in the typical z basis will be 0.5. Some algorithms output like in chemical simulations output real number values that are obtained through expectation value assessment using a large number of circuit runs.

Euclidean networks: class of encryption algorithms used in post-quantum cryptography (PQC).

Fabry-Pérot cavities: equipment used in lasers that combines two parallel mirrors, one of which is semi-reflective. This contributes to the creation of the laser effect in the cavity. The length of the cavity is generally a multiple of the laser light wavelength, at least if we want to emit coherent light with photons having all the same phase. The name of the cavity comes from the French scientists Charles Fabry (1867-1945) and Alfred Pérot (1963-1925).

FBQC: fusion-based quantum computation, a variant of MBQC crafted by PsiQuantum that is based on micro-clusters states with groups of 4 qubits connected together and using Resource State Generators (RSGs). It's replacing measurement of entangled states in MBQC with double measurement of non-connected adjacent qubits to create entanglements between them.

Feedforward: qualifies quantum computing and communications operations involving some quantum measurement and quantum teleportation protocols, where the measurement outcome can be random and will be used in subsequent operations. It is used in error correction protocols, to implement conditional quantum gates and with quantum teleportation, where the state of a qubit is transferred from one location to another, without physically transporting the qubit itself.

Fermi sea: electrons filling the lowest atom orbits or degenerate low-energy states within a solid and at very low temperature near 0 K. It corresponds to low-energy states that do not participate in materials thermal activity.

Fermi-Hubbard model: theoretical model in condensed matter physics that describes the behavior of interacting fermions on a lattice structure. It is named after Enrico Fermi and John Hubbard, who contributed to the development of the model. It helps understand the behavior of electrons in solids, where they interact with each other and move within a crystal lattice. It is used to study systems like strongly correlated materials, high-temperature superconductors, and ultracold atomic gases trapped in optical lattices. Fermi-Hubbard models are typically solved at a small scale with VQE algorithms running on NISQ quantum computers and also on analog quantum computers.

Fermions: particles with individualistic behavior. Two particles of this type cannot be in the same state at the same place. This includes electrons, quarks, half-integer spin composite objects. For example, deuterium, lithium-6, potassium-40 atoms (source: Jean Dalibard). In contrast, integer spin bosons such as photons and some atoms can accumulate in the same state. In a word, bosons are communists and fermions are ultra-liberals.

Fine structure: splitting of an energy level or spectral line into several distinct components that take into account electron spins and relativistic corrections to Schrodinger's wave equation.

Floquet code or Planar Honeycomb Code is a family of quantum error corrections codes created by Matthew B. Hastings and Jeongwan Haah from Microsoft in 2021. It simplifies toric codes with fewer qubits and stabilizers and is adapted to qubits architectures implementing pair-wise qubit measurements like with the elusive Majorana fermions.

Fluxonium: variation of flux superconducting qubit. It has a better coherence time than transmon, above 100 µs but two-qubit gates are more difficult to implement, and this architecture seemingly has not yet been tested beyond 10 functional qubits.

Flying qubits: qubits that can move, as opposed to stationary qubits that do not move. They are usually photons but there's a small branch of flying qubits studying flying electrons.

Fock space: mathematical object of algebra used to describe the quantum state of a set of identical particles whose number is variable or unknown. It is a Hilbert space made up of the sum of the tensor products of Hilbert spaces for the particles that make up the set.

Fock state: defines a group of quantum objects, like photons, who have the same quantum numbers and are indistinguishable. They are defined by their number, a photon number in the case of photons, and their common quantum numbers describing the quantum objects state.

Flux biasing: technique used to control with some direct current the resonant frequency of a frequency-tunable superconducting.

Fourier Transform: mathematical decomposition of a time domain signal into elementary single frequency signals with their frequency, amplitude and phase. It is a complex value function of time with, for each frequency, a magnitude (real part) and a phase offset (complex part) of the sinusoid of this elementary frequency. The inverse Fourier transforms that frequency decomposition function back into its original compound signal.

FPGA: Field Programmable Gate Arrays. Integrated circuit where some or all functions can be dynamically defined and program on-demand. It can have analog and digital features. Modern FPGAs also embed full-fledge processing units (Arm cores, GPUs, neural processing units, networking units). FPGAs are used in qubit control electronics for reading out the signals coming from the resonators attached to superconducting and electron spin qubits. It measures the phase and amplitude of the reflected microwaves after they are converted from analog to digital with an ADC (analog-digital converter) that can be embedded in the FPGA.

Fredkin gate: quantum gate operating on three qubits that inverts the state of the second and third qubit if the first qubit is 1. Also called CSWAP gate (conditional SWAP).

FTQC: Fault-Tolerant Quantum Computer. Error-resistant quantum computer that is based on logical qubits made of many physical qubits and implementing quantum error correction. Fault-tolerance is based on the error correction making sure errors don't propagate to other qubits. Also, errors must be corrected faster than they are created in the system.

FTDQC: Fault Tolerant Distributed Quantum Computation, and extension of the FTQC concept to distribution quantum computing.

Gate-based quantum computing: the broader category of quantum computing systems based on qubits and quantum circuits implementing quantum gates on 1, 2 and 3 qubits at a time.

Gate teleportation: application of a quantum gate on an unknown state while it is being teleported. It enables the creation of two-qubit gates from different systems that are connected through some entanglement resource, enabling distributed quantum computing.

Gaussian Boson Sampling (GBS): variation of a boson sampling experiment that uses continuous variables based Gaussian states photons as input. It is a physical model that is even more difficult to digitally simulate than a boson sampling since the underlying mathematical object is a Hafnian instead of a permanent, that is even more complicated to compute classically.

Gaussian state: describe particular photon states that are classical. The gaussian curve is the form in three dimensions of Wigner's function which describes the phase and amplitude distribution of the photon. It is opposed to non-Gaussian states which are non-classical, with some negative Wigner function values and a non-Gaussian form for the 3D function.

GHZ: means something other than giga Hertz in quantum computing! It is a multi-qubit Greenberger-Horne-Zeilinger superposed state that allows to demonstrate the inexistence of hidden variables in the quantum entanglement of at least three particles and with a finite number of measurements. The concept dates back to 1989 and has been experimentally validated in 1999. The typical experimental GHZ states use three qubits. A large GHZ state with many qubits is a typical large entangle state that could be use in photonic qubit computing based on the MBQC paradigm.

GKP qubits: error corrected qubits according to a method proposed by Gottesman, Kitaev and Preskill that encodes a qubit in a harmonic oscillator. It also works with photon qubits using linear elements for implementing Clifford gates.

Gleason's theorem: according to Andrew M. Gleason's theorem proven in 1957, the functions assigning probabilities to measurement outcomes are projection operators that must be expressible as density operator and follow the Born rule. This determines the way to calculate probabilities and the set of possible quantum states.

GPGPU: General Purpose Graphical Processing Unit, used for simulation, scientific computing and machine learning, like the Nvidia V100, A100 and H100. These are coprocessors which are mostly not anymore used for graphics software or gaming but more for machine and deep learning and scientific computing, thus the "general purpose" nickname addition.

Grotrian diagram: diagram used to show the various electronic energy transitions for a given atom, introduced in 1928 by the German physicist Walter Grotrian. The frequencies of transitions to higher energy levels provide an indication of their source like lasers (in the hundreds of nm wavelengths) or microwaves (in the 4-20 GHz frequency regimes).

Ground state: lowest energy state of an atom, other states being excited states. The hydrogen atom ground state happens when its electron occupies the lowest energy level (with main quantum number n = 1). More generally, is also said of a qubit that is in its ground basis state $|\mathbf{0}\rangle$.

Grover (algorithm): quantum algorithm for finding an element in a non-indexed array or a unique element for which an oracle function returns 1. Theoretically provides a polynomial acceleration which is deemed not that interesting compared to quantum algorithms providing an exponential acceleration.

Gurobi: classical optimization algorithm and software tool. It is today one of the faster classical solvers.

H-bar: see Dirac constant.

Hadamard (gate): quantum gate creating an even superposed state between $|\mathbf{0}\rangle$ and $|\mathbf{1}\rangle$ in a qubit when starting with $|\mathbf{0}\rangle$ or $|\mathbf{1}\rangle$. It is equivalent to a quarter turn around the y axis in the Bloch sphere representation of a qubit.

Hall effect: production of a voltage difference across an electrical conductor that is transverse to an electric current in the conductor and to an applied magnetic field perpendicular to the current. The effect was discovered by Edwin Hall in 1879.

Hamiltonian: operator used to determine the total and potential energy of a system of quantum objects when applied to the quantum object psi function. It is the global operator of the right part of Schrödinger's equation. Hamiltonians are everywhere in quantum physics. Among other places, it is used in analog quantum computers like D-Wave quantum annealers or Pasqal simulators. "Preparing a Hamiltonian" in this kind of computer is equivalent to setting up a matrix of qubits linked together by potentials and which will seek a minimum energy resulting in a balanced Hamiltonian corresponding to the solution of the problem to be solved. The solution is about finding the right combination of qubits states (up/down for quantum annealing) that minimizes the energy of the whole system.

Hamming distance: metric used to compare two binary data strings of equal lengths. It is the number of bit positions in which the two bits are different. For two strings a and b, it is denoted as d(a,b).

Harmonic oscillator: in classical mechanics, system that, when displaced from its equilibrium position, experiences a restoring force proportional to its displacement x with a frequency that does not depend on the amplitude. Quantum physics formalizes the whereabouts of many harmonic oscillators including photons in cavities, superconducting qubits, phonons, diatomic molecules, etc.

Hartree-Fock: approximation method to compute atomic structures using the time dependent Schrödinger's wave equation.

Heisenberg (principle of indeterminacy): fundamental principle of quantum mechanics which postulates that there is a lower limit to the precision with which one can measure two independent parameters relating to the same object such as its speed and position or the energy emitted and the duration of emission.

Heisenberg limit: in quantum sensing, like with interferometry, the optimal rate at which the accuracy of a measurement can scale with the energy used in the measurement. More precisely, not every quantity is a quantum observable that can be measured directly. The estimation of such quantity, however, can be performed by measuring a state whose probability distribution depends on it. To evaluate the accuracy of this estimation, one often considers the variance of the estimated quantity. When using, for instance, an ensemble of photons as the meter probing our parameter of interest, if these photons are allowed to be initially entangled, then this variance is lower bounded by the fundamental Heisenberg limit. As for the standard quantum limit, it implies that the more resources, the more accurate the measurement. However, only quantum probing resources can reach the Heisenberg limit which states that our estimation's standard deviation is at best inversely proportional to the size of the meter, hence here the number of photons. The Heisenberg limit can be reached in quantum sensing with using entangled objects with a precision that scales better at a rate of 1/N instead of $1/\sqrt{N}$ with the standard quantum limit. The Heisenberg limit is reached with using so-called NOON states superposing N (bosonic) objects (like photons) in a state with all the objects being in one or the other of two modes.

Helium 3: a rare isotope of helium that is used in cryogenic quantum computer systems to generate temperatures below 1K as part of dilution refrigeration systems. It is usually produced from tritium in specialized nuclear power plants, including the US Department of Energy's Savannah River nuclear power plant.

Helium 4: a common helium isotope that is also used in cryogenic systems operating at 3K to 10K temperatures.

Heralded single-photons: pairs of single photons can be generated in highly correlated states from using a single high-energy photon to create two lower-energy ones. One photon from the resulting pair is detected to "herald" (or "signal") the other so its state is pretty well known prior to its own detection or whereabouts. The two photons need not be of the same wavelength, but the total energy and resulting polarization are defined by the generation process. Two commonly types of heralded single-photon sources are SPDC (spontaneous parametric down-conversion with line width in the THz range) and SFWM (spontaneous four-wave mixing with line width in the MHz range or even narrower). It's used with QKD.

Heterodyne measurement: method used for extracting information from an oscillating signal along two orthogonal components in phase space like the in-phase and quadrature signals coming out of an I/Q mixer. In this type of measurement, two conjugate operators are measured simultaneously, which create added noise.

Hilbert (space): vector space of real or complex numbers with a Euclidean or Hermitian scalar product, which is used to measure distances and angles and to define orthogonality. It is an n-dimensional extension of the concept of three-dimensional Euclidean space. In quantum mechanics, the state of a quantum is represented by a vector in a Hilbert space with as many dimensions as the number of basic (or observable) states of this quantum. These are geometrical spaces which are used in particular to measure lengths and angles, to make projections on dimensions and to define the orthogonality between vectors. Continuous variable quantum properties belong to infinite dimensions Hilbert spaces.

Hidden variables: interpretation proposals of quantum physics based on the use of (yet) unobservable hypothetical entities what would explain phenomena like entanglement, nonlocality and describe the physical reality. Bell's theorem implies that local hidden variables of certain types cannot exist while being compatible with quantum physics postulates. This relates to the assumption, promoted by Albert Einstein in the famous 1935 EPR paper, that quantum physics is an incomplete theory that does not describe entirely the physical reality.

Homodyne measurement: method used for extracting information encoded as modulation of the phase and/or frequency of an oscillating signal. It compares that signal with a standard oscillation carrying no information. Homodyne detection uses a single frequency where heterodyne detection uses dual frequencies. Since we measure only one characteristic of the signal, it can yield a better precision than with heterodyne measurement which captures two characteristics using two conjugate operators.

HPC: high-performance computers, classical computers in the upper range of available power. A subset of these are supercomputers, which are ranked in the TCP500 list, updated twice a year.

HPQC: High Performance Quantum Computing, a quantum analogue of HPC (High Performance Computing). These are currently theoretical models of quantum mainframes comprising giant matrices of qubits that can be partitioned for shared use by several users.

Hubbard model: physics simulation model of mixed conducting and isolated systems based on a simple Hamiltonian. It is a typical physics problem that is used in testing the capabilities of quantum computers. See also Fermi-Hubbard model.

Hybrid quantum algorithm: an algorithm that combines classical processing running on classical computers and some processing performed on quantum computers, where needed. Variational algorithms are a category of hybrid quantum algorithm.

Hyperfine structure: small splitting of atomic energy levels or spectral lines with electrons with the same quantum numbers into several distinct components that are explained by the interactions between the nucleus and electron clouds.

Indistinguishability: relates to bosons quantum objects that have the same quantum state in a given location and are impossible to separate with any measurement tool.

Indistinguishable photons: see Indistinguishability, photons being a common type of boson.

Integrated Quantum Photonics (IQP): technologies exploiting photons as quantum information carriers and implemented on chips using wafer-scale fabrication, mostly in silicon-based CMOS or with III/V materials like gallium arsenide (GaAs) and indium phosphide (InP). IQP is used in quantum telecommunications and computing. It uses optical waveguides to guide and route single-photons, provides miniaturized split and phase control circuitry, entangled state generation, overall manipulation and sometimes even photons generation and photons detection.

Interference: fundamental phenomenon of quantum physics used with the wave aspect of quantum objects, when several waves can add or annihilate with constructive and destructive interferences.

Invertible computation: involves computations that run both forwards and backwards so that the forward/backward semantics form a bijection. In classical computing, it can correspond to some symmetric logical circuits that can process data forward and backwards with both ends used as inputs and outputs. It's used for example in MemComputing classical processors. The principle was created by Supriyo Datta from Purdue University in Indiana, USA.

Irreversible: said in computing of a calculation that makes it impossible to compute the initial values with using the result of the calculation. This is the case with all two-classical bits gates (NOR, OR, AND). Contrarily, quantum computing gates are mathematically reversible since relying on unitary transforms that, multiplied with their transconjugate, generate an identity operator. In plain language, if you apply a unitary (set of quantum gates) to a set of qubits, you can reverse this computing with the transconjugate of this unitary. Practically, it means playing in reverse order the gates initially applied. This technique is used in the uncompute trick that we describe elsewhere. Qubit measurement is also an irreversible process unless they happen to be in their $|0\rangle$ or $|1\rangle$ state and not entangled with other qubits.

Ion: non-neutral atom, which has a positive or negative electric charge. It is negative if its number of electrons exceeds the number of protons (anions) and positive in the opposite case (cations).

Ion traps: circuit devices used to control trapped ions in trapped ions quantum computers.

IonQ: an American startup from the University of Maryland that pioneered the first commercial quantum computers using trapped ions. Their operational record as of 2021 was 11 qubits with 32 qubits to be made readily available.

I/Q mixer: in phase and quadrature mixer, which adds two pulse signals of same frequency but with different amplitude and phase. The In-phase signal is a sinusoid, and the quadrature signal is a cosine. They have a delay of $\pi/2$ or 90°. When added up in the mixer, the sum of both signals create an arbitrary phase and amplitude signal of the same frequency.

Ising (model): a statistical physics problem that can be simulated and solved using quantum algorithms, especially on quantum annealers like those from D-Wave and quantum simulators ala Pasqal. It models the interactions between two-levels particles (spin, ferromagnetism). Some optimization problems can be mapped on an Ising model.

Isotopes: variations of atoms where the number of protons and electron is the same, sharing the same atomic number, but when the number of neutrons is different. For example, helium can exist in the for He^3 and He^4 with one and two neutrons. Many materials involved in quantum technologies are used with particular isotopes, like Si^{28} in silicon wafer used with electron spin qubits, the reason being the number of neutrons has an influence on atom nucleus spins, that can interfere with their electron spins.

IT: information technologies.

Jaynes-Cummings Hamiltonian: Hamiltonian used to describe the total energy of a system linking a resonator usually implemented as a coplanar waveguide (CPW) resonator with a superconducting circuit.

JJ: “jay-jay”, nickname for Josephson junctions.

Josephson (effect): physical phenomenon happening in a superconducting current loop traversing a thin insulating barrier known as a Josephson junction (JJ) like some non-superconducting metal thanks to the tunneling effect. It enables the creation of a multiple level energy or phase state for the superconducting current. This technique is used in superconducting qubits from quantum systems such as those of IBM and Google. It is also used in quantum sensing with SQUIDs (superconducting quantum interference devices) that are used as very sensitive magnetometers.

JPA (Josephson Parametric Amplifiers): simple amplifiers, using one or two Josephson junctions that are used for the first stage amplification of readout microwaves in superconducting or silicon spin qubits. Their narrow bandwidth prevents their implementation with frequency-domain qubits readout multiplexing.

Kerr effect: when some materials refractive index is modified in a nonlinear (quadratic, second-order nonlinear) manner as a function of the electric field applied to them. Is a variant of Pockels effect.

Ket: vertical vector describing in Dirac’s notation the state of a quantum object, with the symbols | and ⟩ forming a psi vector noted $|\psi\rangle$. It contains complex number amplitudes defining the relative weights in the computational basis. For a qubit, it’s a 2 complex numbers vector. For a register of N qubits, it’s a 2^N size vector of complex numbers defining the amplitudes of each combination of N 0s and 1s, which are orthogonal states in the 2^N state vector Hilbert space.

Kochen-Specker theorem: no-go theorem that states that it is impossible to assign simultaneously values with certainty to all observables in all possible contexts. This simple observation contradicts classical physics, where such an assignment is quite possible. It is the formal proof of quantum contextuality.

Lamb-Dicke regime: situation in which a trapped ion or atom's motion is much smaller than the wavelength of the laser light that is used to control it. This allows precise manipulation of the ion's quantum states. It is crucial for quantum operations, gates, and high-precision measurements in fields like quantum computing and precision metrology.

Larmor frequency: frequency of the Larmor precession (magnetic moment rotation). It is frequently mentioned in papers related to electron spins qubits.

Larmor precession: rotation of the magnetic moment of an object like an electron when it is exposed to an external magnetic field. This rotation happens along the axis of the magnetic field.

Laser: coherent light source invented in 1960 and used in many fields such as CD and DVD players, fiber optic communications, surgery, ophthalmology and dentistry, LiDARs. They are also often found in quantum computing to control cold atoms or manage photon-based qubits as well as in quantum cryptography and telecommunications (QKD & co). Laser means Light Amplification by Stimulated Emission of Radiation. It is a source of coherent light, i.e. it consists of photons of the same polarization, phase and wavelength, and emitted in the same direction in a narrow beam. Light amplification uses a process of stimulated emission in an amplifying active medium made of solid, fiber, liquid, gas or semiconductor which is placed in the center of a resonant optical cavity with a reflecting mirror on one side and a semi-reflecting mirror on the other side, which allows the light beam to exit. The wavelength and power of the light radiation depends on many parameters. The energy comes from an excitation or pumping system: primary laser, laser diode, flash lamp or electric discharge.

Leakage errors: error type corresponding to a qubit drifting and stabilizing in another energy state than the basis states $|0\rangle$ or $|1\rangle$. This can occur for example in the $|2\rangle$ level of a superconducting qubit. It is one category of erasure error that is relatively easy to detect and correct.

Leggett-Garg inequality: mathematical inequality fulfilled by macroscopic physical theories and systems. It says that a macroscopic object which has two or more distinct states, is at any given time in of those states. And it is possible in principle to determine which of these states the system is in without any effect on the state itself, or on the subsequent system evolution. This inequality is violated by quantum systems when superposition and entanglement are put in play like in interference processes.

Lie groups: mathematical objects in groups theory that are frequently encountered in quantum physics and quantum computing. They combine the notions of a group and a smooth manifold. Lie groups are used to describe symmetries and transformations in various mathematical and physical contexts. It is also used in quantum error correction codes.

Lindblad equation: equation describing the time evolution of the density matrix ρ of a quantum system that preserves the laws of quantum mechanics, meaning it preserves the trace and positiveness of the matrix. But the transformation is usually not a unitary due to decoherence. Also named a Lindbladian, a quantum Liouvillian, and in the long form, a Gorini–Kossakowski–Sudarshan–Lindblad equation (GKSL equation, for Vittorio Gorini, Andrzej Kossakowski, George Sudarshan and Göran Lindblad).

Linear algebra: branch of mathematics that is used in quantum physics and quantum computing. It is based on the manipulation of vectors and matrices within Hilbert spaces. In particular, the state of a sets of qubits is represented by vectors in a Hilbert space of size 2^N when N is the number of qubits. Computing with qubits consists in applying linear transformations.

Linear optics: field of quantum mechanics that manipulates photons based on their classical properties: polarization, phase or frequency.

LLM: large language model. These natural language processing models that fuel ChatGPT and the likes. These are classical tools which are not suited for a quantum computing implementation due to the size of the models, which store trillions of parameters.

Locality (principle): in classical physics, principle according to which distant objects cannot have a direct influence on each other. An object can only be influenced by its immediate environment. This principle derived from Albert Einstein's restricted relativity is questioned by quantum mechanics, nonlocality and quantum entanglement observed experimentally since at least 1982 with photons, in Alain Aspect's famous experiment (with Philippe Grangier and Jean Dalibard). But there are various interpretations of quantum physics which explain entanglement without resorting to nonlocality.

Logical Qubit: an assembly of physical qubits implementing hardware and software quantum error correction. Seen from the software developer's point of view, it creates a virtual logical qubit with a very low error rate. The fidelity of logical qubits depends in particular on the number of physical qubits they contain, the quality of the error correction codes and the qubits fidelity stability with the increase in the number of physical qubits.

LSQC: Large Scale Quantum Computing also frequently called FTQC for fault tolerant quantum computing. Category of future fault tolerant quantum computers. These will be based on the use of numerous physical qubits assembled into logical qubits with a very low error rate as seen from the software. Precisely, an LSQC implementing fault-tolerance has error corrections codes with at least two characteristics: it must not propagate errors broadly in the physical qubits and it must be able to implement non-Clifford group qubit gates like the single qubit gate T or the three qubits gate Toffoli. But LSQC definition is not clear yet. It could pertain to a large number of physical qubits (not necessarily arranged in logical qubits) or a large number of logical qubits, way beyond the first generations of FTQC. The jury's out to settle dusts with this terminology.

Magic states distillation: process that converts a set of noisy qubits into a smaller number of qubits with a lower noise. It is particularly useful for non-Clifford group quantum gates that bring universal computing power and exponential speedup. It is one of the ways to create fault-tolerant quantum computers but it has a high overhead cost with physical qubits. It was proposed in 2004 by Emanuel Knill, Sergey Bravyi and Alexei Kitaev.

Magneto-Optical Trap (MOT): device used to cool down and trap a cloud of neutral atoms. It uses a combination of magnetic trapping using two coils and Doppler effect in three orthogonal directions for cooling, the magnetic field strength being progressively adjusted to adapt to the speed of the cooled particles. The technique is used in cold atom interferometry (cold atoms gravimeters) and cold atom computers.

Majorana fermion: an electron-based quasiparticle in superconducting materials that could be used to manage reliable qubits in so-called topological computing. This virtual particle was imagined by Ettore Majorana in 1937. Microsoft intends to build a quantum computer based on these quasiparticles. But their very existence has not yet been really demonstrated.

Manifold: corresponds to the discrete controllable states of a quantum object.

Matrix: mathematical object made of rows and columns of values.

Matter wave: principle of quantum physics enacted by Louis De Broglie in 1924 according to which massive objects can also behave as waves. The De Broglie wavelength of a massive particle is the Planck constant divided by its momentum.

MBE: molecular beam epitaxy, a variety of PVD process (thin-film deposition), used to create a single orderly crystal structure in semi-conductor manufacturing. Is notably used to produce semiconductor quantum dots in III/V materials.

MBQC: Measurement Based Quantum Computing, a quantum computing method invented in 2001 by Robert Raussendorf and Hans Briegel that uses a high number of groups of pre-entangled qubits, called cluster states, embedded in two-dimensional grids in which qubit state readouts modify the grid structure and help create quantum gates. The last measured qubit gives the result of the algorithm. This technique is particularly useful with flying qubits like photons because it can be implemented in a highly parallel way and support the finite depth of quantum gates that these qubits enable.

Mesoscopic: subdiscipline of condensed matter physics that deals with materials of an intermediate size. The size ranges from a couple atoms to a µm.

Metal layers: in semiconductor chips, correspond to the layers containing wires connecting the various transistors and other electronic elements. These layers are surrounded by some insulator (silicon oxide or other). In typical CMOS processors, you have over 12 metal layers of decreasing density as you move father from the logic layer, In superconducting qubits, you have no metal layers on top of the Josephson junctions since it must be as isolated as possible.

Microring resonator: tiny optical waveguides looped back onto themselves in circle or spiral which enable interference phenomena, the creation of delay lines, and various other optical devices used for example in entangled photon generation.

Mode-locked laser: pulse laser generating streams of very short pulses of light formed of wave-packets in the picosecond to femtosecond range. These pulses are generated thanks to the emitted photons being synchronized in phase. A synonym of mode-locked is phase-locked!

Mott insulator: material that are expected to conduct electricity but are insulators, particularly at low temperatures, and under certain conditions which can be controlled, leading to so-called Mott transitions.

Mott transition: change in a condensed matter material's behavior from insulating to metallic generated by multiple factors like and ambient electric field changing the band structure of the material like with some metal oxides.

MVP: Minimum Viable Product. Concept used mostly in startups consisting in creating the simplest form of a product before starting to sell it. Opposite to full-fledged product with tons of R&D and an ever-lasting perfectionist approach.

MINLP: Mixed Integer Non Linear programming, a class of complex problems that can potentially be solved with quantum algorithms. It is about finding the minimum(s) of nonlinear functions and under constraints that aim to respect nonlinear functions. The variables in the equation are a combination of integers and floating-point numbers. The applications are numerous in all cases where one seeks to optimize a constrained function (energy distribution, optimum take-off of an aircraft, optimization of financial portfolio, minimizing risk in insurance or credit, etc.).

Mixed state: quantum objects state that is a classical statistical combination of several pure states. They can be prepared by physically associating several sources of pure states, like with merging two laser beams in one beam. A subsystem of an entangled quantum objects system is also a mixed state. A mixed state is mathematically represented by a density matrix operator, providing all the information that can be obtained about the related quantum system.

Momentum: physical property of an object or particle that for a massive particle is equivalent to its mass multiplied by its velocity. Usually denoted p. A (massless) photon has a momentum equal to their wavelength multiplied by Planck's constant.

Multimode: said of an optical fiber with a larger core (about 50 to 62 µm) where several light beams can be transported, usually with different wavelengths. Light propagation uses bouncing inside the fiber walls. These fibers are used for short distances communications of less than a kilometer and with bit rates reaching 200 GBit/s. The contrary of multimode fibers are monomode fibers. Also said of multimode photons, with an entirely different meaning and a way more complicated one, never explained in plain language by quantum photonicians. Its contrary is single mode photons. A single mode photon has one complex amplitude while a multimode photon is a mixed state of single mode photons with several independent complex amplitudes. If you want to know more, you get to use a complicated mathematical formalism.

Muonic atoms: atoms like hydrogen where the electron is replaced by a muon, which has the same electric charge and spin ½ but a mass 207 times larger and a smaller magnetic moment. Its life expectancy is very small, in the µs range. It decays as X rays and the muon is absorbed by the atom nucleus. Muons are generated in high-energy particle accelerators. They can help study the structure of atom nucleus.

Mutually unbiased bases: it is a concept mostly used in quantum key distribution. Bases is a set of orthogonal vectors in a Hilbert space. Mutually unbiased bases are two bases where the measurement of one of the vectors of one basis will yield a random result on the other base.

NISQ: Noisy Intermediate-Scale Quantum, a name for current and near future gate-based quantum computers, which are intermediate in terms of number of qubits (a few tens to hundreds) and subject to quantum noise that limits their capabilities. This acronym was created by John Preskill.

No-go theorem: theorem that demonstrates that a physical phenomenon is not possible. In quantum physics, famous no-go theorems are Bell's theorem and the Kochen–Specker theorem which con-strain hidden variable theories trying to explain nonlocality and entanglement with an underlying deterministic model featuring hidden states and variables. You also have the no-cloning and no-deleting theorems which prevents the cloning and deletion of a quantum object state.

Non-Clifford gates: said of quantum gates that are outside the Clifford group itself based on combining Pauli gates (half-turns in Bloch's sphere), Hadamard gates (quarter turns) and CNOTs for entanglement. To make things simple, non-Clifford gates enable the creation or arbitrary rotations in Bloch's sphere and their multi-qubits gates derivatives. The single qubit T gate (one eighth turn in Bloch's sphere) is the minimum additional gate, that, combined with the others, enable by approximation the creation of any arbitrary gate and unitary transformation.

No-cloning theorem: prohibits the identical copy of the state of a quantum. Therefore, it is impossible to copy the state of a qubit to exploit it independently of its original. Any copy destroys the original!

Nonlinear optics: field of optics where the optical properties of materials depend on the light amplitude and lead to the creation of new frequencies. Nonlinearity qualifies the response of a medium to an excitation that is generally quite energetic from intense fields, mainly from lasers, especially femtosecond pulsed lasers. In this case, the response of a material to the sum of two electromagnetic fields is not equal to the sum of the response to each individual field. Nonlinear optics can be used to create two-photon quantum gates with continuous variables photons. See also Chi.

Nonlocality: principle allowing a (quantum) object to somewhat and instantaneously influence the state of another (quantum) object at a distance, which can be very large. Contradicts the principle of locality, which means that an object can only influence another object at close range. Photons quantum entanglement at great distances verifies nonlocality when being measured. However, the initial quantum state of both objects is always random. So, it doesn't transmit some predetermined information per se from one place to the other.

NMR: Nuclear magnetic resonance, a type of qubit that was investigated in the 1990s and early 2000s and was then nearly abandoned. The reason is it didn't scale well at all and these were very noisy qubits and difficult to entangle. It was based on exploiting quantized states of atoms nuclei spins. However, the Chinese startup SpinQ is offering a desktop NMR-based quantum computer with 2 to 5 qubits. It is useful only for educational tasks.

Non-classical light: forms of light and electromagnetic fields treated as quantum systems. It contains single photon wave packets, pairs of entangled photons and squeezed states of light.

Non-demolition measurement: see QND.

Non-selective measurement: measurement that is physically done but not yet read. Its outcome is not available either because it wasn't yet used or because it is inaccessible when measurement is done by the environment.

NOON state: many-body entangled state superposing N quantum objects, usually bosons, in two modes. Namely, it superposes all the objects in one mode and all the objects in the other mode. This kind of superposition is used in quantum metrology to obtain a precision reaching the Heisenberg limit, which is better than a standard quantum limit based measurement.

Normalization: in quantum physics, normalization is used in many situations like with scaling wave functions so that the sum of probabilities equal one. This 1 is considered as a normalization constant or constraint. Born's rule is a normalization constraint.

NP (problem class): class of problems whose solution is verifiable in a polynomial term relative to the size of the problem. It includes the so-called exponential or intractable problems, whose solution time is exponential with respect to their size. A quantum computer is supposed to solve some NP problems in a tractable way, meaning, not exponential time.

NP-complete (problem class): decision problem for which it is possible to verify a solution in polynomial time and for which all problems of the NP class are reduced to it via a polynomial reduction. This means that the problem is at least as difficult as all other problems of the NP class. The problems of the traveling salesperson and the knapsack problem are Complete NP problems. The concept dates from 1971 and comes from Stephen Cook.

NP-difficult (problem class): problem to which any problem of the NP class can be reduced by a polynomial reduction. If it is also in the NP class, it is said to be an NP-complete problem. If P ≠ NP, then NP-difficult problems cannot be solved in polynomial time.

Observable: equivalent in quantum mechanics of a physical quantity in classical mechanics, such as position, momentum, spin or energy. In quantum physics, an observable is a mathematical operator used for the measurement of one property using its quantum state description.

ODMR: optically detected magnetic resonance is a quantum sensing testing is a double resonance technique where the electron spin of a crystal defect like a NV center is optically pumped for initialization and readout with a green laser light. It radiates some red light or nothing depending on the cavity electrons spin. It uses the Zeeman effect in unpaired electrons. With NV^- centers, it is used for high-precision magnetometry and medical imaging with a sensitivity ranging from 10^{-9} to 10^{-15} T/√Hz, the unit of magnetometry precision.

On-premises: said of hardware that sits in a customer site or datacenter. It is the opposite of sitting in a datacenter from a cloud vendor. Often simplified as 'on prem'.

Optical molasses: gas of cold neutral atoms whose cohesive strength is of the viscous type. It is cooled with lasers using the Doppler effect, usually with three pairs of lasers in three orthogonal directions.

Optical pumping: technique used to modify the states of atoms by increasing their energy level using polarized photons. Alfred Kastler, invented it in 1950 and was awarded the Nobel prize in Physics in 1966. The technique is used in lasers and quantum sensing. Optical pumping passes through three to four energy levels of atoms (E0, E1, E2, E3). Pumping moves an atom from its fundamental level E0 to E3. A (mechanical) relaxation brings the atom back from the E3 state to E2. In lasers, this generates a population inversion between the E1 and E2 states, so that there are more atoms in the E2 state than in the E1 state. The spontaneous and stimulated emission of photons of E2-E1 energy can then take place. The atom in the E1 state then returns to the E0 state by relaxation.

Orbital angular momentum (OAM): quantum number for electrons, nucleons and photons. With electrons, it describes in a quantized manner the shape and size of the subshell where the electron sits. It has a different meaning with photons. Discovered in 1992 by Les Allen et al from Leiden University, photon OAM is more difficult to visualize than spin angular momentum. With OAM, the photon itself rotates along its propagation axis or vector. One analogy with the Earth is its own rotation (spin angular momentum, defining days) and its rotation around the Sun (orbital angular momentum, defining years). This orbital angular moment is quantified with integers times the reduced Planck constant. It can be any integer! One record OAM number of 10.100. Being quantized, it can lead to superposition and entanglement. It can also be used to encode information on fibers.

P (problem class): problem that can be solved in polynomial time with respect to its size, on a deterministic Turing machine.

Paramp: parametric amplifier using a parametric nonlinearity and a pump wave. Paramps exist for photos in the visible spectrum (these are OPA for optical parametric amplifiers) as well as for microwaves. In this last case, they are used to amplify readout microwaves from superconducting or silicon spin qubits. The most recent breed of paramps are the TWPAs.

Pauli (exclusion principle): postulates that two fermion particles cannot be in the same quantum state. Two electrons or two neutrons cannot be in the same place with the same energy level. If an external force such as gravitation forces them to be in the same place, they cannot have the same energy, i.e., the same speed. If a set of fermions has to be in the same place, they will have to have different velocities. Fermions have half-integer spins. Half-integer of what by the way?

Permanent: real number resulting from n! additions of multiplications of n values of a square matrix n*n. They are used to evaluate the complexity of matrices representing graphs.

Phase Estimation Algorithm: algorithm created by Alexei Kitaev in 1995 and used to find the phase of an eigenvector of a unitary operator U. This algorithm is based on an inverse QFT. It is used as part of period finding in Shor's factoring algorithm and in quantum chemistry algorithms. Also named a QPE for quantum phase estimate.

Phase: an important physical property of quantum objects given they all can behave as waves. It explains interferences between all sorts of quantum objects, like electrons on top of photons. Phase is a continuous quantum property.

Phasor diagram: two-dimensional diagram describing electromagnetic field quadratures positioning the statistic characteristics of a photon source, with X1 and X2 orthogonal axis corresponding to two oscillating electric fields that are out of phase by 90°.

Phonon: collective excitation in a periodic, elastic arrangement of atoms or molecules in condensed matter, specifically in solids and some liquids. In quantum information technologies, it is mostly used with trapped ions to provide a n-to-n connectivity between qubits.

Photoelectric effect: emission of electrons from a material like a metal when electromagnetic radiation above a certain minimum frequency strikes it, independently of its intensity. Formalized by Albert Einstein in 1905.

Photolithography: patterning process in semiconductor manufacturing used to define in which zones matter must be removed or added in subsequent steps. It uses ultraviolet rays illuminating a photomask that exposes a photoresist film or coating. For very high densities, the exposure is done with extreme ultra-violet waves. The related manufacturing tools are now produced by a single company in the world, ASML (the Netherlands).

Photon: quantum of energy associated with electromagnetic waves ranging from radio waves (long waves, low frequencies) to gamma rays (very short waves, very high frequencies) through visible light. Its mass is zero. Its spin is 1 and it is therefore part of the bosons. Photons are absorbed an emitted by atoms during energetic levels changes.

Photon measurement: measurement of a photon where the degree of freedom is the excitation quanta. It can yield a number of superposed photons that we try to detect, but not their characteristics like their phase or frequency, which requires homodyne or heterodyne measurement.

PKI: Public Key Infrastructure, set of roles, policies, hardware, software and procedures used to create, manage, distribute, use, store and revoke digital certificates and manage public-key encryption.

Planck constant: fundamental constant of quantum physics ($h=6.626 \times 10^{-34}$ Js). Created in 1900 with Max Planck's explanation of black body radiation spectrum and then used in most other quantum physics equations, including Schrodinger's wave equation.

Pockels effect: effect used in optical modulators where a medium refraction index changes in a linear manner as a function of the electric field applied to it.

Polarized Beam Splitter (PBS): class of beam splitters that use birefringent materials to split light into two beams of orthogonal polarization states.

Polaritons: quantum quasiparticles with strong interactions between light and matter in semiconductors. It results from the coupling between photons and an electrical polarization wave which occurs in particular in plasmons (oscillations of free electrons in metals), phonons (oscillations of atoms, especially in crystalline structures) and excitons (pairs of electron holes generated by photons in semiconductors).

Postselection: process of conditioning the outcome of a quantum measurement on the value of some other qubit or on a pre-given probability distribution.

POVM: Positive Operator-Valued Measure, quantum measure generalizing Projection-Valued Measures (PVMs) which is useful when the measurement basis is not made of orthogonal states in their Hilbert space. POVMs that are not PVMs are called non-projective measurements. They have many use cases like enhancing quantum states tomography, help detect entanglement and allow unambiguous state dis-crimination of non-orthogonal states, with applications in quantum cryptography and randomness generation.

PQC: Post Quantum Cryptography, cryptography resistant to quantum computers-based codebreaking algorithms. It is based on the use of public keys that are not decomposable with conventional or quantum computers.

PQS: Programmable Quantum Simulator, or analog quantum computers.

Prepare-and-measure: type of quantum key distribution protocol (QKD) that is not relying on quantum entanglement but on sending photons from Alice to Bob. The key is prepared by Alice, using some random number generation source, like a QRNG. It is an actual key distribution while entanglement based protocols generate a random key that comes from correlated measurements of entangled photons by Alice and Bob.

Private Key: key used in private key encryption systems. Keys are exchanged beforehand by the parties using an encryption algorithm, often hash or Diffie-Hellmann algorithms.

Property: physical characteristic of a physical object. In quantum physics, observables are the mathematical operator used to compute properties values using the quantum object state vector. For a photon, it can be for example its phase, polarization and wavelength. In quantum physics, it's not possible to evaluate the values of all properties of quantum systems to describe it, due to Bohr's complementarity principle.

Public key: an encryption system that involves sending a public key to an interlocutor who will use it to encrypt a message sent in the other direction. The elements used to create this public key are used to decrypt the message sent.

It is normally impossible or very difficult to decompose the public key to find the elements that were used to create it. PQCs are based on public keys.

Purcell effect: relaxation or loss of energy of a superconducting qubit through its readout resonator. More generally, it's the enhancement of a quantum system's spontaneous emission rate by its environment, discovered in the 1940s by Edward Mills Purcell with the spontaneous emission rates of atoms when incorporated into a resonant cavity.

Purcell filter: high and low-band filter that reduces the Purcell effect between a superconducting qubit and its readout resonator.

Pure state: quantum state of an isolated quantum system of one or several objects constructed as a linear superposition of the states from its computational basis.

Purification: process applied to a mixed state which integrates it in a larger system to create or reconstruct a pure state. It can be applied to a set of entangled qubits as well. It is used in some error-correcting codes both in quantum telecommunications and quantum computing.

PVD: physical vapor deposition, is a material deposition technique in semiconductor manufacturing where the material to deposit is first turned into vapor and then condenses on the target surface. There are various PVD methods like sputtering that is using ion projection to pull material from a source and deposit it on the target,

PVM: Projective Value Measurement, used in quantum computing, consists in doing a geometrical vector projection of your qubit pure state on any axis in the Bloch sphere.

Q factor: quality factor, dimensionless value defined as the ratio between the energy stored in a resonator and the energy dissipated per oscillation cycle times 2π. With the frequency of the oscillator, it provides an indication on the oscillator lifetime. In a superconducting qubit, it characterizes the stability of its oscillation and determines its T_1 or relaxation time. The greater the Q factor is, the longer the T_1 is. The higher the better, this factor can exceed 10^7. The dissipation comes from cavity losses and depends on the materials and structure of the electromagnetic cavity. Another definition for Q factor for an oscillator is the ratio between the main resonance frequency and its bandwidth.

QAOA: Quantum Approximate Optimization Algorithm created by Edward Farhi in 2014 which is a combinatorial optimization algorithm finding approximate solutions with graph and slice management problems (Max-Cut), various tasks and jobs scheduling problems like the Binary Paint Shop Problem (BPSP), TSP (traveling salesperson problem) as well as for solving 3SAT Boolean satisfiability problems.

QAOA: Quantum Alternating Operator Ansätze (QAOA), the ansatz circuit that is used within a variational algorithm, alternating single qubit rotation gates and CNOT gates. So a QAOA relies on a QAOA… !

QCKA: see Quantum Conference Key Agreement.

QCaaS: quantum computing as a service, a fancy acronym for quantum computing running in the cloud.

QFHE: Quantum Fully Homomorphic Encryption. A method of quantum information encryption allowing to perform processing on encrypted data.

QFT: Quantum Fourier Transform. Quantum variation of the Fourier transform. The classical Fourier transform allows to decompose a signal (as in audio) into frequencies (or frequency spectrum). The QFT does this on a sequence of integers and determines its largest observable frequency.

QIP: Quantum Information Processing, a name sometimes used to information tools based on second-generation quantum technologies. It contains quantum computing, quantum simulation, quantum cryptography and quantum telecommunications.

QHO: quantum harmonic oscillator.

QIR (Quantum Intermediate Representation): an intermediate representation for quantum programs launched in September 2020, serving as a layer between gate-based quantum programming languages and target quantum computers. It can also be used to run code on an emulator. It is supported by the QIR Alliance launched in November 2021 and is part of the Linux Foundation. The Alliance founding members are Honeywell, Microsoft, the DoE Oak Ridge National Laboratory, Quantum Circuits Inc. and Rigetti Computing.

QKD: Quantum Key Distribution, a secure protocol for sending symmetrical keys via an optical link based on quantum entanglement (fiber or satellite). These keys are tamper-proof, or at least an interception of the key is detectable.

QLM: Quantum Learning Machine, name of the Atos quantum emulator appliances using classical hardware (Intel and/or Nvidia).

QMA: Quentin Merlin Arthur, a class of problems that is verifiable in polynomial time on a quantum computer with a probability greater than 2/3. It is the quantum analogue of the "traditional" NP complexity class. QML: Quantum Machine Learning. Branch of quantum algorithms used in machine learning.

QML: Quantum Machine Learning. Class of quantum algorithms implementing machine learning or deep learning techniques.

QND: quantum non-demolition measurement, a sequence of measurements where results are completely predictable using the result of the first measurement. Practically, it stays the same. Let's say we measure the state of a qubit that yields a $|0\rangle$ or a $|1\rangle$. The measurement is a QND one if a new measurement will yield the same $|0\rangle$ or a $|1\rangle$ coming out of the first measurement and so on. In mathematical parlance, it means the measurement observable commutes with itself at different times. How about a destructive measurement of a qubit? It can happen for example with a photon detector which absorbs it. The photon is then entirely destroyed (and converted to some current in the detector) and cannot be measured a second time. Usually, a QND signal is quantum and extremely weak and is obtained with a quantum probe.

QRNG: Quantum Random Number Generator, the optical random number generators used in quantum cryptography, like those of the Swiss IDQ.

QSVT (Quantum Singular Value Transformation): quantum algorithm that performs a polynomial transformation of the singular values of a linear operator embedded in a unitary matrix. Was created by András Gilyén, Yuan Su, Guang Hao Low and Nathan Wiebe in 2018.

Quantization: in quantum physics, happens with quantum objects having some physical properties that are discontinuous and not continuous, like electron energy levels and electron spins.

Quantum accelerator: quantum computer used as a complement to a supercomputer or HPC, usually to run hybrid algorithms like VQE (Variational Quantum Eigensolvers) combining a classical part that prepares the data structure that feeds a quantum accelerator.

Quantum advantage: occurs when a quantum computer executes some processing faster than its optimum equivalent adapted to a supercomputer, with a useful algorithm. This advantage can be declined on another aspect than the duration of the calculation. For example, a quantum energy advantage relates to energy consumption instead of computing time.

Quantum annealing: technique used to find the global minimum of a given objective function over a given set of candidate solutions, based on using quantum fluctuations. It is used to solve combinatorial optimization problems with a discrete search space. This computational process is in D-Wave quantum computers. It doesn't use any thermodynamics effect but rather a magnetically induced annealing process, relaxing a transversal magnetic field controlling the qubits state.

Quantum channel: transformation of a quantum state resulting from any kind of interaction with a quantum environment. It is modelized with a density matrix super-operator. It is useful to modelized subsystems, decoherence, quantum error correction and qubits noise.

Quantum Chaos: branch of physics studying how chaotic classical dynamical systems can be described with quantum theory. It deals with the relationship between quantum mechanics and classical chaos and with the boundaries between classical and quantum physics in modelling chaos.

Quantum chromodynamics: describes the strong interaction, one of the four fundamental forces, that governs the interactions between quarks and gluons and the cohesion of atomic nuclei. Why "chromo"? Because we describe the states of elementary particles with color codes: blue, green and red for particles, then anti-blue, anti-green and anti-red for anti-particles. This theory is based on the quantum field theory. This part of quantum physics is not used in the creation of qubits. It is used for the physics of elementary particles and is verified in large particle accelerators such as the CERN LHC in Geneva.

Quantum circuit: see circuit.

Quantum cognition: descriptive model of the functioning of human knowledge (language, decision-making, memory, conceptualization, judgment, perception) based on the mathematical formalism of quantum mechanics, proceeding mainly by analogy, without going through physical explanations or quantification of the neurosciences, which themselves fall within the "quantum mind" field resulting from the work of Roger Penrose.

Quantum Conference Key Agreement (QCKA): extension of QKD protocols to enable the generation of quantum secret keys among several users. The protocol makes use of N-dimension entangled GHZ states share across the parties.

Quantum dots: we can mention at least three different types of quantum dots: the powders used in LCD screens that convert the blue backlighting LED light into green or red light based on their grain size. Then we have the quantum dots used to generate single photons. Finally, we have quantum dots used to trap electron spins in spins qubits.

Quantum Electro-Dynamic (QED): branch of quantum physics, or QED, which is "*a physical theory that aims to reconcile electromagnetism with quantum mechanics using a relativistic Lagrangian formalism. According to this theory, electric charges interact by photon exchange*" (Wikipedia). This is the basis of the quantum field theory which applies to all elementary particles.

Quantum emulator: a software and/or hardware system using a conventional computer to run and test some software programmed for a quantum computer. This makes it possible to test quantum programs without a quantum computer. The execution speed is not as good as on a quantum computer as soon as you exceed a few tens of qubits. And beyond about fifty qubits, the capacity of classical machines is insufficient to perform it properly. Emulation should not be confused with quantum simulation, which simulates quantum physics phenomena with an analog quantum processor like those using cold atoms. But a quantum emulator can use a digital quantum simulator to physically simulate the underlying qubits used by the emulator. In classical computing, a software emulator usually runs code made for an older and incompatible hardware system. In the case of quantum computing, the historical time is reversed since we are emulating code designed for future quantum computers, particularly using logical qubits with error correction.

Quantum engineering: is about developing quantum technologies in computing, telecommunications, cryptography and/or sensing with a pluridisciplinary approach merging quantum physics and other related sciences and technologies like thermodynamics, cryogeny, electronics, semiconductors, cabling, mathematics, information theory, programming and the likes.

Quantum Fisher Information (QFI): fundamental limit of precision in quantum measurements which are subject to intrinsic limitations due to quantum uncertainty. It quantifies how well a parameter of interest can be estimated in a quantum system and the level of multipartite entanglement in use.

Quantum foundations: branch of science philosophy that aims to build some understanding and description of the real world in quantum physics and, as such, associate it to some ontology.

Quantum gates: operations modifying the state of one or several qubits. Multi-qubit gates (Toffoli, Fredkin, ...) exploit the principle of quantum entanglement. The operations of quantum gates are generated by physical actions on the qubits which depend on their nature. For superconducting qubits, this involves sending microwaves between 5 and 10 GHz via electrical conductors. For trapped ions, these are laser-controlled operations. For electron spins qubits, these are a mix of electrical voltages and microwave pulses. For qubits based on mass particles (electrons, ions, cold atoms), quantum gates act on the qubits but these do not move in space. For flying qubits based on photons or electrons, these circulate and cross quantum gates which modify their state (phase, frequency, or other).

Quantum Hall effect (QHE): or integer quantum Hall effect is a quantized version of the Hall effect which is observed in two-dimensional electron systems at low temperatures and under a strong magnetic field, in which the Hall resistance Rxy has quantized values. It is related to the field of quantum matter. The effect was discovered by Klaus von Klitzing from the MPI in Germany in 1980 and was awarded the Nobel prize in physics in 1985.

Quantum hydrodynamics: studies the hydrodynamic effects of quantum systems such as superfluid elements (helium at very low temperature) or polaritons and associated light fluids.

Quantum Internet: marketing term describing a quantum network enabling quantum telecommunications based on entanglement, particularly to connect quantum systems like quantum computers and quantum sensors. By extension, it also includes quantum key distribution infrastructures that are used to secure information exchange with encryption keys that are shared quantumly between senders and receivers.

Quantum jump: describes the continuous change happening during the measurement of a quantum object. It is not instantaneous.

Quantum kernel: function computing a kernel matrix, given a set of datapoints x and y, it looks like the dot product $\langle f(x), f(y)\rangle$, f being a function mapping data from a low dimensional space (n) to a higher dimensional space (m). Quantum kernel matrices are used in quantum machine learning algorithms for various tasks like SVM, PCA, classification and regressions.

Quantum medicine: in general, fake science and charlatanism based on totally fanciful interpretations of quantum mechanics.

Quantum Non Demolition measurement (QND): type of measurement in which the uncertainty of the measured observable does not increase from its measured value during the subsequent normal evolution of the system. QND measurements are the least disturbing type of measurement in quantum mechanics. In other words, for a qubit, it would mean that after a $|\mathbf{0}\rangle$ or $|\mathbf{1}\rangle$ is measured, subsequent measurements will always yield the same $|\mathbf{0}\rangle$ or $|\mathbf{1}\rangle$ that was obtained in the first place.

Quantum number: variables describing quantum objects physical quantities or variables that are discrete. Electrons have four quantum numbers: principal quantum number (energy level or electron shell), angular momentum also named azimuthal or orbital quantum number describing electron subshell, magnetic quantum number describing the electron energy level within its subshell and spin projection quantum number, being either +1/2 or -1/2, in a given spatial direction.

Quantum postulates: basis of quantum physics formalism. These are postulates and not laws because it describes a mathematical formalism that cannot be proved per se. There are many different presentations of quantum postulates in reference sources (Nielsen & Chuang, Preskill, Cohen-Tannoudji, Wikipedia, ...). Depending on the sources, you'll find 3, 4, 5, 6 or even 7 of them.

Quantum Physical Unclonable Functions (qPUF): quantum based physical identifiers that can be used to create unique and unclonable security keys.

Quantum reservoir computing: specific category of recurrent neural networks used to process time series. It uses a set of neuron weights and links between neurons randomly fixed in the reservoirs, all with nonlinear activation functions. The hundreds of neurons in a reservoir are fed by input data stored in the reservoirs. The activation functions nonlinearity makes this memory evanescent. The training parameters of these networks are located in the weights of the neurons that connect the reservoirs to the output data

Quantum reservoir engineering: set of techniques for managing qubits through their interaction with a "quantum thermal bath" (quantum bath) to reduce energy consumption, reduce the duration of the measurement of the state of the qubit and allow a non-destructive and reversible measurement of this state ("Quantum non-demolition" or QND). Reservoir engineering is used in cat-qubits.

Quantum simulator: system that is able to compute the physics of a many body quantum system. It can be done on a classical computer of a quantum computer. In that case, it refers to analog quantum computers that are capable of simulating quantum objects and solving related problems, particularly in materials physics. By abuse of language, the name is used for classical computers capable of emulating quantum code. In this case, it is preferable to use the term quantum emulator.

Quantum speed limit: concept that sets a limit on how quickly a quantum system can evolve from one state to another, including during measurement. It provides a boundary on the minimum time required for a quantum system to transition between states while still obeying the laws of quantum mechanics. There's a general trade-off between the energy uncertainty and the time it takes for the transition to occur. The quantum speed limit can be expressed in terms of the system's Hamiltonian (or energy operator) and the state's initial and final states. It involves Planck's constant and the difference in energies between the initial and final states.

Quantum state: mathematical object used to compute at a given time the probabilities of a quantum object or set of object property values that would be obtained when measuring it and to predict their evolution over time. It is usually represented by a vector in a Hilbert space (linear, metric and complete). This is however only the case for a pure state. A mixed state is represented by a density matrix. The notion of quantum state is usually the first quantum postulate.

Quantum state tomography: technique used to characterize the quality of qubits and qubits gates or any quantum channel. It is used to experimentally reconstruct a density matrix of a set of qubits. It also requires a lot of classical computing to process the experimental data obtained with repeated state preparation and measurements.

Quantum Steering: quantum measurement phenomenon when one subsystem can influence the wave function of another subsystem by performing specific measurements.

Quantum supremacy: describes a situation where a quantum computer can perform some computation that is inaccessible to the best current supercomputers with the best classical algorithm and in a humanly reasonable time. The computing time differential between quantum computing and classical computing must be several orders of magnitude. It can deal with a useful calculation or not. Thus, the quantum supremacy claimed by Google in October 2019 dealt with a random algorithm that had no practical interest. The term was coined by John Preskill in 2011. Nowadays, the trend is to use the quantum advantage denomination.

Quantum switch: consists in creating a series of qubit transformations that can be implemented simultaneously in different orders, creating an indefinite causal order computing flow.

Quantum teleportation: technique used to transfer the state of one qubit to another location. It is usually performed with three communication links: a pair of previously entangled photons and two classical bit links. It has many uses such as in quantum cryptography (QKD). The no-cloning theorem also says that the state of a teleported quantum disappears from the source after teleportation. It can be used to transmit a rich quantum state of several qubits and can enable distributed quantum computing.

Quantum trajectories: describe the continuous time evolution of quantum systems and their quantum state, for example when executing a quantum gate.

Quantum variational circuits: type of quantum algorithm used to implement machine learning.

Quasiparticles: physical concept which treats elementary excitations in solids like spin waves, as particles. As the particles do not consist of actual matter, they are called quasi particles. Majorana fermions and polaritons are examples of quasiparticles.

Qubit or physical qubit: the elementary unit of information in quantum computing in quantum computers and quantum telecommunication. It stores a quantum state associating two distinct states of a particle or of a quantum system (electron spin, energy level of a superconducting loop, energy level of a trapped atom or ion, polarization or other property of a photon). Its mathematical representation is a vector comprising two complex numbers in a Bloch sphere.

QUBO: Quadratic Unconstrained Binary Optimization problem. It is a generic NP hard combinatorial optimization problem and its related algorithm which can help solve many applications in finance, logistics and other domains. Many classical combinatorial problems like maximum cut, graph coloring and the partition problem, can be turned into a QUBO problem. QUBO problems can be solved on all three quantum computer paradigms (gate-based, annealing, simulation).

Qudits: generic form of qubit that has d possible quantum states instead of two. The approach is rarely used, at least in quantum computers outside research laboratories.

Qunat: another name for qubits based on continuous variables.

Qutrit: it is a form of qubit which instead of having two possible quantum states, has three. It is a special case of qudits.

Rabi (oscillation): oscillations between states of a two-level system excited at a frequency close to its resonance. This phenomenon is observed between two spin states in nuclear magnetic resonance as well as when an electric field acts on the transitions from one electronic state of a system to another for an atom or molecule. The curve describing the oscillation resembles a sinusoidal curve that attenuates over time. Isidor Isaac Rabi is an American physicist of Hungarian origin (1898-1988) who was awarded the Nobel prize in Physics in 1944. Rabi's oscillations can be found almost everywhere, especially in the operation of superconducting qubits with microwave pulses.

Raman cooling: variant of the Doppler effect using the Raman effect used to cool atoms below the limit of Doppler-based cooling, under 1μK. It uses two counter-propagating laser beams. This effect is used in cold atom based interferometry in absolute gravimeters. Also known as Raman sideband cooling.

Raman effect or Raman scattering: shift in wavelength of an inelastically scattered radiation where an incident monochromatic photon energy and momentum are both changed. Discovered by Chandrasekhara Venkata Raman (1888-1970, India), Nobel prize in Physics in 1930. This is a small effect that accompanies the predominant Rayleigh scattering of light (unchanged wavelength). The incident polarized light is scattered at its original frequency (Rayleigh elastic scattering) and with higher and lower frequencies (Raman stokes and anti-stokes anti-elastic scattering).

Raman spectroscopy: determines vibrational and rotational level spacings from the energy (wavenumber) shifts of inelastic scattered light (*aka* Raman scattering). It is used to analyze multi-atoms molecules through their vibrational modes, particularly in organic chemistry.

Raman transition: couples two atomic levels by the absorption of a photon in one Raman beam (pump beam) and by stimulated emission of another one in the other beam (Stokes beam). It is used in cold atom interferometry to split a cold atom cloud into two superposed matter waves of different energy levels and vertical velocity.

Ramsey experiment: technique used to measure the T_2^* of a superconducting qubit, with applying one Hadamard gate, waiting a time t, then applying another Hadamard gate, and measuring the output. The sinusoid curve amplitude slowly decreases around a probability 0.5. T_2^* is obtained when the probability reaches 1/e.

Rayleigh scattering: predominantly elastic scattering of electromagnetic radiation by particles that are much smaller than the radiation wavelength. Elastic scattering happens with incident photons whose direction is changed but not their energy (color or wavelength). It explains why the sky is blue, linked to blue light being more scattered than green and red light, and also polarized.

Realism: in science, philosophical view according to which there exists a reality independently of an observer. The Copenhagen interpretation of quantum physics is non-realist since it believes reality is only what can be observed and measured.

Reduced Planck constant: see Dirac constant.

Reflectometry: technology used with superconducting and electron spin qubits readout. It consists in sending a microwave to the qubit and to analyze the reflected microwave, which can have different phase and amplitude depending on the measured qubit state.

Register: set of bits or qubits. In the case of qubits, it provides an exponentially growing computational base space with the number of qubits.

Relaxation: corresponds to the T_1 or lifetime of a qubit, which defines when the qubit loses its amplitude.

Renyi entropy: generalized version of entropy that can be used as a measure of entanglement. Shannon entropy is a special case of Renyi entropy.

RIE: reactive ion etching, a process used to remove some material on a semiconductor target using accelerated molecular or atomic ions in vacuum.

RSA: a public key encryption system based on the difficulty of factoring a public key formed by multiplying two very large prime numbers. This factorization is theoretically possible with Peter Shor's quantum algorithm. However, it requires a very large number of qubits to break the most common RSA keys at 1,024 or 2,048 bits. For 2048-bit keys, 20 physical million qubits with a 99.9%+ fidelity are required, which is very long-term in quantum computer roadmaps.

Rydberg (atoms): excited state of an atom having one or more electrons and whose principal quantum number n (index of the electron layer in the atom which is an integer between 1 and the number of electron layers in the atom) is very high. These atoms are generally of large size, proportional to n^2, and with very strong inter-atomic interactions. These interactions are used to build entanglement between atoms. These atoms have been used by Serge Haroche's team to detect non-destructively the presence of a photon in a cavity, and thus study quantum decoherence. Hydrogen can also be a Rydberg atom if it is excited with high energy levels.

Sapphire: aluminum oxide crystals (Al_2O_3) that is sometimes used as a substrate instead of silicon for the manufacturing of superconducting qubits chips. In that case, wafers are made of synthetic sapphire.

SAT: class of logic problem or Boolean satisfiability problem, of 0-order logic. It is a decision problem, which, given a propositional logic formula, determines whether there is an assignment of propositional variables that makes the formula true. As when looking for Boolean variables x, y and z that satisfy the equation $(x \vee y \vee z) \wedge (\bar{\mathbf{x}} \vee \bar{\mathbf{y}}) \wedge (\bar{\mathbf{x}} \vee y \vee z)$, $\wedge$ meaning "and", and $\vee$ "or" or "and". $\bar{\mathbf{x}}$ being the negation of x. The problem becomes very complex if the number N of variables becomes very high because to test their combinatorics with brute force, we will have to test 2^N combinations. This problem has been highlighted by Cook's theorem according to which the SAT problem is NP-complete. The SAT problem also has many applications, notably in constraint satisfaction, classical planning, model verification, diagnostics, up to the configurator of a PC or its operating system: we go back to propositional formulas and use a SAT solver.

Scale-out: generic information technology term describing the capacity to expand computing power with several processors connected to the other. This is done in classical server clusters and datacenters, using both hardware (multiple processors on same board, high-speed connectivity between boards and servers, high-speed data storage, ...). Such techniques are envisioned with quantum computing, consisting in connecting different processing units, usually with using photons and entanglement resources.

Scattering: deflection of moving particles by some physical medium or radiations.

Schrödinger (equation, wave function): describes the evolution in time and space of the wave state of a quantum object with a mass like an electron, i.e. the probabilities of finding the object at a given place and time in time.

Schrödinger wave function collapse: in the case of a qubit, happens at the end of the coherence (superposed state) which is generated by its state readout, bringing it back to one of its basis states ($|\mathbf{0}\rangle$ or $|\mathbf{1}\rangle$). This collapse is also caused by the interaction between the qubit and its environment and after qubit measurement.

Second quantization: field of quantum physics that deals with many-body quantum systems. It was introduced by Paul Dirac in 1927 and developed afterwards by Vladimir Fock and Pascual Jordan.

Second quantum revolution: covers advances in quantum physics since the 1980s, when we began to control the properties of individual quanta, at the level of photons (polarization, ...), electrons (spin) and atoms and also use superposition and entanglement. It covers in particular the uses of these properties in cryptography and telecommunications, quantum computing and quantum sensing. The term was created simultaneously in 2003 by Alain Aspect, Jonathan Dowling and Gerard Milburn.

Semi-classical light: describes interactions between quantized matter (atoms, electrons) and classical light fields. Laser light belongs to this category.

Shor (algorithm): integer quantum factorization algorithm invented by Peter Shor in 1994. It would theoretically allow to break RSA public keys by decomposing them into prime numbers.

Silicon 28: Silicon isotope allowing the creation of silicon wafers suitable for the creation of silicon qubits. Silicon 28 has a zero spin that does not affect the spin of the trapped electrons used to manage the qubits. It is purified in Russia and can then be deposited in a thin layer in the gas phase on conventional silicon.

Single mode: said of an optical fiber using a small core (around 9 µm) and transporting a single light beam that doesn't bounce off the inside walls of the fiber. It has low loss and is adapted to long distance transport, usually in the 1,310 nm or 1,550 nm wavelengths. These cables still use multiple wavelengths, with WDM (wave-division multiplexing). Also said of a single mode photon, see Multimode.

SPAC: special purpose acquisition company. A funding mechanism used by IonQ and HQS (Honeywell Quantum Systems) consisting in getting acquired by an investment fund creating a dedicated fund for the company and raising money on both limited partners (individual corporate ventures and the likes) and on the stock market like the NASDAQ.

SPAM: State Preparation And Measurement, a sequence of operations after which the fidelity of qubits is measured. This fidelity reflects that of an initialization sequence, the application of single qubit gates and the measurement of the qubit state.

Spectral lines: lines obtained graphically after decomposing an electromagnetic radiation into frequency components, usually with some spectrography apparatus. You have absorption and emission spectral lines depending on the source of light (indirect, direct). Each line corresponds to the emission or absorption of photons in atoms at particular energy levels, then wavelength and frequency.

Spectral decomposition: mathematically, spectral decomposition of a pure state vector in a Hilbert space is its eigenstates $|i\rangle$ and eigenvalues λ_i. It can be related to the wave-duality aspect of all quantum objects. A quantum object pure state is indeed decomposable in a coherent superposition of elementary waves, the eigenstates.

Spin: quantized angular momentum of elementary (like electrons or photons) or composite particles (like atoms) that cannot be described or explained in classical physical terms. The spin of composite particles is the addition of its components spin. A proton and a neutron have a spin of 1/2. An electron has a spin of +1/2 or -1/2. A photon also has a spin, which relates to its circular polarization. Spin help distinguish fermions who have half integer spins from bosons who have integer spins.

Spintronics: a set of technologies based on the manipulation of electron spin. It is found in memristors as well as in hard disks using giant magnetoresistance (GMR). The latter was discovered by Albert Fert (France) and Peter Grünberg (Germany) independently and the same year, in 1988. This got them the Nobel prize in Physics in 2007.

Spontaneous emission: when an atom emits a photon resulting from the transition of an electron from an excited to a lower energy state.

Spontaneous Four-Wave Mixing (SFWM): photons pairs source category based on pumping nonlinear optical waveguides or cavities.

Spontaneous Parametric Down-Conversion (SPDC): system converting high-energy photons into pairs of photons of lower energy, based on pumping nonlinear optical waveguides (crystals) or cavities. It can be used to create pairs of entangled photons as well as single photons sources.

Squeezed states of light: correspond, in a quadrature or phasor diagram representation, to wave functions which have an uncertainty in one of the quadrature amplitudes (phase or photon number) smaller than for the groundstate corresponding to the vacuum state. It can be generated by different means like a parametric down conversion. In other words, it's a way to increase the measurement precision of one of the photons characteristics at the expense of another characteristic.

SQUID: Superconducting Quantum Interference Device, a magnetometer that measures the direction of current in a superconducting qubit. It is notably used by D-Wave and in some quantum sensors.

Stabilizer gates: quantum gates that are used in error correction systems: CNOT, H (Hadamard) and P (phase).

Standard quantum limit: to estimate a system's parameter, one usually uses light as the meter by making it interact with the system and thereby extracts some information. The standard quantum limit, also known as shot noise, states that the variance of this estimation is larger than the inverse of the square root of the number of times the measurement is made. It limits the precision of quantum sensing using non-entangled quantum states. See Heisenberg limit.

Stark shift or Stark effect: shifting and splitting of spectral lines of atoms and molecules due to the presence of an external electric field. This is the electric-field analogue of the Zeeman effect which is linked to the effect of the magnetic field,

State reduction: consequence of the measurement of the state of a quantum or a qubit, which modifies its (superposed) state into a stable state (not superposed). For a qubit, it is one of the two basic states: excited or non-excited, horizontal or vertical polarization for a photon, spin orientation for an electron, excited state for an ion or a cold atom, etc.

State vector: Hilbert space vector representing a pure state of a quantum object.

Stationary qubits: stationary (or static) qubits, which do not move in a circuit. This is the case of superconducting qubits, trapped ions and cold atoms qubits as well as electron spin qubits. They are opposed to flying qubits that move, like photons.

Stern-Gerlach experiment: deviation of projected atoms placed under an intense magnetic field, which is explained by electron intrinsic angular momentum or spin.

Stimulated emission: when an incident photon is not absorbed by an excited atom but stimulates the atom to emit a second photon with the same wavelength. This principle is used in lasers to amplify light in their cavity.

Sturm–Liouville problem: mathematical problem consisting in solving some second-order differential equations where the unknown is a density function and finding eigenvalues and eigenvectors and satisfying bound limits. Solving Schrodinger's wave equation is a particular case of such a problem.

Superconductivity: the ability of some materials to conduct electricity without resistance. It generally occurs at low temperatures. It is linked to the behavior of electrons in some crystalline structures who happen to gather in pairs, Cooper pairs, who become bosons, and have a collective behavior enabling them to move around within the structure. Superconducting and electron spins qubits use this effect. The first with superconducting loops traversing a Josephson barrier and all of them with cabling and some surrounding electronics.

Superdense coding: technique used to send two bits on a single (optically transmitted) qubit between two points when they are already connected by a pair of entangled photons. It is a communication protocol imagined by Charles Bennett and Stephen Wiesner in 1992 and experimented in 1996 by Klaus Mattle, Harald Weinfurter, Paul Kwiat and Anton Zeilinger. The initial entanglement preceding the transmission of the two bits in the qubits avoids violating Holevo's theorem that a set of qubits cannot carry more information than the equivalent number of classical bits.

Superoperator: linear operator that transforms a linear operator like a density matrix. It must be a CPTP map, completely positive and trace preserving map (see CPTP definition).

Superposition: property of quantum objects and qubits to be able to be in several states at the same time. This can be explained by the wave-like nature of quantum objects. A superposition is a linear combination of quantum eigenstates (the $|0\rangle$ and $|1\rangle$ in the case of qubits).

Surface codes: type of quantum error correction code that is tolerant to high qubits error rates and require a larger number of physical qubits per logical qubits and have a design constraint for physical qubits that must be connected to their immediate neighbors in a 2D structure. This is the QEC architecture chosen by Google.

SVD: singular value decomposition, a mathematical process to factorize any mxn complex values matrix into three matrices: a unitary mxm matrix, a rectangular diagonal mxn matrix (which has no zero values only in its diagonal) and a unitary nxn matrix. SVD is used in QSVT algorithms.

SWAP: quantum gate that inverts the state of two qubits. It is very useful since most qubit geometries don't allow an any-to-any qubit connection. The SWAP gate enables this kind of connection that is mandatory for many quantum algorithms.

Symmetry: refers to situations where physical properties remain unchanged when a specific transformation is applied to a system like spatial rotations, translations, reflections, time inversions, and more. Symmetry is deeply linked to the conservation laws in physics, where the invariance of certain quantities like energy, momentum, and angular momentum under specific transformations leads to the conservation of those quantities. For example, the Schrödinger equation exhibits rotational symmetry. See also antisymmetry. The Noether's theorem establishes a deep connection between symmetries and conserved quantities in physical systems.

T gate: single qubit gate implementing a "quarter turn" phase rotation around the Z axis in the Bloch sphere. It is a very important gate enabling (on top of the three-qubit Toffoli gate) the creation of a universal gate set.

T-count: number of T gates required in a quantum algorithm. It is an important metric since T gates are the most expensive to correct with FTQC.

T-depth: number of circuit layers implementing one or several T gates in a quantum algorithm.

T_1: qubit amplitude coherence time, which indicates the end of coherence of the qubits linked to a loss of amplitude ("energy relaxation"). Aka qubit lifetime.

T_2: phase related coherence or time when some phase shift occurs, i.e. a rotation around the z axis in the Bloch sphere of the qubit state.

Tabu search: metaheuristic local search method used for mathematical optimization. It currently fares better than quantum algorithms using QAOA (for gate-based systems) or QUBO (for annealers/simulators).

TDSE: acronym for time dependent Schrodinger equation.

Tensor: in multilinear algebra and differential geometry, a tensor designates a very general object whose value is expressed in a vector space. In quantum physics and computing, tensors are used to describe the state of a compound quantum object with several quanta or qubits. A qubit is represented by a vector of 2 complex numbers. A register of N qubits is represented by a vector with 2^N complex numbers resulting from the tensor product of N vectors of 2 complex numbers. In a way, the tensor product represents the combinatorial space of the values that a combination of qubits can take. Before entanglement comes into play to mix things up and create non separable vector states, i.e., which cannot be expressed as tensor products of individual quantum states.

TeraQuop: tera quantum operations, a threshold for fault-tolerant quantum computers before a single logical error appears. Now, it depends how you define an error.

Thermodynamics first law: the internal energy of an isolated system is a constant, applying the principle of the conservation of energy. Inside the system, the form of energy can however be transformed.

Thermodynamics second law: the entropy of a closed system cannot decrease. In other words, heat does not flow spontaneously from cold to hot objects. Was formalized by Rudolph Clausius in 1854.

Time crystal: also, DTC for discrete time crystal is a topological state of condensed matter at very low temperature where atoms in their ground state are periodically arranged in both space and time with in a permanently oscillating structure with a given period (for discrete time crystals).

Time domain: deals with the evolution of some value and signal over time. It's frequently opposed to frequency domain where a signal is analyzed by decomposing it into frequencies (mathematically, with a Fourier transform).

Time reversal: clearly a misnomer. It describes situations when from a given point in time, for some physical property like energy, a system presents a symmetry when you evaluate it with look forward or backward at time. It's a mathematical symmetry. You don't change the arrow of time backwards. Time reversal is not a time machine!

Toffoli (gate): also called CCNOT is quantum gate operating on three qubits which modifies the value of the third qubit if the value of the first two is 1.

Topological: topological quantum computing is based on the notion of anyons which are "quasiparticles" integrated in two-dimensional systems. The anyons are asymmetric and two-dimensional physical structures whose symmetry can be modified. This allows the application of topology principles with sets of successive permutations applied to pairs of anyons that are in proximity in circuits. The associated algorithms are based on the concepts of topological organizations of braids or nodes ("braids"). There is an algorithmic equivalence between computation with universal gated qubits and topological qubits.

Transmon: transmission-line shunted plasma oscillation qubit, variation of superconducting qubit with superconducting current oscillating at two different frequencies across a Josephson junction. The difference between these two frequencies corresponds to the energy of the microwave pulses sent to the qubit to drive single qubit gates.

Transpiler: source-to-source compilers used in classical computing and quantum computing to use the universal gate set available in the quantum processor. It is usually associated with an optimizer that optimizes the source code to reduce the number of gates to execute and as a result, reduce errors (particularly with NISQs) and the algorithm execution time.

Transpilation: code conversion and optimization achieved by transpilers.

Transversal gates: relates to quantum error correction and fault tolerance. These are gates implemented with QEC where there is a 1-1 correspondence and link between all qubits from a given corrected qubit with a similar corrected qubit, when they are assembled through concatenation. This mechanism limits the propagation of errors between logical and physical qubits.

Trapped ions: these are ions used in certain types of quantum computers. They are usually trapped magnetically or electrically, and their state is controlled with lasers. Their readout uses a laser excitation and an imager readout of the resulting ions fluorescence. Trapped ions are controlled by ion traps circuits, using electromagnetic waves in the microwave regime and coming from laser beams.

Trotterization: technique used in quantum computing that breaks down the evolution of quantum systems into smaller time steps. This time discretization is used to simulate the time evolution of quantum systems with complex interactions, making it easier to implement it with quantum algorithms.

Tunnel effect: property of a quantum object to cross a potential (or energy) barrier even if its energy is less than the minimum energy required to cross this barrier. This effect is used in D-Wave's quantum annealers to quickly determine the minimum energy of a complex system ("Hamiltonian" implemented as an Ising model).

Two-Level Systems (TLS): other descriptor of quantum systems used to implement qubits. A qutrit is a three-level system.

TWPA (Travelling Waves Parametric Amplifiers) microwave readout amplifiers implemented in a long array of SQUIDs. Their broad bandwidth that can reach 2 GHz with a 15 dB amplification enables up to 20 qubits readout multiplexing. But it depends on the gate speed since the faster the gate, the smaller the microwave pulse bandwidth will be large.

UHV: Ultra High Vacuum, the ultra-high vacuum required to operate certain types of qubits. It is mainly used for cold atoms and trapped ions. Superconducting qubits are integrated in a vacuum cryostat that does not require ultra-high vacuum.

Ultraviolet catastrophe: expression of Paul Ehrenfest, linked to the Rayleigh-Jeans law proposed in 1900 to explain the black body radiation spectrum, which was diverging to infinite values as the temperature was growing, when reaching ultraviolet wavelengths. Planck's law based on quanta solved the problem and got rid of the ultraviolet catastrophe.

Unary gates: single qubit gates. Not to be confused with unitary operations that are the result of the combination of all qubit gates on a given set of qubits. A unitary transformation of the computational state vector is a matrix operator that is equal to its transconjugate. It is a mathematically reversible operation.

Unconventional Computing: computing methods that do not fall under the classical computing principles of Turing and Von Neuman machines. Covers non-traditional tools and methods that include, but are not limited to, quantum computers. It also includes molecular computers and neuromorphic processors.

Unitary operation: linear operation on a vector that preserves its length. In the case of qubits whose vector always has a length of 1, the unitary quantum gates apply on it a transformation that preserves this length and is also reversible. In the representation of qubits in the Bloch sphere, the operation rotates the vector representing the state of the qubit in this sphere.

Universal quantum computer: most generic form of a quantum computer exploiting a universal quantum gate set, and which can both simulate quantum physics and implement any operations of a classical computer.

Universal quantum gates: sets of quantum gates from which all other quantum gates can be reproduced to create any unitary transformation on any number of qubits (by approximation and under a given error rate). It requires a non-Clifford group gate, like a T gate or a Toffoli gate.

Unruh effect: relativistic thermodynamic effect with a black body radiation showing up in vacuum at relativistic speed. Il has some connection with quantum noise.

Variational algorithm: see VQA.

VQA (Variable Quantum Algorithm) generic quantum hybrid algorithm using a classical optimizer that is used to train a parametrized quantum circuit. It could lead to obtain some quantum advantage with NISQ quantum computers. VQA has a broad set of applications: finding ground and excite states (VQE), quantum simulations (VQE), machine learning (QML) and optimizations (QAOA).

VQE (Variational Quantum Eigensolver): hybrid quantum algorithm used in chemical simulation created in 2013. Its main contributor is Alán Aspuru-Guzik, a researcher at the Zapata Computing startup. It is also used in machine learning tasks. VQE was the first proposed VQA.

Wave packets: is a burst of electromagnetic wave that travels as a unit. It is formed by the addition of an infinite number of sinusoidal waves of different frequencies, phases and amplitudes creating constructive and destructive interferences on a small region in space, and destructively elsewhere. Wave packets are used in many quantum technologies such as with microwaves sent to superconducting and electron spin qubits or by femto- and picoseconds lasers. In these cases, their decompositions in frequencies lead to so-called frequency combs.

Wave-particle duality: the property of elementary particles such as electrons, neutrons, atoms and photons to behave as both particles with momentum and waves that can generate interference. It is verified with the famous Young's slits experiment which shows these interferences with both photons and electrons.

Wien's displacement law: describes the relationship between peak wavelength and temperature in black body energy spectrum. Discovered by Wilhelm Wien in 1893.

Wigner function: representation of a quantum state used to measure the level of quantumness of a light pulse. It has the particularity of having negative values for entangled and non-gaussian states. It is usually visualized in a 3D chart with peaks and lows. Also called Wigner quasiprobability distribution or Wigner-Ville distribution. It was created by Eugene Wigner in 1932.

X: quantum gate at a qubit that inverts its amplitude, goes from $|\mathbf{0}\rangle$ to $|\mathbf{1}\rangle$ or from $|\mathbf{1}\rangle$ to $|\mathbf{0}\rangle$ for the basis states.

XY gates: two qubit entangling gate.

Y: single-qubit quantum gate that performs a 180° rotation around the Y axis in the Bloch sphere.

Z: quantum gate to a qubit that applies a sign change to the β component of the qubit vector, i.e. a phase inversion and a 180° rotation with respect to the Z axis. More generally, Z gates is also a denomination for phase change gates.

Zeeman effect: splitting of spectral lines when atoms are placed in a static magnetic field. Explained by the different electron's magnetic moment and/or by the atom nucleus spin for the nuclear Zeeman effect. There are two Zeeman effect, the normal and abnormal effect, differentiated with the off/even spectral rays generated and generated by different orbital angular momentum (normal) and spin (abnormal).

Zeeman cooling or slower: use of the Zeeman effect to cool atoms at a lower temperature than with a simpler Doppler effect. Invented by William D. Phillips (Nobel prize in physics in 1997), it consists in adjusting the atoms resonant frequency with a magnetic field as the atoms are slowing down when implementing the Doppler effect.

ZX calculus: graphical language and formalism used to visualize in quantum programming the notions of entanglement, complementarity, causality and their interactions. It can be used for Measurement Based Quantum Computing (MBQC), the creation of error correction codes and compiler optimization techniques.

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Table of figures

Revisions history

This book improved over time with successive revisions. The first editions from September 2018, 2019 and 2020 were published in French and this sixth one is the third published in English. It is freely downloadable on:

https://www.oezratty.net/wordpress/2023/understanding-quantum-technologies-2023/

Different PDF formats are available: **A4 in full resolution in a single volume** (about 100 MB), **A4 in reduced resolution in three volumes** (to fit under the 32 MB threshold for some ebook readers) and the same in **Letter format** for USA and Canada readers who would like to print it on their own.

After a post-publication proof-reading and correction couple weeks period, I also published it in November 2023 on **arXiv** (https://arXiv.org/abs/2111.15352) and in printed paperback editions on most **Amazon** web sites.

This log describes the various successive editions and minor revisions (in .x numbering although it could be more relevant to use a .0x versioning).

Version and date	Modifications
1.0 (332 pages) September 29th, 2018	First version of this document published in French and consolidating 18 posts published between May and September 2018 on www.oezratty.net.
2.0 (504 pages) September 20th, 2019	Second edition, also in French. New content on superconductors, superfluidity, quantum sensing, quantum supremacy, quantum computing emulation, cryogeny, hybrid algorithms, algorithms certification, quantum teleportation, blind computing. Addition of a glossary and bibliography.
3.0 (684 pages) September 7th, 2020	Third edition, also in French. New content on Maxwell, Schrödinger and Dirac's equations, relativistic quantum chemistry, how research works, lasers and masers, polaritons, extreme quantum, linear algebra, quantum gates classes, quantum error correction codes, cryo-electronics, MBQC, quantum cloud, qubits technologies, unconventional classical computing, quantum hype cycle, quantum foundations and on the influence of science fiction.
4.0 (838 pages) September 27th, 2021	Fourth edition, the first one in English. Main new features vs the 3.0. on top of updates nearly everywhere: New section on quantum physics postulates. More on wave-particle duality and on photon qubits physics. Improvements and extensions on linear algebra, on quantum measurement and quantum memory. New part on vacuum in enabling technologies. Much improved section on algorithms, including data preparation and debugging. Expanded part on QRNG. Went from 265 to over 450 vendors covered in the various sections of the book. More on ethical issues and gender balance. Additional covered countries: Belgium, Portugal, Italy and Abu Dhabi and nice ecosystems maps for the USA and the UK. Added an index with company names, people and some scientific terms and many terms in the glossary (from 179 to >280).

From October 2021 to January 2022	The two-volumes printed version of the book was made available to purchase at an affordable price on most Amazon sites. Added a new graph explaining Dirac $\langle A\|B\|C\rangle$ notations in the measurement section. Added a roadmap for managing the energetic footprint of quantum computing. Added American Binary (Ambit) in the quantum cryptography vendors section that starts page 1036. Small update related to D-Wave gate-based quantum computing announcement in the quantum annealing section that starts page 312. Added Energetics of quantum technology, Quantum postulates, QML, QLM and scale-out in the glossary. Corrected wrong naming for Atos QLM (instead of Atos QML or Atos aQML). Update on Origin Quantum and their cloud quantum emulation offering. Added StarX Electronics in the inventory of enabling technology vendors and Chipiron in quantum sensing imaging. Added Two-Level Systems in glossary. Added a chart from Joseph Bardin describing the various microwave and other signals used to drive superconducting, electron spin and trapped ions qubits. Added Algorithmic qubit, anharmonic oscillator, cQED, CQED, fluxonium, Q-factor, Dirac and Reduced Planck constants, homodyne detection and Universal quantum computer in the glossary. Updates on IBM and its 127 qubits processor announced in November 2021, the related qubits fidelities chart with the best-in-class IBM 27, 65 and 127 QPUs. Some updates on IQM and OQC (on Amazon Braket). Updates on Kipu Quantum and on Quantinuum (new name for HQS/CQC merger). Added a mention on Alexia Auffèves quantum energy initiative. Updates on Quandela, Qu&Co (acquired by Pasqal) and Rahko (acquired by Odyssey Therapeutics). Added Mikhail Lukin in the quantum computing physicists section. Added Black Quant and Runa Capital in the quantum investors section.
5.0 (1,132 pages in single volume version) September 2022	Many updates everywhere, including… **Why**: new intro, rearranged and updated the part on Moore's law. Removed the long abstract from the previous version. **History and scientists**: updated scientists with more stuff on Marc Benioff, David Deutsch, David Hilbert, Daniel R. Simon, Erwin Schrodinger and new bios for Andrew G. White, Gaston Floquet, Bruce Kane, Daniel Kleppner, Daniel Loss, Frank Wilczek, Gerhard Rempe, Herbert Walther, James Clarke, Jeff Kimble, Jian-Wei Pan, Leo Kouwenhoven, Menno Veldhorst, Steven Girvin, Rob Schoelkopf, Roy J. Glauber, Jay Gambetta, Alexandre Blais, Robert Raussendorf, Maciej Lewenstein, Ronald Walsworth and Philip W. Anderson. More details on how research works, papers published and evaluated. **Quantum physics 101**: added a table listing the various quantum physics postulates versions, added nuclear quantum numbers in quantization, added a part on quantum matter, including on quantum batteries, time crystals and skyrmions. **Gate-based quantum computing**: various updates on quantum gates, added illustrations on quantum computing dimensionality, better differentiation between quantum emulation and quantum inspired software. **Quantum computing engineering**: simplified qubit type descriptions, update content on exotic qubits, better explained how error rates are measured, many updates on quantum error

	corrections, definitions of FTQC, universal QC and LSQC. Described in detail the whereabouts of the Quantum Energy Initiative. Added a definition of quantum switch. **Quantum computing hardware**: added vendor investment comparisons per type of qubit, types of use cases per qubit type, inventory of scalability challenges per type of qubit, rearranged and standardized the presentation of each qubit type with history, science, qubit operations, research and vendors. More schematics. More science on quantum annealing. Genealogy of superconducting qubits. Updated content on IBM, Google, Rigetti, IQM, OQC and added Atlantic Quantum, Baidu and Toshiba in superconducting computers vendors. Comparisons of different types of quantum dots spin qubits. Added Diraq in quantum dots spin qubits vendors, XeedQ in NV centers qubits vendors, It's Q, Quantum Source Labs and TuringQ in photonic qubits vendors and Crystal Quantum Computing and Planqc in neutral atoms qubit vendors, and Hon Hai / Foxconn in the trapped ions vendors list. Added coherent Ising machine in quantum photonics systems. Archer Materials, Atom Computing, Bleximo, BosonQ Psi, Nord Quantique and QBoson. **Quantum enabling technologies**: rearranged the section on control electronics. Added ICE, Maybell Quantum and FormFactor, and updated myCryoFirm in cryostats vendors, Active Technologies, Keysight, QuantrolOx and Scalinq in control electronics vendors, Qubic Technologies, Raditek, QuinStar Technology, RF-Lambda, Wenteq Microwave Corp, Holzworth Instrumentation, apitech, Analog Quantum Circuits, CryoHEMT and Silent Waves in cryoelectronics vendors, CryoCoax, XMA and Rosenberger Group in cable and filtering vendors and Alcyon photonics, Teem photonics and Scintil Photonics in photonic enabling technologies vendors, AnaPico, Diatope, HiQeTe Diamond, Orsay Physics, QuantTera and Quantum Diamant in other enabling technologies vendors. Added a new part on fabs, processes and manufacturing tools. Added here many manufacturing tools vendors like BESI, PlasmaTherm, Picosun Group, NanoAcademic Technologies and QuantCAD. Updates on raw materials. **Quantum algorithms**: added a part on tensor networks. Significant updates on quantum machine learning. **Quantum software development tools**: updates on emulation software, restructured and updated the part on benchmarking. **Quantum business applications**: updated all vertical case studies lists. Added Arclight Quantum, Allosteric Bioscience, Artificial Brain, ColibrITD, Dirac, Foqus, GenMat, Good Chemistry Company, Ingenii, Qbraid, QEDma, Qoherent, Quanscient, Quantagonia, Sanctuary, SavantX, Tinubu Software and Turing in the software and tools vendors inventory. Created a section on IT service vendors working on quantum technologies. Added DN-Quantum Computing, Kvantify, Plantagenet Systems, Protiviti, Psi-Ontic, Quantum Computing Engineering, Quanvia, quGeeks, Quant-X Security & Coding, Qubitech and Unitary Zero Space. Updated information on AegiQ, Azurlight Systems, Cogniframe, exaQ.ai, Horizon Quantum Computing, HQS Quantum Simulations, Multiverse Computing, and Nomidio, Phasecraft, OTI Lumionics, Q.ant, Quantum Computing Inc, QunaSys, Q-Ctrl, Strangeworks and Terra Quantum. **Quantum enabling technologies**: I singled out this part and moved it to the second volume. Content has been sporadically updated. **Quantum telecommunications and cryptography**: added a part on quantum photon sources and detectors in the QKD section, improved description of trusted nodes and repeaters, rearranged and enriched the quantum interconnect and telecommunication part. Update on PQC with NIST 2022 selection results. Added Abelian, ComScire in QRNG vendors, Bohr Quantum Technology, Photoniq and Entanglement Networks in quantum telecommunications, NodeQ, Patero, QANplatform, Quantum Collective, SandboxAQ, Synergy Quantum and ThinkQuantum in quantum telecommunications and cryptography vendors. Updated information on Post-Quantum, IDQ, Qnu Labs, QphoX and, Qunnect. **Quantum sensing**: added new parts on quantum sensing taxonomy and on quantum pressure sensing. More details on quantum thermometers. Added OK Quantum and Zero Point Motion in quantum gravimeters and accelerometers, Improved the technical description of a quantum gravimeter. QuSpin, Siloton, QLM Technology and Mag4Health in imaging sensors and qdm.io and Elta Systems in quantum magnetometers. Updated information on Chipiron.

	Quantum technologies around the world: updates in investments data, new part disappeared startups, SPACs, government spending and quantum national initiatives rationales. Added Finland, Norway, Hungary, Ireland, and Qatar in countries overview and Intqlabs startup in UAE. Updated nearly all other countries quantum activities. Added a map of Canada's quantum ecosystem. **Quantum technologies and society**: added EFEQT in scientific education and on quantum technologies marketing. **Bibliography**: reshuffled the quantum events section, added an events timeline. **Glossary**: added Ansatz, Bell state, Chi (nonlinearity order), Circuit, Coulomb blockade, crosstalk, CVD, Fermi sea, Floquet code, EBL, electron gas, FTDQC, Flux biasing, Gaussian Boson Sampling (GBS), Hall effect, Heterodyne and Homodyne measurements, I/Q mixer, Jaynes-Cummings Hamiltonian, JPA, Leggett-Garg inequality, MBE, Magneto-Optical Trap, mesoscopic, metal layers, microring resonator, Mott insulator and Mott transition, Mutually unbiased bases, no-go theorem, normalization, on-premises, ODMR, paramp, photolithography, Purcell effect, Purcell filter, purification, PVD, Quantum Hall effect, QHO, QND, QSVT, QUBO, quantum steering, quantum switch, Ramsey experiment, relaxation, Renyi entropy, RIE, Sapphire, surface code, SVD, time crystals, time reversal, TWPA, Stark shift, Unruh effect and Zeeman cooling. Added over 900 figure captions with sources and sorting out how figures are referenced in the text (mostly).
5.1 (1,132 pages in single volume version) From October to December 2022 updates.	Several corrections in the precursors and founders sections with errors in Einstein's photoelectric and Dirac's relativistic equations and other hidden details. Added Nobel prize mentions for John Clauser, Alain Aspect and Anton Zeilinger. Updates on Alain Aspect's experiment and its explanation. Integrated some corrections suggested by André Konig. Various improvements in the part on quantum error correction. Modifications in the FTQC/LSQ nomenclature with suggestions from Alastair Abbott and Tristan Meunier and thoughts about the progressive growth of logical qubits fidelities. Presentation improvements in quantum physics simulation algorithms. Various edits in satellite QKD, China funding, Google Sycamore, NV centers frequency analyzers in sensing. Updates in insurance use cases. Added the class NISQ in quantum complexity classes. Added quantum amplitude estimation in the algorithm toolbox. Precisions on transpilers and transpilation in the gates section and with the glossary. Added T gate, T-count and T-depth in the glossary. Added IonQ, Rigetti and D-Wave quarterly revenue in the investor section. Added a new partnership between Switzerland and the USA. Updated the record "non-Shor" integer factoring algorithms in the quantum cryptoanalysis threat section. Updated chart on algorithms per computing paradigm and UK industry ecosystem Updates on Keequant, Orca Computing, Quantopticon, Quantum Brilliance, QuiX, Q.ANT, Qunnect and XeedQ using content from the Optica Conference in November 2022. Updates on M Squared and Exail (formerly ixBlue) in the quantum sensing part. Added LuxQuanta in QKD vendors. Added Siquance in the silicon qubits startups.

6.0 (1,364 pages in single volume version) September 28th, 2023	Sixth edition, and third in English. 274 new or updated figures, over 1330 new references. **Why**: rewrote much of this part, mainly the “Why quantum computing” section with a rearrangement of the section on Moore’s law and transistor density story. Added a one page table with common misconceptions and their explanation and location in the book. **History and scientists**: added Immanuel Bloch in quantum physicists. Sorted the hall of fame into subcategories. Rewrote and completed the Research for dummies part, adding my own experience on publishing a paper in a peer-reviewed journal, and using ChatGPT. **Quantum physics 101**: added some explanation of the quantum nature of continuous quantum variable systems, added measurement techniques of nuclear spin, added the first quantum battery vendor, Planckian in the quantum matter section, reorganized the part on superconductivity and added news about LK99. **Gate-based quantum computing**: various minor corrections on linear algebra mathematics. Added a space vs time advantage comparison. Updated some illustrations. **Quantum computing engineering**: updates and new illustrations in the quantum error correction and quantum error mitigation part, various updates in the energetics with a new chart on quantum computing power decomposition, and economics part. **Quantum computing hardware**: updates on qubit figures of merit, many updates for major superconducting vendors (IBM, Google, …), added ARQUE and SemiQon and QSi in silicon qubit vendors, added neQxt, Quantum Art and Qudora Technologies in trapped ions qubit vendors, NanoQT, GDQLABS and QUANTier in neutral atoms qubit vendors, Quantum Transistor and SaxonQ in vacancies qubits, Quanfluence and Rotonium in photonic qubits. Added a part on how Majorana qubits operate. **Quantum enabling technologies**: added Zero Point Cryogenics and Ulvac Cryogenics in cryostats vendors. Added Ohtama, QuEL and T0.Technology in electronics vendors, ParTec in other enabling technologies vendors, and Onnes Technologies in fabs/characterization vendors. Added ConScience in foundries. **Quantum algorithms**: new segmentation of algorithms and better semantics about problems, algorithms and models, arithmetic part of Shor’s algorithm, PDEs (partial differential equations), added VQLS and QSVD in NISQ algorithms, added a long part on NISQ algorithms, new part on algorithms hybridization. Better explanations on complexity classes. **Quantum software development tools**: new parts on abstraction levels, code factoring, securing quantum computing, and resource estimators. Clarifications on the difference between emulations and simulations. Added Wolfram quantum framework in emulators. **Quantum business applications**: added a framework to assess quantum computing case studies, added a part on climate change, rewrote many parts particularly on healthcare, chemistry/energy, including a detailed overview of the famous FeMoCo case study, and finance applications. Added Beyond Limit, BlueQubit, First Quantum, Haiqu, Kunfeng Quantum Technology, InstaDeep, MolKet, Qbrain, QCentroid, QPerfect, Qruise, QSIMPLUS, Quantagonia, QAR-lab and Qausal.ai in the software and tools vendors list. Added Applied Quantum Computing, AngelQ, QuantumBCS, Qdeeptech and Reply Sigma in the IT service vendors list. **Unconventional computing**: rewrote many parts, restructured the optical computing part. Added Quantum Silicon Inc in the classical computing space. **Quantum telecommunications and cryptography**: changed some schematics and added some new ones, added a small part on quantum computing cryptography, added descriptions for Temporal entanglement, Quantum Secure Direct Communication, Quantum Pseudotelepathy, Quantum Conference Key Agreement, added a part on PQC and blockchain, added Alea Quantum Technologies in QRNG vendors. Added vendors in QKD/PQC: CAS QuantumNet, Craft Prospect, Quantum Bridge, levelQuantum, Wisekey, Seal SQ, Xiphera, Cryptolab, HEaaN CryptoLab, CyferAll, Icarus Quantum, LQUOM, QTI, Qtlabs and Q-Bird. Added MemQ and PhotonicsQ in interconnect vendors. Added BTQ Technologies and ChainMakers in vendors.

	Quantum sensing: extension and news schemas about sensing taxonomy and measurement precision, added Delta G, Euqlid, GLOphotonics and Guoyao Quantum Radar Technology in vendors. **Quantum technologies around the world**: added a part on patents, added Brazil, Pakistan and South Africa. Updates on most other countries and related charts. Country map for Germany. Market map for China. **Quantum technologies and society**: minor updates, particularly on the quantum hype phenomenon. **Quantum fake sciences**: minor updates. **Resources**: a couple additions in events and books. **Glossary**: added antisymmetry, Born-Oppenheimer approximation, cap table, chip, chipset, chiplet, contextuality, dark state, dark state protocols, dressed state, Fermi-Hubbard model, Gate teleportation, Gurobi, HPC, Lamb-Dicke regime, Lie groups, LLM, non-selective measurement, postselection, prepare-and-measure, QAOA, Quantum Conference Key Agreement (QCKA), Quantum Fisher Information, Quantum jump, Quantum Kernel, Quantum Speed Limit, Quantum trajectory, Tabu Search, TDSE, TeraQuop, Trotterization.
6.1 (1,364 pages in single volume version) September 30th, 2023	Various mostly indiscernible refinement text edits. Added some scientist's names in the hall of fame sections at the beginning. IonQ's Tempo's 64-qubit announcement and more on IonQ's touted case studies. Added minor edits in the complexity classes section, mentioning that these refer to worst case scenarios that may not be applicable in real-life applications. Tweaked the case study evaluation framework.
6.2 (1,364 pages in single volume version) October 4th, 2023	Added a mention of the Quantum Position Verification protocol in quantum telecommunications. Corrected DRMG into DMRG in various places. Added additional bibliographical references here and there.
6.3 (1,364 pages in single volume version) October 10th, 2023	Changed or updated several illustrations into vector format for better reading and printing. Updates on Australia. Added IQM's 20 qubits fidelities in the scatter plot chart in the QEC section. Added a mention on the 2023 Nobel prizes in physics, and attolasers in the lasers section in enabling technologies. Added a mention of Quantum Italia and Redstone VC in investment funds. Other added additional bibliographical references here and there.
6.4 (1,364 pages in single volume version) October 13th, 2023	Other added additional bibliographical references here and there. Various minor edits. Updates on the economics of quantum computers, page 300. Added Quoherent in the topological qubits vendors. Various logo updates in quantum computing charts.
6.5 (1,366 pages in single volume version) October 16th, 2023	Small improvements in the quantum memory part, page 278. Updated the Grotrian diagram of NV center qubits and related explanations, page 441. Updating some company logos, particularly in the hardware chart page 327. Some updates in quantum sensing and quantum radars. Added Erasure error and Leakage error in glossary. Added Cryotech Instruments and Qlibri in enabling technology vendors.

6.6 (1,366 pages in single volume version) October 17th, 2023	Some updates in quantum physics history and timelines. Other added additional bibliographical references here and there. Mention of IQM's Q-Score of 8. Version submitted to Amazon KDP for paperback edition in three volumes.
6.7 (1,366 pages in single volume version) November 12th, 2023 version submitted to arXiv	Again, added additional bibliographical references here and there. Some pagination cleanup. Some historical updates on Eistein, Hadamard, Heisenberg and Schrodinger. Modified schematics of quantum numbers and properties for atom/ions, electrons, nucleons and photons. Updates on NV center qubit operations. Latest news on C12, Quantum Motion, QuEra, InfinityQ, IQM, Atom Computing, Oxford Ionics, Infineon, Photonic and Quandela. Updated charts of pure/mixed states, RSA, USA ecosystem map. Corrections about Shor's algorithm description and quantum algorithms inputs/outputs. Added QDC.ai, Quantum AI and Inspiration-Q in quantum computing software vendors, Qurv, Vector Atomic and ZebraKet in quantum sensor vendors and g2-zero in photonics enabling technologies vendors. Update information on Algorithmiq in quantum software vendors. Added Atom Quantum Labs in cold atom computing vendors.
6.8 (1,366 pages in single volume version) April 7nd, 2024	Added a reference explaining why Albert Einstein didn't get the Nobel prize in physics for his work on relativity. Corrected Lov Grover's biography. Added the SX gate in the quantum gates description. Added the new IBM Layer Gate Fidelity benchmark (in the quantum volume section). Update on Photonic and their architecture. News on Orca Quantum Computing, Quandela and Multiverse. News on Alibaba, Baidu, Infleqtion, IBM, OQC, IBM, Riverlane, Harvard/QuEra, Rigetti, Origin Quantum, Quantinuum and Qphox. Updated the qubit fidelities chart in the error correction section. Updates on supercomputers in France. Added FermionIQ in software vendors. Updated enabling technology vendor map. Update on the energy consumption table per qubit type (Figure 269). Updated complexity classes chart and Simon's algorithm chart. Changed Wainvam into Kwan-Tek in magnetometer sensor vendors and added QuantumDiamonds in quantum sensing vendors. Update on Aliro Quantum. Updated software vendor map. Update on Riverlane. Corrected global investment chart (Australia being there twice). Small updates in the quantum sensing part (Safran, Cryolock becoming QuantX lab). Added a map of the Netherlands research ecosystem. Small updates on the Denmark and Finish quantum ecosystems. Small edits in the quantum hype section. Added band gap, break-even, cavity, counterfactual models and feedforward in glossary.

I update the book on a regular basis as I find editorial issues, mistakes, misspellings, and the likes. You can submit me any comment, correction, suggestion or even request for digging into some untapped topic (olivier@oezratty.net).

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