Understanding Quantum Technologies

Sixth edition – 2023

Volume 1/3: history, quantum physics, quantum computing basics and engineering

Olivier Ezratty

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Olivier Ezratty advises and trains various public and industry organizations in the development of their innovation strategies in the quantum technologies realm. He brings them a rare 360° understanding of the scientific, technology, market and ecosystems dimensions of this burgeoning and complex domain.

He covered many other topics since 2005, like digital television, Internet of things and artificial intelligence. As such, he carried out various strategic advisory missions of conferences or training in different verticals and domains such as **media and telecoms** (Orange, Bouygues Telecom, TDF, Astra), **finance and insurance** (BPCE, Société Générale, Swiss Life, Crédit Agricole, Crédit Mutuel-CIC, Generali), **industry and services** (Schneider, Camfil, Vinci, NTN-STR, Econocom, ADP, Air France, Airbus) and the **public sector** (CEA, Météo France, Bpifrance, Business France).

He became a quantum technologies specialist in 2018 with many complementary activities:

- **Author** of the reference book **Understanding Quantum Technologies** (September 2021, 2022 and 2023) following three previous editions in French in 2018, 2019 and 2020. The 2021, 2022 and 2023 editions are also available in paperback version on Amazon.
- **Trainer and teacher** on quantum technologies for **Capgemini Institut** and for **CEA INSTN**. In September 2021, he took in charge an elective curriculum on quantum technologies for **EPITA**, an IT engineering school in France.
- **Speaker** in a large number of quantum technology events since 2018 such as the Q2B Paris organized by QC Ware, France Quantum and other events, on top of presentations at Société Générale, BNP, Crédit Agricole, Michelin, Adéo, L'Oréal, FIECC, IHEDN, Business France, CentraleSupelec, Avolta Partners, IHEDN, etc.
- **Producer** of two series of podcasts on quantum technologies along with Fanny Bouton (in French): a monthly « Quantum » on tech news (since September 2019) and Decode Quantum, with entrepreneurs and researchers since March 2020, with a total of over 100 episodes as of July 2023.
- **Cofounder** of the **Quantum Energy Initiative** with Alexia Auffèves (CNRS MajuLab Singapore), Robert Whitney (CNRS LPMMC) and Janine Splettstoesser (Chalmers University, Sweden).
- **Expert** for **Bpifrance** to evaluate quantum collaborative projects and startups.
- **Ambassador** for France 2030 since February 2022, the French government innovation strategy plan.

He also lectures in various universities such as CentraleSupelec, Ecole des Mines de Paris, Telecom Paristech, Les Gobelins, HEC, Neoma Rouen and SciencePo, on artificial intelligence, entrepreneurship and product management (until 2020) and on quantum technologies (since 2018), in French and English as needed. He is also the author of many open source ebooks in French on entrepreneurship (2006-2019), the CES of Las Vegas yearly report (2006-2020) and on artificial intelligence (2016-2021).

Olivier Ezratty started in 1985 at **Sogitec,** a subsidiary of the Dassault group, where he was successively Software Engineer, then Head of the Research Department in the Communication Division. He initialized developments under Windows 1.0 in the field of editorial computing as well as on SGML, the ancestor of HTML and XML. Joining **Microsoft France** in 1990, he gained a strong experience in many areas of the marketing mix: products, channels, markets and communication. He launched in France the first version of Visual Basic in 1991 and Windows NT in 1993. In 1998, he became Marketing and Communication Director of Microsoft France and in 2001, of the Developer Division, which he created in France to launch the .NET platform and promote it to developers, higher education and research, as well as to startups. Olivier Ezratty is a software engineer from **Centrale Paris** (1985), which became CentraleSupelec in 2015.

Credits

Cover illustration: personal creation associating a Bloch sphere describing a qubit and the symbol of peace (my creation, first published in 2018) above a long list of over 400 scientists and entrepreneurs who are mentioned in the ebook.

This document contains nearly 1,000 illustrations. I have managed to give credits to their creators as much as possible. Most sources are credited in footnotes or in the text. Only scientists' portraits are not credited since it's quite hard to track it. I have added my own credit in most of the illustrations I have created. In some cases, I have redrawn some third-party illustrations to create clean vector versions or used existing third-party illustrations and added my own text comments. The originals are still credited in that case.

Table of contents

Foreword

Quantum technologies hold the promise of major disruptions in computing, communications and sensing. While 2023 marks the beginning of the era of “Quantum Utility” according to IBM, with a term coined for the unveiling of their new processor, scientific and technological challenges to their large-scale deployment are still important, and it is quite difficult for public decision makers, users, investors, professionals, and the public at large to anticipate when these will happen. This is of paramount importance for companies to stay competitive, for governments to position their country in this technology race, or for students to make decisions about their career. While some quantum devices are already in use with practical impact, e.g., sophisticated microscopes taking benefit of the exquisite sensitivity of the spin of point defects in diamonds, other technologies will take years if not decades to reach the markets.

But the situation is changing fast and the evolutions that come with each new edition of Olivier Ezratty’s *magnum opus* make me realize each year that the quantum revolution is happening at a faster rate than anybody could have anticipated. Some in the public might be disappointed because applications are further away “at the corner of the street” but the truth is that the scientific and technological challenges are significant, and the community is addressing them step by step. I’m often asked whether there is not too much “hype” in the field. I don’t think so, particularly when I am comparing quantum technologies with other sectors. This is the beginning of market recognition, for a sector which impact is slowly being assessed properly. At Quantonation, we are making investments since 2018 and have funded 25 companies in Quantum Tech and Deep Physics, and our vision of a future powered by Quantum Technologies is consolidated every day.

But to make proper assessments and keep control of the quantum narrative, we need deep experts who have a proper understanding of all the facets of the technology, from the fundamentals of the science to its applications, including questions about their deployment, their funding, how to teach them, and more. There is a need for a multidisciplinary collaboration involving scientists, engineers and users capable of taking a forward-looking posture. And here enters my friend Olivier Ezratty, the author of this most wonderful book “Understanding Quantum Technologies”, who embodies multidisciplinarity. He has the unique ability to listen, question, gather facts, and synthesize his learnings in a book that stands out as unique in the whole world, as far as I know.

I first met Olivier when I started Quantonation back in 2018. From the start I was impressed by his extremely methodic approach that he had applied with success on an earlier publication on artificial intelligence, and his very unique ambition. The book was first published in French, later in English, and it grew with the field he was “decoding” to use the title of Olivier’s famous podcast with Fanny Bouton on quantum technologies. The book has gotten only better with time, with thorough updates and new chapters about exciting topics. Olivier has also been among the very first supporters of the not-for profit that I co-founded and chaired, Le Lab Quantique. Le Lab Quantique is proud to promote “Understanding Quantum Technologies”, an instrument that will benefit its ecosystem building mission.

Since the last edition, the field of quantum science has been greatly honored by the Nobel Prize in Physics being awarded to the John Clauser – Alain Aspect – Anton Zeilinger trio, the “godfathers of the second quantum revolution”. Olivier Ezratty’s book is an indispensable instrument to read how this revolution is happening and how it will impact our world.

Christophe Jurczak, Partner at Quantonation, Paris and co-founder, Le Lab Quantique

Why

Welcome to the 6th edition of "Understanding Quantum Technologies", a unique book for multiple reasons! Its history, its content, its density, and its purpose. The first edition started as a compilation of a series of 18 posts that I published in French between June and September 2018. After two enriched editions in French in 2019 and 2020, I switched to English in the fourth edition in September 2021, the fifth in September 2022 and this sixth edition in September 2023.

This book is a kaleidoscope for quantum technologies with a 360° perspective encompassing historical, scientific, technological, engineering, entrepreneurial, geopolitical, philosophical, and societal dimensions. It is not a quantum for dummies, babies, or your mother-in-law book. It mainly targets information technologies (IT) specialists and engineers who want to understand what quantum physics and technologies are about and decipher its ambient buzz, all participants to the quantum ecosystem from researchers to industry vendors and policy makers, and at last scientific students who would like to investigate quantum technologies as an exploratory field.

This book is also a very large review paper with over 3,301 bibliographical references (+1,209) totaling 105,632 pages and 214 presentations containing 10,180 slides, all in 4,771 footnotes. The book bears a lot of specificities compared to the existing quantum literature. While being rather technical in many parts, it tries to explain things and translate the complex quantum lingua in other tech's lingua, particularly for IT and computer science professionals. It looks at the history of quantum science and ideas and pays tribute to key people, from the past and the present.

Figure 1: metaphoric view of my position when updating this book. Charles Chaplin in "Modern Times", 1936.

It investigates rarely covered aspects of quantum technologies and quantum engineering like various enabling technologies (cryogenics, electronics, materials design, semiconductors, cabling and lasers, manufacturing technique), their energetic dimension and what raw materials are used and where they come from. I also cover quantum matter. I even explain how research works in general and in the quantum realm and its various codes and practices. The book can also be viewed as an integrated collection of several books, which also covers quantum sensing, telecommunications, and cryptography. I also created many precisely crafted custom illustrations that I use in my teachings and training. In a way, it may be a demonstration of how quantum engineering could be viewed as a discipline.

Another differentiation is in the tone, relaxed when possible and calling out the nonsense when necessary. It is abundant, particularly when some analysts and vendors are fueling the quantum hype. As quantum technologies are more commonplace, these are still largely misunderstood by general audiences as well as by many IT professionals, and by many people writing about it. One striking example shows up when some folks explain that thanks to quantum cryptography, quantum computers will help make cryptography more secure!

Governments' technology ambitions and industry vendors have elevated quantum technologies to the rank of strategic sectors in many developed countries, even more than artificial intelligence. Most governments have launched their national quantum plans, starting with Singapore, the UK, China, USA, Germany, Japan, Australia, France, Russia, Israel, Taiwan, India and the Netherlands. The worldwide quantum technologies race is on. Countries are embattled to acquire or preserve their technological sovereignty, like if it was the last chance to achieve it, particularly for those countries who felt they lost the digital battle against the USA and Asia (mostly China, South Korea and Taiwan).

Also, like many deep techs, quantum technologies are dual-use ones, with both civilian and military use cases, increasing the strategic stakes.

While it has not yet reached the volume and funding of other sectors such as artificial intelligence or the digital cloud, the quantum startup and small business ecosystem continues to expand worldwide. In this book edition, I mention about 650 such companies in many different categories (hardware, software, telecommunications, cryptography, sensing, enabling technologies, services). In most cases, hardware is in the deep tech realm if not in hard tech territory, with many still at an applied research stage with a rather low technology readiness level. Being still very uncertain, this market remains quite open to opportunities for scientists and creative innovators, while in other markets like with semiconductors and large consumer Internet players, the game looks like it is less open.

Quantum technologies are also surrounded by a fair share of hype. A few scientists, their laboratory's communication department, startups and large vendors frequently exaggerate the impact of their work. Many companies also integrate "quantum" into their positioning if not branding in many fancy ways. Either in a totally artificial way or based on using technologies from the first quantum revolution. Transistors, lasers, and image sensors are quantum, so most digital technologies can claim to be quantum. Therefore, we must learn to distinguish the old (first quantum revolution related) from the new (second quantum revolution related). The hype shows up also when analysts are pretending that quantum computing is ready for business, misleading customers about the maturity of the technology. However, even stronger bs shows up elsewhere, with false science-based quantum medicine and other charlatanism. I showcase it in a unique section dedicated to quantum hoaxes and scams, starting page 1266.

This book has another special flavor. It is the result of an unprecedented human adventure at the heart of the quantum ecosystem. I started the journey back in 2016. I had then decided to select the theme of quantum computing for my usual techno-screening activities, ranging from preparing conferences and training to writing educational ebooks for professionals. I was joined by my friend **Fanny Bouton** to run a popularization conference on quantum computing in Nantes. She brought and still brings a different perspective, including some science fiction derived inspirations. This led to the conference **Le quantique, c'est fantastique** on June 14th, 2018 (video) and to numerous subsequent presentations. On top of that, we launched two series of podcasts (mostly in French) covering quantum tech news and with interviews with researchers, entrepreneurs, investors and users. We also worked on gender balance and contributed as early as possible to this sector feminization and attract new talents. Fanny took an interesting turn in 2020, starting to work on **OVHcloud**'s startup program. She was instrumental in embarking this European cloud vendor in the quantum adventure and is their quantum lead since 2022. We both went from an observer role to a very different one.

In this journey that is still going on, we've had the opportunity to meet with top researchers and entrepreneurs, first in France, and then internationally. This list keeps growing.

It started with **Alain Aspect** (IOGS), **Philippe Grangier** (IOGS), **Daniel Esteve** (CEA), **Patrice Bertet** (CEA), **Maud Vinet** (CEA), **Tristan Meunier** (CNRS Institut Néel), **Eleni Diamanti** (CNRS LIP6), **Iordanis Kerenidis** (CNRS IRIF), **Pascale Senellart** (CNRS & UPS C2N and Quandela), **Elham Kashefi** (CNRS LIP6 and VeriQloud), **Alexia Auffèves** (CNRS Institut Néel in Grenoble and now MajuLab in Singapore), **Philippe Duluc** and **Cyril Allouche** (Atos), **Xavier Waintal** (CEA), **Robert Whitney** (CNRS LPMMC), **Théau Peronnin** (Alice&Bob), **Georges-Olivier Reymond** and **Antoine Browaeys** (Pasqal) and many others afterwards. We also toured almost all quantum **startups** in France. And of course, **Christophe Jurczak** from Quantonation, who kindly wrote this book foreword. Our outreach then expanded internationally, particularly in Canada, the USA, the UK, Austria and the Netherlands. I had the opportunity to discuss with **Artur Ekert**, **Peter Knight**, **Tommaso Calarco** and many startup founders, from **PsiQuantum**, **IQM**, **ParityQC**, **ProteinQure**, **Qilimanjaro**, **Qblox**, **Quantum Brilliance**, **Quantum Machines**, **AQT**, **SEEQC**, **Jay Gambetta**, **John Martinis**, **Jerry Chow** and **Hanhee Paik** from IBM, **Simone Severini** from AWS and so on.

In short, during these years, we have been "embedded" in the scientific and entrepreneurial ecosystem. We also applied one of Heisenberg's principles derivatives, namely that a measurement device may influence the measured quantity. It was and remains a beautiful adventure with real people, passions, convictions, ups and downs, and in the end, a nice result with French and European research and entrepreneurship in quantum technologies that are more dynamic and better positioned than a few years ago.

You may wonder why this book is free and what is its underlying business model. Are you the product like we say with free Internet services? Well, not at all. I have published all my books like this since 2006 and fared well so far (on entrepreneurship, artificial intelligence and other technology and science related topics). I favor distribution breadth over revenue. It makes knowledge easily accessible to broad audiences, particularly with students. Also, being distributed in digital format, books are easy to correct and update (see Figure 1). It is quite practical when you mention hundreds of people and organizations, and deal with complicated scientific matters. Afterwards, I sell my time in a rather traditional way with speaking, training, expertise, and consulting missions. The business model is simple: the (very) long version is free and the (too) short versions are charged. Since the people who don't have time usually have money and the other way around, it works quite well even if it may be counterintuitive in the first place. In the end, my pride is to meet young professionals in the quantum ecosystem who thank me for the book, which contributed to them landing in quantum.

A complex domain in search of pedagogy

After having swept through many areas of science and deep techs, I can definitively position quantum physics and quantum computing at the complexity scale apex. Quantum physics is difficult to apprehend since relying on counter-intuitive phenomena like wave-particle duality and entanglement, and on a mathematical formalism that is not obvious to most people, particularly with most IT specialists and developers, one of the key audiences for this book. It is still an open challenge to first understand, then translate this scientific field lingua into natural language for most people, even with a strong engineering background. And you know, I also don't have a PhD in quantum physics (private joke)!

You probably heard about the rehashed quote from **Richard Feynman** who pointed out that when you study quantum physics, if you think you understood everything, you are making a fool of yourself. **Alain Aspect** confirms this, always expressing doubts about his own understanding of the nonlocality of quantum entanglement that he experimented with photons in his famous 1982 experiment which led him to be awarded the Nobel prize in physics in 2022.

Explaining quantum technologies is thus a new and difficult art. When reading quantum physics books, you discover a mathematical formalism and many terms like observables, degeneracy, gentle measurement, unitary and projector, operators and the likes and wonder how they relate to the physical world. Sometimes, it takes quite a while before being able to make the connection, whenever possible! On the other hand, you hear simplistic descriptions of quantum physics, noticeably on superposition and entanglement, and quantum computing, some coming from quantum computing vendors themselves[1]. And you have the infamous dead and alive cat that is the best fake news in quantum physics.

Once you think you understand it after having created a mental view of how it works, your explanations become quickly inaccessible for the profane. How do you avoid this side effect? Probably with finding analogies and using more visual tools to explain things than too much mathematics. I try this approach in many sections of this book, but, still, mathematics are useful in many parts. Also, to make sure it does not lose its scientific soundness in the process, many parts of this book have been fact-checked and proof-read by quantum scientists. I'd say, still not enough. You'll be the judge.

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[1] See the interesting point in What Makes Quantum Computing So Hard to Explain? by Scott Aaronson, June 2021.

But you know what? Some parts in this document contain stuff that I write but do not understand well. Or sometimes, I understand it well but when I review it later, my understanding is gone (like for the topics on the right of the complexity scale in Figure 2). Quantum scientists sometimes feel the same.

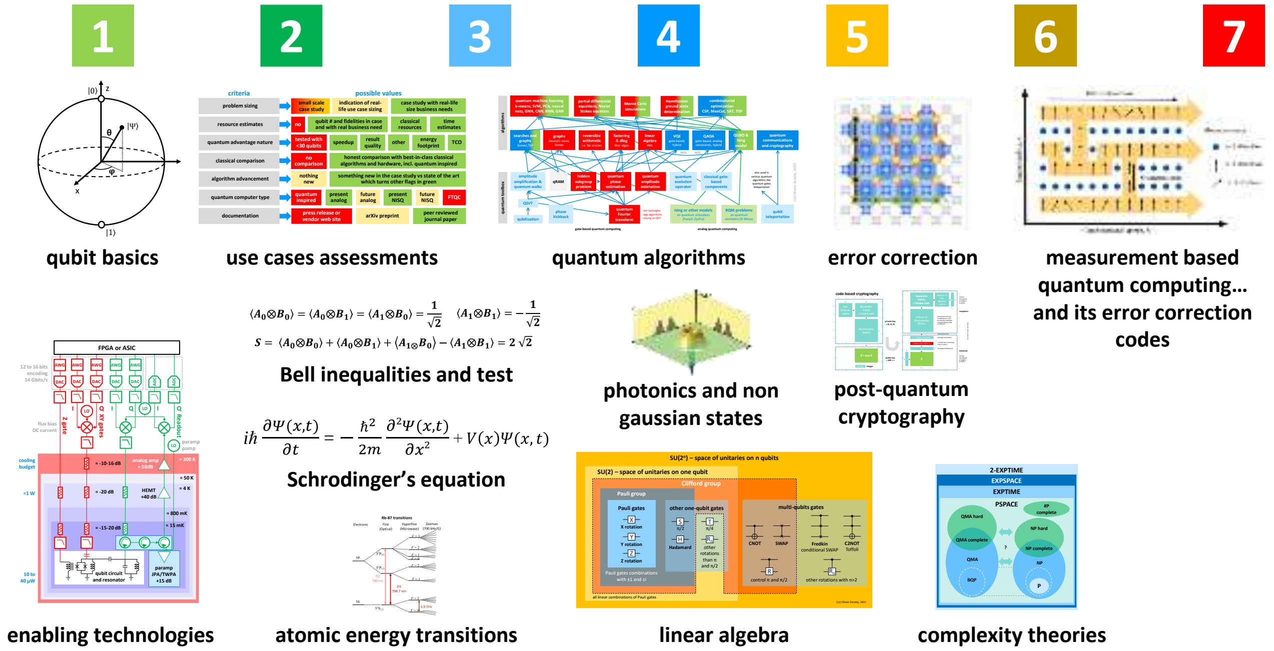


Figure 2: a scale of complexity in quantum physics and technologies, from the easy (left) to the very difficult (right), at least, as far as I am concerned. I could have added qRAM here but there was not enough room in the chart! (cc) Olivier Ezratty, 2023.

This book frequently responds to questions like what, why, where and how? Has Moore's empirical law really stalled (page 11)? What being "quantum" mean for a product or technology (page 7)? Do we really have objects sitting simultaneously at two different locations (page 117)? Why parallel opposite vectors in the Bloch sphere representing a qubit state are mathematically orthogonal (page 186)? Why and where density matrices are useful (page 173)? What are pure and mixed states describing in the physical world (page 171)? Why superposition and entanglement are the two sides of the same coin (page 119)? Why do we need to cool many qubit systems at very low temperatures (page 359)? How are cryostats operating (page 563)? What is the energy consumption of a quantum computer (page 284)? How much data sits in a quantum register (page 190)? How is data loaded in a quantum program (page 867)? What data is generated by quantum algorithms and how is it decoded (page 871)? Are quantum computers made for big data applications (no, page 867)? How can you compare such and such quantum computer technology and qubit type (page 327)? What is your preferred one (none)? Which one can scale best (all have limitations)? Can NISQ bring some commercial value (not yet, page 910)? Can analog quantum computing compete with gate-based models (page 930)? Is the Shor integer factoring algorithm a serious threat for your cybersecurity (page 711)? When will we have a "real" quantum computer (page 266)? Will quantum computers save the world (healthcare, climate change, …) (no)? Have we really achieved quantum supremacy (no, page 1017)? What is the difference between quantum supremacy, advantage and utility (page 1017)?

And on and on… What is the real speedup of quantum algorithms (it depends on a lot of factors, page 949)? How to analyze a quantum computing case study (page 1025)? Are there quantum computing case studies in production (not really)? Will a quantum Internet replace the existing Internet (no, page 761)? Can quantum telecommunications enable either faster than light communications or high-throughput data links (no, page 119)? How are classical computing technologies competing with quantum computers (page 668)? Why are some quantum random number generators not that random (page 717)? Why can entanglement improve quantum sensors precision (page 800)? Is China going to kill us (metaphorically) with their (not so) huge R&D investments in quantum technologies (no, page 1205)? Have they really invested \$15B in quantum technologies? (no, page 1205).

Can Europe take its fair share in this new market (page 1128)? Oh, and if I'm in an organization... what should I do (page 1227)? Am I late in the game by doing nothing? Should I stay or should I go? Why are some people overselling the capabilities of quantum computers (page 1031)? Will governments build dangerous weapons with quantum technologies (not really, page 1085)?

So, you may understand why I am kind of annoyed when I am invited to present the whole field of quantum technologies in a half an hour session!

To properly address this broad laundry list of questions, this book is positioned above the average media coverage of quantum computing, as well as analyst reports, and below classical scientific publications that are generally largely inaccessible to non-specialists, or to specialists from other domains.

A new technology wave

Quantum computing stays on top of the various applications of the second quantum revolution. Quantum sensing is more exotic and fragmented, and quantum telecommunications and cryptography are less fascinating. Why is quantum computing becoming an important topic? Firstly, because large IT companies such as IBM, Google, Intel and Microsoft are making headlines with impressive announcements that we must, however, take with a grain of salt, with a lot of hindsight, and decipher calmly. There's also the obvious impact of Peter Shor's factoring algorithm. It drives fuzzy and I'd say unfounded fears on the future of Internet security and for your own digital privacy.

Above all, it is linked to the broad impact that quantum technologies could have on many scientific fields and digital markets. It may theoretically make it possible to solve problems belonging to classes of complexity that even the largest giant supercomputers will never be able to tackle with. The other reason for this sudden interest is that we are still at the beginning of the story. New leaders will show up. A new ecosystem is being built. This is a field where there are still enormous scientific and technological challenges to overcome. It is a land of opportunities for science, technology, and innovation. Like with quantum physics, we are in a highly indeterministic world.

It is quite difficult to evaluate the feasibility of large-scale quantum computing. For most scientists, we are still many decades away from it. Some believe it will never show up. Others are more optimistic. The main enemy is quantum decoherence and qubits errors happening during computing, and which are difficult to avoid and correct. The plan is to fix that with quantum error corrections and logical qubits made of physical qubits. It then becomes, at least, a physical scalability issue with a bunch of complex engineering issues related to cooling, cryo-electronics, cabling, classical computing, miniaturization, as well as fundamental thermodynamic and energetic dimensions. In the end looms an overarching question: how many quantum objects can we control in an entangled state? It is a very interesting living case study of how mankind builds upon scientific progress and addresses the most difficult challenges around. For this respect, it is on par with controlling nuclear fusion. The joke being, who is going to be first? Nobody really knows for sure.

Reading guide

Here is a tentative to prioritize which parts of this book you could read according to your business and scientific level (Figure 3). Physicists can find a state-of-the-art tour covering all dimensions of quantum technologies beyond the field they have already mastered.

Computer scientists, engineers and students in various scientific fields are the core target audience for this book, as it presents, popularizes and contextualizes the various scientific, mathematical and engineering concepts used in quantum technologies.

The required mathematical and computer basics level is at the bachelor's degree level for most parts. Afterwards, it can also depend on your age since many of these concepts were not in current programs a couple decades ago unless you were already specialized in quantum physics.

	Book sections	Quantum physicists	Computer scientists and developers	Students in sciences (STEM)	Non technical audiences	Business audiences
Volume 1	Why					
	History and scientists					
	Quantum Physics 101	known	optional			
	Gate-based Quantum Computing					
	Quantum Computing Engineering					
Volume 2	Quantum Computing Hardware					
	Quantum Enabling Technologies		optional			
	Unconventional computing					
	Quantum Telecommunications and Cryptography					
	Quantum Sensing					
Volume 3	Quantum Algorithms					
	Quantum Software Development tools					
	Quantum Computing Business applications					
	Quantum Technologies around the world					
	Corporate Adoption					
	Quantum technologies in society					
	Quantum Fake Sciences					

Figure 3: Understanding Quantum Technologies parts and audience relevance. (cc) Olivier Ezratty 2021-2023.

Non-technical and decision-makers can still read the sections dealing with usages as well as with how countries are faring and societal issues.

Here's another view of the table of contents (Figure 4) showcasing the overall logic between the lower « physics » layers and the upper hardware, software and solutions layers.

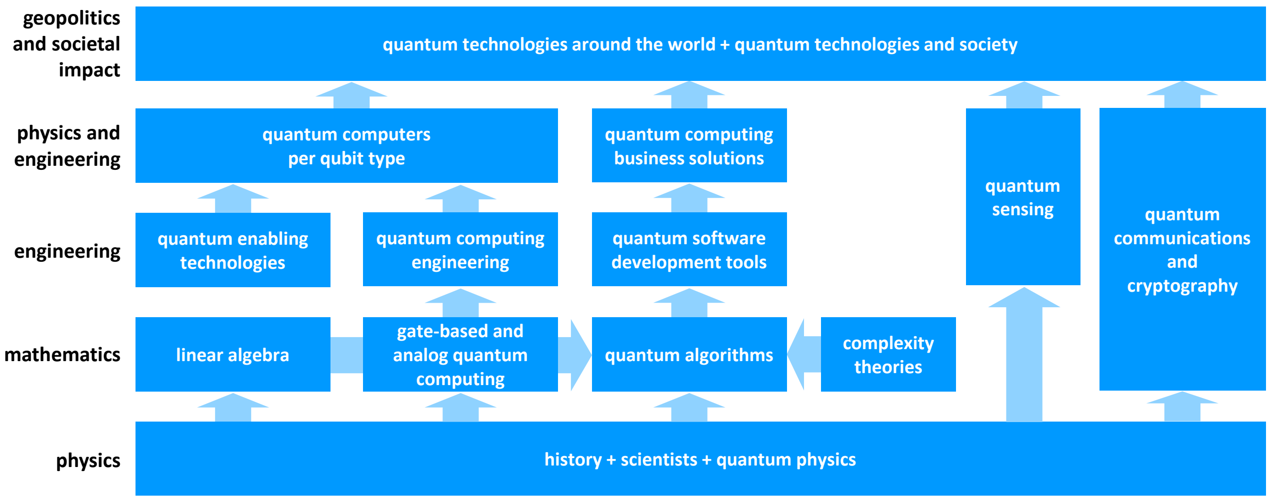


Figure 4: how the topics covered in Understanding Quantum Technologies are related with each other. (cc) Olivier Ezratty.

Let's also mention one of the reasons why a curious mind may like quantum technologies: they encourage you to explore many scientific disciplines, even human and social sciences, like a scientific Pandora's box as described in Figure 5. On top of that, learning quantum science is probably more efficient than Sudoku or crosswords to train your brain muscle as it ages!

physics
electromagnetism
quantum physics
quantum matter
thermodynamics
fluids mechanics
photonics

engineering
materials design
electronics engineering
cryogenics

mathematics
linear algebra
groups theory
analysis
complexity theories

computer science
information theory
algorithms design
programming
classical computing
telecommunications

human sciences
philosophy
epistemology
sociology
technology ethics
economics of innovation
R&D policy making
geopolitics
startups ecosystem

Figure 5: the many scientific domains to explore when being interested in quantum technologies. That's why you'll like this book if you are a curious person. (cc) Olivier Ezratty, 2021-2023.

First and second quantum revolutions applications

Quantum physics has been implemented since the post-war period in almost all products and technologies in electronics, computing, and telecommunications.

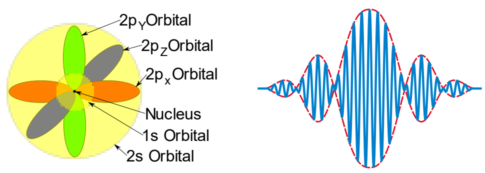
This corresponds to the **first quantum revolution**. It includes transistors, invented in 1947, which use the field effect and are the basis of all our existing digital world, photovoltaic cells which rely on the pairs of electron holes created by incident photons, and lasers which also exploit the interaction of light and matter and are used in a very large number of applications, particularly in telecommunications and optical storage (CD audio, DVD and the likes, which are now mostly outdated).

first quantum revolution
manipulating
groups of quantum particles
photons, electrons and atoms interactions

transistors, lasers, fiber optics, GPS
photovoltaic cells, atom clocks
medical imaging, digital photography and video
LEDs, LCD TV quantum dots

1947-*

second quantum revolution
manipulating
superposition and **entanglement**
and/or **individual particles**



quantum computing
quantum telecommunications
quantum cryptography
quantum sensing

1982-*

Figure 6: first and second quantum revolution definition and related use cases. (cc) Olivier Ezratty, 2020-2023.

Many medical imaging solutions rely on various quantum effects, including nuclear magnetic resonance imaging (MRI). LEDs are also based on quantum effects. GPS relies on atomic clocks synchronization.

Quantum dots used in high-end LCD displays and Smart TVs also use variations of the photoelectric effect[2]. The list is long, and we will not detail all these use cases (Figure 6)!

The **second quantum revolution** covers the technologies combining all or part of the ability to control individual quantum objects (atoms, electrons, photons), use quantum superposition and/or entanglement. We owe the names of the first and second quantum revolutions to Alain Aspect, Jonathan Dowling and Gerard Milburn in 2003[3]. The first and the two following ones created it simultaneously and independently. In the United States, the paternity is attributed to the latter, while in France, it is attributed to the former! Who knows why?

The scope of the second quantum revolution covers various recent applications of quantum physics that integrate quantum computing, quantum telecommunications, quantum cryptography and quantum sensing. Said simply, it is about improving our digital world performance and security, and to increase the precision of all sorts of sensors.

- **Quantum computing** is the broad domain of using quantum physics to find solutions to various computing problems. It includes various computing paradigms like gate-based computing, quantum annealing and quantum simulations. Hundreds of pages are covering this topic in this book, from hardware to software.
- **Quantum cryptography** is a mean of communicating inviolable public cryptography keys thanks to quantum physics phenomena and rules, like photon entanglement and the no-cloning theorem. It relies either on fiber optic communications or on space links with satellites as China has tested with its Micius satellite since 2017. Even though some researchers are proposing to use new quantum computer cryptography schemes, most quantum cryptography plans rely on using quantum key distribution using photonic links.
- **Quantum telecommunications** enables distributed computing, connecting quantum computers enabling qubit to qubit distant entanglement, and, potentially, quantum sensors, which can be implemented to improve their accuracy. This field still in the making could become the base for a very secure quantum Internet and quantum cloud infrastructures. We cannot exploit it to transmit classic information faster than today[4]. However, it can be used to distribute quantum processing on several quantum processors. It could provide a mean to "scale-out" quantum computers when "scale-in" approaches reaches its limits. This requires a lot of engineering, particularly to convert solid qubits into photon qubits deterministically and leverage shared entanglement resources.
- **Post-quantum cryptography** is a different field which is intended to replace current classical cryptographic solutions with new solutions that are supposed to be resistant to attacks carried out by future quantum computers. It does not belong to the second quantum revolution per se but is rather a consequence of it.

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[2] Alexi Ekimov (1945, Russian), Louis Brus (1943, USA), and Moungi Bawnedi (American-Tunisian-French) were awarded the Nobel prize in chemistry for the discovery and synthesis of quantum dots in October 2023.

[3] See Speakable and unspeakable in quantum mechanics by John S. Bell, June 2004 edition (289 pages) which contains a preface by Alain Aspect on the second quantum revolution, dated February 2003, pages 18 to 40. We find the expression in Quantum technology: the second quantum revolution by Jonathan P. Dowling and Gerard J. Milburn, June 2003 (20 pages) as well as in Quantum Technology Second Quantum Revolution by Jonathan Dowling, 2011 (60 pages). Dowling's writings make a very large inventory of various quantum technologies embedded in this second quantum revolution. The Second Quantum Revolution: From Entanglement to Quantum Computing and Other Super-Technologies by Lars Jaeger, 2018 (331 pages) is a broader overview of the different sides of the second quantum revolution.

[4] But..." *Entangled states cannot be used to communicate from one point to another in space-time faster than light. Indeed, the states of these two particles are only coordinated and do not allow to transmit any information: the result of the measurement relative to the first particle is always random. This is valid in the case of entangled states as well as in the case of non-entangled states. The modification of the state of the other particle, however instantaneous it may be, leads to a result that is just as random. Correlations between the two measurements can only be detected once the results have been compared, which necessarily implies a classical exchange of information, respectful of relativity. Quantum mechanics thus respects the principle of causality*". Source: https://fr.wikipedia.org/wiki/Intrication_quantique.

- **Quantum sensing** makes it possible to measure most physical dimensions with several orders of magnitude better precision than existing classical sensing technologies, even existing atomic clocks. It is a vast scientific field that is the subject of numerous research projects and industrial solutions. It includes ultra-precise atomic clocks[5], cold atom accelerometers and gyroscopes that use atomic interferometry, SQUIDs (superconducting based) and NV center magnetometers. Micro gravimeters measure gravity with extreme precision, enabling discoveries of underground anomalies like holes, water, and various materials. This domain also includes various advanced medical imaging systems with higher precision and non-destructive imaging and measurement tools[6]. A dedicated section of this book is covering quantum sensing, starting page 327. The diversity of quantum sensing solutions or prospect solutions is staggering.

You may have heard about a third quantum revolution. It is a misnomer that you can quickly forget given it adds a "nuclear" quantum revolution before the two from Aspect/Dowling/Milburn[7].

Why quantum computing?

The main reason why quantum computers are being built is to solve complex problems that are and will stay inaccessible to classical computers. This happens for example when these problems solutions scale exponentially in computing time on classical machines. In extreme cases, computing times on conventional computers for exponential problems, even with the most powerful supercomputers of the moment, could largely exceed the age of the Universe, estimated at 13.85 billion years. We cannot be that patient.

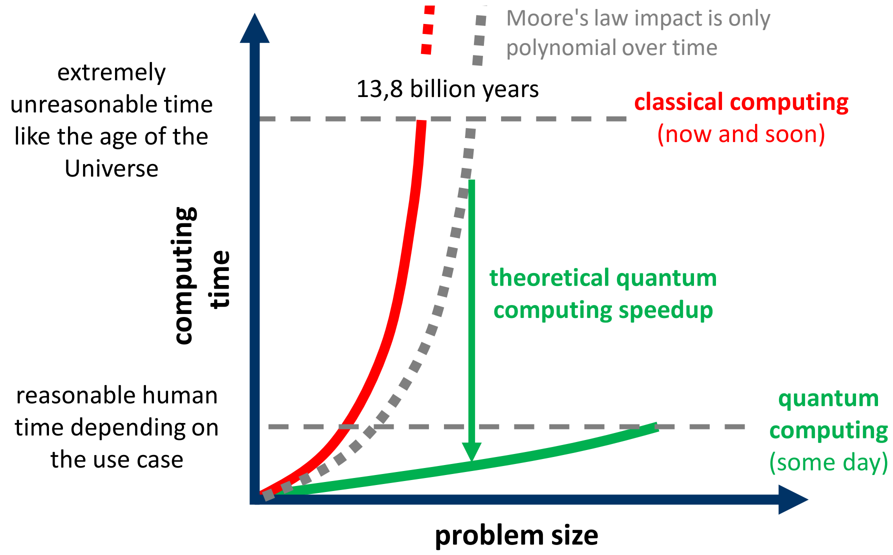

Figure 7: simplified view of the quantum computing theoretical promise. Before delivering this promise, quantum computers may bring other benefits like producing better and more accurate results and/or doing this with a smaller energy footprint. (cc) Olivier Ezratty, 2022.

The dream with quantum computers it to solve these problems in times that scale differently, polynomially, or even at a lower scale (linearly, logarithmically, …) and, preferably, down to reasonable times with regards to the business needs.

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[5] See for example this NIST work on an atomic clock based on rubidium, the element most frequently used in atomic clocks. NIST Team Demonstrates Heart Of Next-Generation Chip-Scale Atomic Clock, May 2019.

[6] See Quantum camera snaps objects it cannot 'see', by Belle Dume, May 2018. This is a variant of Diffraction Free Light Source for Ghost Imaging of Objects Viewed Through Obscuring Media by Ronald Meyers, 2010 (22 pages). Yanhua Shih (University of Maryland) US Army Research Laboratory, has been working on the subject since 2005. Quantum Imaging by Yanhua Shih, 2007 (25 pages). Also, see Quantum Imaging - UMBC (47 slides).

[7] See "3rd Quantum Revolution": Can The Radical Potential Of Quantum Be Reclaimed? by Lucy Rose Sollitt, The Quantum Inside, December 2022.

Problems that scale polynomially on classical hardware are said to be of lesser interest for quantum computing although some algorithms are supposed to bring some potential useful speedups in that area too. The promise of quantum computing is to address this need. But a big warning and legal disclaimer is needed here: it is still an undelivered *promise*! Turning this promise into reality is one of the most difficult, challenging and exciting goals in science and technology development.

Quantum computing promise

Typical exponential problems are combinatorial optimization searches and chemical simulations. Their size is usually expressed as a quantity like a number of steps for solving a travelling salesperson problem. Exponential problems are said to be "intractable" because their classical computation time evolves in crazy proportions, exponentially, with their size.

Optimization problems are first in sight, such as the above-mentioned famous traveling salesperson problem, with its contemporary equivalents applied to product delivery or autonomous vehicles routing. Today, you optimize your route with Google Maps or Waze, based on traffic conditions. Traffic conditions are variable, and your actual journey time is not always what was planned nor optimal. With fully autonomous fleets, it may theoretically be possible to optimize the individual path of each vehicle based on their departure and destination locations. Conventional algorithms could work with a limited number of vehicles, but beyond a few hundred vehicles and trips, traditional computing capacities would be largely saturated. Quantum computing may then come to the rescue at some point, provided they can handle very large problems with hundreds of thousands of vehicles, which is not that obvious as we will see later in this book. It should work in real time, which would not necessarily be a given, even with very powerful quantum computers.

Physics and molecular simulations come next, themselves governed by many-body quantum physics equations. Showcased chemical quantum simulations algorithms are usually about finding the minimum energy configuration of a system, its ground state. But other algorithms are looking at how molecules interact, i.e. docking, at chemical pathways, and at the way molecules are vibrating or rotating. Rest assured, this will not go so as far as simulating an entire living being or even a living cell. It will already be a fantastic feat if and when we are able to simulate some simple de-novo protein folding in a better way than what DeepMind AlphaFold 3 is doing today with deep learning, the next step being protein interactions simulations[8]. Physics simulations also deal with material designs based on the understanding of crystal structures or how magnetism works operates.

Machine learning is another field of interest with training and inferences of machine learning and neural networks models. It is now within the reach of conventional computers equipped with GPGPUs (general purpose GPUs) such as Nvidia's V100, A100, H100 and H200 and their tensor processing specialized units, optimizing matrices-based operations. Obtaining a quantum advantage is less obvious in this field, particularly since machine learning must usually be trained with large data sets. Nowadays, however, using quantum computing for machine learning happens to potentially bring another benefit which is creating better solutions instead of bringing some speedup.

Integer factorization comes last, which is of particular interest to the NSA and their peers to break RSA-type public-key encryption security. There is no business case for this unless you want to spy on somebody. We will dig into this in details starting page 705 in the cryptography threats assessment section.

Quantum computing will however not become a “jack of all trade” solution. It won’t become a replacement tool but more a complement to current High-Performance Computers (HPC).

[8] The competition from classical machine learning is still significant and growing. See Scientists are using AI to dream up revolutionary new proteins by Ewen Callaway, Nature, September 2022.

Many, if not most of today's classical computing problems and software are not at all relevant use cases for quantum computing. Most businesses data processing tasks will remain classical.

From an economical historical perspective, the consequence is that quantum computing will probably not be a Schumpeterian innovation. It will not entirely replace classical legacy technologies. It will complement it. It is an incremental instead of being a replacement technology.

You probably won't have a quantum desktop, laptop or smartphone to run your usual digital tasks although quantum technologies can be embedded in these devices like quantum sensors and quantum random number generators. Quantum computers will be hidden from users and sit in cloud data centers, like Nvidia GPGPUs racks. This will be even amplified by the progress we can anticipate with wireless telecoms. When and if quantum computers scale, some year after 2030, we'll probably use 6G or 7G networks with even better latency and bandwidth! Of course, it is still hard to anticipate the usages brought by quantum computers when they will scale.

Let's still boil in the fact that, as we'll see later, quantum computers are not excellent at handling big data, nor are they adapted for any form of real-time computing. This makes it less relevant to use a local quantum processor, as it makes sense today to have local neural networks capacities to handle your in-camera image recognition processing and voice recognition in smartphones. Less data means more relevance for distant quantum computation implemented in the cloud.

Business cases are investigated for different markets such as finance, insurance and even marketing. Many businesses have complex optimization problems to solve. Like with most technology-driven disruptions, businesses will progressively discover quantum computing use case as its market and related skills grow.

But we will avoid putting the cart before the wheel. Contrarily to what is usually said, we do not lack algorithms and use cases. What is missing is the hardware to execute it. All these promises are dependent on the ability to create large scale and fault-tolerant quantum computers, which are years if not decades away. In the interim, we may end up having quantum systems able to deliver other benefits like producing better and more accurate results and/or doing this with a smaller energy footprint, but not with some exponential speedup.

> "*Building a quantum computer is a race between humans and nature, not between countries*"
> Lu Chaoyang, China
> December 2020.

Moore's law

One strong motivation to build quantum computers is the perception that classical technology progress may be stalling. The end of Moore's law is looming. Classical computing progress seems to have reached hard limits and a disruptive approach is needed.

Gordon Moore's law was a sort of exponential regression used to predict the rate of growth of the number of transistors in a chip, doubling every 24 or 18 months [9]. Gordon Moore's paper was written when he was working at Fairchild Semiconductors, only 5 years after the production of the first integrated circuit and 6 years before Intel created its first microprocessor, the 4004. It was an era of relatively fast technological progress.

The Moore's law nickname was created after Moore's paper was published, by Carver Mead, a Professor at Caltech and friend of Gordon Moore, who passed away in 2023. Moore's law was based on a sampling made with only five data points ranging from 1960 to 1965 as shown in Figure 8, in the very early years of the history of integrated circuits production.

[9] See Cramming more components onto integrated circuits by Gordon Moore, Electronics, Volume 38, Number 8, April 19, 1965.

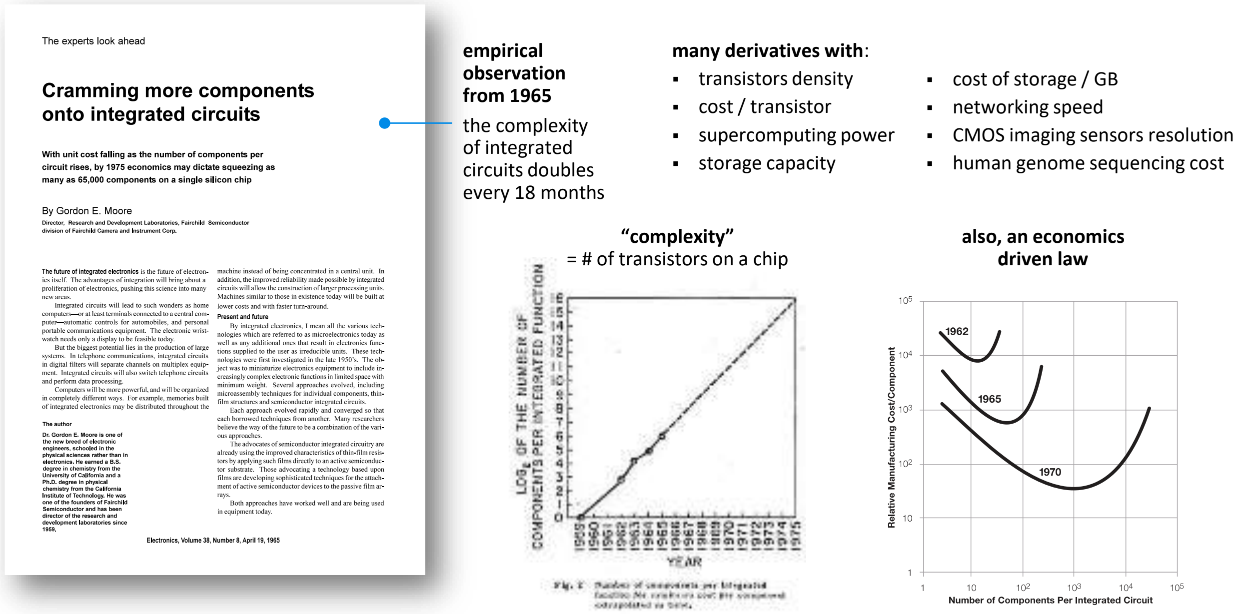


Figure 8: Gordon Moore's original 1965 paper dealt with both transistor number per chip trends and an economic driven law. Source: Cramming more components onto integrated circuits by Gordon Moore, Electronics, Volume 38, Number 8, April 19, 1965. (cc) 2023.

Integrated circuits were invented by Jack Kilby from Texas Instrument in 1958 and first produced in 1960. The progress was both in number of transistors, cost per transistor and surface density. A regular wafer was only one inch large when nowadays, they are 12 inches large (30 cm) and can accommodate hundreds if not thousands of chips depending on their size, or just one large chip, like Cerebras' giant CS-2 wafer-scale chip manufactured by TSMC.

Gordon Moore's empirical law application, or "More than Moore" as its successor is now labelled, would have a marginal impact on computing times for complex problems as dotted in Figure 7. Whatever the progress, it would not bring the capacity to solve exponential problems in non-exponential times. The addition of a single qubit theoretically doubles quantum computers power, both in terms of internal memory space and computing parallelism capacity, even though one could argue that adding a single functional qubit to a quantum computer appears to be exponentially difficult with the number of qubits.

So, why does Moore's law seem to have reach its limits? As a matter of fact, it hasn't yet, when looking at the trend plotted in Figure 9. The number of transistors per chip is still increasing. The progress that is not literally associated with Moore's law and that has stalled is elsewhere, with the single-thread performance, chip clock rates, power per chip and also, their number of logical cores.

Dennard scale is the real law that came to an end, a while ago, around 2006. It stopped progress in the three mentioned areas (thread performance, clock, power). **Robert Dennard**'s (1932, American) scale established in 1974 that, forecasted that, as the transistors density increased, the power consumed per unit area of the chips was to be stable. As shown in Figure 10, this happened since the transistor's voltage and current could decrease with their density, while increasing the clock frequency. Starting with 65 nm integration in 2006, this rule was broken, coming from unwanted leakage current between source and drain regions caused by depletion areas interpenetration.

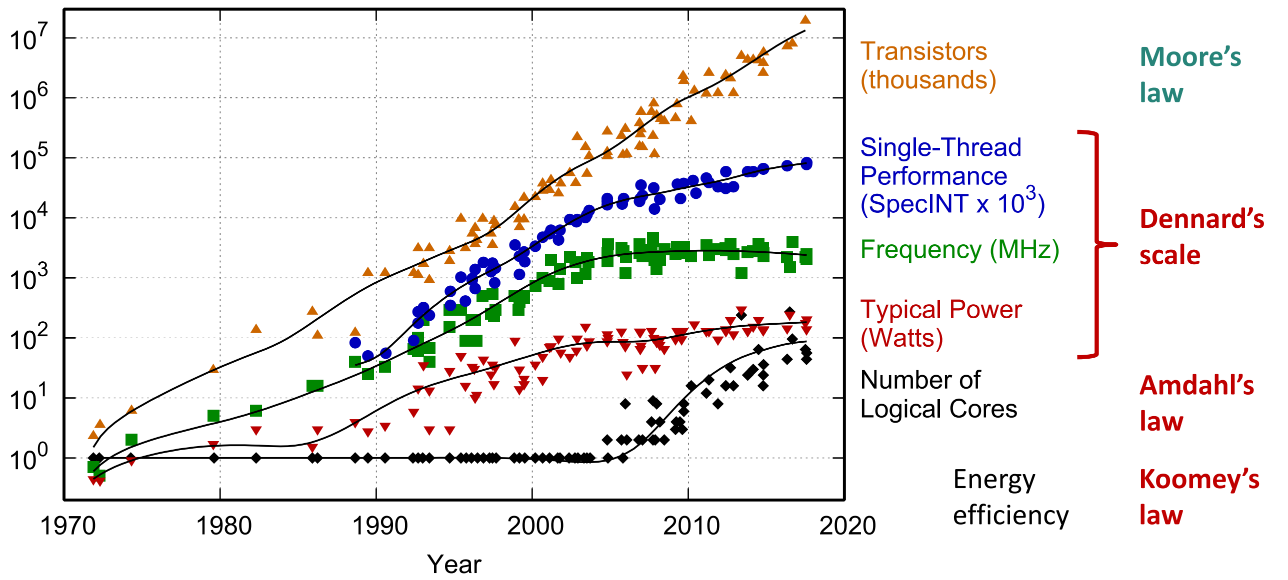


Figure 9: 42 years of microprocessor technology trends. Source: [Karl Rupp](), 2018. (cc) Olivie Erratty, 2023.

That's why, among other phenomena, your laptop computer is also heating your legs when you use it in public transportation or in your coach[10]. As a result, this "heat barrier" limited the capacity to increase processor clock speed beyond 5 GHz. It can reach 6 GHz with liquid cooling[11]. The transistors current leaks started to grow and power consumption soared. This is what prevents the growth of processors clock. At the beginning of the 2000s, Intel planned in its roadmaps to raise their CPU clock frequency up to 20 GHz.

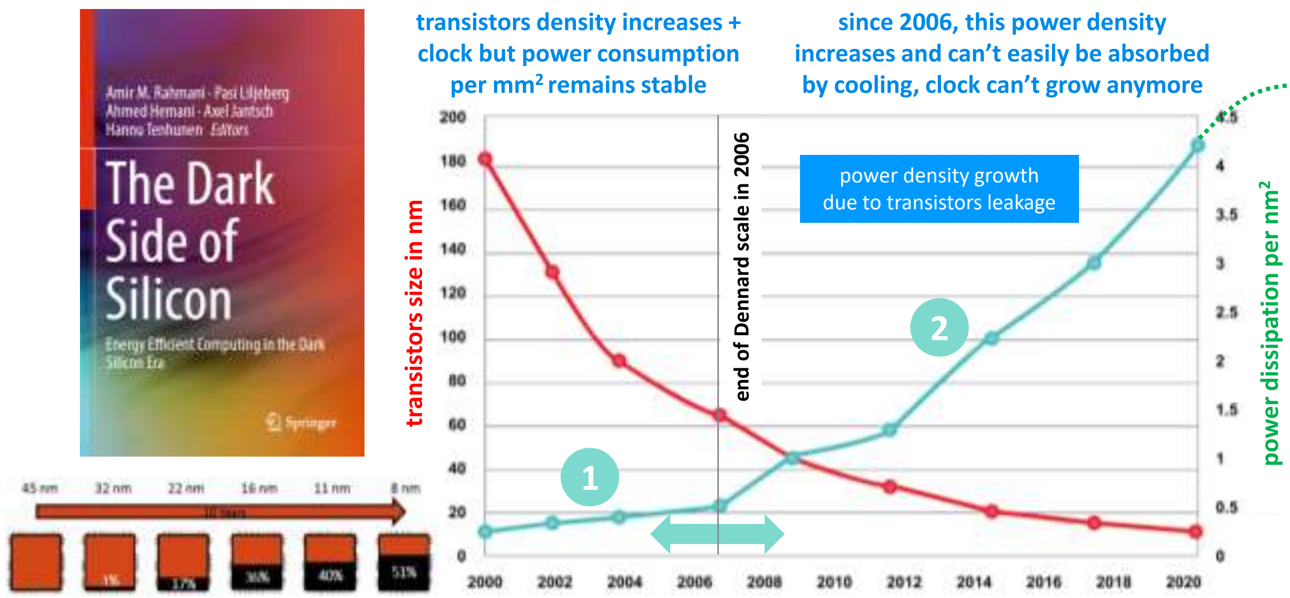


Figure 10: Dennard's scale which explains the dark silicon phenomenon where all CMOS chips components cannot be used simultaneously. Compilation (cc) Olivier Ezratty. 2020-2023.

[10] Another phenomenon is the tunnel effect happening at the thin grid oxide level, that is reduced with using high-dielectric constant oxides ("high k dielectric").

[11] See on this subject [Minimum Energy of Computing, Fundamental Considerations]() by Victor Zhirnov, Ralph Cavin and Luca Gammaitoni, 2014 (40 pages) which compares the energy efficiency of living things and electronics.

Intel then stopped playing this game and instead entered the multicore realm (Figure 11). However, in June 2021, Intel released a new microprocessor for high-end laptops running at a 2.9 GHz base clock but with a 5 GHz turbo mode for a single core, the 4-core i7-1195G7, etched in 10 nm, and with a 28W TDP[12].

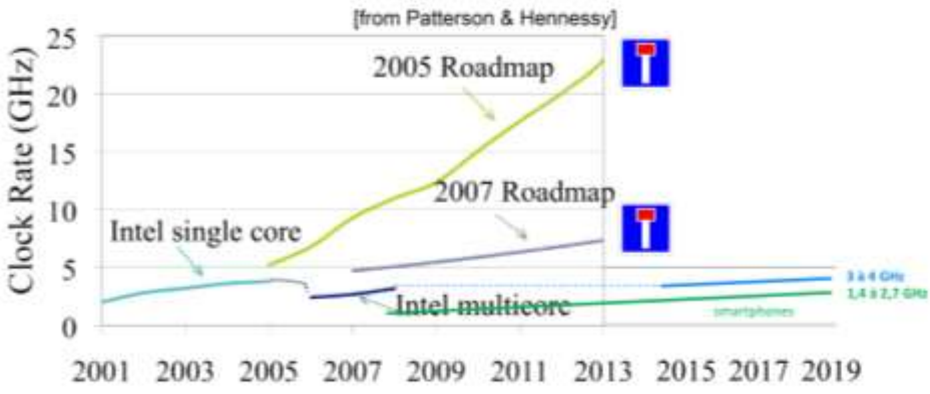


Figure 11: how CMOS chips clock was supposed to increase… and didn't. Source: [High Performance Computing - The Multicore Revolution]() by Andrea Marongiu (41 slides), 2019. Additions: Olivier Ezratty.

The semiconductor demand switched in 2007 towards low-power multi-functions chips for smartphones. This opened a boulevard for Arm core-based processors and growth for corporations like **Qualcomm**.

Koomey's law empirical law proposed in 2010 by **Jonathan Koomey** observed that the available computing power per consumed kW increased steadily, doubling every 1.57 years between 1946 and 2009. However, this doubling slowed down to 2.6 years after 2000, due to the end of Dennard's scale. It indirectly explained why multicore architectures are limited in number of independent cores.

Dark silicon is a rarely mention phenomenon associated to the end of Dennard's scale. As the chips get too hot, it becomes difficult to use it entirely. Various methods are then combined to circumvent this inconvenience: on-demand cores or functions deactivations according to usage needs, a shutdown of certain portions or cores, a voltage drop, a selective clock frequency adjustment per core of simply, a low clock speed (Nvidia GPGPU's run at 1 GHz).

Multi-core architectures enabled parallel processing but with limits formalized by **Amdahl's law,** which describes the upper limits of parallel computing systems acceleration.

This is what is used in the Arm core-based processors of smartphone chips, whose cores do not use the same clock rates, in the so-called big.LITTLE architectures created in 2011, and replaced with the more flexible DynamIQ architecture in 2017 [13].

Some other laws are also applicable in the science-fiction domain when you reach quantum limits[14].

Transistor density evolution

The semiconductor industry had to cope with many limitations when improving transistor density, Landauer barrier, the heat barrier, some unwanted quantum effects, the reticle size limits and the resolution of etching manufacturing techniques (Figure 12).

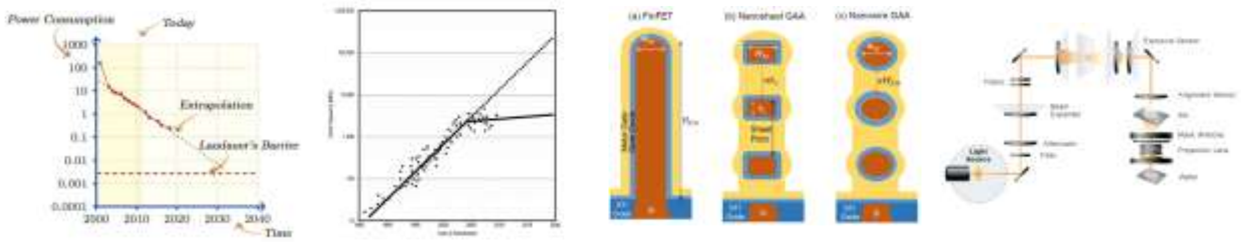
Landauer barrier defines the minimum energy required to erase a bit of information. It is a very low theoretical barrier contested by some physicists. And it can be circumvented as we will see with the technique of adiabatic and reversible computing that is covered page 682. It was created by **Rolf Landauer's** (1927-1999, researcher at IBM in 1961).

[12] Thermal dissipation power.

[13] There are many other techniques to improve classical processors energy efficiency. See for example Energy Efficient Computing Systems: Architectures, Abstractions and Modeling to Techniques and Standards by Rajeev Muralidhar et al, AWS and Melbourne University, July 2020 (35 pages).

[14] See Ultimate physical limits to computation by Seth Lloyd, 2000 (22 pages).

some CMOS density technical challenges



Landauer barrier lower power consumption limit

heat barrier limiting processor clocks

quantum effects dielectric <=6 atoms thick (light blue)

reticule size limit chipsets die size limit

Figure 12: some of the key CMOS density technical challenges to overcome by the semiconductor industry. Two sources: [Reversible Circuits: Recent Accomplishments and Future Challenges for an Emerging Technology]() by Rolf Drechsler and Robert Wille, 2012 (8 pages) and [On the Vertically Stacked Gate-All-Around Nanosheet and Nanowire Transistor Scaling beyond the 5 nm Technology Node]() by Hei Wong et al, Nanomaterials, 2022 (15 pages).

Quantum effects are undesirable phenomenon appearing with a tunnel effect showing up in the thinner grid oxide.

Etching resolution is getting smaller to enable the manufacturing of more precise and smaller features in transistors, particularly below 10 nm nodes. Lithography etching systems are using extreme ultraviolet, coming from **ASML**. Etching resolution indeed depends on the wavelength of the light used to project a mask on a photoresist. Lowering the transistors size requires increasing this frequency to decrease the wavelength, and thus go from the current deep ultraviolet to extreme ultraviolet. It took more than 10 years to develop these EUV lithography systems. It is in production since 2019 in TSMC and Samsung 5 nm nodes fabs. One of key benefits of EUV etching is to reduce the usage of the costly multiple patterning process to improve lithography resolution. ASLM's latest EUV lithography generation is dubbed High-NA (for high numerical aperture). A bit like in photography, High-NA optics will convey more light onto masks and silicon targets and will be required for nodes under 3 nm. It requires both new UV optics, but also new light sources. And the EUV machines are much bigger and costly. These machines will be deployed around 2024. The generation after High-NA would be Hyper-NA but even ASML is doubting it will be economically viable[15].

Reticles size corresponds to the optical systems used in lithography whose size is physically limited, especially optically. It is explained in Figure 13, coming from **ASML,** the world leader in semiconductor lithography. This limit has been reached with the largest recent processors.

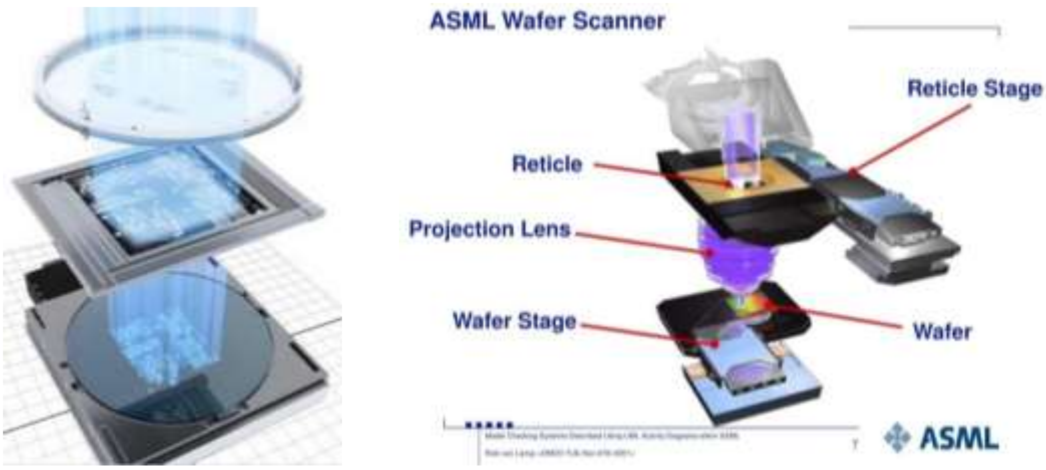


Figure 13: reticle used in photolithography and its related optics, explaining the size limitation of dies in semiconductor manufacturing.

[15] See [Hyper-NA after high-NA? ASML CTO Van den Brink isn't convinced](), Bit Chips, September 2022.

Other scaling solutions were found including using vertical transistors like the traditional FinFET technology that has been in use for more than 10 years, that is now expanded with nanowires and nanosheets techniques as shown in Figure 14[16], multi-die packaging associating multiple chips in a single packaging. The FD-SOI technology from CEA-Leti and STMicroelectronics adds an isolated layer of silicon oxide on silicon wafers, that limits the effects of transistor leakage and enables better operations at high frequencies with energy savings. It is particularly used in radio-frequency front-end chips in smartphones.

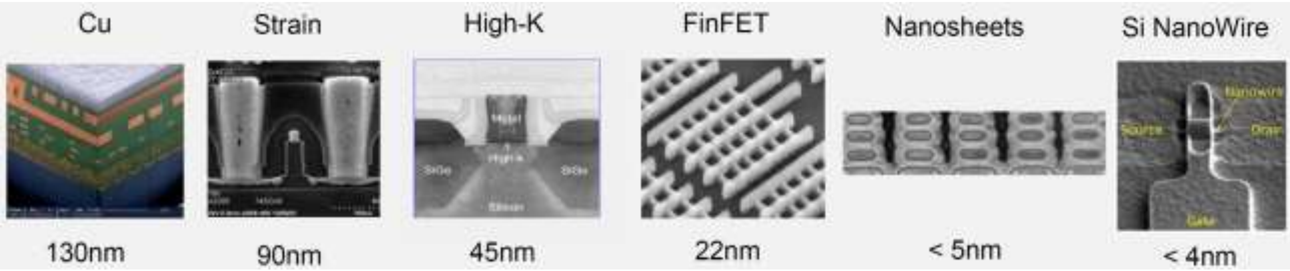


Figure 14: the various CMOS transistor technologies used as density increased.

Transistor density fake news. After 2006, transistor density continued to grow. You've heard about these successive generations of 28 nm, 14 nm, 10 nm, 7 nm, 5 nm, 3 nm and now 2 nm transistor sizes. In May 2021, IBM announced it had prototyped 2 nm nanosheet-based chips, manufactured by Samsung, and also using EUV lithography[17]. In December 2022, the company announced they could scale as low as 1 nm thanks to using ruthenium for chip interconnects[18]. In July 2021, Intel announced a new density scale using angstrom sized transistors, with 20Å and 18Å by 2025 (meaning... about 2 nm, given 1 Å = 0.1 nm). TSMC announced the production of 2 nm chips in 2022.

Unfortunately, this is all fake news, probably one of the most significant in the digital industry, and it has been going on for over 10 years. Seriously! These transistors have no features with these announced sizes. This is a marketing trick from the whole semiconductor industry.

This is shown in Figure 15 with a table consolidated by the IEEE late 2022. It describes all the transistor feature sizes for the "nodes" labelled 3 nm (2022) down to 0.5 nm (planned for 2037)[19]. What you discover here is that the metal pitch between 3 nm transistors is of 24 nm with "3 nm" transistors and will go down to 16 nm for "0.5 nm" transistors. In these generations of transistors, the smallest features are with the nanosheet thickness, which is 4 nm. So how does the semiconductor industry justify this marketing labelling of nodes? One is the real labelling is too complicated, with G48M24 for gate pitch and metal pitch sizes for "3 nm" densities. The second is that this fake density corresponds to the density power increase of these chips. Lastly, starting in 2031, density progress will come from stacking several layers of transistors on top of the other. It will probably not avoid the heat barrier and the dark silicon phenomenon. In the end, the only feature that is below the 1 nm threshold is the gate oxide thickness but it is a vertical, not an horizontal feature.

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[16] See Beyond CMOS, Superconductors, Spintronics, and More than Moore Enablers by Jamil Kawa, Synopsys, March 2019 (43 slides), a good presentation describing the various ways to improve the power of components including cold CMOS, semiconductors operating at liquid nitrogen temperature levels (-70°C) and superconducting Josephson effect based transistors.

[17] See IBM Introduces the World's First 2-nm Node Chip by Dexter Johnson, IEEE Journal, May 2021.

[18] See The path to 1 nanometer chips and beyond by Mike Murphy, IBM, December 2022.

[19] As mentioned in Wikichip technology node, "Since around 2017 node names have been entirely overtaken by marketing with some leading-edge foundries using node names ambiguously to represent slightly modified processes. Additionally, the size, density, and performance of the transistors among foundries no longer matches between foundries. For example, Intel's 10 nm is comparable to foundries 7 nm while Intel's 7 nm is comparable to foundries 5 nm".

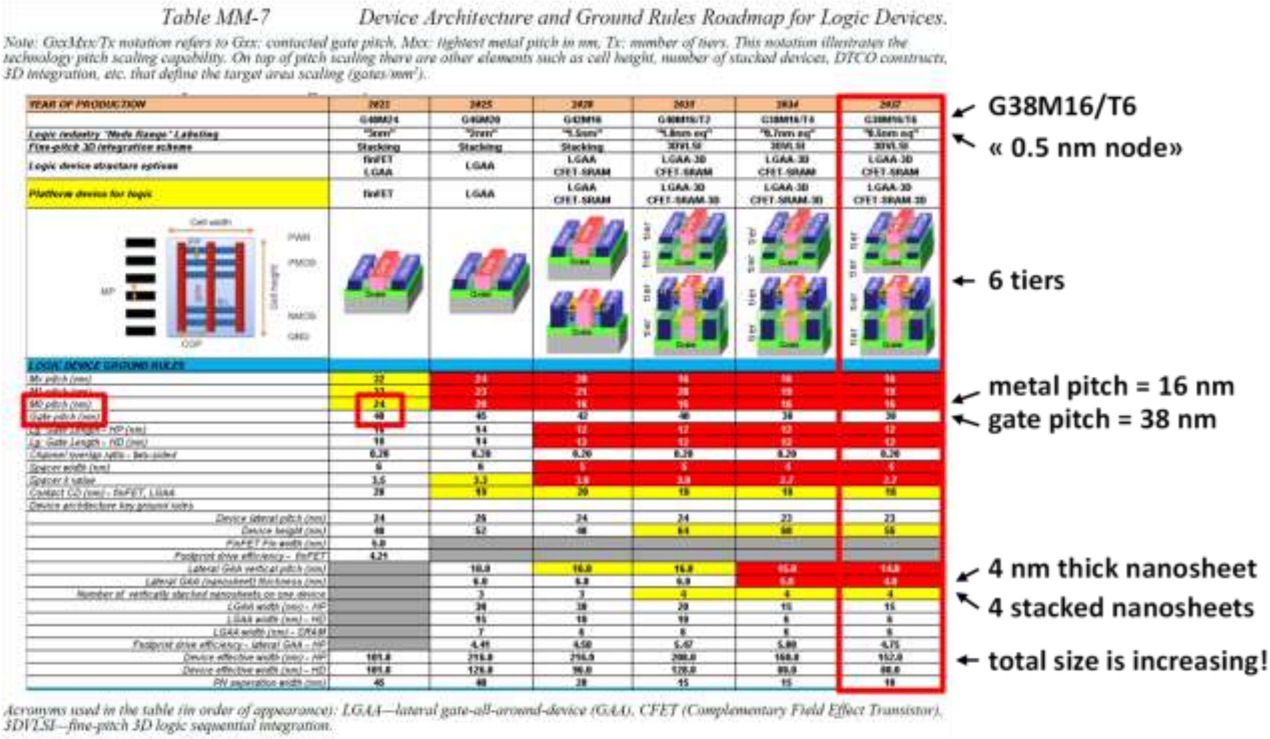

Figure 15: the real transistor feature sizes per generation showing that 3 nm, 2 nm and below do not correspond to any real size in transistor designs in horizontal features. Transistor size is not significantly changing from one generation to the other, validating the "end of Moore's law" claim. The right way to describe these nodes would be a number scheme like G48M24T1 with a gate pitch of 40 nm, a metal pitch of 24 nm and one layer of transistors for 3 nm nodes. The 0.5 nm in the table above would become G38M16T6. It is of course more complicated than 0.5 nm! There are still features between 2.7 and 6 nm which are "spacers" between transistor features. Source: International Roadmap for Devices and Systems 2022 update "More Moore", 2022 (39 pages). (cc) Olivier Ezratty, 2023, for the annotations.

Classical computing technology developments

The semiconductor industry used some other techniques to increase classical computing power, and I won't mention all these here.

Domain Specific Architectures consists in encoding in silicon various features to make it more efficient both in speed and energy consumption. Most smartphone and laptop chips have been using and improving this technique for a while, embedding features like GPU cores, tensor cores for machine learning computing, audio and video codec DSPs, security units, input/output units and the likes. Multiple features are integrated in single die chips aka "system on chip" (SoC).

2.5 and 3D packaging is another path used to integrate multiple features in small packaging associating specialized chips manufactured with different techniques (CPU, GPU, fast SRAM cache memory, storage, RF, photonic links features) and connected through high-speed links and buses (Figure 17).

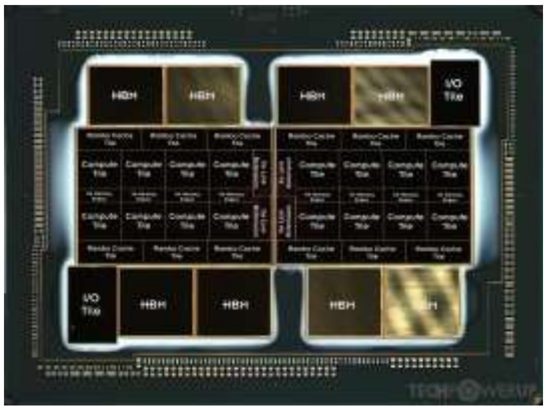

Figure 16: Intel Ponte Vecchio processor with its chiplet with 47 chips including cache, compute, HBM memory and I/O chips. Source: Intel. 2023.

One example is the Intel Ponte Vecchio processor used in the DoE Frontier Aurora supercomputer build with a chiplet containing 47 active chips[20] (Figure 16).

[20] See Intel's Take on the Next Wave of Moore's Law Ann B. Kelleher explains what's new 75 years after the transistor's invention by Samuel K. Moore, IEEE Spectrum, December 2022.

Memory. One key technological development is to make sure memory is as close as possible to processing units, including in-memory processing[21].

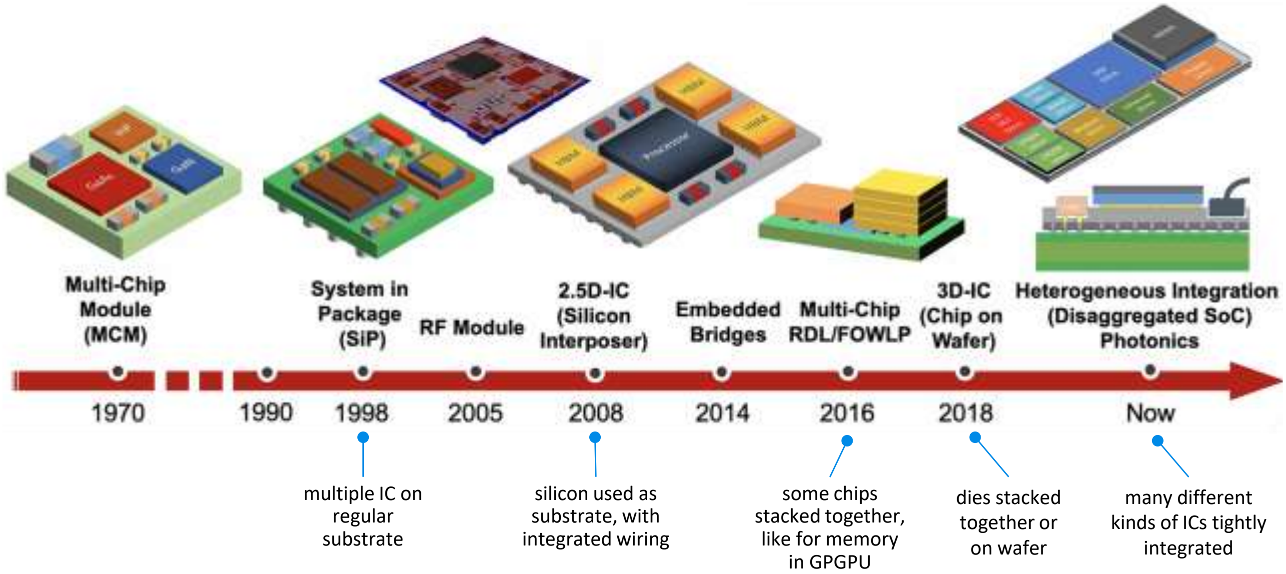

Figure 17: various 2D to 3D chips integration techniques into chiplets. Source: An introduction to chiplet-based architectures by John Park, Chipscale Review, 2020 (4 pages).

SIMD for single instruction multiple data processing which is used in GPUs and GPGPUs (general purpose GPUs) handling matrix multiplications in parallel. This is the technique used by **Nvidia** among others, starting with the V100 in 2017, the A100 in 2020, the H100 in 2022 and H200 in 2023.

The A100 had 54.4 billion transistors superseded closely in size by the **Graphcore** GC200 with its 59.4 billion transistors and 1,472 cores. The H100 launched in 2022 has 80 billion transistors, consolidating two adjacent chips in a single package. Then came the Nvidia GH200 which embeds CPU arm cores, removing the need for a traditional CPU-to-GPU PCIe connection. They use Nvidia NVLink-C2C chip interconnects, increasing the bandwidth between GPU and CPU by 7x compared with the latest PCIe technology and reduces interconnect power consumption by more than 5x. The H200 launched in November 2023 has 141 GB of HBM3E memory with a 4.8TB/s bandwidth.

SSD storage with PCIe connectivity has accelerated computing by an order of magnitude compared to classical hard disks. In your laptop, you can reach a 3 GB/s data transfer speed compared to about 100 MB/s with a hard drive. The integration levels in 3D NAND flash chips are similar to CMOS transistors with pitches that can go down to 12 nm. But since all memory is not used simultaneously, these chips used stacks of transistors. The current record is 232 layers of memory with 1,000 layers in sight by 2030[22].

Neuromorphic processors which mimic biological neuron features with integrated memory and processing using memristors[23]. They can be implemented with spintronics electronics, that imitate how brain cells work with their own memory[24].

[21] See Energy Efficient Computing Systems: Architectures, Abstractions and Modeling to Techniques and Standards by Rajeev Muralidhar et al, July 2020 (35 pages) which makes a good inventory of the various ways to save energy with classical computing. And Processing-in-memory: A workload-driven perspective by S. Ghose et al, IBM Research, 2019 (19 pages).

[22] See Improvement of memory performance of 3-D NAND flash memory with retrograde channel doping by Deepika Gupta et al, July 2023 (6 pages).

[23] One famous work with neuromorphic processor is the Loihi project from Intel. See Intel's Neuromorphic Chip Gets A Major Upgrade Loihi 2 packs 1 million neurons in a chip half the size of its predecessor by Samuel K. Moore, IEEE Spectrum, October 2021.

[24] See the review paper Quantum materials for energy-efficient neuromorphic computing: Opportunities and challenges by Axel Hoffmann, Julie Grollier et al, April 2022 (24 pages).

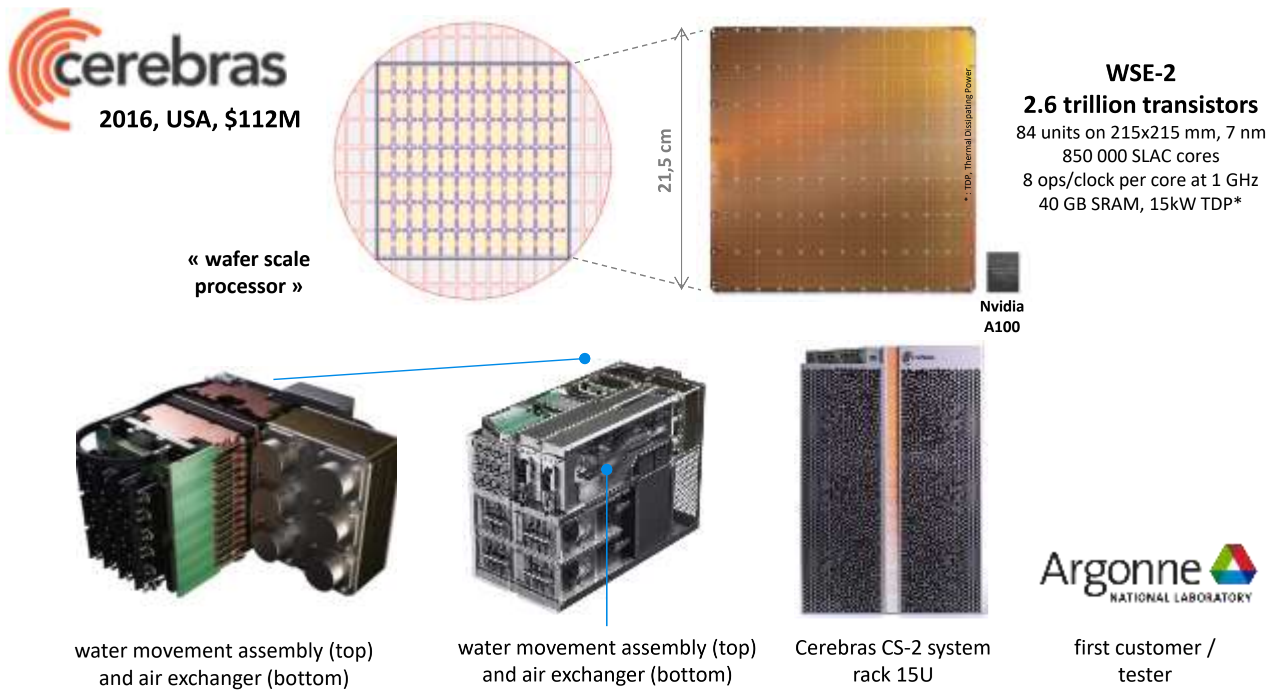

Figure 18: the impressive Cerebras wafer-scale chip. Source: Cerebras.

Chip size record can reach 21.5 cm x 21.5 cm. It was achieved in 2019 by **Cerebras** (USA), fitting the chip in an entire 300 mm wafer, which circumvents the reticle size limit by being etched in several runs, for its 84 main processing units connected by metal layers. The second version of this chip launched in 2021 contains 2.6 trillion transistors and 40 GB of cache SRAM memory and has a memory bandwidth of 20 PB/s, allowing it to significantly accelerate neural networks training. A single chip can accommodate multi-trillion parameters large language models (LLMs) ala ChatGPT.

This massive Cerebras chip, shown in Figure 18, burns about 15 kW per hour which are evacuated by a specific water-cooling system in their CS-2 15U system. Manufacturing techniques generate defects and more than a couple percent of the 850,000 processing units are defective and are short-circuited during software execution[25]. In September 2022, Cerebras announced its own cluster computer using up to 192 CS-2 systems and in November 2022, the Andromeda cluster with 16 CS-2 and 1 exaflops of computing power, fed by 284 AMD 64-core EPYC CPUs[26]. It is competing aggressively against Intel/Nvidia and AMD-based supercomputers that are currently dominating the HPC landscape.

Unconventional computing

In a dedicated part starting page 855, we will evaluate some the other avenues considered to overcome the current limitations of classical computing, which may provide some power or efficiency gains positioned between classical and quantum computing. These belong to the broad category of "unconventional computing".

This includes **superconducting computing** operating at low temperatures (investigated in the USA and Japan), **digital annealing** computing (proposed by **Fujitsu**), **reversible** and/or **adiabatic computing** that could reduce energy consumption and circumvents Dennard's scale end, **probabilistic computing** as well as different breeds of **optical computing** (Figure 19).

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[25] With its D1 chipset presented in July 2021, Tesla chose another approach. Engraved in 7 nm, it has a computing capacity of 22.6 TFLOPS FP32, with 50 billion transistors and a 400W TDP. It contains 354 computing units with 1.25 MB SRAM per unit. They assemble these D1 in 25-chipsets tiles, consuming 15 kW, exactly like a Cerebras chipset.

[26] I am surprised I have not yet heard about Cerebras computers being used for quantum code emulation and tensor networks works.

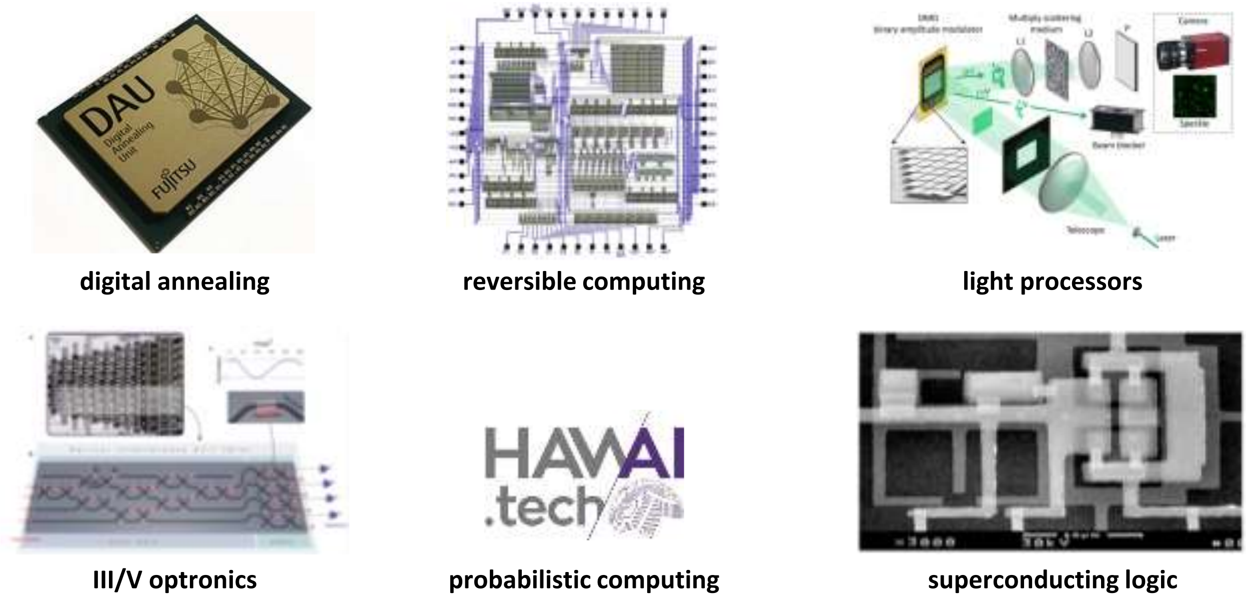


Figure 19: various unconventional computing approaches besides quantum computing. (cc) Olivier Ezratty with uncredited images.

I also delve into some of the inner workings of supercomputers and specialized processors to better understand their strengths and weaknesses. When comparing quantum computers to classical computers, we are better off with knowing both sides of the equation, not just the loud new kid in town!

These are sort of backup solutions, should science fail to create scalable quantum computers. It will also complement quantum computing used in the context of hybrid computing. Interestingly, some unconventional computing avenues, such as superconducting electronics, are potentially enabling technologies for scaling certain types of quantum computers.

However, at this point, none of these solutions seem positioned to solve intractable problems although some of these are claiming they have this capacity, which is quite hard to fact-check at large scales.

The history of technology is about exploring multiple branches. Some do not succeed. Some help each other. Also, some can suddenly wake up after being frozen for decades. The game is open!

Why... key takeaways

- This book is unique in its shape, structure and length. It covers quantum technologies with a 360° approach. It is more scientific than most broad-reach publications, outside research review papers. It is a good appetizer for those who want to investigate the matter whatever the angle. It contains an extensive bibliography with over 3,200 scientific papers. It tries to answer many commonplace questions that are not well addressed in broad audiences scientific publications.
- All existing digital technologies are already based on quantum physics. They are part of the "first quantum revolution" including transistors, lasers and the likes, leveraging our control of light-matter interactions with large ensembles of quantum objects (electrons, atoms, photons). So, your laptop, smartphone, digital camera, television and other digital objects are already "quantum". The "second quantum revolution" corresponds to a new generation of technologies that are using a variable mix of superposition, entanglement and individual quantum objects control. It usually contains quantum computing, quantum telecommunications, quantum cryptography and quantum sensing. Quantum matter applications are a new addition to this list.
- Quantum technologies are at the crossroads of many scientific domains encompassing physics, mathematics, computing, social and economics sciences and the likes. It creates new educational and pedagogy challenges that must be addressed in innovative ways and customized according to various audiences. This book targets broad audiences with a technical background, including computer science engineers, but also quantum physicists and quantum information scientists who want to have a look at what is happening broadly in the field and its burgeoning ecosystem.
- Quantum computing is based on a promise to solve so-called intractable problems whose (classical) computing complexity grows exponentially with their size. These can't be solved with classical computing, whatever happens with Moore's law. But we are not there yet since there are several enormous challenges to overcome to scale quantum computers beyond what can be done today. In the interim, some marginal improvements may come with noisy intermediate scale computers, including better and more precise solutions in several domains. Analog quantum computers may be first to bring a moderate quantum advantage.
- Moore's law has not ended yet, particularly with regards to the number of transistors per chips. Classical computing still strive compared to existing and future quantum computers when dealing with large volumes of data.
- Other new technologies may compete with quantum computing, belonging to the broad "unconventional computing" category. Only a very few of these could also bring some exponential computing capacity. Most others bring other benefits compared with classical computing like in the energy consumption domain. Some of these technologies like superconducting electronics and adiabatic/reversible computing could also be helpful as enablers of quantum computing scalability, particularly with superconducting qubits.

Common quantum computing misconceptions

Misconceptions	Explanation	Where to go
Quantum computing is fast or instantaneous.	While quantum algorithms may be faster than their classical equivalent, in a quantum advantage regime, they are usually quite slow and can last hours, days if not months and more.	Page 276 and page 949.
Quantum computing speedups are explained by the Hilbert space of a qubit register (2^N).	Qubit registers vector state indeed scales exponentially with the number of qubits but it doesn't alone explain the various speedups obtained with quantum algorithms. These speedups depend on other factors like the type of qubit gates used and the size of entangled states in the qubit register.	Figure 181, page 193.
Quantum computers will soon break RSA keys and all Internet cybersecurity.	Progress is currently too slow to envision this. We'd need between 350,000 cat-qubits or 22 million more regular qubits to break an RSA-2048 key. We are many years if not decades off this mark at best.	Page 713.
Quantum computing will enable larger big data applications.	Not really. Quantum computers are quite slow to feed with data compared to classical computers, by several orders of magnitude. This could slightly improve with the use of quantum memory that is not available at all.	Page 867.
Quantum computing will accelerate the advent of artificial general intelligence (AGI).	We don't know yet how to build an AGI, whatever the algorithm. Large languages models are nearly passing the Turing test with their huge training data sets that wouldn't fit in any quantum computer. Quantum machine learning algorithms don't seem to advance the field of symbolic artificial intelligence to implement reasoning. Also, scaling and data loading issues are not solved yet.	Page 893.
Quantum computing is bound to progress following some Moore's exponential law.	While qubit numbers are making progress and fit more or less some exponential trend, it is not the case with qubit fidelities and many other figures of merit.	Figure 279, page 331.
Quantum computing is business ready now.	Many analysts and vendors would like you to believe it but it is not yet the case. Most gate-based quantum computers are either too noisy or have too few qubits to be useful and bring some quantum advantage. Analog quantum computers seem however closer to providing some quantum advantage.	Page 1025.
Quantum computing will (soon) help fix climate change.	No way! It may enable some research in innovative chemical engineering but this will require large scale fault-tolerant quantum computers which are decades away. We'd better fix climate change with classical methods in the meantime.	Page 1089.
China is investing between \$15B and \$25B in quantum technologies and is beating all other geographies.	This is not true. China is investing less than the USA and the European Union. Serious estimates are \$4B over 10 years. The \$15B to \$25B mark comes from a misleading 2017 statement on the Hefei lab buildup, which has fewer than 600 researchers.	Page 1205.
Private investment means it will work.	In many situations, investors' money can't make difficult scientific tasks easier to solve although large startups and a few large industry vendors are better equipped to innovate in an integrated and pluridisciplinary way.	Page 1133.
Quantum communication will replace the Internet with faster transmission speed.	Current quantum communications technologies can't enable this. Fast data communication could still be possible between two quantum computers connected by an entanglement based quantum communication link.	Page 761.
Quantum computers will save energy.	We are not so sure about it, particularly with large scale fault-tolerant systems. This is an area of investigation and optimizations.	Page 284.

History and scientists

After having set the stage, we'll make a historical detour to discover the origins of quantum physics. As with any scientific and technological endeavor, it is above all a great human story. I pay tribute here to the many scientists who, step by step, made all this possible and are still working on it for those who are still in this world.

Nanoscopic physics. Quantum physics deals with atomic and sub-atomic level particles and with the interactions between electromagnetic waves and matter (Figure 20). It differs from classical Newtonian physics, which predictably governs the dynamics of macrophysical objects, beyond a few microns and up to the size of planets and stars. Classical physics is governed by Newton's laws for matter, by Maxwell's laws for electromagnetic fields and associated forces and by statistical physics which describes continuous media such as gases and fluids and from which the principles of thermodynamics are derived.

When the speed of objects becomes close to the speed of light or when we reach large object's mass, the theory of relativity comes in, explaining the curvature of space-time and modelling the impact of gravity. It helps describe extreme phenomena such as black holes or neutron stars. It allows us to interpret the History of the Universe, but not entirely. But relativistic electrons are also hidden in our body's atoms and in many elements on earth as we'll quickly discover with the weird field of relativistic quantum chemistry.

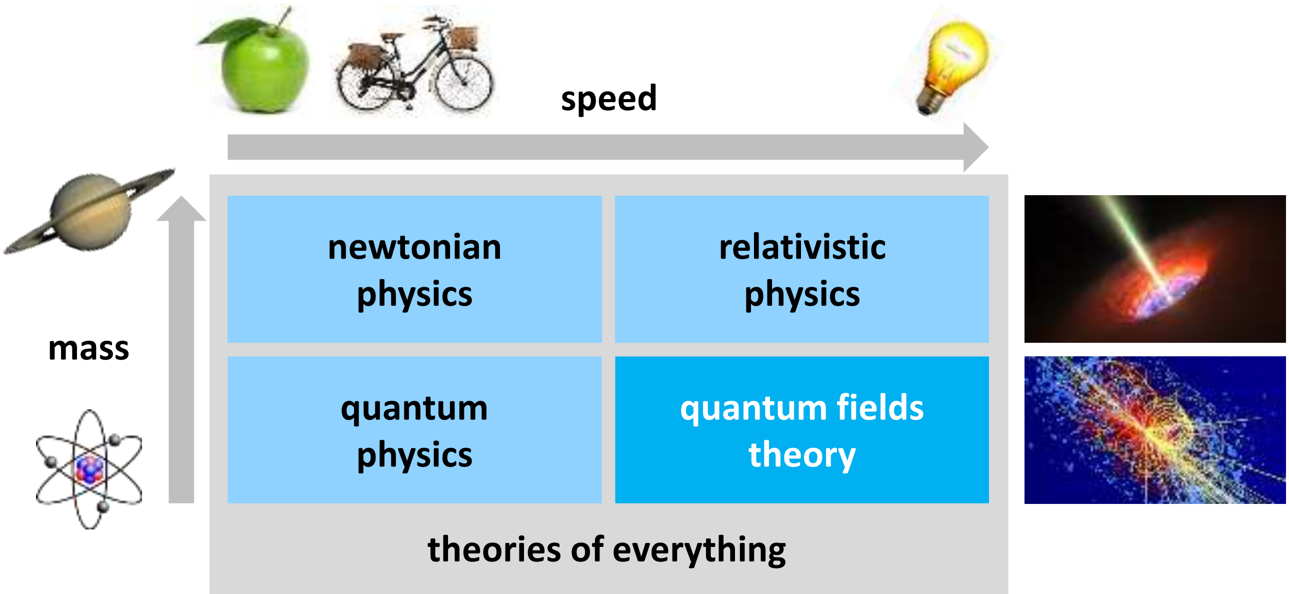


Figure 20: high-level classification of the branches of physics. (cc) Olivier Ezratty, 2020.

The fourth domain of physics in this quadrant is quantum fields theory. It describes the physics of high-speed elementary particles, such as those observed in particle accelerators like quarks and the famous Higgs boson. Richard Feynman is one of the founders of quantum electrodynamics, a subset of quantum field theory.

In a way, quantum physics was a means to unify classical matter physics and electromagnetic waves physics. It helps describe how matter was organized at the atomic and electrons levels and how these interacted with quantized electromagnetic waves, aka photons, including visible light.

Unification is still in the making. Physics is still not yet complete nor unified. Some observable physical phenomena still resist it. We do not know how to explain the origins of gravitation and we are still looking for the dark matter and energy that would explain the cohesion of galaxies and the

Universe current expansion. Scientists would like to explain everything, but some knowledge may never be accessible such as the shape and form of the Universe before the Big Bang.

The so-called theory of everything (ToE) or unification theory sought after by some physicists would be a formalism unifying all the theories of physics and in particular relativity, gravity and quantum physics. This very serious field of physics is still in the making[27]. Numerous proposals emerge and sorting it out is not easy[28].

Connecting the dots. This part will help you memorize who's who in the History of quantum physics and quantum computing. It will also cover some important science basics such as the Maxwell and Schrödinger equations that I will try to explain in layman's terms, at least for readers having basic sciences knowledge. Explaining quantum computing inevitably starts with some quantum physics 101 explanations. Some of its basics, although sometimes quite abstract, must be understood. I still always try to connect the dots between quantum physics and quantum computing from a practical basis. It is a vast puzzle. I will add its pieces one by one and even though the puzzle may not be fully completed, you'll get a picture enabling you to become well educated on quantum computing.

Experiments and theories. Quantum physics took shape in 1900. Like almost all sciences, it is the result of the incremental work of many scientists with interactions between experimentation, theories and mathematical creativity. Sometimes, quantum physics is better explained with its underlying mathematical models than with incomplete physical interpretations. Representation models such as the broad field of linear algebra play a key role in describing quantum states and their evolution in space and time. Linear algebra is also an essential tool to understand how quantum computer qubits are manipulated and measured. Even if we can trace the beginning of quantum physics to Max Planck's 1900 quanta discovery, it was based on earlier work from many other scientists who devised about the particle or wave nature of light, on the discovery of electromagnetism and atoms. Quantum physics is a human adventure that brought together immense talents who confronted each other and evolved step by step their understanding of the nanoscopic world. New generations of scientists have always questioned the state of the art built by their predecessors[29]. Physicists conducted numerous experiments, elaborated theories, and then verified it experimentally, sometimes with several decades of latency. They also had to pour philosophy into their work to interpret the deep significance of their discoveries, and quantum physics was not an exception. Despite its constant enrichment, quantum physics has shown an astonishing robustness to stand the test of time and with extreme precision, in the 10^{-12} range.

[27] The American-Japanese physicist Michio Kaku estimates that some theory of everything will be finalized by 2100. See Michio Kaku thinks we'll prove the theory of everything by 2100, April 2019. Michio is at the origin of string theory. He defines very well the connection between the different branches of physics and this theory of everything in A theory of everything?. But for many reasons too long to explain here, he happens to be very optimistic in his prediction particularly given the inaccessible sheer size and scale of particle accelerators that would be needed to expand the existing field of high-energy particle physics! It creates a crisis with physicists in that field who wonder what to do next.

[28] This is the case of the Wolfram Physics Project launched in April 2020 by Stephen Wolfram, a prolific Anglo-American physicist, mathematician and computer scientist. Building on his 2002 book "A new kind of science", the author's idea is to explain everything, the world, physics, the universe, whatever, with cellular automata, graphs and fractals. The world would be discrete on a small scale, including time. His Physics Project focuses on the unification of physics with the same set of tools. See the hundred pages presentation of the project, the white paper which contains a section on quantum physics. Physicists' views on this theory are more than circumspect. The paper does not develop a theory that would be verifiable with an experimental approach as was the case for quantum physics (superposition, wave function, wave function collapse, atomic transition spectral lines, ...). Wolfram's theory was critically analyzed in 2002 by Scott Aaronson in a 14-page review, particularly about his Bell's inequalities interpretation, and in A New Kind of Science by Cosma Rohilla Shalizi of Carnegie Mellon University, who does not mince his words. The same "hammer/nail explains everything" approach was created by a team of scientists who describe the Universe physics laws self-learning capabilities with a giant neural network approach, in The Autodidactic Universe by Stephon Alexander, Jaron Lanier, Lee Smolin et al, 2021 (79 pages).

[29] Max Planck's cynically explained in 1950 the evolution of science with the death of old generation of scientists: "*A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die and a new generation grows up that is familiar with it*".

Misrepresentations. Many quantum physics scientists are famous even for general audiences, even though their work has been overly simplified. Schrödinger's famous cat and Heisenberg's indeterminacy principle are commonplace... even when their underlying details are quite different from their related clichés. Schrödinger's key work is his non-relativistic particles wave equation, not the 10 lines he wrote in 1935 on his eponymous cat thought experiment that is usually grossly misinterpreted!

Like life in general, science is a great relay race, with many players. Hundreds of other less-known contributors have also grown the field and must be recognized. Sometimes, genius scientists were so prolific that we forget their contributions. This is the case of John Von Neumann who is better-known for his “Von Neumann model” that is the cornerstone of classical computing and for his contribution to the development of EDVAC in 1949, the first stored program-based computer, rather than for his huge contribution to quantum physics mathematical formalism with density matrices and quantum measurement. It depends on the field you are working in, classical computing or quantum physics.

You won't find here inventors or entrepreneurs *a la* Steve Jobs or Elon Musk, even though the founders of startups like D-Wave, IonQ, Rigetti and PsiQuantum are among the entrepreneurial pioneers of this burgeoning industry, all being high-level scientists with a PhD!

Hall of fame. The History of 20^{th} century quantum physics is embodied in the mythical Fifth **Solvay Conference in 1927,** held at the Institute of Physiology in Brussels (Figure 21). It brought together the greatest mathematicians and physicists of the time including almost all the historical founders of quantum physics with Max Planck, Albert Einstein, Niels Bohr, Louis de Broglie, Erwin Schrödinger, Max Born, Werner Heisenberg and Paul Dirac[30]. All this happened as the foundations of 20^{th} quantum physics theories were fairly well laid out. 17 of its 29 participants got a Nobel Prize, 6 of which before the congress (names underlined in green) and the others afterwards (in blue). It was probably one of the largest concentrations and density of scientific brains per square meter in the history of mankind!

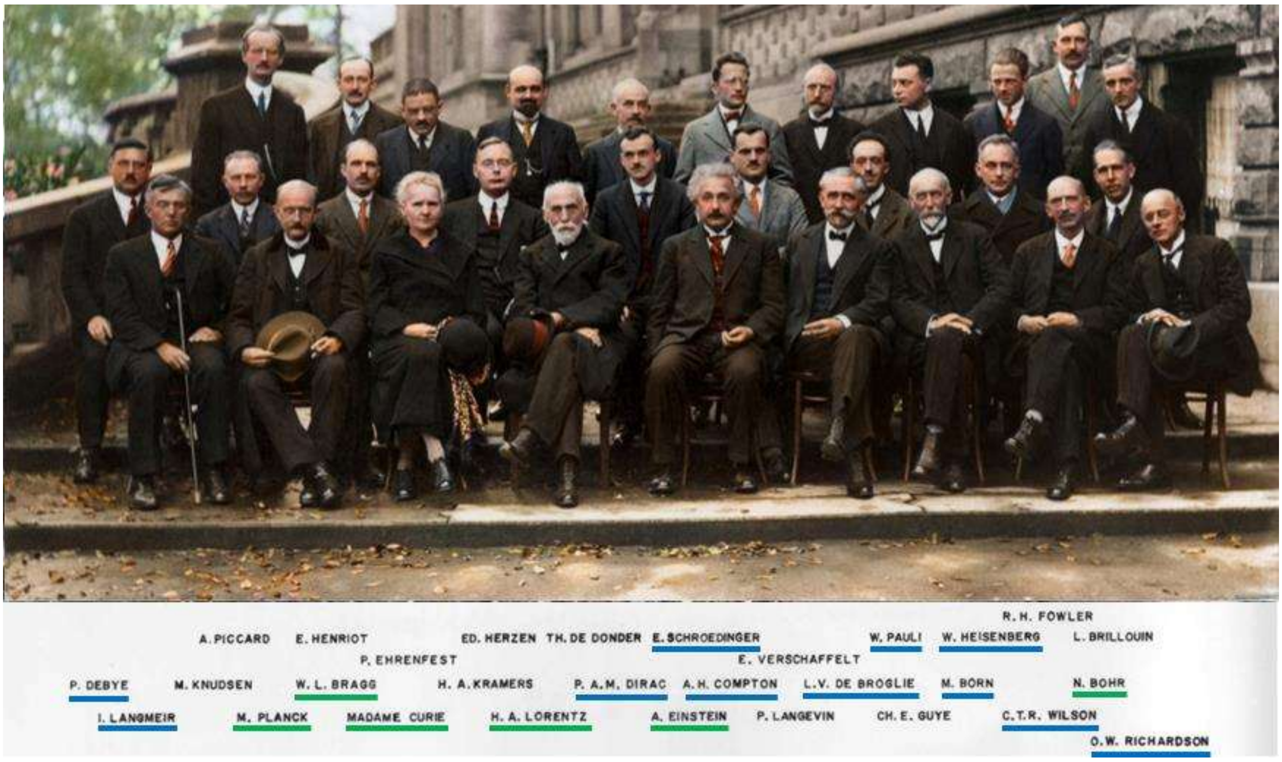


Figure 21: the famous Solvay 1927 conference photo with its 17 Nobel prizes (6 back then, and 11 after the conference). Photo credit: Benjamin Couprie, Institut International de Physique de Solvay.

[30] Only fathers and no mother! Marie Curie was present but was not specialized in quantum physics. She worked on radioactivity.

Solvay conferences on physics have been held every 3 to 4 years since their creation in 1911 by the entrepreneur and chemist **Ernest Solvay**. The 1927 congress's topic was electrons and photons, which are at the heart of quantum physics. Half of these conferences are dedicated to quantum physics, the other on different branches of physics. The 28th edition was held in May 2022 and gathered a contemporary hall of fame of quantum scientists from quantum physics to quantum information science.

The major contributions of early scientists in quantum physics are generally arranged in chronological order, with some indication of who influenced whom.

Precursors

We begin with the classical physicists and mathematicians of the 18th and 19th centuries who laid the scientific groundwork that allowed their 20th century successors to formalize the foundations of quantum physics[31] (Figure 22).

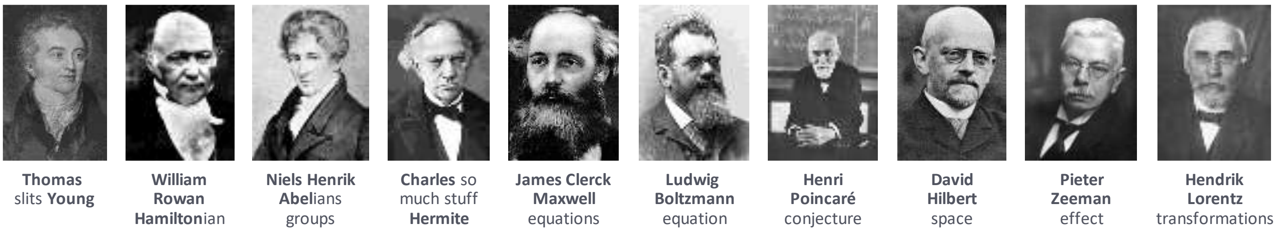


Figure 22: precursor scientists who laid the ground particularly in the electromagnetic fields and mathematics domains.

It is roughly organized in scientific contributions chronological order.

Thomas Young (1773-1829, English) was one of the great sciences and arts polymaths of his time, working in optics, medicine, linguistics, Egyptology, and music. He determined that light behaved like a wave, which he proved with the double-slit experiment around 1806, illustrated in Figure 23, that now bears his name. When reducing the size of both slits, it generates interference fringes creating alternating light and dark zones related to the wave nature of light. We had to wait till Albert Einstein's work in 1905 to determine that light was also made of particles.

His experiment used red filtered sunlight going through a first slit. Contemporary experiments use coherent laser light sources. This experiment is one of the cornerstones leading much later to the creation of the electromagnetism theory by James Maxwell. The slit experiment was implemented with electrons in 1961, with a similar result, illustrating the electron wave-particle duality, devised first by Louis de Broglie in 1924. It was then also done with atoms in 1991 and with various molecules starting in 2002.

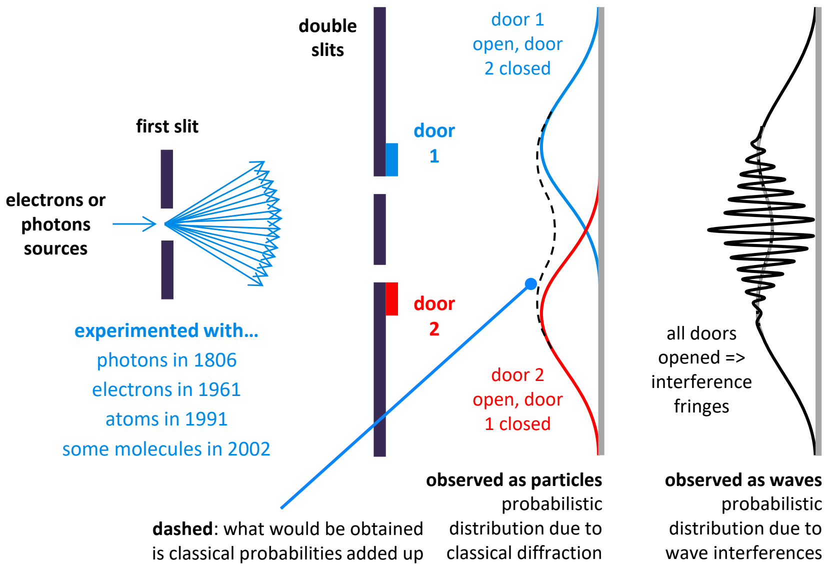


Figure 23: the double-slit experiment principle (cc) Olivier Ezratty, sources compilation.

31 I do not always indicate the source of the diagrams used in this text. These are part of common scientific knowledge that are now in the public domain.

Thomas Young also worked on the principles of refraction and human trichromatic vision as well as in fluid mechanics, including on the notion of capillarity and surface tension. As an Egyptologist, Thomas Young contributed to the study of the hieroglyphs of the famous Rosetta Stone, which was later used by **Jean-François Champollion** to decipher the whole stone texts. Champollion was then sponsored and helped by a certain **Joseph Fourier**. Yes, the mathematician and physicist!

William Rowan Hamilton (1805-1865, Irish) was a mathematician and astronomer. He invented around 1827 a set of new mathematical formulations of the laws of physics incorporating electromagnetism. In quantum mechanics, we often speak of Hamiltonians or Hamiltonian functions. These are mathematical operators used to evaluate the total energy of a system of elementary particles including their kinetic and potential energies. This energy is evaluated over time.

Schrödinger's 1926 wave equation describes the evolution of a system's Hamiltonian over time. Among other domains, this concept is used in analog quantum computing with quantum simulators and quantum annealers, like with D-Wave's systems. We'll have the opportunity to cover this in detail in this book, starting page 332.

Hamilton is also behind the creation of quaternions in 1843 which generalize complex numbers, with using i, j and k as imaginary numbers with $\mathrm{i}^2 = \mathrm{j}^2 = \mathrm{k}^2 = \mathrm{ijk} = -1$. It can be used to compute three-dimensional rotations and have some applications in quantum computing like for the representation of two-qubit entanglement and of single qubit gates from the Pauli group, in topological quantum computing. This is an exotic domain that we won't cover in this book.

Niels Henrik Abel (1802-1829, Norwegian) is a prolific mathematician at the origin of the so-called Abelian groups. His work focused on the semi-convergence of numerical series, sequences and series of functions, the convergence criteria of generalized integrals, the notion of elliptic functions and integrals (used in cryptography) and the resolution of algebraic equations including his proof of the impossibility of solving general quintic equations.

He died way too early at the age of 26 from tuberculosis while visiting Paris and meeting his fiancée! Along with William Rowan Hamilton, Charles Hermite and Emmy Noether, he is one of the main 'suppliers' of the mathematical foundations used in quantum mechanics.

The adjectives "Abelian" and "non-Abelian" are associated with anyons, the quasiparticles that are the basis of topological quantum computing.

Why do these concepts of quantum mechanics invented long after his death refer to this mathematician? Mainly because the distinction between Abelian and non-Abelian is linked to their commutative mathematical representation. A system with A and B is Abelian when A*B = B*A or non-commutative and non-Abelian when A*B is not equal to B*A. The most common non-commutative operations are non-square matrices multiplications. The multiplication of a matrix $(p \times q) * (q \times p)$ will give a matrix $(p \times p)$ whereas in the other direction, $(q \times p) * (p \times q)$ will generate a matrix $(q \times q)$, q and p being here a number of rows and columns.

Non-commutativity is frequently found in quantum physics and particularly with quantum measurement. The order in which quantum objects properties are measured may influence the results because the used measurement operators are non-commutative. In some cases, though, operators are commutative, like with the Measurement-Based Quantum Computing (MBQC) technique that we will have the opportunity to describe later when dealing with photon-based quantum systems.

Charles Hermite (1822-1901, French) was another prolific 19th century mathematician. He worked on numbers theory, quadratic forms, the theory of invariants, orthogonal polynomials, elliptic functions and algebra. His main works were concentrated on the 1848-1860 period. We owe him the notion of Hermitian functions and matrices, which are widely used in quantum physics and quantum computing. A Hermitian matrix is composed of real numbers in the diagonal and can be complex in the rest, and is equal to its transconjugate.

Namely, their transpose matrix whose value of complex numbers has been inverted (i becomes -i and vice-versa) as shown in Figure 24.

Quantum measurement operations in quantum physics and computers are defined by Hermitian matrices.

$$A = \begin{bmatrix} 2 & i & -2i \\ -i & 1 & 3 \\ 2i & 3 & -1 \end{bmatrix}^{\dagger} \qquad \bar{A} = \begin{bmatrix} 2 & -i & 2i \\ i & 1 & 3 \\ -2i & 3 & -1 \end{bmatrix} \qquad A = \overline{(A)}^{*}$$

transposed matrix

Hermitian matrix
matrix equals its transconjugate

Figure 24: how a Hermitian matrix is constructed.

Achille Marie Gaston Floquet (1847-1920, French) was a mathematician who developed mathematical analysis in the theory of differential equations. His name is used in Floquet codes (quantum error correction codes) and we also find him in the physics of quantum matter.

James Clerk Maxwell (1831-1879, Scottish) created in 1865 the theory of electromagnetic fields, combining an electric field and a magnetic field orthogonal to the direction of wave propagation as in the diagram *below*, and moving at the speed of light. This theory explains light-light interactions such as reflection, diffraction, refraction and interferences. Maxwell's work built on and improved the formalism created by Michael Faraday, Carl Friedrich Gauss, and André-Marie Ampère.

Maxwell's equations illustrate that when they are constant, electric, and magnetic fields are independent, and in variable regime (with a wavelength λ), it becomes interdependent ($\vec{E}$ and $\vec{B}$), one generating the other and vice-versa, hence the notion of electromagnetic waves and fields (Figure 25).

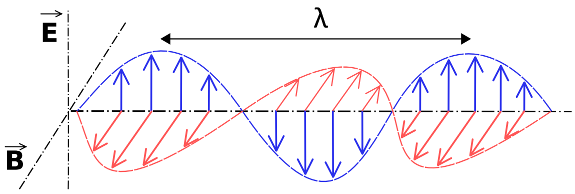


Figure 25: electromagnetic wave electric and magnetic fields components.

In Maxwell's equations, the electromagnetic field is represented by an electromagnetic tensor, a 4x4 matrix whose diagonal is zero and whose half of the components describe the electric field and the other half the magnetic field. These four dimensions correspond to space (3) and time (1).

In fact, there are four main Maxwell equations[32]:

- The **Maxwell-Gauss** equation describes how an electric field is generated by electric charges. At each point in space, the electric field is directed from positive to negative charges in directions depending on the charges space position (Figure 26).

electric field

divergent, measures field variation orientation

$$\mathrm{div}\left(\vec{E}\right) = \frac{\rho}{\epsilon_0}$$

charges distribution

vacuum dielectric permittivity

Figure 26: Maxwell-Gauss equation describing the electric field created by electric charges.

[32] See these well done and visual explanations of Maxwell's equations: A plain explanation of Maxwell's equations.

- The **Maxwell-flux** equation states that a magnetic field is always generated by a dipole with positive and negative charges that are connected and inseparable. Mathematically, this translates into the fact that the divergence of the magnetic field is zero and that there is no magnetic monopole (Figure 27).

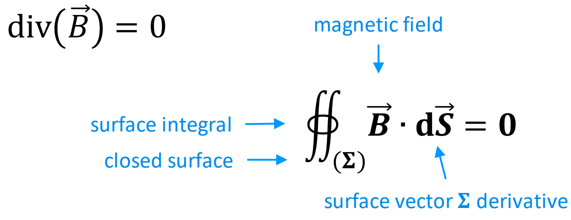


Figure 27: Maxwell-flux equation.

Namely, that there is no magnetic field line that escapes to infinity as we have with an electric field.

- The **Maxwell-Faraday** equation describes how the variation of a magnetic field creates an electric field. This is the principle used in electric alternators. The rotational operator using a nabla sign ∇ corresponds to a differential vector operation. It is equal to the first derivative of the magnetic field over time (Figure 28).

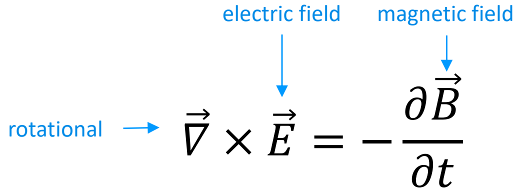


Figure 28: Maxwell-Faraday equation connecting the magnetic and electric fields.

- The **Maxwell-Ampere** equation states that magnetic fields are generated by electric currents or by the variation of an electric field. This interdependence between magnetic fields and varying electric fields explains the circulation of self-sustaining electromagnetic waves. In Figure 29 are the equation is the rotational magnetic field.

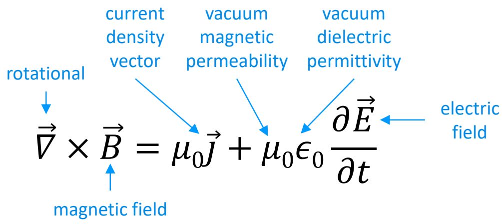


Figure 29: Maxwell-Ampere equation connecting magnetic field to electric field

As with Schrödinger's equation, Maxwell's equations have several variations, which may be confusing. Maxwell first published twenty equations with twenty unknown variables in 1865. In 1873, he reduced them to eight equations. In 1884, **Oliver Heaviside** (1850-1925, English) and **Willard Gibbs** (1839-1903, American) downsized the whole stuff to the four partial differential vector equations mentioned above. These four vector equations are reduced to two tensor equations for electromagnetic waves propagated in vacuum (Figure 30). The non-interaction with other elements explains the independence in this equation between electric and magnetic fields.

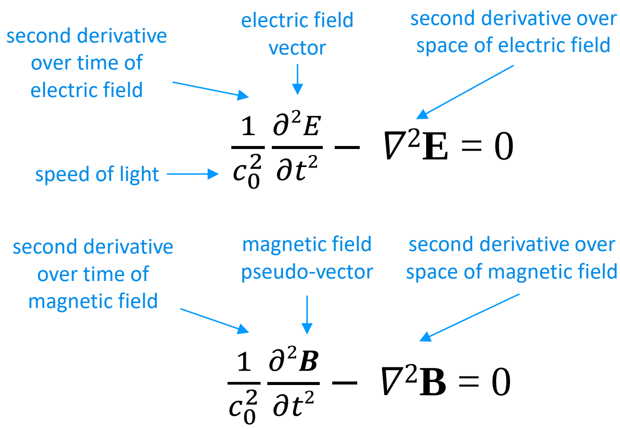
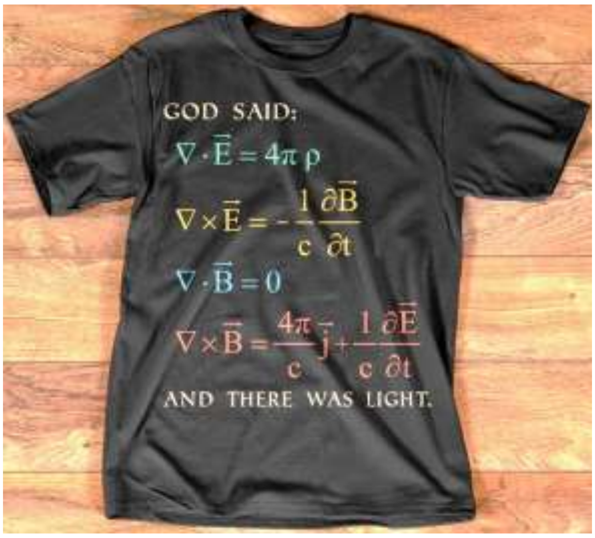


Figure 30: Maxwell's equations in vacuum.

Maxwell predicted that electromagnetic waves were travelling at the speed of light.

Electromagnetic waves were only experimentally discovered after Maxwell's death, by **Heinrich Hertz** (1857-1894) between 1886 and 1888. Hertz also discovered the photoelectric effect in 1887. Maxwell's description of electromagnetic waves had a phenomenal impact on electromagnetic telecommunications and optronics. It also served as a foundation for the first quantum physics laws developed by Max Planck in 1900 which led progressively to the quantization of the electromagnetic waves.

Maxwell is also at the origin of the **Maxwell-Boltzmann** statistical law of gas distribution. It models the particle velocity distribution of a perfect gas. It does not take into account the interactions between particles and is not applicable to extreme conditions, such as very low temperatures.



In particular, it is replaced by the **Bose-Einstein condensate** statistic for bosons (integer spin particles such as helium 4, which can be gathered in the same quantum state and energy level) and by the **Fermi-Dirac** statistic for fermions (particles with half-integer spins such as electrons or helium-3, which cannot cohabit in the same quantum and energy state).

Maxwell is the designer in 1867 of the so-called **Maxwell's demon** thought experiment which would make possible the reversibility of thermodynamic exchange processes and invalidate the second law of thermodynamics.

It rests on two boxes containing two different gases where a gas at two different temperatures is separated by a hole and a closure controlled by a "demon". When the door is opened, the gases mix.

Once mixed (see Figure 31), the demon would control which molecules could go from one box to another, taking advantage of the natural kinetic energy of the gases. This would allow in theory and after a certain time to return to the previous equilibrium in a non-equilibrium situation (on the right in Figure 31).

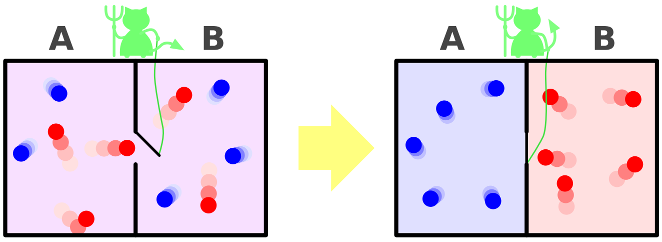


Figure 31: Maxwell's demon principle. Source: Wikipedia.

It took several decades to find the fault, notably via Léo Szilard in 1929 and Léon Brillouin in 1948. Initially, the explanation was that the demon needs to consume some energy to obtain information about the state of the gas molecules to sort them out. Therefore, energy is consumed to modify the stable equilibrium obtained to mix the gases.

The "up to date" explanation is somewhat different. The energy cost comes from resetting the demon's memory, which ultimately consists of a single bit of information[33].

All this had repercussions on the notion of the energy value of information and led, much later, to the creation of the field of information thermodynamics, i.e., the study of the energetic and entropic footprints of information, particularly in quantum computing.

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[33] Here is the detailed explanation by Alexia Auffèves (CNRS Institut Néel / MajuLab): we can understand the operation of resetting a bit of memory by considering an ultimate Carnot engine, consisting of a single particle that can be located to the left or right of a certain thermostated volume. Left = 0, Right = 1 There are two possible operations. The first one is compression. The particle is initially to the left or to the right of the volume that contains it (we don't know) and we compress the said volume so that at the end it is necessarily on the left. It is an initialization operation where the bit is reset to state 0. As with any compression, you have to pay, here in this ultimate case, the work to be expended is kT log 2. This is Landauer's famous work, which sets an energy bound to all logically irreversible operations. The second operation is relaxation. In the beginning, we know whether the particle is on the left or on the right. We position a wall, a pulley with a mass at the end and let the trigger operate while extracting an elementary work equivalent to kT log 2. This is a Szilard machine. These two manipulations were performed experimentally in 2011 at ENS Lyon. It shows the energy footprint of information and are the ultimate solution to Maxwell's demon paradox. See Information and thermodynamics: Experimental verification of Landauer's erasure principle by Antoine Bérut, Artyom Petrosyan and Sergio Ciliberto, Université de Lyon and ENS Lyon, 2015 (26 pages).

This field was then investigated by **Rolf Landauer**, known for his study of irreversible information management circuits heat generation, and by **Charles Bennett** and **Gilles Brassard**, the co-inventors of the QKD based BB84 protocol, which we will discuss later, and then by **Paul Benioff**, who was at the origin of the idea of gate-based quantum computing.

We finally owe Maxwell the creation of color photography in 1855, that was implemented in 1861, based on the three primary colors of human vision.

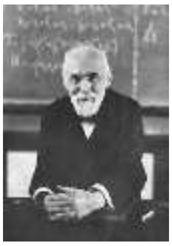
Maxwell's electromagnetic field equations have very well survived the test of time. It is still the basis of classical optics and quantum optics. Even when studying quantized light, researchers and students still rely on Maxwell's equations and their subsequent derivations created since then.

Ludwig Boltzmann (1844-1906, Austrian) was a physicist, the father of statistical physics, defender of the existence of atoms, facing a strong opposition from scientists until the beginning of the 20th century, and creator of equations describing fluid and gas dynamics in 1872. He is also at the origin of the probabilistic interpretation of the second law of thermodynamics, which establishes the irreversibility of physical phenomena, particularly during thermal exchanges.

Irreversibility is associated with the creation of entropy which measures the level of disorder in a system. Boltzmann's equation *aka* the Boltzmann-Planck equation relates the entropy *S* of an ideal gas to the multiplicity W, i.e., an integer number of microstates corresponding to the gas's macrostate: $S = k_B \log(W)$ where k_B is Boltzmann constant equal to 1.380649×10^{-23} J/K.

A number of microstates correspond to the different possible arrangements of molecular position and kinetic energy at a particular thermodynamic state.

Boltzmann tried his hand at philosophy while defending the existence of atoms. Depressed, he died by committing suicide.



Henri Poincaré (1854-1912, French) was a mathematician and physicist, precursor of the theory of relativity and gravitational waves. We owe him a probabilistic function that bears his name, and which is the optical equivalent of the Bloch representation that we will see later, which mathematically describes the state of qubits. He is also the author in 1904 of the mathematical conjecture that bears his name and that was demonstrated in 2003 by Grigori Perlman in Russia. It is relative to the 3-sphere, the hypersphere that bounds the unit ball in a 4-dimensional space.

He was also a philosopher of sciences and one of the last « universal scientist » covering many branches in mathematics, physics and philosophy. He was also a first cousin of Raymond Poincaré (1860-1934), president of France during the First World War, a lesser-known figure than Georges Clémenceau who was then Prime Minister and drove the war efforts against Germany and with allies from the UK and the USA.

David Hilbert (1862-1943, German) is yet another prolific mathematician who, at the end of the 19^{th} century, was the creator of the mathematical foundations widely used in quantum physics, in particular his so-called Hilbert spaces using vectors to measure lengths, angles and define orthogonality. They are used to represent the state of quantum objects and qubits with vectors and complex numbers with an inner product, distances and an orthonormal basis (see Figure 32). Still, his work had nothing to do with quantum physics, which was not yet formulated at the time.

His work was used by Paul Dirac in 1930 and John Von Neumann in 1932 to lay the groundworks of quantum physics mathematical foundations like the Dirac Bra-Ket notation and the Von Neumann quantum measurement formalism.

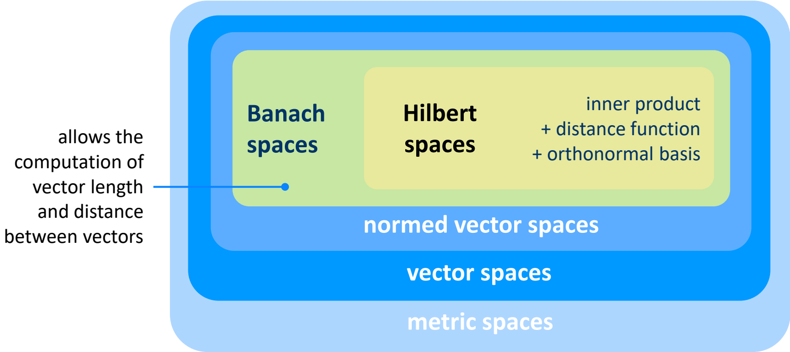


Figure 32: a Hilbert space is a vector space with an inner product. It enables the measurement of vector distances, angles, and lengths. Source: compilation Olivier Ezratty, 2022.

Pieter Zeeman (1865-1943, Dutch) was a physicist, Nobel prize in Physics in 1902 with Hendrik Lorentz, for the discovery of the effect that bears his name between 1896 and 1897. The Zeeman effect occurs when excited atoms are exposed to a magnetic field. This affects their emission or absorption spectrum, that displays many discrete spectral lines. The effect is observed with spectroscopy, which breaks down light rays of different wavelengths with a prism.

In his experiment, spectral lines are broken down into an odd number of lines (normal Zeeman effect, as shown in Figure 33 for cadmium atoms) or an even number of lines (abnormal Zeeman effect). The decomposition depends on the intensity of the magnetic field passing through the analyzed atoms. There is also a nuclear Zeeman effect explained by the spin of atom nucleus.

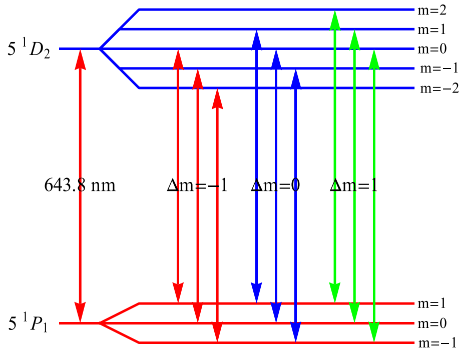


Figure 33: normal Zeeman's effect energy transitions. Source: Lecture Note on Zeeman effect in Na, Cd, and Hg by Masatsugu Sei Suzuki and Itsuko S. Suzuki, 2011.

It is matched by a polarization of the generated light whose nature and intensity depends on the orientation of the magnetic field relative to the light beam as shown here. The Zeeman effect can be explained by Pauli's exclusion principle, elaborated in 1925, and by the transitions in the energy level of the electrons in the same atom layer and having different orbital angular momentum (normal) and spin (abnormal). In astronomy, the Zeeman effect measurement is used to evaluate the intense magnetic fields in stars as well as within the Milky Way. The nuclear Zeeman effect is used in magnetic resonance spectroscopy in MRI scanners.

Hendrik Antoon Lorentz (1853-1928, Dutch) was a physicist who worked on the nature of light and the constitution of matter and made the link between light and Maxwell's electromagnetism equations. We owe him the Lorentz transformations that explain the results of Michelson-Morley's experiments between 1881 and 1887 which showed that the speed of light is stable, whatever the reference frame. With Henri Poincaré and George Francis FitzGerald (1851-1901, Irish), he was a key contributor to the theory of relativity formalized later by Albert Einstein between 1905 and 1915.

Let's also add **Joseph Larmor** (1857-1942, Irish/British) who, among other various contributions, was one of the first to associate electric charges with electron particles in 1894. We also own him the notion of Larmor precession, the rotation of the magnetic moment of an object when it is exposed to an external magnetic field, discovered with protons in 1919 and later extended to electrons.

Founders

The foundations of quantum physics started with Max Planck's black-body explanation with energy quanta and, then took shape over three and a half decades, roughly until 1935 (Figure 34). It involved the successive contributions from Einstein, Bohr, De Broglie, Schrödinger, Born, Heisenberg, Dirac, and Von Neumann to mention only the best-known contributors who were all theoreticians and not experimentalists.

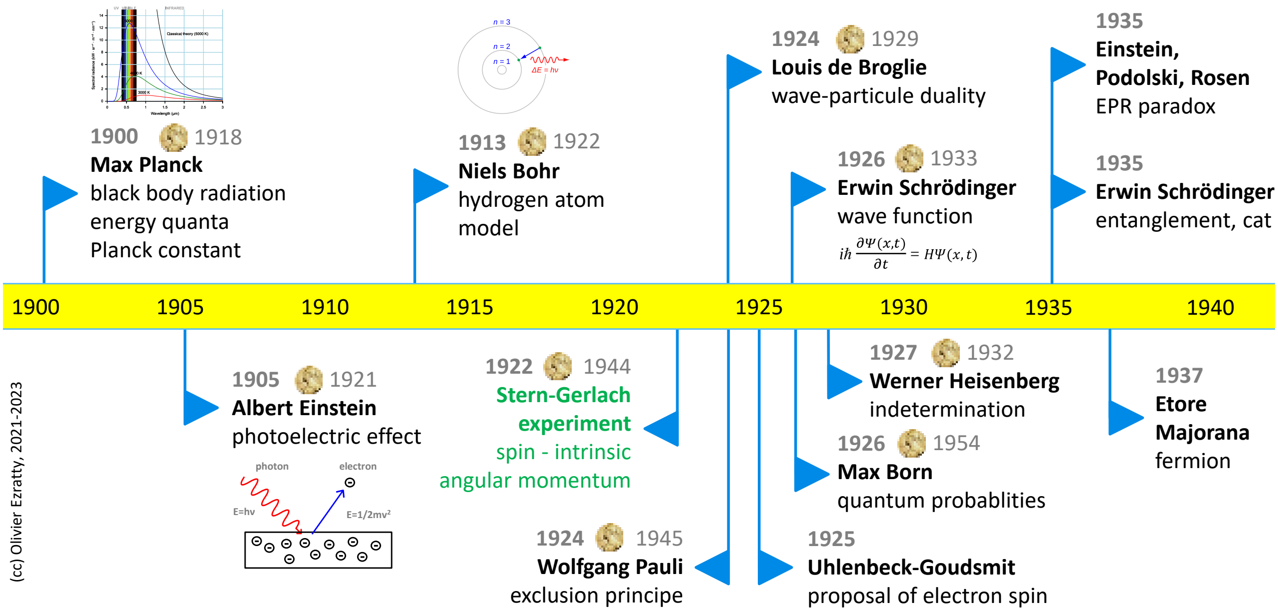


Figure 34: quantum physics foundational years timeline. In green, experimentalist works, in black, theoretician works. The gold coins represent a Nobel prize. (cc) Olivier Ezratty, 2021-2023.

Figure 35: the key founders of quantum physics in the first part of the 20th century. (cc) Olivier Ezratty, 2020-2023.

Things were relatively quiet during World War II as lots of scientists were focused on creating the atomic bomb in the USA under the umbrella of the then very secret Manhattan project while Europe was not the best place in the world for travel and international scientific collaborations.

German scientists who initially led quantum physics became isolated or emigrated to the USA or the UK because they were Jews, like Albert Einstein or Max Born.

Here is a broader tour of the great physicists and mathematicians who laid the foundations of quantum physics. They are all Europeans who, some of whom emigrated from Europe to the United States before World War II (Figure 35).

Max Planck (1858-1947, German) was a physicist, initially specialized in thermodynamics. In 1900, he developed the first basis of quantum physics, hypothesizing that energy exchanges between light and matter are made by discrete quanta. This radiation is not continuous but varies by thresholds, in steps of a certain amount of energy, hence the term "quantum" and "quantum physics" or “quantum mechanics”. His theory allowed him to roughly explain for the first time the enigmatic radiation of black bodies, that absorbs all incident magnetic radiation (Figure 36).

Examples of black bodies are a closed cavity like an oven, a heated metal that becomes red, orange, then white depending on the temperature, or a star like our own Sun. The spectrum of electromagnetic waves emitted by a black body depends only on its temperature and not at all on its material. The higher the temperature is, the more the electromagnetic spectrum emitted by the black body slides towards higher frequencies on the left, therefore towards purple and ultraviolet. The theory solved the ultraviolet catastrophe.

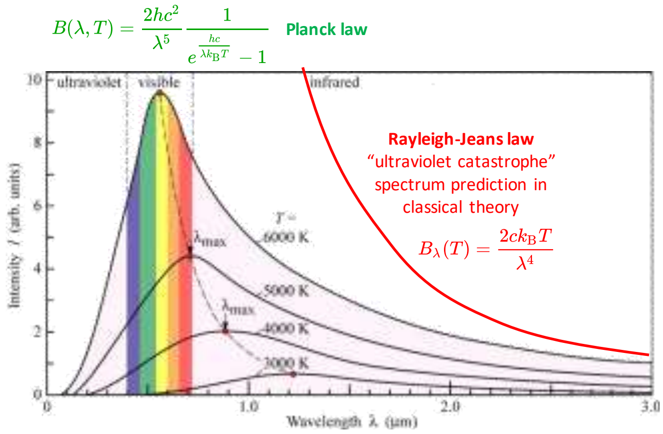


Figure 36: black-body spectrum and the ultra-violet catastrophe.

This so-called ultraviolet catastrophe, an expression **Paul Ehrenfest** (1880-1933, Austrian) created later in 1911, happened with the Rayleigh-Jeans law also proposed in 1900, which was trying to predict the shape of the light spectrum with the black body temperature. It was diverging to infinite values as the temperature was growing. Planck’s law solved the problem and avoided the ultraviolet catastrophe. He found his spectrum equation empirically and only then, a related explanation based on harmonic oscillators and energy quanta exchanged between the radiation and the black body “wall”. For this work, Max Planck was awarded the Nobel prize in Physics in 1918.

We also owe him the constant which bears his name (h) and which is used in his blackbody radiation equation. The Planck constant (6.626×10^{-34} Js) was then used in the equation according to which atomic state energy shifts equals to the radiation frequency multiplied by Planck's constant. The constant appears in most quantum physics equations (De Broglie, Schrödinger, Dirac, etc.).

When an electron changes its orbit in a hydrogen atom, it emits or absorbs an electromagnetic wave whose energy is equal to Planck's constant multiplied by the emitted light frequency. More generally, a system can evolve only with multiples of Planck’s constant. Despite the numerous experimental validations carried out a few years later, Max Planck expressed until his death a lot of doubts about the very principles of quantum physics!

Planck is also at the origin of several infinitesimal constants as shown in Figure 37: Planck time, which is $t_P=10^{-44}$s and Planck distance which is $l_P=1.616255*10^{-35}$m. Planck's time is the time it would take for a photon to travel the Planck distance.

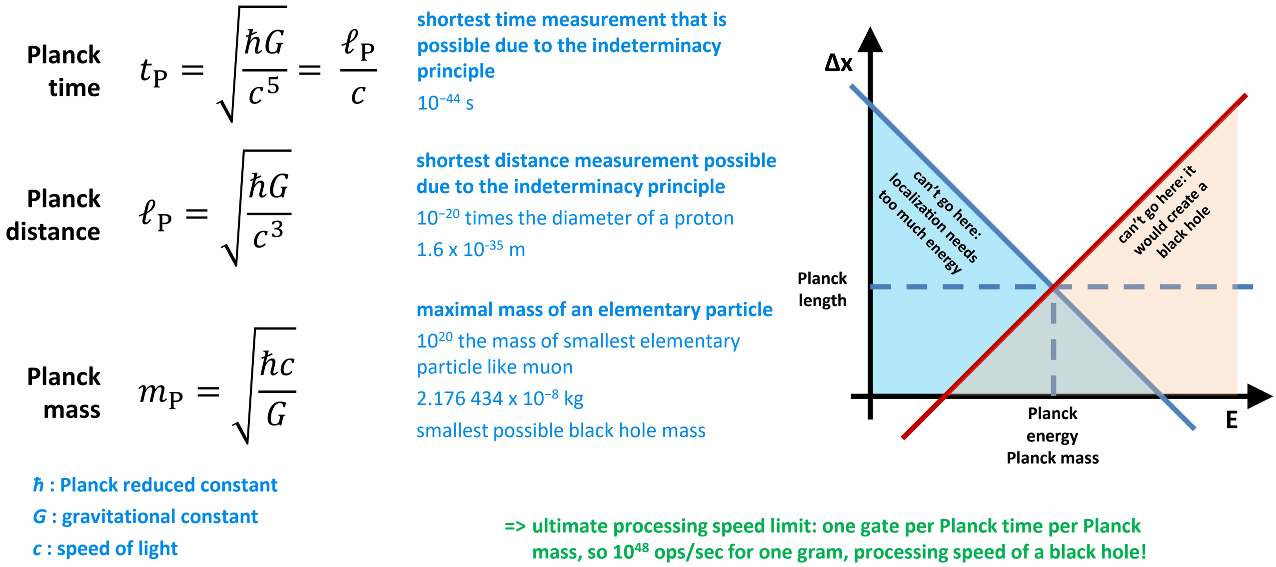


Figure 37: Planck time, distance and mass constants (cc) Olivier Ezratty, 2021-2023.

These are the dimensions of the infinitely small below which any observation would be impossible. The length of Planck l_P is so small that a photon used to observe it would have such a high frequency and energy that it would generate a black hole around it and would therefore become unobservable!

At last, Planck mass is the maximum mass of an elementary particle. A particle with this mass and the size of Planck's distance would be a black hole. These are quite extreme physics. In today's classical cosmology, Planck's wall corresponds in the history of the expansion of the Universe to the moment when 10^{-43}s after the big bang, its size would have been 10^{-35}m, which is respectively the Planck time and Planck distance. Needless to say that the experimental conditions of the big bang are difficult to reproduce. It doesn't prevent some physicists to try to simulate it digitally[34].

Albert Einstein (1879-1955, German then American) got his Nobel prize in physics in 1921 for his interpretation of the photoelectric effect in 1905, which became one of the foundations of quantum mechanics after Planck and before De Broglie, Heisenberg and Schrödinger. Einstein determined that Planck's quanta were elementary grains of energy $E = h\nu$ (Planck constant × frequency) with a momentum of $p = h\nu/c$ [35]. These were named "photons" in 1926 by **Gilbert Lewis** (1875-1946, American). Symbolically, 1905 is also the year of Jules Verne's death.

Symmetrically to what Louis de Broglie would later do with electrons, he hypothesized that a photon behaves both as a wave and as a particle.

This was coming out of just one out of his four 1905 "annus mirabilis" papers sent between March and June to Annalen der Physik, the others being on special relativity, Brownian motion and mass-energy equivalence, published when he was just 26. This was on top of his own 24 pages PhD thesis on a theoretical method to calculate molecular sizes using fluid mechanics and hydrodynamics.

With the photoelectric paper, he reconciled the corpuscular theories of **René Descartes** (1596-1650, French, in 1633) and **Isaac Newton** (1642-1726, English, in 1704) with the wave-based theories of **Christiaan Huygens (**1629-1695, Dutch, in 1678) to describe light.

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[34] See A new algorithm that simulates the intergalactic medium of the Universe in seconds is developed by the Instituto de Astrofisica de Canarias, May 2022.

[35] In On a Heuristic Viewpoint Concerning the Production and Transformation of Light, 1905.

This was followed by the works from **Augustin-Jean Fresnel** (1788-1827, French), **Léon Foucault** (1819-1868, French, who measured first the speed of light), **Hippolyte Fizeau** (1819-1896, French, who co-discovered the Doppler effect) and of course **James Clerk Maxwell**.

The photoelectric effect corresponds to the capacity of a photon to dislodge an electron from a generally inner orbit of an atom and to create some electric current[36] (Figure 38).

It is exploited in the cells of silicon-based photovoltaic solar panels. It also explains photosynthesis in plants, which is the metabolic starting point of glucose production.

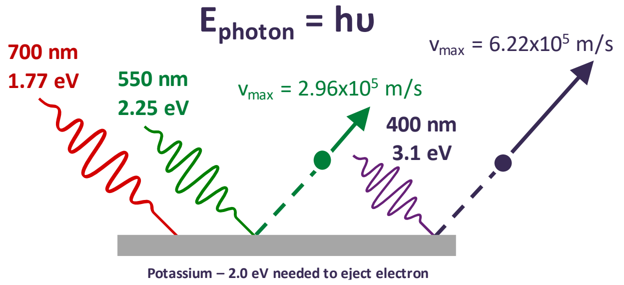

Figure 38: the photoelectric effect.

In addition to Max Planck's work on black body radiation, Einstein's interpretation was based on the earlier work of **Heinrich Hertz** (1857-1894, German) who discovered in 1887 that light can extract an electron out of metal, and **Philipp Lenard** (1862-1947, German) who, in 1902, studied the photoelectric effect and determined that it is only triggered at a certain frequency for the projected light. The latter was awarded the Nobel prize in Physics in 1905. Becoming a fervent Nazi and opposed to Einstein by scientific rivalry and then by explicit anti-Semitism, he mostly disappeared from quantum physics hall of fame.

Einstein's photoelectric effect equations were then verified by the experiments of **Robert Andrews Millikan** (1868-1953, American) between 1909 and 1914. It enabled him to measure the electric charge of a single electron, which earned him the Nobel prize in Physics in 1923.

Of course, Einstein is also at the origin of the special and general theories of relativity. He didn't obtain a Nobel Prize for his work on relativity despite its considerable impact on science. This is due, among other things, to his theories being based on earlier work from **Hendrik Antoon Lorentz** and **Henri Poincaré** as well as the contribution of his former professor **Hermann Minkowski** (1864-1909, German) who created the four-dimensional space-time notion in 1908[37].

On top of many other contributions in quantum physics, Einstein predicted the photons stimulated emission effect in 1917, that would later lead to the creation of lasers. He also predicted in 1925 a particular behavior of matter, the Bose-Einstein condensate, which occurs when gases are cooled to very low temperatures. Atoms are then in a minimum energy quantum state showing particular physical properties. This is the case of superfluid helium, discovered in 1938, which is superfluid at very low temperatures, i.e., it can move without dissipating energy. Bose is the name of **Satyendra Nath Bose** (1894-1974, India) with whom Einstein had worked during the 1920s and to whom we owe the "bosons", which verify the characteristics of Bose-Einstein's condensates.

Bosons include elementary particles without mass such as photons and gluons but also certain atoms such as deuterium or Helium 4 as well as certain quasiparticles such as the superconducting electron pairs that are Cooper's pairs.

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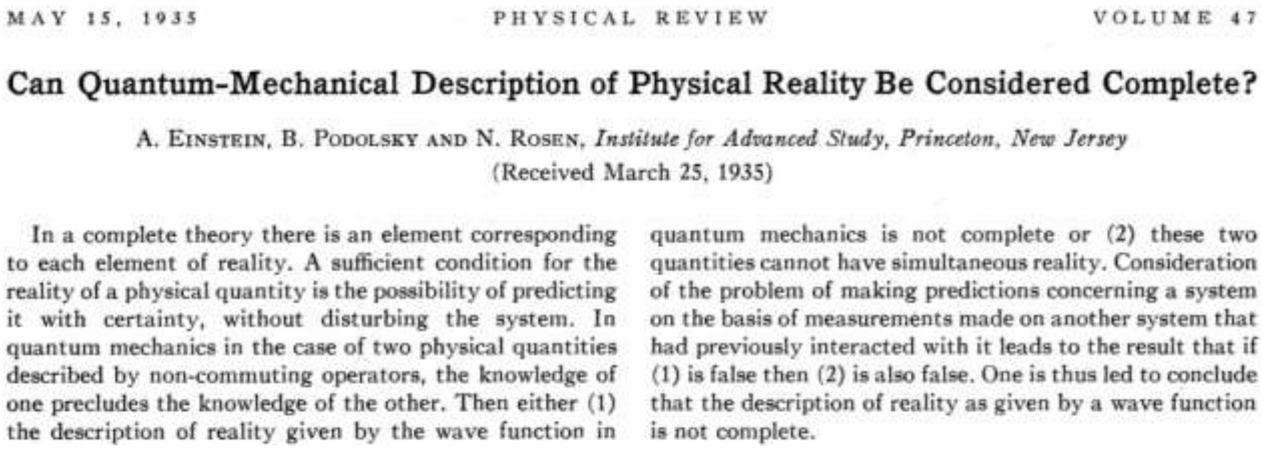
[36] The electron layers of the atoms are numbered from 1 to N, their quantum number. One also starts the numbering by K (first layer close to the nucleus with a maximum of 2 electrons) then L (8 electrons maximum), M (with a maximum of 18 electrons but in practice 8), etc. The photoelectric effect mainly concerns the layers K and L. The ejected electron is then replaced by an electron of external orbit, which generates a new photon, in X-rays or in fluorescence, according to the energy of the incident photon. This then emits an X-ray photon due to the energy differential between electronic layers or an electron called "Auger" from the name of Pierre Auger. This phenomenon was discovered around 1923 by the latter and by Lise Meitner. Another variant of the photoelectric effect is the Compton effect, when the high energy of an incident photon in gamma rays will release an electron from the valence layer and generate another photon. Finally, when the energy of the incident photon is even higher, the interaction takes place at the nucleus of the target atom and generates an electron and a positron.

[37] See another historical explanation in The dramatic story behind general relativity's Nobel Prize snub by Robert Friedman, Advanced Science News, August 2022.

We will see a little later that it is a question of the spin sum of these particles that determines the fact that they are bosons as opposed to fermions.

Albert Einstein also contributed to the philosophical-scientific debates on quantum physics realism, confronting Niels Bohr. He focused on the fact that quantum physics did not seem to completely describe the physical world with its probabilistic bias. Einstein wanted to find a realistic interpretation of quantum physics. He could not be satisfied with a probabilistic description of the state of electrons and other quantum objects. He could not find sufficient the interpretation of quantum physics according to which the observer and the measurement "make" the real world. He thought that the real world exists independently of measurements and observers.

The debate between Albert Einstein and Niels Bohr revolved around various thought experiments on determinism discussed during the 1927 Solvay Congress.


MAY 15, 1935 PHYSICAL REVIEW VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*
(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Figure 39: the famous EPR paper from Albert Einstein, Boris Podolsky and Nathan Rosen published in 1935.

It culminated later, in 1935, with the famous **EPR paradox** paper (Figure 39), named after its authors Albert Einstein, Boris Podolsky and Nathan Rosen. The paper raised the question of the incompleteness of quantum mechanics at the time[38].

It sought to explain the nonlocality of the correlated quantum state measurement results of entangled particles which was a consequence of Schrödinger's wave function. It was not yet physically observed as of 1935[39]. For the EPR paradox paper authors, the quantum theory based on Schrödinger's wave function was either incomplete or two quanta could not be instantaneously synchronized at a distance at measurement time. Their measurement outcome being random and correlated, entangled quantum objects had to convey with them a sort of "information switch" indicating where the random measurement should land. A physical theory is complete if each component of reality has a counterpart in the theory that makes it possible to predict its behavior, such as some tuning happening at the source when entangled quanta are created, and transmitted to each one with some hidden variables that would determine the outcome of their measurement. This underlies the notion of determinism, a principle that is absent in Schrödinger's wave function which is entirely probabilistic in nature.

[38] See Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?, by Albert, Einstein, Boris Podolsky and Nathan Rosen, Physical Review, March 1935 (4 pages). The real author of the paper was Boris Podolsky and Albert Einstein thought it didn't really express his own views. While the thought experiment in EPR's paper didn't explicitly mention entanglement, a term that was created later the same year by Erwin Schrodinger, it related to two particles A and B that could interact for some time and not after it, that can now be considered as being entangled particles having some common past.

[39] Einstein's view was that classical and relativistic physics act locally. Gravity is local and is transmitted at the speed of light. All physical theories before quantum physics were local or EPR-local. Remote actions all involve a delay, usually coupled with attenuation with distance as it is the case for gravity.

Einstein thought that quantum physics was an incomplete theory that didn't describe reality precisely enough. Einstein was then often credited with the idea that there were hidden variables. It seems, however, that he never mentioned them in his writings despite what John Stewart Bell later said. The EPR paper ends with indicating that it should be possible to build a complete theory of quantum mechanics[40] (Figure 40). Hidden variables are a consequence rather than a hypothesis in the EPR paradox paper.

The New York Times.

NEW YORK, SATURDAY, MAY 4, 1935.

EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

Figure 40: the New York Times coverage of the EPR paper on May 4th, 1935, which infuriated Albert Einstein!

The explanation of entanglement by "hidden variables" comes rather from Louis de Broglie with his pilot wave hypothesis elaborated in 1927, an idea later pursued by David Bohm in the 1950s[41]. With his "inequalities", John Stewart Bell demonstrated in 1964 that the existence of such hidden local variables was incompatible with the principles of quantum mechanics. Alain Aspect et al 1982 experiment on photon entanglement did invalidate Bell's inequalities and the existence of hidden variables compatible with quantum mechanics postulates. In the end, Einstein could not finish his work on his theory of general relativity which was, for him, as incomplete as quantum mechanics. In particular, he wanted to reconcile quantum mechanics and gravity.

Be careful with the simplistic views that Einstein was "against" quantum mechanics, had it all wrong or did not believe in it[42]. He first questioned the principle of indeterminacy in 1927 and 1930, then estimated that the theory was incomplete to explain entanglement, with the EPR paradox paper published in 1935, and finally, he opposed the lack of realism of quantum theory. This incompleteness is still being discussed more than 80 years later. The origins of entanglement and nonlocality are still not physically explained. It is only observed physically and described mathematically[43]. This remains an open debate as scientists continue to ponder the different possible interpretations of quantum physics. This is part of the field of quantum foundations and quantum physics philosophy that we cover later in this book, page 1238.

Niels Bohr (1885-1962, Danish) was a physicist, Nobel prize in Physics in 1922, who created in 1913, aged 28, a descriptive model of the hydrogen atom with its nucleus made of a proton and an electron rotating around the nucleus on precise orbits corresponding to levels of kinetic energy, multiple of $h/2\pi$, h being Planck's constant and $n = 1, 2, 3$ and so on. This model explained hydrogen spectral lines observed in the experiments of **Johann Balmer** (1825-1898) in 1885, **Theodore Lyman (**1874-1954) in 1906 and **Friedrich Paschen (**1865-1947) in 1908 (Figure 41, *right*).

It also explained why electrons didn't crash on atom nucleus! Niels Bohr followed the work of **Ernest Rutherford** (1871-1937) who discovered in 1911 the structure of atoms with their positively charged nucleus, thanks to its protons, and their electrons revolving around the nucleus. The latter, with whom Niels Bohr was doing his post-doc in 1911, relied himself on **Hantaro Nagaoka** (1865-1950, Japan)

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[40] The 1935 New York Times article was published thanks to a "leak" provoked by Boris Podolsky, the youngest of the EPR 3 gang.

[41] See Albert Einstein, David Bohm and Louis de Broglie on the hidden variables of quantum mechanics by Michel Paty, 2007 (29 pages) which sets the record straight on Albert Einstein's position on the subject of hidden variables. The author, born in 1938, is a physicist and a philosopher of science.

[42] This story is well told in Einstein and the Quantum - The Quest of the Valiant Swabian by A. Douglas Stone, 2013 (349 pages).

[43] See the abundant Einstein Bohr debates and Interpretations of quantum mechanics pages on Wikipedia, from which the table on the next page is taken.

who predicted in 1903 the structure of atoms with a positively charged nucleus and negatively charged electrons revolving around it, called the "Saturnian model".

Electrons had been discovered by **Joseph John Thomson** (1856-1940, English) in 1897 by analyzing the rays emitted by a cathode in a cathode ray tube (CRT), deflected by an electric field as well as by a magnetic field, and detected by a layer of phosphorus. He was awarded the Nobel prize in Physics in 1906.

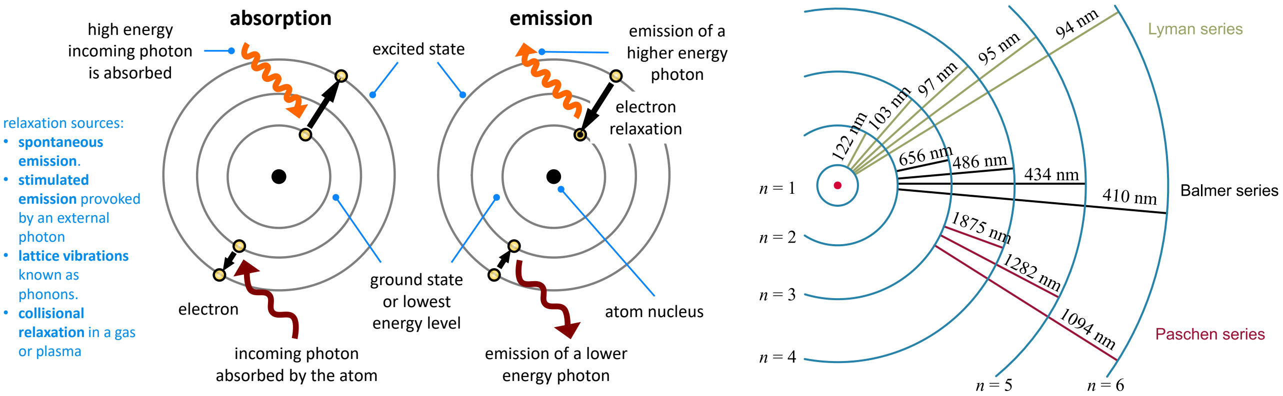

Figure 41: the Bohr atomic model. Source: Wikipedia and other open sources. 2023.

Ernest Rutherford had also imagined the existence of neutrons, which was not verified experimentally until 1932 by **James Chadwick** (1891-1974, English). **Marie Curie** (1867-1934, Polish and French) had discovered polonium and radium in 1898 and some effects of radioactivity but not the existence of neutrons. According to Niels Bohr, electrons emit or absorb a photon when they change orbit. Subsequently, Louis de Broglie's work on wave-particle duality interpreted that the orbits of the electrons were an integer multiple of their associated wavelength.

Interpretation	Year	Author(s)	Deterministic?	Ontic wavefunction?	Unique history?	Hidden variables?	Collapsing wavefunctions?	Observer role?	Local dynamics?	Counterfactually definite?	Extant universal wavefunction?
Ensemble interpretation	1926	Max Born	Agnostic	No	Yes	Agnostic	No	No	No	No	No
Copenhagen interpretation	1927	Niels Bohr, Werner Heisenberg	No	Some	Yes	No	Some	No	Yes	No	No
de Broglie–Bohm pilot wave theory	1927–1952	Louis de Broglie, David Bohm	Yes	Yes	Yes	Yes	Phenomenological	No	No	Yes	Yes
Quantum logic	1936	Garrett Birkhoff	Agnostic	Agnostic	Yes	No	No	Interpretational	Agnostic	No	No
Time-symmetric theories	1955	Satosi Watanabe	Yes	No	Yes	Yes	No	No	No	No	Yes
Many-worlds interpretation	1957	Hugh Everett	Yes	Yes	No	No	No	No	Yes	Ill-posed	Yes
Consciousness causes collapse	1961–1993	John von Neumann, Eugene Wigner, Henry Stapp	No	Yes	Yes	No	Yes	Causal	No	No	Yes
Many-minds interpretation	1970	H. Dieter Zeh	Yes	Yes	No	No	No	Interpretational	Yes	Ill-posed	Yes
Consistent histories	1984	Robert B. Griffiths	No	No	No	No	No	No	Yes	No	Yes
Transactional interpretation	1986	John G. Cramer	No	Yes	Yes	No	Yes	No	No	Yes	No
Objective-collapse theories	1986–1989	Ghirardi–Rimini–Weber, Penrose interpretation	No	Yes	Yes	No	Yes	No	No	No	No
Relational interpretation	1994	Carlo Rovelli	No	No	Agnostic	No	Yes	Intrinsic	Possibly	No	No
QBism	2010	Christopher Fuchs, Rüdiger Schack	No	No	Agnostic	No	Yes	Intrinsic	Yes	No	No

Figure 42: the various interpretation of quantum physics. Source: [Interpretations of quantum mechanics](), Wikipedia.

Together with Werner Heisenberg, Pascual Jordan and Max Born, Niels Bohr is at the origin of the so-called **Copenhagen** interpretation of quantum physics which is based on three key principles[44] :

- The description of a wave-particle is realized by its wave function, and no other "hidden" local information or variable can be used to describe its state. We must accept the wave function probabilistic used to describe a quantum state.
- When a quantum state measurement is performed, its composite wave function of several states is reduced to the wave function of one of the possible states of the quantum with a probability defined by Born's rule (we'll see that later). This is the collapse of the wave function.
- When two properties are linked by an uncertainty relationship, the two properties cannot be measured with a greater precision than that allowed by the uncertainty relationship (Heisenberg principle of indeterminacy). Moreover, when we measure the position of a particle, we affect its motion, and vice versa. It comes from the bare fact that speed and position do not have any meaning before measurement in quantum physics. Variables linked through an indetermination link are conjugate with regards to actions which can change only by quantum leaps.

This is the main interpretation of quantum mechanics. There are many other interpretations available, listed in Figure 42. We will have the opportunity to detail the Copenhagen interpretation in the philosophy of quantum physics part already mentioned page 1238.

Note that Niels Bohr's son, **Aage Niels Bohr** (1922-2009, Danish), was awarded the Nobel prize in Physics in 1975 for his work on the structure of atom nucleus[45]!

Emmy Noether (1882-1935, German) is the creator of the theorem that bears her name in 1915 at the University of Göttingen in Germany and which says that if a system has a continuous symmetry property, then there are corresponding quantities whose values are conserved in time[46]. At the origin of the field of abstract algebra, it is a foundation to Lagrangian mechanics, precursor of Hamilton's formalism. At that time, she could not teach at the University because this role was forbidden to women. Her theorem was only published in 1918 and she could not officially teach until 1919. She did not receive a salary from the University until 1923. Her theorem links conservation principles and symmetries (Figure 43). It is one of the foundations of particle physics. Her work helped Albert Einstein to refine the foundations of the theory of general relativity he developed in 1915[47]. She died relatively young, at 53.

$$\frac{d}{dt}\left(\sum_{a} \frac{\delta L}{\delta \frac{dq_a}{dt}} \delta q_a\right) = 0$$

Figure 43: Emmy Noether's main equation.

Arthur Holly Compton (1892-1962, American) was a physicist who got the 1927 Nobel prize in Physics for the discovery in 1922/1923 of the effect which demonstrates that photons can have momentum and behave as particles (Figure 44). His experiment makes a photon interact with a free electron around an atom, validating the photoelectric effect theories of Planck and Einstein. The Compton effect is a variant of this effect, applied to X and gamma rays which are high energy photons.

[44] See also Richard Webb's Seven ways to skin Schrödinger's cat, 2016 which describes the different schools of thought in quantum physics. See also other interpretations of quantum physics in Ethan Siegel's The Biggest Myth In Quantum Physics Starts With A Bang in Forbes, 2018.

[45] See Quantum Model of the Atom by Helen Klus, 2017.

[46] See In her short life, mathematician Emmy Noether changed the face of physics Noether linked two important concepts in physics: conservation laws and symmetries by Emily Conover, 2018. She created a second important and more general theorem that is the basis of gauge fields theories in quantum fields theory.

[47] See Women in Science: How Emmy Noether rescued relativity, by Robert Lea, February 2019.

Compton scattering deals with the reception of an X or gamma photon which has an energy higher than that of the ejected electron. The X ray photon is slowed down and deflected with a lower energy and becomes a scattered photon. This is also called an elastic shock.

The Compton effect is used in X-ray radios. X-rays are emitted during electronic transitions between the atomic layers K, L and M (the first around the nucleus of the atom). The emission angles of the ejected electron and the re-emitted photon depend on the incident photon energy level.

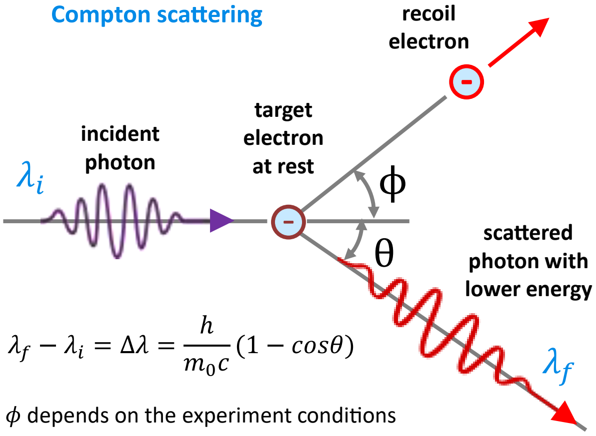

Figure 44: Compton scattering phenomenon.

Otto Stern (1888-1969, German-American) and **Walther Gerlach** (1889-1979, German) respectively conceived in 1921 and together realized in 1922 in Frankfurt the famous Stern-Gerlach experiment which discovered the intrinsic angular momentum (or spin) quantization in a magnetic field using a beam of electrically neutral silver atoms as shown in Figure 45 [48]. In the experiment, this momentum came from the 47th electron spin from heated silver atoms[49].

It showed that these atoms have a quantized angular dipole that deflects the beam in a given direction upward or downward. It later became known as particle spins a bit later, in 1925, per the work of George Uhlenbeck and Samuel Goudsmit. The experiment also did show that spin measurement along a given direction was incompatible with being done in another direction, corresponding to the notion of observables complementarity.

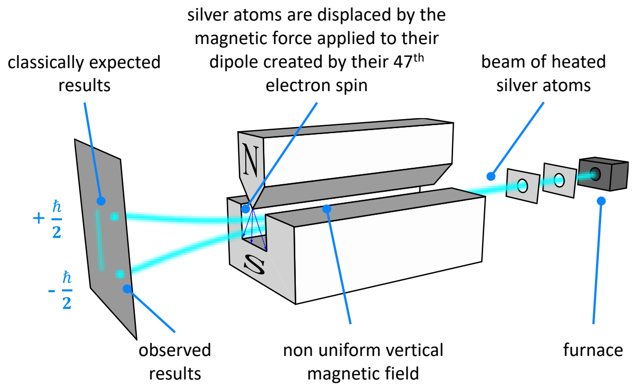

Figure 45: the Stern-Gerlach experiment where an atomic stream of silver is deviated in two discrete directions by a magnetic field. Image source: Wikipedia. 2023.

Jacques Salomon Hadamard (1865-1963, French) was a mathematician who worked on complex numbers, differential geometry and partial differential equations (PDEs), particularly during the 1920s. He also became interested in the creative process of mathematicians with studying the creative process of hundreds of colleagues. His name was given to the Hadamard single qubit gate used in quantum computers and quantum algorithms which creates a superposed state between $|0\rangle$ and $|1\rangle$.

[48] The X, Y and Z components of the electron spin measured in the Stern-Gerlach experiment are complementary variables. Measuring one of the three variables prevents from doing so with the two others.

[49] See The Stern-Gerlach Experiment, Translation of: "Der experimentelle Nachweis der Richtungsquantelung im Magnetfeld" by Martin Bauer, January 2023 (5 pages).

It is a special case of the Hadamard transforms which are square matrix operations with 2^n complex or integer values on each side as described in Figure 46. The single qubit quantum gate named after Hadamard is a transform of Hadamard of type H_1. It changes the amplitude of a qubit by a 90° rotation around the Y axis of the Bloch sphere as we will see starting page 194.

$$H_0=1$$

$$H_1=\frac{1}{\sqrt{2}}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$$

$$H_2=\frac{1}{2}\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1\\ 0 & 0 & 1 & 0\end{bmatrix}$$

$$H_3=\frac{1}{2^{3/2}}\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1\\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0\end{bmatrix}$$

Figure 46: Hadamard matrices of various dimensions.

This superposition is one of the enablers of computing parallelism in quantum computing, in addition to the principle of entanglement which links the qubits together and is one of the sources of quantum exponential acceleration. Superposition is only responsible for a potential polynomial acceleration.

Louis de Broglie (1892-1987, French) was a mathematician and physicist who, in 1923 and 1924, extended the particle-waves duality, then only applied to photons, to massive particles, mainly electrons, and also atoms, protons and neutrons[50]. According to this principle, elementary particles behave like particles (with a position, a trajectory and possibly a mass) and like waves (potentially delocalized and scattering in all directions and generating interference) depending on the circumstances.

This is the case of electrons which have a mass and can interfere with each other. Louis de Broglie turned this duality into an equation: λp=h, where λ is a wavelength, p is a quantity of motion and h is Planck's constant (Figure 47).

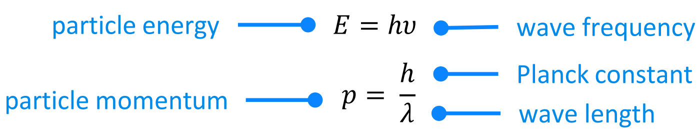

Figure 47: De Broglie wave-particle equation with electrons.

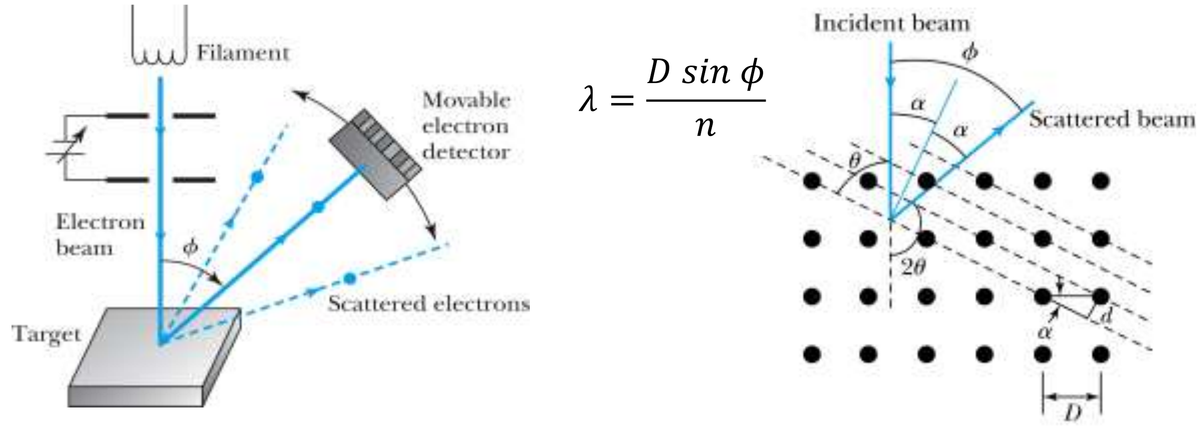

Figure 48: electron wave-particle diffraction experiment. Source: Wave Properties of Matter and Quantum Mechanics I (48 slides).

This earned him the Nobel prize in Physics in 1929. He is the main French contributor to quantum physics during the inter-war period. The wave-particle duality of electrons was confirmed in 1927 as shown above in Figure 48 with a nickel crystal based diffraction experiment by **Clinton Davisson** (1881-1958) and **Lester Germer** (1896-1971) from the Bell Labs in the USA, who shared a Nobel prize in physics in 1937.

[50] Louis de Broglie's brother, Maurice de Broglie (1875-1960), was also a physicist. He had studied X-rays and spectrography. Both brothers were members of the Academy of Sciences in France.

The electron wavelength is in the picometer range and is much smaller than visible photon wavelengths. It explains why electron microscopes have a better resolution than classical optical microscopes. **George Paget Thomson** (1892-1975) from the University of Aberdeen in Scotland did a similar experiment also in 1927. However, the Young double-slit experiment done with electrons was realized much later, in 1961, by **Claus Jönsson** (1939, German).

The confirmation of the wave-particle duality was then verified for neutrons much later in 1988 by **Roland Gähler** and **Anton Zeilinger**[51] and for atoms in 1991 by **Olivier Carnal** and **Jürgen Mlynek**, using double-slit diffraction and by **Mark Kasevich** and **Steven Chu**, who created the first cold atom interferometer using a light-beam splitter based on Raman transitions. It became the basis of atom interferometry used in quantum absolute gravimeters, using an equivalent of a Mach-Zehnder interferometer replacing light with so-called matter-wave made of atoms[52]. It is even verifiable with molecules of several atoms.

Wolfgang Pauli (1900-1958, Austrian/American) is at the origin of the principle of exclusion which bears his name elaborated in 1925 and according to which two electrons cannot have the same quantum state in an atom. He first discovered in 1924 the atom nucleus spin, used to explain the hyperfine structure of atomic spectra, i.e., the existence of very close spectral lines observed during their excitation. It cannot be explained by quanta and energy levels of the electron layers in the atoms.

In 1925, he formulated the exclusion principle according to which electrons in the same system (an atom) cannot be simultaneously in the same quantum state, a principle that was later extended to all fermions, i.e., half-integer spin particles (electrons have a spin ½ but fermion atoms can have 3/2, 5/2, 7/2 and even 9/2 spins, like ^{40}K). For example, the two electrons in a helium atom are in the same shell and must have an opposite spin. On top of that they are entangled since they are indissociable.

He then proposed between 1925 and 1927 that the electron has an additional theoretical degree of liberty, on top of the first three quantum numbers describing the state of an electron in an atom, with the mathematical formalism to describe it using the famous Pauli matrices and operators. The first electron quantum number is the energy level of the electron in the atom (the layer where it is located), the second is the azimuthal quantum number or orbital quantum number (which defines the electron sub-shell) and the third is the magnetic quantum number (which describes a discretized orientation of the subshell and makes it possible to distinguish the orbitals of the electron in the atom)[53].

This fourth degree of freedom was identified in 1925 by **George Uhlenbeck** (1900-1988, The Netherlands/USA) and **Samuel Goudsmit (**1902-1978, The Netherlands/USA) as an intrinsic angular momentum or electron spin[54]. But we currently do not have an image or physical representation of what the spin is[55]. Electron spins are used in silicon qubits that we cover later, starting page 349. He also conjectured the existence of the neutrino in 1930, which was experimentally proven in 1956, and on worked on quantum electrodynamics. He was awarded the Nobel prize in Physics in 1945.

[51] See Single- and double-slit diffraction of neutrons by Anton Zeilinger et al, Review of Modern Physics, 1988 (7 pages).

[52] In this setup, the Mach-Zehnder beamsplitter is replaced by a series of three lasers pulses creating a superposition of two atomic energy states driving a diffraction effect, then a mirror effect and at last for a recombination of split wave packets.

[53] The second and third electron quantum numbers were introduced by Arnold Sommerfeld (1868-1951, German). Among others, Wolfgang Pauli and Werner Heisenberg were his PhD students. The alpha constant or fine structure constant is also called the Sommerfeld constant per his work from 1916! See Electron spin and its history by Eugene D. Commins, May 2012 (28 pages).

[54] George Uhlenbeck and Samuel Goudsmit were students of Paul Ehrenfest (1880-1933, Austria/the Netherlands). His laboratory had welcomed some illustrious future physicists such as Enrico Fermi, Robert Oppenheimer, Werner Heisenberg and Paul Dirac. Ehrenfest was a specialist in statistical physics. In particular, he contributed to the understanding of phase changes in matter.

[55] See How Electrons Spin by Charles T. Sebens, California Institute of Technology, July 2019 (27 pages) which provides a good background on electron spin's physical interpretations, particularly with regards to electron's size. Pauli did demonstrate in 1924 that if the electron spin corresponded to an angular momentum, the electron's rotation would exceed the speed of light.

Otherwise, 137 is a number that played a weird role in Pauli's life. It turns out that 1/137 is a value that roughly corresponds to the fine-structure constant, a ratio that is found in many places in quantum physics and compares data of the same dimension[56]. It is for example the ratio between the velocity of an electron in the lower layer of a hydrogen atom and the speed of light or the probability of emission of the absorption of a photon for an electron (complete list). "137" is a sort of "42" of quantum physics. Wolfgang Pauli died after some pancreatic cancer surgery, while his hospital room number was 137!

Erwin Schrödinger (1887-1961, Austrian) is a physicist who was awarded the Nobel Prize in 1933 for the creation of his famous wave function in 1926, *aka* Schrödinger equation, which describes the evolution in time and space of the quantum state of a massive quantum particle and the probabilities of finding the quantum at a given place and time. Schrödinger's equation is a variant of the Newtonian mechanics equations that define the total energy of an object as the sum of its kinetic energy and its potential energy. We describe this equation in detail in a dedicated section page 111.

Erwin Schrödinger also created his famous alive and dead cat in a box thought experiment[57]. The story was hidden for a while and revived in the early 1980s, particularly after Alain Aspect's experiment[58].

In the Schrödinger original scenario, an opaque box contains a vial of poison, the opening of which is caused by the disintegration of a radioactive radium atom generating alpha particles ("alpha decay"), made of two protons and two neutrons, that are detected by a Geiger counter. Since radium has a 50/50 chance of disintegrating at its mid-life, the cat has a 50/50 chance of being alive and dead, at deadline. When opened, it is either alive or dead.

As long as the door is not opened, the cat is said to be superposed in the alive and dead states and entangled with the radium atom state. This story has been repeated ad-nauseam since 1935. But his thought experiment was created to show the absurdity of the measurement postulate, the wave function collapse and Born's rule. Unfortunately, the contrary has been memorized, as pictured in Figure 49.

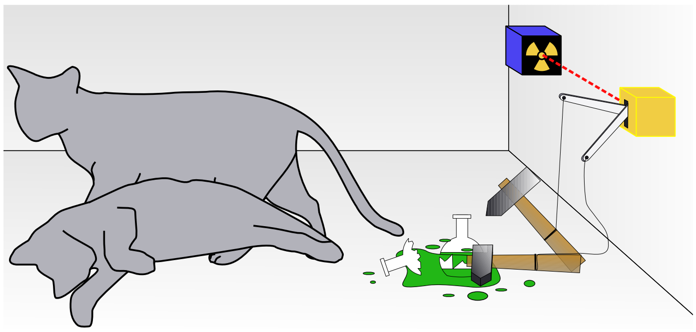

Figure 49: the infamous Schrodinger's cat thought experiment.

The caveat is that a cat can't be superposed in two states because it is a macroscopic object of a size well beyond the quantum/classical limit. It is either alive or dead, never both. These are exclusive states. On top of that, the radium atom disintegration as well as the cat's death are both irreversible processes. They can't be implemented as linear superpositions of waves. When the cat is dead, he's not in a superposition. He's just plain dead[59].

---

[56] The fine-structure constant was measured with a precision of 2.0×10^{-10} in 2020 using cold atoms interferometry. See Determination of the fine-structure constant with an accuracy of 81 parts per trillion by Léo Morel, 2020 (36 pages).

[57] The Cat Thought Experiment was published in a series of three papers in 1935, shortly after the publication of the EPR paradox paper by Einstein, Podolsky and Rosen. See The Present Status of Quantum Mechanics by Erwin Schrödinger, Die Naturwissenschaften, October 1935 (26 pages). The history of the cat occupies only nine lines in this long document which deals with superposition, measurement, and entanglement. That's even where Schrödinger coined the term entanglement in the first chapter "*The Lifting of Entanglement. The Result Depends on the Will of the Experimenter*". Schrödinger translated himself the German word Verschränkung into entanglement. The cat that appears only three times in all and for all is therefore anecdotal but that is what everyone has remembered. Which is quite normal: the rest is much less easy to apprehend!

[58] See Is Schrödinger's Cat Alive? by Mani L. Bhaumik, October 2022 (11 pages).

[59] See Simple no-go proof on observing real Schroedinger's cats by Guang Ping He, April 2023 (5 pages) which demonstrates this.

We can consider that the cat's death is provoked by a not yet read measurement when the box is closed, corresponding to a non-selective measurement as described page 215. The cat state uncertainty is a classical one, not a quantum one. The cat is in a maximally "mixed state" where the uncertainty of its death is classical, not in a "pure state" where it would be quantum (we define these notions starting page 171). If you used a webcam inside the box and made sure it didn't influence the radium half-life period, you could track the cat state all along, from alive to dead or alive to alive, which are the only two possible paths and observe the absence of superposition[60].

This thought experiment was intended to highlight two things. First, that superposition and entanglement only applied to the infinitely small and not to macroscopic objects. History retained the principle of superposition and not this difference between the microscopic and macroscopic worlds. Second, that there was and still is an uncertain limit between the quantum and classical worlds. Schrodinger's thought experiment also dealt with the entanglement between the radium atom and the cat. Could this entanglement work with a macro-object[61]? The paper containing this thought experiment was about entanglement and that was forgotten. Also, this paper's publication was the one generating the publication of the EPR paradox piece by Einstein et al. We should remember that Schrödinger's wave function and the notion of states superposition only make sense at a microscopic scale. Let's leave that poor cat alone in his dreams!

Max Born (1892-1970, German) is a physicist and mathematician who developed the mathematical representation of quantum in a matrix form. We owe him in 1926 the statistical explanation of the probability of finding an electron in a given energy state from its wave function, elaborated by Schrödinger the same year. This principle is applied to qubits, where the sum of the square of the probabilities of the two states of the qubit is equal to 1, given the probabilities are complex numbers.

In 1925, he created the non-commutativity relation of two conjugate quantities, one being the Fourier transform of the other (the commutator $[X, P] = XP - PX = i\hbar I$, where X is a position and P a momentum and I, the identity). It led to the creation of the indeterminacy principle. Max Born also created the first version of the adiabatic theorem with Vladimir Fock in 1928. He got the Nobel prize in physics in 1954. Fun fact, the British singer Olivia Newton-John is his grand-daughter[62]. And he was also the PhD Director of Robert Oppenheimer, the future Director of the Manhattan project.

Werner Heisenberg (1901-1976, German) is a physicist, Nobel prize in Physics in 1932, to whom we owe in 1927 the creation of the famous principle of uncertainty, or rather indeterminacy, according to which one cannot accurately measure both the position and the velocity of an elementary particle, or, more generally, two arbitrary unrelated quantities. He is at the origin, with Max Born and Pascual Jordan in 1925, of the quantum matrix formalism describing physical quantities.

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[60] The Schrodinger text says exactly: "*It is also possible to construct very burlesque cases. Imagine a cat locked up in a room of steel together with the following hellish machine (which has to be secured from direct attack by the cat): A tiny amount of radioactive material is placed inside a Geiger counter, so tiny that during one hour perhaps one of its atoms decays, but equally likely none. If it does decay then the counter is triggered and activates, via a relais, a little hammer which breaks a container of prussic acid. After this system has been left alone for one hour, one can say that the cat is still alive provided no atom has decayed in the mean time. The first decay of an atom would have poisoned the cat. In terms of the ψ−function of the entire system this is expressed as a mixture of a living and a dead cat*".

[61] You can apply this thought experiment to the baking of the half-cooked chocolate. As long as you don't take it out of the oven after the mandatory baking of 9 minutes, but with an oven with an unknown power, you don't know if it is well done or not, and run it through the middle before you take it out. It is in a state of superposition between undercooked, well done and overcooked. On the other hand, if it is overcooked, it will be difficult to go back, like Schrödinger's half-dead cat in case he died. Overcooking as well as the death of the cat are irreversible. It is therefore not a true superposition of quantum states. But here, I have no clue about how the oven and the half-baked chocolate are entangled. It's about statistical physics and thermodynamics, not quantum physics even though we could conjecture the existence of some cookie action at a distance! Cheers!

[62] See Olivia Newton-John's grandfather Max Born was friend of Albert Einstein by Matthew Alice, 1995.

The indeterminacy principle is a consequence of this formalism. It was described mathematically in a simplified manner in 1927 by **Earle Hesse Kennard** (1885-1968, American) in the famous equation in Figure 50, where the product of the standard deviation of position and velocity is greater than half the Dirac (or reduced Planck) constant.

$$\Delta x \, \Delta p \geqslant \frac{\hbar}{2}$$

Figure 50: Heisenberg-Kennard inequality, as formulated by Earle Hesse Kennard.

This principle can be used to improve the accuracy of a measurement of any quantity by lowering the accuracy of another quantity characterizing a quantum[63]. These quantities can be for example an energy level, a position, a wavelength, or a speed.

One consequence of Heisenberg's indeterminacy principle is that all particles in the Universe are in permanent motion. If they were stable, we would know their position (fixed) and their velocity (zero), violating the indeterminacy principle.

Another consequence is that a perfect vacuum could not exist because the value and evolution of the magnetic and gravitational fields that pass through it would be stable, violating once again Heisenberg's indeterminacy. This explains the astonishing vacuum quantum fluctuations we discover a little further starting in page 155. The no-cloning theorem of a qubit state also derives from the principle of indeterminacy.

For some, this indeterminacy principle is a simplified interpretation of the corpuscular nature of matter. It leads to the question of the position and velocity of an electron when it has no precise position. According to the Copenhagen interpretation of quantum mechanics, we shouldn’t try to determine where the electron is located.

In practice, quantum particles are not classical physical particles and therefore their velocity and position cannot be measured. They can only be described by their (Schrödinger) wave function and position probabilities. More generally, in the infinitely small, the measurement device influences the measured quantity. One example illustrates this phenomenon at the macroscopic level: if you illuminate an insect with sunlight and a magnifying glass to better observe it, you may burn it! The same happens with a photon that is used to detect an electron, in the Heisenberg microscope thought experiment, as shown in Figure 51. It will change the speed and position of the electron.

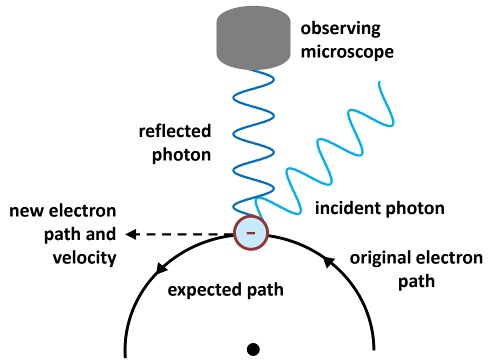

Figure 51: Heisenberg microscope thought experiment. [Source](#).

Finally, like many of the colleagues of his time, Werner Heisenberg was interested in the links between science, quantum mechanics and philosophy, and as early as 1919. He was an assistant to Niels Bohr between 1924 and 1927, before leaving for the University of Leipzig. Max Born was also one of his professors.

During World War II, he was asked with other German scientists to work on the Reich’s atomic bomb project. Later revelations did show that he was not very active on this project and did not believe it was an achievable goal. He even discussed it with Niels Bohr during the war, in September 1941 in Copenhagen, which was then occupied by the Germans[64]. It even led to the creation of the play “Copenhagen” by Michael Frayn which was published in the UK in 1998.

[63] This measurement technique is used in "quantum squeezing" which is integrated in the latest version of LIGO for the measurement of gravitational waves: NIST Team Supersizes 'Quantum Squeezing' to Measure Ultra Small Motion, 2019.

[64] See A historical perspective on Copenhagen by David C. Cassidy, American Institute of Physics, 2000 (5 pages)

Paul Dirac (1902-1984, English) is a mathematician and physicist among the founders of 20th century quantum physics. He is credited with the 1928 electron spin equation, which is one of the foundations of relativistic quantum physics (*below*). His equation is a kind of variant of Schrödinger's equation for free relativistic particles, fermions (electrons, protons, neutrons, quarks, neutrinos) which are half-integer spin particles. Relativistic particles are those moving at a speed close to the speed of light, which contains electrons if lower shells of heavy atoms.

In Dirac's equation (Figure 52), the wave function ψ of the electron includes four components of complex numbers that integrate time and space. Dirac's equation enabled him to predict the existence of a particle that was later be called the positron, an opposite of the electron with a positive charge[65].

$$\left(\beta mc^2 + c\sum_{n=1}^{3}\alpha_n p_n\right)\psi(x,t) = i\hbar\frac{\delta\psi(x,t)}{\delta t}$$

Figure 52: Dirac's relativistic wave-function equation.

Dirac formalized the quantization of the free electromagnetic field in 1927. He also introduced in 1939 the bra-ket notation, known as Dirac's notation, which simplified the notation and manipulation of quantum states and operators in linear algebra (example: $\langle\phi|\psi\rangle$). The Dirac constant also named reduced Planck constant is the Planck constant h divided by 2π, also called "h-bar" for its italicized strikethrough h symbol: $\hbar$. This Dirac constant is used in the Schrödinger wave function.

Paul Dirac was awarded the Nobel prize in Physics in 1933, at the age of 31. The Nobel Prizes of the early 20th century were frequently awarded to young scientists, which seems to be out of fashion since then! The youngest Nobel prize in physics was awarded to Lawrence Bragg, who won it at the age of 25 in 1915 for his discovery of X-ray refraction at the age of 22[66].

In which case do we have to deal with relativistic particles, in particular with electrons? It is generally considered that an electron becomes relativistic when the total of its mass and kinetic energy is at least twice the rest mass.

This ratio corresponds to the Lorentz factor. It represents a speed of at least 86% of the speed of light (Figure 53). But relativistic phenomena may occur before that speed is reached. In Newtonian equivalent, the speed of an electron around the nucleus of a hydrogen atom is about c/137. With electrons from heavy atoms inner shells, this velocity can exceed c/2.

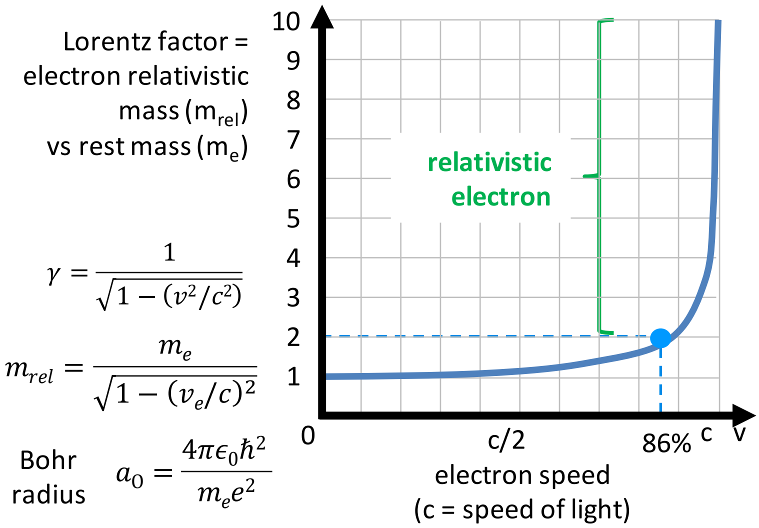

Figure 53: relativistic electrons and Lorentz factor. Relativistic atomic phenomena are said to start at 85% of the speed of light. They occur with inner layers electrons in relatively heavy elements.

[65] Positrons were discovered experimentally by Carl Anderson in 1932. He was awarded the Nobel prize in physics in 1936.

[66] Paul Dirac was distinguished by his shyness and parsimonious oral expression in meetings or during meals. So much so that his Cambridge colleagues had defined the "dirac" unit as the most concise way to express himself in a meeting, namely, at the rate of a single word per hour. His behavior was equivalent at the Solvay Congresses he attended, notably that of 1927. However, he must have broken a record in his speech accepting his Nobel Prize at the end of 1933. It is still six pages long! Half, however, of the 12 pages of the speech of Erwin Schrödinger, also winner of the Nobel prize in physics that year. Another anecdote: Dirac was married to one of the sisters of Eugene Wigner, Nobel prize in physics in 1963 and famous for his function and also his "friend" paradox.

This affects the position of relativistic electrons in the low orbits of heavy atoms such as lanthanides, which belong to the rare earths. The Bohr radius that defines the average orbital of an electron decreases inversely proportional to the apparent mass of the electron. Because the electron's apparent mass increases, this Bohr radius is smaller for relativistic electrons. This modifies the structure of the electron orbitals of heavy atoms and the transition energy levels between orbitals that absorb or emit photons.

This explains the color of gold and silver, due to relativistic modification of orbits of electron layers between which transitions occur due to the absorption of photons. Blue is absorbed in the case of gold, explaining its yellow color. Without the relativistic effect, gold would be white. This has a lot of implications in the chemistry of these materials and with their crystal organization[67].This quantum relativistic effect also explains why mercury is liquid at room temperature[68]. All this gives rise to a field of chemistry called relativistic quantum chemistry[69]. It also explains why the size of atoms is not proportional to their number of protons and electrons[70] .

Particles also become relativistic in **particle accelerators** such as the CERN LHC near Geneva (the largest in the world), the ESRF in Grenoble (European Synchrotron Radiation Facility, specialized in the generation of "hard", very high-frequency X-rays) or the SOLEIL light synchrotron located in Saint-Aubin near Saclay just next to the CEA, also in France, or its equivalent from PSI in Switzerland.

The SOLEIL synchrotron uses electrons accelerated to a relativistic speed and inverters that generate beams of light 10,000 times denser than sunlight[71] . Equivalent instruments exist such as the Advanced Photon Source at the Argonne National Laboratory from the US Department of Energy near Chicago.

Free Electron Lasers (**FEL**) exploit relativistic electron sources. These are lasers generating coherent light (spatially and temporally, the emitted photons have the same frequency, phase and in that case, also polarization) and exploit relativistic electron sources from synchrotrons (Figure 54).

The interaction between these electrons and a strong alternating magnetic field makes it possible to generate coherent light in electromagnetic frequency ranges from infrared to X-rays, through visible light and ultraviolet. The FEL are used to explore all sorts of matter, particularly in biomedical research like with X-rays crystallography.

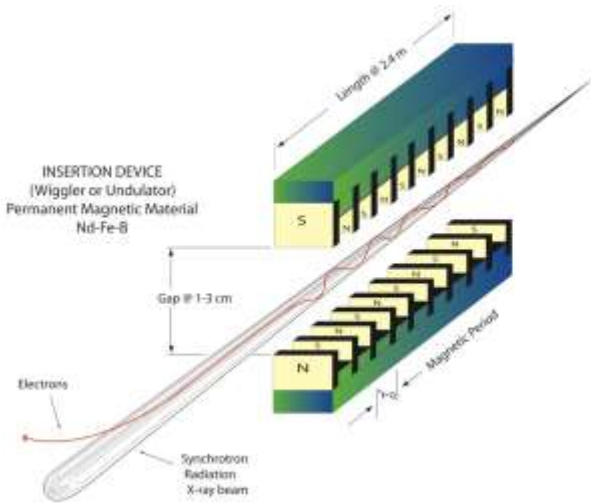

Figure 54: free-electron laser.
Source: X-ray diffraction: the basics by Alan Goldman (31 slides).

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[67] See more examples in Relativistic Effects in Chemistry More CommonThan You Thought by Pekka Pyykko, 2012 (24 pages).

[68] See Why is mercury liquid?Or, why do relativistic effects not get into chemistry textbooks? by Lars J. Norrby, 2018 (4 pages).

[69] See Relativistic quantum chemistry by Trond Saue, 2019 (110 slides) and An introduction to Relativistic Quantum Chemistry by Lucas Visscher (107 slides). The mathematical formalism of relativistic quantum chemistry is well documented in the voluminous Introduction to Relativistic Quantum Chemistry by Kenneth Dyall and Knut Faegri, 2007 (545 pages).

[70] See this periodic table of elements with an indication of the sizes of the atoms.

[71] See the conference Electrons relativists as light sources by Marie-Emmanuelle Couprie, Synchrotron Soleil, 2011 (1h25). Electrons circulate in the synchrotron at a speed close to that of light. SOLEIL powers more than 25 analytical instruments covering the spectrum from infrared to X-rays, with numerous applications in precision microscopy, including a microscopy using very well collimated and polarized white light. These instruments can be used to analyze the three-dimensional structure of organic molecules such as complex proteins, such as the glycoproteins that surround viruses. This even allows one to study how these proteins combine with those of the attacked cells, or ribosomes, which are used to produce the proteins in the cells, are also analyzed.

Finally, relativistic particles can be found in **astrophysics** and, for example, in cosmic ray sources as well as in relativistic plasma jets produced at the center of galaxies and quasars[72].

Vladimir Fock (1898-1974, Russian) was a theoretician physicist who worked on quantum physics, the theory of gravitation and theoretical optics. We own him the Fock space, representation and state, used in quantum photonics to represent the state of bosons many-body systems having the same quantum state. He co-created the Klein-Gordon equation in 1926, the relativist version of Schrödinger's equation for zero spin massive particles, the adiabatic theorem with Max Born in 1928 and the Hartree–Fock quantum simulation method in 1930. He also worked on quantum electrodynamics and quantum foundations.

Pascual Jordan (1902-1980, German) was a physicist who collaborated with Max Born and Werner Heisenberg and contributed to laying the mathematical foundations of quantum mechanics, especially in matrix computation. Like Philipp Lenard, he was somewhat forgotten because of his membership in the Nazi Party during the 1930s, although he was rehabilitated after the World War II thanks to the help of Wolfgang Pauli. He became interested in the philosophical notion of free will.

Linus Pauling (1901-1994, American) was a biochemist known to have co-founded the scientific fields of quantum chemistry and molecular biology. He had the opportunity to meet in Europe the founders of quantum physics like Erwin Schrödinger and Niels Bohr in 1926-1927. He described chemical bonds over a period between 1928 and 1932 and the hybridization of orbitals which explains the geometry of molecules. He published reference book "The Nature of the Chemical Bond" in 1939.

He was awarded the Nobel Prize in Chemistry in 1954 and the Nobel Peace Prize in 1962 for his political activism in favor of nuclear disarmament. He is at the origin of computational chemistry, which makes it possible to numerically simulate the structure of molecules and which we discuss in the section on quantum applications in healthcare page 1037.

James Chadwick (1891-1974) is an English physicist who was responsible for the discovery of neutrons in 1932, which earned him the Nobel prize in Physics in 1935. This discovery was late compared to quantum physics and the discovery of electrons. Nuclear physics has indeed progressed in parallel with quantum physics, which was mainly concerned with the interactions between electrons and photons. Before the discovery of neutrons, scientists thought that the nucleus of atoms contained protons and electrons.

John Von Neumann (1903-1957, Hungarian, then American) was a polymath and an extremely prolific mathematician. He participated in the creation of the mathematical foundations of quantum mechanics, notably in the "Mathematical Foundations of Quantum Mechanics" published in 1932. He transposed the main principles of quantum mechanics into models and equations of linear algebra. He devised the key mathematical principles behind quantum measurement models.

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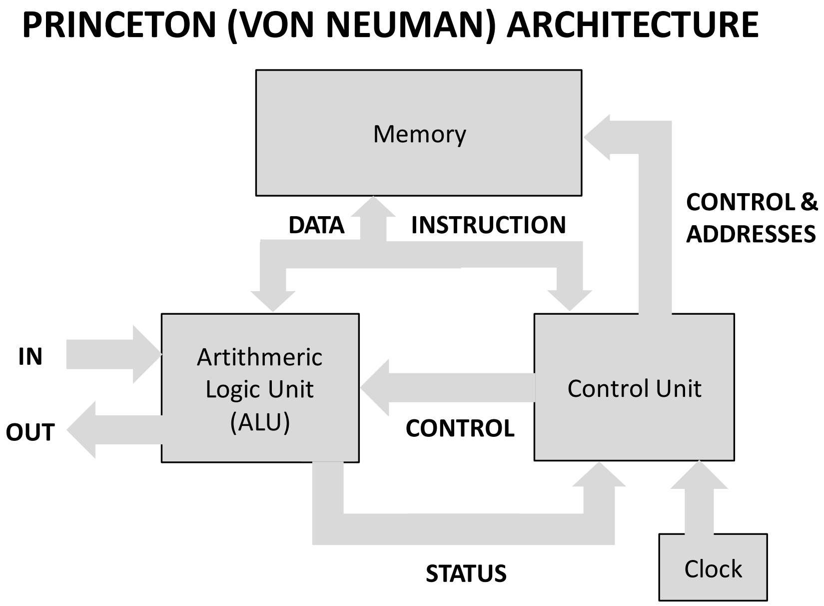
[72] Dirac's equation is linked to the **Klein-Gordon equation (**1926) which applies to bosons such as elementary gluon particles and pions, particles having integer or zero spin. Relativistic quantum mechanics is a broad field of physics, used in particular in elementary particles physics. I have not yet found any use cases of this branch of physics in current quantum technologies. See the main foundations of relativistic quantum mechanics in Relativistic Quantum Mechanics by David J. Miller, University of Glasgow, 2008 (116 slides).

This deals, for example, with the representation of quantum states as a position in a Hilbert space, the observables which are projections into Hilbert spaces and the indeterminacy principle which can be explained by the non-commutativity of measurement operators. These principles are also named Birkhoff-von Neumann *quantum logic*, in connection with their seminal paper published in 1936[73].

Von Neumann also affirmed that the introduction of hidden variables to incorporate determinism was a lost cause because it would contradict other (verified) predictions of quantum physics. Three years before Einstein/Podolsky/Rosen's EPR paper!

We owe him the creation of the notion of entropy (by Von Neumann), in 1932, which is associated with the notions of operators and density matrices that he created in 1927 and which describe the state of a multi-partite quantum system. He participated in the Manhattan project in the USA.



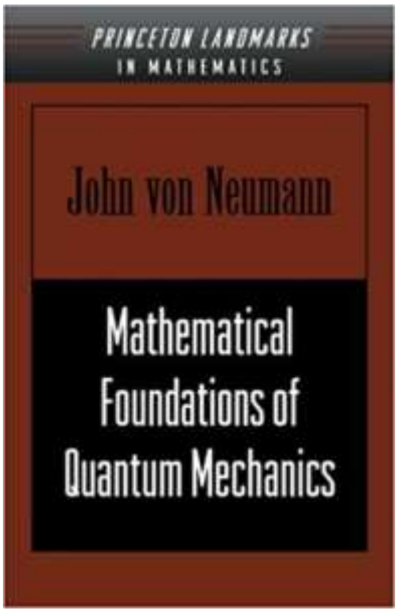


Figure 55: the Von Neuman Princeton architecture which still defines classical computing.

He modelled explosions and lenses for compressing plutonium in A-bombs. He is also responsible for the basic concepts in game theory and classical computers that are still in use. Almost all computers use a Von Neumann architecture with memory, registers, control unit, computing unit, inputs and outputs as shown in Figure 55. What a contribution!

Boris Podolsky (1896-1966, Russian then American) wrote the EPR paradox paper with Albert Einstein and Nathan Rosen in 1935 on quantum entanglement and questions of nonlocality of the properties of entangled quanta. He was a specialist in electrodynamics which deals with the analysis of electric and electromagnetic fields. He emigrated to the USA and, according to Russian archives, was a post-war KGB informant on the American atomic program between 1942 and 1943. His code name was... " Quantum".

Nathan Rosen (1909-1995, American then Israeli) is the third EPR paradox author when working as an assistant to Albert Einstein in Princeton. After moving to Israel in 1953, he created the Institute of Physics at Technion University in Haifa. He was mainly working on astrophysics and relativity theory. He devised the concept of wormholes, a theoretical link between different points in space and time. He also thought neutrons were built out of a proton coupled to an electron.

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73 See The Logic of Quantum Mechanics by Garrett Birkhoff and John Von Neumann, 1936 (22 pages).

Ettore Majorana (1906-circa 1938, Italian) imagined the existence of a fermion in 1937 based on Dirac's equations, an elementary particle that would be its own anti-particle. The Majorana fermion naming is also abusively applied in condensed matter physics to quasiparticles having similar properties. Their existence was discovered in 2012 and verified in 2016 and then in 2018, even if it is still disputed by many physicists and two related 2018 papers had to be retracted in 2021.

These Majorana quasiparticles (or “Majorana Zero Modes”) could make it possible to design universal quantum computers called topological computers that can handle very efficient error correction codes requiring a small number of physical qubits. This is the exploration path chosen by Microsoft after the work of Michael Freedman and Alexei Kitaev in the late 1990s. Ettore Majorana is said to have committed suicide after a depression, because he could hardly stand the pressure of his genius! But his disappearance remains enigmatic because his body has never been found!

Alonzo Church (1903-1995, American) was a mathematician who was a key contributor to the foundations of theoretical computer science and on the notion of computability. Among other things, he created the lambda calculus in 1936, a universal abstract programming language which inspired the creation of LISP. He also created the so-called Church-Turing thesis. For this last one, any automatic calculation can be carried out with a Turing machine. Church and Turing also proved an equivalence between being λ-computable and Turing computable.

Many variations of the Church-Turing thesis were elaborated after them to extend the broad field of complexity theories. For example, the extended Church-Turing thesis states that the computation time of a problem is equivalent at worst to a polynomial depending on the size of the problem. It is not demonstrable.

What about the others, known, unknown or less famous from the 1927 Solvay Congress? Two participants deserve to be mentioned who had some connections with quantum physics.

Léon Brillouin (1889-1969, Franco-American) who is less known in France because of his expatriation to the USA during World War II contributed to advances in quantum physics between the two World Wars. In particular, he brought quantum mechanics closer to crystallography. He especially discovered the phenomena of diffraction of waves traversing crystals, called Brillouin scattering.

And then, finally, **Hendrik Anthony Kramers** (1894-1952, Dutch) who assisted Niels Bohr in the creation of quantum theory. Many of the participants were not quantum physics scientists. They were invited because the Belgium organizers tried to have a stable proportion of Belgians, French, Germans and English participants. Were there, for example, **Émile Henriot** and **Marie Curie** who were focused on radioactivity, way before it could be explained by the formalism of quantum physics, **Paul Langevin** (with whom Marie Curie had had an affair in 1910, after the accidental death of her husband Pierre Curie in 1906), as well as a good number of chemists.

What was striking during this prolific period was the way the social network of physicists worked, without smartphones and the Internet. They had many encounters, cross-University tenures, meetings, letter exchanges and conferences. It was slow according to today’s references, but the results were still astounding.

To conclude this part, Figure 56 reminds us how young the founders of quantum physics were when they published their seminal work in the key years from 1900 to 1935. Back then, scientific research didn’t work the same way. They also were frequently awarded Nobel prizes at less than 40! Nowadays, most of the times, you must wait until you are at least 50 if not 70.

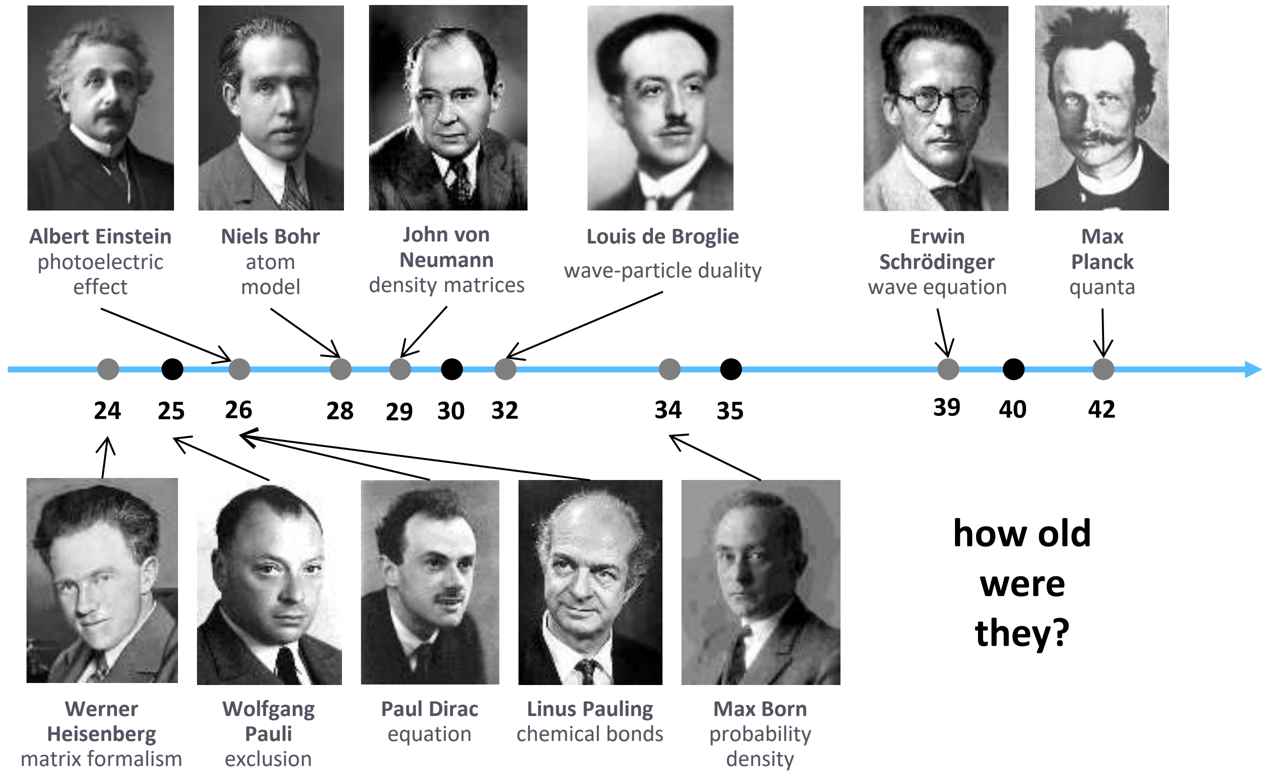


Figure 56: how old were quantum scientists when they were awarded the Nobel prize in physics? (cc) Olivier Ezratty, 2021.

Post-war

As mentioned before, quantum physics developments seemed to slow down between 1935 and 1960. Physicists were then busy with nuclear physics. The Manhattan project mobilized an amazingly large number of physicists like John Von Neumann and **Enrico Fermi** (1901-1954, Italian American, Nobel prize in physics in 1938) whose contributions were centered in nuclear physics (first nuclear reactor in 1942). Pre-war, he was also a key contributor to quantum physics with his work on statistical physics, leading to the Fermi-Dirac ideal gas statistics, fermions and Fermi sea (1926), Fermi's golden rule (1927) and his work on neutrinos (1934).

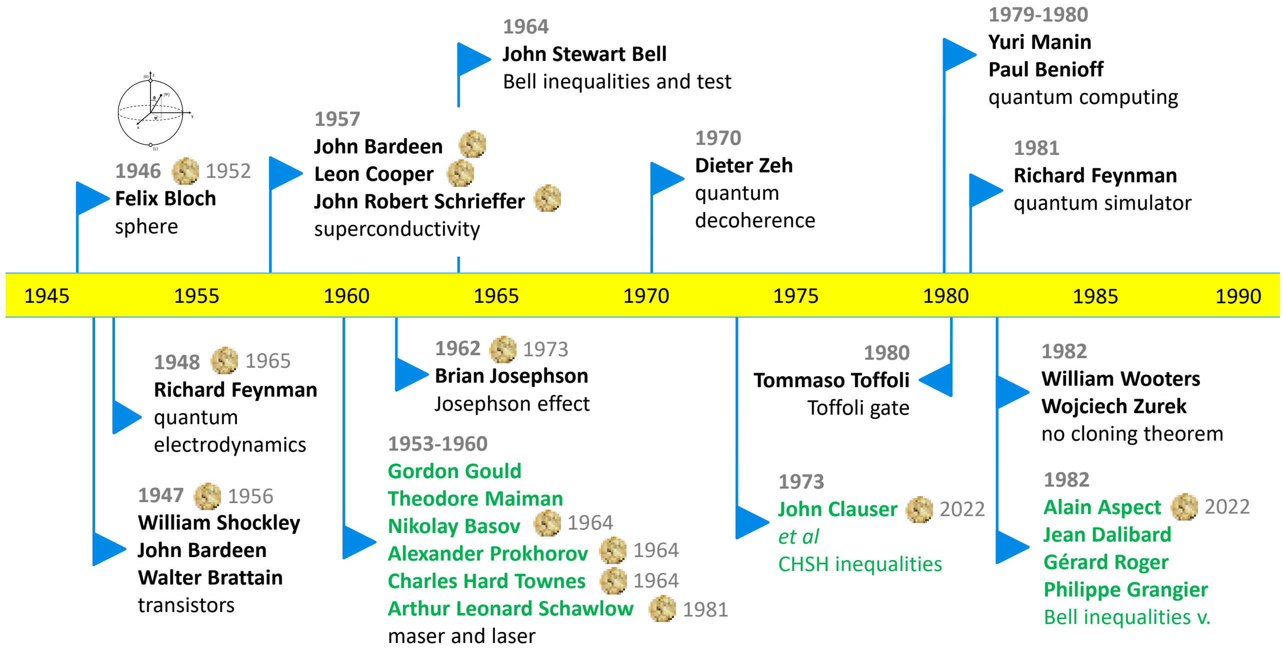


Figure 57: timeline of key events in quantum physics after World-War II. (cc) Olivier Ezratty, 2020-2023.

This led to the creation of several research labs (Los Alamos, Oak Ridge, etc.) which are now DoE labs and host strong academic research in quantum technologies.

Quantum physics still led, after World War II, to an incredible wealth of technologies that revolutionized the world (Figure 57). We can mention three important branches resulting from the applications of the first quantum revolution: **transistors**, invented in 1947 by William Shockley, John Bardeen and Walter Brattain from the Bell Labs[74], **masers** and **lasers** invented between 1953 and 1960 by Gordon Gould, Theodore Maiman, Nikolay Basov, Alexander Prokhorov, Charles Hard Townes and Arthur Leonard Schawlow, only a few of whom received the Nobel Prize associated with these discoveries, **photovoltaic cells** that convert light into electricity, and the **GPS**.

Transistors and lasers are the basis of much of today's digital technology. All our digital devices are already quantum! The field of quantum optics started in the early 1960s with the laser invention and Roy J. Glauber's work, with his seminal work in 1963 on light classification where he formalized the coherent states generated by lasers, *aka* Glauber states.

The post-war period was also dominated in quantum physics by advances made on superconductivity with the BCS theory in 1957 and the Josephson junction in 1962, and by the theoretical work of John Stewart Bell in 1964.

We then have the verification of entanglement by Alain Aspect's experiment in 1982. 1980 and 1981 are other key dates which mark the symbolic beginnings of quantum computing, imagined by Yuri Manin and Paul Benioff (gate-based quantum computing) and Richard Feynman (quantum simulation).

The term **second quantum revolution** covers advances from the 1990s and later, when the quantum properties of individual particles could be controlled at the level of photons (polarization, ...), electrons (spin) and atoms or ions, as well as superposition and entanglement. This led to the emergence of quantum cryptography and quantum telecommunications, in addition to the premises of quantum computing. The original definition of this second quantum revolution is however not as precise[75].

Felix Bloch (1905-1983, Swiss then American) is a physicist who created the geometrical representation of a qubit state in a sphere, Bloch's sphere was elaborated in 1946 in a paper on nuclear magnetism, his main specialty. Like other physicists of his time, he contributed to the Manhattan project, although quite shortly. He was awarded the Nobel prize in Physics in 1952 for his work on nuclear magnetic resonance and magnons conceptualization. He was also the first director of the international particle physics laboratory CERN in 1954.

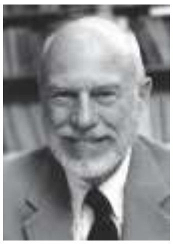
[74] Transistors are based on many quantum phenomena, particularly the electronic structure of atoms in semiconductors crystals that was discovered during the 1930s and creates forbidden energy levels named band gaps (found by Sir Alan Herries Wilson, UK, in 1931), the impact of defects in crystals leading to doping and the tunneling effect due to the wave-particle duality of electrons. It also uses the field effect, which modulates the electrical conductivity of a material by the application of an external electric field. It was invented by Julius Edgar Lilienfeld (1882-1963, Austro-Hungarian and American) who got a related patent granted in 1926 using copper-sulfide semiconductor materials. It corresponds to what we today call a "Field Effect Transistor" (FET). The first transistor invented in 1947 was made of germanium, not silicon. See The Transistor, an Emerging Invention: Bell Labs as a Systems Integrator Rather Than a 'House of Magic' by Florian Metzler, October 2020 (57 pages) which shows the flow of discoveries that led to the creation of the first transistor by the Bell labs in 1947. This first computer using transistors was the TRADIC Phase One computer that was built in 1954.

[75] The second quantum revolution expression was created simultaneously and independently in 2003 by Alain Aspect and by Jonathan Dowling and Gerard Milburn. The latter is also known to be one of the three protagonists of the KLM model of photon-based quantum computing, created in 2001 jointly with Emanuel Knill and Raymond Laflamme.

Chien-Shiung Wu (1912-1997, Chinese then American) was a scientist who contributed to the development of nuclear physics and to the Manhattan project, with her gaseous diffusion process used for separating uranium 238 from uranium 235. She also contributed to the development of quantum physics by conducting the first experiment related to the synchronization of photon pairs and entanglement in 1949, before Alain Aspect's experiment in 1982[76].

This experiment was different and was based on the measurement of the angular correlation of gamma ray photons (with very high-frequency and high-energy) generated by the encounter of electrons and positrons.

Hugh Everett (1930-1982, American) is a physicist who created the formulation of relative states and a global wave function of the Universe integrating observations, observers and tools for observing quantum phenomena. He met Niels Bohr with other physicists in Copenhagen in 1959 to present his theory. He was politely listened to, but his interlocutors said that he understood nothing about quantum physics.



Everett was also a contributor to the connections between the theory of relativity and quantum physics, especially around quantum gravitation. He is credited with the hypothesis of multiple or multiverse worlds, or many-worlds interpretation, explaining quantum entanglement and nonlocality. It is in fact coming from **Bryce DeWitt** (1922-2004, American) who interpreted his work in 1970. DeWitt also worked on the formulation of quantum gravity theories.

John Wheeler (1911-2008, American) supervised Hugh Everett's thesis. He was a specialist in quantum gravitation. He worked in the field of nuclear physics, notably in the Manhattan project, on the first American H-bombs and on very high-density nuclear matter found in neutron stars. He popularized the term black hole in 1967. He imagined a delayed-choice experiment to decide when a quantum object decides to travel as a wave or as a particle, which was later implemented in the 2000s.

He collaborated with Niels Bohr and among his PhD students were Richard Feynman and Wojciech Zurek!

Richard Feynman (1918-1988, American) is one of the fathers of quantum electrodynamics starting with seminal papers published starting in 1948, which earned him the Nobel prize in Physics in 1965. Before that, he was one of the contributors to the Manhattan project where, among other things, he calculated with his both Hans Bethe the yield of nuclear explosions. He is also at the origin of the quantum explanation of helium superfluidity at very low temperature in a series of papers published between 1953 and 1958.

He theorized in 1981 the possibility of creating quantum simulators, capable of simulating quantum phenomena, which would be useful to design new materials and molecules in various fields like chemistry and biotechs[77]. He was also known for his great presentation skills.

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[76] See The Angular Correlation of Scattered Annihilation Radiation, Wu and Shaknov, 1949.

[77] See Simulating Physics with Computers submitted in May 1981 to the International Journal of Theoretical Physics and published in June1982. Later, in Quantum Mechanical Computers by Richard Feynman, published in 1985 (10 pages), he described a model of gate-based quantum computing and how it could be equivalent to a quantum simulator solving a given Hamiltonian. See this related work in The Efficiency of Feynman's Quantum Computer by Ralph Jason Costales, Ali Gunning and Tony Dorlas, Dublin Institute for Advanced Studies, September 2023 (6 pages).

Roy J. Glauber (1925-2018, USA) was a theoretical physicist, teaching at Harvard and at the University of Arizona. He got the Nobel prize in Physics in 2005 for his foundational work on the quantum theory of optical coherence. He is considered to be a pioneer of non-classical light description and of the quantum optics field, with his work in 1963, describing the various types of light (coherent, not coherent, ...). He also worked in the field of high-energy particle physics, which we don't cover in this book since out of scope of the "second quantum revolution".

Philip W. Anderson (1923-2020, USA) was a theoretical physicist who contributed to the theories of localization (*aka* "Anderson localization" according to which extended states can be localized by the presence of disorder in a system), antiferromagnetism and quantum spin liquid, symmetry breaking leading to the creation of the Standard Model, superconductivity (at high-temperature, pseudospin approach to the BCS theory, Anderson's theorem on impurity scattering in superconductors).

He created the "condensed matter physics" naming. He got the Nobel prize in physics in 1977 for his work on the electronic structure of magnetic and disordered systems. He worked at the Bell Labs and was also a teacher at Cambridge University, UK.

John Stewart Bell (1928-1990, Irish) relaunched research in quantum mechanics in the 1960s on the notion of entanglement. We owe him the Bell inequalities that highlight the paradoxes raised by quantum entanglement. Bell's 1964 theorem indicates that no theory of local hidden variables - imagined by Einstein in 1935 - can reproduce the phenomena of quantum mechanics [78]. He was rather pro-Einsteinian in his approach and favorable to a realistic interpretation of quantum physics [79].

His Bell inequalities define the means to verify or invalidate the hypothesis of the existence of hidden variables explaining quantum entanglement. Bell's inequalities were violated by the experiments of **Alain Aspect** in 1982, demonstrating the inexistence of these local hidden variables. Prior to this experiment, Bell's inequalities had been formulated for pairs of entangled photons by **John Clauser** (1942, American, 2022 Nobel prize in physics), **Michael Horne** (1943-2019, American), **Abner Shimony** (1928-2015, American) and **Richard Holt** in 1969 with their so-called CHSH inequalities with some experimental settings proposals[80]. John Bell's work was completed in 2003 by **Anthony Leggett** (1938, Anglo-American, Nobel prize in physics in 2003 for his work on superfluid helium) with his inequalities applicable to hypothetical non-local hidden variables[81]. Anthony Leggett was also an initial key contributor to what led to the creation of superconducting qubits. Anton Zeilinger (1945, Austrian) managed to experimentally violate these inequalities in 2007. According to Alain Aspect, however, this did not call into question the non-local hidden variable model proposed by David Bohm.

Quantum technologies physicists

This story now provides an overview of key contributors to the physics of quantum computing. They are often specialized in condensed matter, such as for superconducting qubits, and in photonics.

[78] See this explanation of Bell's theorem in a paper by Tim Maudlin on the occasion of the 50th anniversary of the theorem: What Bell Did, 2014 (28 pages). And Bell's original document: On the Einstein-Podolsky-Rosen paradox, John S. Bell, 1964 (6 pages). In 1964, Bell worked at the University of Wisconsin.

[79] See What Bell Did by Tim Maudlin, 2014 (28 pages) which describes the EPR paradox and Bell's contribution.

[80] See Proposed experiment to test local hidden-variable theories, 1969 (5 pages).

[81] See Nonlocal Hidden-Variable Theories and Quantum Mechanics: An Incompatibility Theorem by Anthony Leggett, 2003 (25 pages).

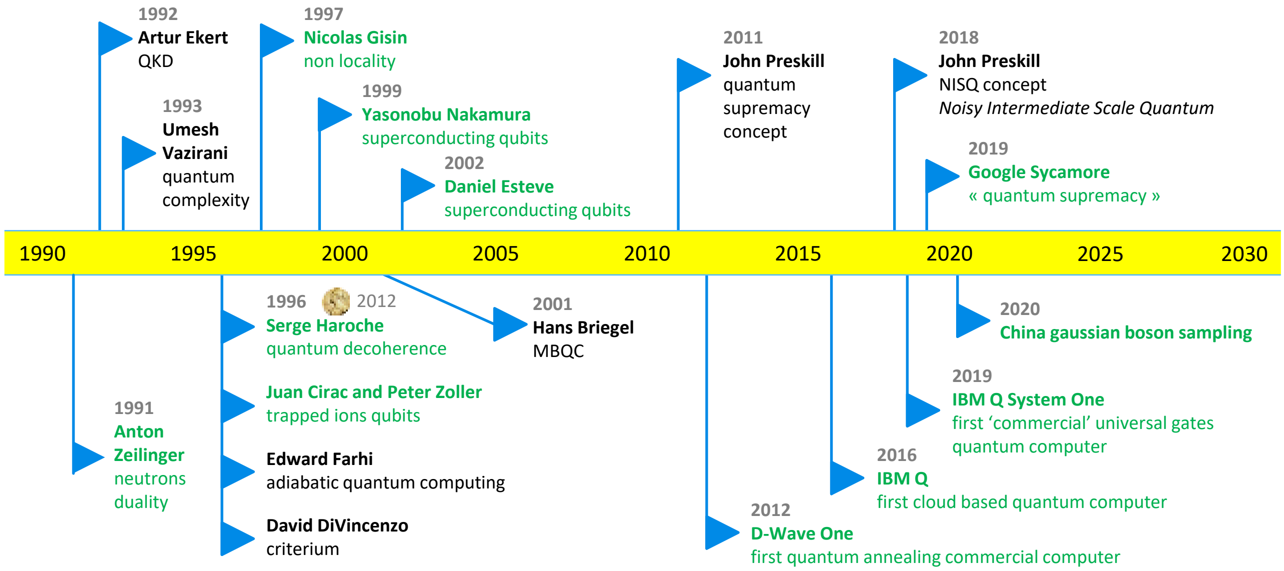

Figure 58: quantum computing key events timeline from 1990 to 2020. In green, experimentalists and experiments. (cc) Olivier Ezratty, 2020-2023.

I highlight many European and French physicists, particularly those I have had the opportunity to meet for the last three years in my journey in the quantum ecosystem. This inventory is both objective and subjective. Objective because it includes a broad and worldwide hall of fame in the field. Subjective because I have added a good dose of physicists I know. It creates a measurement bias which is easy to understand in social science as well as in quantum physics.

Starting in the 2023 edition, I have clustered these physicists in broad categories, the first ones being the generalists who worked in different fields. See also a simplified time of the recent period in Figure 58.

Generalists

Alain Aspect (1947, French, 2022 Nobel prize in physics) observed violations of Bell's inequalities with a series of experiments conducted between 1980 and 1982 at the Institut d'Optique (Orsay University in the southern suburb of Paris with Jean Dalibard, Philippe Grangier and Gérard Roger. Taking the principles of quantum physics for granted, it validated the nonlocality of quantum properties[82]. One other option is you need to reject these principles and use a local variable model to explain the phenomenon. But it is not the only one[83].

[82] Alain Aspect's experiments were using calcium atoms as source of photons, using some laser excitement and an atomic cascade generating pairs of entangled photons in the visible spectrum at 551 nm and 423 nm. There were actually several experiments: in 1981 with Philippe Grangier and Gérard Roger with one way polarizers, 1982 also with Grangier and Roger with two-channels polarizers and also 1982, with Jean Dalibard and Gérard Roger, using variable polarizers based on acousto-optical 10 ns switches. These could act faster than light propagation between the polarizers (40 ns) and even than the photons time of flight between the source and each switch (20 ns). See Experimental Test of Bell's Inequalities Using Time-Varying Analyzers by Alain Aspect, Gérard Roger and Jean Dalibard, PRL, December 1982 (4 pages).

[83] You have superdeterminism-based theories promoted by Carl H. Brans, Sabine Hossenfelder and Tim Palmer that are based on the hypothesis of superdeterministic hidden variables theory and could still violate Bell's inequalities, but also the CSM ontology which pertains that the Psi function is lacking information on the measurement context, like described in Why ψ is incomplete indeed: a simple illustration by Philippe Grangier, October 2022 (2 pages).

After two first experiments in 1981 and 1982 with one and two way polarizers, the third one avoided any potential synchronization between the polarizers, using a 50 MHz random optical switch on both sides, feeding two orthogonal polarizers and photon detectors, using different angles (Figure 59)[84] [85].

From 1988 to 2015, other experiments were conducted elsewhere and implemented loophole-free Bell tests, first closing individual loopholes and then, in 2015, closing them altogether. It confirmed then that there were no local variables explaining entanglement and validated one key nonlocality condition: having a long distance between the photon analyzers to avoid any interactions made possible by special relativity.

It avoided detection loopholes with high-efficiency photon detectors on top of escaping 'memory loopholes', which was already obtained by Alain Aspect et al in their seminal 1982 experiment[86]. A Loophole-free Bell inequality violation experiment was even done with superconducting circuits in 2023[87].

After his work on photon entanglement, Alain Aspect shifted gear on cold atoms control with lasers, starting with helium. This led to the creation of a promising field of quantum computing in France, using cold atoms, embodied by the startup **Pasqal**, whose scientific director is Antoine Browaeys, a former PhD student of Alain Aspect who also worked with Philippe Grangier. Along with other scientists, Alain Aspect is also a member of Eviden/Atos Scientific Council and in the scientific board of **Quandela**. He teaches quantum physics, notably in MOOCs created for Ecole Polytechnique and distributed by Coursera.

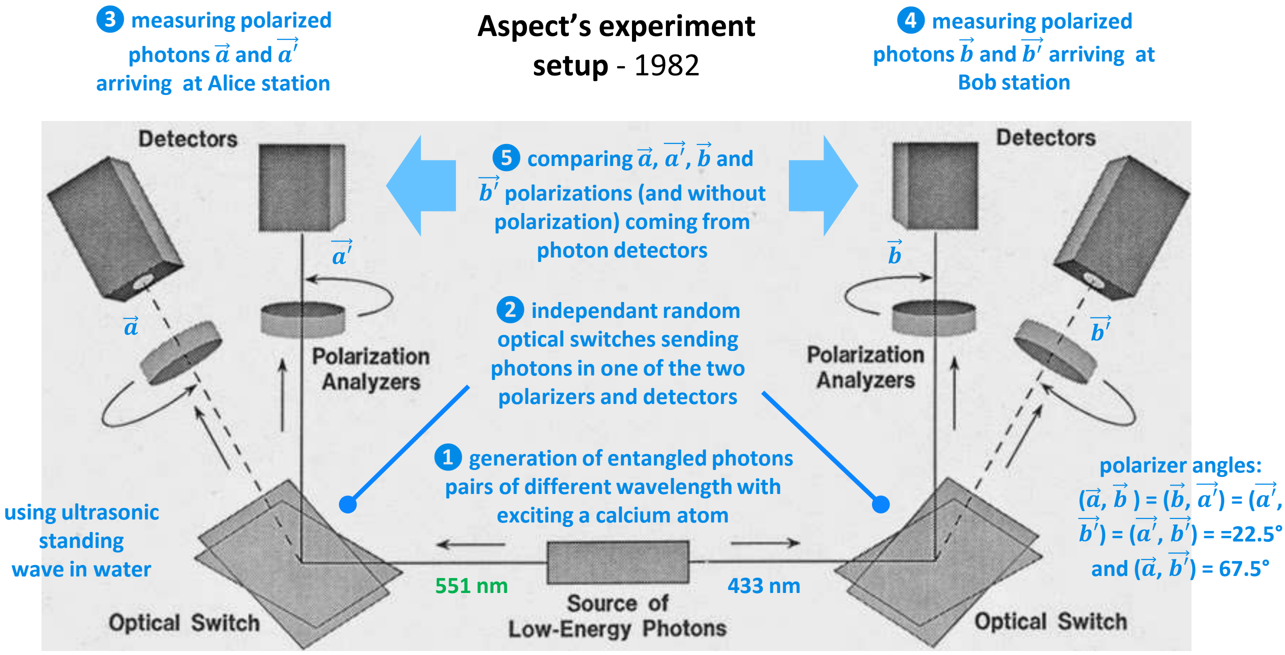

Figure 59: Alain Aspect et al 1982 Bell inequality test experiment setup. Comments, Olivier Ezratty, 2021-2023.

[84] See Experimental tests of Bell's inequalities: A first-hand account by Alain Aspect by William D. Phillips, December 2022 (14 pages) in which Alain Aspect is interviewed by two eminent physicists including another Nobel prize in physics awardee.

[85] See Alain Aspect's experiments on Bell's theorem: A turning point in the history of the research on the foundations of quantum mechanics by Olival Freire Junior, December 2022 (22 pages).

[86] See Experimental loophole-free violation of a Bell inequality using entangled electron spins separated by 1.3 km by B. Hensen et al, ICFO and ICREA in Spain and Oxford, UK, August 2015 (8 pages) and also A strong loophole-free test of local realism by Lynden K. Shalm et al, September 2016 (9 pages).

[87] See Loophole-free Bell inequality violation with superconducting circuits by Simon Storz, Paul Magnard, Jean-Daniel Bancal, Nicolas Sangouard, Alexandre Blais, Andreas Wallraff et al, Nature, May 2023 (8 pages).

Philippe Grangier (1957, French) was a PhD student of Alain Aspect with whom he worked on the 1982 experiment with Gérard Roger and Jean Dalibard. He is one of the world's leading specialists in quantum cryptography, especially on CV-QKD. He was involved in the creation of the associated startup, Sequrnet, in 2008 and closed in 2017, probably created a little too early in relation to the needs of the market. He is also invested in cold atoms control with lasers at IOGS (Institut d'Optique).

At last, he cocreated the CSM ontology of quantum foundations with Alexia Auffèves and Nayla Farouki, starting in 2013 and with a series of 7 foundational papers published between 2015 and 2019. CSM ontology is quickly covered in the Quantum Foundations section starting page 1238. He currently runs the European Flagship coordination project QUCATS.

Jean Dalibard (1958, French) is a research physicist at the ENS and teacher at the Polytechnique and the Collège de France. He is a specialist in quantum optics and interactions between photons and matter[88]. He participated with Philippe Grangier in the set-up of Alain Aspect's experiment in 1982 when he was a contingent scientist at the Institut d'Optique. He created the magneto-optical trap (MOT) system in 1987 that is used to cool neutral atoms using a mix of variable magnetic fields and lasers.

Anton Zeilinger (1945, Austrian, 2022 Nobel prize in physics) is a physicist who advanced the field of quantum teleportation in the 2000s. He also proved in 1991 the wave-particle duality of neutrons. He was also the first to demonstrate qubit teleportation in 2009. He is a specialist in quantum entanglement, having proved that it is possible to entangle more than two quantum objects or qubits. He created theoretical and experimental foundations for quantum cryptography.

With two colleagues, he also developed the GHZ (Greenberger-Horne-Zeilinger) entangled state, which enables yet another demonstration of the inexistence of hidden variables which would explain quantum entanglement of at least three particles and with a finite number of measurements.

The concept was created in 1989 and was validated experimentally in 1999. Anton Zeilinger also supervised the thesis of **Jian-Wei Pan**, who became later the quantum research czar in China with the development of many advances, particularly in quantum communications and photonics.

Frank Wilczek (1951, American) is a professor of physics at MIT and the chief scientist at the Wilczek Quantum Center in Shanghai. He was awarded in 2004 the Nobel Prize in Physics, shared with David Gross and H. David Politzer, for his work on the theory of strong interaction and quantum chromodynamics. He is known for his work on quasiparticles and anyons in 1982. He also predicted the existence of time crystals in 2012 (covered page 150).

John Preskill (1953, American) is a professor at Caltech. Among many other contributions, he is the creator of quantum supremacy notion in 2011 and of NISQ in 2018, the Noisy Intermediate-Scale Quantum, qualifying current and future noisy quantum computers. He is a regular speaker at conferences where he reviews the state of the art of quantum computing[89]. He's now involved with Amazon and their cat-qubits superconducting project revealed in December 2020.

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[88] See in particular his lesson on cold atoms at the Collège de France which describes well how atoms are cooled at very low temperatures with lasers.

[89] See his presentation that provides an overview of the state of the art of quantum computing Quantum Computing for Business, John Preskill, December 2017 (41 slides).

Jian-Wei Pan (1970, China) is the leading quantum physics scientist in China. He is a professor and Executive VP at USTC (University of Science and Technology of China) and a member of CAS (China Academy of Science). He did his PhD in Vienna under the supervision of Anton Zeilinger. He and his team are famous for premiere experiments on photons quantum entanglement in 2004, quantum key distribution over a satellite (2017), with boson sampling (2019) and superconducting qubits (2021).

Dieter Zeh (1932-2018, German) is the discoverer of the quantum decoherence phenomenon in 1970. It marks the progressive end of the phenomenon of superposition of quantum states, when particles are disturbed by their environment and their amplitude and phase is modified. The notion of decoherence is key in the design of quantum computers. The objective is to delay it as much as possible resulting from the interaction between quanta and their environment[90].

Wojciech Zurek (1951, Polish) is a quantum decoherence physicist who contributed to the foundations of quantum physics applied to quantum computers. We owe him the no-cloning theorem, which states that it is impossible to clone a qubit identically without the resulting qubits then being entangled. He is also at the origin of the concept of quantum Darwinism which would explain the link between the quantum world and the macrophysical world.

Maciej Lewenstein (1955, Polish) is a theoretical physicist, specialized in quantum optics of dielectric media and cavity quantum electrodynamics, teaching at ICFO in Spain. He worked with many leading worldwide scientists including Roy J. Glauber (Nobel in Physics in 2005) at Harvard, Thomas W. Mossberg, Andrzej Nowak, Bibb Latané, Anne L'Huillier (CEA, France), Peter Zoller and Eric Allin Cornell (Nobel in Physics in 2001 for his work on Bose-Einstein condensates in 1995), in the USA, France, Spain, Poland and Germany.

His contributions span an incredible number of fields like the physics of ultra-cold gases, quantum information, quantum optical systems, quantum communications, quantum cryptography, quantum computers, mathematical foundations of quantum physics, tensor networks and entanglement theory, laser-matter interactions atto-second physics, quantum optics (cQED), atoms cooling and trapping, non-classical states of light and matter and quantum physics foundations.

Cold atoms

Claude Cohen-Tannoudji (1933, French) is a former student of Ecole Normale Supérieure (ENS Paris) where he followed the teachings of mathematicians Henri Cartan and Laurent Schwartz and physicist Alfred Kastler. He was awarded the Nobel prize in Physics in 1997 at the same time as Steven Chu, who was later Secretary of Energy during Barack Obama's first term. This Department (DoE, Department of Energy) is one of the federal agencies most invested in quantum technologies, notably because they operate the largest supercomputers in the country.

Claude Cohen-Tannoudji owes his Nobel Prize to his work on atoms laser cooling which made it possible to reach extremely low temperatures, below the milli-Kelvin[91]. Alain Aspect once worked in his team.

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[90] Dieter Zeh is notably the author of On the Interpretation of Measurement in Quantum Theory in 1970 (8 pages).

[91] See his Nobel lecture.

Alain Aspect says that he discovered quantum physics with reading the reference book on quantum physics by Claude Cohen-Tannoudji, Bernard Diu and Franck Laloë published in 1973[92]. It totals over 2,300 pages. So, this book is quite small in comparison. And, maybe more accessible!

Serge Haroche (1944, French), Nobel prize in Physics in 2012, is a founder of Cavity Electrodynamics (CQED) which describes the interaction between photons and atoms in cavities. He used it to create cold atom based qubits. **Jean-Michel Raimond**[93] and **Michel Brune** were among his key collaborators. Serge Haroche was the first to measure the phenomenon of quantum decoherence (loss of superposition) in an experiment in 1996. This experiment was conducted at the ENS with rubidium atoms. Serge Haroche is also a member of Atos Scientific Council.

CQED was later applied in the field of superconducting qubits with Circuit Electrodynamics (cQED), where atoms are replaced by an artificial atom made with a Josephson junction and the cavity by a planar microwave resonator. Serge Haroche is one of the most circumspect scientists on the future of quantum computing, at least for universal gate computing. He believes more in the advent of quantum simulation[94].

Other scientists brought key contributions in atoms science. **Daniel Kleppner** (1932, American) was the first to create a Bose-Einstein condensate with Rubidium atoms in 1995, and then in 1998 with hydrogen. **Herbert Walther** (1935-2006, German) did pioneering work in cavity quantum electrodynamics and also with trapped ions. He created the Max Planck Institute of Quantum Optics in 1981. **Gerhard Rempe** (1956, German) developed cavity quantum electrodynamics with the control of neutral atoms using microwaves, in connection with **Jeff Kimble** (1949, American, Caltech).

Christophe Salomon (1953, French) is a physicist specialized in photonics and cold atoms, research director at the LKB (ENS Paris). He is particularly interested in quantum gases superfluidity (Bose-Einstein condensates) and in time measurement with cesium atomic clocks. He did a thesis in laser spectroscopy and then did a post-doc at the joint JILA laboratory between NIST and the University of Colorado. He is also a member of the Academy of Sciences since 2017.

Immanuel Bloch (1972, Germany) is the scientific director at the Max Planck Institute of Quantum Optics, Garching and professor for experimental physics at the Ludwig-Maximilians University (LMU) in Munich. He is a leading experimentalist in the cold atom domain, his work covering quantum gases in optical lattices, the first realization of a quantum phase transition from a weakly interacting superfluid to a strongly interacting Mott insulating state of matter based on based on a theoretical proposal by Peter Zoller and Ignacio Cirac.

[92] This book is published in three tomes that were last revised in 2019. The first one is Quantum Mechanics, Volume 1: Basic Concepts, Tools, and Applications. The second deals with Angular Momentum, Spin, and Approximation Methods and the third one with Fermions, Bosons, Photons, Correlations, and Entanglement. These are classical quantum physics student textbooks.

[93] See his interesting conference Quantum Computing or how to use the strangeness of the microscopic world, Jean-Michel Raimond, 2015 (1h36mn). See also his presentation material (56 slides).

[94] See Quantum Computing: Dream or Nightmare? by Serge Haroche and Jean-Michel Raimond, Physics Today, 1996 (2 pages) who expressed their skepticism about quantum computing. Serge Haroche continues to convey this skepticism.

Mikhail Lukin (USA) is a Russian born quantum physics professor at Harvard. He's a prolific scientist with a skyrocketing h-index of 179 (as of October 2023), working on quantum optics, quantum control of atomic and nanoscale solid-state systems, quantum sensing, nanophotonics and quantum information science. He is behind many feats in cold atoms physics with record two-qubit gate fidelities of 99.5% obtained in 2023 with 60 atoms as well as in the NV centers field, being the inventor of NV centers based magnetometry.

He cofounded QuEra (USA) that develops a cold atoms gate-based quantum computer, reaching 256 qubits as of 2021 (in analog mode). He is also a cofounder and scientific advisor of QDTI (USA).

Marie-Anne Bouchiat (1934, French) is a specialist in rubidium atoms physics and their control by optical pumping. This is the basis for the creation of quantum computers based on cold atoms. Her daughter **Hélène Bouchiat** (1958, French) is also a physicist, specialized in condensed matter at the LPS laboratory of the University Paris-Saclay and member of the Académie des Sciences since 2010, like her mother who has been there since 1988.

Antoine Browaeys (c. 1970, French) is a CNRS research director leading the quantum optics-atom team in the Charles Fabry Laboratory at Institut d'Optique specialized in the control of cold atoms. He is a pioneer in the control of individual cold atoms and on their use in quantum simulators and digital quantum computers. He is also a cofounder and the scientific director of Pasqal, a startup designing a cold atoms computer that will be first used as a quantum simulator, and then, as a universal gates quantum computer. He was awarded the CNRS silver medal in 2021.

Elisabeth Giacobino (1946, French) is a specialist in laser physics, nonlinear optics, quantum optics and superfluidity, particularly in relation to the control of cold atoms. She worked at the CNRS in the ENS LKB (Laboratoire Kastler-Brossel). She is a member of the scientific selection committee of the European Quantum Flagship and also for the ANR (Agence Nationale de la Recherche).

Hélène Perrin (c. 1975, French) is CNRS research director working at the Laboratoire de Physique des Lasers (LPL) from Université Sorbonne Paris Nord, working on Bose-Einstein condensates and cold atoms control. Together with Pascal Simon, she drives the Quantum Simulation SIM project, a cold atom-based quantum simulator. She also gives lessons on quantum computing. She did her PhD thesis with Christophe Salomon at the ENS LKB in Claude Cohen-Tannoudji's group. At CEA-Saclay, she also worked on fractional quantum Hall effect. Since 2022, she is the director of QuanTIP, the Paris region quantum ecosystem network.

Francesca Ferlaino (1977, Italian) is a typically European researcher, having worked in many laboratories from different countries. She is research director at the IQOQI in Innsbruck, Austria, where she leads the Dipolar Quantum Gases laboratory. She is a specialist in cold atoms and erbium-based Bose-Einstein condensates.

Trapped ions

Wolfgang Paul (1913-1993, Germany), not to be confused with Wolfgang Pauli, is a physicist who conceptualized trapped ions in the 1950s. He got the Nobel prize in physics in 1989. We owe him the traps that bear his name and are used to control trapped ions. He shared his Nobel prize with **Hans Georg Dehmelt** (1922-2017, Germany) who codeveloped these traps with him. The physicists **Juan Ignacio Cirac** (1965, Spanish) and **Peter Zoller** (1952, Austria) theorized, designed and tested the first trapped ion qubits in 1996, based on the work of Wolfgang Paul.

David Wineland (1944, American) is a Boulder-based NIST physicist known for his advances in trapped ions and their laser-based cooling in 1978. He also created in 1995 the first single quantum gate operating on a single atom. He was awarded the Nobel prize in Physics in 2012 jointly with Serge Haroche for his advances in atoms and ions laser cooling, a technique he first experimented in 1978, followed by the first quantum gate applied to a trapped ion in 1995 and the entanglement between four trapped ions in 2000.

Rainer Blatt (1952, Austrian and German) from the University of Innsbruck is an experimental physicist specialized, among other things, in trapped ions qubits. He was the first to entangle the quantum states of two trapped ions in 2004 and then with eight ions in 2006. He co-founded Alpine Quantum Technologies (AQT), whose ambition is to create and commercialize a trapped ions based quantum computer. He also works at TUM in Munich, Germany and is the coordinator of the Munich Quantum Valley since 2021.

Christopher Monroe (1965, American) is an American physicist known for his work on trapped ions and for co-founding IonQ in 2015, one of the two best funded quantum startups worldwide with PsiQuantum. He worked on trapped ions with David Wineland at the NIST Maryland laboratory. He demonstrated the ability to entrap ions, create ions-based quantum memory and create analog quantum simulators. He also ran a laboratory at the University of Michigan in the early 2000s.

Tracy Northup (c. 1975, Austria) is a researcher working on trapped ions and optical cavities, one of the major branches of quantum computing. She leads the Quantum Interfaces Group laboratory at the University of Innsbruck, which is one of the most active in the field of trapped ions, a major Austrian specialty.

Perola Milman (c. 1975, French) is a specialist in the theory of quantum computing and in particular with trapped photons and ions. In particular, she has demonstrated the entanglement capacity of molecules. She is a lecturer-researcher at the Laboratory of Quantum Materials and Phenomena of the University Paris Diderot. She is a professor of quantum theory of light and on quantum entanglement.

Superconductivity

Brian Josephson (1940, English) is a physicist from the University of Cambridge. He was awarded the Nobel prize in Physics in 1973 at the age of 33[95], for his prediction in 1962 of the effect that bears his name when he was only 22 years old and a PhD student at the University of Cambridge. The Josephson effect describes the passage of current in a superconducting circuit through a thin insulating barrier a few nanometers thick, using tunneling effect, and the associated threshold effects.

Below a certain voltage, the current starts to oscillate (Figure 60). It is generated by electrons with opposite spins organized in Cooper pairs named after Leon Cooper who discovered it in 1952. These pairs behave as bosons.

These electrons pairs have opposite spins (magnetic polarity). The system behaves as a resistance associated with a loop inductance, the oscillation being controllable by a magnetic field and having two distinct energy states. Superconductivity was discovered in 1911 by **Heike Kamerlingh Onnes** (1853-1926, the Netherlands). This is the basis of superconducting qubits and their quantum gates!

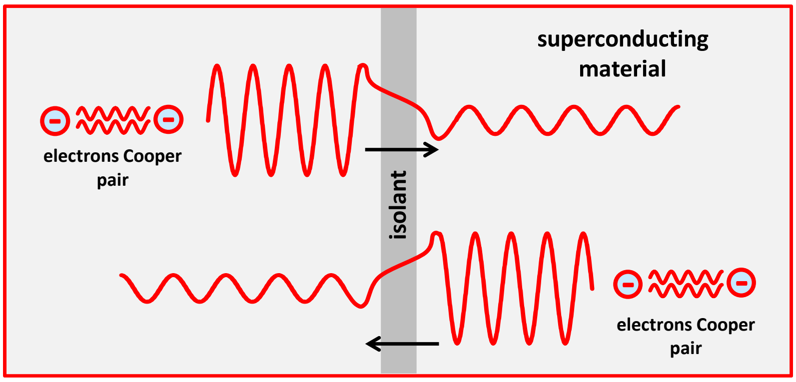

Figure 60: Josephson effect and Cooper pairs of opposite spin electrons.

Daniel Esteve (1954, French) is a physicist in charge of the CEA's Quantronics laboratory in Saclay, France, launched in 1984 with Michel Devoret and Cristian Urbina, and part of the IRAMIS laboratory. He contributed to the development of transmon superconducting qubits. He created a first operational qubit in 1997, the quantronium, followed by another controllable prototype in 2002, with Vincent Bouchiat. He continues to work on improving the quality of superconducting qubits.

Michel Devoret (1953, French) is a telecom engineer turned physicist, co-founder of the Quantronics laboratory with Daniel Esteve at the CEA in Saclay between 1985 and 1995, which is one of the world pioneers of superconducting qubits. He is a professor at Yale University since 2002. He was a co-founder of the American startup QCI with his Yale colleague Rob Schoelkopf (1964, USA), which he left in 2019/2020. He preferred to be entirely dedicated to research.

He worked several times with John Martinis, when John was a PhD student in UCSB, then when he was a post-doc at CEA in Saclay in the early 2000s, and at last at the University of California Santa Barbara (UCSB), where they wrote together a review paper in 2004 on superconducting qubits[96].

[95] Brian Josephson shared the 1973 Nobel prize in physics with two scientists who had worked before him in the same field: Leo Esaki (1925, Japan, still alive in early 2020) for his discovery of the tunnel effect in semiconductors in 1958 and Ivar Giaever (1929, Norway, also still alive) who found that this effect could occur in superconducting materials in 1960.

[96] In Implementing Qubits with Superconducting Integrated Circuits by Michel Devoret and John Martinis, 2004 (41 pages).

Steven Girvin (1950, USA) is a professor of physics at Yale University, specialized in condensed matter physics, and Director of the Co-design center for Quantum Advantage, at Brookhaven University since 2020. He is a key contributor to works on circuit quantum electrodynamics (cQED) and superconducting qubits. At Yale, he works with Robert Schoelkopf and Michel Devoret on the various engineering problems associated with superconducting qubits.

Rob Schoelkopf (1964, USA) a physicist and director of the Yale Quantum Institute. Along with Steve Girvin and Michel Devoret, he made key advances in superconducting qubits. He particularly worked on single-electron devices, being the inventor of the Radio-Frequency Single-Electron Transistor. He also created the field of circuit quantum electrodynamics (cQED) with Andreas Wallraff and Alexandre Blais who were respectively Yale post-doc and PhD student around 2002-2004.

In 2007, with Steven Girvin, he engineered a superconducting communication bus to store and transfer information between distant qubits on a chip. In 2009, their team, also including Alexandre Blais and Jay Gambetta, demonstrated the quantum processor running some quantum computation, with two qubits[97].

John Martinis (1958, American), is a physicist from UCSB who famously worked at Google between 2014 and 2020 where he led the hardware team in charge of superconducting qubits up to creating the Sycamore processor and its related "quantum supremacy experiment", published in Nature in October 2019. After his thesis at Berkeley on superconducting qubits, he did a post-doc in Daniel Esteve's Quantronics laboratory at the CEA in Saclay.

In September 2020, he started to work with Michelle Simmons at SQC in Australia. He also created Quantala in 2020, a quantum computing company selling IP and protecting his own patents.

Jay Gambetta (1979, USA) is the scientist leading as a VP since 2019 IBM's research team working on superconducting qubits quantum computers after running the IBM team that created and launched IBM Quantum Experience, Qiskit and the IBM Quantum System One in 2019. He joined IBM in 2011. After a thesis in quantum foundations and non-Markovian open quantum systems done in Australia in 2004, he focused on developing superconducting qubits, first in a post-doc tenure at Yale University and then at the Institute for Quantum Computing in Waterloo. He also worked on quantum validation techniques, quantum codes and applications.

Andreas Wallraff (German) is a Professor for Solid State Physics at ETH Zurich after having obtained degrees in physics from the London Imperial College and RWTH Aachen in Germany and worked at the Jülich Research Center also in Germany, Yale University in the USA and the LKB in France. He is specialized in the coherent interaction of single photons with quantum electronic circuits and quantum effects as well as on hybrid quantum systems combining microwave control, superconducting circuits and semiconductor quantum dots.

[97] See Demonstration of Two-Qubit Algorithms with a Superconducting Quantum Processor by L. DiCarlo, Rob Schoelkopf et al, 2009 (9 pages).

Alexandre Blais (Canada) is a Professor in the Department of Physics and Director of the Université de Sherbrooke's Institut Quantique. He is one of the key contributors to the development of circuit quantum electrodynamics (cQED) that enable the creation of superconducting qubits. He is also a cofounder of Nord Quantique, a Quebec startup developing bosonic code qubits. Like Jay Gambetta, he did a post-doc at Yale, the US epicenter of the early developments of superconducting qubits.

Irfan Siddiqi (1976, American-Pakistani) is one key contributor to advancements in superconducting qubits. He did his PhD and post-doc at Yale, working initially in aluminum hot-electron bolometers for microwave astronomy and then, high frequency measurement techniques for superconducting qubits. He developed the Josephson Bifurcation Amplifier that uses the non-dissipative and nonlinear nature of the Josephson junction to create high gain and minimal back action readout of qubits.

This led to the creation of superconducting parametric amplifiers and Josephson traveling wave parametric amplifiers. He then moved at Berkeley University and the DoE Lawrence Berkeley National Laboratory. He works on quantum electrodynamics, quantum error correction, multi-partite entanglement generation and single photon detection. He runs there the Advanced Quantum Testbed, an integrated research platform on superconducting qubits and enabling technologies.

Patrice Bertet (c. 1976, France) is part of Daniel Esteve's team at CEA-SPEC. He did his thesis at Serge Haroche on Rydberg atoms and then went to Delft University. He participated in the early days of superconducting qubits (quantronium at CEA and TU Delft). He then worked on QED (quantum electrodynamics) circuits based on cavities and then on transmon qubits. He is working on the association of superconducting qubits and the measurement of their state with electron spins, notably based on NV centers, which can also be used for quantum memories.

Audrey Bienfait (c. 1990, France) is a former PhD student of Patrice Bertet at CEA-SPEC who is now doing her research at ENS Lyon in the team of **Benjamin Huard** (1979, French). She was awarded the Bruker Prize 2018 for her thesis on electron paramagnetic resonance or "ESR - Electron Spin Resonance" in quantum regime and the Michelson Postdoctoral Prize 2019 in March 2020 for her work on the entanglement of superconducting qubits via phonons.

Shi Yaoyun (1976, Chinese) is a professor at the University of Michigan and also leading the Alibaba Quantum Laboratory which develops fluxonium superconducting qubit computers. He created various records of quantum simulation on server clusters that we will describe in this book. He earned a computer science PhD from Stanford. He also worked on quantum cryptography and certifiable randomness.

Spin qubits

Daniel Loss (1958, Swiss) proposed in 1998 with David DiVincenzo to use electron spins in quantum dots to create a quantum computer. He currently is the Co-Director and founding member of National Center on Spin Qubits (NCCR SPIN) that gathers the University of Basel, EPFL and IBM Zurich, an initiative from the Swiss Nanoscale Center SNI. He is the Director of the Center for Quantum Computing at the University of Basel. After a PhD in theoretical physics at the University of Zurich in 1985 he was a post-doc in the group of Anthony J. Leggett in the USA and at IBM Research. After a stint in Vancouver, he went back to Switzerland.

He works on condensed matter physics and spin-dependent and phase-coherent phenomena in semiconducting nanostructures and molecular magnets with applications in quantum computing.

Bruce Kane (c. 1958, American) is a researcher at the Joint Quantum Institute from the University of Maryland (a JV with NIST). While he was doing research at UNSW, he presented in 1998 the “donors spin” model, a spin-based qubit concept based on using individual phosphorous atoms in pure silicon lattice structures. This is the principle on which Michelle Simmons works at both UNSW and her startup SQC.

The jury’s still out to demonstrate that this technology can scale among the various spin qubits proposals.

Menno Veldhorst (1984, Dutch) is a group leader at QuTech. He got his PhD in 2012 on superconducting and topological hybrids at the University of Twente. He then worked on silicon quantum dots at UNSW where he demonstrated in 2015 the first two qubit operations in silicon. At QuTech, he works on silicon and silicon/germanium (SiGe) qubits to build scalable quantum computers. His team is currently pioneering work on SiGe/Ge qubits with qubits manipulation in arrays up to 16 quantum dots. He proposed a crossbar array architecture to create logical qubits.

Leo Kouwenhoven (1963, Dutch) is a quantum physicist who got his PhD at TU Delft in 1992 and became a professor there in 1999. He led experimental results on the potential "signatures" of Majorana fermion quasiparticles in 2012 and later on their “definitive” existence in 2018. The related Nature paper had to be retracted in 2021 due to experimental data mismanagement and reporting. From 2016 till 2022, he was a researcher at Microsoft Research. He left Microsoft in 2022 and has returned to his home based at QuTech and the Kavli Institute of Nanoscience from TU Delft.

Lieven Vandersypen (1972, Belgian) started as a mechanical engineer and a PhD at Stanford, then went to IBM in Almaden, California, where he became interested in MEMS. He demonstrated the use of Shor's algorithm for factoring the number 15 with NMR qubits, and then became a researcher at TU Delft University in the Netherlands and in its QuTech spin-off, which he currently runs. He is a pioneer of electron spin qubits. In this capacity, he works notably with Intel, and is testing their FinFET-based qubit chips at QuTech with Intel, which invested $50M in QuTech in 2015.

Michelle Simmons (1967, British-Australian) is a physicist from the University of New Wales in Australia (UNSW), working on silicon spin qubits. She is the director of CQC2T (Centre of Excellence for Quantum Computation and Communication Technology) from UNSW. She is also the co-founder of SQC (Silicon Quantum Computing), the leading quantum computing Australian startup ($66M), a spin-off from her university and from QQC2T.

In 2019, her team built the first two-qubit gate between phosphorous atom qubits in silicon, operating in only 0.8 ns. It became a full-fledged 10 qubit processor in 2022. She is using STM (scanning tunneling microscopes) to position phosphorus dopants in the silicon substrate.

Andrew S. Dzurak (Australian) is the Director of the Nanotechnology Fabrication Unit at UNSW's Australian National Fabrication Facility from the CQC2T research center. This facility’s white room is used to manufacture silicon qubits chips. Andrew Dzurak is a pioneer of silicon qubits since 1998. He is leading research at CQC2T on silicon qubit control and reading. He created the first phosphorus-based silicon double qubits in 2015. He was a lead scientist for SQC, founded by Michelle Simmons, but seemingly left the company in 2021.

He created Diraq in 2022, a startup dedicated to the creation of scalable quantum computers using quantum dot silicon spin qubits.

Andrea Morello (1972, Italian) is one of the star researchers at UNSW in Australia. He is Program Manager of the ARC Centre of Excellence at CQC2T and leads the Fundamental Quantum Technologies Laboratory at UNSW. During his studies, he attended the Laboratoire National des Champs Magnétiques Intenses of the CNRS in Grenoble. Today he is one of the specialists in silicon-based qubits. He is also a quantum engineering teacher at UNSW.

His team was the first to demonstrate coherent control and readout of an individual phosphorus atom electron and nuclear spin in silicon and held for many years the record for the longest quantum memory time of 35.6 s in a single solid-state qubit.

James Clarke (c. 1971, American) launched Intel's quantum computing research efforts and the Director of Quantum Hardware at Intel since 2015. He's also behind Intel's partnership with QuTech in The Netherlands. He is currently focused with his team of about 100 researchers and engineer on creating scalable quantum computers with silicon and SiGe qubits. He started working at Intel as a process engineer in 2001 after having studied and worked on organic chemistry (PhD in Harvard and post-doc at ETH Zurich).

Maud Vinet (1975, French) started as physics engineer and was granted a PhD in physics from Grenoble University. She then spent 20 years working in silicon technologies development and transfer for the semiconducting industry. She led the silicon qubit project at CEA-Leti in Grenoble. Since 2016, CEA-Leti was focused on silicon spin qubits leveraging the strong relationships between fundamental science and technology in Grenoble ecosystem. In November 2022, Maud Vinet launched Quobly (formerly Siquance) with Tristan Meunier (CNRS) and François Perruchot (CEA-Leti).

Maud was driving QLSI, the European Quantum Flagship research project on silicon spins qubits, awarded in March 2020, after obtaining with **Tristan Meunier** (1977, French, at CNRS Institut Néel) and **Silvano de Franceschi** (1970, Italian, at CEA IRIG) an ERC Synergy grant funding of 14M€ in 2018 for the QuCube silicon qubit project.

Before her journey in quantum computing, she had previously contributed to the industrialization of the FD-SOI technology with CEA and STMicroelectronics[98], GlobalFoundries and IBM.

Anne Matsuura (c. 1970, Japanese-American) is a physicist who is leading the Quantum & Molecular Technologies team from the Intel Quantum Research Laboratory since 2014. She leads the American's efforts in the creation of superconducting and silicon qubits quantum computers, with an overall vision of the hardware architecture. Her impressive career starts with a thesis at Stanford in synchrotrons, then in US Air Force labs and In-Q-Tel (the CIA investment fund). She also directed the European Theoretical Spectroscopy Facility in Belgium.

[98] FD-SOI = Fully-Depleted Silicon on Insulator. The technology uses on the one hand a layer of silicon oxide insulator and on the other hand, channels of undoped silicon between the drain and the source, limiting leakage between the latter two.

NV centers

Jean-François Roch (1964, French) is a quantum physics professor at ENS Paris Saclay. He is a pioneer of the usage of NV centers in many applications, particularly in quantum sensing, including for studying matter and magnetism at very high-pressure, which could be helpful for the discovery of high-temperature superconducting materials. He conducts these researches in partnership with Thales and with the CEA. He also led the founding Wheeler delayed choice experiment in 2006.

Ronald Walsworth (c. 1972, American) is a pioneer in the usage of NV centers for quantum sensing in various fields, from life science to physics and astrophysics like for the detection of dark matter. He leads the Walsworth group at the University of Maryland and is the founding director of the UMD Quantum Technology Center. Several startups emerged from his lab like qdm.io, Hyperfine.io (MRI) and QDTI (which he both cofounded).

He also launched the Quantum Catalyzer quantum startups accelerator (Q-CAT) that creates quantum startups from scratch. He got a PhD in physics from Harvard in 1991.

Photonics

Jürgen Mlynek (1951, German) is a physicist specialized in optronics and interferometry. He was the coordinator of the strategic advisory board behind the launch of the European Flagship project on quantum in 2018. We owe him, as mentioned in connection with Louis De Broglie, the experiment validating the wave-particle duality of atoms carried out using helium in 1990 with Olivier Carnal at the University of Konstanz.

Jean-Michel Gérard (1962, French) is a physicist from the CEA IRIG laboratory in Grenoble and director of the joint PHELIQS laboratory (PHotonics, ELectronics and Quantum Engineering) from UGA (University of Grenoble) and CEA. He works in particular on the creation of single photon sources based on quantum dots as well as single photon detectors based on superconducting nanowires and OPO laser diodes.

Andrew G. White (c. 1970, Australian) is a leading Australian quantum scientist who is the Director of the University of Queensland Quantum Technology Laboratory. He is most known for his work in quantum photonics, including a first demonstration of an optical CNOT entangling gate realized in 2004 and based on the Knill, Laflamme and Milburn (KLM) protocol and linear optics. He is also very eclectic, having also worked on nuclear physics and marine biology. He's a scientific advisor for Quandela.

Pascale Senellart (1972, French) is a physicist, CNRS research director at the C2N laboratory. She designed and invented a process for manufacturing sources of unique and indistinguishable photons used in quantum telecommunications and computing. These are GaAsAl semiconductor quantum dot trapped in a multi-layered 3D structure, powered by a laser and directly feeding an optical fiber. She co-founded the startup Quandela in 2017 with **Valérian Giesz** (CEO) and **Niccolo Somaschi** (CTO and Chairman) who were a PhD student and a post-doc in her team.

Quandela is selling these photon sources and is creating photon qubit-based quantum computers. She is their scientific advisor. Pascale Senellart also launched the Quantum hub of the University Paris-Saclay in November 2019, which brings together public and private research laboratories as well as higher education institutions. She was awarded the CNRS Silver Medal in 2014.

Christine Silberhorn (1974, German) is a researcher and professor working on photon-based quantum computing at the University of Paderborn located between Dortmund and Hanover. She leads there the Integrated Quantum Optics group. Her laboratory designs and manufactures integrated optronics components, entangled photon sources and quantum array systems. Her team designed a system to convert photon qubits between infrared and visible wavelengths. She also works on optical quantum memories. She was awarded the Leibnitz prize in 2011.

She cofounded It'sQ in 2022, a quantum photonic computing startup and is one of the very few lead researchers in Germany who created a quantum computing hardware company. It turned into a different venture named QCDESIGN focusing on qubit control and error correction architecture and she left the company.

Fabio Sciarrino (1978, French Italian) is the director of the Quantum Information Lab at the Sapienza University of Rome and specialized in photonics. His team is at the origin of many advances in the field, notably in boson sampling, a key experiment in the path of photon-based quantum computers. He collaborates with Quandela's team and the C2N of Palaiseau (Pascale Senellart).

Jacquiline Romero (c. 1985, Philippines) is a quantum optics physicist doing research in Australia at the University of Queensland, after completing her PhD in Glasgow, UK. She is working on optical neuromorphic architectures and on dense encoding of information in photons using several of their characteristics in addition to the usual polarization.

Jelena Vucokic (c. 1975, Serbian) is a research professor at Stanford, working in quantum photonics. She directs the Nanoscale and Quantum Photonics Lab and the Q-FARM (Quantum Fundamentals, ARchitecture and Machines initiative), an interdisciplinary quantum laboratory. She contributes to developments in photonics for the development of optical quantum computers. She did her PhD at Caltech in 2002.

Stefanie Barz (c. 1980, German) is a quantum optics professor and researcher at the University of Stuttgart. Her interests include quantum cryptography and quantum telecommunications. She worked in particular on blind computing with Elham Kashefi and Anne Broadbent. She leads the SiSiQ project funded by the German Ministry of Research with €3.6M of European funding, which aims to create quantum communication infrastructure with silicon photonics.

Sophia Economou (c. 1980, Greek-American) is an Associate Professor in the Department of Physics at Virginia Tech College of Science. She previously worked at the US Naval Research Laboratory. She is a physicist specialized in the control of quantum dot semiconductor spins and their spin-photon interfaces. She is also a creator of advanced molecular simulation algorithms on quantum computers.

Quantum communications and cryptography

Nicolas Gisin (1952, Switzerland) is a physicist specialized in quantum communication. He demonstrated quantum nonlocality with an experiment in 1997 over a 10 km distance, extending the performance achieved in the laboratory by Alain Aspect in 1982. He co-founded IDQ in 2001, a Swiss startup initially specialized in quantum random number generators using photons passing through a dichroic mirror. It was acquired by SK Telecom in 2018.

Artur Ekert (1961, Polish and English) is a quantum physicist known to be one of the founders of quantum cryptography. He had met Alain Aspect in 1992 to talk to him about this inspiration after discovering the latter's experiments. This is a fine example of step-by-step inventions, one researcher inspiring another! He was the director of the Singapore Center for Quantum Technology from 2007 to 2020. He is also a teacher at Oxford University and a member of Atos's Scientific Council.

Stephanie Wehner (1977, German) is a physicist working on quantum communication protocols, based at the University of Delft in the Netherlands. She coordinates the "Quantum Internet Alliance", one of the projects of the European Quantum Flagship, which plans to deploy a quantum key distribution (QKD) Internet network running in mesh mode. She started her professional life in cybersecurity, detecting system flaws. She is also producing many quantum tech MOOCs.

Eleni Diamanti (1977, Franco-Greek) is a leading specialist and experimenter in the development of photonic resources for quantum cryptography, also working on quantum communication complexity. She's a CNRS research Director and faculty at LIP6 laboratory from Paris-Sorbonne University. She is the vice-director of the Paris Centre for Quantum Technologies (PCQT) since April 2020. She is also involved in many European projects around quantum key distribution, like the Quantum Internet Alliance and OpenQKD.

She is a recipient of a European Research Council Starting Grant. At last, she's a cofounder and a scientific advisor with Julien Laurat for the startup Welinq, created in 2022 with Tom Darras as CEO, which creates cold atom based quantum memories for quantum computer interconnects and quantum repeaters.

Sara Ducci (1971, French) is another teacher-researcher at the same Laboratoire Matériaux et Phénomènes Quantiques (MPQ) where she co-founded in 2002 a team in charge of nonlinear optical devices. She is working on producing pairs of entangled photons sources based on III-V semiconductors. She is also interested in the characterization (state measurement...) and manipulation of photons. At last, she teaches quantum physics at Ecole Polytechnique.

Sébastien Tanzilli (France) is the director of the InPhyNi physics laboratory in Nice and also the CNRS national quantum program director. He works on quantum cryptography with continuous or discrete keys (CV-QKD and DV-QKD), in fundamental quantum optics as well as in hybrid quantum systems for the study and realization of quantum communication networks. He was also the president of the GDR-IQFA, a community of quantum physics researchers in France (IQFA = Information Quantique, Fondements & Applications) from its creation in 2011 until 2021.

Virginia D'Auria (Italy) is a researcher working on quantum optics transmission systems using continuous and discrete variables and DV/QV hybridization. Having worked at the ENS LKB in Paris, she also worked on photon detectors. Since 2010, she is part of the photonics group of InPhyNi and works on discrete and continuous variable quantum communications compatible with optical fibers of telecom operators.

Frédéric Grosshans (1976, French) is a CNRS researcher at LIP6 from Université Paris-Sorbonne, specialized in QKD, repeaters and quantum networks. He was the creator with Philippe Grangier of the continuous variable QKD. He is also the co-director with Nicolas Treps (from LKB) of the Quantum Information Center Sorbonne of the Alliance Paris-Sorbonne launched in September 2020, which federates quantum research and training of several Parisian quantum groups.

Some other key names here are **Samuel L. Braunstein** and **Stefano Pirandola** (University of New York), **Renato Renner** (ETH Zurich) and **Dirk Englund** (MIT).

Other domains

Jacqueline Bloch (1967, French) is a research director at CNRS (PI) in the Centre de Nanosciences et de Nanotechnologies (C2N) lab from CNRS and Université Paris-Saclay, working on polaritons, quasiparticles coupling light and semiconductor matter, mainly built in gallium arsenide (GaAs). These have potential applications in the creation of quantum simulators based on polariton arrays as well as for quantum metrology.

Marcus Huber (Austria) is a research group leader at the IQOQI in Vienna, working on quantum entanglement, qubit state measurement and quantum thermodynamics in general. In addition to the IQOQI, he has also worked at the Universities of Bristol, Geneva and Barcelona. He is a great advocate of the open publication of research work, being at the origin of the Quantum-Journal.org website, a kind of arXiv for quantum science.

Alexia Auffèves (1976, French) is a CNRS research director and the director of Singapore's CNRS MajuLab international laboratory since January 2023 after having conducted her research for over 15 years in Grenoble at CNRS Institut Néel. She started as an experimentalist, doing he PhD thesis creating "Schrodinger's cats" with cold atoms at ENS LKB in Paris with Serge Haroche. She then became a theoretician focused on quantum energetics, with a strong interest in photonics and spins. She created the CSM ontology with Philippe Grangier and the philosopher Nayla Farouki[99].

She also launched and coordinated QuEnG (Quantum Engineering Grenoble), the Grenoble quantum ecosystem, which became the QuantAlps federation in January 2022. She cofounded the Quantum Energy Initiative in August 2022 with **Robert Whitney** (a physicist from CNRS LPMMC in Grenoble), **Janine Splettstoesser** (Chalmers University, Sweden) and Olivier Ezratty. She also runs with Robert Whitney the Grenoble/Singapore Quantum Energy Team ($|QET\rangle$).

[99] See Contexts, Systems and Modalities: a new ontology for quantum mechanics by Alexia Auffèves and Philippe Grangier, 2015 (9 pages). See also the associated Wikipedia page. This work has been articulated on a total of seven papers released between 2015 and 2019.

Jason Alicea (American) is a Professor of Theoretical Physics at Caltech University's IQIM (Institute for Quantum Information and Matter). He is specialized in condensed matter physics and topological phase of matter which could lead on creating non-Abelian anyons and Majorana fermions, a qubit type mainly explored by Microsoft.

Alexei Grinbaum (1978, Franco-Russian) is a researcher at CEA-Saclay in Etienne Klein's LARSIM laboratory. He works on the quantum foundations and quantum physics philosophy[100]. He is notably the author of the book "Les robots et le mal" (Robots and evil) published in 2018. He is particularly interested in the ethics of science, its acceptance by society and responsible innovation.

Quantum information science and algorithms creators

Let's continue this long "hall of fame" with some of the main contributors to the creation of quantum information science and algorithms. It is a relatively new discipline that emerged in the early 1980s.

Theory

Paul Benioff (1930-2022, American) proposed in 1979/1980 the concept of a reversible and non-dissipative quantum Turing machine using 2D lattices of spins ½, based on earlier work from Rolf Landauer on the thermodynamics of computing and Charles Bennett on reversible computing [101]. It was a semi-classical machine concept that didn't yet exploit entanglement and interferences. His work was extended by the "universal quantum computer" concept from David Deutsch in 1985.

Yuri Manin (1937-2023, Russian and German) is a mathematician who proposed the idea of creating gate-based quantum computers, in his 1980 book "Computable and Uncomputable", then in the USSR.

Then, **Richard Feynman** devised in 1981 the idea of a quantum simulator. Feynman and Benioff were participants of the famous "Physics & Computation" conference in 1981 that was co-organized by IBM and the MIT at the MIT Endicott House[102].

It brought together several well-known scientists in quantum information technology such as Tommaso Toffoli and Edward Fredkin (Figure 61).

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[100] See Narratives of Quantum Theory in the Age of Quantum Technologies by Alexei Grinbaum, 2019 (20 pages).

[101] See The computer as a physical system: A microscopic quantum mechanical Hamiltonian model of computers as represented by Turing machines by Paul Benioff, Journal of Statistical Physics, June 1979, published in May 1980 (30 pages). Paul Benioff was then in a visiting stay at the Centre de Recherche Théorique from CNRS in Marseille, France while being affiliated with the DoE Argonne National Laboratory in the USA. The paper was followed by Quantum Mechanical Hamiltonian Models of Turing Machines by Paul Benioff, October 1981 and June 1982, also in the Journal of Statistical Physics (32 pages). This theoretical system was based on using a two-dimensional lattice of spin ½ systems (today, it would be electron spins based qubits). Back in the 1980s, the very notion of qubits was not yet in the radar. It appeared much later, in 1995. In Benioff's model, a quantum gate was a Hamiltonian transformation of individual spins that was driven by the Turing quantum machine.

[102] See How a 1981 conference kickstarted today's quantum computing era by Harry McCracken, FastCompany, May 2021.

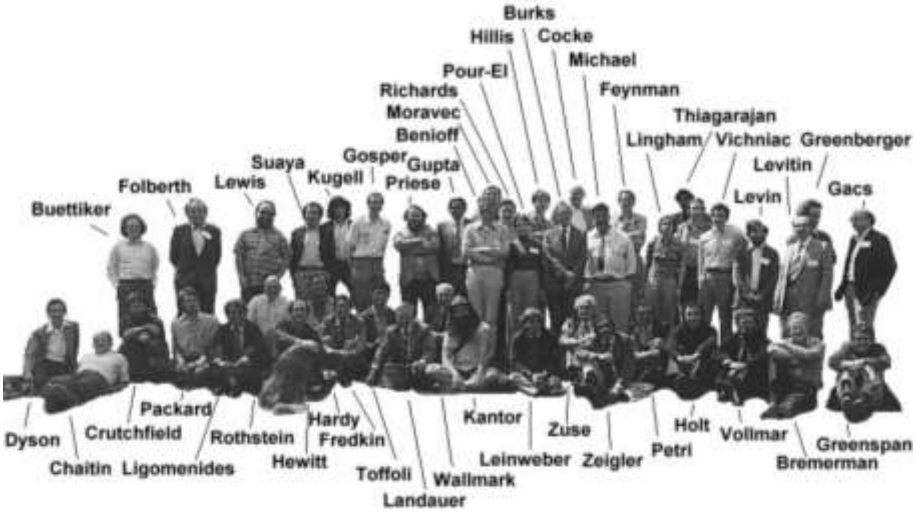

Figure 61: participants of the first quantum computing conference in 1981. Source: [Simulating Physics with Computers](#) by Pinchas Birnbaum and Eran Tromer (28 slides).

Rolf Landauer was also among them. It was for this conference that Richard Feynman published his famous paper "Simulating Physics with Computers" which created the concept of quantum simulation[103]. Figure 62 shows a timeline how the concepts of quantum computing came out.

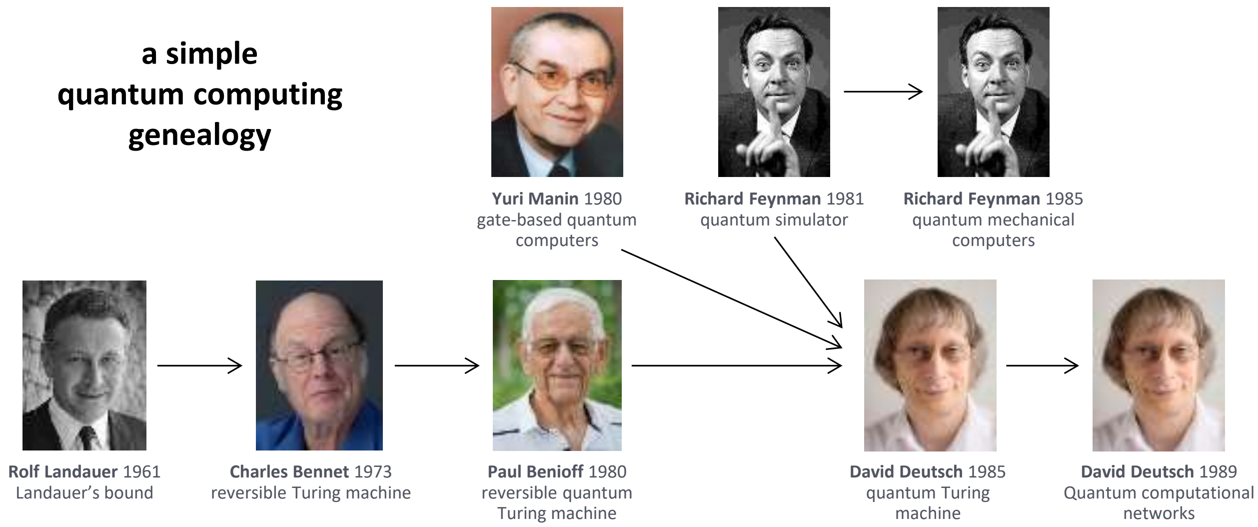

Figure 62: quantum computing genealogy to remind us that other scientists than Richard Feynman have to be remembered for their contribution. (cc) compilation Olivier Ezratty, 2022.

Alexander Holevo (1943, Russian) is a mathematician working in quantum information science and who devised the 1973 Holevo theorem according to which we cannot retrieve more than N bits of useful information from a register of N qubits[104]. This is the consequence of the wave packet reduction that reduces the qubit state to its basis states $|0\rangle$ and $|1\rangle$ after measurement. He also developed the mathematical basis of quantum communications.

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[103] See [Simulating Physics with Computers](#) by Richard Feynman, 1981 (103 pages).

[104] This theorem indirectly validates the fact that it is difficult to do "big data" with a quantum computer in the sense of storing and analyzing large volumes of information. On the other hand, Grover's algorithm makes it possible to quickly find a needle in a haystack, as we will see later.

David DiVincenzo (1959, American) was a researcher at IBM and the creator of the criteria that define the minimum requirements for a quantum computer with universal gates. He is now a researcher and professor at the University of Aachen in Germany. He is a member of the Atos Scientific Council, along with Alain Aspect, Serge Haroche, Artur Ekert and Daniel Esteve, among others.

Gil Kalai (1955, Israeli) is a professor of mathematics at the Hebrew University of Jerusalem and at Yale University. His main ambition is to demonstrate mathematically that it will be impossible to create real universal quantum computers, due to their error rate, even with error correction codes and the notion of logical qubits that assemble physical qubits. He also questioned the reality of the October 2019 Google supremacy performance in several of his writings and conference talks.

Robert Raussendorf (c. 1975, German) is well known for having invented one-way quantum computing and measurement-based quantum computing (MBQC) along with **Hans Briegel** (1962, German) in the early 2000's. He is an Associate Professor at the Department of Physics and Astronomy of the University of British Columbia. He did his thesis at the Ludwig Maximilians University in Munich, Germany in 2003 on MBQC.

Elham Kashefi (1973, British Iranian) is a research director at CNRS in France, in the LIP6 laboratory from Sorbonne University. She is also the co-founder with Marc Kaplan of VeriQloud, a secure quantum telecommunications startup, and teaching quantum information science at the University of Edinburgh. Originally a mathematician and computer scientist, she became a specialist in quantum communication protocols and quantum algorithms, around topics like code verification and blind quantum computing.

She created the BFK blind computing protocol in 2009 with Anne Broadbent and Joe Fitzsimons (who created Horizon Quantum Computing in Singapore). With her team at LIP6, she is at the origin of the creation of a site on the zoo of quantum communication protocols[105]. And as this was not enough, she is also versed in Quantum Physical Unclonable Functions (QPUF), physical identifiers of quantum and tiltable objects, a topic we briefly cover in this book in page 775. In November 2022, Elham Kashefi was appointed as Chief Scientist for NQCC, the UK National Quantum Computing Center, and will chair its Technical Advisory Group.

Anne Broadbent (Canadian) is a mathematician from the University of Ottawa specialized in quantum computing, quantum cryptography and quantum information. She was a student of Alain Tapp and Gilles Brassard at the Université de Montréal. She created the BFK blind computing protocol in 2009 along with Elham Kashefi and Joe Fitzsimons.

John Watrous (Canadian) is a researcher working at the University of Waterloo, Canada, specialized in quantum algorithms and complexity theory. He demonstrated some complexity classes equivalencies like QIP is in EXP and QIP=PSPACE. He also worked on cellular automata. He had previously collaborated with Scott Aaronson. He is the author of the voluminous The Theory of Quantum Information, 2018 (598 pages).

105 See the Protocol Library wiki.

Algorithms

Tommaso Toffoli (1943, Italian then American) is an engineer known for the creation, at the beginning of the 1980s, of the quantum gate bearing his name, a conditional gate with three inputs that is widely used in quantum programming. After working at MIT, he became a Boston University professor, where he has served since 1995. Like Stephen Wolfram, his interests include cellular automata and artificial life.

Edward Fredkin (1934, American) is a professor at Carnegie Mellon University. He is the author of the two-way conditional swap quantum gate (SWAP). He is also the designer of the concept of reversible classical computer with Tommaso Toffoli at MIT. He is also a prolific inventor far beyond quantum computing and is the originator of vehicle identification transponders and automotive geonavigation.

He is also a promoter of the notion of "digital philosophy" which reduces the world and its functioning to a giant quantum program, a theory he shares with Seth Lloyd, an idea that has been revived by Elon Musk who believes that the Universe is a gigantic program and that we live in a simulation. Is the "automatic" respect of elementary physical laws a "program"? A thorny philosophical and semantic question!

David Deutsch (1953, Israeli and English) is a physicist from the Quantum Computing Laboratory at Oxford University in the UK. He devised in 1985 the idea of creating a universal quantum computer using a quantum Turing machine which led him to create in 1989 the gate-based circuits programming model, completing Yuri Manin's and Paul Benioff's 1980 ideas[106]. He is also the author of a search algorithm, with two variants, a first one from 1985 and a second one in 1992 that he co-created with Richard Jozsa.

Umesh Vazirani (1945, Indian-American) is a professor at the University of Berkeley. He is one of the founders of quantum computing, with his paper co-authored in 1993 with his student Ethan Bernstein, Quantum Complexity Theory. He is also the creator of the Quantum Fourier Transform (QFT) algorithm, which was used less than a year later by Peter Shor to create his famous integer factoring algorithm that served as a spur to funding research in quantum computing in the USA. The QFT is a founding algorithm used in many other quantum algorithms.

Edward Farhi (1952, American) is a theoretical physicist who has worked in many fields, including high-energy particle physics, particularly at the CERN LHC in Geneva and then at MIT. He worked with Leonard Susskind on unified theories with electro-weak dynamical symmetry breaking. He and Larry Abbott proposed a model in which quarks, leptons, and massive gauge bosons are composite. He is the creator of adiabatic quantum algorithms and quantum walks. He also introduced with Peter Shor the concept of quantum money in 2010.

[106] See Quantum theory, the Church-Turing principle and the universal quantum computer by David Deutsch, 1985 (21 pages). This is a foundational paper describing a lot of concepts, including the unitaries used in single qubit gates, the notion of quantum computing complexity, etc. It was also followed by Quantum computational networks by David Deutsch, September 1989 where networks correspond to series of gate operations. Back then, the very name of qubit didn't exist yet, and was created only in 1995.

Peter Shor (1959, American) is a mathematician who became the father of the algorithm of the same name in 1994 which allows the factorization of integers into prime numbers, based on quantum Fourier transforms (QFT). Before that, he created the first quantum discrete-log algorithm (dlog) and, later, the famous nine-qubit flip error and phase error correction algorithm for quantum computers called the "Shor code"[107]. We indirectly owe to him the whole movement of post-quantum cryptography (PQC).

PQC is about creating cryptography codes resisting public keys breaking using the Shor algorithm and other quantum algorithms... with quantum computers that do not yet exist. Peter Shor created his famous factorization algorithm while working at Bell Labs. He has been teaching applied mathematics at MIT since 2003.

Daniel R. Simon (American) is the creator of another search algorithm in 1994, bearing his name. Precisely, his quantum algorithm solves the hidden subgroup problem (HSP) using an oracle based model, providing an exponential acceleration compared to classical computing[108]. Daniel Simon worked at Microsoft Research when he created his famous algorithm. He later worked on cybersecurity research until his retirement, always with Microsoft Research.

Lov Grover (1961, Indian-American) is a computer scientist who created the seminal quantum algorithm in 1996 that is said to be a search algorithm in a database but has many more use cases as we will see in the quantum algorithms part of this book (page 870). He got a bachelor's degree from IIT Delhi in 1981 and a PhD from Stanford University in 1984, both in electrical engineering. He worked at the Bell Labs starting from 1984 and was also a visiting professor at Cornell University. He became an independent researcher in 2008.

Scott Aaronson (1981, American) teaches information science at the University of Austin in Texas. He is a leading expert in quantum algorithms and complexity theories. He is notably at the origin of a quantum algorithm used for boson sampling, a way to demonstrate some quantum advantage for photonic based experiments. Bosons are integer spin particles such as photons, while particles such as electrons, neutrons and protons are fermions, with a spin 1/2.

Dorit Aharonov (1970, Israeli) is a quantum algorithms researcher. She received her PhD in Computer Science in 1999 at the Hebrew University of Jerusalem on "Noisy Quantum Computation" and then did a post-doc at Princeton and Berkeley. She is credited with the "quantum threshold theorem" co-demonstrated with Michael Ben-Or which states that below a certain error rate threshold, error correction codes can be recursively applied to obtain an arbitrarily low error rate of logical qubits.

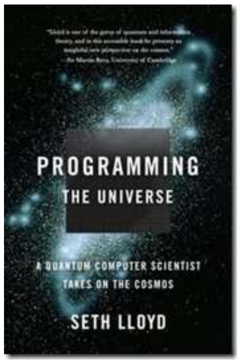
This is a very theoretical mathematical approach that does not take into account the way noise is also scaling as we increase the number of qubits. Dorit Aharonov's uncle is **Yakir Aharonov** (1932, Israeli), a physicist who had worked with David Bohm, among others.

[107] See the excellent The Early Days of Quantum Computation by Peter Shor, August 2022 (10 pages) where Peter Shor recount the history of the early years of quantum computing and how he discovered his various algorithms with try and error.

[108] See On the power of quantum computation by Daniel Simon, 1994 (11 pages) also updated in 1997.

Aram Harrow (American) is a prolific specialist in quantum algorithms. He teaches both quantum physics and quantum computing at MIT. At MIT, he is surrounded by Peter Shor and Charles Bennett. He is the co-author of the HHL quantum algorithm used to solve linear equations which he created jointly with Avinatan Hasidim and Seth Lloyd[109] . He is also interested in the creation of hybrid classical/quantum algorithms.

Seth Lloyd (1960, American) is a professor at MIT who is a prolific contributor to quantum information and quantum algorithms. He is the initiator of Quantum Machine Learning, of the concept of qRAM (quantum random access memory), of continuous variables gates-based quantum computing (1999), of quantum radars (2008). He's also the L in the famous HHL quantum linear equation solving algorithm and worked on quantum error correction codes and quantum biology.



In his 2006 book, Programming the Universe, Lloyd contends that the universe itself is one big quantum computer producing what we see around us, and ourselves, as it runs a cosmic program. According to Lloyd, once we understand the laws of physics completely, we will be able to use small-scale quantum computing to understand the universe completely as well. In about 600 years.

Seth Lloyd was laid off from MIT in 2019 then put on leave, then on disciplinary actions for a period of five years starting in 2020 because he had not informed his management of some Jeffrey Epstein originated funding.

In 2016, he created Turing (2016, USA) with Michele Reilly, a software company working on hybrid classical-NISQ software solutions using AI and quantum machine learning techniques.

Alán Aspuru-Guzik (circa-1978, American) is a research director at the University of Toronto, formerly at Harvard, who, among other things, created various quantum chemistry algorithms, a topic we will cover in the section dedicated on quantum algorithms. He is also the co-founder of the Zapata Computing, a startup developing quantum computing software frameworks, particularly in chemical simulation.

Frédéric Magniez (French) is the Director of the CNRS IRIF laboratory mentioned above. He also did run a Chair at Collège de France in Spring 2021. His research focuses on the design and analysis of randomized algorithms for processing large datasets, as well as the development of quantum computing, particularly algorithms, cryptography and its interactions with physics. In 2006, he founded and led the national working group for quantum computing, bringing together 20 research groups.

Iordanis Kerenidis (c. 1980, Greek) is a director of research from CNRS at IRIF (Institut de Recherche en Informatique Fondamentale), Paris, France, working on cryptography, quantum communication, quantum complexity theories and quantum machine learning, his latest specialty. He did his thesis at MIT under the supervision of Peter Shor and worked in the same office as Scott Aaronson and also worked at Berkeley with Umesh Vazirani. He is part of the founding team of QC Ware.

[109] See Quantum algorithm for linear systems of equations, 2009 (24 pages).

There he leads the R&D in quantum algorithms. He also co-leads the Paris Quantum Ecosystem (PCQC) with Eleni Diamanti. He was one of the members of the parliamentary mission on quantum technologies led by MP Paula Forteza between April 2019 and January 2020.

Ryan Babbush (circa-1989, American) is a Google researcher working on quantum simulation algorithms. His goal is to create commercial quantum chemistry solutions. In a February 2020 presentation, he did show that chemical simulation with Google's Sycamore 53 qubits processor could not use more than 12 qubits because of its high error rate.

Maria Schuld (c. 1989, German) is a senior researcher and software developer at Xanadu since 2017, based in South Africa at the University of KwaZulu-Natal in Durban where she got her PhD in quantum machine learning and was then a post-doc after a short internship at Microsoft Research in the USA. She is a key contributor to the development of quantum machine learning algorithms, particularly in the field of pattern recognition.

Cristian Calude (1952, Romanian/New Zealander) and **Elena Calude** (Romanian/New Zealander) are researchers from the Institute of Information Sciences, University of Albany in Auckland, New Zealand. They work on quantum algorithms, hybrid quantum algorithms and complexity theories.

Ewin Tang (2000, American) published in July 2018 a paper demonstrating a classical recommendation algorithm as efficient as an algorithm designed for D-Wave quantum computers by Iordanis Kerenidis and Anupam Prakash in 2016[110]. They responded by finding a flaw in the reasoning. On close inspection, the quantum algorithm would scale better in some extreme conditions. She was 18 years old at the time. Ewin Tang is now a computer scientist at the University of Washington.

Over time, I realized how this list can be incomplete. I should also mention key researchers like **Barbara Terhal** (QuTech), **Garnet Chan** (Caltech), **Craig Gidney** (Google AI), to just name a few.

Error correction

Michael Freedman (1951, American) is a mathematician who founded and did run the Microsoft Station Q laboratory in Santa Barbara, California. He now works at Google. He is one of the fathers of topological quantum computing along with Alexei Kitaev. He was also awarded the Fields Medal in 1986 for his work on the Poincaré conjecture, later demonstrated in 2006 by Grigori Perelman.

Alexei Kitaev (1963, Russian and American) is with Michael Freedman one of the fathers of the topological quantum computer concept in 1997, investigated by Microsoft. He was a researcher at Microsoft Research in the early 2000s and is now working at Caltech University and with Google. He has also done a lot of work on error correction codes, including the creation of toric codes, surface codes and magic states distillation (with **Sergey Bravyi**) and the Quantum Phase Estimate algorithm, used in Shor's integer factorization algorithm.

[110] See A quantum-inspired classical algorithm for recommendation systems, Ewin Tang, July 2018 (32 pages) and Major Quantum Computing Advance Made Obsolete by Teenager by Kevin Harnett, July 2018.

Andrew Steane (1965, English) is a Professor of Physics at Oxford University. He created the so-called Steane quantum error correction code in 1996. This code corrects flip and phase errors on a single qubit. Looking at how it works provides good insights on the inner workings of quantum error correction codes, although this particular code will probably not be used when we will have scalable quantum computers. Other more sophisticated QEC codes are investigated like color codes, surface codes and Floquet codes.

Daniel Gottesman (1970, American) is a physicist from the Perimeter Institute in Waterloo, Canada. He did his PhD thesis at Caltech under the supervision of John Preskill. He is known for his work on quantum error correction codes (QEC) and is co-author of the famous Gottesman-Knill's theorem according to which a quantum algorithm using only Clifford gates can be efficiently simulated (meaning, polynomially) on a classical computer.

Clifford group quantum gates are based on half and quarter-turn rotations (of the qubit in the Bloch sphere), Hadamard gate and the C-NOT conditional gate. This theorem thus indirectly proves that a basic gate set is insufficient to generate an exponential quantum advantage. We need to add a T gate to make it possible to approximate any arbitrary unitary transformation, meaning, any move within the Bloch sphere for single qubit operations. This is particularly important for the Shor algorithm.

Mazyar Mirrahimi (circa 1980, Iranian) is a mathematician who moved to quantum physics. He is currently the director of Inria's Quantic laboratory, which specializes in error correction codes and quantum algorithms, among other topics. He did his post-doc with Michel Devoret at Yale University. Back in 2013, he published a seminal paper on cat-qubits.

These are physical qubits using a cavity and a superconducting qubit that self-corrects some errors, starting with flip errors. These cat-qubits are used by the startup Alice&Bob as well as by Amazon, as announced in December 2020.

Zaki Leghtas (Morocco/France) is a researcher based in France in Mazyar Mirrahimi's team and is also specialized in error correction codes and systems. He is notably one of the creators of cat-qubits mentioned above. These are supposed to enable the creation of logical qubits with fewer than 100 physical qubits. He worked in Michel Devoret's laboratory at Yale University before joining Inria's Quantic team in 2015. He is also affiliated with ENS and Mines ParisTech.

Other names worth mentioning here are **Shruti Puri** (Yale), **Sergey Bravyi** (IBM), **Dave Bacon** (Google), **Liang Jiang** (Caltech), **Robert Calderbank** (Duke University), **Gilles Zemor** (University of Bordeaux), **Matthew B. Hastings** and **Nicolas Delfosse** (Microsoft, then IonQ), **Jean-Pierre Tillich**, **Anthony Leverrier** and **Christophe Vuillot** (Inria).

Other domains

Kristel Michielsen (circa-1969, Belgian) is a physicist working at the University of Aachen in Germany and at the Jülich Supercomputing Centre (JSC) where she leads the Quantum Information Processing (QIP) research group. She has contributed to numerous works in quantum computing both in physics and algorithms. She created the QTRL scale, for Quantum Technology Readiness Level, that is used to evaluate the level of maturity of quantum technologies and which we will discuss in the section dedicated to practices in research.

Matthias Troyer (1968, Austrian) is Professor of Computational Physics at ETH Zurich. He joined Microsoft Research in Redmond at the beginning of 2017. He is one of the creators of the Q# language for quantum programming and of the open source framework ProjectQ launched in 2016 by ETH Zurich. He is particularly interested in chemical simulation with quantum computers. He received his PhD from ETH Zurich in 1994.

Krysta Svore (c.1978, American) is currently the general manager of quantum software at Microsoft. She has a Ph.D. in Computer Science from Columbia University. Her contribution in quantum information science covers a broad range of topics: MBQC, quantum machine learning, contributing to the creation of the LIQUi|> quantum programming language, surface codes, fault-tolerance quantum computing.

Benoît Valiron (1980**,** France) is a researcher at the CNRS LIR laboratory from Université Paris-Saclay and teaching quantum programming and algorithms, including at CentraleSupelec. This quantum programming specialist is the co-author of the open source quantum programming language Quipper, which he contributed to create while being at the University of Pennsylvania.

Bettina Heim (c. 1980) is a Microsoft developer specializing in quantum software. She is responsible for the development of the quantum programming language Q# compiler, promoted by Microsoft since 2017 and which is part of their Quantum Development Kit, currently running on quantum emulators on traditional processors and now supported on third party hardware proposed on the cloud, including IonQ and Honeywell trapped ion based quantum processors.

Cyril Allouche (French) has been leading Atos R&D efforts in Quantum Computing since its beginning in 2015. Cyril Allouche are the "implementers" of the quantum vision of Thierry Breton, CEO of Atos until 2019. His work encompasses developing the aQASM (Atos Quantum Assembly Language) quantum programming language and the myQLM quantum programming emulator running on regular personal computers and servers.

Here we are. We've covered a whole lot of people and certainly missed many who should be on this hall of fame list! I'll update it whenever required. We will encounter many of these scientists in this book.

Research for dummies

As I investigated the broad quantum science and technology landscape, I learned more on how fundamental and applied research was operating. I did not know much about it before this adventure. Working in the 'digital world', as a developer, marketer and in the entrepreneurial ecosystem doesn't necessarily make you look deeply into the inner workings of research. I discovered many aspects that I am detailing here, particularly with regards to practices, lingua-franca, careers and evaluations.

I went up to publishing a review paper in a peer-reviewed journal, helping me discover all the creation, editing, feedback from and to referees, publishing process and the aftermath with getting contacted by predatory journals and conferences.

If you're a researcher, this is very basic stuff that you already know well. For others, it will clarify some of vague understanding you might have on how research works. If you have a master's degree and are looking to get a PhD, it may be useful for your future endeavors.

Long-term

The first key point is the long-term approach in quantum technologies. It can also be found in other branches of physics and so-called deep-tech and hard-tech related sciences. Time scales are measured in decades. It starts with intuition, creativity, passion, rigor, and hard work. These ideas are not always broadly adopted right away. There's always some resistance with the current scientific establishment.

This long-term history can be observed in condensed matter physics. Brian Josephson devised the Josephson junction in 1962. IBM tried to use it unsuccessfully to build superconducting computers. Anthony Leggett made significant discoveries in the early 1980s which led to the creation of the first superconducting qubits in the early 2000s and to Google and IBM's superconducting machines between 2016 and 2020. And we're not done there since this technology's scalability has not yet been proven.

Alain Aspect's work leading to his 2022 Nobel prize in physics award started in the mid-1970s and culminated with his 1982 experiment. Back then, it had no immediate industry application. Fortunately, he was well supported by many laboratories, particularly to build the necessary instrumentation. His work led to the creation of many of the branches of quantum technology. For example, Artur Ekert was inspired by Alain Aspect's work to advance the field of quantum cryptography in the early 1990s.

All of this cannot be meticulously planned in advance. Research serendipity must prevail. Commercialization comes later, through meetings between specialists from different and complementary disciplines. Innovators are either the researchers themselves, or others, engineers and entrepreneurs, who know how to detect research work having some business potential. Hence the importance of bringing them together in ecosystems of innovation. However, in its current shape, the quantum startup ecosystem is mostly made of researchers turned into entrepreneurs.

This generates its share of misunderstandings with public authorities and policy makers. They are tempted to over-evaluate and measure the performance of basic research, if not to fund it, using only criteria from the business world. On the other hand, and this is particularly true for quantum technologies, research work requires peer reviews. This may give the impression that researchers are both judge and jury. To prevent this from driving decision-makers and people suspicious, research work must honestly be translated in layman's terms. This should encourage researchers to communicate with broader audiences than their peers. It requires leadership. Scientists must be more involved there, particularly in those times where people are more and more skeptical of science and innovation.

Scientific papers

Fundamental research is by default an open world. Scientists from all over the world publish their work in scientific papers. Science is about building knowledge and transmitting it. Every researcher benefit from the work of other researchers.

Preprints. It is commonplace to see research being published first in open access on the **arXiv** website that is managed by Cornell University. It contains articles pre-prints that have not yet gone through peer reviewing nor have been subsequently published in peer-reviewed journals. The volunteers who manage arXiv still check the basic quality of the submitted papers. In order to publish on arXiv, you need to get the endorsement of at least one researcher who regularly publishes preprints on arXiv. It enables authors to collect comments on their work. Papers can be easily updated to take into account feedback and embed various corrections. Their quantity and quality depend on the author's fame, the topic and the number of researchers who master it[111]. It also depends on the way researchers communicate about their papers in social networks. It allows fast turnaround for debates between scientists[112], sometimes in a controversial way[113]. arXiv papers must still sometimes be taken with a grain of salt. There are other sites for pre-prints like arXiv, with for example **TechRxiv** on engineering and computer science, **engrXiv** on engineering (with some papers related to quantum technologies), **ChemRxiv** in chemical engineering and **medXxiv** in health sciences.

Peer-reviewed publishing. Between about 9 to 18 months after a preprint, the paper may be published in a peer-reviewed journal. If the delay is too short, it may mean the journal is a predatory one. It is usually published mostly as is, includes some revisions suggested by the "referees" of the review committees, or even with a change of title. In these cases, the version published on arXiv is not necessarily the most recent. It is sometimes updated. The benefits are openness and free access. The peer-reviewed version is usually different from the preprint one. It contains more or less revisions based on the referee's feedback and may even have a new title. Sometimes, papers are directly published on peer-reviewed journals while skipping the preprint stage. It happens frequently with major Google papers and some IBM papers.

Quantum technologies peer-reviewed[114] journals include **Nature** and its various thematic variations like **Nature Communications**, **Science**, **Physical Review X**, **Physical Review Research (PRR), Physical Review Letters (PRL)**, **Physical Review A** and **Physical Review B**, **Quantum Journal**, **Quantum Science and Technology**, **Journal of Applied & Computational Mathematics, International Journal of Quantum Information, Quantum Engineering, Advanced Quantum Technologies, Quantum Information Processing, IEEE Journal of Quantum Electronics, and** IEEE **Transactions on Quantum Engineering** (Figure 63).

Figure 63: some key quantum physics peer-review publications.

[111] See Comment bien lire et comprendre une étude scientifique par Gary Dagorn, Mathilde Damgé et Bessma Sikouk, May 2021. It provides a lot of insights on how to read a scientific paper. You can translate this article in French in your browser. Also look at Ten simple rules for reading a scientific paper by Maureen A. Carey, Kevin L. Steiner and William A. Petri Jr, July 2020.

[112] Like with Reply to arXiv:2203.14555 by Margaret Hawton, May 2022 (1 page) that is a reply to A Comment on the "Photon position operator with commuting components" by Margaret Hawton and A. Jadczyk, March 2022 (4 pages). See also Is the Moon there if nobody looks: A reply to Gill and Lambare by Marian Kupczynski, September 2022 (8 pages) which is typical of the debates going on with quantum foundation topics and on the nature of reality.

[113] See Matters Arising: Distributed quantum sensing with mode-entangled spin-squeezed atomic states by Liam P. McGuinness, February 2023 (5 pages).

[114] In peer-reviews journals, the reviewers are unknown to the paper authors. They provide some feedback on the paper and expect a paper update. The authors provide an updated version and comments that are either accepted or rejected by the reviewer. It can lead authors to modify their claims and even their paper title. When everything's finalized, the paper can be published. Nowadays, the initial paper published on arXiv is also updated to reflect these changes. There is also a special double-blind review process where the authors are unknown from the reviewers to avoid any reviewer bias. I have bumped only once on such a case in quantum technologies, on a QML algorithm: On the universal approximability and complexity bounds of deep learning in hybrid quantum-classical computing, 2021 (15 pages).

Fortunately, in this field, there are only a few predatory journals that have a very poor peer-reviewing process and charge researchers. I am currently spammed by the **Scirea** publishing group to publish my EPJ-A paper, which doesn't make any sense. You never publish a paper in two distinct peer-reviewed journals!

In most scientific fields, including quantum science, there are many publications but not enough skilled reviewers. This job is sometimes done by PhD students. Sometimes, innovative papers are locked by reviewers, particularly when they are cross-discipline, which is frequently the case with quantum science and is a problem when publications are over-segmented.

PhD theses are easier to retrieve and are generally published freely. These are usually good sources of bibliographical information. Beyond the main thesis goal that is to advance science in a usually narrow domain, it generally starts with making an inventory of the state of the art, like in review papers. Review papers present a state of the art of a field. Their bibliography is generally impressive, sometimes as long as the paper itself. They are a good starting point to study a subject, especially if the paper is not too old. I provide links to many such review papers, particularly on specific qubit types. If the author's pedagogy is good, it can be very useful for learning on your own. A bibliography generally allows you to go deeper into the subject by discovering the need-to-know fundamental texts.

Authors. Several authors are usually mentioned in scientific papers, up to a very large number. In general, beyond three authors, the first is the one who was the owner and done the bulk of the work. As illustrated in Figure 64, it is usually a PhD student or a post-doc. He/she has processed the experience and written a large part of the document, but this may depend on countries, laboratories and thesis supervisors. The last one is the thesis or research laboratory supervisor[115] . In the latter case, the penultimate author is the thesis director who supervised the work. In between are the other contributors, experimenters or simple reviewers. They may work in a single or in several organizations. It can be a mix of public research laboratories associated with one or several industry vendors.

Some papers have a very large number of authors. It is typical from the papers published by Google AI which can have upwards of 80 coauthors, which means about half of their whole team. They probably all contributed to the published work but certainly not equally[116].

Well crafter papers don't forget to mention the respective contribution of all the authors, like in the example shown in Figure 65. It also mentions reviewers (not those from a peer-review publication), research funding source, any potential competing interest, how the research data can be accessed and the availability of any supplemental material, that is now usually placed at the end of papers in their pre-print format. These supplemental materials can contain technical details and can be very interesting, like for example, describing the experimental setup and its hardware and/or software engineering.

The other extreme case is a paper having only a single author. It means first that it is probably not a PhD student, otherwise his PhD supervisor would be a coauthor, or the author is a PhD but he lost the support of his/her supervisor for whatever reason, which is a bad omen and very rare. It may also be a Master student who is looking for a PhD position. You can look at whether the author works in a research institution and his CV. Finally, you can assess the author's network if he/she mentions and thanks reviewers or contributors.

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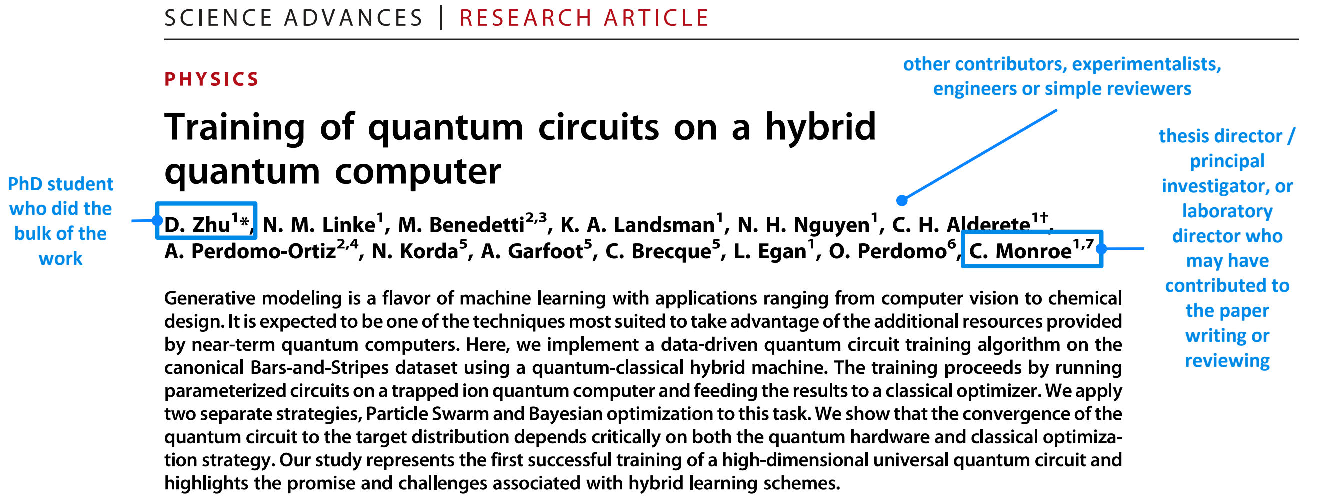
[115] This is the case of these hundreds of publications with the famous Didier Raoult who is cited as the last contributor, as laboratory director but not necessarily thesis director.

[116] I found out this extreme case in Search for a massless dark photon in $\Lambda_c^+ \rightarrow p\gamma'$ decay by BESIII Collaboration, August 2022 (8 pages) with 573 authors from 75 research organizations, in China. For just 8 pages! But the world record seems to be the Higgs boson CERN paper from 2015 with 5,154 authors, probably nearly all the CERN and partners collaborators in the project. See Combined Measurement of the Higgs Boson Mass in *pp* Collisions at $\sqrt{s} = 7$ and 8 TeV with the ATLAS and CMS Experiments by G. Aad et al, CERN, PRL, May 2015 (35 pages).


SCIENCE ADVANCES | RESEARCH ARTICLE

PHYSICS

Training of quantum circuits on a hybrid quantum computer

D. Zhu[1*], N. M. Linke[1], M. Benedetti[2,3], K. A. Landsman[1], N. H. Nguyen[1], C. H. Alderete[1†], A. Perdomo-Ortiz[2,4], N. Korda[5], A. Garfoot[5], C. Brecque[5], L. Egan[1], O. Perdomo[6], C. Monroe[1,7]

Generative modeling is a flavor of machine learning with applications ranging from computer vision to chemical design. It is expected to be one of the techniques most suited to take advantage of the additional resources provided by near-term quantum computers. Here, we implement a data-driven quantum circuit training algorithm on the canonical Bars-and-Stripes dataset using a quantum-classical hybrid machine. The training proceeds by running parameterized circuits on a trapped ion quantum computer and feeding the results to a classical optimizer. We apply two separate strategies, Particle Swarm and Bayesian optimization to this task. We show that the convergence of the quantum circuit to the target distribution depends critically on both the quantum hardware and classical optimization strategy. Our study represents the first successful training of a high-dimensional universal quantum circuit and highlights the promise and challenges associated with hybrid learning schemes.

Figure 64: typical presentation of scientific paper's co-authorship.
Source: [Training of quantum circuits on a hybrid quantum computer](#) by D. Zhu, Christopher Monroe et al, 2019 (7 pages).

The author may be already an established on like John Preskill, Peter Shor, Seth Lloyd or Scott Aaronson, so no problem. Other cases with no attached institution, record or network may mean that the author may be working on some fringe theories in a very isolated fashion, particularly if there is no mention of any help or thanks to anybody for the research. Sometimes, all the authors have the same last name[117]. It may be a family business!

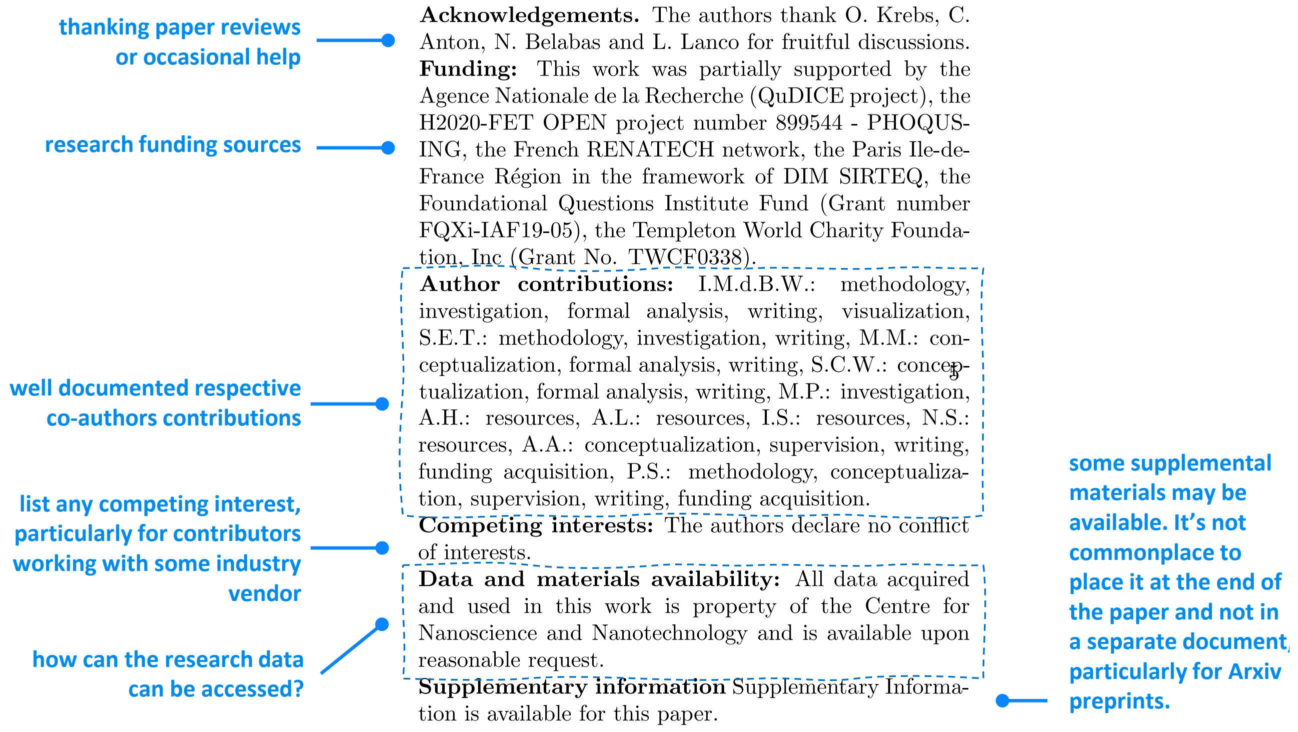

Acknowledgements. The authors thank O. Krebs, C. Anton, N. Belabas and L. Lanco for fruitful discussions. **Funding:** This work was partially supported by the Agence Nationale de la Recherche (QuDICE project), the H2020-FET OPEN project number 899544 - PHOQUSING, the French RENATECH network, the Paris Ile-de-France Région in the framework of DIM SIRTEQ, the Foundational Questions Institute Fund (Grant number FQXi-IAF19-05), the Templeton World Charity Foundation, Inc (Grant No. TWCF0338). **Author contributions:** I.M.d.B.W.: methodology, investigation, formal analysis, writing, visualization, S.E.T.: methodology, investigation, writing, M.M.: conceptualization, formal analysis, writing, S.C.W.: conceptualization, formal analysis, writing, M.P.: investigation, A.H.: resources, A.L.: resources, I.S.: resources, N.S.: resources, A.A.: conceptualization, supervision, writing, funding acquisition, P.S.: methodology, conceptualization, supervision, writing, funding acquisition. **Competing interests:** The authors declare no conflict of interests. **Data and materials availability:** All data acquired and used in this work is property of the Centre for Nanoscience and Nanotechnology and is available upon reasonable request. **Supplementary information** Supplementary Information is available for this paper.

Figure 65: typical credits at the end of a scientific paper. Source: [Coherence-powered work exchanges between a solid-state qubit and light fields](#) by Ilse Maillette De Buy Wenniger, Maria Maffei, Niccolo Somaschi, Alexia Auffèves, Pascale Senellart et al, April 2022 (17 pages). This is the typical requirement for some peer-reviewed publications like Nature.

In many countries, such as the USA, it is common practice to mention authors with the initials of their first and middle names initials. It does not make it easy to search them online, especially for Chinese authors. This is particularly the case when there are many contributors.

[117] See [Fidelity and Entanglement of Random Bipartite Pure States: Insights and Applications](#) by George Biswas, Debasish Biswas and Anindya Biswas, August 2023 (6 pages).

I try to quote authors with their first name when they are easy to be found.

Fringe science preprints. **viXra** (arXiv in reverse order) is an arXiv for the preprints that will never be published in peer-reviews publication and are too fringe to be accepted on arXiv (vixra.org/quant). An author only visible on viXra and even not on arXiv is really fringe[118] [119].

In the thousand footnotes in this book, I take the liberty of not using the cryptic description convention that is used in the abundant bibliographies of some scientific publications, sometimes using authors, publication references but not the paper title (Figure 66)!

[39] M. V. Burdastyh, S. V. Postolova, T. Proslier, S. S. Ustavshikov, A. V. Antonov, V. M. Vinokur, and A. Y. Mironov, Scientific Reports **10**, 1471 (2020).
[40] C. Müller, J. H. Cole, and J. Lisenfeld, Reports on Progress in Physics **82**, 124501 (2019).
[41] W. A. Phillips, J. Low Temp. Phys. **7**, 351 (1972).

Figure 66 : why (t.h.) these long bibliographies do not contain any title? And this paper is from 2023!

I use a clear title convention followed by first author/authors, sometimes their research laboratories or companies, publication date and then number of pages or slides, which helps you identify immediately the volume and depth of the referenced documents. The many footnotes in that book may be cumbersome, but they prevent you from looking at bibliographical references at the end of the document, which is never very practical whether you read a paperback or electronic version of the document. When I don't mention all a paper's contributors, I use the expression "*et al*" which is an abbreviation of the Latin "*et alia*", meaning "*and the others*".

I'm usually selecting the first and last authors, then in the middle those I happen to know some way or the other, as described in Figure 67.

[693] See Error Propagation in NISQ Devices for Solving Classical Optimization Problems by Guillermo González-García, Rahul Trivedi, and J. Ignacio Cirac, MPI and MCQST, PRX Quantum, December 2022 (17 pages) and Quantum entanglement can be a double-edged sword, December 2022.

[694] See Shadow Distillation: Quantum Error Mitigation with Classical Shadows for Near-Term Quantum Processors by Alireza Seif, Liang Jiang, March 2022 (16 pages).

[695] See Virtual Distillation for Quantum Error Mitigation by William J. Huggins et al, Google AI, PRX, 2021 (25 pages).

[696] See Crucial leap in error mitigation for quantum computers by Monica Hernandez and William Schulz, Lawrence Berkeley National Laboratory, December 2021, referring to Randomized Compiling for Scalable Quantum Computing on a Noisy Superconducting Quantum Processor by Akel Hashim, Irfan Siddiqi et al, 2021 (12 pages).

Figure 67: bibliographical references as presented in this book. I find it more practical although it doesn't seem to be orthodox.

Frauds and retractions. It doesn't seem that quantum research is prone to significant paper-milling or even to papers being retracted[120]. On the **RetractionWatch** database, you can find only a few retracted papers in quantum physics, mostly coming from China and India (102 items with "quantum" in the title). It includes the famous retracted papers from The Netherlands and Denmark on Majorana fermions in 2021 and 2022. Another notorious case is the retraction of Ranga Dias's paper on ambient room temperature superconductors by Nature in 2022[121] [122] [123]. Sometimes, even preprints in arXiv are retracted, but are not accounted for in statistics.

[118] Like Dark Matter Fluid Interpretation of Quantum Entanglement by Zhi Cheng, March 2023 (8 pages).

[119] See The Ecology of Fringe Science and its Bearing on Policy by HM Collins, A Bartlett and LI Reyes-Galindo, June 2016 (27 pages).

[120] See The fight against fake-paper factories that churn out sham science, Nature, March 2021.

[121] See Retraction Note: Room-temperature superconductivity in a carbonaceous sulfur hydride, Nature, September 2022.

[122] See Allegations of Scientific Misconduct Mount as Physicist Makes His Biggest Claim Yet, March 2023.

[123] See Breakthrough or bust? Claim of room-temperature superconductivity draws fire by Robert F. Service, Science, October 2021.

Writing a paper

As of September 2023, I had published three papers on arXiv on top of the English editions of this book[124] [125] [126], but submitted none of them to a peer-reviewed journal. However, I had obtained a lot of feedback before and after publishing these papers and thank those who helped me craft these review papers at their end. Then came my EPJ-A paper which helped me craft this new part on the tools and methods used to prepare a paper[127].

Papers are usually final or interim reports of a research project led by a PhD or postdoc under the supervision of a thesis director. It can be theoretical, experimental, or mixing both aspects. It must advance the given science field in a demonstrative way. Most of the time, the advance is incremental and sometimes, it can be potentially disruptive. Still, you will not find here any advice on how to conduct good research since I am not a researcher nor have a PhD. Others do that well[128].

I'll investigate the software technical aspects of publishing. So far, I have always written all my papers and books with Microsoft Word, using a template and the likes, even up to developing some VBA productivity enhancement macros with over 3,000 lines of code, like for creating footnotes, index entries, updating it and the likes. But the standard in research is LaTeX although Word submissions are supported by most publications. When you've been accustomed to Word or similar tools, getting accustomed to LaTeX feels like coming back into the early 1980s, before the advent of "WYSIWYG", for "what you see is what you get", when graphical editing interfaces came up with Pagemaker, Word and other software of this period. LaTeX is still preferred in the research world since it is a programmable and flexible open source markup language with a rich software ecosystem.

Here are the tools I have been using to prepare my first LaTeX paper, on a Windows PC:

MiKTeX, a LaTeX compiler, that you don't use directly and is used by your LaTeX editor. You install it on your computer before installing a front-end LaTeX editor of your choice.

KILE, a LaTeX editor with some WYSIWYG features that relies on MiKTeX (Figure 68). It helps you edit the .tex file, compile it into a PDF version that you can see on the right of your screen. When it works, clicking on the .tex file finds the corresponding location in PDF and vice versa. One initial problem is to get rid of all the warnings and errors generated by your .tex file. Others prefer **Lyx**. Those working on a paper collaboratively frequently use the online service **Overleaf**.

Figure 68: the user interface of KILE to edit a LaTeX file. It can synchronize your source code on the left and the position in the PDF view on the right. 2023.

[124] See Where are we heading with NISQ? by Olivier Ezratty, May-June 2023 (50 pages).

[125] See Is there a Moore's law for quantum computing? by Olivier Ezratty, March 2023 (32 pages).

[126] See Mitigating the quantum hype by Olivier Ezratty, January-February 2022 (26 pages).

[127] See Perspective on superconducting qubit quantum computing by Olivier Ezratty, The European physical Journal-A, May 2023 (18 pages). I wrote this review paper on behalf of Thomas Ayral (Eviden/Atos) and Thomas Duguet (CEA) for a special edition of this journal dedicated to quantum computing.

[128] See Quick thoughts on research by Michael Nielsen, January-July 2023.

Jabref, a bibliography creation tool, that helps refine the bibliography .bib file associated to your .tex file (Figure 69). For example, it will find various meta data of the incomplete entries in your .bib file, like the DOI URL, all the author names and the likes. As a result, the bibliography in your LaTex file will look professional.

Another popular tool here is **Zotero**. Both are open source.

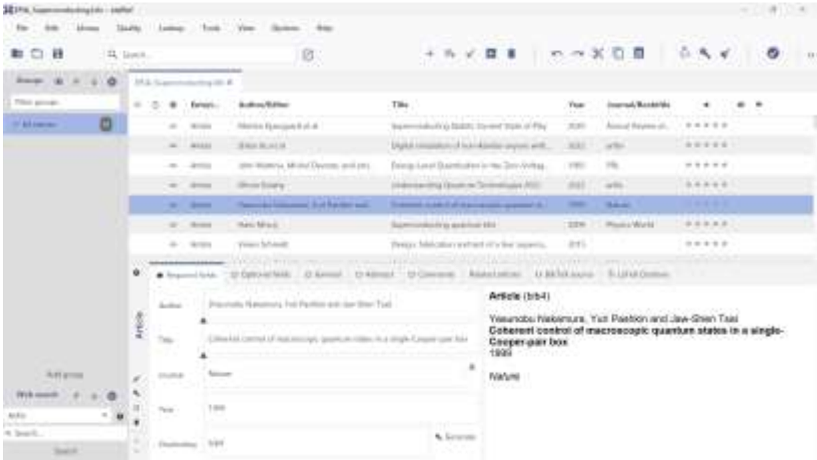


Figure 69: the user interface of Jabref to refine a bibliography file for LaTeX. 2023.

Spyder, a Python coding environment to create some scatter plots that I couldn't create with Excel (Figure 70). It works locally on your laptop. I used it to plot qubit fidelities against qubit numbers with different scales, imported from a CSV text file exported from Excel and to showcase the different related vendors. I export the plots in SVG files to make sure their rendering is vector based in Word and the book PDF.

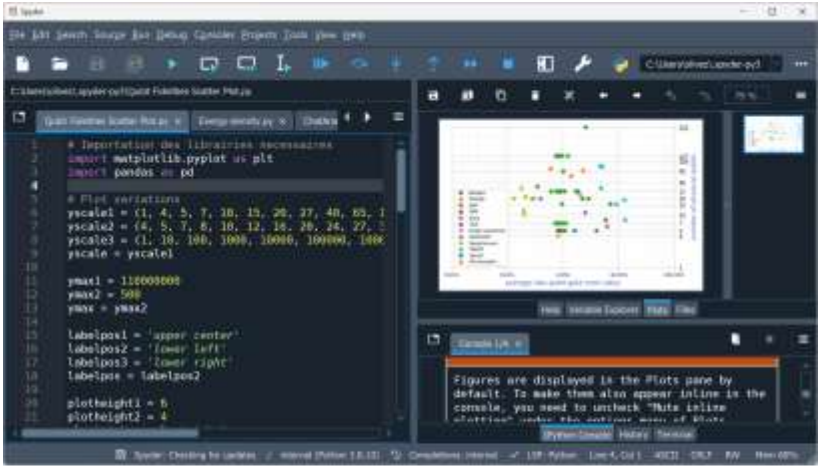


Figure 70: the Spyder Python editor to create scatter plots. 2023.

Several aspects are important in any paper:

Title. It must be concise and catchy, and preferably understandable without being a Ninja in the related domain.

Introduction. This is what 90% of your readers will see from your paper. It must contain all the key messages. It describes the domain, a given problem in the domain, a summary of the work and related results, put into context and a broad perspective on its related impact and applications (Figure 71).

Please, avoid using the now nearly parodic "*hold the promise*" expression when talking about quantum computing[129]. Find something else! Or release the promise!

> *Introduction* — Quantum computers hold the promise of finding useful applications in combinatorial optimization, quantum chemistry, cryptography decryption, and quantum simulations of many-body quantum systems.

Illustrations. This is the second part that is mostly looked at by readers. It frequently contains XY plots or more sophisticated plots. Take care of adding readable and understandable labels in the plot. Illustrations also have a legend that makes the chart and plot easy to understand. All variables must be described!

[129] See Dissipative mean-field theory of IBM utility experiment by Emanuele G. Dalla Torre and Mor M. Roses, August 2023 (4 pages) and also in Phase transition in Random Circuit Sampling by A. Morvan et al, Google AI, April 2023 (39 pages).

Discussion. It is a wrap up of the paper that lays out the conclusion of the presented work. It should contain key quantitative outcomes from the experiments.

In the case of quantum algorithms, for example, it would be nice to remind the reader in a simple way what is the quantitative gain in speedup vs classical algorithms. Charts showing this data are usually unreadable by non-expert eyes.

Length. A paper length depends on its goals and contents. Some only have 2 to 4 pages, and others can be much longer. Google's 2019 supremacy paper with all its supplementary materials was 70 pages long. Long papers are usually review papers of a given field with many contributing authors.

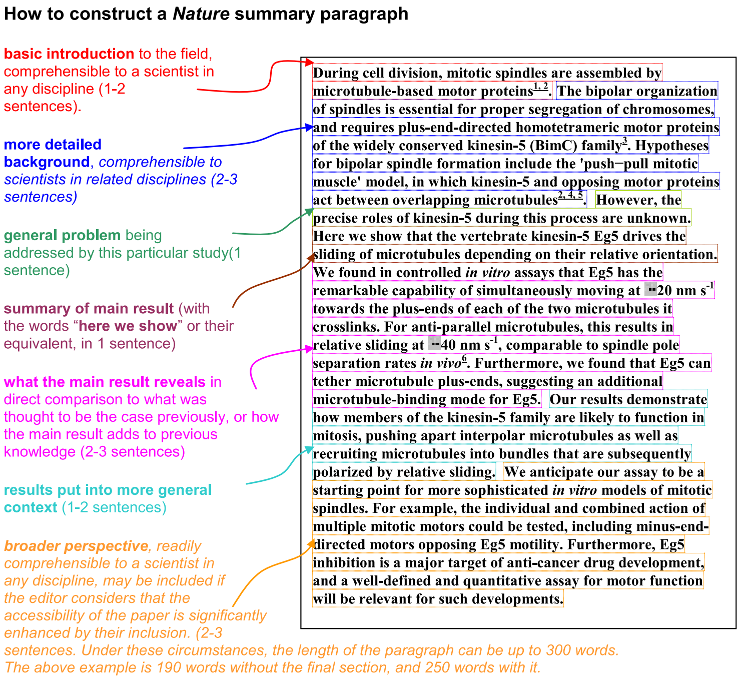


Figure 71: Nature introduction guidelines.
Source: [Summary/abstract writing](), Nature (1 page). 2023.

Bibliography. It is important to source all what you say that belongs to existing knowledge and state-of-the-art in what you cover. This is a commonplace work done in the early stages of a thesis. It is good to reference key review papers from your domain. You usually don't escape quoting the most famous papers like John Preskill's 2018 NISQ paper if you write about quantum computing, but adding other papers who add some in-depth perspectives on the most famous papers is also useful for the reader.

Data and code. Research results and datasets can be published in various platforms like **Zenodo**, which was developed under the European OpenAIRE program and is operated by CERN. The deposits can contain research papers, the experiments data sets, research software, detailed reports and any other digital artefacts. Using this sort of service is becoming common practice, to make sure experiments are reproducible. Other services like **OSF** (Open Science Framework) also promote open research practices. Otherwise, if the only thing to publish is code, it can be done on **Github**.

Publishing a paper

We havve already covered how to publish a pre-print on arXiv. How about a peer-reviewed journal? Well, you go on their website and submit your paper. Each paper provides guidelines and a process like PRX Quantum[130]. I went through a similar process with EPJ-A for my first peer-reviewed paper.

You submit your paper and propose a couple referee names. The paper can be rejected right away if it doesn't fit the quality standards of the publication. Otherwise, it will be approved for reviewing by some of the referees you have proposed or others, depending on the scope of your paper. It is typical to have about three referees. After a couple weeks, the publication sends you the referee's written feedback, which is most of the time anonymous. You respond to this feedback in two ways: with a revised version of your document considering the feedback, with a ready to publish version and a version using colored markups, and a formal and polite point by point response to all the feedback. After the reviewers accept the corrected paper, it will go through some editing process by the publication permanent staff.

[130] See Information for Authors, PRX Quantum.

They may recommend some changes in figures labelling, make sure all figures are referenced in the text and to order the figures against how they are mentioned in the text. Sometimes, it can also relate to currency, numbering and units' consistency.

Then, your paper is ready for publication. You may need to select your publishing "business model". Basically, you can pick an open source publishing model, but you are the one who pays for it, for example, $3,450 for a PRX Quantum paper, or your research organization already has an agreement with the publisher. The other option is with readers paying the access to your paper either per paper or on a subscription basis, and then publishing becomes free for you as an author. You can usually publish on arXiv the version that was submitted to the publication but not the edited version that contains adjustments linked to the referee's feedback. Other models exist like with the Quantum Journal where the publishing fee is set on a voluntary basis, with a regular tag at $450.

Communicating on a paper

Nowadays, it is a common practice for researchers to communicate on their preprint or peer-reviewed papers in social networks. I see such news on Twitter (now X) and LinkedIn. When you follow many quantum physics or quantum computing researchers, you get good "head-up" news about interesting papers. Also, some researchers make the effort to summarize the key points of their papers and ask for comments.

Scientific publications can be discovered by following the RSS feeds of arXiv, reference specialized papers, in addition, from scientific news feeds of online media or popular scientific press. I also discover new interesting papers with scanning scientific conferences presentations[131].

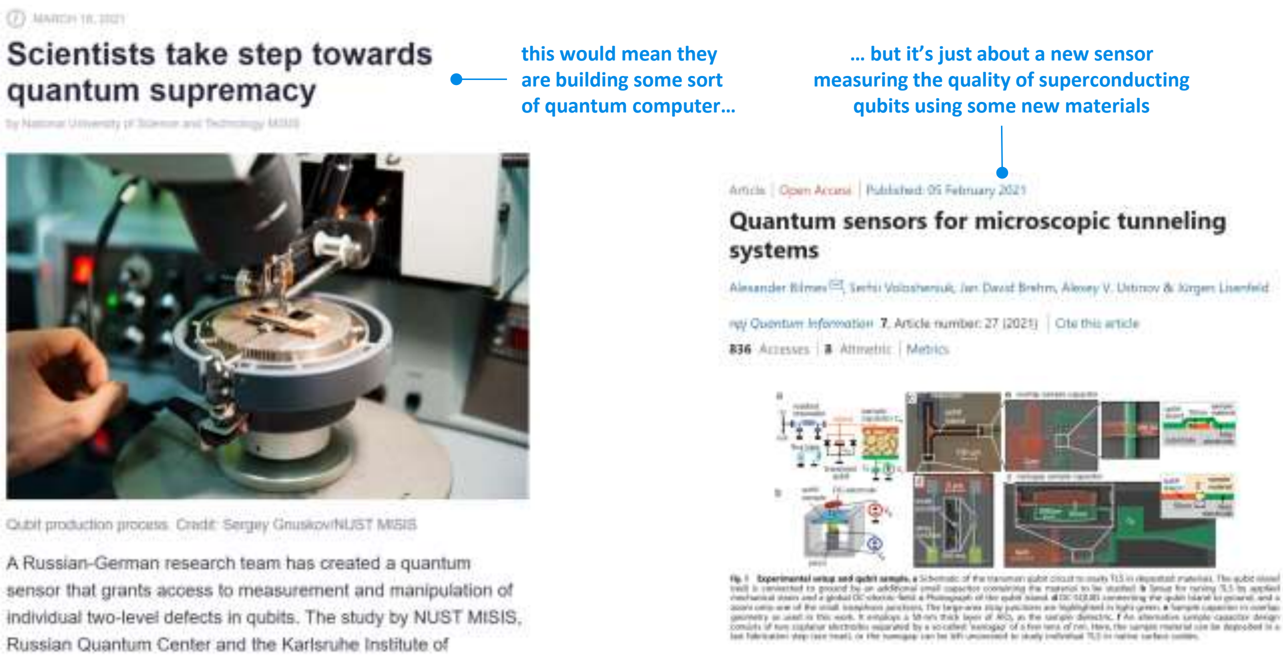


Figure 72: example of a scientific paper presented with outrageous claims by its lab communication department. Sources: Scientists Take Step Towards Quantum Supremacy, MISIS, March 2021 and Quantum sensors for microscopic tunneling systems by Alexander Bilmes et al, February 2021 (6 pages).

[131] Here is an example with a list of many IEEE superconducting technologies presentations.

In the case of quantum technologies, the "tech" media often broadcasts scientific news dressed-up with sensationalism and exaggerations. This often stems from the propensity of laboratory communicators or sometimes researchers themselves to make shortcuts between their work and its potential usage that may be very long-term[132]. It is even stronger when the communication comes from a large company such as Google or when the article was written by the laboratory's communication branch (example Figure 72). The job of the technology screener consists in sorting this out. When your local non-English speaking media broadcasts such information, it is often necessary to start by identifying the original paper which is possibly quoted at the end of the article. Sometimes, you discover blatant translation errors that entirely twist the scope of the covered scientific advance.

Predatory and fake conferences

There are really many conferences and workshops organized throughout the world on the myriad of different special domains of interest in quantum science and technologies. We provide a list of the key ones at the end of this book, starting page 1296. The conferences and workshops have different tracks; keynotes and tutorials delivered by invited speakers, sometimes panels, and also poster sessions and presentations, open to PhD students and postdocs.

When you publish your first papers in peer-reviewed journals, you discover another artefact from this ecosystem: predatory and fake conferences[133]. They invite you as pre-selected as "invited speakers". These conferences are predatory when they are imitating an existing conference with a slightly different name. But they take place in weird locations, most of the time, second-grade hotels. They charge you as invited speakers when usually you are not charged, and the most prestigious ones will even cover the travel expenses of invited speakers. These conferences can even be entirely fake, meaning, they collect the money from invited speakers and vanish in a parallel world with your money.

You can spot the weirdness of these events by looking at the invited speakers and organizing committee pedigree and countries of origin. Sometimes, the advertised invited speakers have even not been contacted nor accepted any participation.

Here is an example with a real conference, Magnetism and Magnetic Materials organized by IEEE and the AIP publishing group, and its predatory version, Magnetic Mat, coming from India and orgznized in Portugal, with a speaker registration fees of $749. You also have many Optics, Photonics, and Lasers Conferences organized throughout the world. I was even invited to speak at one of these in Rome, May 2024, based on my superconducting qubits review paper who is clearly not in that field.

So, beware!

Papers analysis and classification

I have been updating this book on a yearly basis since 2018. Starting in 2020, I used arXiv and scientific publications are key sources for the updates. As a general rule, when I discover the existence of an article, I search for it on Google Search with the name followed by "filetype:PDF" and I find it free of charge in more than 90% of the cases on arXiv or on the ResearchGate site, the researchers' reference social network. I also now use the **Unpaywall** Chrome extension which automatically finds an open access PDF version of the paper when you are on a journal web site. It either opens the PDF of the published version when access is open, or automatically finds the equivalent arXiv version if not.

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[132] The example below comes from Scientists take step towards quantum supremacy by National University of Science and Technology MISIS, March 2021. The supremacy from the article title is very far away considering the paper is about some sensing technology to measure the efficiency of some superconducting qubit.

[133] See Inside a "Fake" Conference: A Journey Into Predatory Science by Ruairi J Mackenzie, Technology Networks, July 2019.

The bulk of my work consists in classifying the collected information: what is it about and how does it fit into the web of quantum technologies? As far as I know, no artificial intelligence can automatize this process[134]. This classification task is a tedious one and you can be easily misled with reading a paper title or press release too quickly[135].

What is the actual progress made with regards to the state of the art? You can rely on these typical recommendations: read the introduction and not just the abstract, identify the problem that the authors are trying to solve and how they are advancing the state of the art, look at the data and identify any missing data, and read the conclusion. If you can't decipher the paper content, make a search of other more generalist web sites mentioning it.

In general, a paper presenting a breakthrough that will allow a quantum computer to operate at room temperature or ahead of all others is often in reality a simple incremental evolution in the development of a particular type of qubit or a "very low TRL" exotic quantum object that is even not yet a controllable qubit. After reading it, it looks like your tiny hairy dog after the shower! In many cases, quantum science-related papers are inaccessible, requiring solid mathematical and/or physics background. Even quantum science specialists have a hard time interpreting many papers.

You also frequently come across a set of Russian dolls concepts with unknown concepts referring to other unknown concepts, and so on. This is some sort of involuntary humor of scientific complexity[136]. However, hopefully, some papers do not use too much jargon and manage to deal with a big fundamental question by making it understandable to many specialists in their discipline and well beyond.

How can you check the whole thing, particularly given the specialists in your own network have not yet had the time to do so? You either need to be patient, do it on your own, or look for someone who has done the job. For big news related to quantum computing, one can wait for the next post from Scott Aaronson or a laconic tweet from John Preskill.

Finally, I use arXiv as soon as I come across a startup that defines in too broad terms what it does without any technology specifics. It is so commonplace now! A search starts with finding the startup scientific founder, then with identifying their research work that they are probably willing to package in their freshly created startup. In their bibliography process, researchers also look at **Google Scholar** and on **SciRate**, where discussions take place around pre-print papers published on arXiv.

We must recognize our limits and understand that we're not protected from believing scientific hoaxes like the famous one created by **Alan D. Sokal** in 1996.

[134] Various tools attempt to automate this sorting work, such as In Layman's Terms: Semi-Open Relation Extraction from Scientific Texts by Ruben Kruiper et al, May 2020 (13 pages). It is currently applied to the field of biology.

[135] Here is one interesting example with a post from James Dargan in The Quantum Insider which wrongly described the European LSQuanT project as an initiative to provide quantum computing solutions to the transportation industry. Wrong! It is a project related to fundamental quantum physics and digital simulation of quantum transport, a condensed matter phenomenon! See LSQuant: Novel Initiative Created To Improve Quantum Transport Methodologies, May 2021.

[136] Here are some interesting examples of papers whose title refers to several cryptic concepts: The Franke-Gorini-Kossakowski-Lindblad-Sudarshan (FGKLS) Equation for Two-Dimensional Systems by Alexander A. Andrianov et al, April 2022 (27 pages), Floquet integrability and long-range entanglement generation in the one-dimensional quantum Potts model by A.I. Lotkov et al, October 2021-April 2022 (24 pages), Probing Lorentz-Invariance-Violation Induced Nonthermal Unruh Effect in Quasi-Two-Dimensional Dipolar Condensates by Zehua Tian et al, May 2022 (12 pages), Emergent quantum mechanics of the event-universe, quantization of events via Denrographic Hologram Theory by Oded Shor et al, August 2022 (12 pages) and Emergent Sasaki-Einstein geometry and AdS/CFT by Robert J. Berman et al, Nature Communications, January 2022 (8 pages) which I found has some connections with Exploring uberholography by Dmitry S. Ageev, August-September 2022 (14 pages) which deals with some quantum error correction code. To some extent, this complexity can be fun. See also Variational quantum algorithm for measurement extraction from the Navier-Stokes, Einstein, Maxwell, Boussniesq-type, Lin-Tsien, Camassa-Holm, Drinfeld-Sokolov-Wilson, and Hunter-Saxton equations by Pete Rigas, September 2022 (144 pages) which requires a significant mathematical background, Quantum Pontryagin Neural Networks in Gamkrelidze Form Subjected to the Purity of Quantum Channels by Nahid Binandeh Dehaghani et al, March-June 2023 (6 pages) and See Multiself-loop Lackadaisical Quantum Walk with Partial Phase Inversion by Luciano S. de Souza et al, May 2023 (16 pages).

It merged social sciences and quantum gravity and was published in a social science publication, not a quantum physics one [137]. Also, take care about papers published on April fool's day[138].

Hopefully, quantum scientific publications are way more serious than most of the quantum hype that is conveyed by general news with their amazing amplification capabilities. You'll read time and again that quantum computing will drive autonomous cars, create quantum intelligent robots, reduce CO_2 emissions, cure cancers, help Tesla or Hyundai build top-notch batteries or that quantum communications will teleport your data faster than light around the Earth. Most of these assertions will flourish when the IBMs and Googles of this world make fancy announcements or after your government launches its own "billion dollars" national quantum plan. But they are at least unproven if not entirely false. Who's going to reveal it to you?

Can ChatGPT help?

ChatGPT and the LLM (large language models) frenzy was the technology buzz of late 2022 and 2023 and it is no surprise that the quantum ecosystem was curious about it. A misplaced one was about predicting that quantum computing would make it easier to build large LLMs. It will not since quantum computing is not relevant for big data applications as we'll uncover in this book.

The right consideration is about getting some help when learning and doing research on quantum science. Machine learning is already used to conduct quantum research in many areas, like for the tuning of the frequencies used in tunable frequencies superconducting qubits with Google, or in designing complicated quantum photonics experiments[139] [140] [141].

Some early ChatGPT experiments were sending mixed signals about LLMs. Chris Ferrie was picky when he denounced the inexactitudes of ChatGPT trying to explain what quantum computing is[142], when ChatGPT was just parroting what is being repeated ad nauseam in the literature. Scott Aaronson – who now works for OpenAI - had ChatGPT passed his final course exam, and it fared relatively well with a 69/100 grade when his students get an average of 74.4/100[143]. I tested OpenAI ChatGPT 3.5 and Google Bard to get answers to the questions I laid out starting page 3 and they fared relatively well although not responding with much details or supporting data and links.

When you know how ChatGPT is constructed and its limits, you can still use it on a daily basis as an extension to your regular search tools. It can help you segment topics, differentiate two algorithms, list characteristics of a system and so on. It works better on intemporal science since it is not frequently updated.

A recent survey conducted by Nature did even show that 25% of researchers were using LLMs to write responses to research grant RFPs[144]. This survey included researchers in all disciplines with an apparent LLM usage peak with computer science researchers. We could expect that quantum physics researchers would be less keen than quantum information science researchers to use LLMs in writing RFP responses.

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[137] See Transgressing the Boundaries: Towards a Transformative Hermeneutics of Quantum Gravity by Alan D. Sokal, 1996 (39 pages).

[138] Like Spontaneous Human Combustion rules out all standard candidates for Dark Matter by Frederic V. Hessman and J. Craig Wheeler, April 2023 (6 pages).

[139] See Terry vs an AI, Round 1: Heralding single-rail (approximate?) 4-GHZ state from squeezed sources by Terry Rudolph, March 2023 (4 pages).

[140] See Computer-inspired Quantum Experiments by Mario Krenn, Manuel Erhard and Anton Zeilinger, February 2020 (17 pages).

[141] See Digital Discovery of 100 diverse Quantum Experiments with PyTheus by Carlos Ruiz-Gonzalez et al, October 2022 (44 pages).

[142] See ChatGPT's explanation of quantum computing is bullsh*t by Chris Ferrie, January 2023.

[143] See GPT-4 gets a B on my quantum computing final exam! By Scott Aaronson, April 2023.

[144] See AI and science: what 1,600 researchers think by Richard Van Noorden and Jeffrey M. Perkel, Nature, October 2023 (5 pages) and ChatGPT use shows that the grant-application system is broken by Juan Manuel Parrilla, Nature, October 2023.

Other use cases are created like using ChatGPT to construct new circuits like variational algorithms ansatzes (we'll cover these notions later in the book)[145]. Indeed, once all the existing formalized knowledge from a scientific field is used to train an LLM, the possibilities are limitless.

Roles

In most countries and in all disciplines, several roles can be distinguished in research organizations.

Doctoral students are students who are undertaking a doctoral thesis (PhD, for Philosophy Doctorate, for any science). It lasts from three to five years depending on the country. This thesis completes a higher education program in the University.

Post-docs or post-doctoral researchers are researchers who, after having obtained their PhD, conduct research in a laboratory under a fixed-term contract. They sometimes do several post-docs in different locations, frequently out of their originating country. It is the anteroom of a full-time research position.

Researchers have a full-time tenure in a research organization whether in the industry or with government funded research organizations. In many countries, they are also civil servant researchers recruited through some open competitions process .

Habilitation to Direct Research (HDR in France) allows a tenured researcher to direct the thesis of one or more doctoral students as a thesis director and to obtain a university professorship. The rules vary from country to country, such as having completed two doctoral theses and having published internationally recognized work in one's field[146].

Research Directors are researchers with the possibility to autonomously determine the field of their research work. They supervise several doctoral students and post-docs when they are successful with finding the related public and/or private funding. They are also selected by competition in research institutions. Depending on the country and research organization, there are several grades in the function, linked to advancement over time and merit.

Principal Investigators are lead researchers who are in charge of the preparation, conduct, resources allocation and administration of a research grant for which they are the project lead researcher and main holder. Sometimes, a PI is synonym of laboratory director or research group leader.

In addition to these roles, let's not forget the **laboratory technicians** who set up the experiments and about whom less is said and the **engineers** who can play a role in the creation of many scientific instruments.

h-index

The h-index, named after its creator Jorge Hirsch in 2005, is an index that quantifies a researcher's productivity and scientific impact. It is based on the level of citations of his scientific publications in peer-reviewed journals. It is a bit like a PageRank for a website, but a simpler one. It is an integer corresponding to the number of papers h that have each obtained more h citations in other papers (Figure 73).

The level of h-index can be used as a quantitative data for obtaining a position as a resident researcher (10-12), professor (>18) or member of an academy of science (>45).

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[145] See Unleashing the Potential of LLMs for Quantum Computing: A Study in Quantum Architecture Design by Zhiding Liang et al, University of Notre Dame, Purdue University, Peking University, University of California, Berkeley, Georgiatech, University of Wisconsin, July 2023 (9 pages).

[146] This habilitation replaced the Doctorat d'Etat in 1984 in France. The HDR is considered to be a diploma. It is awarded on free application by the research commission of the Universities which deliberates in the form of a jury.

As with any composite index[147], it generates side effects: a race to "publish or perish" papers of little incremental value, cross-referencing between researchers, self-citation, an abundance of co-authors[148], etc.

The discrepancy of h-index is quite high with researchers with a Nobel prize in physics with low index like with John Clauser (29, Nobel in 2022) and Brian Josephson (22, Nobel in 1973) and very high index like Anton Zeilinger (139, Nobel in 2022) or David Wineland (122, Nobel in 2012).

Some alternatives indexes have been proposed like the recent h-frac, but not yet adopted[149]. It remains, however, an interesting indicator of the influence of researchers and their production volume. On average, the h-index of a researcher in physics is close to the length of his career since his PhD. It obviously evolves over time. It is full of flaws like all quantitative indicators. For example, the basic h-index does not distinguish between the main author and the co-authors. Hence the abundance of authors cited in many papers, some of them having made only marginal contributions.

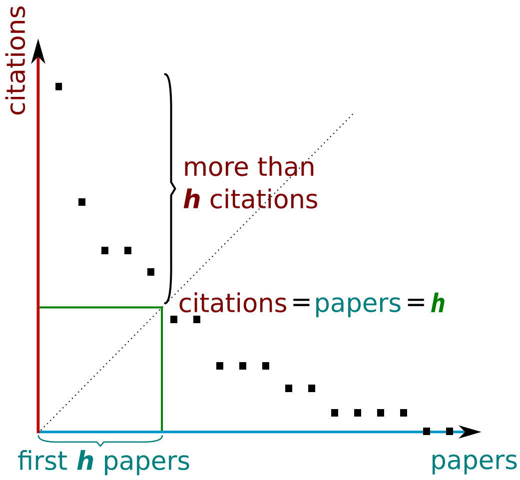

Figure 73: h-index explained graphically.

The index is usually calculated from **Google Scholar** data, but it is sometimes found calculated only on the SemanticScholar website. The most serious index is provided by the Website of **Science** because its database is the cleanest.

Poster sessions

In a scientific conference, a "poster session" is usually a part of the conference dedicated to the presentation of researchers' projects during a break, in a dedicated area.

Researchers display a poster describing their research work and talk with conference participants as they stroll through the conference exhibition area during dedicated breaks. It is an exercise in humility reminding what Jehovah's witnesses are doing in the streets.

Figures of merit

This common expression broadly describes a set of specifications and the success metrics to be achieved to bring a given technology to fruition. DiVincenzo's qubit technology criteria can be considered a figure of merit for success for quantum computing. It usually provides a roadmap and set of goals for researchers and technology vendors.

International

Nowadays, all modern countries have crafted their "quantum national plan" with a certain willingness to better control their sovereignty. It is like being the first with the atomic bomb during World War II.

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[147] The Shanghai ranking list of universities comes to mind.

[148] In this paper from Google, we have no less than 85 co-authors: [Implementing a quantum approximate optimization algorithm on a 53-qubit NISQ device] by Bob Yirka, February 2021 (19 pages). It's a bit too much and we can wonder about their all contributions!

[149] See [The h-index is no longer an effective correlate of scientific reputation] by Vladlen Koltun and, David Hafner, Intel Labs, February 2021 (26 pages). Among other things, the authors found out that the correlation between h-index and scientific awards in physics is declining. They propose an alternative index named [h-frac], for h-fractional, that improves the correlation between the index and other scientometric measures like scientific awards. It allocates citations fractionally and evenly among all coauthors of scanned papers to avoid the phenomenon of low-contribution hyperauthors.

But let's remember that international collaboration between researchers is intense. Most of those I met in French laboratories collaborate with colleagues either in Europe within the framework of Europe 2020 projects, the European Flagship or for some ERCs. They also collaborate with researchers outside the European Union, particularly in Asia (Japan, Singapore), as well as in the USA, UK, Switzerland and Australia[150].

Quantum science knowledge is quite open and is rather well shared on a global scale. This is encouraged by many international scientific conferences where knowledge is being built, researchers get to know each other, and joint projects are being launched.

This is one of the reasons why I don't believe in the existence of a supposed quantum computer whose capabilities would defy understanding and which would be hidden in the basement of a secret NSA datacenter to break all the RSA keys of the Internet.

Scientific nationalism in quantum technologies finally comes into play further downstream of research, when it comes to transforming it into industrial advantage. Technologies often have their "magic sauce", as in semiconductor manufacturing processes. This has always been the case in digital technologies.

Technology Readiness Level

This technology readiness level notion is commonly used in deep techs. It describes the level of maturity of a technology with a scale from 1 to 9 (Figure 74). It follows a relatively standardized classification initially created by NASA in 1975[151], then used by the European Union and various other organizations. It was initially mainly used in the aerospace, defense and energy industries.

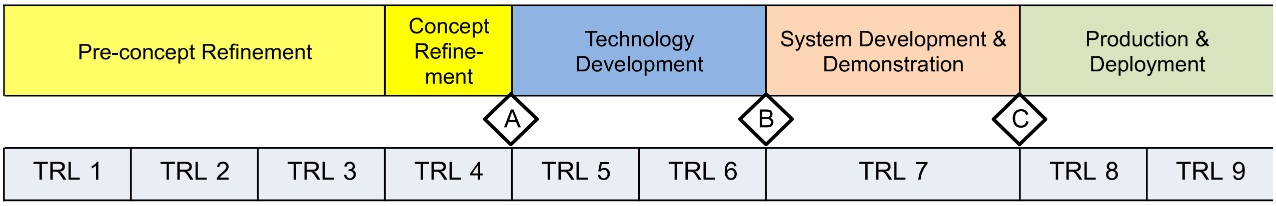

Figure 74: the scale of technology readiness level.
Source: Some explanations on the TRL (Technology readiness level) scale, DGA, 2009 (15 pages).

This scale can have several use cases. It is used to assess the level of risk and maturity for an investor in a startup. Very advanced deep techs are also the playground of TRL and quantum technologies are no exception.

The TRL scale has 9 levels[152]:

- **TRL 1**: basic principles are described or observed, at the theoretical or experimental stage.
- **TRL 2**: technological concepts are formulated and not yet necessarily tested.
- **TRL 3**: proof of concept is carried out in a laboratory, at the level of the technical process.
- **TRL 4**: the technology is validated in the laboratory as a whole.
- **TRL 5**: a technology model in a production grade environment is created.
- **TRL 6**: a technology prototype is demonstrated in an environment representative of the intended use case.

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[150] This can also take the form of CNRS International Mixed Units such as those established in Japan and Singapore.

[151] See Technology Readiness Levels at 40: A Study of State-of-the-Art Use, Challenges, and Opportunities by Alison Olechowski et al, 2015 (11 pages) which is the source of the diagram.

[152] See Technology Development Stages and Market Readiness by Surya Raghu, June 2017 (35 slides).

- **TRL 7**: a prototype is evaluated in an operational environment.
- **TRL 8**: a complete system has been evaluated and qualified.
- **TRL 9**: a complete system is operational and qualified in production.

The relevance of the solution to market needs is missing at this scale, but it is a marketing rather than a technical consideration[153]. Most of the time, it more or less coincides with TRL levels 7 to 9 since reaching this scale requires funding and finding customers willing to test the solution.

Kristel Michielsen has proposed a scale suitable for quantum computing, the QTRL, for the Quantum Technology Readiness Level in Figure 75. Her assessment of some technologies can be argued. For example, she positions D-Wave's quantum-annealed computers in TRL 8 and 9. This is commercially correct since these computers are well marketed. This being said, if they are well available physically, it is not proven that they are of much use at the moment.

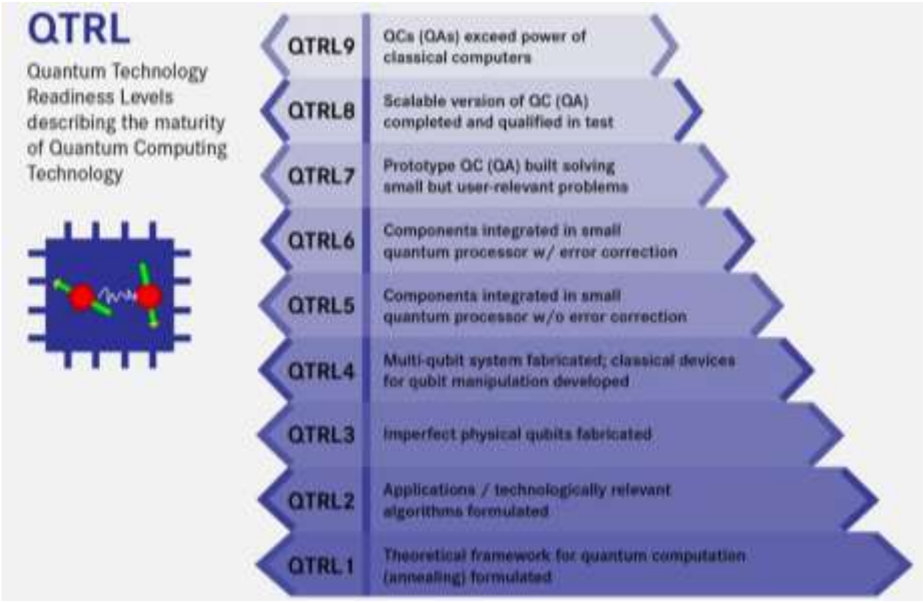

Figure 75: the quantum TRL scale, created by Kristel Michielsen. Source: Simulation on/of various types of quantum computers by Kristel Michielsen, March 2018 (40 slides).

The TRL scale can be extended with two additional market development steps as shown in Figure 76.

The specificity of quantum technologies is that many hardware startups are created with very low TRLs. This is particularly true for those who are starting to design qubits using technologies that have not yet been proven, even in the labs.

In quantum technologies, the notions of "MVP" (minimum viable product) are very different from the classical digital world. It is based on scientific rather than functional metrics. We have many such startups around in quantum technologies because of the famous FOMO (fear of missing out) syndrome with investors.

This shows up with investors who fear of missing the future golden goose or unicorn. They are ready to overinvest in companies they perceived will be the future market champion. This explains for example the level of funding for startups like **Rigetti** and **PsiQuantum** or the new SPAC funding mechanism (special purpose acquisition company) implemented by **IonQ**, **Rigetti** and **D-Wave** and the quantum business spin-off from **Honeywell** and its merger with **CQC** which became **Quantinuum** in December 2021.

[153] See TRL, MRL, POC, WTF? by Massis Sirapian, France Defense Innovation Agency, April 2019.

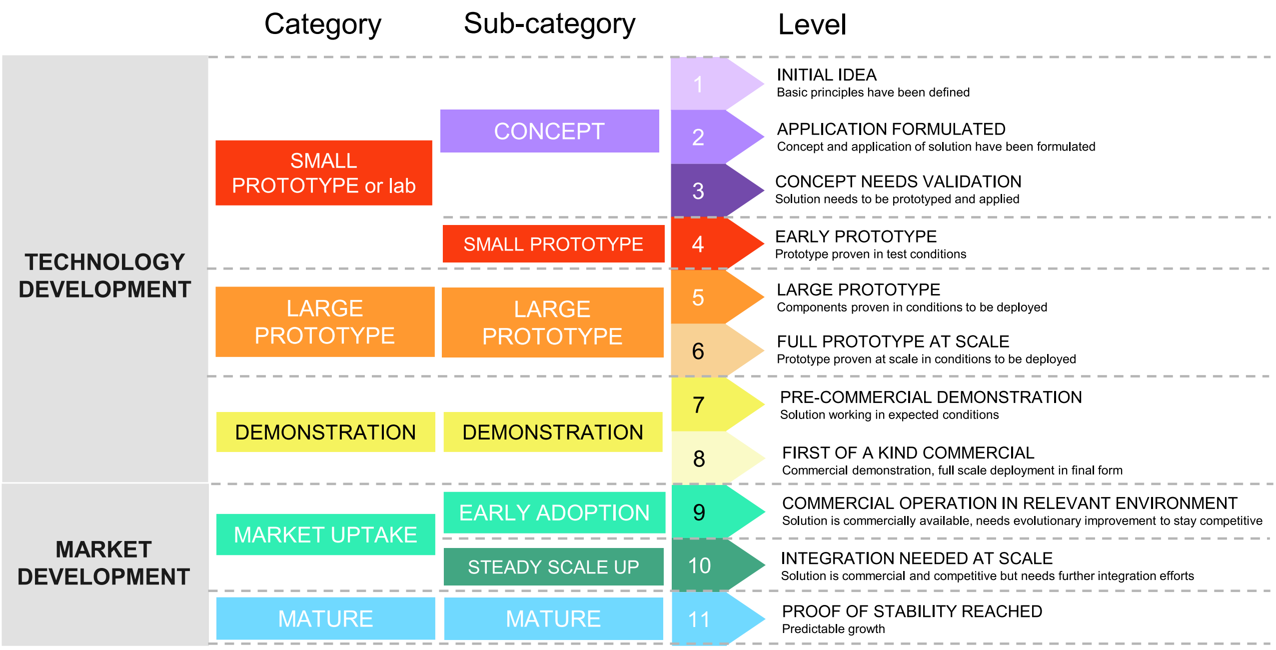

Figure 76: this other TRL scale has 11 levels, adding levels 10 and 11 for integration at scale and proof of stability reached. Source: Ammonia Technology Roadmap, IEA, October 2021 (168 pages), page 108.

Quantum physics history and scientists key takeaways

- A first wave of 19th century scientists laid the groundwork that helped create quantum physics afterwards (Young, Maxwell, Boltzmann, mathematicians). The photoelectric effect, black body spectrum and atoms emission or absorption spectrum were not explained with the current theoretical frameworks.
- Starting with Max Planck, a second wave of scientists (Einstein, De Broglie, Schrodinger, Heisenberg, Dirac, Born, Von Neumann) created quantum physics to describe light/matter interactions, energy quantification and wave-particle duality. It solved most of the 19^{th} century unexplained physics experiments.
- These scientists were theoreticians while many lesser-known researchers were experimentalists with landmark discoveries (superconductivity, electron interferences, Stern-Gerlach experiment, ...). Quantum physics also relies on a significant body of mathematics like linear algebra and group theory.
- After World War II, all digital technologies (transistors, lasers, telecommunications) were based and are still based on quantum physics, as part of what is now called the first quantum revolution.
- Since the 1980s and thanks to advances in individual quantum objects control and the usage of quantum superposition and entanglement, new breeds of technologies were created, most of them belonging to the “quantum information science” field and being part of the second quantum revolution. Many of these research programs were funded by governments after Peter Shor’s integer factoring algorithm was created.
- While the first quantum revolution was driven by research coming mostly out of Europe, the last wave comes out of all countries across several continents (North and South America, Europe, Asia/Pacific).
- This book also describes how research works in general and particularly in quantum physics and information science. It explains how scientific papers are written and communicated, how researchers are evaluated, and how quantum technologies readiness level can be assessed.

Quantum physics 101

Let's now look at the fundamentals of quantum physics in a more structured way. Several years of undergraduate and graduate studies are usually necessary to master quantum physics notwithstanding its rich mathematical foundations. This part will provide some background knowledge that will help you better understand the various quantum information systems and technologies exposed in the remainder of this book.

Quantum physics appeared at the beginning of the 20th century to explain the dynamics of elementary particles, particularly to study how **photons**, **electrons** and **atoms** behave and interact[154] (Figure 77). Quantum physics also deals with elementary particles from the standard model like quarks and neutrinos, but it is usually out of the scope of the "second quantum revolution"[155]. In some cases, we still care about atom nucleus spins, which relate to proton spins, itself linked to its quark constituents. Nucleus spin plays a role in various quantum technologies like with NV center and donor spins qubits. We also care about it with electron spin-based qubits since nucleus spin can have a detrimental impact on electron spins handling qubits information. It relates to the kinds of isotopes of carbon and silicon that are used in carbon nanotubes and silicon wafers used to create electron spin qubits.

Although sometimes mysterious and said to be "incomplete" by some, quantum physics has gone through the test of time and experiments for over a whole century. Thousands of experiments have validated the underlying postulates, theory and mathematical formalism behind it even though we still cannot describe the physical nature of quantum entanglement, of the electron spin and of the wave-particle duality phenomenon.

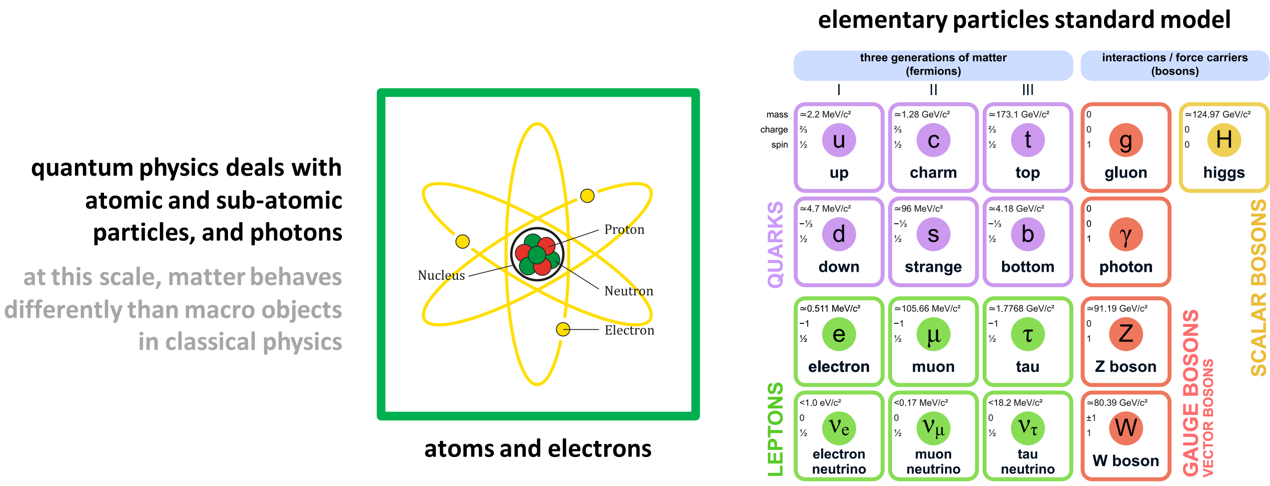

Figure 77: what particles are we dealing with quantum physics? All of them, but in the second quantum revolution, we mainly use electrons, photons, atoms and their nucleus spin. Source: Wikipedia.

Quantum physics first helped explain various observations such as the **black-body radiation** (solved by Max Planck in 1900), the **photoelectric effect** (solved by Albert Einstein in 1905) and the **sharp spectral lines** observed with excited atoms like hydrogen (solved by Niels Bohr and its atom model in 1913).

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[154] As a reminder, here are the dimensions of elementary particles: 10^{-10}m for an atom, 10^{-15}m for the diameter of a hydrogen atom nucleus, thus of a single proton, and 10^{-18}m for that of an electron. As written in Quantum Theory Needs No 'Interpretation' by Christopher A. Fuchs and Asher Peres, 2000 (2 pages). "*We have experimental evidence that quantum theory is successful in the range from 10^{-10} to 10^{15} atomic radii*". A 10^{25} scale of operations is significant and not marginal!

[155] See Neutrinos as Qubits and Qutrits by Abhishek Kumar Jha et al, March 2022 (30 pages) which makes a proposal to use neutrinos for quantum computing, without taking care of the related engineering problems. It's very hard to contain and control neutrinos!

Later on, in the mid 1920's, quantum physics was built upon a **mathematical formalism** using multi-dimensional Hilbert spaces and vectors. It centered around the **Schrödinger wave equation** which describes how a massive particle like the electron behaves over space and time, using complex number probability amplitudes and differential equations over time and space. These provide a probabilistic insight on the outcome of the measurement of a particle's energy, momentum, and many other physical properties.

Quantum mechanics differs from classical physics with demonstrating how and why quantum particles energy, momentum, angular momentum and other metrics are restricted to discrete values (**quantization**), objects behave as particles or waves depending on the context (**wave-particle duality**), and there are limits to how accurately the value of a physical quantity can be predicted prior to its measurement, given a complete set of initial conditions (**indeterminacy principle**).

It also refers to **state superposition** which is at the basis of qubit operations and one of the sources of the quantum computers processing parallelism and **entanglement** which is a direct consequence of superposition applied to several quantum objects and is used with multi-qubits quantum gates and is also related to quantum communications and cryptography. Quantum objects **no-cloning** is a particular aspect of quantum physics that limits what we can do with qubits and how memory is managed. At last, **quantum tunnelling effect** has some impact in quantum technologies, like with the Josephson junctions used in superconducting qubits and with D-Wave quantum annealers (Figure 78).

We will see later, page 110, that there is an exception to this definition with **continuous variable quantum systems** which check all the above items at the exception of the discretization of their observable properties.

Quantum physics explains other physical phenomena belonging to the broad **quantum matter** category which can be of macroscopic scale compared to atomic and electron particle scales, like **superconductivity** which plays a key role in superconducting qubits, **superfluidity,** used with liquid helium in dilution refrigerators and **quantum vacuum fluctuation** and its role in quantum decoherence. It also enabled the creation of **lasers**, used in many places like for controlling cold atom and trapped ion qubits and for all photonic based quantum computing and telecommunications. At last, **polaritons** are sets of interactions between light and semiconductors which could become useful in quantum sensing and quantum simulation. The quantum objects bestiary also includes **skyrmions** and **magnons**!

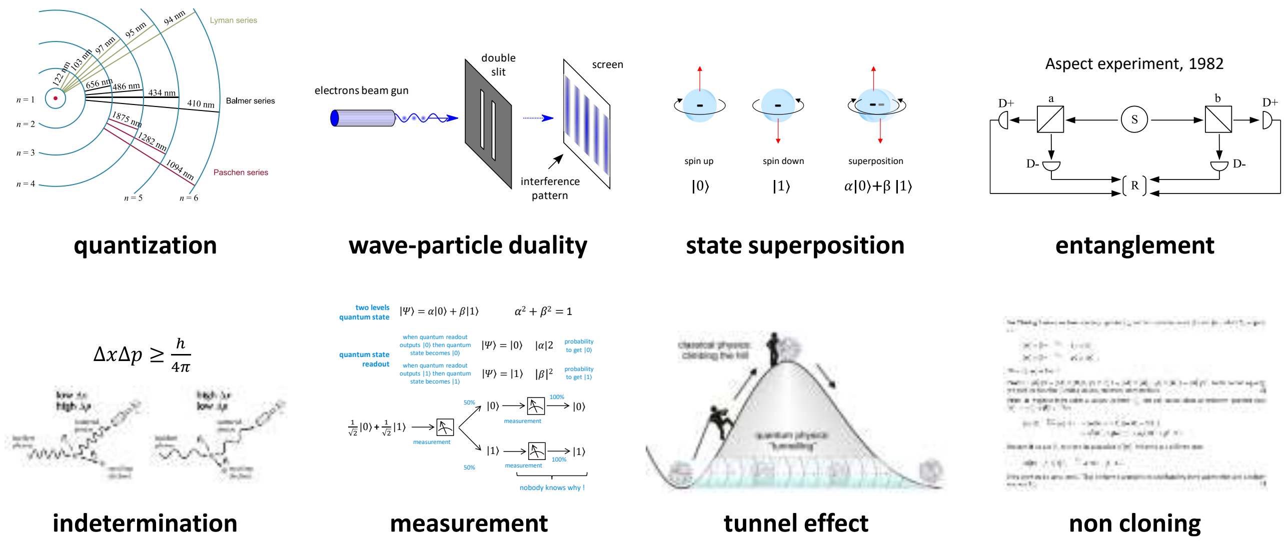

Figure 78: eight key dimensions of quantum physics that we are dealing with. (cc) compilation Olivier Ezratty, 2021-2023.

Postulates

Quantum physics formalism is based on a set of postulates that follows[156]. Why are these postulates and not laws? Mainly because they describe a mathematical formalism that cannot be proved per se.

One of the other reasons is that quantum physics does not rely on an ontology describing the physical objects it is based upon. I'll try whenever possible to connect these postulates with some physical meaning. If all of this seems gibberish for you, skip it!

Postulate I - Quantum state: the state of an isolated physical system is represented, at a given time t, by a state vector $|\psi\rangle$ (psi) belonging to a Hilbert space H called the state space with vectors of length 1, using complex numbers. This is the canonical definition of a quantum state. The $|\psi\rangle$ vector contains the knowledge we can have of a quantum system, represented by the values taken by its measurable and compatible properties. A broader definition of a quantum state is the ensemble of values taken by compatible physical properties of a system made of one or several quantum objects. These compatible properties must be measurable simultaneously or in any order. The $|\psi\rangle$ vector is a mathematical object that helps determine and predict over time the probabilistic distribution of the various values of the quantum object compatible properties. The immediate consequence of this first postulate is the notion of superposition where a linear combination of several $|\psi\rangle$ vectors can form another valid quantum state. For a generic qubit, its quantum state defines its amplitude and phase as we'll see later in the Bloch sphere description. $|\psi\rangle$ is then a vector in a two-dimensional Hilbert vector space combining the $|0\rangle$ and $|1\rangle$ basis states with their related complex amplitudes.

Postulate II - Physical quantities: are related in quantum physics with observables that are mathematical operators $\hat{A}$ acting on the $|\psi\rangle$ vector as $\hat{A}|\psi\rangle$. With the quantum matrix formalism, $\hat{A}$ is a Hermitian (linear) matrix operator acting on the state vector $|\psi\rangle$ to evaluate quantized or continuous physical properties of quantum objects. This operator is a self-adjoint matrix, with the implication that several consecutive measurements generate the same (vector) result. A projector operator like a Pauli matrix σ_x, σ_y or σ_z used to measure a qubit state is a specific case of an observable operator.

By the way, let's clearly define properties and their variations:

Properties correspond to a quantum system's various observables. For a photon, it can be, for example its phase, polarization, and wavelength. In quantum physics, it is not possible to evaluate the values of all properties of quantum systems to describe it, due to Bohr's complementarity principle. Properties can also be continuous like a quantum object momentum or position.

Exclusive property values are the possible results of a quantum measurement of a quantized property. The classical examples are vertical and horizontal polarization for a photon or spin up or down for an electron spin along a projection axis. These are mutually exclusive since it corresponds to two results of a physical measurement. Mathematically speaking, two properties are exclusive if their projector operators (*aka* observables...) are orthogonal. Otherwise, these are non-exclusive properties.

Compatible properties of a quantum system can be measured in any order or simultaneously[157]. In that case, their observable operators A and B commute (AB=BA), or their commutator is equal to zero ([A,B]=AB-BA=0)[158]. Compatible properties have commuting observables.

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[156] Source: Wikipedia.

[157] The notion of properties compatibility must not be confused with complementarity. There is complementarity between incompatible properties, like position and momentum! Incompatible observables are related to conjugate variables, defined by one being a Fourier transform of the other and Heisenberg's indeterminacy principle being consequently applied to both these variables measurement. See Bohr's Complementarity and Kant's Epistemology by Michel Bitbol and Stefano Osnaghi, 2013 (22 pages) which lay out well these different concepts.

[158] Compatible properties are well explained in Mathematical Foundations of Quantum Mechanics: An Advanced Short Course by Valter Moretti, 2016 (103 pages).

Measuring a complete set of commuting observables (CSCO) constitutes the most complete measurement of a quantum system.

Incompatible properties aka conjugate variables cannot be measured simultaneously and their observable operators A and B do not commute (AB <> BA or [A,B] $\neq$ 0). This is a particularity of quantum mechanics.

However, revealing one property value with a measurement doesn't exclude revealing another property afterwards. But it is not possible to obtain exact knowledge of both properties at the same time (in the probabilistic sense and following Born's rule). At least one will be totally probabilistic. For a single particle, one example of incompatible properties or observables are two different spin components (X and Y or X and Z). After measuring the X spin component, a Z measurement will yield a random result. Also, the energy and position of an electron are incompatible properties.

Postulate III - Measurement: is the result of a physical quantity measurement with an observable operator A. The measurement result is one of the observable operator eigenvalues. We define eigenvalues later starting page 167 and cover the related mathematical formalism in the measurement section of this book starting page 209. This postulate is sometimes embedded or associated with the previous one. The observable operator doesn't generate a measurement result per se. It helps create a probabilistic distribution of the possible measurement outcomes of a property given what is mathematically known of the quantum object state vector. When applied to a quantum object vector, it creates another state vector along the eigenvectors of the observable operator. It can then serve to create a series of real numbers describing the probabilities of the various exclusive values a given property can take. The **expectation value,** or **predicted mean value**, is the average value of repeated measurements that would be obtained with the physical implementation of the observable. We'll come back to this later starting page 209. The measurement postulate is also named the Von Neumann measurement postulate.

Postulate IV - Born rule: when the physical quantity A is measured on a system in a normalized state $|\psi\rangle$, the probability of obtaining an eigenvalue α_n for discrete values or α for continuous values of the corresponding observable A is given by its squared amplitude of the related wave function. It is a projection on the corresponding eigenvector. This is related to Max Born's probability rule. A quantum state can be generally represented by a density operator, which is a square matrix, nonnegative self-adjoint operator ρ normalized to be of trace 1. The average expected value of A in the state ρ is $tr(A\rho)$, the trace (sum of diagonal matrix values) of the observable operator applied to the density matrix[159]. This postulate is sometimes merged with the measurement postulate. This postulate is associated with the principle of spectral decomposition. For a single qubit, the Born rule is simple to describe with $|\alpha|^2$ being the probability of getting a $|0\rangle$ and $|\beta|^2$ of getting a $|1\rangle$ when the qubit state is described as $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ with α and β being complex numbers. And due to probabilities normalization, $|\alpha|^2 + |\beta|^2 = 1$.

Postulate V - State collapse: only one result is obtained after a quantum measurement. Two sequential measurements based on the same observable operator will always output the same value. For a qubit, after we measure its state, whatever it is, we get a $|0\rangle$ or a $|1\rangle$ and this becomes the new qubit state after measurement.

159 There are variations of this postulate for various quantum spectrum (discrete and nondegenerate, discrete and degenerate, continuous and non-degenerate). Degenerate spectrum is defined in the glossary.

Postulate VI - Time evolution: the time evolution of the state vector $|\psi(t)\rangle$ is governed by the Schrödinger wave equation[160]. We don't directly deal much with time evolutions to understand quantum computing with qubits and gates, but it still plays a key role in quantum annealing and quantum simulation and, behind the scenes, in gate-based computing, with qubits decoherence, quantum noise, quantum error corrections mechanisms and measurement.

There is also a **Composition** postulate, which defines the notion of tensor product applied to separable composite quantum systems. *Aka* "Composite Systems" with John Preskill's axioms. We'll talk about it abundantly when covering linear algebra starting page 165 and qubit registers starting page 190.

There are indeed many variations of these postulates in shape, form, name and number, which ranges from 4 to 9 depending on the source[161] (Figure 79). Quantum State can become State Space and Physical Quantities become Unitary Dynamics[162]. John Preskill lists five 'axioms', considering that postulates are axioms since they are not contradicted experimentally[163]. There is not really a single "bible" of quantum postulates even when reading quantum physics founders writings (Bohr, Heisenberg and others) who didn't agree on all of it. I have consolidated below a table with some of these variations of postulates. Imagine if there were various versions of the Bible with 5, 7, 9, 10 and 12 commandments!

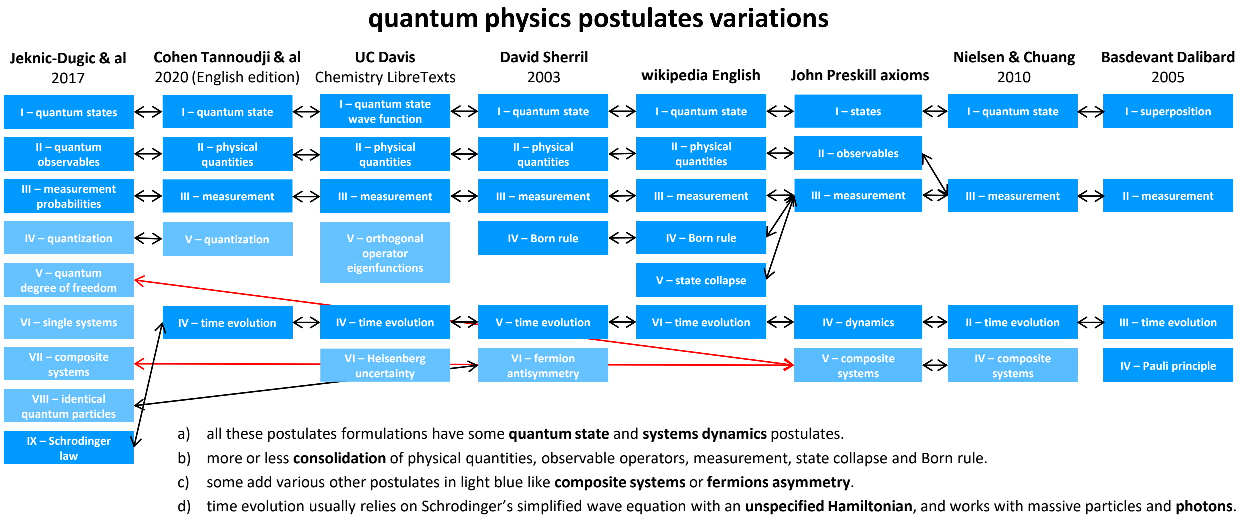

Figure 79: a compilation of various inconsistent lists of quantum postulates and axioms. (cc) Olivier Ezratty, 2022.

Mostly covered in linear algebra section starting page 165, the main related quantum physics mathematical tools are:

- **Linear algebra**: complex numbers, eigenvectors, eigenvalues and eigenstates.
- **Functional analysis**: Hilbert spaces, Hermitian matrices, linear operators, spectral theory.
- **Differential equations**: partial differential equations, separation of variables, ordinary differential equations, Sturm–Liouville problems, eigenfunctions.
- **Harmonic analysis**: Fourier transforms and series.

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[160] As a result, the postulates are applicable for massive non-relativistic particles. Relativistic massive particles time evolution is described by the Dirac and Klein-Gordon equations while photons are covered by Maxwell's equations and their various derivations.

[161] 9 postulates are listed in Axiomatic quantum mechanics: Necessity and benefits for the physics studies by J. Jeknic-Dugic et al, 2017 (23 pages).

[162] In Quantum mechanics distilled by Andy Matuschak and Michael Nielsen on the Quantum Country site.

[163] See Lecture Notes for Ph219/CS219: Quantum Information Chapter 2 by John Preskill, California Institute of Technology, July 2015 (53 pages).

Quantization

In quantum physics, material or immaterial quantum objects have some physical properties that are discontinuous and not continuous like distances in classical physics. This frequently corresponds to the orbits of electrons around atomic nuclei which are defined in a discrete way, to atom energy levels, but also deals with electrons, atom nucleons and nucleus and photons various properties (Figure 86). The particles from the standard model (quarks, gluons, neutrinos, …) that are studied in high energy particles physics (HEP) also have their quantized properties, but we won't deal with it here.

Principle

There is a correspondence between the discontinuous energetic transitions of electrons in orbit around atoms and the related absorbed or emitted photons. Quantization shows up in other various places like crystals. Atoms also form harmonic oscillators and vibrate at quantified amplitudes in crystalline structures, according to a model Einstein developed in 1907. You'll find many quantum oscillators all over the place, like in superconducting qubits.

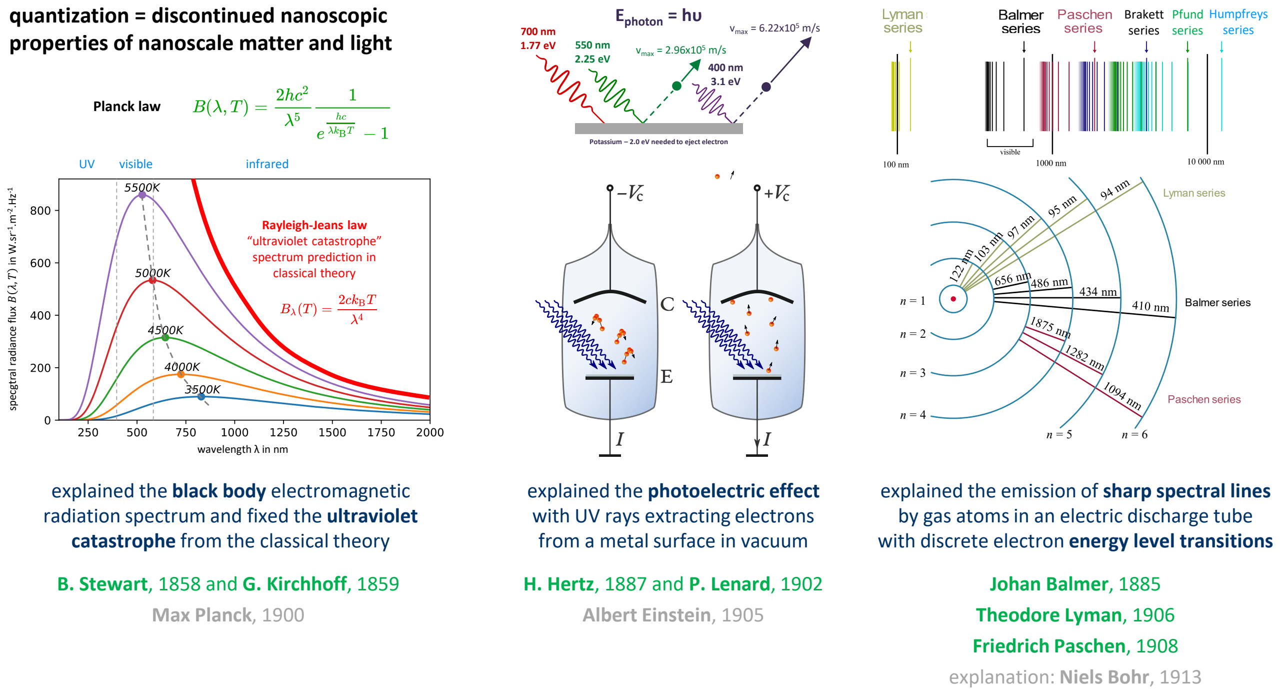

Figure 80: the three fundamental 19th century electro-magnetic waves experimental results which were later explained by quantum physics, all explained by quantization of the electro-magnetic wave field. (cc) Olivier Ezratty 2023 compilation. Various sources.

Quantization was a way to progressively explain experiments done beforehand, the first being the blackbody radiation spectrum (Figure 80). This one marked the beginnings of quantum physics.

Before explaining black body spectrum, let's recall the three kinds of spectrum that can be usually found experimentally and are pictured in Figure 81.

- A **continuous spectrum** comes from a hot and dense body like the sun, a heated solid or a perfect such body *aka* black body. It contains light in all visible frequencies that come from the random excitement of atoms in the examined body.
- An **absorption spectrum** is usually made of a continuous source of light traversing an absorbing medium like a cold gas. The resulting spectrum will be a continuous one with black lines corresponding to the frequencies absorbed by the medium.
- An **emission spectrum** is created by some rarified hot gas. It shows discrete spectrum lines corresponding to photons emitted by the excited gas atoms at specific frequencies.

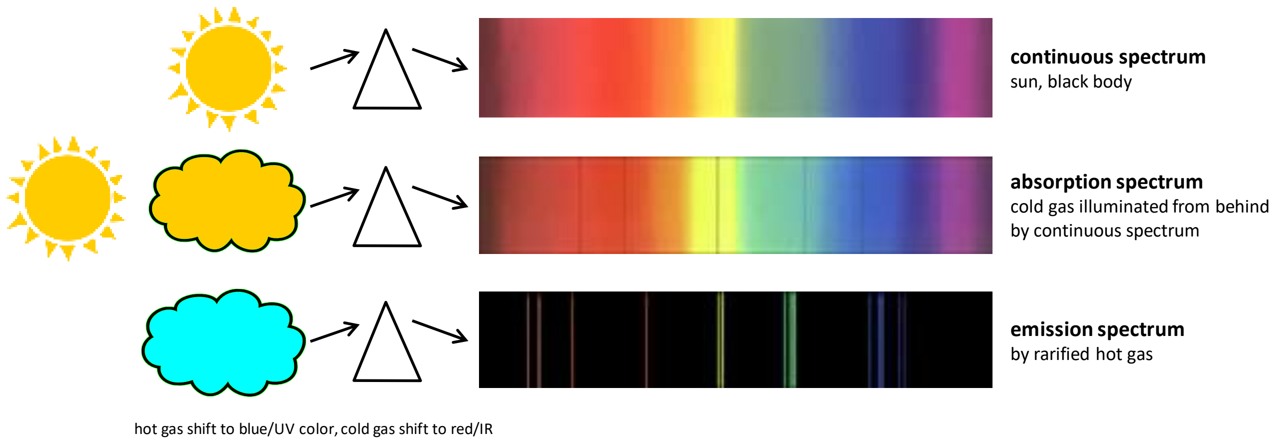

Figure 81: differences between continuous spectrum, absorption spectrum and emission spectrum.

Black bodies were theorized by **Gustav Kirchhoff** in 1859. These are ideal physical bodies in thermal equilibrium that absorb all incident electromagnetic wave radiations and reflects or transmits none. Since it absorbs all wavelengths, it is supposed to be black, although stars like the Sun are good approximations of black bodies and are not black at all. In usual experiments, a black body has a little hole that emits radiations which are analyzed by a spectrograph. The challenge which took four decades to be resolved was to evaluate the spectrum of the cavity's radiation.

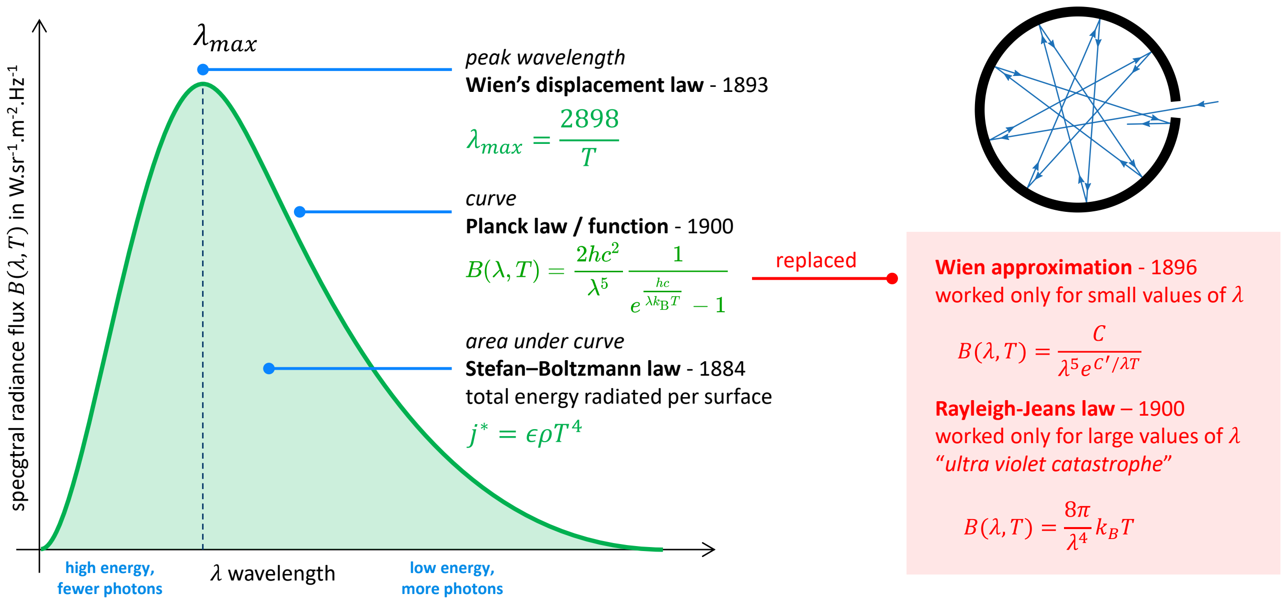

Figure 82: blackbody spectrum explanations over time. The sr in the radiance units decomposition is a steradian or square radiant, a unit of solid angle. Compilation (cc) Olivier Ezratty, 2021.

It was first discovered that the spectrum didn't depend on the body radiation and only on its temperature T and wavelength λ (lambda). It also proved that thermal radiation was an electromagnetic one. Hot objects like lightbulbs and heated metals are close to black bodies. As the temperature increases, the black body color, corresponding to the spectrum peak shown in Figure 82, shifts from red to blue. There were various attempts to explain the blackbody radiation with thermodynamics and oscillators and to predict the spectrum curve.

Before Planck's work, Stefan-Boltzmann's law (1884) described the relation between temperature and total energy radiated per surface area ($\epsilon \rho T^4$) and Wien's displacement law (1893) described the relationship between peak wavelength and temperature. These two laws worked well. **Wilhelm Wien** (1864-1926, Germany) even won the 1911 Nobel prize in Physics for this discovery.

Predicting the spectrum curve didn't work so well. First, Wien devised another law in 1896, Wien's approximation or radiation law that didn't work well with large wavelengths. The Rayleigh-Jeans formula created in 1900 didn't work for small wavelengths, leading to the so-called ultra-violet catastrophe. It was based on Boltzmann's statistical methods.

To make a better curve prediction, Max Planck guessed that the energy of the oscillators in the cavity was quantized and was a multiple of some quantity with the formula E = nhν, n being an integer, h being Planck's constant and ν the wave frequency. With this discretization, oscillators couldn't afford to have many energy quanta for high energy levels. Thus, their number decreased as the frequency increased instead of growing exponentially as in Rayleigh-Jeans law.

There was however no clear explanation on the origin of these quanta. The second step was Albert Einstein's work on the photoelectric effect in 1905, explaining how light and electrons interacted in quantized form. He guessed that the energy from an electromagnetic field is not spread over a spherical wavefront but is localized in individual directional quanta, which were later described as wave packets with a speed (of light) and length. But light quantization can show up in many other photon's characteristics: their polarization, their frequency, their phase and other various characteristics.

Electrons quantum numbers

The Niels Bohr's atomic model elaborated in 1913 helped describe the electron energy transitions within atoms that explained the various hydrogen emission spectrums experimentally discovered by **Johan Balmer** in 1885, **Theodore Lyman** in 1906 and **Friedrich Paschen** in 1908, corresponding to transitions starting from the second, first and third atom electron layers. These are known as Balmer series, Lyman series and Paschen series. Other transitions from the fourth, fifth, sixth and seventh layers were later discovered in the infrared by **Frederick Sumner Brackett** in 1922, **August Herman Pfund** in 1924, **Curtis Judson Humphreys** in 1953, **Peter Hansen** and **John Strong** in 1973.

But other energy transitions like those from the **Zeeman** effect could only be explained by the existence of other quantum numbers.

During the 1920s, a better understanding of the quantum nature of electrons was achieved. It was progressively discovered that electrons in atom shells had four quantum numbers, as shown in Figure 83:

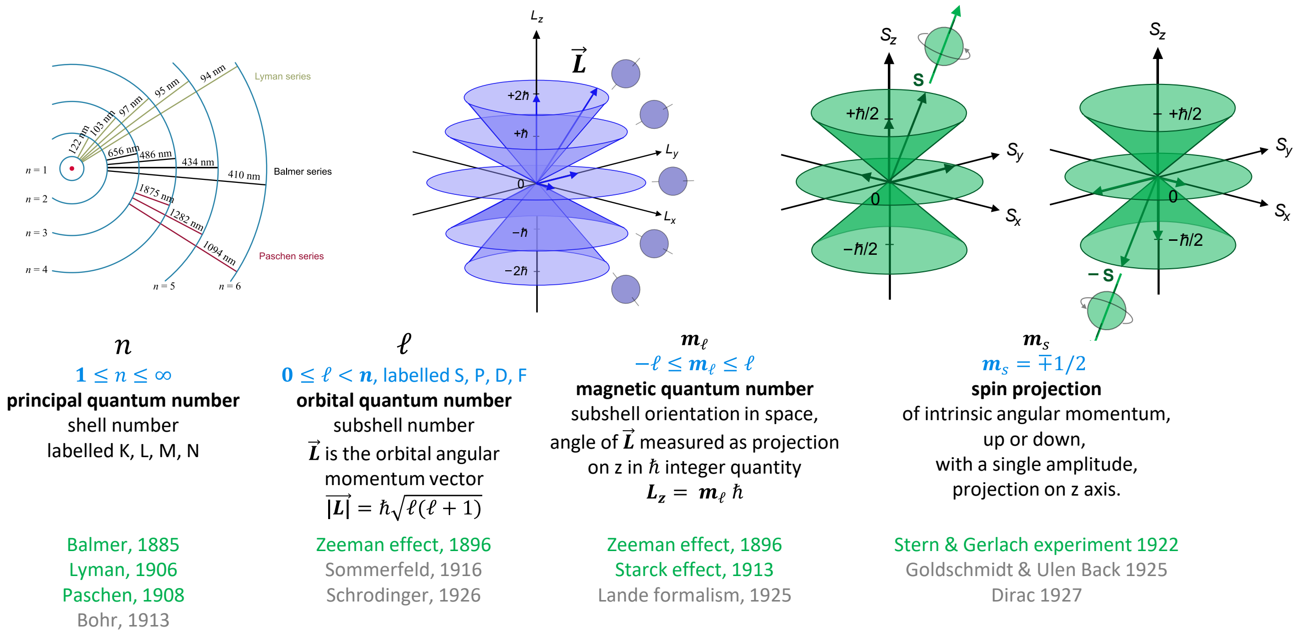

Figure 83: the four electron quantum numbers explained visually. (cc) Olivier Ezratty, 2023.

- n = **principal quantum number** corresponding to their energy level or electron shell in the atom electron shells, numbered from 1=K, 2=L, 3=M to n, n being very high for so-called Rydberg (high-energy) states close to atom ionization[164]. This number may correspond to some energy levels used in cold atoms and trapped ions qubits. It corresponds to the rows shown in Figure 84.
- ℓ = **orbital angular momentum** numbered from 0 to n-1 or letters (s, p, d, f, g, h, i, etc.) also named azimuthal or orbital quantum number, which describes the electron subshell and quantifies its amplitude. It corresponds to different types of orbitals around the atom and to the columns as shown in Figure 84.
- m_l = **magnetic quantum number** describing the orbital orientation in space, within its subshell. Its value or angle is also quantized and is an integer between $-\ell$ and ℓ.
- m_s = **spin projection quantum number** being either +1/2 or -1/2, in a given spatial direction (usually x, y or z in an orthonormal basis), called spin component, also named intrinsic angular momentum. This is the property used in so-called electron spin or silicon qubits. But… what is the unit of the spin? It is rarely mentioned but the spin unit is the Dirac constant $\hbar$, which equals the Planck constant h divided by 2π. It is an intrinsic property which doesn't depend on the situation like temperature. What physical property is it describing? It seems it doesn't describe a physical rotation of the electron around an axis. It may still be linked to some field rotation[165].

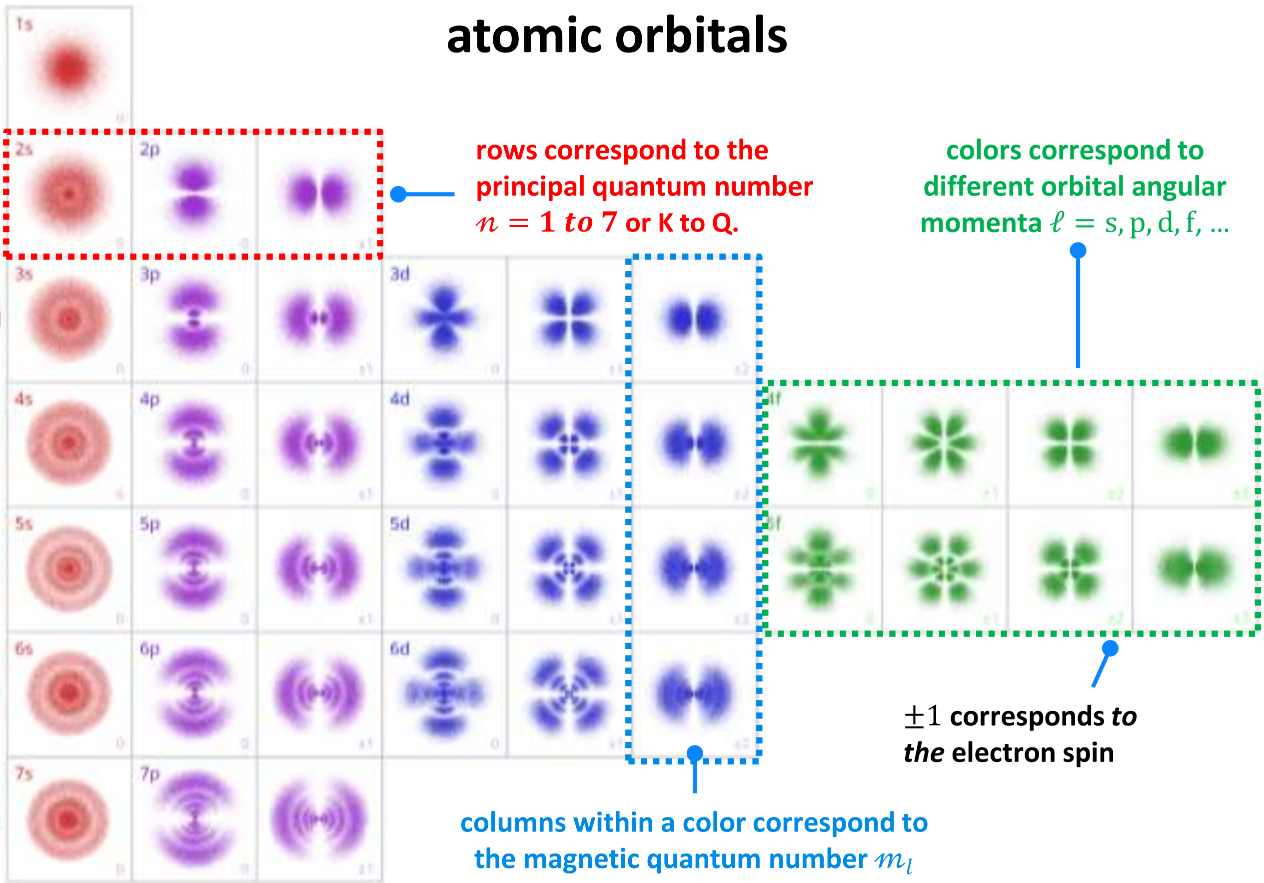

Figure 84: visual representations of the electron atomic orbitals of the hydrogen atom, corresponding to their principal number n, orbital angular momentum ℓ and magnetic quantum number m_l. Source: Keith Enevoldsen adapted by Olivier Ezratty, 2023.

[164] The principal quantum number is limited to 7 for non-excited atoms and is theoretically illimited with excited atoms. A record of n=766 was observed with hydrogen atoms in interstellar medium.

[165] See What's everything made of? by Charles Sebens, Caltech, October 2019 and The fundamentality of fields by Charles T. Sebens, September 2022 (32 pages).

It is key to understand the effect of these various electron quantum numbers in many fields like with NV centers and silicon spin qubits, quantum dot photon sources and many others.

Nucleons and nucleus quantum properties

Electrons have a spin but also atom nucleus and their nucleons constituents that are neutrons and protons. An atom nucleus has Z protons corresponding to the element atomic number and N neutrons which add up to a total of A=Z+N nucleons. Protons and neutrons have a spin of $1/2$. The nucleus has a half-integer spin ($1/2, 3/2, 5/2, \ldots$) when the number of neutrons plus the number of protons is odd, an integer spin ($1, 2, 3, \ldots$) when the number of neutrons and protons are both odd and no spin at all when its number of neutrons and protons are both even.

Nucleus spins are either something we need to avoid like in silicon qubits produced with ^{28}Si, the silicon isotope with a null spin, or that we use to store qubit information like in NV centers and SiC cavities and electron donor qubits based on atoms like phosphorus where there is a coupling between some atom nucleus spins and some free electrons.

But how is it possible to have a zero spin when you add-up the spins of protons and neutrons which are positive? Let's take a pause and provide some answers. This is due to the way a nuclear spin is calculated.

Nucleons are composite quantum objects made of quarks that have quantum properties similar to electrons quantum numbers, but with different possible value bounds and meanings:

- n = **nucleon shell number** or layer with integer values ranging from 0 to 6. It is bounded and there is no equivalent of Rydberg states in nucleus with large principal quantum number. The nuclear shell model is the equivalent of the atomic Bohr model related to electron shells.
- ℓ = **orbital angular momentum** which is also quantized with an integer value starting at 0, the angular momentum itself being $L^* = \hbar\sqrt{\ell(\ell+1)}$. ℓ is labelled s, p, d, f, g, h, i like with electrons.
- m_l = **magnetic quantum number** with integer values ranging from $-\ell$ to ℓ. In each nucleon shell, nucleons of the same type tend to regroup by pairs with opposite magnetic quantum numbers.
- m_s = **spin quantum number**, being either +1/2 or -1/2, in each spatial direction, also named intrinsic angular momentum. A nucleon spin s equal to 1/2 is the size of the vector $\vec{s}$. The spin angular momentum is $S^* = \hbar\sqrt{s(s+1)}$ with s=1/2.

A nucleon total angular momentum is a vector $\vec{j} = \vec{\ell} + \vec{s}$ and $j = \ell + s$ in scalar representation with ℓ being the nucleon orbital angular momentum and $s = 1/2$ its intrinsic angular momentum or spin. The scalar representation is a good approximation of the vector representation since nucleons move in an average magnetic field orienting them in a similar direction. In the end, the atom **nuclear spin** is the sum of its nucleon's total angular momentum j.

As atomic nucleus size grows, nucleus shells are filled progressively. Filled layers have a number of neutrons or protons called "magic numbers" (2, 8, 20, 28, 50, 82, 126) as shown in Figure 85. Atoms with entirely filled layers of either neutrons or protons are more stable. Nucleons pair in orbits with projections $\pm m_\ell$ such that their momenta cancel. The notion of layer magic number applies separately for protons and for neutrons. For example, ^{116}Sn (selenium) has a magic number of 50 protons and ^{54}Fe (iron) has a magic number of 28 neutrons. Filled shells have a total angular momentum of zero since made of pairs of neutrons or protons with opposite projections of total angular momentum. That's why an even number of protons and neutrons lead us to have a zero nuclear spin. And when you have both a magic number of neutrons and protons, your nucleus is doubly magic like with ^{40}Ca and ^{208}Pb and has exceptional stability. All this refers to atoms and nucleus in their ground state.

On top of these numbers, nucleus have a parity that is $\lambda = (-1)^{\ell}$ where ℓ is the total orbital angular momentum of the nucleus. Its value corresponds to the symmetrical or asymmetrical structure of the nucleus.

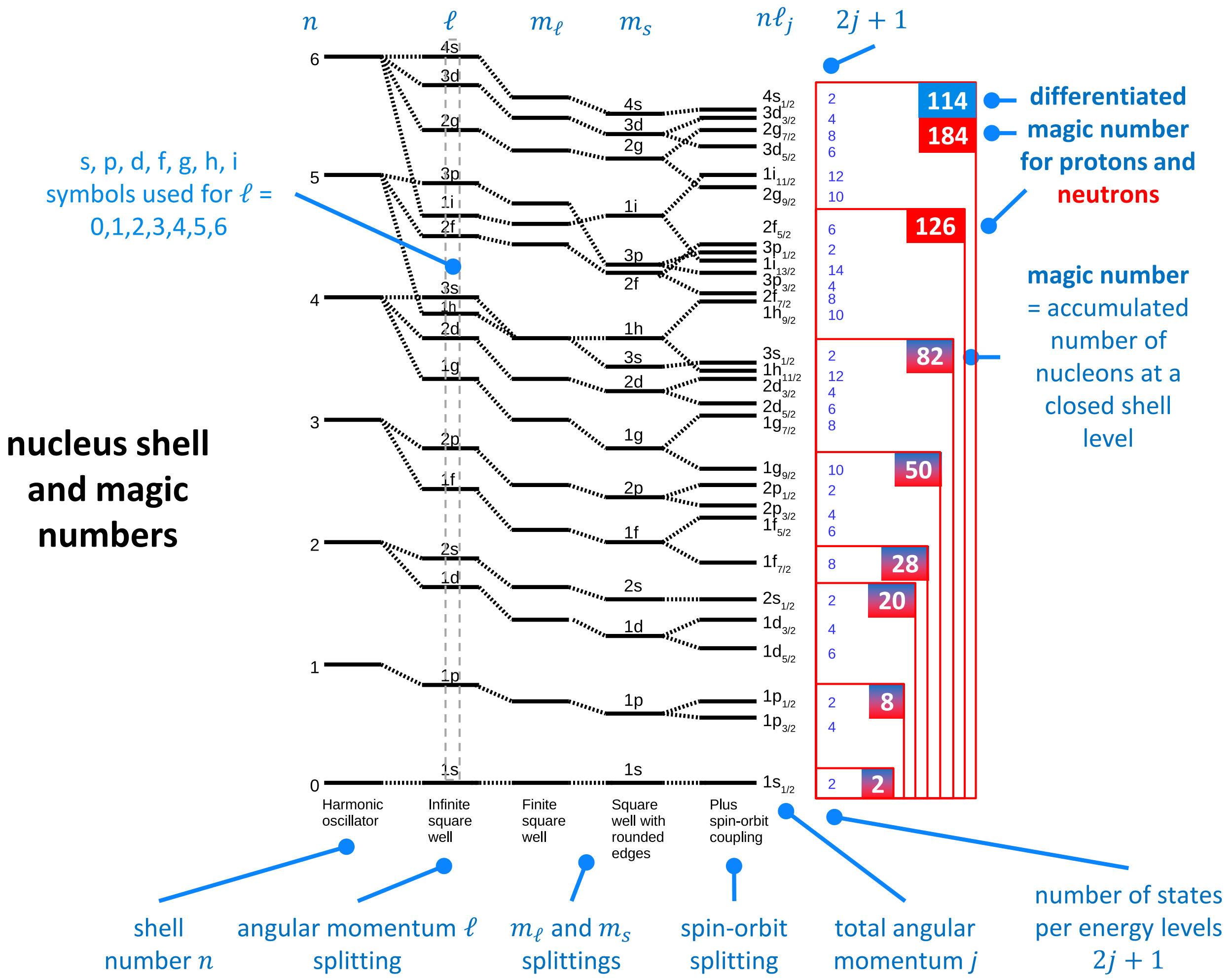

Figure 85 : nucleus shells and magic numbers. Source: Particle and Nuclear Physics Handout #3 by Tina Potter, 2022 (124 slides). (cc) Olivier Ezratty, 2023 for annotations.

The cohesion of the atom nucleus comes from the strength of the nuclear force that binds nucleons together. It is countered by Coulomb's force that creates a repulsion between same charge particles like protons. The relative value of the nuclear force and the Coulomb repulsion force explain nuclear fusion for small atoms and fission for large atoms. Iron sits in the neutral zone in the elements table in terms of abundance. This iron peak is explained by the fact that lighter elements were created by ordinary stellar nucleosynthesis and heavier elements by explosive nucleosynthesis in star supernovas.

Two other notions are related to atom's nucleus and are frequently mentioned elsewhere in this book:

Hyperfine structure are small differences in otherwise degenerate (equivalent, equal) energy levels in atoms, molecules and ions that are explained by the electromagnetic multipole interaction between the nucleus and electron clouds. In atoms, hyperfine structure come from the energy of the nuclear magnetic dipole moment interacting with the magnetic field generated by the atom electrons and the energy of the nuclear electric quadrupole moment in the electric field gradient due to the distribution of charge within the atom.

Spin–orbit coupling or spin–orbit interaction is a relativistic interaction of a particle's spin with its motion inside a potential. One example is the shifts in an electron's energy levels that due to electro-magnetic interaction between the electron's magnetic dipole, its orbital motion and the electrostatic field of the atom nucleus.

Multiple methods are available to detect a nuclear spin:

Nuclear Magnetic Resonance (NMR) spectroscopy exposes the sample to a strong external magnetic field which creates some energy level splitting created by the alignment of the atoms nuclear spins align with the field. A sample is then irradiated by a radiofrequency (TF) field around the Larmor precession frequency of the searched elements, in the hundred MHz range. This is the frequency of the nucleus spin vector rotation in a cone around the axis of the ambient magnetic field. A receiver captures the transmitted RF signal, amplifies it and passes it through a quadrature demixer fed by a reference frequency tone. It down converts the signal to a lower frequency and decomposes it into its in-phase and quadrature which is then converted into digital format with ADCs (analog-to-digital converters) before being analyzed digitally. NMR is used in molecular chemistry to detect various properties of organic molecules and proteins which happened to include hydrogen, carbon (^{13}C) and phosphorus (^{31}P)[166].

Electron Spin Resonance (ESR) which is also known as electron paramagnetic resonance (EPR) focuses on the spins of unpaired electrons rather than nuclei and indirectly provide information about the nuclear spins through hyperfine interactions between the nuclear and electron spins. It can use NV center sensors[167].

Nuclear Quadrupole Resonance (NQR) works with heavy nuclei having a quadrupole moment and detects the interaction between the quadrupole moment and an applied electric field gradient.

Mössbauer Spectroscopy utilizes the recoilless emission and absorption of gamma-ray photons by a nucleus. It is highly sensitive to changes in the nuclear environment, such as shifts in energy levels due to nuclear spin interactions. It is particularly used in geology.

Optical Pumping and Magnetic Resonance (OPMR) excites an atom nucleus with laser light and then probes the resonant behavior of the spin transitions with magnetic fields and more laser light.

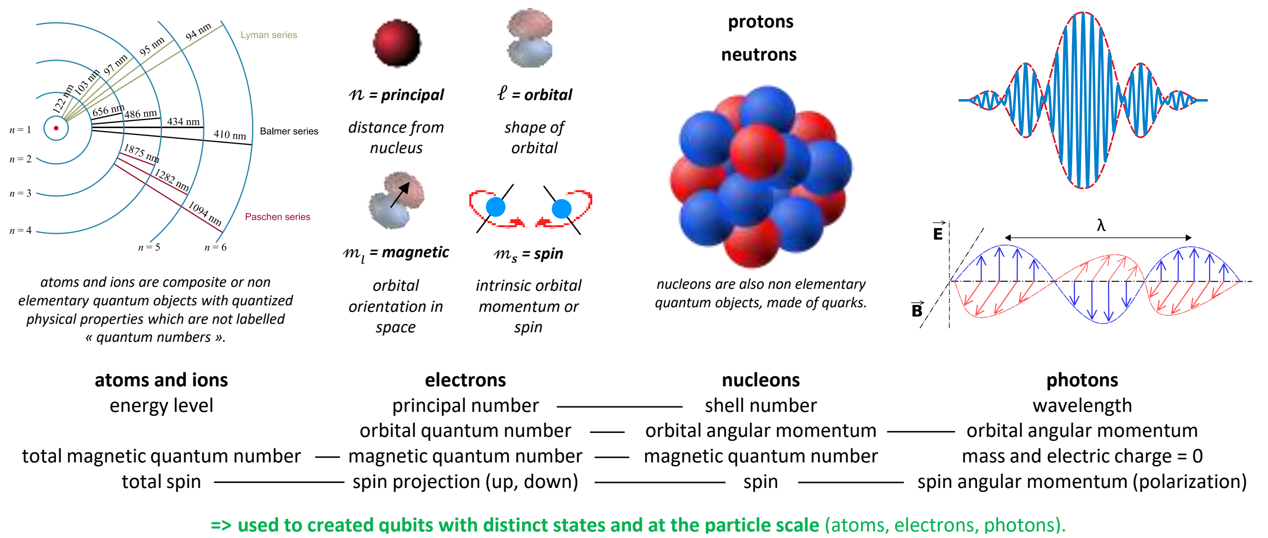

Figure 86: quantized properties of atoms, electrons, nucleons and photons, and some correspondence between atoms, electrons, nucleons and photons. Only elementary particles like electrons and photons have "quantum numbers", atoms and nucleons being composite quantum objects made of electrons and quarks only have "quantum properties". (cc) Olivier Ezratty, 2022-2023, with some Wikipedia images sources.

Scattering Experiments provides information about the nuclear spin and magnetic properties of atoms by analyzing how the scattered particles interact with the atomic nuclei.

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166 See Introduction to Nuclear Magnetic Resonance Spectroscopy by Dean L. Olson, University of Illinois, 2007 (40 slides) which describes the practical implementation of NMR.

167 See In situ electron paramagnetic resonance spectroscopy using single nanodiamond sensors by Zhuoyang Qin et al, Nature Communications, October 2023 (8 pages).

Photon quantum numbers

Photons also have their quantum numbers, but they are different than with electrons and nucleons. We describe it in the section dedicated to photon qubits, starting page 518.

In quantum information systems, we use quantum objects which can usually have two different separable states that can be initialized, modified, and measured. Even superconducting loops in superconducting qubits rely on two systems levels clearly distinct for the oscillating current flowing through their Josephson effect insulator.

Quantum continuous variables

There are actually two different sets of quantum variables that characterize quantum objects:

Discrete quantum variables like electron or nucleon spin, photon polarization or number, and atoms energy levels which are restricted to a set of values which can be numbered but also infinite, like the theoretical level of energy of electrons in an atom. It corresponds to all the above text in this "quantization" part. These variables are in a finite dimension Hilbert size like, as we will see later, 2^n for n qubits. Such quantum variables are used in gate-based quantum computing and also with some quantum key distribution techniques, labelled "discrete variable quantum key distribution" (DV-QKD).

Continuous quantum variables like a quantum particle position and impulsion and the amplitude of some electromagnetic field. These variables are in a Hilbert space of infinite dimension. They are used in continuous variables quantum computing like with Xanadu's qumodes which is one breed of analog quantum computing, and also with continuous variable quantum key distribution (CV-QKD). These variables relate to the notions of quadrature, phase space and squeezing that we will describe in various places in this book, particularly when dealing with photons. Photons can have both discrete (Fock numbers) and continuous variables (quadratures).

So, how can we have "quantum" objects with properties that are not "quantized" in the first place? Is that a misnomer to name them "quantum"? Not that much. Continuous quantum variables systems still belong to the quantum world and its mathematical framework with Hilbert spaces, quantum states and operators. They exhibit wave-particle duality, superposition, entanglement, probabilistic measurement and are subject to the Heisenberg indeterminacy principle, which by the way is nearly always illustrated with continuous variables like a particle position and momentum. Continuous quantum variables are also quantum in opposition to the macroscopic analog world where all these quantum phenomena are not observed like the wave-particle duality.

Wave-particle duality

We often read and hear that quantum objects like photons and electrons are both waves and particles. They behave differently depending on the way they are observed. In some experiments, these quantum objects behave like classical waves, are not localized in space and generate interferences when added together, a bit like colors can mix (photons) and sounds can mix (acoustic waves). In other experiments, they behave as classical particles and can be localized in space and have a kinetic momentum and mass[168]. Another interpretation is that quantum objects act as a particles when observed and as waves when not observed and interact with each other. Various experiments such as Young's double-slit experiment show that both photons and electrons behave both as particles and as waves depending on the context and measurement system, generating interference fringes when observed as waves. You can observe the path of a quantum object or the interferences it creates, but not both simultaneously (Figure 87). This is the Bohr's principle of complementarity according to which it is not possible to apply observables simultaneously in terms of particles and waves.

[168] Usually, it is impossible to observe these two behaviors simultaneously although there are some exceptions.

It shows up in the Young slit experiment: if we let the quantum object traverse both slits, it behaves like a wave and creates interferences. If we detect the quantum object in each of the slits, which is practically implemented with closing one of the slits, it creates a measurement-based decoherence and the quantum object behaves and is observed as a particle. The classical probabilities of particle observation do not add up to make for the interferences observed with the wave observation. This wave-particle duality is also interpreted thanks to the quantum physics mathematical formalism that relies on vectors that can add up linearly like waves. It led to a still unsolved mystery, the "which-way" question. When interference fringes appear on the screen, by superposition of paths coming from the two slits, which path did the single photon or electron take?

Wave-particle duality is used in many quantum computers to make physical qubits such as trapped ions interact with energy in the form of photons emitted by lasers. Qubits can also interfere with each other thanks to interferences. It is also used in cold atom matter-waves in absolute gravimeters.

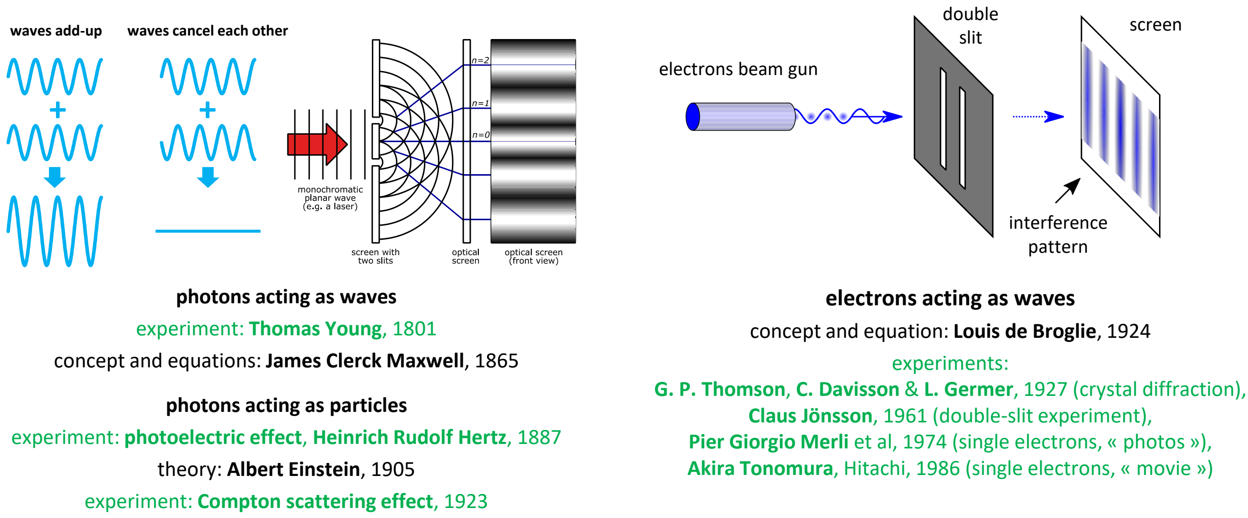

Figure 87: wave-particle duality with photons and electrons theory and experiments. (cc) compilation Olivier Ezratty, 2023.

Historically, **Thomas Young**'s famous slit experiment did demonstrate that photons acted as waves. It was later formalized by **James Clerck Maxwell** electromagnetic equations in 1865. Photons particle behavior was first observed with the photoelectric effect discovered by **Heinrich Rudolf Hertz** in 1887 and later formalized by **Albert Einstein** in 1905 and confirmed by the Compton scattering experiment in 1923. Electrons wave behavior was theorized by **Louis de Broglie** in 1924 and shown experimentally in several steps, first by **Georges Paget Thomson**, **Clinton Davisson** and **Lester Germer** in 1927 with a crystal diffraction setup, then by **Claus Jönsson** with a double slit experiment in 1961 and then, with experiments showing how single electrons could interfere with themselves, first by **Pier Giorgio Merli** et al in 1974 and then by **Akira Tonomura** from Hitachi in1986[169].

Schrödinger's wave equation

Wave-particle duality led Erwin Schrödinger to create his famous wave equation which describes a massive non-relativistic quantum object with a wave function defining probabilities of finding a particle at a particular position in space and time, as shown in Figure 88.

Here's how to understand the components of this equation and their implications:

- Its **unknown** is the wave function of the particle $\psi(x,t)$ that describes its probabilistic behavior in space and time. x indicates the position of the particle in space, with one, two or three

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[169] See The double-slit experiment with single electrons by John Steeds, Pier Giorgio Merli, Giulio Pozzi, GianFranco Missiroli and Akira Tonomura, 2023, Physics World, 2003 (2 pages).

dimensions depending on its constraints, and t is the time. This function returns a complex number that encodes the wave amplitude and phase.

- The full Schrodinger wave equation illustrates the principle of **energy conservation**. The item to the left of the equation describes the total energy of the particle at a given time and place. The elements on the right are the particle kinetic energy and its potential energy. As indicated in the quantum physics postulates part starting page 100, the Schrodinger's Hamiltonian, which is a time-dependent unitary matrix operator, is expressed differently with photons and with relativistic massive particles.

- The **wave function square** is equal to the probability of finding the particle at location x at time t. For an electron, which is the most commonly analyzed particle with this equation, it is an indication of the probability of finding it at a given distance from the nucleus of the atom around which it orbits. Logically, as a result, the sum of the probabilities of finding the particle somewhere is equal to 1. This is called a normalization constraint (shown in Figure 89). One of its derivatives is the Max Born function that we will see later. The modulus of a complex number is the size of its vector. If z = a + ib, the modulus $|z|$ of z is thus the square root of the sum of the squares of a and b, see *below*.

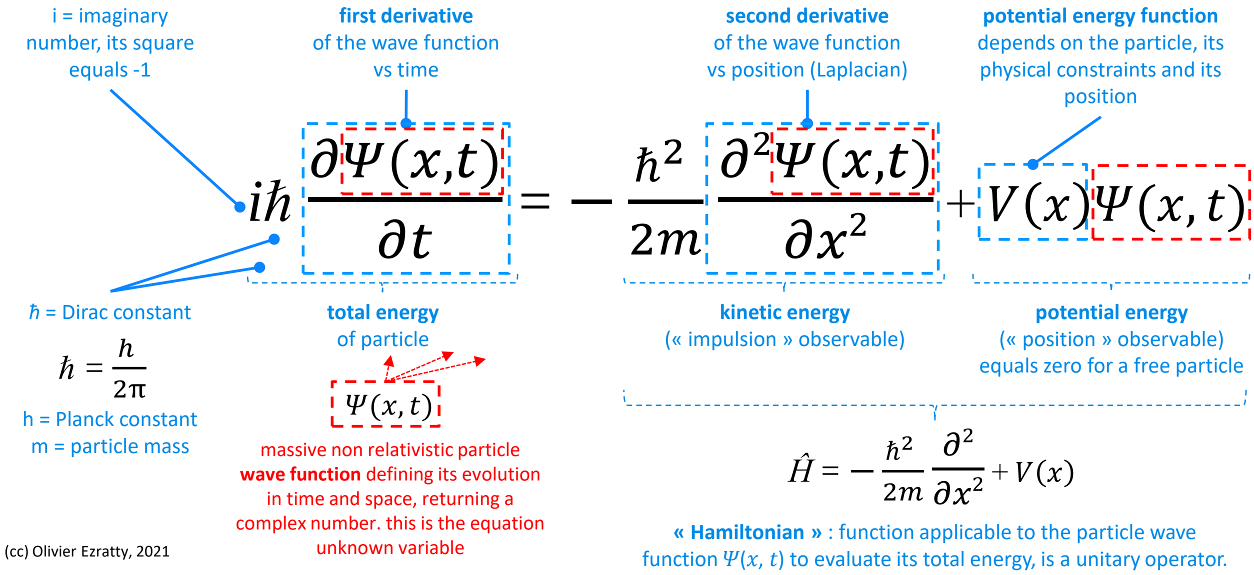


Figure 88: the famous Schrodinger's wave equation explained in detail (cc) Olivier Ezratty, 2021.

- It is a **partial differential** equation, i.e., it connects its components via derivative functions, in this case of first degree (a slope on a curve) and second degree (an acceleration). The particle wave function appears three times in the equation: to the left of the equation with a first derivative on the time of the wave function, to the right with a second derivative on its position and with a simple multiplication with the function V(x).

- The **potential energy of the particle** is defined by the function V(x) which depends only on the particle position in space and its physical constraints, in particular electromagnetic ones. When a particle is free and moves without constraints, this function returns zero. This function V(x) is the main variable of Schrödinger's equation.

- Schrödinger's equation is **analytically solved** in a limited number of cases such as for the electron of a hydrogen atom, a free particle, a particle in a potential well or box or a quantum harmonic oscillator. In the most complex cases, the resolution of the equation requires non-analytical methods, raw calculation and simulation. It is one of the fields of application of quantum simulators and gate-based quantum computers to solve the Schrödinger equation in cases where analytical methods are not available. Any micro or macro-object has a Schrödinger wave function, all the way to the entire Universe. But the equation only makes practical sense for nanoscopic objects.

$$\mathbf{z} = \mathrm{a} + \mathrm{ib} \quad |\mathbf{z}| = \sqrt{a^2 + b^2}$$

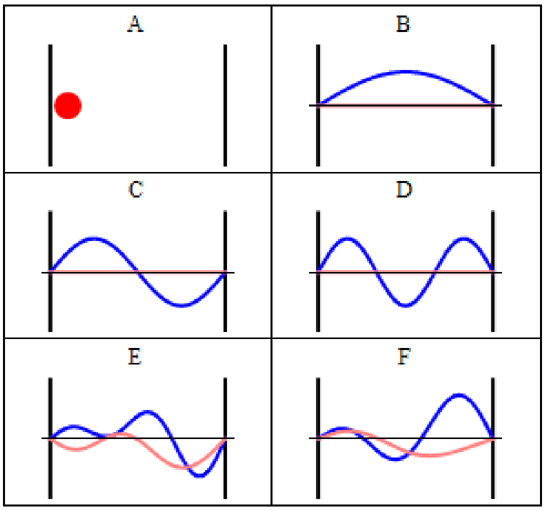
complex number module
size of its vector in two dimentional space

$$|\psi(x,t)|^2$$

wave function module square
probability to find the particule at position x at time t.
If it's time independant, the system is in a stationary state.

$$\int_{-\infty}^{+\infty} |\psi(x,t)|^2 \, dx = 1$$

integral of the propability to find the particle in any position equals 1



example of V(x) potential energy constraints in cavities

Figure 89: constraints of the Schrodinger's equation (cc) Olivier Ezratty, 2021.

- The equation is **linear** over time. This means, among other things, that any combination of solutions of the equation becomes a new solution of the equation. This makes it possible to decompose a wave function into several elementary wave functions that are called the "eigenstates" of the quantum object. They correspond to the different energy levels of the particle that are discrete when the particle is constrained in space, like the electrons in an atom. One can indeed in this case derive the notion of quantification of the particle states from the Schrödinger equation (demonstration). The linearity of this equation has a lot of consequences like superpositions, entanglement as well as the no-cloning theorem.
- The operator which acts on the right side and accumulates the second derivative and the potential energy function is called a **Hamiltonian,** which describes the total energy of the system. We find this expression in the quantum annealing calculation with D-Wave and with quantum simulators.
- This equation is a **general postulate** that has been experimentally validated in a very large number of cases. Its interpretation has given rise to much debate, namely, is it a simple probabilistic model or does it describe reality? We deal with this in the chapter on quantum physics foundations starting page 1238.
- The generic Schrödinger equation presented so far is said to be **time dependent**. This equation is presented in various ways depending on the needs and annotations. The second derivative of the wave function on the position of the particle is sometimes presented with the nabla sign squared (∇^2).

A nabla operates a derivative on a scalar or vector function. The ∇^2 operates a second derivative, also called Laplacian. The most concise form of Schrödinger's equation is in Figure 90, with a Hamiltonian operator on the left (Ĥ) and the energy operator on the right (Ê), both of which apply to the particle wave function $\psi(x,t)$.

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + \mathrm{V}\right]\Psi = i\hbar\frac{\partial}{\partial t}\Psi$$

$$\hat{H}\psi(x,t) = \hat{E}\psi(x,t)$$

Figure 90: concise versions of Schrodinger's wave equation.

- The **time-independent** form of Schrödinger's equation applies to particles in a stationary state[170]. In this version of Schrödinger's equation, the energy operator E is a simple constant, a real number (Figure 91).

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V(\mathrm{r})\right]\Psi(\mathrm{r}) = \hat{E}\psi(\mathrm{r})$$

Figure 91: time-dependent version of the Schrodinger's equation.

- The Schrödinger equation is **symmetric** or **antisymmetric** depending on the particle type. When applied to two quantum objects r_1 and r_2, $\psi(r_1, r_2) = \psi(r_2, r_1)$ when the equation is symmetric (meaning, the wave equation is not differentiated by the given particles order) and $\psi(r_1, r_2) = -\psi(r_2, r_1)$ when it is antisymmetric. The first case corresponds to bosons which can be indistinguishable and "live" together and have a zero or integer spin and the second, to fermions, which can't cohabit with the same quantum state at the same location and have half-integer spins. All this is a consequence of Pauli's exclusion principle.
- The $\psi(x, t)$ function must be a **continuous function** and "filled" everywhere in space. Its value is bounded by 0 and 1, with no infinite value anywhere. It also has a single value, even in the case of superposition. In that case, the $\psi(x, t)$ is a linear superposition of two Psi functions and is itself a psi function. A quantum superposition is just another wave function.

For a system with several quantum objects, the wave function describes the quantum system state, or quantum state. According to the Copenhagen interpretation of quantum physics, the wave function from the Schrödinger equation contains the best description possible of a quantum system.

If electrons and photons both can behave as waves, they have not the same wavelengths. Indeed, a photon with an energy of 1 eV (electron-volt) has a wavelength λ of 1240 nm (in the infrared spectrum) while an electron with the same energy has a much shorter wavelength of 1.23 nm (in the X-ray spectrum). This short wavelength explains why we use electron microscopes to probe matter with a much better resolution than light-based microscopes.

Relativistic particles obey to Dirac and Klein-Gordon wave equations while photons are described with Maxwell's equations combined with a formalism coming from the so-called second quantization which regroups superposed photons, use photon numbers, and creation/annihilation operators.

Let's mention at last **quantum trajectories** which describe the time evolution of quantum systems and their quantum state, which can be solved using the time-dependent Schrödinger's equation[171]. They are related to the **quantum jump** phenomenon, which describes the continuous change happening for example during the measurement of a quantum object[172].

Delayed choice experiment

John Wheeler proposed various thought experiments between 1978 and 1984 to determine if light choses its path with sensing the experimental devices. The Wheeler's delayed-choice or which-way experiment asked the question: when does a quantum object decide to travel as a wave or as a particle?

---

[170] According to Wikipedia: "*A standing wave is the phenomenon resulting from the simultaneous propagation in opposite directions of several waves of the same frequency and amplitude, in the same physical medium, which forms a figure, some elements of which are fixed in time. Instead of seeing a wave propagating, we see a standing vibration but of different intensity at each observed point. The characteristic fixed points are called pressure nodes.* ».

[171] See Overview: Dynamics with Quantum Trajectories by Robert E. Wyatt, The University of Texas at Austin, 2008 (24 slides).

[172] See To catch and reverse a quantum jump mid-flight by Zlatko K. Minev, Rob J. Schoelkopf, Mazyar Mirrahimi, H. J. Carmichael, Michel H. Devoret et al, 2019, Nature (29 pages).

It led to various experiments like the 1999 quantum eraser but the most decisive experiment was conducted by a team of French researchers in 2006 as shown in Figure 92 [173].

They generated pulses of single photons with an NV centers source created by Jean-François Roch, a pioneer in this domain, that were sent through a first beam splitter (BS_{input}) and a delay line of 48 meters. Then, the two beams traversed a dynamic-controlled beam-splitter by electro-optical modulator driven (BS_{output}) by a quantum random number generator (QRNG).

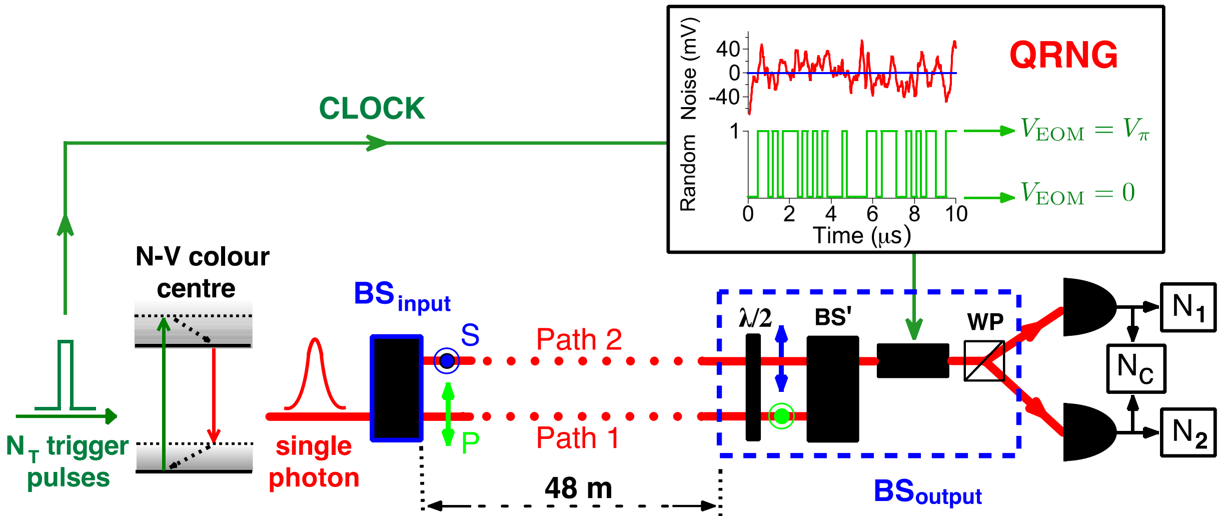


Figure 92: delayed choice experiment and its quantum eraser. Source: Experimental realization of Wheeler's delayed-choice GedankenExperiment by Vincent Jacques, Frédéric Grosshans, Philippe Grangier, Alain Aspect, Jean-François Roch et al, 2006 (9 pages).

At last, two photon detectors (N_1 and N_2) could determine if the photon behaved as a particle (no interference due to the inactive beamsplitter) or as a wave (with interferences due to the activated beamsplitter).

The QRNG clock was near the photon source, but the QRNG was positioned close to the dynamic beamsplitter. The experiment determined that the wave/particle behavior of the photons in the interferometer was dependent on the choice of the measured observable at the end of the photon journey, not the beginning. And even when that choice was made at a position and a time sufficiently separated from the entrance of the photons in the interferometer. Although it is still debated, it does not require a backward in time effect explanation[174].

Other more delayed-choice sophisticated experiments are regularly done. A Chinese team demonstrated a generalized multipath wave-particle duality implemented by a large-scale silicon-integrated multipath interferometers[175]. A delayed choice for entanglement swapping experiment was also achieved in 2023 where entanglement is produced a posteriori, using two pairs of entangled photons, after the entangled particles have been measured and may no longer exist[176].

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[173] See Experimental realization of Wheeler's delayed-choice GedankenExperiment by Vincent Jacques, Frédéric Grosshans, Philippe Grangier, Alain Aspect, Jean-François Roch et al, 2006 (9 pages). The experiment used a single photon source using NV centers. The experiment has been reproduced many times since then with many variations. See for example A generalized multipath delayed-choice experiment on a large-scale quantum nanophotonic chip by Xiaojiong Chen et al, 2021 (10 pages) which is based on a nanophotonic component.

[174] See Delayed choice experiments: An analysis in forward time by Marijn Waaijer and Jan van Neerven, July 2023 (22 pages).

[175] See A generalized multipath delayed-choice experiment on a large-scale quantum nanophotonic chip by Xiaojiong Chen et al, 2021 (10 pages).

[176] See Experimental delayed-choice entanglement swapping by Xiao-song Ma, Stefan Zotter, Johannes Kofler, Rupert Ursin, Thomas Jennewein, Časlav Brukner and Anton Zeilinger, Nature Physics, 2012 (17 pages).

Large objects wave behavior

Figure 93: C_{60} fullerene molecule.

The wave-particle duality was verified with atoms in 1991 in interferometry experiments involving lasers and classical optics. A Young double-slit experiment was also carried out in Austria in 2002 with fullerene molecules (C_{60}, formed of 60 carbon atoms as in Figure 93[177], but also with a 70 atoms variant) and in 2012 with molecules containing 58 and 114 atoms, the latter named $F_{24}PcH_2$ being made of fluorine, carbon, oxygen, hydrogen and nitrogen[178]. Figure 94 shows the shape of the molecule. In 2019, the same kind of experiment was done with a slightly more complex molecule, a polypeptide of 15 amino acids which serves as an antibiotic, gramicidin A1[179] .

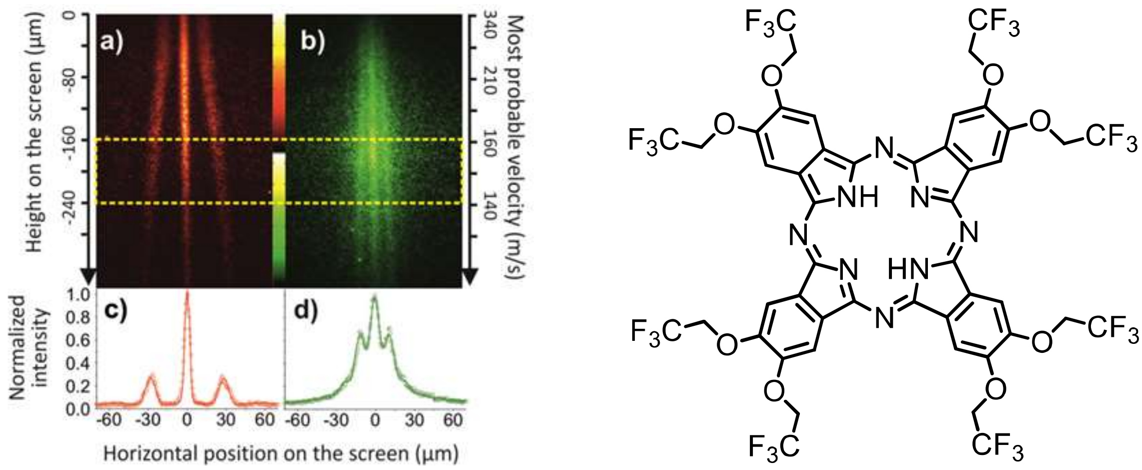

Figure 94: $F_{24}PcH_2$ made of fluorine, carbon, oxygen, hydrogen and nitrogen. Sources: Real-time single-molecule imaging of quantum interference by Thomas Juffmann et al, 2012 (16 pages) and Highly Fluorinated Model Compounds for Matter-Wave Interferometry by Jens Tüxen, 2012 (242 pages).

In 2021, other experiment led to the creation of larger quantum objects, sized 100 and 140 nm, and cooled at ultra-low temperatures[180]. Nowadays, experiments are done with even larger systems made of thousands of atoms[181].

Photon's wave-particle duality

On the other hand, photons can behave under certain conditions like particles. When they reach an atom, they can transmit it some kinetic motion. This is what makes it possible to generate the somewhat counter-intuitive physical phenomenon of atoms laser cooling using lasers and a Doppler effect. Temperature is related to the movement of atoms in their gaseous, liquid or solid medium. Lowering the temperature means slowing down the movement of atoms.

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[177] See Quantum interference experiments with large molecules by Olaf Nairz, Markus Arndt and Anton Zeilinger, 2002 (8 pages).

[178] See Real-time single-molecule imaging of quantum interference by Thomas Juffmann et al, 2012 (16 pages). See also the video of the experiment. Highly Fluorinated Model Compounds for Matter-Wave Interferometry by Jens Tüxen, 2012 (242 pages) describes the experimental device for the verification of the wave-matter duality of large molecules.

[179] See A natural biomolecule has been measured acting like a quantum wave for the first time, November 2019, which refers to Matter-wave interference of a native polypeptide by Armin Shayeghi et al, October 2019 (10 pages).

[180] See How Big Can the Quantum World Be? Physicists Probe the Limits by Philip Ball, Quanta Magazine, July 2021, Real-time optimal quantum control of mechanical motion at room temperature by Lorenzo Magrini et al, July 2021 (36 pages) and Quantum control of a nanoparticle optically levitated in cryogenic free space by Felix Tebbenjohanns et al, Nature, July 2021 (26 pages).

[181] See Experimental challenges for high-mass matter-wave interference with nanoparticles by Sebastian Pedalino, Markus Arndt et al, January 2023 (10 pages).

A Doppler effect is used to do this. The moving atoms are illuminated with a laser whose frequency is tuned just below the energy absorption level of the atoms as explained in Figure 95.

The atoms moving towards the light will absorb the photons because these have an apparent frequency that is higher than the absorption level. This reduces the kinetic energy of the atoms receiving the photon.

The photons moving in the other direction will not absorb them because the apparent frequency of the incident photon is below the absorption level, so it is unable to change the energy state of the atoms.

Thanks to the random movement of the atoms in all directions, after a certain time, the overall temperature drops. This phenomenon slows down once the velocity of the atoms falls below a certain threshold, which explains the Doppler effect attenuation ("Doppler shift").

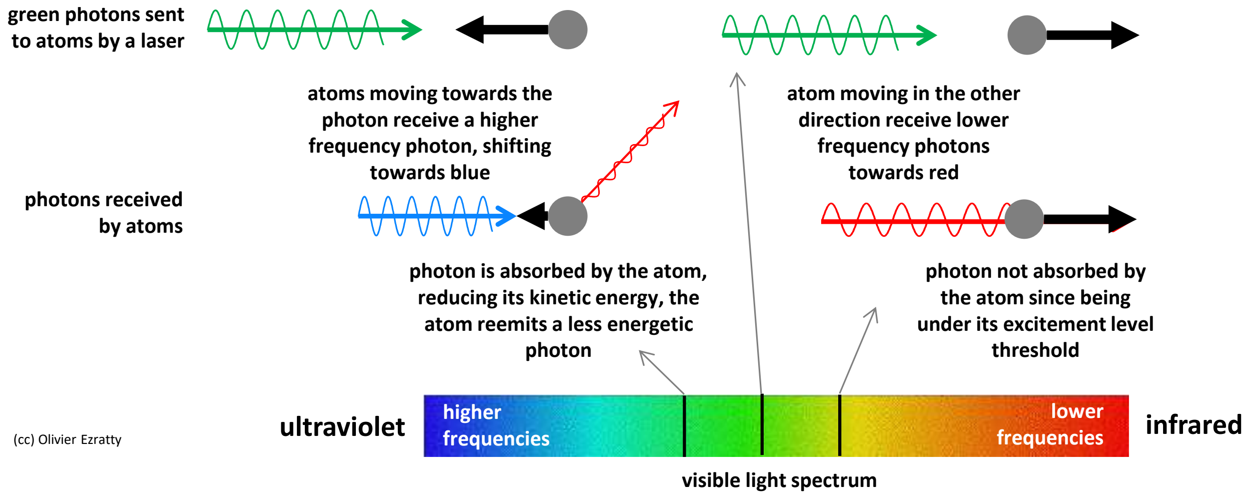

Figure 95: explanation of Doppler effect with photons, (cc) Olivier Ezratty, 2021.

These techniques are used to cool atoms to temperatures close to absolute zero. It is used to prepare cold atoms and trapped ions used in certain types of quantum computers, often in combination with magnetic and/or electronic traps to control the atoms position[182].

The record low temperature was reached in 2019 with 50 nK, achieved by researchers from JILA, the joint laboratory of NIST and the University of Colorado[183].

Superposition and entanglement

Superposition and entanglement are directly related to the wave nature of quantum objects and to the linearity of the underlying mathematical models expressed in quantum physics postulates.

Superposition

The strawman's version of superposition is that quantum objects can be simultaneously in several states or locations, such as the direction of electron spin, upward or downward, the linear polarization of photons, horizontal or vertical, or the frequency, phase or energy of an oscillating current in a superconducting loop crossing the barrier of a Josephson junction.

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[182] Doppler measurement is also used to evaluate the speed at which stars and galaxies move away from each other and to evaluate the rate of expansion of the Universe. Other atoms laser-based cooling methods crafted to reach lower temperatures include Sisyphus cooling first proposed by Claude Cohen-Tannoudji in 1989 and using two counter-propagating lasers using orthogonal polarization, evaporative cooling using magneto-optical traps (MOT) and optical molasses with 3D Doppler effect.

[183] See JILA Researchers Make Coldest Quantum Gas of Molecules, February 2019. The 50 nK record was obtained with laser cooling of a gas containing 25,000 potassium-rubidium molecules.

It is not correct according to canonical interpretations of quantum mechanics. It is more related to quantum objects behaving as waves when not being measured.

Superposition is also a mathematical consequence of quantum postulates and wave-particle duality. It results from the fact that a linear combination of solutions to the Schrödinger equation is also a solution to this equation (Figure 96).

Several quantum states of a given quantum object can be added together or superposed. Superposition explains the interferences obtained with electrons in the 1961 double-slit experiment.

A quantum object is not per se in a superposition of various states. It has a single and predictable quantum state described by a probability distribution of given observables. Measuring this property can provide different values according to the probability distribution. That's all.

According to the Copenhagen interpretation of quantum physics, one should not try to give a physical meaning to superposition before any measurement. In a classical physics interpretation, superposition could be explained by a very high frequency of quantum state changes. It is considered to be totally inaccurate for specialists, but it is still an intuitive way to figure out how superposition looks like in the physical world.

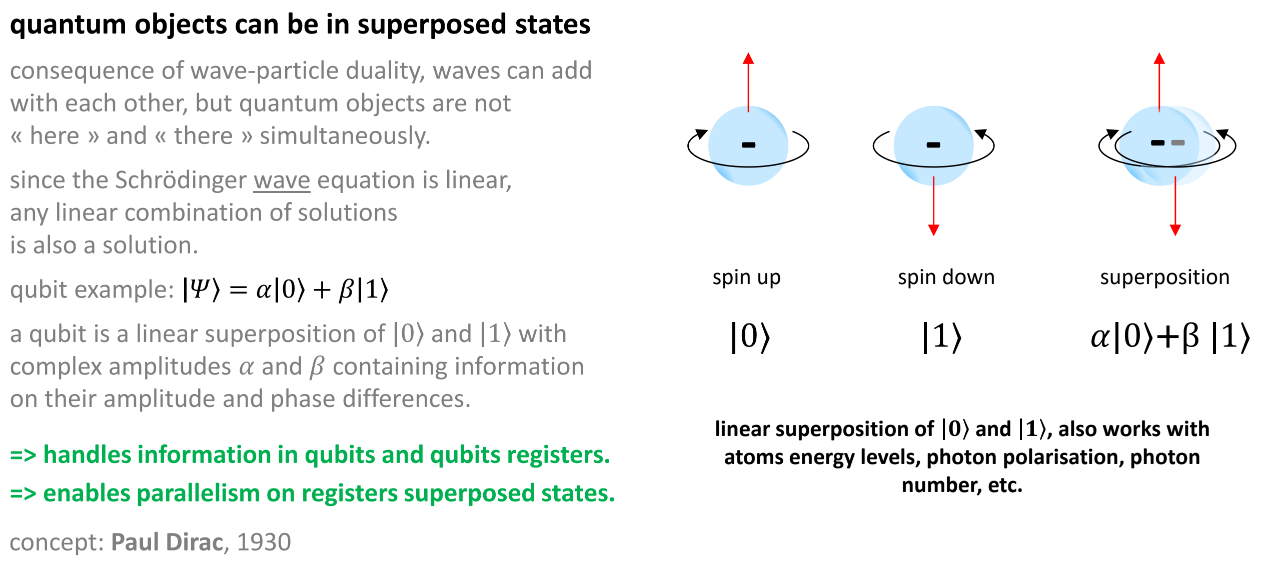

Figure 96: electron spin superposition. (cc) Compilation Olivier Ezratty, 2021-2023.

Superposition can happen with various weird situations, as we'll later see. For example, you can create superposition between several photon Fock-states, meaning, superposing 0 photon, 1 photon and 2 photons, or even photon frequencies. You can even superpose temperatures[184] and thermodynamic evolutions with opposite time arrows[185] which can challenge your willingness to visualize what's it all about!

In quantum computing, superposition shows up with qubits, allowing which have an internal "value" linearly combining their basis states $|0\rangle$ and $|1\rangle$ with complex amplitudes instead of having one of the two values as with classical bits. This mathematical view is expanded to N qubits whose internal state is characterized by 2^N complex amplitude values. This contributes to the massive parallelism enabled by quantum computers. It looks like it should enable some exponential computing capacity but it is not the case. As we'll investigate later in this book, superposition alone is not sufficient.

[184] See Quantum Superposition of Two Temperatures by Arun Kumar Pati and Avijit Misra, December 2021 (7 pages).

[185] See Quantum superposition of thermodynamic evolutions with opposing time's arrows by Giulia Rubino, Gonzalo Manzano and Časlav Brukner, November 2021 (10 pages).

We also need entanglement and some specific quantum gates to really bring some exponential acceleration. In the case of a single quantum object, superposition is a combination of states corresponding to several exclusive states of an observable. Coherence is another name describing a superposition. And decoherence is a phenomenon that destroys superposition, particularly with quantum measurement.

Entanglement

The simplest way to describe an entangled state of two quantum objects is to say these have a correlated state, whatever the distance between them. They form sort of a single object. You measure one object, then the other, and the related results will be correlated. This can be checked with tests done on a system repeatedly prepared in a similar way, using a so-called Bell test.

Entanglement can also be formalized with a mathematical viewpoint based on superposition. The mathematical representation of a quantum system AB made of two subsystems A and B is the tensor product of the two subsystems, meaning, a large vector or matrix: $H_{AB} = H_A \otimes H_B$. We will describe the shape of the matrices representing quantum systems a little later. In that case, the system AB can be described by or decomposed with its individual parts A and B. There are, however, situations where you can linearly combine several of these composite quantum states, which becomes a new quantum state. In many cases, such a composite state cannot anymore be mathematically decomposed as the tensor product of two states. The composite quantum state becomes mathematically inseparable. That is when entanglement shows up! Entanglement is a direct consequence of superposition applied to multi-object systems.

Entangled quantum objects cannot be considered as separate objects. With a pair of entangled quantum objects, a measurement made on one quantum will instantly influence the other quantum, without waiting for a delay in the transmission of information at the speed of light between the two quanta. This is the principle of the nonlocality of quantum properties that disturbed Einstein in 1935 and spurred his famous EPR paper with Rosen and Podolsky.

Using qubit's representation that we'll describe later, classical entangled two-qubit states are Bell pairs, like $\frac{|00\rangle+|11\rangle}{\sqrt{2}}$ or $\frac{|01\rangle+|10\rangle}{\sqrt{2}}$. You see that they are a simple linear combination of separable states ($|00\rangle$ and $|11\rangle$ or $|01\rangle$ and $|10\rangle$). If you measure the first qubit in both cases, you have an even 50%/50% chance to get a $|0\rangle$ or a $|1\rangle$. When you measure the second qubit, you then have a 100% chance to get respectively the same value of the opposite values $|1\rangle$ or $|0\rangle$. But you can't decide in advance what is the first measurement outcome (on Alice's side). So, you observe some synchronicity between two measurements but no determinism on the first readout value.

It is all about having two simultaneous synchronized random values. It is described as the "no-signaling principle": there is no statistical difference between a "first" or "second" measurement of entangled pairs, meaning Bob doing the measurement before or after Alice and Alice did not send any actual pre-determined information to Bob when doing the measurement on her side.

But that is a mathematical representation of entanglement. You might wonder how these composite objects are created in the real world. Of course, some physical interaction must be created to entangle electrons, atoms and/or photons[186]. Photons can be prepared to be entangled with being generated by some excitement of atoms like calcium which generates photon couples of different wavelengths but with some correlated properties like their polarization. Neutral atoms can be entangled with exciting them with lasers, raising their energy levels to a so-called Rydberg state, which then creates links with neighbor atoms.

[186] See How Do You Create Quantum Entanglement? by Chad Orzel, Forbes, February 2017 which explains entanglement creation in plain language.

Electron spins are entangled with lowering a potential energy barrier between them. Quantum objects of different types can also be entangled, like photons with atoms or electron spins with photons[187]. Also, the two electrons of a helium atom are entangled since they have interdependent opposite spins due to Pauli's principle. You can even generate entanglement between different properties of a same particle[188] and simple molecules[189].

These entangled quantum objects are not linked by chance. They usually had a common past or some past interactions. For example, two entangled photons can be produced with a birefringent mirror and separated by dichroic mirrors, creating two photons of orthogonal polarizations. The action on one of the two photons has an impact on the other photon as demonstrated by Alain Aspect in his famous 1982 experiment. But the values that are generated are completely random! It is not defined at one end and transmitted to the other end. It is a random value that can be uncovered at two different places with some quantum measurement.

A 2019 experiment conducted at the University of Glasgow has even allowed to photograph a representation of the state of entangled photons[190]. Some other proposals also exist to see entanglement with your own eyes but has not yet been implemented [191] [192].

Nevertheless, we are still able to entangle quantum objects that do not necessarily have a common past[193]. Bell's inequalities were first validated with photons in the visible spectrum. It has been extended to other parts of the spectrum, of course in the infrared bands that are used in fiber optics and free space quantum communications and even in the X-ray band[194] [195]. It has also been done with all sorts of qubits (superconducting, silicon spin, trapped ions, neutral atoms). An EPR experiment was even realized in 2023 with two times about 700 atoms in a BEC (Bose Einstein Condensate)[196].

Despite its randomness, entanglement is a very powerful resource. It helps generate random secret keys for two parties with the QKD (quantum key distribution) protocols. It powers quantum computing with creating interdependencies between qubits. Multi-qubits quantum gates conditionally link them together. Once entangled, qubits have inseparable quantum states. Without it, no useful quantum algorithm could work. But quantum entanglement does not mean we can transmit some useful information faster than light since the entangled objects' properties are random.

The Bell theorem states that "*quantum mechanics is either incomplete or non-local*". Based on the linearity of quantum mechanics, a **Bell inequality test** or Bell experiment (see glossary, page 1307 and Figure 97) looks at the statistical correlation between the states of two quantum objects, with an experiment done a large number of times with the same settings.

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[187] In 2017, researchers in Warsaw were able to entangle a photon with billions of rubidium atoms. See Quantum entanglement between a single photon and a trillion of atoms, 2017.

[188] See Generation of intraparticle quantum correlations in amplitude damping channel and its robustness by Animesh Sinha Roy et al, March 2023 (9 pages).

[189] See On-demand entanglement of molecules in a reconfigurable optical tweezer array by Connor M. Holland, Yukai Lu and Lawrence W. Cheuk, Harvard, Science, December 2023 (15 pages). It is trapping two-atom bialkali CaF (calcium monofluoride) molecules with optical tweezers.

[190] See Scientists unveil the first-ever image of quantum entanglement by Paul-Antoine Moreau, July 2019.

[191] See What does it take to see entanglement? par Valentina Caprara Vivoli, Pavel Sekatski et Nicolas Sangouard, February 2016 (7 pages).

[192] See Proposal for witnessing non-classical light with the human eye par A. Dodel, Nicolas Sangouard et al, Avril 2017 (9 pages).

[193] See Qubits that never interact could exhibit past-future entanglement by Lisa Zyga, July 2012.

[194] See Entangled X-ray Photon Pair Generation by Free Electron Lasers by Linfeng Zhang et al, August 2022 (13 pages).

[195] See Production of Entangled X-rays through Nonlinear Double Compton Scattering by T. D. C. de Vos et al, November 2023 (20 pages).

[196] See Einstein-Podolsky-Rosen Experiment with Two Bose-Einstein Condensates by Paolo Colciaghi, Yifan Li, Philipp Treutlein, and Tilman Zibold, PRX, May 2023 (10 pages).

When the statistical correlation test S is equal or near to $2\sqrt{2}$ which corresponds to the Tsirelson bound, an upper limit to statistical correlations between distant events detected on two entangled objects, the Bell inequality test is passed. It demonstrates indirectly that the entangled objects correlated measurement can't be explained by a hidden variable.

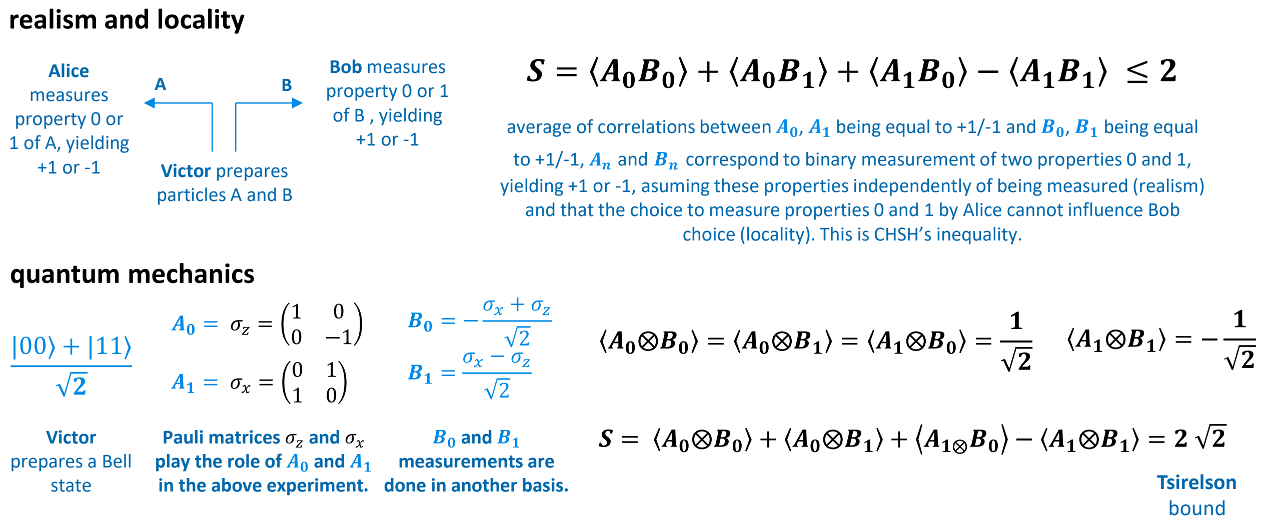

Figure 97: the famous statistical rules computing the test statistics S which separates classical realism from quantum mechanics entanglement, when it matches the Tsirelson bound which has other values for entanglement states with more than one inputs for each quantum object. Source: Wikipedia. 2023.

This test was extended with the **Mermin inequalities test** created by David Mermin in 1990 to extend Bell's inequalities test to the entanglement of a higher number of quantum object like a GHZ state with three or more qubits[197]. These tests are very costly as you increase the number of correlated quantum objects. Another variation is to conduct a state tomography for a set of qubits as described page 212. Again, its cost grows exponentially with the number of qubits, which explains why most qubit tomographies are not done beyond 6 qubits.

In science at the frontier of science fiction, some imagine exploiting quantum entanglement to analyze a quantum state inside a black hole[198]! This is beyond the scope of this book[199]!

Indetermination

Heisenberg's principle of indeterminacy or indetermination states that one cannot accurately measure both the position and velocity of a particle or two complementary quantities describing a quantum object state. It is mathematically described as an inequality showing that the multiplication of both precisions can't be lower than the Planck constant divided by 4π. Surprisingly, this inequality was not created by Werner Heisenberg but devised and demonstrated by **Earle Hesse Kennard** in 1927 as he was doing a sabbatical at the University of Göttingen. It is even named the Kennard inequality or Heisenberg-Kennard inequality[200].

The indeterminacy principle has another consequence: one cannot observe at the same time a quantum object in its particle state and in its wave state, per the principle of complementarity enacted by Niels

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[197] See Extreme quantum entanglement in a superposition of macroscopically distinct states by David Mermin. PRL, 1990 (no free access). Daniel Greenberger, Michael A. Horne, and Anton Zeilinger imagined a four entangled particles thought experiment in 1990. David Mermin then simplified it to use only three particles, which became the basis of the simplest GHZ state also labelled $|GHZ_3\rangle$.

[198] See Can entangled qubits be used to probe black holes? by Robert Sanders, 2019.

[199] Superposition also happens within benzene C_6H_6 with two carbon-carbon links with their neighbors, using one or two electrons.

[200] See The Uncertainty Principle, Stanford Encyclopedia of Philosophy, 2001 (14 pages).

Bohr around 1928, that we already mentioned in the wave-particle duality section. It also explains vacuum quantum fluctuations that we cover later in page 155.

For purists, the notions of particle speed and position are even meaningless for electrons. Its characterization is based on its wave nature and its probabilistic description via Schrödinger's wave function. Don't even try to understand where it is at a given place and time.

When it deals with velocity and position or waves, Heisenberg's indeterminacy principle is closely related to a characteristic of Fourier transforms: a nonzero function and its Fourier transform cannot both be sharply concentrated, so, precisely measurable. The more concentrated a signal is in the time domain, the more spread out it is in the frequency domain and vice versa. We have here a mathematical balance between a pulse length precision and its spectral analysis precision.

Since complementary (or incompatible) properties can't both be measured with an arbitrary precision, we can improve one property measurement precision by decreasing the measurement precision of the complementary property. It is being implemented with the so-called photons squeezing technique. This technique is implemented in the latest LIGO (USA) and VIRGO (Italy) huge interferometers that are used to measure gravitational waves generated by huge astrophysical phenomena like dual black hole collapses. These instruments increase the precision of photons time arrival in the interferometer at the price of a greater imprecision in the number of photons[201].

Measurement

Measuring quantum object properties follows a very different path than with classical physics due to the back action induced by quantum measurement on the measured system and to its probabilistic dimension.

With classical mechanics, you can usually predict over time the results of the measurement of macro-objects properties (dimension, speed, position) based on their dynamics. In quantum mechanics, given the knowledge of the position of the measured object, one cannot measure precisely its momentum.

More generally, the knowledge we have about two non-commuting observables is bounded such that we can never assign them a well define value simultaneously, due to the Heisenberg uncertainty principle.

Moreover, a quantum measurement readout requires some interaction with a macroscopic object that automatically selects one specific outcome. In strawman language, quantum measurement is in the eye of the beholder! Measuring the same initial state several times can lead to different outcomes. However, even if each measurement yields a probabilistic result, when repeated several times, their statistical distribution is not probabilistic. It corresponds to the knowledge that can be obtained from the evaluated quantum state created experimentally in a similar way several times.

Before measurement, a single isolated quantum object is said to be in a pure state, represented by a vector in a Hilbert space, or its "Psi" (ψ) vector. It is a superposition, or linear combination of basis states or one of the object basis states, like "ground state" or "excited". When a quantum object is measured against one observable, the state of the quantum object becomes one of the observable basis states, like a spin direction up or down or a discrete energy level. The quantum object collapses in a probabilistic way into one of the available basis states. If we conduct another measurement, we'll always get the same result being the basis state that was obtained beforehand in the first measurement. This is also called "*Schrödinger wave function* collapse" or "*wave packet collapse*" which however works only with so-called projective measurement, as defined by John Von Neumann.

[201] See Squeezing More from Gravitational-Wave Detectors, December 2019. Kip Thorne (1940, USA), Rainer Weiss and Barry C. Barish got the Nobel prize in physics in 2017 for their contributions to the creation of the LIGO detector and the observation of gravitational waves.

With a photon of intermediate polarization between horizontal or vertical linear polarization, it will become a horizontally or vertically polarized photon after its polarization measurement.

In quantum computing, this principle of reduction is implemented when measuring the state of a qubit. It modifies its value by collapsing it to the basis states $|0\rangle$ or $|1\rangle$.

The outcome is probabilistic with a chance of retrieving a $|0\rangle$ or a $|1\rangle$ depending on the qubit state. However, when the quantum state is a basis state, say $|0\rangle$ or $|1\rangle$ for a qubit, its measurement should return this basis state in 100% of the cases and is therefore not probabilistic but deterministic. This works, however, only in a perfect world without any quantum noise. Even when a qubit is in a basis state, its measurement doesn't return a perfect basis state 100% all the time. You get a % that is inferior to 100% and corresponds to the readout qubit fidelity. It turns a basis state measurement into a probabilistic one.

The subtle information contained in a qubit that is represented by a complex number or a two-dimensional vector is reduced to $|0\rangle$ or $|1\rangle$ at the time of its measurement. It becomes a classical bit. A single measurement is then making us lose all the wealth of information contained in the qubit. We turn the equivalent of two floating point numbers into a single bit! However, this measurement is supposed to happen only at the end of quantum algorithms. During computing, the whole wealth of qubit internal information is leveraged, particularly with the creation of interferences between qubits. All this is illustrated in Figure 98. We will come back to the meaning of α and β complex numbers in the next section on qubits.

This reduction should occur theoretically only at the end of computing. During computing, qubits are modified by quantum gates preserving the richness of their information, the combinatorial nature of their values based on superposition and entanglement. However, quantum measurement is to be implemented during computing with systems implementing quantum error corrections.

The subject of quantum measurement is quite broad. In a forthcoming more detailed section page 209, we will cover several additional concepts such as projective (Von Neumann) measurement, non-selective measurement, weak measurement, gentle measurement and non-destructive measurement.

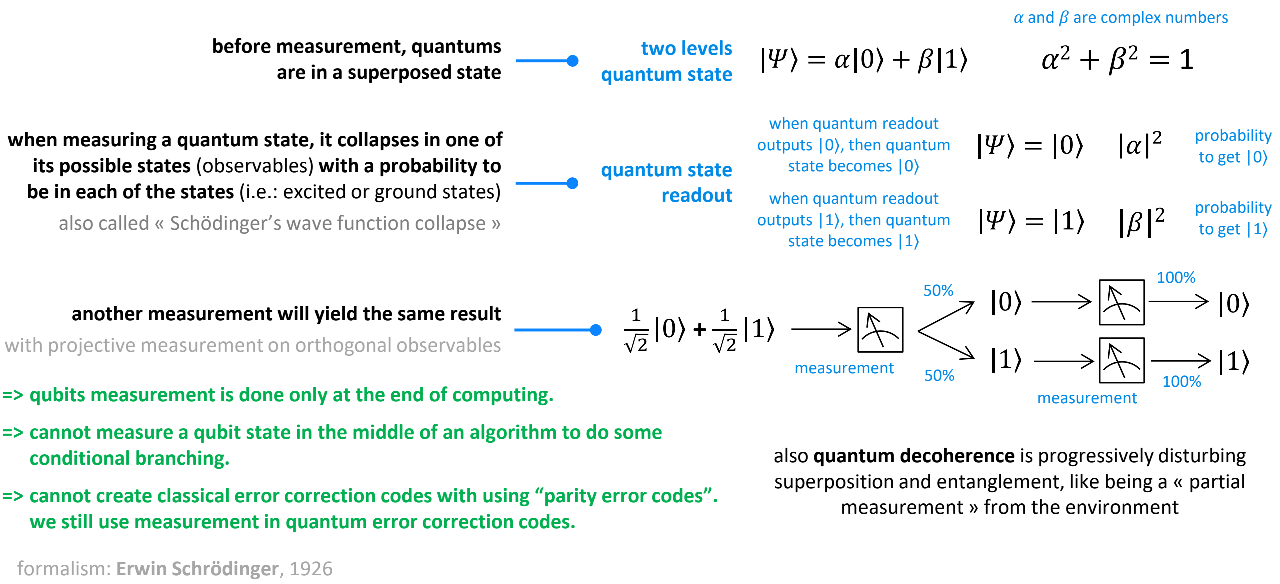

Figure 98: quantum measurement explained with qubits, (cc) Olivier Ezratty, 2021.

No-cloning

The no-cloning theorem prohibits the identical copy of the state of a quantum object onto another quantum object. The theorem is mathematically demonstrated in six lines in Figure 99.

The theorem was demonstrated in 1982 by William Wootters, Wojciech Zurek and Dennis Dieks[202]. The article is still not available in open source on a site such as arXiv, self-applying the no-cloning theorem[203]!

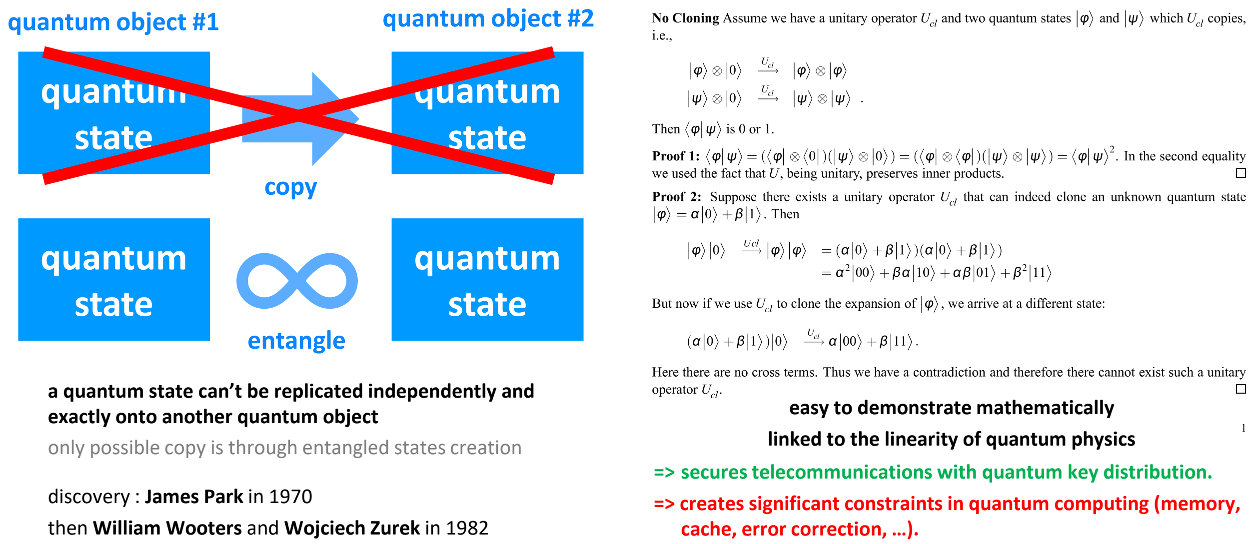

Figure 99: no-cloning theorem demonstration, source: C/CS/Phys C191 No Cloning, Teleportation Fall 2005 Lecture 6, Benenti et al. 2023.

Therefore, it is impossible to copy the state of a qubit to exploit it independently of its original, contrarily to a classical bit that can be copied from/to memory and from/to storage. It also prevents quantum computers from implementing a Von Neumann computing model with separate processing and memory.

In quantum computers, qubits can be duplicated via quantum gates and entanglement, but the resulting qubits are entangled and therefore somehow synchronized, inseparable and... random. Reading the copy destroys the original by projecting the state of the two qubits to the 0 or 1 closest to their initial state and in a probabilistic way.

This has a direct impact on the design of quantum algorithms and on the error correction codes of quantum computers. These error-correction codes use the trick of projective measurement on a different computational basis as we'll see later.

A derivative of no-cloning is non-deleting. In the case of a qubit, it means it is impossible to reset a qubit from an entangled set of two qubits $|\psi\rangle$, meaning to transform $|\psi\rangle|\psi\rangle$ into $|\psi\rangle|0\rangle$.

Tunnel effect

The wave-particle nature of matter allows it to cross physical barriers also known as energy walls in some circumstances, depending on the wall thickness and quantum object wavelength. The transmitted wave is usually attenuated after crossing the barrier and its strength depends on the wavelength with regards to the barrier length and composition (Figure 100).

This phenomenon was first accidentally unveiled by **Henri Becquerel** in 1896 when he discovered radioactivity. It did show up with uranium salts decaying, producing alpha rays comprised of two neutrons and two protons. This phenomenon was explained later thanks to quantum physics and wave-particle duality by **George Gamow** (1904-1968, Ukrainian-Russian-American) in 1928.

[202] See A single quantum cannot be cloned by William Wootters and Wojciech Zurek, Nature, 1982.

[203] A summarized version if available in The no-cloning theorem by William K. Wooters and Wojciech H. Zurek, Physics Today 2009 (2 pages).

Just before in 1927, the German physicist **Friedrich Hund** (1896-1997, German) created the formalism explaining electron based tunneling effect.

The tunnel effect is used in superconducting Josephson junctions and exploited in D-Wave quantum annealers where it is used to converge a system of spin qubits ("Hamiltonian", with a given total energy level) towards an energy minimum corresponding to the resolution of a complex combinatorial problem or a search for energy minimum as in chemistry or molecular biology.

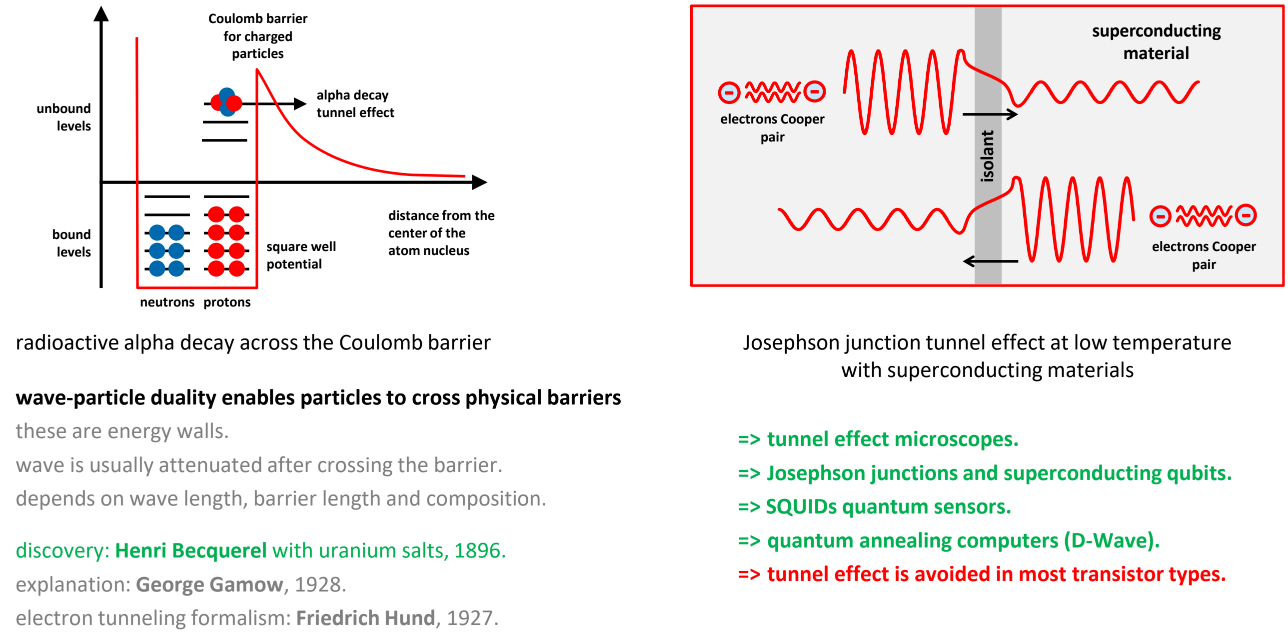

Figure 100: overview of the tunnel effect and its use cases, (cc) Olivier Ezratty, 2021-2023.

But contrarily to what I wrote in previous editions of this book, the tunnel effect is not exploited in transistors. Most transistors make use of the field effect which was patented by Julius Edgar Lilienfeld in 1926. It is implemented in MOSFETs (metal-oxide-semiconductor field effect transistor) and in CMOS (complementary-metal-oxide-semiconductor, that use variants of MOSFETs). These transistors use a metal gate deposited on a silicon-dioxide (SiO_2) and now a "high-K dielectric" as the gate dimension is decreasing with higher densities, to reduce the tunnel effect. The gate voltage determines the transistor conductivity.

Quantum matter

Quantum matter refers to materials or assemblies of few atoms which, for specific conditions, physical observables such as magnetism, electronic state or optical properties are only described by advanced quantum physics. They are at the crossroads of statistical physics. Iconic quantum materials are superconducting materials in which, below a certain temperature, electrons behave collectively as a sort of fluid dubbed "quantum fluid". Other known quantum fluids in physics are superfluid helium, Bose-Einstein condensates, polariton condensates and ultra-cold neutral atoms. They all exhibit quantum mechanical effects at a macroscopic collective level. These phenomena are usually reported at very low temperatures, close to -273°C, and sometimes high pressures but some of them start to emerge in less drastic conditions.

Definitions

Given all standard matter such as metal, semiconductor or insulator rely on quantum description, starting with electrons quantum numbers and the atomic structure, how are quantum materials and quantum matter being accurately defined? Where is the frontier? Well, there's no real consensus on this, a bit like how postulates are formalized in quantum physics.

One of the reasons is that quantum materials range from yet untested theoretical concepts to lab-based experiments, up to industry applications like with graphene. It is an entirely new research field that is still in the making with a lot of fundamental research.

It is also a field that is hard to dig into, even more than many other fields that are covered in that book, like quantum error correction. So, forgive some of the vagueness of this part. I have not really understood *all* the sentences I wrote here!

The simplest definition I found is "*materials where electrons do interesting things*". Then, I opened quantum matter's Pandora's box and found many other definitions.

The US Department of Energy created its own definition in 2016 with "*solids with exotic physical properties that arise from the interactions of their electrons, beginning at atomic and subatomic scales where the extraordinary effects of quantum mechanics cause unique and unexpected behaviors*"[204].

A more precise definition was proposed by Philip Ball in 2017 which is based on materials where electrons are operating collectively as quasiparticles and are frequently confined in some 2D geometries like graphene sheets, with derivatives in 3D assemblies of graphene sheets with small angle rotations called **magic angle**, creating the new field of **twistronics**[205] [206] [207] [208].

Quantum materials are also grouped as **strongly correlated materials** where magnetism is important and their behavior is "*dictated by quantum mechanical correlations between electrons*", and **topological materials** where some symmetry of the material lattice provides protected electronic states on the surface or in the bulk of the crystal[209] [210].

And I didn't try to find any semantic nuance between quantum matter and quantum materials! In another source [211], quantum matter deals with "*novel phases of matter at zero temperature with exotic properties*". It adds:

"*The main ways of characterizing and manipulating quantum matter are with entanglement, symmetry, and topology:*

***Entanglement** is the quantum property of correlated physical attributes among particles (position, momentum, spin, polarization).*

[204] Seen in Basic Research Needs for Quantum Materials, DoE, 2016 (4 pages), with a slightly simpler one "*solids with exotic physical properties, arising from the quantum mechanical properties of their constituent electrons; such materials have great scientific and/or technological potential*" seen in Quantum Materials for Energy Relevant Technology by the DoE Office of Science, 2016 (170 pages).

[205] In Quantum materials: Where many paths meet by Philip Ball, MRS Bulletin, 2017 (8 pages), Magic angle, a new twist on by Pablo Jarillo-Herrero and Senthil Todadri, MIT, January 2021 (12 pages).

[206] See Magic-Angle Multilayer Graphene: A Robust Family of Moiré Superconductors by Jeong Min Park, Pablo Jarillo-Herrero, December 2021 (15 pages). This could lead to interesting superconducting effects.

[207] See Magic-angle twisted symmetric trilayer graphene as a topological heavy-fermion problem by Jiabin Yu et al, PRB, July 2023 (65 pages).

[208] See Evidence for Dirac flat band superconductivity enabled by quantum geometry by Haidong Tian et al, Nature, February 2023 (15 pages).

[209] See The 2021 Quantum Materials Roadmap by Feliciano Giustino et al, February 2021 (93 pages).

[210] See Introduction to Quantum Materials by Leon Balents, KITP, 2018 (51 slides).

[211] See Quantum Matter Overview by Melanie Swan, Renato P. dos Santos and Frank Witte, April 2022 (23 pages).

Symmetry *refers to features of particles and spacetime that are unchanged under some transformation, seen as the property of a system looking the same from different points of view (a face, a cube, or the laws of physics) and its partner, symmetry breaking (in phase transitions)*[212].

Topology *is the property of geometric form being preserved under deformation (bending, stretching, twisting, and crumpling, but not cutting or gluing). Physical systems may have global symmetric and topological properties that remain invariant across system scales*". Usually, obtaining some topological order requires cooling at very low temperatures like with superconducting materials.

As stated before, quantum matter is characterized by being based on collective excitations. These excitations are composite entities that are analogous in their behavior to a single particle[213] named quasiparticles.

It can be quasiparticles that are assemblies of several fermions, mostly electrons and holes, like two electrons in Cooper pairs explaining superconductivity, polaritons, excitons and vortex magnetic phenomena like skyrmions, etc... It can also be collective excitations of bosons like phonons in crystal lattices. There are over 30 identified quasiparticles classes including some that are very exotic and less talked about like the Bogoliubon (a quasiparticle found in superconductors) and the wrinklon[214].

Philip Ball proposes a classification of these quasiparticles in seven categories[215]:

- **Cooper pairs** of electrons in classical superconductivity (high-temperature superconductivity with cuprates requires a more complicated explanation). We cover their various use cases with superconducting qubits and sensors.
- **Relativistic Dirac fermions** such as many-electron excitations in Dirac semimetals and in graphene[216]. Graphene has many applications in sensing and electronics. There was even a European Union Graphene Flagship program launched in 2013 with 1B€.
- **Weyl fermions** are massless fermions related to Dirac fermions whose existence was predicted by Herman Weyl in 1929 and discovered in 2015 at Princeton[217]. These fermions are massless, have a high degree of mobility and are quasiparticle excitations in Weyl semimetals. Topological semimetals could be used in low-consumption spintronic and magnetic memory devices and ultrafast photodetectors[218].

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[212] Classical matter phases transitions are traditionally described with Lev Landau's symmetry breaking model elaborated in 1937. It describes in a simplified way what happens at phase transitions (like gas↔liquid and liquid↔solid) with the evolution of a symmetry-breaking order parameter (OP) named η (eta). It also describes various types of ordering phenomena like ferromagnetic, ferroelectric, ferroelastic or other types of electronic orders like Mott or insulator-metal transitions systems. In most cases, quantum matter is described by a "topological order" that can't be explained by Landau's model. Some examples include topological insulators, topological semimetals, fractional quantum Hall states, quantum spin liquids and Fermi liquids. A Mott transition is a particular type of topological phase transition. Mott insulators are materials that are expected to conduct electricity but are insulators, particularly at low temperatures, and under certain conditions which can be controlled, leading to so-called Mott transitions.

[213] Source: Webster. Quasiparticle were first defined by Lev Landau in the 1930s.

[214] Source: https://en.wikipedia.org/wiki/List_of_quasiparticles.

[215] See Quantum materials: Where many paths meet by Philip Ball, 2017 (8 pages).

[216] See the thesis Relativistic Phases in Condensed Matter by Thibaud Louvet, 2018 (165 pages).

[217] See After 85-year search, massless particle with promise for next-generation electronics discovered by Morgan Kelly, 2015.

[218] See Topological Semimetals by Andreas P. Schnyder, 2020 (32 pages).

- **Laughlin quasiparticles** proposed by Robert Laughlin in 1983 and who received the Nobel in Physics in 1998 for his theoretical explanation of the fractional quantum Hall effect, together with Horst Störmer and Daniel Tsui, who discovered the effect experimentally. They relate to the "fractional quantum Hall effect" (FQHE, discovered in 1982) in a 2D "electron gas" placed in a magnetic field. It involves electron quasiparticles behaving like if they had a fractional charge, such as 1/3, 2/5 or 3/7, 1 being the charge of a single electron. One use case is to create an electron interferometer[219].
- **Majorana fermions** are hypothetical particles proposed in 1937 by Ettore Majorana, which are their own antiparticles. They exhibit some quantum phenomenon in devices like superconducting nanowires. Their existence is still questioned since, when observed, the related phenomenon measurement can easily be confused with ambient noise. Theoretically, Majorana fermions could lead to creating topological qubits quantum computers with a better resistance to quantum noise and errors and some better scalability potential in the fault-tolerant regime (FTQC).
- **Anyons** are hypothetical particles proposed by Frank Wilczek in 1982. Anyons have quantum statistics positioned in a continuum between fermions (1/2 spin) and bosons (integer spin). They could show up in quantum spin liquids[220]. These quantum spin liquids which can show up in magnetic materials where electron spins are not orderly aligned but are entangled. The first spin liquids were experimentally detected in 2020[221]. It could help to create innovative electronic memories. This state of matter was envisioned in 1973 by Philip W. Anderson[222].
- **Skyrmions** take the form of vortex-like topological quasiparticle excitations of spins in some magnetic materials. They were envisioned in 1962.

We could still add here various classes and subclasses of quantum materials:

- **Spin glasses** where electron spins freeze in a disordered fashion at some non-zero temperature. It leads to the notion of **quantum glasses**[223].
- **Plasmons** which are collective oscillations of electrons on the surface of a conductor that can interact with photons. It could also help create energy savings and faster data storage solutions.
- **Topological insulators** are materials whose bulk part is insulating and whose surface (2D or 3D) presents counterpropagating spin channels with no charge current [224] [225]. It could for example lead to the creation of new breeds of energy-saving and fast-switching transistors[226].

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219 See Realization of a Laughlin quasiparticle interferometer: Observation of fractional statistics by F. E. Camino, Wei Zhou and V. J. Goldman, 2005 (25 pages).

220 See A Field Guide to Spin Liquids by Johannes Knolle and Roderich Moessner, 2018 (17 pages).

221 See Scale-invariant magnetic anisotropy in $RuCl_3$ at high magnetic fields by K. A. Modic et al, October 2020 (32 pages).

222 See Quantum Spin Liquids by C. Broholm et al, May 2019 (21 pages).

223 See the review paper Quantum Glasses by Leticia F. Cugliandolo and Markus Müller, Sorbonne Université CNRS LPTHE and Paul Scherrer Institute, August-September 2022 (23 pages).

224 See the review Topological Insulators by M. Z. Hasan and C. L. Kane, Review of Modern Physics, 2010 (23 pages).

225 See Topological Insulators by Yoichi Ando, University of Cologne, July 2023 (10 pages).

226 See A Beginner's Guide to Topological Materials The stuff of 2016's Nobel prize in physics could become the logic in future computers and consumer electronics by Charles Q. Choi, IEEE Spectrum, June 2021

- **Quantum wires** are conducting wires with quantum confinement effects modifying the transport properties, mostly when the wires have a diameter of a few nanometers, event down to a single atom[227].

  They are usually called nanowires. Carbon nanotubes are a class of quantum wires (shown in Figure 101).

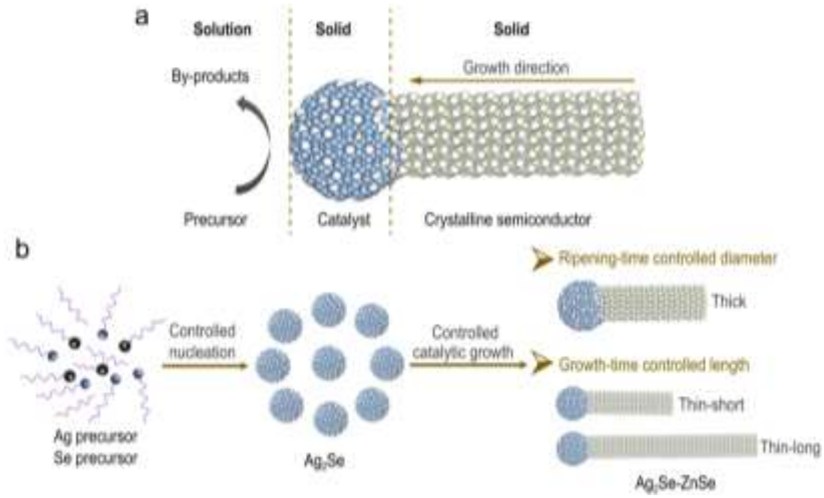

Figure 101: quantum wires. Source: On demand defining high-quality, blue-light-active ZnSe colloidal quantum wires from Yi Li et al, National Review Science, April 2022 (29 pages).

- **Spin-torque materials** that are already used in some low-power non-volatile magnetic memories (STT-RAM or STT-MRAM).
- **Time crystals** which we'll cover later, and it is the source of a lot of headaches.
- **Wigner crystals** are another very weird phenomenon. Predicted by Eugene Wigner in 1934 (the same Wigner of the Wigner function used in quantum photonics), it consists in crystals made of electrons, of course also at very low temperatures.

  They were experimentally observed in 2018 by an Israeli-US-Hungarian team in one dimension at 10 mK using carbon nanotubes for their measurement[228] and 2020 in 2D at 80 mK by a team from ETH Zurich (as shown here on the right in Figure 102) [229].

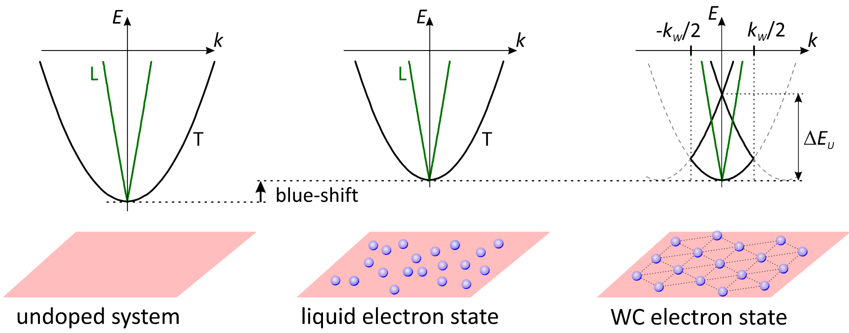

Figure 102: Wigner crystals. Source: Observation of Wigner crystal of electrons in a monolayer semiconductor by Tomasz Smoleńsk et al, 2020 (26 pages).

- **Quantum batteries** are still theoretical devices that would be more efficient than traditional batteries with a shorter recharging cycle.

[227] See one recent example of quantum nanowire in On demand defining high-quality, blue-light-active ZnSe colloidal quantum wires from Yi Li et al, National Review Science, April 2022 (29 pages).

[228] See Imaging the electronic Wigner crystal in one dimension by I. Shapir et al, Science, 2019 (38 pages).

[229] See Observation of Wigner crystal of electrons in a monolayer semiconductor by Tomasz Smoleńsk et al, 2020 (26 pages).

- **Quantum dots** that are used in LCD screens and are not considered as being quantum materials since their behavior is explained by single electrons and classical quantum light/matter exchanges. They are made of powder with tiny compound grains of different sizes between 2 and 6 nm which are used to down-convert the blue light coming from LEDs into red and green light, creating a better balanced coverage of primary colors, as shown in Figure 103[230]. The main problem is to replace cadmium that is a pollutant. These LCD screens quantum dots must not be confused with the quantum dots used in silicon qubits to trap single electrons and control their spin as well as the quantum dots used in unique photon sources like the ones from Quandela.

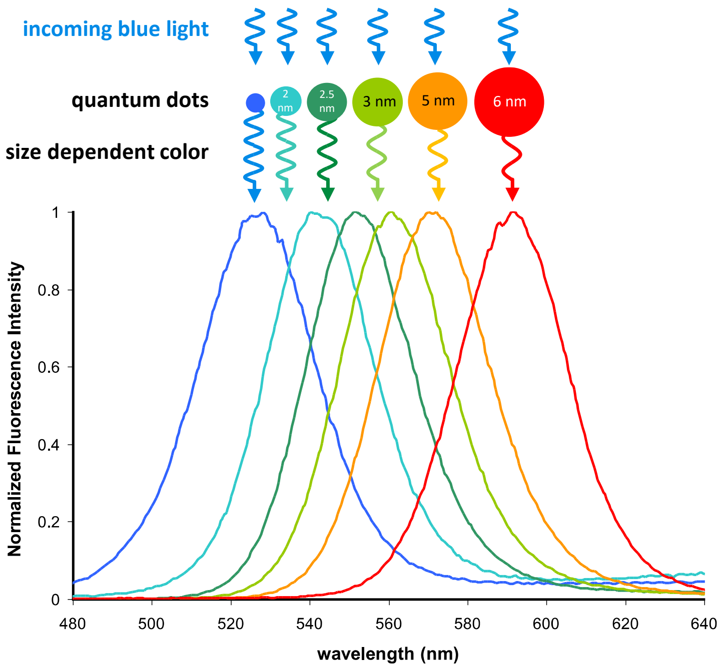

Figure 103: quantum dots used in LCD screen and lighting. Source: Olivier Ezratty, 2023, inspired from: Nanomatériaux et nanotechnologies : quel nanomonde pour le futur? by Pierre Rabu, 2018.

Some other concepts related to quantum matter mandate some explanations:

Many quantum matter species happen in crystals. And there are a lot of types of crystals classified by their crystallographic order! There are 230 crystallographic space groups organized in 7 crystal systems named triclinic, monoclinic, orthorhombic, tetragonal, trigonal, hexagonal and cubic and subclasses with primitive centering, centered on a single face, body centered and face centered (Figure 104)[231].

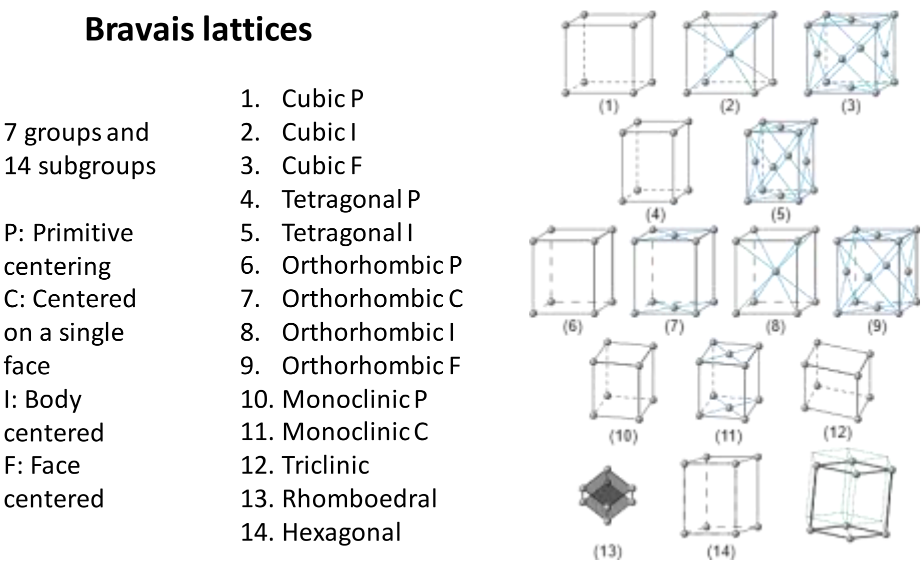

Figure 104: Bravais lattices and crystal structure classification. Source: Wikipedia.

230 It was first discovered at the end of the 1970s by Alexei Ekimov in Russia and explained in 1982 by Alexander Efros, also from Russia. From The Quantum Dots Discovery. See Advances in Quantum-Dot-Based Displays by Yu-Ming Huang et al, 2020 (29 pages), schema from Quantum dots and their potential impact on lighting and display applications by Paul W. Brazis, 2019 (18 pages).

231 See Crystal Systems and Space Groups by Paul D. Boyle, University of Western Ontario (44 slides) and Cristallographie et techniques expérimentales associées (in English) by Béatrice Grenier, 2014 (67 slides).

One key notion in crystallography is chirality which describes how crystal structures break spatial symmetry and are not identical to their mirrored structure[232]. There are also 1,651 magnetic space groups which describe magnetism configurations at the atom level in crystal lattices[233] (Figure 105).

Another key notion in quantum matter is time reversal symmetry. A time reversal symmetry means that the material looks the same when looking at a time scale backwards and forward.

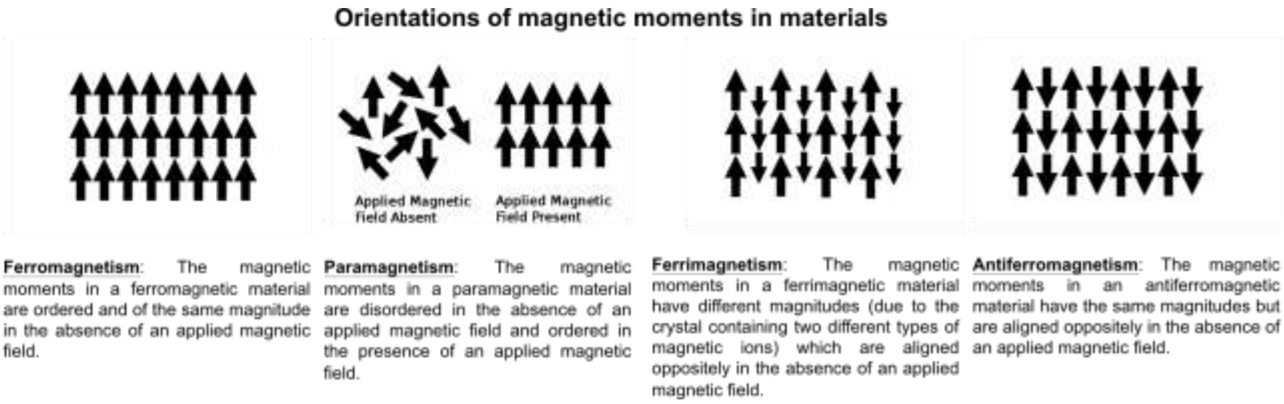

Figure 105: ferromagnetism, paramagnetism, ferrimagnetism and antiferromagnetism explained. Source: Wikipedia.

Reversing time means looking backwards in time only from a mathematical standpoint, not physically reversing time. There's no way you can change the arrow of time backwards. Time reversal is not a time machine! Figure 106 presents an inventory of some physical properties that change or do not change with time reversal. Some experiments of time reversal are also implemented with photons, but the related symmetries doesn't mean these systems can really go back in time like in science fiction[234 235 236].

time reversal symmetry

do not change with time reversal	changes with time reversal
position of particle in space	time of events
particle acceleration in space	particle velocity
force on the particle	particle linear momentum
particle energy	electric vector potential
electric potential and field	magnetic field
density of electric charge	electric current density
energy density of the EM field	power / rate of work done

Figure 106: time reversal symmetry explained.

Superconductivity

Superconductivity occurs when under a low-level temperature, some conducting materials no longer oppose resistance to electric current. With usual electric current, electrons move from atom to atom and transform part of their kinetic energy into heat related to the movement of the atoms hit by electrons, also known as the Joule effect.

[232] See A Chirality-Based Quantum Leap by Clarice D. Aiello and many al, November 2021 (93 pages) described in Chirality and the next revolution in quantum devices by César Tomé López, Mapping Ignorance, May 2022. See also Topology and Chirality by Claudia Felser and Johannes Gooth, May 2022 (27 pages) which makes a good classification including chiral and topological matter.

[233] See Magnetic Group Table, Part 2 Tables of Magnetic Groups, by Daniel B. Livin, 2014 (11,976 pages). I hope the author found some way to automatize the production of all these pages! See also Exhaustive constructions of effective models in 1651 magnetic space groups by Feng Tang et al, March 2021 (25 pages) and Structure and Topology of Band Structures in the 1651 Magnetic Space Groups by Haruki Watanabe et al, August 2018 (43 pages).

[234] See Physicists made Light move Simultaneously Forward and Backward in Time using 'Quantum time flip', Guardian mag, December 2022.

[235] See Experimental demonstration of input-output indefiniteness in a single quantum device by Yu Guo et al, October 2022 (18 pages).

[236] See Experimental superposition of time directions by Teodor Strömberg et al, November-December 2022 (15 pages).

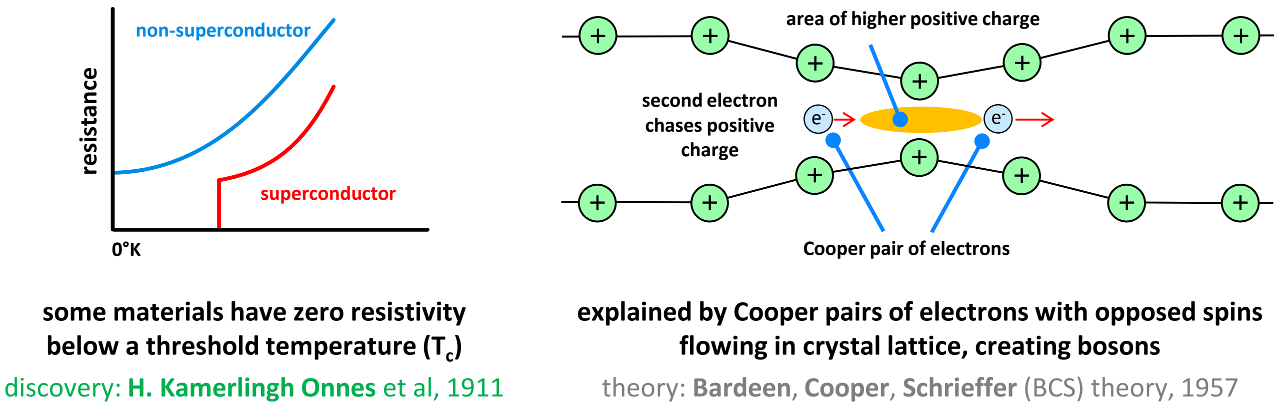

Figure 107: superconductivity explained. (cc) Olivier Ezratty, 2020-2023.

Discovery. Superconductivity was discovered experimentally in 1911 by **Heike Kamerlingh Onnes** (1853-1926), Cornelis Dorsman, Gerrit Jan Flim and Gilles Holst at the University of Leiden in the Netherlands, with solid mercury at 4.2K. Kamerlingh Onnes also discovered that a magnetic field whose intensity depends on temperature could make the superconducting effect disappear[237]. The critical temperature T_C of a superconductor is the maximum temperature at which it shows superconductivity behavior.

Explanation. Superconductivity interpretation was formulated much later, in 1957 and achieved by **John Bardeen**[238], **Leon Neil Cooper** and **John Robert Schrieffer** of the University of Illinois. They built the so-called **BCS theory** which explains how superconductivity works[239]. Electrons arrange themselves in pairs, called Cooper's pairs, circulating between atoms without friction (Figure 107). The structure of the atoms of the conductive metal is also modified. Waves of atoms occur that follow and accompany the movement of Cooper's pairs. These are specific breeds of phonons. Cooper's pairs are electrons of opposite spins forming composite bosons (ensemble with zero spin), allowing them to have the same quantum state[240] [241].

Elements. About 50 chemical elements are superconducting at low temperature but the superconductivity temperature and pressure thresholds are very variable (Figure 108). The superconducting effect is maximum for atoms that have a large number of valence electrons, i.e., in the last orbital layer, with the highest quantum number. In general, metals that are superconductors are poor conductors in their normal state and most good conductors like copper, gold and silver are not superconductors. Superconductivity is possible with composite alloys such as germanium, titanium and niobium alloys or copper-based materials (as cuprates). This is particularly the case with aluminum and mercury.

[237] See this detailed presentation: Superconductivity and Electronic Structure by Alexander Kordyuk, 2018 (145 slides).

[238] John Bardeen holds two Nobel prizes in physics, one in 1956 for the invention of the transistor with William Shockley and Walter Brattain and the other for the interpretation of superconductivity in 1972 with Leon Neil Cooper and John Robert Schrieffer. Cooper co-created the BCS theory at the age of 27 and won the corresponding Nobel Prize at the age of 42. Born in 1930, he is still with us today.

[239] An accurate timeline of the discovery of the principle of superconductivity is provided in the presentation 50 Years of BCS Theory "A Family Tree" Ancestors BCS Descendants, by Douglas James Scalapino, John Rowell and Gordon Baym, 2007 (52 slides). See also the excellent book The rise of superconductors by P.J. Ford and G.A. Saunders 2005 (224 pages) which tells the story of the discovery and then interpretation of superconductivity. Before the BCS theory, many physicists had broken their teeth on the explanation of superconductivity: Albert Einstein, Niels Bohr, Lev Landau, Max Born, Felix Bloch, Léon Brillouin, John Bardeen (co-inventor of the transistor), Werner Heisenberg and Richard Feynman.

[240] Cooper's pairs can also be formed with atoms as with helium 3, a fermion, in its superfluid state named a fermionic condensate.

[241] See Theory of Superconductivity by Carsten Timm, 2020-2023 (150 pages).

The most common superconducting materials are aluminum and a niobium and titanium alloy, used in superconducting wires in MRI imaging systems and superconducting qubit cryostats[242]. Titanium becomes superconducting at 390 mK, aluminum at 1.2K, indium at 3.4K and niobium at 9.26K.

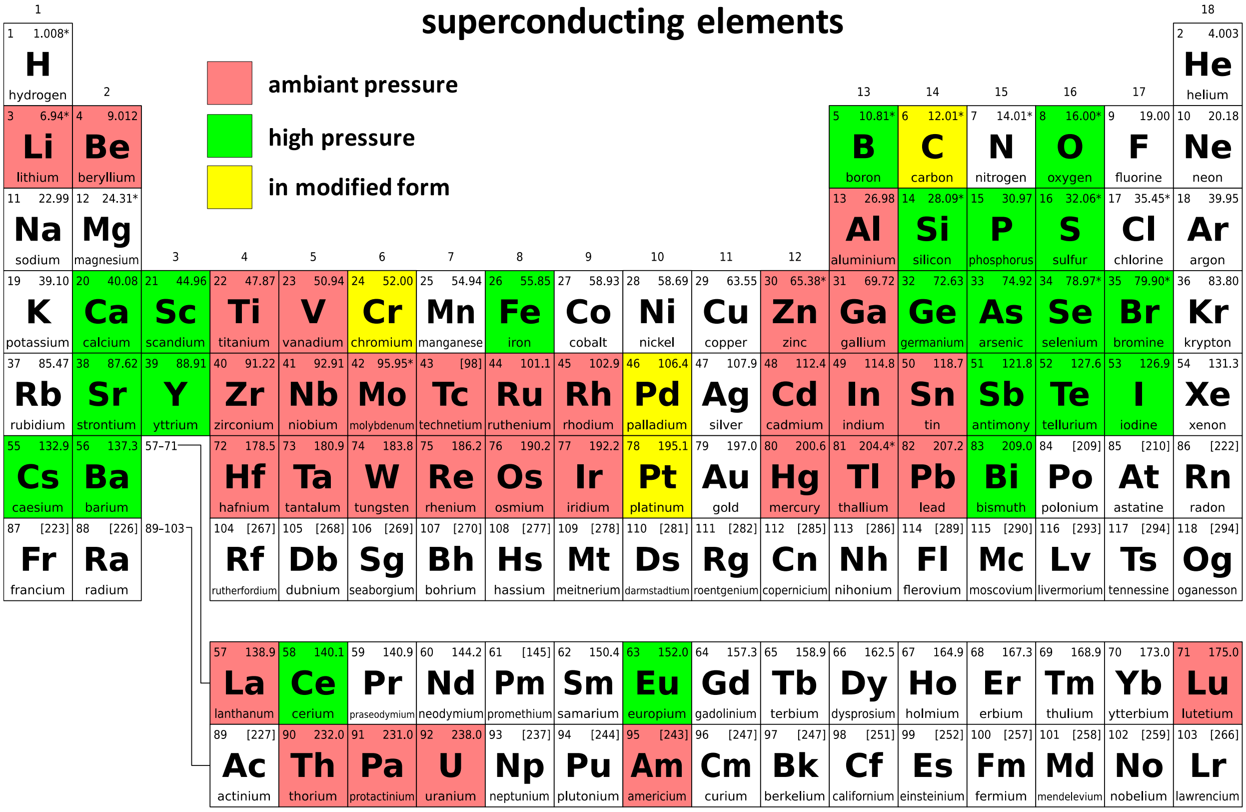

Figure 108: table of chemical elements with those which are superconducting. Source: Wikipedia and various other sources.

Later experiments and extrapolations on the persistence of circulating currents injected into macroscopic superconducting rings found that the lower bound of these permanent currents was around 10^5 years.

Meissner effect. Superconductivity explains unexpected phenomena such as the levitation of magnets above superconductors immersed in liquid nitrogen. Superconducting ceramics, discovered since 1986, can be used in this striking experiment. The magnetic field is then expelled from inside the superconducting material (Figure 109). This is the Meissner effect, discovered in 1933 by **Walther Meissner** (1882-1974, German).

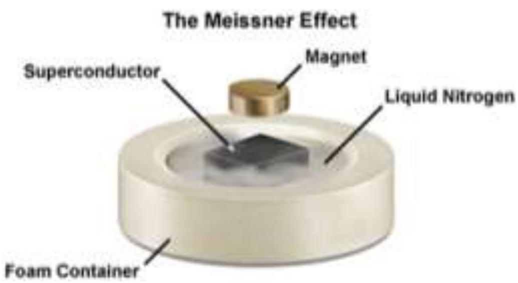

Figure 109: the Meissner effect.

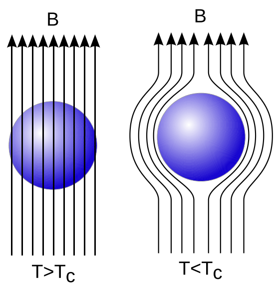

Figure 110: Meissner effect explanation.

242 See Superconductivity 101. The superconducting properties of the niobium-titanium alloy were discovered in 1962. It is widely used in the cooling of MRI scanners but also in many scientific instruments, notably in the ITER experimental nuclear fusion reactor at Caradache. The Periodic Table of Elements comes from Wikipedia.

Type I and Type II. The first classification of superconducting relates to the way they react to an ambient magnetic field[243] (Figure 111).

Type I superconductors are characterized by having a single critical magnetic field, denoted as B_C (C for "critical field") as shown in Figure 110. Below this critical field, type I superconductors expel all magnetic flux from their interior as in the Meissner effect and exhibit perfect diamagnetism (being pushed out of regions with stronger magnetic fields). They completely screen out external magnetic fields. Type I superconductors are typically s-wave superconductors with isotropic behavior. Meissner effect only works with type I superconductors.

Type II has an intermediate phase between the classical metallic phase and the superconducting phase that allows the magnetic field to pass partially, with two critical magnetic fields: the lower critical field (B_{C1}) and the upper critical field (B_{C2}).

Below B_{C1}, type II superconductors behave similarly to type I superconductors, expelling magnetic flux.

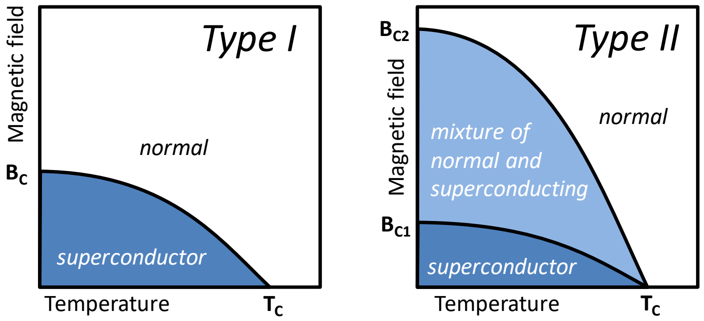

Figure 111: type I and II superconductors characteristics. Source: [Critical Magnetic Field](), undated.

Between B_{C1} and B_{C2}, they allow partial penetration of the magnetic flux into the material while still exhibiting superconducting behavior. Above B_{C2}, the superconducting material transitions into the normal state and loses all of its superconducting properties.

There is even a proposal for a Type III superconductor which show up in thin films with different wave functions mechanisms, "*described by a topological gauge theory and corresponds to an inhomogeneous network of condensate droplets getting connected by tunneling pairs percolation and is destroyed by vortex liberation instead of pairs breaking*"[244]!

s-wave, d-wave and p-wave. Superconducting materials are also classified in three kinds, dealing with the symmetry of the wave function describing the Cooper pairs. It has implications on the anisotropy of various properties and to the materials applications.

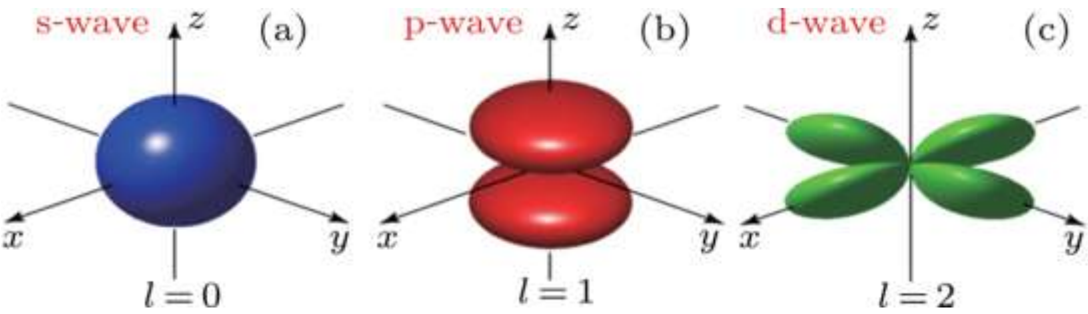

Figure 112: representation of the Cooper's pair wave function for s-wave, p-wave and d-wave superconductors. 2023.

In s-wave (spheric) superconductors, the electron pairs have an angular momentum of zero. Their wave function is symmetric under a 360-degree rotation, thus the spherical representation shown in Figure 112 and them being isotropic, i.e., their properties being the same in all directions. It allows

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243 Type I and II superconductors are mathematically and quantumly explained by the **Ginzburg–Landau** theory created in 1950. See [Theory of Superconductivity]() by Carsten Timm, TU Dresden, February 2022 (150 pages).

244 See [Type-III Superconductivity]() by M. Cristina Diamantini, Carlo A. Trugenberger, Sheng-Zong Chen, Yu-Jung Lu, Chi-Te Liang, and Valerii M. Vinokur, University of Perugia and National Taiwan University, Advanced Science, March 2023 (7 pages).

the electrons to move freely without scattering and leads to the observed zero resistance and expulsion of magnetic fields shown in the Meissner effect.

d-wave (degenerate) superconductors have electron pairs with an angular momentum of 2. Their wave function changes sign after a 180-degree rotation. Their behavior can be anisotropic with properties varying depending on the direction in the material. Certain high-temperature superconducting materials such as the cuprates are usually of the d-wave type.

p-wave (polar) superconductors have electron pairs with an angular momentum of 1. They also exhibit anisotropic behavior, similar to d-wave superconductors, with different anisotropy patterns.

There are also a lot of so-called "unidentified Superconducting objects" with weird characteristics like palladium hydride (52K to 61K), multi-walled carbon nanotubes (6.8K), moiré graphene sheets (twistronics) and doped graphite.

Use cases. Here are the several known industry use cases for superconductors.

MRI scanners use large superconducting magnets that are cooled with liquid helium. Scanners are encased in a protective coating to constrain the magnetic field inside the scanner. The niobium-titanium coil wiring is enveloped in a copper matrix[245] (Figure 113).

Figure 113: MRI principle.
Source of illustration on the right: Helium Reclaiming Magnetic Resonance Imagers by Dan Hazen, MKS Instruments (5 pages).

CERN LHC is also using this combination in large physics instruments like the LHC in Geneva with 1,200 tons of cables including 470 tons of NbTi (niobium-titanium), the rest being copper, in cables totaling 21 km. Superconductivity creates a current of 11,850 A generating a powerful magnetic field of 8.33 tesla creating a centripetal force holding the accelerated particles. These magnets are cooled by 10,000 tons of superfluid helium-4 at 1.9K. Their cables are made of niobium-titanium filaments surrounded by copper. The whole unit power is 40MW with an electricity consumption estimated at 750 GWh per year according to CERN. It is the largest and most powerful refrigerator in the world!

Transportation. The Chuo Shinkansen Maglev high-speed train in Japan, which has been undergoing trials since 2013 and is expected to reach a commercial speed of 505 km/h uses a superconductive based magnetic suspension with a rather expensive infrastructure. Power consumption per passenger/kilometer is three times that of traditional Shinkansen, but it is still competitive with airplanes. A 286 km Tokyo-Nagoya line is planned for commercial service in 2027.

Electric motors and generators. Superconductivity has also been studied to improve the efficiency of electric motors and generators with HTS Synchronous Motors (High-Temperature Superconductors). It allows a reduction of motors size and efficiency improvements.

[245] Nuclear magnetic resonance imaging.

It is based on superconducting materials that only require liquid nitrogen cooling, but some systems still use helium-based cooling. Studies began in the 1980s and these engines and generators are beginning to be deployed in the military navy and in wind power generation, notably at **ASMC**, **Sumitomo Electric**[246] and with the European **EcoSwing** project, which involves Sumitomo's cryostat division.

Electric cables. Superconducting cables have also been introduced to transmit electricity without power loss and greater capacity to meet the ever-increasing demand. They are offered by the French cable manufacturer **Nexans** (Figure 114), which installed one in Long Island. Their 600 m underground cable has been in operation since 2008. It can supply electricity to 300,000 homes[247]. But it is rather complex to implement and was not seemingly replicated in many places. The project cost was $46.9M.

Figure 114: Nexans superconducting cable.

Superconducting qubits exploit the Josephson effect that we have already described in another section. It is usually based on aluminum and niobium. This technology is also used in variations of SQUIDs (superconducting quantum interference device) in quantum sensing. Josephson junctions have a relationship between voltage and frequency which enables the creation of various sensors. It can convert a voltage to frequency as well as a frequency to voltage (with the inverse AC Josephson effect using a microwave impulse). We also find it in the type II niobium-titanium based superconducting cables used for reading the state of superconducting and electron spin qubits.

We may wonder why scientists are not using high-temperature superconductors to build superconducting qubits? There are various reasons, one being the need to avoid thermal photons that can destroy the qubit coherence. Also, superconductors used in a Josephson junction must be of the s-wave type due to their robustness against magnetic fields, well controlled energy levels and transition frequencies used in qubit control, their ease of manufacturing on thin films, and electrons isotropic pairing symmetry which simplifies the design of the qubits.

Superconducting electronics. Superconductivity could also be used to create processors operating at low temperatures and capable of operating up to 700 GHz, much faster than current server processors running at a peak 4 to 5 GHz[248]. An MIT team announced in July 2019 a proposal for a technique to create spiking neurons with superconducting Josephson effect circuits using nanowires[249]. This is still a research field with very few industry applications at this point. We'll investigate this field in a specific section on unconventional computing. Superconducting electronics could be very useful to create and analyze the microwaves used in superconducting and electron spin qubits.

High-Temperature Superconductors (HTS) also named HTC (for "high T_c" or high-critical temperature superconductors). Scientists from IBM began discovering superconducting metal alloys above 77K (-196°C) in the late 1980s, the temperature of liquid nitrogen.

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[246] See Design of MW-Class Ship Propulsion Motors for US Navy by AMSC by Swarn S. Kalsi, 2019 (50 slides).

[247] Information Source: Long Island HTS Power Cable, Department of Energy, 2008 (2 pages). In addition to Nexans, the cryogenic system was supplied by Air Liquide.

[248] See Superconductor ICs: the 100-GHz second generation by Darren Brock, Elie Track and John Rowell of Hypres, 2000 (7 pages).

[249] See A Power Efficient Artificial Neuron Using Superconducting Nanowires by Emily Toomey, Ken Segall et Karl Berggren, 2019 (17 pages).

Most of them are cuprates alloys (copper-based). A record was achieved in 2019 with a molecule combining lanthanum and hydrogen (LaH_{10}, illustrated in Figure 115) and at -23°C, thus a near-ambient temperature. In the latter case, however, it works at a huge pressure of 218 GPa, representing more than 2 million times the atmospheric pressure, which is 101,325 Pa [250]. Other records were broken with metallic hydrogen in 2020 by CEA, operating at 17°C and at an even greater pressure of 400 GPa[251]. Another record of 15°C with 270 GPa was achieved in the USA also in 2020, using a carbonaceous sulfur hydride[252]. A less impressive 2022 record was created in China with clathrate calcium hydride (CaH_6) being superconducting at 215K and 172 GPa[253].

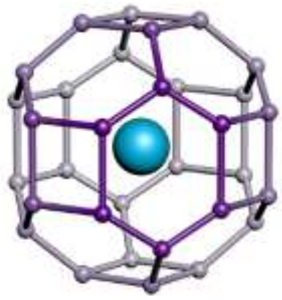

Figure 115: LaH_{10} high superconducting temperature molecule.

You always see this trade-off between superconducting temperature and pressure. At this very high pressure, practical use cases are not easy to implement! But at lower temperatures, interesting used cases arise like with single photons detectors[254]. Hence the willingness to use quantum simulators or computers to run superconductivity quantum equations and identify materials that would be superconducting at room or near-room temperature[255].

We then have the quest for ambient pressure and temperature superconductors. Some were announced in the past. Like with Indian Institute of Science (IISc)'s questionable silver nanoparticles in a gold grid (not cheap)[256] [257] or Ranga Dias's papers in Nature and PRL that were retracted in 2022 and 2023[258].

LK99 which appeared in July 2023 made the news as being a potential ambient room and pressure superconducting material. The copper-lead-phosphorus-oxygen material was discovered by a team of South-Korean researchers in July 2023[259] [260]. It drew a strong interest since, if working as advertised, it would be the Holy Grail of superconductivity and have significant industry applications as showcased above in this text. Patents were deposed and a startup even already created. How do you determine that a material is superconducting? Many tests must be done on AC susceptibility, temperature-dependent critical field and critical current, single-particle tunnelling gap, jump in specific heat at T_C, Josephson tunnelling, AC Josephson effect, Meissner effect and so on (Twitter source). The original papers were quite sketchy in the first place. The superconducting effect was limited to very weak currents (<1V, <500mA). Also, LK99 is of d-wave type.

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[250] See Quantum Crystal Structure in the 250K Superconducting Lanthanum Hydride by Ion Errea, July 2019 (20 pages).

[251] See Here comes metallic hydrogen - at last! by Jean-Baptiste Veyrieras, May 2020. Another record was broken in 2019 with YH_6 (yttrium hybrid) at a pressure of 110 GPa. See Anomalous High-Temperature Superconductivity in YH6 by Ivan A. Troyan et al, 2019 (36 pages).

[252] See Room-temperature superconductivity in a carbonaceous sulfur hydride by Elliot Snider et al, Nature, October 2020 (14 pages).

[253] See High-Temperature Superconducting Phase in Clathrate Calcium Hydride CaH_6 up to 215 K at a Pressure of 172 GPa by Liang Ma et al, PRL, April 2022 (not open access).

[254] See Single-photon detection using high-temperature superconductors by I. Charaev et al, August 2022 (8 pages).

[255] Another fancy solution consists in lowering the room temperature as described in Novel approach to Room Temperature Superconductivity problem by Ivan Timokhin and Artem Mishchenko, April 1st, 2020 (4 pages).

[256] See Scientist claimed to obtain superconductivity at ambient temperature by Amit Malewar, August 2018.

[257] See A Superconductor Scandal? Scientists Question a Nobel Prize-Worthy Claim by Shannon Hall, Scientific American, August 2018.

[258] See 'A very disturbing picture': another retraction imminent for controversial physicist, Nature, July 2023.

[259] See The First Room-Temperature Ambient-Pressure Superconductor by Sukbae Lee et al, July 2023 (22 pages).

[260] See Superconductor $Pb_{10-x}Cu_x(PO_4)_6O$ showing levitation at room temperature and atmospheric pressure and mechanism by Sukbae Lee et al, July 2023 (18 pages).

Very quickly, other laboratories in China and the USA, tried to reproduce the experiment[261]. It did generate very mixed to bad results. A scientist from the Lawrence Berkeley National Laboratory used a classical DFT-based simulation to explain the potential origins of the material superconductivity[262]. One China team proposed a way to improve the performance of the material[263]. Another China team couldn't replicate room temperature superconductivity at ambient temperature[264] and another one could make it but only at 100K, which is still kind of cold[265]. Another experiment did show the effect of ferromagnetism explaining the observed levitation, but not superconductivity. But the material was produced with sintering instead of annealing which may explain the different results[266]. Other papers explained that the material was only diamagnetic, which is commonplace[267] and that the near zero resistivity at 387K was explainable by some Cu_2S impurities[268]. At last, the Korean academy temporarily invalidated the work[269]. You can also find a lot of more or less valid scientific information about LK99 in various community sites[270] [271] [272] and even an estimation of the quantum computing resources that would be needed to simulate LK99[273].

Just after LK99 was announced, a weird US company, Taj Quantum, specialized in securing communications with a Blockchain, announced the validation of a patent for a type II ambient temperature superconducting material using graphene doped with aliphatic hydrocarbons, created by John Wood and Paul Lilly[274]. Well well.

The end conclusion in August 2023 was that LK-99 is "*an insulator with a resistance in the millions of ohms, too high to run a standard conductivity test. It shows minor ferromagnetism and diamagnetism, but not enough for even partial levitation*"[275]. Case closed!

This showcases the problem with today's scientific communication with the media and other influencers broadcasting news before scientists can carefully examine new claims. The timing discrepancy between these two categories of players is a given, teaching us to always be careful about extraordinary claims. But when the scientific community is mobilized to fact check claims, it can make miracles in a very short time.

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[261] See LK-99: The Live Online Race for a Room-Temperature Superconductor (Summary), 2023.

[262] See Origin of correlated isolated flat bands in copper-substituted lead phosphate apatite by Sinéad M. Griffin, July 2023 (14 pages).

[263] See The Cu induced ultraflat band in the room-temperature superconductor $Pb_{10-x}Cu_x(PO\$_4)_6O_4$ (x=0,0.5) by Kun Tao et al, August 2023 (7 pages).

[264] See Semiconducting transport in $Pb_{10-x}Cu_x(PO_4)_6O$ sintered from Pb_2SO_5 and Cu_3P by Li Liu et al, July 2023 (12 pages).

[265] See Observation of zero resistance above 100K in $Pb_{10-x}Cu_x(PO_4)_6O$ by Qiang Hou et al, August 2023 (7 pages).

[266] See Ferromagnetic half levitation of LK-99-like synthetic samples by Kaizhen Guo et al, August 2023 (10 pages).

[267] See Absence of superconductivity in LK-99 at ambient conditions by Kapil Kumar et al, August 2023 (14 pages).

[268] See First order transition in $Pb_{10-x}Cu_x(PO4)_6O(0.9<x<1.1)$ containing Cu_2 by Shilin Zhu et al, August 2023 (7 pages).

[269] See Academic body invalidates superconductor research results, August 2023.

[270] See Will the LK-99 room temp, ambient pressure superconductivity pre-print replicate before 2025?, Manifold.

[271] See Room-temperature superconductor discovered, RedIt.

[272] See The First Room-Temperature Ambient-Pressure Superconductor, comments on PubPeer.

[273] See How hard is it to model LK99 on a quantum computer? by Glenn Jones and Evan Sheridan, Phasecraft, August 2023.

[274] https://tajquantum.com/art-t2sc/ and https://tajquantum.com/11710584-2/.

[275] See LK-99 isn't a superconductor — how science sleuths solved the mystery by Dan Garisto, Nature, August 2023.

Superfluidity

Superfluidity is yet another quantum physics phenomenon to cover here. It occurs only with superfluid helium which, at ambient pressure, never freezes, no matter how low the temperature can be.

Superfluid liquid has zero viscosity and flows without any loss of kinetic energy. When poured into a recipient, it tends to rise by capillary action on its rim and flow out of it. It can even pass through very fine capillaries (Figure 117).

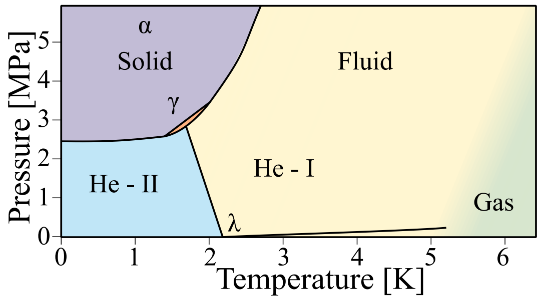

Figure 116: superfluidity. Source: Wikipedia.

Helium was first liquefied in 1908 at 4.2K by Heike Kamerlingh Onnes, the discoverer of superconductivity in 1911. Its superfluidity was highlighted independently in 1938 by **Pyotr Kapitsa** (1894-1984, USSR), **John Frank Allen** (1908-2001, USA) and **Don Misener** (1911-1996, USA)[276].

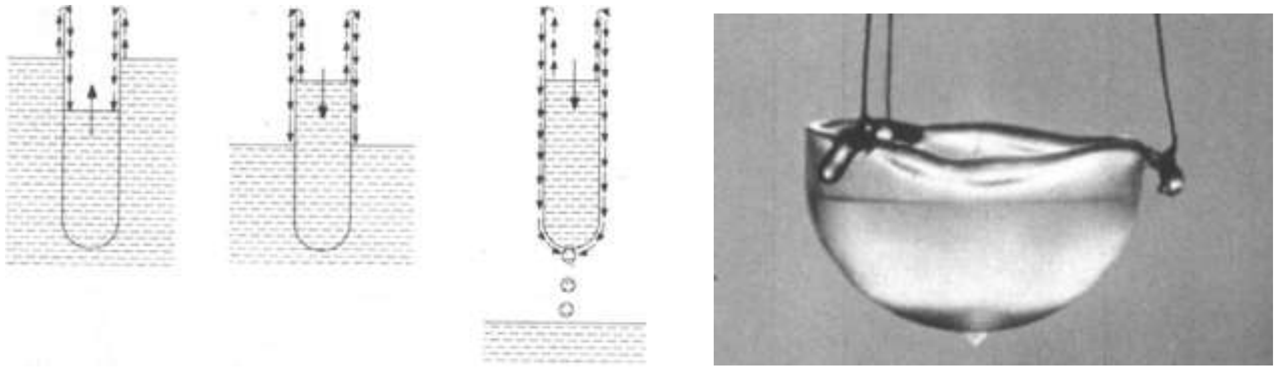

Figure 117: visualization of the superfluidity phenomenon. Source: Helium 4 (14 slides).

There are two isotopes of helium: ^{3}He with a single neutron, which is the least abundant in nature, and ^{4}He, with two neutrons, the most common. The latter is a boson, with an integer spin, giving it different properties from helium 3, which is a fermion with a half-integer spin. At low temperature, ^{4}He behaves like Bose-Einstein condensates since being bosons. ^{3}He behaves differently, being fermions, and assemble in pairs similar to electron Cooper pairs.

It becomes superfluid at lower temperatures than ^{4}He, at around 1 mK in the absence of a magnetic field (see the phase diagram in Figure 116), vs. 2.17K for ^{4}He.

Its superfluidity was only discovered in 1973[277]. The different properties of ^{3}He and ^{4}He are used to operate the dilution cryogenics systems that equip many quantum computers whose operating temperature is between 10mK and 1K. We will study this in detail in this book, starting page 562. Superfluid helium could also be used to create low dissipation mechanical resonator-based dark matter detectors[278].

Industrial demand for helium is spread across many industries: medical imaging for MRI systems magnets cooling, then microelectronics industries (Figure 118).

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[276] See Viscosity of Liquid Helium below the λ-Point, Pyotr Kapitsa, Nature (1938) and Flow of liquid helium II, Joan F. Allen, Don Misener, 1938 (1 page). Pyotr Kapitsa was awarded the Nobel Prize in 1978 for his work in the field of low temperatures.

[277] David Morris Lee (1931), Douglas Dean Osheroff (1945) and Robert Coleman Richardson (1937-2013) were awarded the Nobel prize in physics in 1996 for their discovery of helium-3 superfluidity.

[278] See HeLIOS: The Superfluid Helium Ultralight Dark Matter Detector by M. Hirschel et al, September 2023 (9 pages).

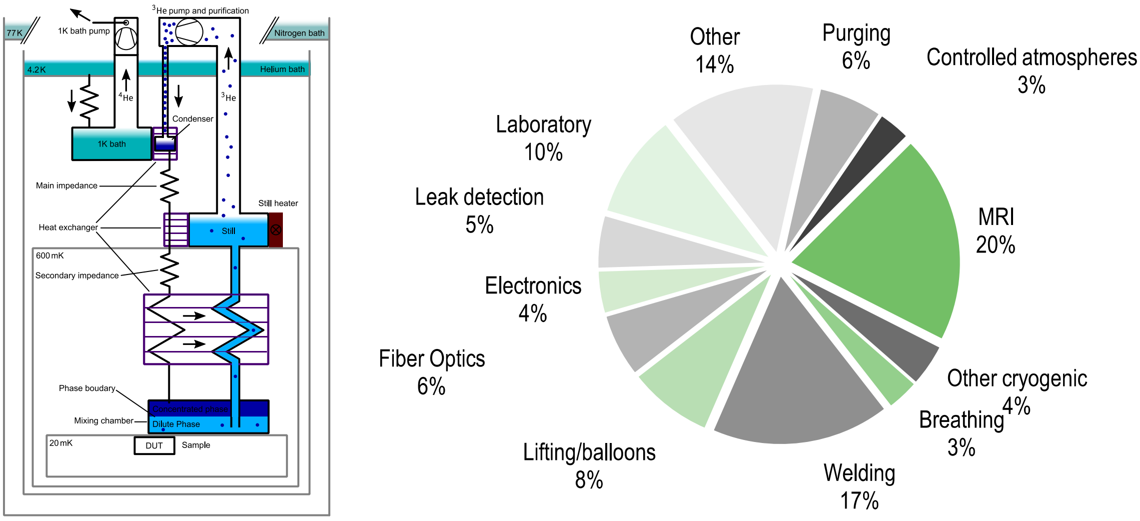

Figure 118: Sources: left diagram: [Wikimedia](), right diagram: [Edison Investment Research](), February 2019, referring to [Kornbluth Helium Consulting]().

Bose-Einstein Condensates

Bose-Einstein condensates is a particular state of matter. These are extremely low-density gases of bosons cooled down to very low temperatures, at the lowest energy level we can set matter in, below solid state. ^{4}He is the most famous element that was experimented in this matter state (Figure 119).

It took a while between the work of Bose and Einstein in 1924 and the experimental discovery of BECs in 1995 by **Carl Wieman**, **Wolfgang Ketterle** and **Eric Cornell** with rubidium 87 at 170 nK. It was cooled with laser-based Doppler effect and magnetic evaporating technique.

BECs play an important role in quantum technologies. They led to the control of individual atoms that are used in quantum simulators and in quantum gravimeters, although at a higher temperature and with atoms being exploited as individual quantum objects in the case of computing. Together with superfluids and supersolids, BECs belong to the field of quantum hydrodynamics.

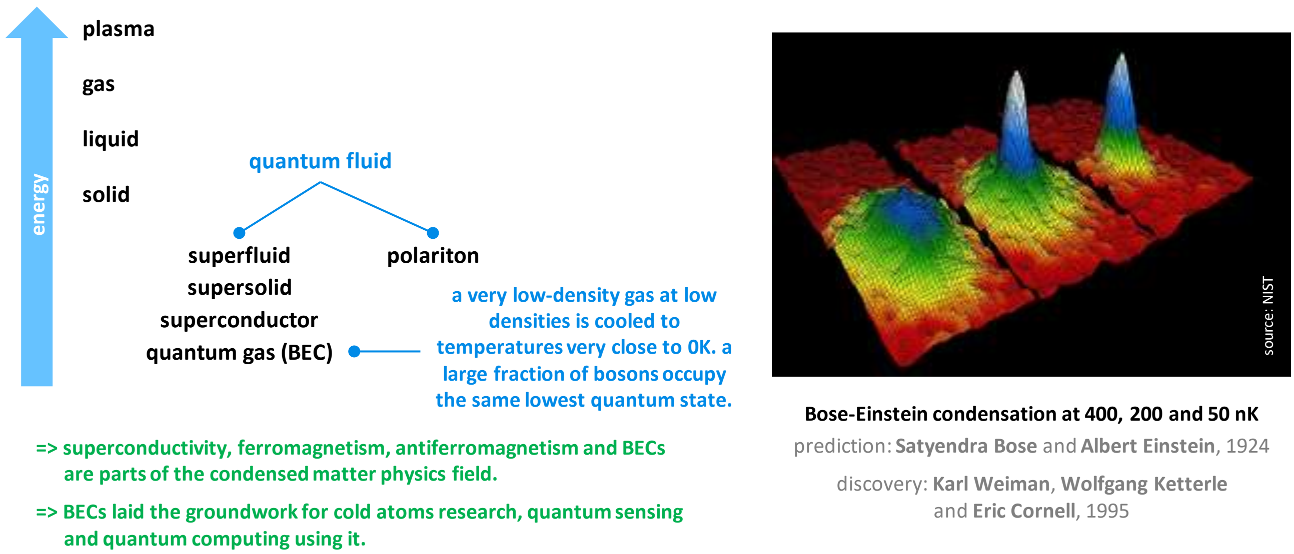

Figure 119: Bose-Einstein condensates positioned within the various states of matter. (cc) Olivier Ezratty, 2021-2023.

BECs are even experimented in space. The NASA Atom Laboratory (CAL) has been studying ultra-cold gases in microgravity conditions and with temperatures below 100 pK, including Bose-Einstein condensates with rubidium-87 and potassium-41 in the ISS since May 2018[279].

Very low temperature can be the playing field of various phenomenon, including the association of atoms in many-body objects like trimers (three atoms) as described by Efimov physics[280].

Supersolids

Supersolidity is another weird quantum state of matter showing up at ultracold temperatures, when atoms behave as a crystal and as a superfluid at the same time. This is made possible with crystal lattice with holes (like in an NV center)[281]. The vacancies behave quantumly as bosons and can switch position in a quantum manner like a Bose Einstein Condensate. It is a vacancies quantum tunnelling phenomenon.

This state of matter was predicted in 1969[282] and it was first demonstrated, although debated for a long time, in 2004 with ^{4}He at a pressure of about 60 bar and below 170 mK[283]. The related fundamental research is going on in various places in the world like in the USA, Innsbruck[284], Pisa[285], Stuttgart, Warsaw, Geneva, and Paris. It is now possible to create supersolids with ultracold dipolar quantum gases of highly magnetic lanthanide atoms like erbium and dysprosium. The supersolidity effect can be controlled by a magnetic field.

There are no known practical applications of this phenomenon to date although it could lead to new forms of quantum simulation systems like the ones using cold atoms.

Polaritons

Polaritons is a field of quantum physics that is rarely mentioned in the context of quantum technologies. It mostly belongs to fundamental research but could be of interest in various fields such as quantum computing and quantum sensing.

Polaritons are quantum quasiparticles in the domain of strong coupling between light and matter. They result from the coupling between photons and an electrical polarization wave.

These waves occur in particular in plasmons (oscillations of free electrons in metals), phonons (oscillations of atoms, especially in crystal structures) and excitons (pairs of electron holes generated by photons in semiconductors[286]). The materials can be atoms gas, massive classical semiconductors, thin films inserted in optical cavities or superconducting Josephson junctions.

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279 See NASA's Cold Atom Laboratory: Four Years of Quantum Science Operations in Space by Kamal Oudrhiri et al, NASA, May 2023 (13 pages).

280 See Efimov Physics: a review by Pascal Naidon and Shimpei Endo, RIKEN, October 2016-October 2022 (97 pages).

281 See Heating a dipolar quantum fluid into a solid by J. Sánchez-Baena, C. Politi, F. Maucher, F. Ferlaino and T. Pohl, Nature Portfolio, April 2023 (6 pages).

282 By David J. Thouless (1934-2019, British, 2016 Nobel prize in physics) and, independently, by Alexander Andreev (1939, Russian) and Ilya Mikhailovich Lifshitz (1917-1982, Russian). See The flow of a dense superfluid by David J. Thouless, 1969 (25 pages) and Quantum theory of defects in crystals by Alexander Andreev and Ilya Mikhailovich Lifshitz, 1969 (7 pages).

283 See Probable observation of a supersolid helium phase by E Kim and M H W Chan, 2004, The enigma of supersolidity by Sébastien Balibar, Nature, 2010 (7 pages) and the review paper Saga of Superfluid Solids by Vyacheslav I. Yukalov, 2020 (26 pages).

284 Research in Austria is led by Francesca Ferlaino from the University of Innsbruck, IQOQI.

285 See The supersolid phase of matter by Giovanni Modugno, 2020 (37 slides).

286 The name of polariton was created by Joseph John Hopfield (1933, American) in 1958 and at that time concerned polariton excitons. See Theory of the Contribution of Excitons to the Complex Dielectric Constant of Crystals by Joseph John Hopfield, 1958 (14 pages). Hopfield is also known in the field of neural networks in AI with his "Hopfield networks".

Excitation photons have a wavelength corresponding to the resonance frequency of the associated medium, often in the visible light or infrared ranges. Polaritons have mixed properties of photons dressed by electronic excitations. They behave like bosons (having an integer spin) that can occupy the same quantum state and operate in groups, such as superconducting currents forms with paired electrons named Cooper pairs or Bose-Einstein condensates (BEC).

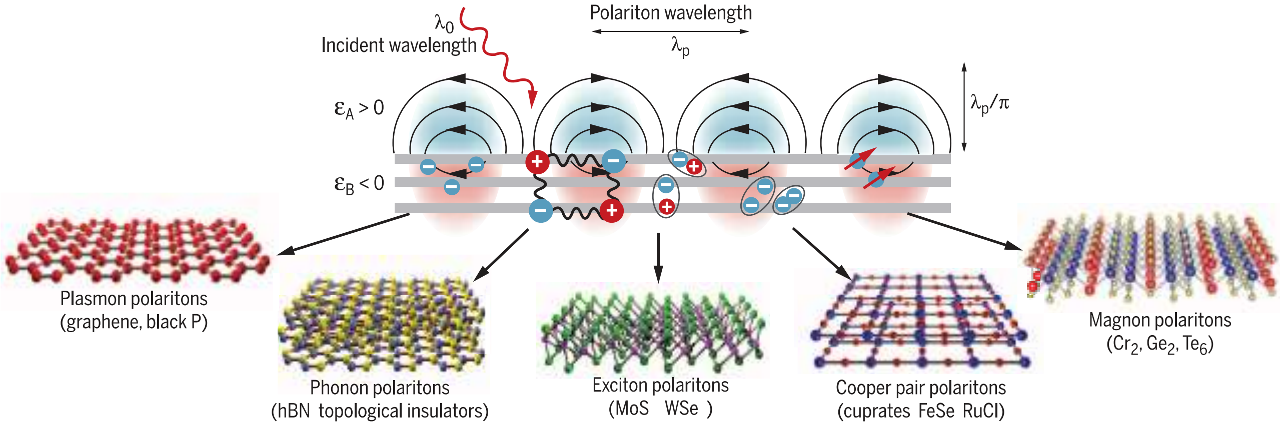

Figure 120: various forms of polaritons.
Source: Polaritons in van der Waals materials by D. N. Basov et al, 2016 (9 pages) which makes a good inventory of different types of polaritons and their fields of application.

Depending on the interaction scale, polaritons operate in a semiclassical or quantum regime. In the first case, the electromagnetic field interacts with a macroscopic polarization field. The polariton field then has the properties of a classical field but its elementary quantum is the result of a dipole-photon "wrapping" that can only be described by quantum mechanics. In the second case, the electromagnetic field interacts with a single polarization field quantum that has been isolated in one way or another, such as a superconducting qubit or an exciton in a quantum box. We are then in the quantum regime of strong coupling, known as the "Jaynes-Cummings Hamiltonian", where the energy levels are discrete and each level correlates to a given number of excitation quanta in the system. Cavity-excited polaritons are generally in the first regime.

In polaritons, semiconductor matter receives photons that excite it. It then emits photons to get out of its excited state, all of this in a very fast iterative cycle, the photons circulating in a closed circuit in the cavity. In practice, electromagnetic and polarization fields co-propagate in the medium in an identical way, notably in polarization and frequency, and with a fixed phase relation (without phase shift or with a 180° phase shift, i.e., π). Polaritons are particularly interesting for generating strong nonlinearities which are searched in photonics[287].

Thanks to the degenerate states in which polaritons can be prepared and to the fact that they interact with each other, polaritons constitute an out-of-equilibrium quantum fluid called "light quantum fluid", or “quantum fluids of light”[288], often abusively referred to as "liquid light".

Polaritons can thus generate surface waves and propagation phenomena typical of quantum fluids such as superfluids.

Polaritons also interact with each other, which is not the case for photons in vacuum[289] .

[287] See also this very dense review paper Quantum Fluids of Light by Iacopo Carusotto and Cristiano Ciuti, 2013 (68 pages).

[288] See Quantum Fluids of Light by Iacopo Carusotto, November 2022 (8 pages).

[289] See the pedagogical presentation Swimming in a sea of light: the adventure of photon hydrodynamics by Iacopo Carusotto, 2010 (28 slides). Presentation realized with the help of, among others, Elisabeth Giacobino and Alberto Bramati from CNRS. See also the very well-illustrated presentation Quantum fluids of light by Jacqueline Bloch, February 2020 (58 slides).

We can experimentally control the spatial distribution of the density, phase and velocity of these fluids of light[290].

There are many variants of polaritons which depend on the nature of the electronic excitation of the matter (Figure 120):

- **Phonon-polaritons** resulting from the coupling between an infrared photon and an optical phonon caused by the mechanical oscillation of two adjacent ions of opposite charge in a crystalline structure. This oscillation produces an oscillating electric dipole moment. This phenomenon was discovered by **Kirill Tolpygo** (1916-1994, Russian) in 1950 and, independently, by **Kun Huang** (1919-2005, Chinese) in 1951. One application of phonon polaritons are thermal emitted and imagers[291].
- **Exciton-polaritons** result from the coupling of a photon with an exciton in a semiconductor cavity. An exciton is a quasiparticle consisting of an electron-hole pair connected by Coulomb forces, generated by excitation photons (Figure 121). The notion of exciton was created by **Yakov Frenkel** (1894-1952, Russian) in 1931. Like all types of polaritons, these have two energy bands: the high and low polariton. It is a general property of the strong coupling regime between electric dipole and electromagnetic field. Here, the level is high when the photon and the semiconductor are excited and in phase, and low when they are in opposite phase.

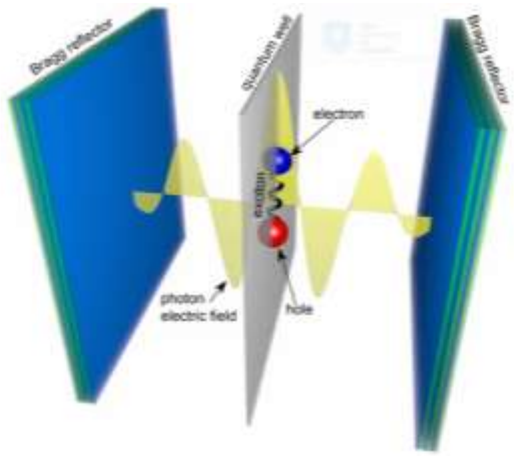

Figure 121: exciton-polariton.
Source: [Polariton: The Krizhanovskii Group](). University of Sheffield.

  Researchers are trying to create transistors using polariton-exciton as well as on single quantum control[292].

- **Surface plasmon polaritons** (SPP) result from coupling surface plasmons and photons. A plasmon is a quantized oscillation of high-density electron gases. A surface plasmon is a coherent electron oscillation occurring at the interface between two different materials, often a metal and a dielectric or between metal and air. A surface plasmon polariton is an oscillation caused by an incident photon (Figure 122).

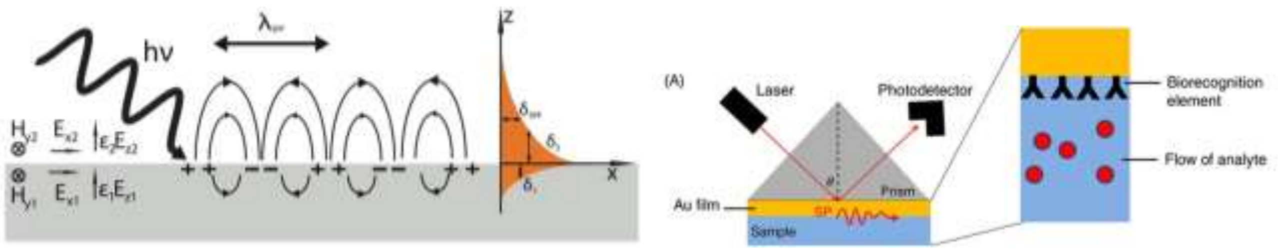

Figure 122: surface-plasmon polariton phenomenon. Source: [Wikipedia]().

[290] Source: description of the ANR project: Quantum Light Fluids - QFL launched in 2016.

[291] See Surface phonon polaritons for infrared optoelectronics by Christopher R. Gubbin et al, January 2022 (23 pages).

[292] The "polariton blockade" mechanism allows in principle to manipulate excitonic cavity polaritons at the single quantum scale. See Towards polariton blockade of confined exciton-polaritons by Aymeric Delteil, 2019 (4 pages).

SPPs are used in optical quantum sensors for temperature and for the detection of the concentration of different components by refractivity and then spectroscopy, like in medtechs (detection of various organic molecules and of interactions between proteins), biological analyses (toxins, drugs, additives) or for the detection of gases[293].

SPRs (Surface Plasmon Resonance Plasma) can be much more powerful than near-infrared spectroscopy sensors such as those from Scio[294] . They measure the polarized light reflected from a laser diode in terms of intensity, angle, wavelength, phase and polarization.

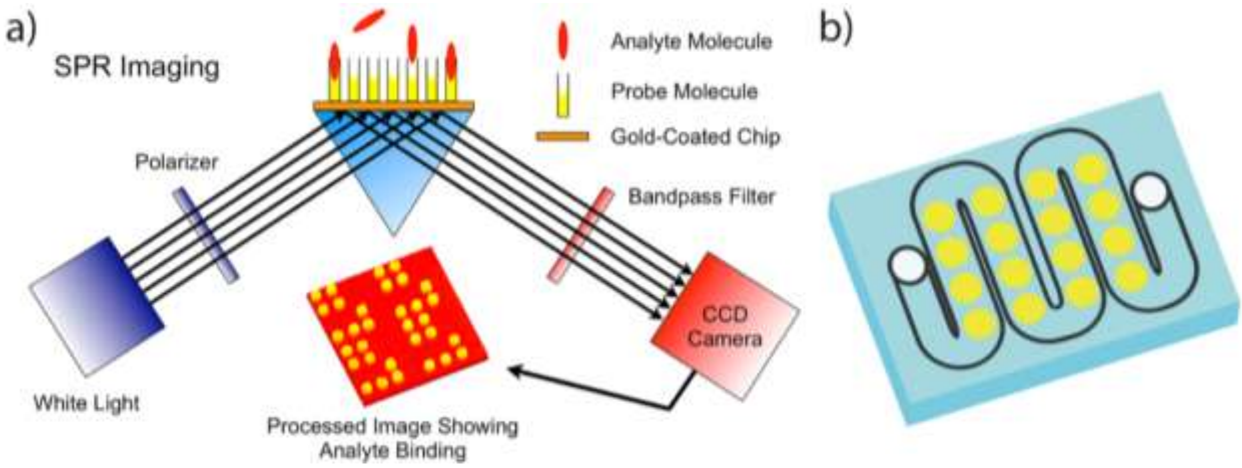

Figure 123: surface plasmon resonance plasma.
Source: [Surface Plasmon Resonance (SPR)] by Lifeasible.

As in many biological analysis systems (Figure 123), it is possible to create 2D matrices (micro-arrays) integrating many detection molecules and to detect a lot of components in the sample to be analyzed[295].

SPRs are commonly marketed by companies such as **Cytiva** (USA), **Carterra** (USA), **Horiba** (Japan)[296], **IBIS Technologies** (Netherlands), **Lifeasible** (USA), **Polaritons Technologies** (Switzerland) and **XanTec** (Germany).

- **Cavity polaritons** are a variant of the polariton excitons where the photon is trapped in a micro-cavity, and the exciton is confined in a quantum well. They are made of III-V semiconductors like indium, arsenic and gallium.

  Photons trapping is often performed using two Bragg mirrors facing each other to create an optical cavity using layers of dielectrics to reflect light very efficiently and of all wavelengths. These mirrors are fabricated from molecular beam epitaxy allowing coherent crystal growth on a gallium arsenide (GaAs) crystal substrate. The result is monocrystalline and can contain more than a hundred layers of different alloys, with thicknesses ranging from 5 nm to 50 nm, controlled to the

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[293] The general principle of this instrument is to use a laser diode to illuminate a gold surface at an angle (via a mechanically controllable angle) and to capture the reflected beam with a detector. The gold surface is coated with a specific molecule ("biorecognition element" in the diagram) that tends to associate itself with a molecule that we want to detect (in the liquid phase "flow of analyte"). The molecules detected can be peptides, polypeptides, proteins, enzymes, vitamins, DNA or RNA sequences, or antibodies (in particular for cancers diagnosis). The association modifies the reflectivity of gold and allows the detection of the target molecule.

[294] See Recent advances in Surface Plasmon Resonance for bio sensing applications and future prospects by Biplob Mondal and Shuwen Zeng, August 2020 (31 pages). The second author is from the Limoges XLIM laboratory in France.

[295] See Surface Enzyme Chemistries for Ultra sensitive Microarray Biosensing with SPR Imaging by Jennifer B. Fasoli et al, 2015 (10 pages) where the associated illustration comes from.

[296] Horiba's European research center is located in Palaiseau next to the C2N of the CNRS, Télécom Paris, Thales and the Institut d'Optique. Horiba is specialized in spectrometers and various other optical instruments like near-IR photoluminescence characterization of InGaAs/GaAs quantum dots. They acquired Yvon Jobin, a French optical instruments manufacturer in 1997.

nearest atomic monolayer[297]. These microcavity polaritons were discovered in 1992 by Claude Weisbuch (France)[298].

- **Intersubband-polaritons** result from the coupling of an infrared or terahertz photon with an intersubband excitation. They can be used to create infrared detectors.
- And then **Bragg-polaritons** (Braggoritons), **plexcitons** (plasmons + excitons), **magnon polaritons** (magnon, spin waves in ferromagnetic materials + photons) and **similaritons** (amplified photons in optical fibers).

In short, all these "*-ons" are the result of the interaction between photons and different forms of matter, noticeably electrons. What does this have to do with quantum computing? Polaritons are used in various optical devices related to photon qubits, including photon transport and single photon detectors.

They could eventually allow the creation of photon qubits that can interact with each other. This is what emerged from an MIT and Harvard publication by Vladan Vuletić and Mikhail Lukin in 2018 which demonstrated the interaction of three photons in an atom placed in a Rydberg state, constituting a "Rydberg polariton"[299]. Another research project in Singapore uses polariton excitons to create photon qubits with the particularity of being able to operate at room temperature, using single-qubit gates and $\sqrt{\text{SWAP}}$ two-qubits gates[300].

Microcavities polaritons can be used to create quantum simulators[301]. They are implanted in III-V semiconductor structures as 2D arrays. One field of application is the simulation of gravitational structures such as a Hawking radiation on the horizon of a black hole. And why not, to simulate the operation of a dilution refrigerator associating helium 3 and 4 at very low temperature.

Polaritons are also the field of topological behaviors of matter and are perhaps an alternative way to the Majorana fermions to create error corrected qubits. These are longer term pathways than the qubit technologies studied in this book, but worthy of interest.

Other applications, already mentioned, target the very diverse field of quantum sensing, including optomechanical systems[302].

In France, polaritons are the specialty of Cristiano Ciuti (UPC MPQ), **Elisabeth Giacobino** (CNRS LKB), **Jacqueline Bloch** (CNRS C2N[303]), **Alberto Bramati** (ENS LKB), **Alberto Amo** (PhLAM CNRS Lille), **Le Si Dang** and **Maxime Richard** (CNRS Institut Néel Grenoble).

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[297] See Cavity polaritons for new photonic devices by Esther Wertz, Jacqueline Bloch, Pascale Senellart et al, 2010 (12 pages).

[298] See Observation of the coupled exciton-photon mode splitting in a semiconductor quantum microcavity by Claude Weisbuch et al, 1992 (4 pages).

[299] See Physicists create new form of light by Jennifer Chu, 2018 referencing Observation of three-photon bound states in a quantum non linear medium by Qi-Yu Liang et al, 2018 (5 pages).

[300] We will define this type of quantum gate in a dedicated section of this book. See Quantum computing with exciton- polariton condensates by Sanjib Ghosh and Timothy C. H. Liew, October 2019 (6 pages). Tim Liew is a researcher at the joint MajuLab laboratory between CNRS and the National University of Singapore.

[301] See Microcavity Polaritons for Quantum simulation by Thomas Boulier, Alberto Bramati, Elisabeth Giacobino, Jacqueline Bloch et al, May 2020 (21 pages) as well as Polaritonic XY-Ising machine by Kirill P. Kalinin, Alberto Amo, Jacqueline Bloch and Natalia G. Berloff, 2020 (12 pages).

[302] See Enhanced Cavity Optomechanics with Quantum-well Exciton Polaritons by Nicola Carlon Zambon, Zakari Denis, Romain De Oliveira, Sylvain Ravets, Cristiano Ciuti, Ivan Favero and Jacqueline Bloch, February-September 2022 (22 pages).

[303] The clean room of the C2N in Palaiseau, France, allows the prototyping of a whole bunch of nanostructures. The semiconductors used to manage polaritons are moreover manufactured with techniques similar to the single photon sources of Pascale Senellart's team, also from the C2N, and the associated startup, Quandela.

Magnons

Quantum matter also includes **magnons**, a category of quasiparticles that take the form of quantized spin waves in magnetic materials, usually crystalline lattices. Magnons were conceptualized by **Felix Bloch** in 1930 and experimentally detected in 1957 by **Bertram Brockhouse** (1918-2003, Canadian). These objects which behave as bosons could be used in quantum information systems.

Current physics experiments are done at the control low-level like with controlling these magnons with microwaves[304] or measured with superconducting qubits[305]. Magnons can also be used at low temperature to create some topological materials[306] and even for some species of SiC-based spin qubit control[307].

Skyrmions

Order is not restricted to the periodic atomic array of a crystal and can also be associated with magnetic order in a solid where spins align parallel to each other in ferromagnets and antiparallel in antiferromagnets. More complex magnetic nanostructures are skyrmions that form mesoscopic magnetic vortex with particle-like properties[308].

Then, how do you distinguish between magnons and skyrmions which are both magnetic quasiparticles? Magnons are quantized dynamic magnetic excitations that travel through magnetic materials while skyrmions are static.

The skyrmion naming comes from **Tony Hilton Royle Skyrme** (1922-1987) who in 1961 formulated a nonlinear field theory of massless pions in which particles can be represented by topological solitons and hopfions[309]. Skyrmions existence in magnetic materials was predicted in 1989 by Bogdanov et al[310]. In 2008, **Sebastian Mühlbauer** discovered skyrmions in MnSi crystals at the Munich reactor using neutrons[311].

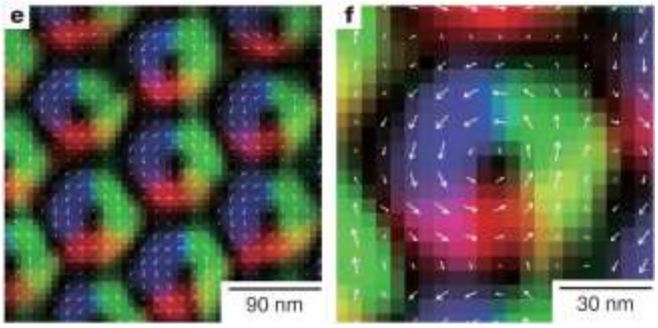

Figure 124:visualizing a skyrmion. Source: Real-space observation of a two-dimensional skyrmion crystal by X. Z. Yu et al, 2010, Nature (5 pages).

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[304] See Floquet Cavity Electromagnonics by Jing Xu et al, Argonne Lab and University of Chicago, October 2020 (9 pages).

[305] See Dissipation-Based Quantum Sensing of Magnons with a Superconducting Qubit by S. P. Wolsk et al, University of Tokyo, September 2020 (6 pages).

[306] See Topological Magnons: A Review by Paul McClarty, 2021 (21 pages).

[307] See Nonlinear magnon control of atomic spin defects in scalable quantum devices by Mauricio Bejarano et al, August 2022 (17 pages).

[308] I found these insights on skyrmions in the presentation Introduction to Contemporary Quantum Matter Physics Lecture 11: Skyrmions I by Marc Janoschek and Johan Chang, 2021 (26 slides) and Part II (24 slides). See also the review paper The 2020 skyrmionics roadmap by C Back et al, 2020 (38 pages).

[309] See Topological transformation and free-space transport of photonic hopfions by Yijie Shen et al, Advanced Photonics, January 2023 (6 pages).

[310] See Thermodynamically stable "vortices" in magnetically ordered crystals. The mixed state of magnets by A. N. Bogdanov and D. A. Yablonskii, 1989 (3 pages).

[311] See Skyrmion Lattice in a Chiral Magnet by S. Mühlbauer et al, Science, 2009 (44 pages) which also mentions hedgehogs or instantons, composed of two merons. An endless story. These skyrmions are observed at a critical temperature of 29.5K. And Instantons: thick-wall approximation by V. F. Mukhanov and A.S. Sorin, June 2022 (12 pages).

Then, researchers in Japan and South-Korea implement real-space imaging of a two-dimensional hexagonally arranged skyrmion lattices spaced by 90 nm in a thin film of $Fe_{0.5}Co_{0.5}Si$ and exposed to a magnetic field of 50–70mT, using Lorentz transmission electron microscopy[312].

This helicoidal structure can also be 3D and create superposition of various magnetic skyrmion states (Figure 124). This could lead to the creation of new ultra-high-density memories[313] particularly with the room-temperature Néel skyrmions that can be made with thin-film systems[314], to in-memory processing architectures[315], to create QRNGs[316], in low-power spintronic applications[317] [318] and in a new breed of qubits with skyrmions in magnetic nano disks bounded by electrical contacts, where static electric and magnetic fields control the skyrmions quantized energy levels corresponding to their helicity. You may probably then need to find a way to entangle them[319]!

Topological matter

The very concept of topological quantum states leading to topological matter was discovered with a specific insulating phenomenon that can be explained by the **quantum Hall effect**, with electrons moving through a strong magnetic field and accumulating in some parts of the material depending on its shape. This electron conductivity is quantized, as discovered in 1980 by **Klaus von Klitzing** (Germany) who was awarded the Nobel prize in physics in 1985. This "integer" quantum Hall effect was later completed by the discovery of the fractional quantum Hall effect by Tsui et al. in 1982 in two-dimensional electron systems in semiconductor devices, followed by the theoretical discovery of the entangled gapped quantum spin-liquid state of integer-spin "quantum spin chains" by **Frederick Duncan** and **Michael Haldane** in 1981, who was awarded the Nobel prize in physics in 2016 along with **David J. Thouless** and **J. Michael Kosterlitz**[320].

In 2005, **Eugene Mele** and **Charles Kane** predicted that topological insulation could happen in graphene sheet submitted to strong spin-orbit coupling creating the quantum Hall effect without any applied magnetic field[321]. This phenomenon is named the "quantum spin Hall effect" and relates to the Kane-Mele invariant[322]. It was demonstrated to occur in wafers of mercury telluride. It was

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312 See Real-space observation of a two-dimensional skyrmion crystal by X. Z. Yu et al, 2010, Nature (5 pages).

313 See for example Skyrmion-Electronics: Writing, Deleting, Reading and Processing Magnetic Skyrmions Toward Spintronic Applications by Xichao Zhang et al, 2019 (80 pages).

314 See Mobile Néel skyrmions at room temperature: status and future by Wanjun Jiang et al, 2016 (15 pages) and Observation of Robust Néel Skyrmions in Metallic PtMnGa by Abhay K. Srivastava et al, Advanced Materials, December 2019 (5 pages).

315 See Skyrmion Logic-In-Memory Architecture for Maximum/Minimum Search by Luca Gnoli et al, January 2021 (15 pages) and Robust and programmable logic-in-memory devices exploiting skyrmion confinement and channeling using local energy barriers by Naveen Sisodia et al, May 2022, UGA, CNRS and CEA (11 pages).

316 See Single skyrmion true random number generator using local dynamics and interaction between skyrmions by Kang Wang et al, Nature Communications, 2022 (8 pages).

317 See The skyrmion switch: turning magnetic skyrmion bubbles on and off with an electric field by Marine Schott et al, CNRS Institut Néel, UGA and CEA IRIG, 2016 (31 pages).

318 See Magnetic Skyrmion Transistor Gated with Voltage-Controlled Magnetic Anisotropy by Seungmo Yang, Jong Wan Son, Tae-Seon et al, Advanced Materials, December 2022 (8 pages).

319 See Skyrmion qubits: A new class of quantum logic elements based on nanoscale magnetization by Christina Psaroudaki and Christos Panagopoulos, Caltech and NTU Singapore, PRL, August 2021 (11 pages) and also Universal quantum computation based on nanoscale skyrmion helicity qubits in frustrated magnets by Jing Xia et al, April 2022 (7 pages).

320 See Topological Quantum Matter by F. Duncan M. Haldane, Nobel Lecture, December 2016 (23 pages).

321 See Quantum spin Hall effect in graphene by Charles Kane and Eugene Mele, University of Pennsylvania, 2005 (4 pages).

322 See Topological Insulators and the Kane-Mele Invariant: Obstruction and Localisation Theory by Severin Bunk and Richard J. Szabo, 2019 (81 pages) and Quantum spin Hall effect: a brief introduction (34 slides).

experimented by **Shou-Cheng Zhang** et al from Stanford University in 2007[323]. The same year, the first 3D topological insulator was discovered by **Zahid Hasan** from Princeton[324].

Since then, over 20 topological insulators materials were discovered and there are probably hundreds of them[325] (Figure 125). A French American research team devised in 2020 a machine learning model to detect such topological insulators out of an initial database of 4,009 candidates[326]. Again, spintronics are a potential use case of topological insulators to create power-saving electronics where the on/off of a bit would be an electron spin instead of the on/off path of an electron stream.

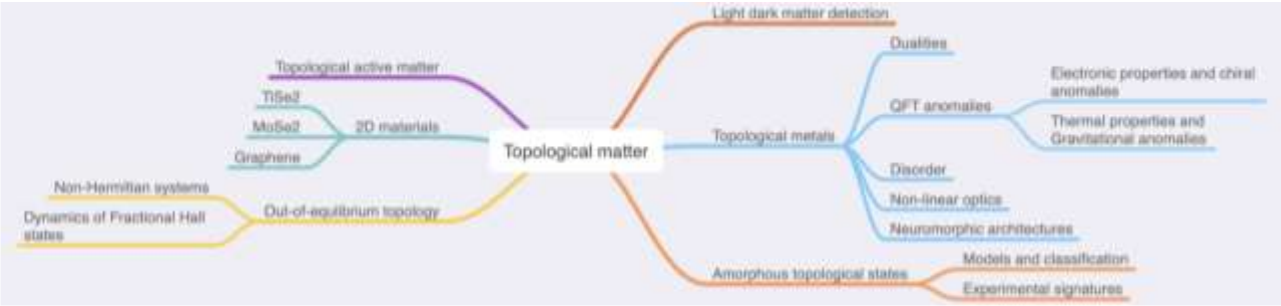

Figure 125: a classification of topological matter. Source: Research Lines - Theory of Topological Matter by Adolfo Grushin, CNRS.

In topology, an invariant can be described by a single winding number which describes the type of structure with its domain walls, vortices and vector order. It related to the **Chern number**. This number changes over quantum phase transitions. These are other various physics concepts to consider, way beyond what I can do at this point in my quantum journey[327].

It is interesting to note that some materials can showcase 3D topological behavior at ambient temperature, like bismuth-selenide (Bi_2Se_3). It is a semiconductor and a thermoelectric material that has a topological insulator ground-state. It could be used in targeted cancer treatments and X-ray to mammography[328]. You can also potentially build **magnetic monopoles** quasiparticles, breaking the convention that magnetism always shows up with dipoles[329].

You are certainly willing to "visualize" the different types of topological materials identified. I found this nice and highly detailed table showing their great diversity in a review paper, in Figure 126.

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323 See Quantum Spin Hall Insulator State in HgTe Quantum Wells by Markus Koenig, Shou-Cheng Zhang et al, October 2007 (16 pages).

324 See A topological Dirac insulator in a quantum spin Hall phase (experimental realization of a 3D Topological Insulator) by D. Hsieh, Zahid Hasan et al, Princeton University, 2009 (12 pages).

325 See Topological phases of amorphous matter by Adolfo G. Grushin, January 2021 (45 pages) which describes the physics of topological phases and Introduction to topological Phases in Condensed Matter by Adolfo G. Grushin (28 pages) which provides some background information on the way to classify topological matter.

326 See Detection of Topological Materials with Machine Learning by Nikolas Claussen et al, ENS Paris, Princeton, June 2020 (15 pages).

327 See Topological Materials : Some Basic Concepts by Ion Garate, 2016 (35 slides), Core Concept: Topological insulators promise computing advances, insights into matter itself by Stephen Ornes, 2016 and Topological phases by Nicholas Read, Physics Today, 2012 (6 pages).

328 See Topological insulator bismuth selenide as a theranostic platform for simultaneous cancer imaging and therapy by Juan Li and al, 2013 (7 pages).

329 See Emergent magnetic monopoles isolated using quantum-annealing computer by Los Alamos National Laboratory, Physorg, July 2021, which refers to Qubit spin ice by Andrew D. King, Science, July 2021 (18 pages) which simulates a new topological material with a D-Wave quantum annealer.

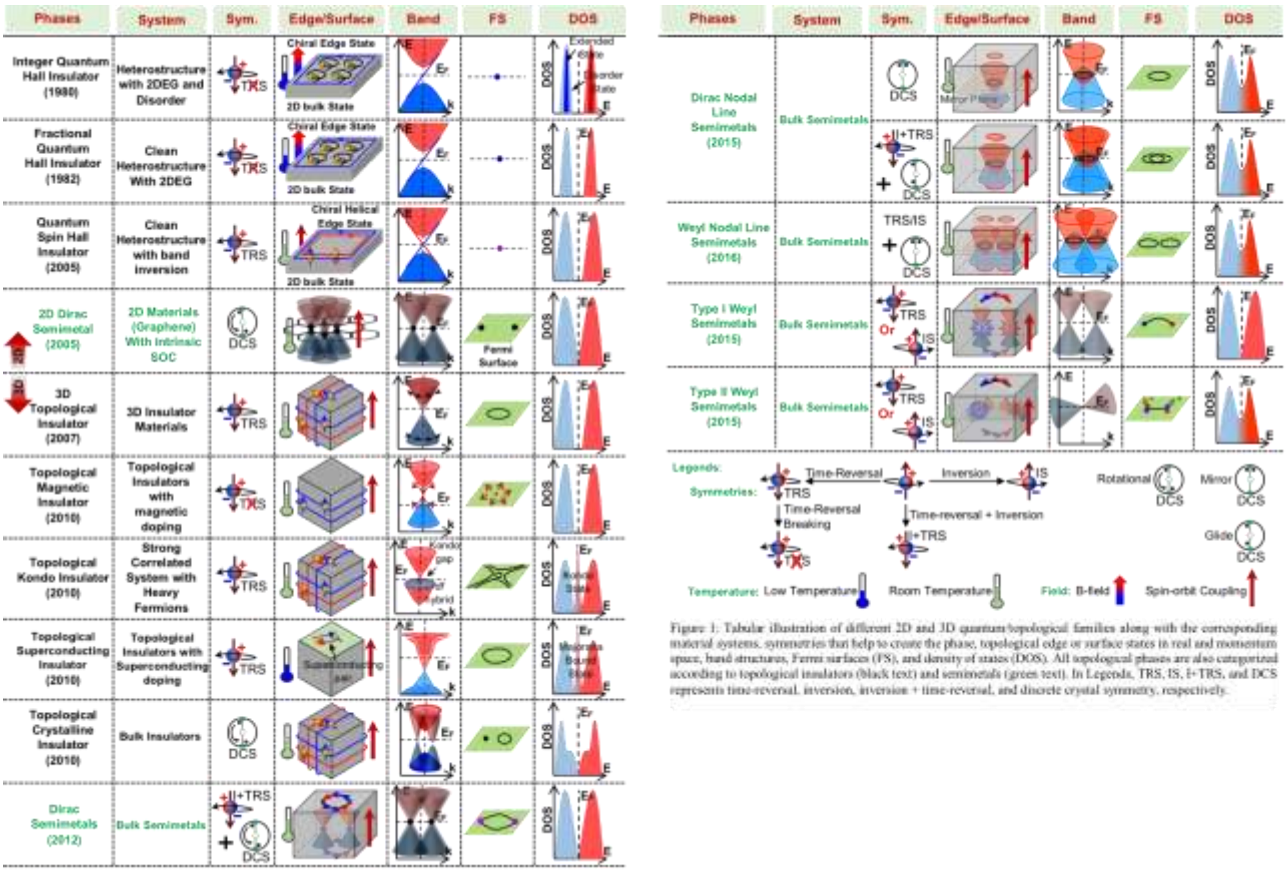

Figure 126: a table with a classification of various topological materials in 2D and 3D and indicating time reversal and operating temperature. Source: [Topological Quantum Matter to Topological Phase Conversion: Fundamentals, Materials, Physical Systems for Phase Conversions, and Device Applications]() by Md Mobarak Hossain Polash et al, February 2021 (83 pages).

Topological matter can have several applications related to light-matter interactions in the Terahertz regime. It can help create waveguides, optical isolators and diodes who are more resistant to their environment perturbations in the recent field of **topological photonics** which is related to polaritons[330].

We even have **topological lasers**[331], which can for example consolidate multiple sources in a coherent way, leading to even more powerful lasers, using a topological insulator vertical-cavity surface-emitting array (VCSEL)[332].

Then of course, one key application of topological matter is topological qubits, often associated with Majorana fermions sought after by Microsoft. But topological qubits are way more diverse with many competing definitions and architectures. For example, you also can count with Fibonacci anyons[333].

330 See Roadmap on Topological Photonics by Hannah Price et al, Journal of Physics, 2022 (63 pages), the well illustrated presentation Introduction to Topological Photonics by Mikael C. Rechtsman, Penn State, AMOLF Nanophotonics Summer School, June 2019 (42 slides), Topological photonic crystals: a review by Hongfei Wang et al, 2020 (23 pages) and Topological photonic crystals: physics, designs and applications by Guo-Jing Tang et al, January 2022 (60 pages).

331 See Topological lasing, PhLAM Laboratory, Lille France.

332 See Topological-cavity surface-emitting laser by Lechen Yang et al, Nature Photonics, 2021 (6 pages) and Topological insulator vertical-cavity laser array by Alex Dikopoltsev et al, Science, 2021 (5 pages).

333 See Fibonacci Anyons Versus Majorana Fermions: A Monte Carlo Approach to the Compilation of Braid Circuits in $SU(2)_k$ Anyon Models by Emil Génetay Johansen and Tapio Simula, 2021 (23 pages).

Time crystals

Time crystals are beasts we hear a lot about since mid-2021, when Google announced it had created such artefact in its Sycamore processor[334]. It shed some light on this weird phenomenon that was devised in a 2012 paper by Frank Wilczek from the MIT (and 2004 Nobel prize in physics awardee) and by another paper by him and Alfred Shapere from the University of Kentucky[335].

This thing is somewhat linked to the history of the search for a perpetuum movement, an isolated object supposed to keep in motion indefinitely. It was dismissed by the French Academy of Science in 1775 due to the limits of friction and, later, to the second law of thermodynamics[336].

In classical crystals, atoms are periodically arranged in space structured according to one of the 230 structured already described. In time crystals, these atoms are periodically arranged in both space and time. It simply means that their structure is in a permanent oscillating mode with a given period, for so-called discrete time crystals[337] [338]. But the scientific description of the phenomenon is the less explicit "spontaneous time symmetry breaking". Then, you quickly lose grounds with common wisdom[339].

Time crystals do not lose energy to the environment. They are the stage of motion without energy. It is a type or phase of non-equilibrium matter. But they are still initially driven, sometimes even out of their equilibrium level. Some real time crystals were first observed in lab experiments, starting in 2017 with some constantly rotating ring of charged ions spin (which by the way, shows some signal damping, in Figure 127)[340]. It can also happen with some continuous change of spin for some particles, when the change periods is up to 100 times longer than the system drive period. It was tested in 2021 by a QuTech team in The Netherlands using controllable ^{13}C nuclear spins in diamond structures[341].

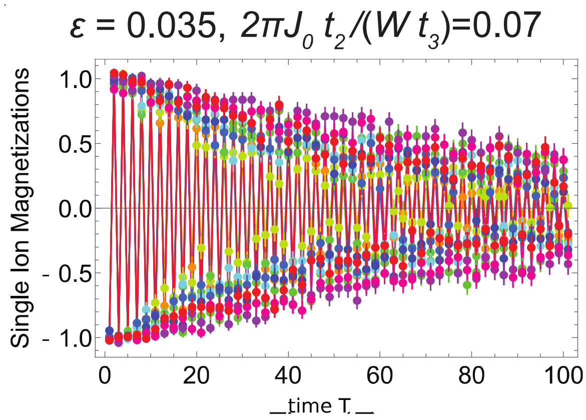

Figure 127: time crystal oscillations over time.
Source: Observation of a Discrete Time Crystal by J. Zhang, Christopher Monroe et al, September 2016 (9 pages).

Things get complicated when you learn that time crystals have also been experimented with superconducting qubits like with the Google 2021 experiments and other subsequent ones with a continuous line of 57 qubits in a 65 qubits IBM QPU[342]. How could a series of connected superconducting qubits become a "crystal" per se?

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[334] See Eternal Change for No Energy: A Time Crystal Finally Made Real by Natalie Wolchover, July 2021 referring to Observation of Time-Crystalline Eigenstate Order on a Quantum Processor by Xiao Mi et al, Google, July 2021 (24 pages) and Realizing topologically ordered states on a quantum processor by K. J. Satzinger et al, Google AI, April 2021 (6 pages).

[335] See Quantum Time Crystals by Frank Wilczek, MIT, 2012 (6 pages) and Classical Time Crystals by Alfred Shapere and Frank Wilczek, PRL, 2012 (5 pages).

[336] See A Decade of Time Crystals: Quo Vadis? by Peter Hannaford and Krzysztof Sacha, April 2022 (8 pages) and A Brief History of Time Crystals by Vedika Khemani et al, Harvard, October 2019 (79 pages).

[337] There are also continuous time crystals that were observed first in 2022 in Germany. See Observation of a continuous time crystal by Phatthamon Kongkhambut et al, February-August 2022 (13 pages).

[338] See Formation of Tesseract Time Crystals on a Quantum Computer by Christopher Sims, Purdue University, May 2023 (12 pages).

[339] There's even an acronym for this, TTSB which means time translation symmetry breaking.

[340] See Observation of a Discrete Time Crystal by J. Zhang, Christopher Monroe et al, September 2016 (9 pages).

[341] See Many-body–localized discrete time crystal with a programmable spin-based quantum simulator by J. Randall et al, Qutech, Science, November 2021 (7 pages).

[342] See Realization of a discrete time crystal on 57 qubits of a quantum computer by Philipp Frey and Stephan Rachel, January 2022 (12 pages).

They may behave as a continuously oscillating system but are not a single crystal since they are a complex assembly of Josephson junctions, capacitances, resonators and microwave drives mixing various elements (aluminum, aluminum-oxide, niobium, titanium…).

So why all this fuss around time crystals and how could they become useful? Some think they may be useful to create some form of quantum memory. Others mentions way to improve NMR spectroscopy, AMO-based quantum simulation (AMO standing for Atomic, Molecular, and Optical physics), to stabilize Schrodinger-cat states against local perturbations, enhance metrological bandwidth while maintaining sensitivity like with the measurement of magnetic fields, as a frequency standard or for beyond-SQL (standard quantum limit) quantum sensing[343].

Quantum batteries

Quantum matter research is leading some labs to investigate the possibility of creating innovative batteries for energy storage relying on some quantum phenomenon including entanglement[344].

Work in this field started around 2012 with some fundamental research by Robert Alicki and Mark Fannes from Poland and Belgium on how much work could be stored and extracted from quantum batteries[345]. Quantum batteries could store energy in high energy states of quantum objects and extracted efficiently. Some of these batteries rely on various quantum principles, some of them being not far from classical quantum photonics. This is a different field than classical batteries whose design could be improved with using quantum computers, as covered page 1061 in this book.

All the papers I've found in that field are very theoretical and quite far from practical batteries. The main benefit of these quantum batteries seems to be fast charging, with the caveat of fast discharging, which is quite inconvenient[346]. I have not found yet any quantum battery that would improve energy density in a real documented manner with full-stack product packaging, one of the main showstoppers for various use cases like for long distance electric vehicles or aerial vehicles. So, you're far from buying your next Tesla equipped with a 1,000-mile range quantum battery[347], particularly given most quantum batteries experiments run in ultra-cold environments to avoid quantum decoherence[348]. So, what do we have in-store here? Mainly scientific work with very low TRLs[349] [350].

Scientists from Australia and Italy are working on an **organic battery** with fast charging using a process called superextensive scaling of absorption, meaning that the larger the system is, the faster it absorbs energy[351].

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[343] See Colloquium: Quantum and Classical Discrete Time Crystals by Michael P. Zaletel, Mikhail Lukin, Christopher Monroe, Chetan Nayak, Frank Wilczek, Norman Y. Yao, May 2023 (29 pages).

[344] See Colloquium: Quantum Batteries by Francesco Campaioli, Marco Pollini et al, August 2023 (36 pages).

[345] See Extractable work from ensembles of quantum batteries. Entanglement helps by Robert Alicki and Mark Fannes, Physical Review E, November 2012 (4 pages).

[346] See Sizing Up the Potential of Quantum Batteries by Sourav Bhattacharjee, Indian Institute of Technology, April 2022.

[347] Despite what you can read in Quantum technology could make charging electric cars as fast as pumping gas by Institute for Basic Science, March 2022 that is linked to Quantum charging advantage cannot be extensive without global operations by J.-Y. Gyhm et al, PRL, April 2022 (13 pages).

[348] See Quantum batteries - The future of energy storage? by James Q. Quach, Giulio Cerullo, and Tersilla Virgili, October 2023 (9 pages).

[349] See The battery capacity of energy-storing quantum systems by Xue Yang et al, February-July 2023 (12 pages) which is a very fundamental research work with no practical data.

[350] See Performance of quantum batteries with correlated and uncorrelated chargers by Mohammad B. Arjmandi et al, Iran, July-November 2022 (9 pages).

[351] See Superabsorption in an organic microcavity: Toward a quantum battery by James Q. Quach et al, Heriot-Watt University, 2022 (9 pages).

It is based on a thin active layer of a low-mass molecular semiconductor named LFO (Lumogen F-orange) that is dispersed into a polymer matrix that is sandwiched between two dielectrics made of 8 and 10 pairs of Brag mirrors, creating a microcavity. The battery cell is then controlled by a laser in the 500 nm red-light range, a noncollinear optical parametric amplifier, beam splitters and delay lines and a detector. In a word, we could say it is a "light" battery, absorbing energy as light, and rendering it as light, in a different wavelength. Like in many other papers of this kind, it is quite difficult to infer the practicality of these quantum batteries (Figure 128).

If researchers are not overselling it, the news media are doing it, touting "*batteries with one million miles autonomy*"[352].

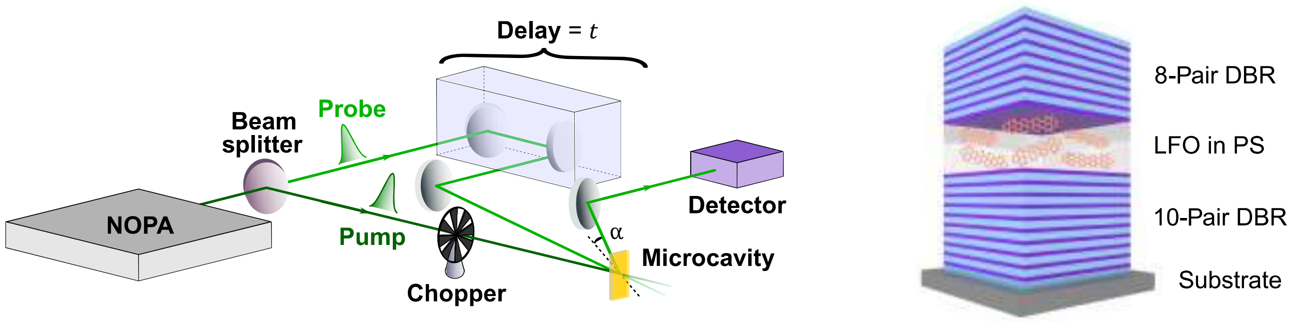

Figure 128: source: Superabsorption in an organic microcavity: Toward a quantum battery by James Q. Quach et al, Heriot-Watt University, 2022 (9 pages).

This comes from another paper, authored by Canadian scientists and an engineer from Tesla which proposes an improved Li-Ion battery that could last 1.5 million miles over its lifespan but, of course, not with a single recharge[353]. And it is not even a quantum battery.

In another approach, other scientists from Australia are looking at ways to store energy in light-induced spin state trapping in spin crossover materials[354]. And a team from Italy and Korea wants to use micromasers to store energy[355]. Researchers in China are algo working quantum batteries fundamental research[356].

Another paper from a Korean American German Singaporean team describes quantum batteries as isolated quantum systems undergoing unitary charging protocols (unitary in the mathematical sense)[357]. With ensembles of such batteries, some collective effects enhance work extraction or boost the charging power thanks to entanglement between the component quantum batteries. The described system is based on an Otto engine which can serve as an engine and as a refrigerator.

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[352] See How quantum batteries could lead to EVs that go a million miles between charges, The Next Web, June 2022.

[353] See A Wide Range of Testing Results on an Excellent Lithium-Ion Cell Chemistry to be used as Benchmarks for New Battery Technologies by Jessie E. Harlow, J.R. Dahn et al, 2019 (15 pages).

[354] See UQ discovery paves the way for faster computers, longer-lasting batteries, June 2022 referring to Toward High-Temperature Light-Induced Spin-State Trapping in Spin-Crossover Materials: The Interplay of Collective and Molecular Effects by M. Nadeem, Jace Cruddas, Gian Ruzzi and Benjamin J. Powell, May 2022 (55 pages). A similar spin-based approach is described in Quantum advantage in charging cavity and spin batteries by repeated interactions by Raffaele Salvia et al, April 2022 (14 pages).

[355] See Micromasers as Quantum Batteries by Vahid Shaghaghi et al, April 2022 (6 pages).

[356] See Remote-charging and anti-aging quantum battery by Wan-Lu Song et al, August 2023 (7 pages).

[357] See Charging Quantum Batteries via Otto machines: The influence of monitoring by Jeongrak Son et al, May 2022 (16 pages). Hard to understand what are the characteristics of this kind of battery and how it performs compared to classical Li-ion batteries!

In some other work from US and Japanese researchers, we are closer to classical battery designs. It is about using lithium-dopped samarium nickelate, a quantum crystalline material with strongly correlated electron systems[358] (Figure 129). Lithium ions are usually the main compound of batteries electrolytes.

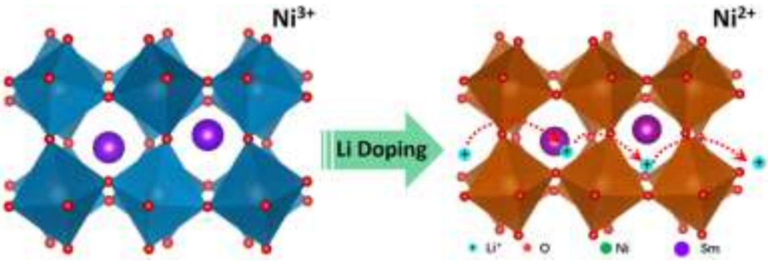

Figure 129: lithium-dopped samarium nickelate quantum battery. Source: Strongly correlated perovskite lithium ion shuttles by Yifei Sun et al, 2018 (6 pages).

The quantum crystal structure improves the conduction of these ions that could also be sodium ions. It could enable better electrolytes but another effect of the structure where additional electron modifies the material conductivity could be used in neuromorphic synapses for storing neural networks connections weights.

Other research work deal with microscopic batteries which don't seem to be useful for energy storage[359]. They can help better understand the thermodynamics of qubits manipulation and provide innovative insights on how to fight decoherence and noise[360].

Higher TRLs can be found with rather classical batteries that would use topological semi-metallic porous carbon materials as potential more efficient anodes for Li-Ion, sodium-ion and potassium-ion batteries. Other topological materials could be useful for supercapacitors. Topological materials could also be useful to create more efficient catalyzers for water electrolysis, with the production of hydrogen in sight coming from renewable originated electricity[361].

Planckian

Planckian (2023, Italy, 2.7M€) is seemingly the world's first startup willing to create a macroscopic solid state quantum battery also named a Dicke quantum battery and is "*powered by entanglement*".

It spun out of the University of Pisa and Scuola Normale Superiore and is run by Michele Dallari (CEO), Marco Polini (Chief Scientist) and Vittorio Giovannetti (Executive Scientific Advisor)[362] [363].

Extreme quantum

Beyond the basics of quantum physics, many other branches of quantum physics deserve to be examined in this book. They can have various impacts on quantum technologies, noticeably on quantum sensing. They are also used in cosmology. Finally, they are unfortunately used by many false sciences and scams that we will discuss in the section dedicated to quantum hoaxes, starting page 1266.

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[358] See Quantum material is promising 'ion conductor' for research, new technologies by Emil Venere, Physorg, 2018. Pointing to Strongly correlated perovskite lithium ion shuttles by Yifei Sun et al, 2018 (6 pages).

[359] Like with IBM Quantum Platforms: A Quantum Battery Perspective by Giulia Gemme et al, April 2022 (13 pages) which is using an IBM superconducting processor to store energy in qubits. It's actually using the Armonk processor which has exactly one qubit. A similar experiment done in China is described in Optimal charging of a superconducting quantum battery by Chang-Kang Hu et al, August 2021 (4 pages).

[360] Like with Coherence-powered work exchanges between a solid-state qubit and light fields by Ilse Maillette De Buy Wenniger, Maria Maffei, Niccolo Somaschi, Alexia Auffèves, Pascale Senellart et al, April 2022 (17 pages).

[361] See Topological quantum materials for energy conversion and storage by Huixia Luo, Peifeng Yu, Guowei Li and Kai Yan, Nature Review Physics, July 2022 (14 pages).

[362] See Quantum Work Capacitances by Salvatore Tirone et al, Scuola Normale Superiore Pisa, NEST and University of Illinois Urbana-Champaign, November 2022 (15 pages).

[363] See High-Power Collective Charging of a Solid-State Quantum Battery by Dario Ferraro, Michele Campisi, Gian Marcello Andolina, Vittorio Pellegrini, and Marco Polini, PRL, 2018 (8 pages).

Quantum field theory

Quantum Field Theory (QFT[364]) is a branch of quantum physics that deals with the physics of elementary particles in the relativistic realm, including their creation or disappearance during various interactions, such as electron and positron pairs. These phenomena are generally reproduced in particle accelerators[365].

QFT also covers the mechanisms of condensed matter such as Bose-Einstein condensates or superfluid helium and more generally, the behavior of quasiparticles, complex collective behaviors such as Cooper's (electron) pairs in superconducting materials.

QFT combines elements of quantum mechanics, special relativity, and classical notions of electromagnetic fields. It is based on a mathematical formalism that is even more difficult to assimilate than the one of non-relativistic quantum physics.

It exploits the notion of Lagrangian and Lagrangian integrals over time describing the evolution of fields and the interactions between the fields of several particles.

QFT is used to explain or modelize the fine structure of the hydrogen atom (corresponding to close spectral lines not explainable by classical quantum energy jumps), the existence of particle spin (which explains these spectral lines), the spontaneous emission of photons by atoms during their return to their fundamental state and the mechanisms of radioactivity.

The foundations of QFT were created by many scientists starting in 1928: **Paul Dirac**, **Wolfgang Pauli**, **Vladimir Fock** (1898-1974, Russian), **Shin'ichirō Tomonaga (**1906-1979, Japanese), **Julian Schwinger (**1918-1994, American), **Richard Feynman** and **Freeman John Dyson (**1923-2020, American[366]). Shin'ichirō Tomonaga, Julian Schwinger and Richard Feynman received the 1965 Nobel prize in Physics for their work on quantum electrodynamics which is part of QFT.

In the early 1950s, they solved the problem of infinite energy values generated by the initial QFT models by using an adjustment technique called **renormalization**.

Physicists are still struggling to integrate the theory of general relativity into the QFT, preventing it from becoming a "theory of the whole" or unified theory explaining all known physical phenomena in the Universe.

QFT is a theoretical framework, among others, that is applied in three main areas:

- In the physics of **high-energy particles** explored in particle accelerators such as the CERN LHC. It has been supplemented on this point by the standard model that we will see below.
- In the **physics of condensed matter** with superconductivity, superfluidity and the quantum Hall effect. This is the framework of **QED** (quantum electrodynamics), launched by Paul Dirac in 1928, which studies in particular the production of positrons and positron/electron interactions (attraction, annihilation, pair creation, Compton effect). The **CQED** (cavity QED) sub-branch studies the relations between matter and photons in optical cavities. It is used by condensed matter physicists working on superconducting qubits.
- In **cosmology** to contribute to modeling the origin and evolution of the Universe as well as certain mechanisms of interaction between black holes and quantum fields.

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[364] Later on, we'll use the QFT acronym with another meaning, Quantum Fourier Transform!

[365] See The History of QFT, a Stanford site, which summarizes the history of QFT.

[366] It also gave rise to the notion of the Dyson sphere, which dimensions the level of technological control of energy sources by extraterrestrial civilizations, with a sphere capturing the totality of a star's energy.

Quantum vacuum fluctuation

One of the consequences of QFT is the notion of quantum vacuum fluctuation, also called vacuum energy. Based on Heisenberg's principle of indeterminacy that quantum fields are in perpetual fluctuation, QFT models zero-point fluctuations or vacuum energy, which is the minimum energy level of quantum systems.

In this framework, Heisenberg's principle can be considered as a generalized predicate. According to these models, total vacuum cannot exist. Elementary fluctuations lead to spontaneous electromagnetic waves creation, given all fields are fluctuating.

One scenario devised by Paul Dirac is the creation of pairs of virtual electron and positron particles, which rapidly annihilate each other, generating photons in the process. But this is not the only solution to his equations. It can come from electromagnetic fields moving at the speed of light.

Under the influence of a surrounding electromagnetic field, this leads to a polarization of the vacuum. The latter even leads to make the vacuum birefringent, its refractive index depending on the polarization of the light that gets through it. The phenomenon is however potentially observable only with some very intense electromagnetic fields.

Theoretical models initially indicated that this vacuum energy would be infinite on the scale of the Universe. They were then corrected using the renormalization method, already mentioned above. These elementary vacuum fluctuations would explain the spontaneous emission of radiation by the electrons in the atoms as well as the spontaneous radioactivity[367].

The concept of vacuum energy originated with **Max Planck** in 1911 when he published an article containing an energy equation for a medium containing a fixed constant, a kind of energy floor for this medium, without being able to interpret it. It was not until 1916 that the chemist **Walther Nernst** (1864-1941, German[368]) interpreted this constant as the energy level of the vacuum in the absence of any radiation. It happens when you cool down a black body to a very low temperature, below a couple millikelvins (mK).

According to the QFT, the Universe is a vast soup containing constantly fluctuating fields, both fermions (leptons and quarks) and bosons (force fields like gluons mediate the strong force that stick together the quarks that are the elementary constituents of protons and protons, and photons, and the cohesion between nucleons is coming from a residual force from strong interactions). This notion of minimum energy level is a modern version of the notion of ether - a not completely empty void - which dominated 19th century physics, notably for James Clerk Maxwell. The electromagnetic bath in which the vacuum is immersed, supplemented by the energy of the vacuum, would give vacuum some viscosity properties. Still, these theories are less complete than classical quantum mechanics. One of the solutions is to assume that fermions have a negative vacuum energy and bosons have a positive vacuum energy, both balancing each other. But this has not been demonstrated experimentally, particularly with non-relativistic energy particles.

Some link could be found between vacuum energy and the dark energy of the Universe as well as gravity[369]. This is very speculative. It could help explain the 73% of the energy contained in the Universe, sometimes called dark energy. Its density is very low, at 10^{-13} Joules/cm^3.

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[367] In addition to these elementary fluctuations, vacuum is constantly traversed, even in the remotest regions of space, by electromagnetic waves, not to mention the effects of gravitation. The Universe is thus filled with radiations including the cosmological background noise which is a remnant of the big bang, having a temperature of 2.7K. It is the same in a vacuum-packed box because all matter emits radiation.

[368] Walther Nernst played a key role in launching the Solvay Congresses from 1911 onwards.

[369] See Casimir cosmology by Ulf Leonhardt, February 2022 (41 pages).

There are different ways to verify the existence of quantum vacuum fluctuations. The best-known is related to the Casimir effect that we will study in the next part. Recently, French and German scientists have also managed to interact with this quantum vacuum fluctuation in a semiconductor[370].

Casimir effect

The physicist **Hendrik Casimir** (1909-2000, Dutch) predicted in 1948 the existence of an attractive force between two parallel electrically conductive and uncharged plates[371]. He obtained his PhD in 1931 at the University of Leiden in the Netherlands. He also visited Niels Bohr in Copenhagen and was a research assistant to Wolfgang Pauli in 1938. The Casimir effect is interpreted as being related to the existence of quantum vacuum energy. The experiment imagined by Casimir uses parallel mirrored metal surfaces that are as perfectly flat as possible. They create a Fabry-Perot cavity similar to the one that is used in lasers.

The Casimir effect is commonly attributed to quantum fluctuations in vacuum. Temporary changes in the energy level at points in the space between the two mirrors would spontaneously generate pairs of very short-lived particles and antiparticles and photons associated with their annihilation. These vacuum fluctuations take place in and out of the volume of the cavity.

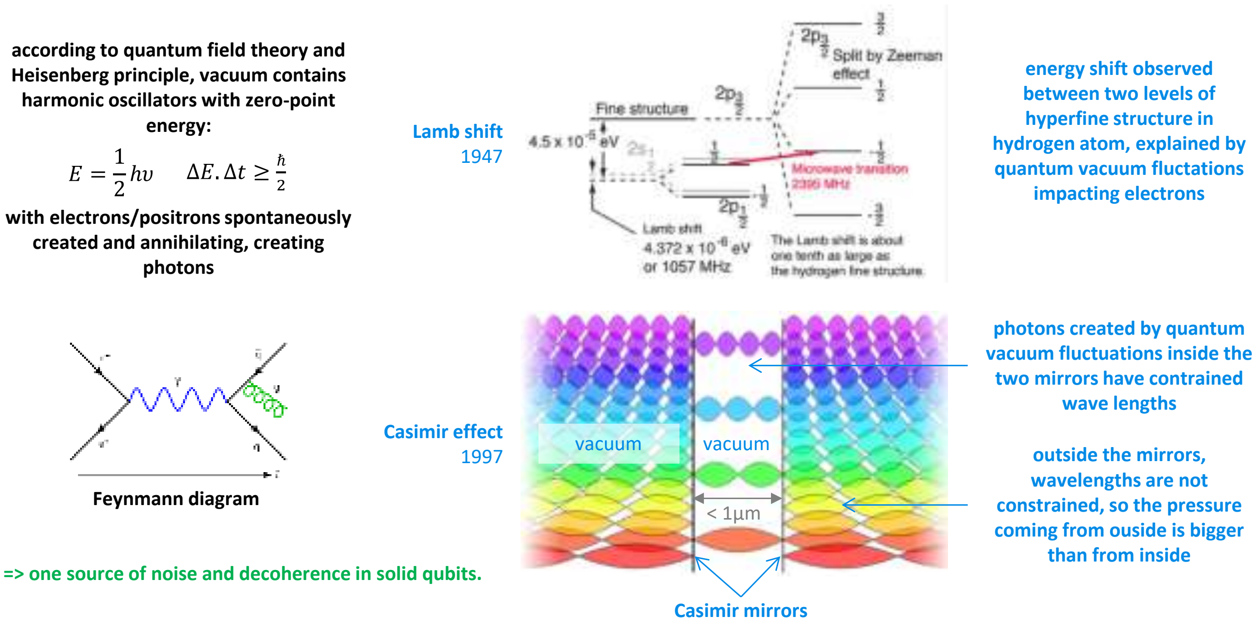

Figure 130: vacuum fluctuations measurement. Sources: The Lamb Shift and The Casimir Effect by Kyle Kingsbury, 2014 (82 slides).

Because of the interference effect induced by the cavity, fluctuations at certain frequencies are reduced. The density of electromagnetic energy in the cavity is thus lower than the density of energy outside the cavity as shown in Figure 130. These are spontaneous quantum fluctuations.

The effect cannot be explained by the simple pressure that is higher on the outside than the pressure between the two plates. In detail, the wavelengths of the photons generated by the vacuum outside the plates can be of any size and especially long while inside the plates, these wavelengths are constrained by the distance between the plates and can only be 1/n of this distance.

[370] See Understanding vacuum fluctuations in space, August 2020 and Electric field correlation measurements on the electromagnetic vacuum state by Ileana-Cristina Benea-Chelmus, Jérôme Faist et al, 2018/2020.

[371] See On the attraction between two perfectly conducting plates by Hendrik Casimir, 1948 (3 pages) and Electromagnetic vacuum fluctuations, Casimir and Van der Waals forces by Cyriaque Genet, Astrid Lambrecht et al, 2004 (18 pages).

The spontaneous electromagnetic spectrum of the vacuum is therefore wider outside the plates than inside, creating a stronger pressure inside than inside, which therefore tends to make the plates move closer together, but very slightly[372].

For two parallel mirrors of surface A and a distance L between the two mirrors, the force of attraction between the two mirrors follows the formula on the right. In practice, L is between 0.2 µm and 5 µm and is usually 1 µm. This is a "macroscopic" scale.

$$F_{Cas} = \frac{\hbar c \pi^2 A}{240 L^4}$$

According to Heisenberg's principle, which is used to explain the effect, energy and time can be linked by the formula on the right. It shows indirectly that during a very short time, a small amount of energy can be created.

$$\Delta E . \Delta t \geq \frac{\hbar}{2}$$

The macroscopic accumulation of these operations is annihilated, making it possible to avoid a violation of the energy conservation principle. So, be uber-skeptic when hearing anyone claiming they can harvest energy from vacuum to produce free electricity.

The experiments are not necessarily 100% conclusive and the data generated do not fit perfectly with the models unlike many classical quantum mechanics experiments. The reason for this is that it is difficult to obtain perfect surfaces.

The first experiments validating the Casimir effect were carried out almost 50 years after the definition of this effect[373]. The first one is that of **Steve Lamoreaux** (American) in 1996, using parallel plates.

His measurement gave a result that was 5% off the predictions. The precision instruments used then detected a force of one billionth of a Newton. The model was improved in other experiments carried out in 1998 and again in 2012 using an electrode geometry combining a plane and a polystyrene sphere with a diameter of 200 µm and covered with gold (Figure 131)[374]. The differences between the models and the measurements decreased to 1%, which remains significant in physics.

The Casimir effect could explain several other commonly observed physical phenomena such as the electron's abnormal magnetic moment and the Lamb shift. The first phenomenon describes a drift of this magnetic moment with respect to Dirac's equations.

The second comes from **Willis Eugene Lamb** (1913-2008, American), Nobel prize in Physics in 1955, who had done his thesis under the supervision of Robert Oppenheimer. Lamb shift is an energy gap observed between two levels of fine structure of the hydrogen atom, two very close energy levels.

The effect is explained by the perturbations coming from vacuum fluctuations and affecting the electron in these two neighboring energy levels, creating the spontaneous generation of photons that are rapidly absorbed by the electron.

The effect was discovered in 1947 by Willis Eugene Lamb and interpreted the same year by **Hans Bethe** (1906-2005, German) for the hydrogen spectrum using the idea of mass renormalization. It was used in the development of post-war quantum electrodynamics.

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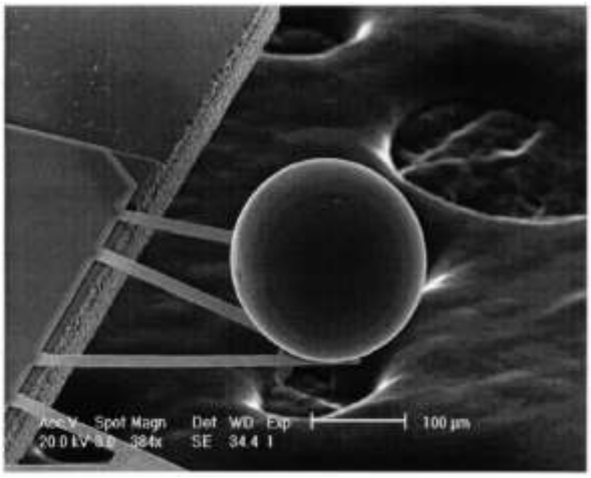
[372] See a good panorama of the Casimir effect with The Casimir effect and the physical vacuum by G. Takács, 2014 (111 slides). See also The Casimir Effect by Kyle Kingsbury, 2014 (82 slides) which describes well the experimental devices for the evaluation of the Casimir effect and evokes some cases of use in MEMS. And then Zero-Point Energy and Casimir Effect by Gerold Gründler, 2013 (47 pages), which casts the history of the Casimir effect, going back to Planck's work in 1911.

[373] The experimental difficulty consists in cancelling out all the other forces between the two plates and they are all much larger than the Casimir effect, particularly electrostatic and van der Waals forces.

[374] See Physicists solve Casimir conundrum by Hamish Johnston, 2012 which refers to Casimir Force and In Situ Surface Potential Measurements on Nanomembranes by Steve Lamoreaux et al, 2012 (6 pages).



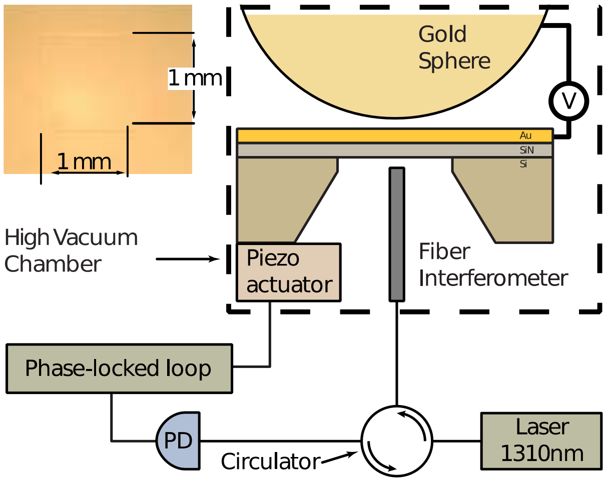


Figure 131: vacuum source measurement with a dynamic Casimir effect.
Sources: The Casimir Effect by Kyle Kingsbury, 2014 (82 slides) and Casimir Force and In Situ Surface Potential Measurements on Nanomembranes by Steve Lamoreaux et al, 2012 (6 pages).

The polarization of vacuum explains part of this shift at 27 MHz for a total of 1,057 MHz[375]. The calculation uses the **fine structure constant** α (about 1/137) which describes the contribution of vacuum energy to the electron's anomalous magnetic moment. The α constant is also used to quantify the strength of the electromagnetic interaction between elementary charged particles[376].

There is also a **Dynamic Casimir Effect** (DCE), discovered by **Gerald Moore** in 1969. It generates pairs of particles by the movement of the mirrors used in the Casimir experiment[377].

As with the Casimir Effect, the energy observed is infinitesimal. For the energy to be significant, the mirrors would have to move at relativistic velocities, which is not very practical. And there is no problem with energy conservation, the necessary energy being provided by the mirror movement. The vacuum simply serves as a nonlinear medium!

The interpretation of the Casimir effect is still debated. Some physicists explain it by other mechanisms than vacuum energy. They rely on the **van der Waals** (1837-1923, another Dutch) forces, where atoms attract or repel each other depending on their distances[378]. However, this infinitesimal force works at a microscopic scale, where the Casimir effect operates at a macroscopic scale.

French physicists are quite active in the field, and, in particular **Astrid Lambrecht,** formerly director of the INP of the CNRS, the Institute of Physics which oversees the physics laboratories of the CNRS[379].

[375] This phenomenon of vacuum polarization in the Lamb effect is described in The Vacuum Polarisation Contribution to the Lamb Shift Using Non-Relativistic Quantum Electrodynamics by Jonas Frafjord, 2016 (61 pages).

[376] See Universal rotation gauge via quantum anomalous Hall effect by Alexey Shuvaev et al, November 2022 (6 pages) which measured with record precision the alpha fine structure constant.

[377] See Electro-mechanical Casimir effect by Mikel Sanz, Enrique Solano et al, 2018 (10 pages).

[378] See The origin of Casimir effect: Vacuum energy or van der Waals force? by Hrvoje Nikolic, 2018 (41 slides) and the even more skeptic The Casimir-Effect: No Manifestation of Zero-Point Energy by Gerold Gründler, 2013 (15 pages) and All wrong with the Casimir effect by Astrid Karnassnigg, 2014 (3 pages). Then, The Casimir effect: a force from nothing by Astrid Lambrecht, 2007 (5 pages).

[379] See The Casimir effect theories and experiments by Romain Guérout, Astrid Lambrecht and Serge Reynaud, LKB, 2010 (28 slides) and Casimir effect and short-range gravity tests, LKB, 2013 (15 slides). Astrid Lambrecht chaired the Casimir RNP group, which brought together researchers from around the world working on the Casimir effect. The group was active between 2009 and 2014.

The Casimir effect could be of interest in quantum metrology to create sensors and in particular NEMS/MEMS. Others are investigating the teleportation of small chunks of energy between qubits but not to the point of implementing real energy harvesting[380].

These theories on quantum vacuum fluctuation and the Casimir effect are also fraudulently exploited by the creators of so-called machines capable of capturing vacuum energy, which collect nothing at all in practice. The fluctuation-dissipation theorem ensures that quantum vacuum fluctuations does not violate the second principle of thermodynamics. No energy can be recovered thanks to these fluctuations! Forget it. It was the conclusion of a report from the DIA in 2010[381].

For example, you have a certain **David Lewis Anderson**, who started the **Anderson Institute** in 1990, who claims to be able to use the Casimir effect to travel back in time and create a "free" electricity generator[382] (Figure 132). In other cases, the Casimir effect is exploited in a scientific but borderline way to imagine science fiction scenarios like ways to cross wormholes[383].

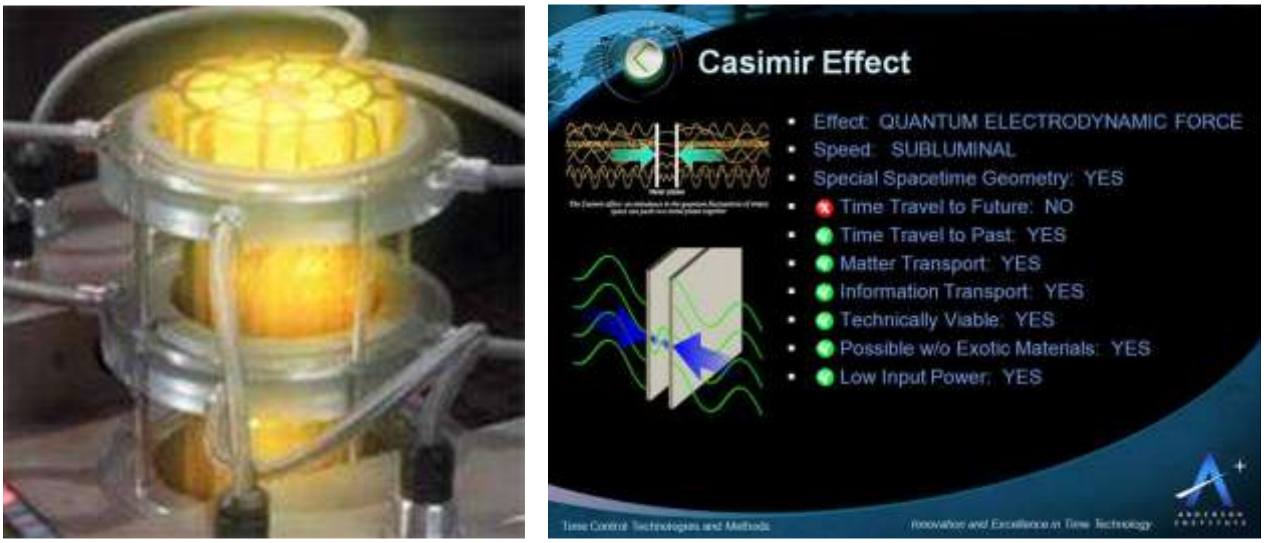


Figure 132: Anderson Institute claims about using the Casimir effect. Forget it!

The **NASA** even explored the idea to use sails and vacuum fluctuation to propel a space vessel between 1996 and 2002, to no avail. It was one of the ideas explored as part of the fancy Breakthrough Propulsion Physics Program, which was awarded a tiny budget of $1.2M and later cancelled.

Unifying theories

The quest for a **unified theory** has occupied many physicists for nearly a century. Its goal would be to consolidate all the physics theories and in particular, quantum physics, relativity and gravity into a single formalism. In addition to the QFT, a very large number of explanatory and unifying theories of physics have been developed.

No such theory is considered today as being complete. Figure 133 shows a rough map showing how these different theories are related.

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[380] See The Quest to Use Quantum Mechanics to Pull Energy Out of Nothing by Charlie Wood, Wired, May 2023.

[381] See Concepts for extracting energy from the quantum vacuum, Defense Intelligence Agency, 2010 (58 pages).

[382] Its website seems to be inactive since 2012. See this radio interview from 2019 with the guy who defies the laws of bullshit in his talk. It shows how an interviewer lacking some scientific background can be fooled by a good talker. In See Is Time Travel Real? 2019 and the Anderson Institute website.

[383] See One Theory Beyond the Standard Model Could Allow Wormholes that You Could Actually Fly Through - Universe Today by Matt Williams, August 2020, mentioning Humanly traversable wormholes by Juan Maldacena and Alexey Milekhin, August 2020.

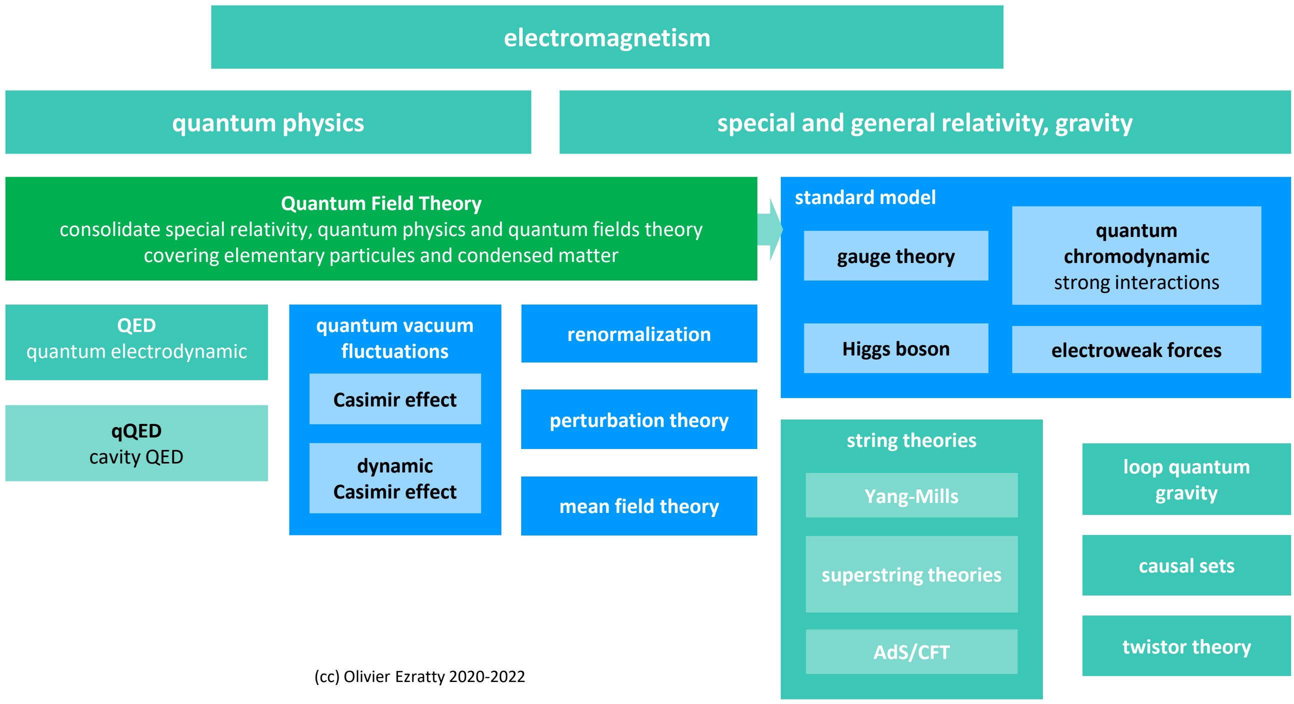

Figure 133: vague classification of quantum physics theories and unification theories. (cc) Olivier Ezratty, 2020-2022.

Quantum chromodynamics provides a description of the strong interactions binding quarks together via gluons to form particles called hadrons, namely, protons and neutrons. Murray Gell-Mann (1929-2019, American, Nobel prize in Physics in 1969) and Georges Zweig (1935, Russian then American, former PhD student of Richard Feynman) each proposed the existence of quarks in 1963. Quantum chromodynamics is an extension of the quantum field theory developed in 1972 by Murray Gell-Mann and Harald Fritzsch[384].

Standard model describes the architecture of known elementary particles and their interactions. It models the fundamental weak and strong electromagnetic forces. It only lacks gravity to be complete. This model predicted the existence of quarks, these massive particles forming neutrons and protons, in addition to other elementary particles such as the famous Higgs boson whose existence was proven at CERN's LHC in 2012. The expression "standard model" was created in 1975. It relies on a gauge theory because of its mathematical symmetries.

It is not the first of its kind because Maxwell's electromagnetism is also a gauge theory, between magnetic and electric fields. The standard model particles do not cover the famous dark matter whose nature is not yet known. It could be made of particles such as WIMPs (weakly interacting massive particles) or axions (which would have a mass equivalent to 10^{-11} of the electron). Various projects have been launched to detect axions using a way to convert them into photons of various energies (X-rays, microwaves, …).

String theory combines general relativity and quantum physics to propose a quantum explanation of gravity, using a new massless particle, the graviton. According to this theory, elementary particles are tiny strings, open or closed, with vibration types defining the nature of the particle. Their size is of the order of magnitude of 10^{-35} m, the Planck length. According to this theory, the Universe would be a set of vibrating strings.

[384] See the review paper 50 Years of Quantum Chromodynamics by Franz Gross et al, 2022 (729 pages).

The graviton would join the three other forces of nature intermediated by particles without mass: electromagnetic waves mediated by photons, strong interactions mediated by gluons that link quarks together in protons and neutrons and weak interactions mediated by W and Z bosons that govern atomic nuclei and in particular radioactivity[385] [386]. String theory essentially covers bosons of all kinds.

Superstring theory is an extension of string theory that adds fermions to the code theory model that focused on bosons. It tries to consolidate the description of all forces in a single unified theory. It quantifies gravity and ties it to other forces. It is based on the notion of supersymmetry which extends the standard model by making each type of boson correspond to a type of fermion. The theory took shape in 1943 with Werner Heisenberg in the form of the S-matrix theory, and then was reborn in 1984. It uses 10 dimensions to describe physics, far beyond the four classical dimensions (three for position and one for time). It also uses the notion of "branes" which describes point particles in these multidimensional spaces. However, this theory is not unique since there are five variants, which some people try to unify in the **M-theory**, which is based on 11 dimensions. A never-ending story!

A brief history of quantum gravity:

1952 Flat space quantization (Rosenfeld, Pauli, Fierz, Gupta,. . .)
1959 Canonical structure of general relativity (Dirac, Bergmann, Arnowit, Deser, Misner)
1964 Penrose introduces the idea of spin networks
1967 Wheeler-DeWitt equation
1974 Hawking radiation and black hole entropy
1984 String theory
1986 New variables for general relativity (Ashtekar, Sen)
1988 Loop representation and solutions to the Wheeler-DeWitt equation (Jacobson, Smolin)
1989 Extra dimensions from string theory
1995 Hilbert space of loop quantum gravity, geometric operators
2000' Spin foam models, group field theory, loop quantum cosmology,. . .

Figure 134: history of quantum gravity. Source: The philosophy behind loop quantum gravity by Marc Geiller, 2001 (65 slides).

Loop quantum gravity theory is another tentative to explain gravity with a quantum model. It discretizes the effects of gravity by presenting space as a meshed structure with quantized areas and volumes of space, and gravitational field quanta connected to each other by links characterized by a spin (that has nothing to do with usual particles spin)[387]. For this theory created in the 1980s, the Universe would be a gigantic spin foam. Its main promoters are **Carlo Rovelli** (Center for Theoretical Physics in Marseille) and **Lee Smolin** (Perimeter Institute for Theoretical Physics in Waterloo[388]). The seeds of the theory date back to 1952, with many intermediate stages as described in Figure 134. It is, above all, a mathematical and topological model.

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[385] A proton has two up quarks and one down quark. A neutron has two down quarks and one up quark. An up quark can disintegrate in a down quark, a positron and a neutrino via a W boson and a down quark can disintegrate in an up quark, one electron, one antineutrino and a W boson. A quark has a size close to that of an electron, about 10^{-16} cm. Radioactivity emits alpha rays via strong forces, particles comprising two protons and two neutrons (helium 4 atom without electron), beta rays generated by weak forces which are electrons or positrons and finally gamma rays which are photons of very high energy level.

[386] See Graviton detection and the quantization of gravity by Daniel Carney et al, CERN, August 2023 (17 pages) which deals with a proposal to detect gravitons.

[387] It is reminiscent of the recent theory of the whole built by Stephen Wolfram and published in 2020.

[388] See Lee Smolin Public Lecture Special: Einstein's Unfinished Revolution, 2019 (1h13mn) where he describes the shortcomings of quantum mechanics.

It does not seem to formulate an experimental validation method even though it is used to model that the big bang was coming after a big bounce in a cyclical phenomenon with contractions and expansions. It may be possible to detect some fossil signatures of these phenomenon.

These are only a few of the many theories being devised. Some amateurs also try to create their own theory of the whole, without usually obtaining any feedback from the scientific community[389].

Quantum physics 101 key takeaways

- Quantum physics is based on a set of postulates and a strong linear algebra mathematical formalism. Surprisingly, there are many variations of these postulates. There is not a single bible or reference for these, illustrating the diversity of pedagogies and interpretations in quantum physics. One big underlying question is "what is reality". But although deemed incomplete, the theory has been validated by an incredible number of experiments and with extreme precision.
- Quantum physics describes the behavior of matter and light at nanoscopic levels, but it can in some conditions extend to larger objects like molecules or even artificial atoms like superconducting current, Bose Einstein condensates and the likes. It deals not only with atoms, electrons and photons which are used in quantum information technologies but also with all elementary particles from the standard model (quarks, ...). We however don't use this level of granularity in quantum technologies.
- Quantumness comes from the quantification of many properties of light and matter that can take only discrete values, from the wave-particle duality of massive (atoms, electrons) and non-massive (photons) particles, and from its consequences like superposition and entanglement. Atoms, electrons, nucleons and photons have several quantum numbers describing their properties. However, quantum objects can have continuous variables. By the way, a cat cannot be both alive and dead since it is not a nanoscopic quantum object. Forget the cat and instead, learn Schrodinger's equation!
- The Heisenberg indetermination principle states that it is impossible to measure with an infinite precision quantum objects properties that are complementary like speed and position. You can use this principle to improve measurement precision in one dimension at the expense of the other. It is used in photons squeezing, itself applied in the LIGO giant gravitational waves interferometer and in other quantum photonics fields and sensing.
- Quantum matter and fluids are showing up with composite elements associating light and matter, or with superfluidity and superconductivity where boson quantum objects can behave like a single quantum object. You find there a wealth of strange phenomenon such as skyrmions, magnons, topological insulators and quantum batteries. They could lead to a new chapter in the second quantum revolution.
- Quantum physics also explains weird effects like vacuum quantum fluctuation, although it doesn't violate the second principle of thermodynamics, nor can it lead to the creation of some free energy sources.
- Most of quantum physics phenomena as described in this section have or will have some use cases in quantum information science and technologies.

[389] See, for example, the Unified Theory Research Team website, which announced the publication in September 2020 of a theory model of the whole called MME for Model of Material and Energy. The site claims that its model, which is presented as an algorithmic approach, can explain everything, from the functioning of all particles to the bricks of life. The team behind this project includes two Pierre and Frédéric Lepeltier from France. The first has been the CEO of the Unified Theory Research Team for 32 years.

Gate-based quantum computing

As a computer scientist, you may have skipped all the previous parts to get here right away. One can indeed understand how quantum computers operate without delving too deeply into quantum physics beyond grasping its basic mechanisms. Some mathematical knowledge is however required on trigonometry and linear algebra, including vectors, matrices, and complex numbers which I cover here[390].

The first basic element of a quantum computer is its inevitable qubit. You've probably already heard about this mysterious object having "simultaneously" the values 0 and 1. As a result, you've been told that a set of N qubits create an exponential 2^N superposed state that explains the power of quantum computing. Unfortunately, most explanations usually stop there, and you then end up wondering how it works to make some calculation. What data comes in and out of a quantum computer? How is it programmed? How do you feed it with data and code? Where is it useful? This book is there to provide you with some educated answers to all these critical questions.

Adopting a "bottom-up approach", we will describe the logical and mathematical aspects of qubits, qubit registers, quantum gates and measurement[391]. When possible, we'll draw parallels with traditional computing. In the following part, we'll look at quantum computer engineering and hardware and describe the complete architecture of a superconducting qubits quantum computer as an example.

In a nutshell

Before digging into qubits, qubit registers and the likes, Figure 135 shows a tentative to summarize the key elements of gate-based quantum computing that we'll cover in detail afterwards. It shows how physics and mathematics are intertwined. It is completed by a simple glossary in Figure 136.

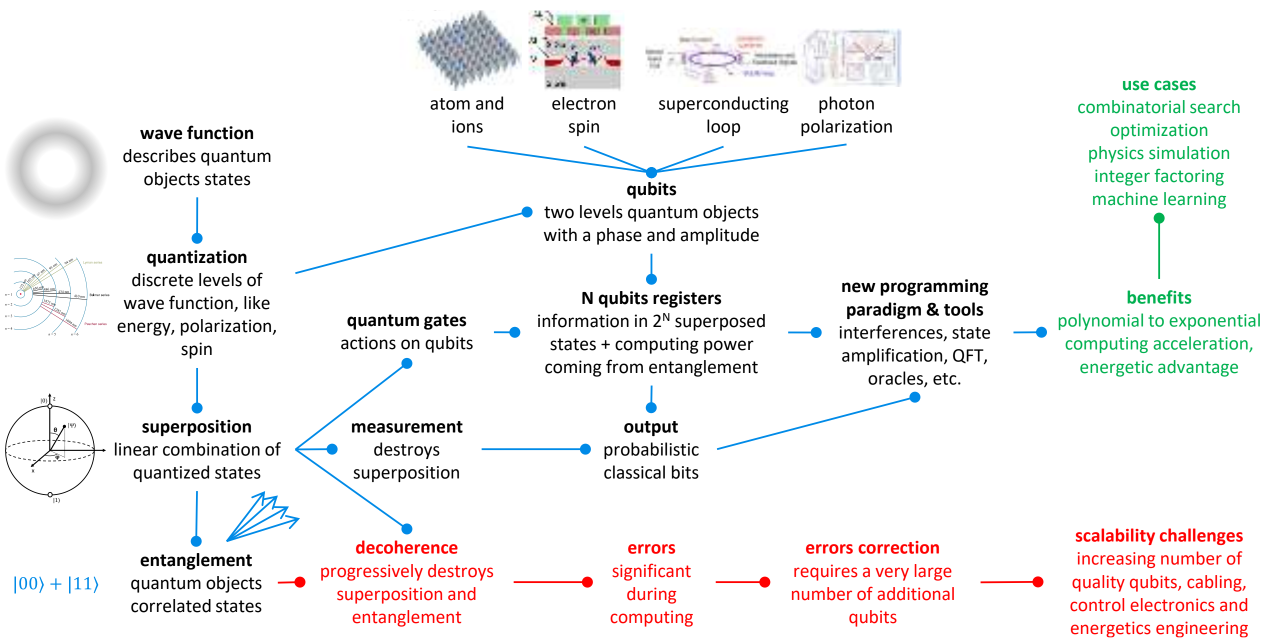

Figure 135: a single schematic to describe quantum physics and quantum computing. (cc) Olivier Ezratty, 2021.

[390] Complex numbers were created by the polymath Girolamo Cardano (1501-1576, Italian) and the Algerian mathematician Raffaele Bombelli (1526-1572, Italian) between 1545 and 1569. They were used to solve polynomial equations associating cubes and squares that kept Italian mathematicians busy since the end of the fifteenth century. See A Short History of Complex Numbers by Orlando Merino, 2006 (5 pages).

[391] The term qubit, for 'quantum' and 'bit', appeared in 1995 in Quantum coding by Benjamin Schumacher, PRA, April 1995 (34 pages).

Wave function
mother equation of quantum physics, created by Erwin Schrödinger. It describes particles properties probabilities in space and time with a complex number. This equation is specific to non-relativistic massive particles like electrons. We also use photons in quantum computing, whose properties are defined by Maxwell's electromagnetic equations and the second quantization equations (Glauber states, Wigner function, Fock states, etc.).

Quantization
properties of quantum objects, having discrete, not continuous and exclusive values. It enables the creation of qubit physical and logical objects having two levels.

Superposition
qubits are quantized quantum objects having two basis computational states |0⟩ and |1⟩. These can be combined linearly, thanks to the linearity over space of Schrödinger's wave equation. Solutions of this equation can be linearly combined with complex numbers. Thus, a wave adding two solution waves is still a solution. This doesn't mean the qubit is really simultaneously in two states.

Entanglement
often presented as a situation where several quantum objects have properties that are correlated. Actually, entanglement is the consequence of superposition of multiple qubit states. This is the phenomenon that provides both a real theoretical exponential acceleration to quantum computing but also enables conditional relations between qubits. Without it, qubits would be independent and no useful computing could be done.

Qubits
mathematical objects with two levels 0 and 1. It's described by two complex number amplitudes. But due to normalization and getting rid of their global phase (we'll explain all of that), they are described by two real numbers for their amplitude and phase. Physical qubits are based on massive (electron, controlled atoms, superconducting currents) or non massive quantum objects (photons) and one of their quantum properties or observables (spin, energy level, current direction of phase, polarity).

Registers
physical and logical assemblies of several qubits. With N qubits, they can handle computing on a space 2^N computational basis states together represented by complex number amplitudes. Each basis state is one of the possible combinations of N 0s and 1s. Computing power comes from entanglement.

Quantum gates
logical operations exerted on qubits. We have single qubit gates which are changing single qubit states and several qubit gates conditionally changing one or two qubits based on the state of a control qubit, and leveraging entanglement. Gates are the only mechanism used to feed a quantum register with data and instructions. These are not separated as in classical computing based on a Von Neumann / Turing machine model.

Programming paradigms
quantum programming is based on very different paradigms than classical programming. In a nutshell, it's analog-based. We play with interferences, states amplification, quantum Fourier transforms and the concept of oracles.

Measurement
the way to extract information from qubits. Unfortunately, you can't read the two real numbers describing the qubit state nor the combination of qubit registers computational basis states. You get just classical 0s and 1s for each qubit. Quantum algorithms toy with the wealth of superposition and entanglement during computing to recover a simple result at the end. Measurement is also used during quantum error corrections. Since qubit measurement output is probabilistic, you generate a deterministic output with running your algorithm several times (up to several thousand times) and computing an average of the obtained results.

Output
for a register of N qubits, you get N 0s and 1s. But these are probabilistic results. You usually need to run your algorithm several times and compute an average of the results to get a deterministic result. Noise and decoherence are additional reasons why you need to do this several times.

Benefit
an acceleration of computing time com-pared to the best classical computers. Accelerations can be from polynomial to exponential. The benefit can also be economic like with the energetic cost of quantum computing that many expect to be fairly low compared to classical computing.

Use cases
quantum computing will not replace most use cases of classical computing. It brings value for complex combinatorial problems, optimization problems, quantum physics simulation, some machine learning problems and at last, fast integer factoring.

Decoherence
the enemy with quantum computing. This is when qubit states is degraded, both for superposition and entanglement. It results from the interactions between the qubits and their environment despite of all the care implemented to isolate it.

Errors
result of decoherence and other perturbations affecting the qubits. Other sources of errors are the imprecision of the control electronics driving qubit gates. Qubit phase and amplitude is degraded over time. Existing error rates are many order of magnitude higher that with classical computing. These are the reasons why we don't have yet quantum computers with a very high number of functional qubits.

Error corrections
set of techniques used to correct these errors. It requires assembling so-called logical qubits made of a great number of physical qubits. The needed ratio at this point is ranging from 30 to 10,000 physical qubits to create a logical qubit. The ratio depends on the qubit quality and technology but also on the target logical qubit fidelity (from 10^{-8} to 10^{-15} error rates).

Scalability challenges
assembling these huge logical qubit is the mother of the challenges with quantum computing. It's not easy to assemble that many qubits and keeping them stable, limit their decoherence and the likes. On top of that, assembling a great number of qubits creates huge engineering challenges with cryogenics cooling power, thermal dissipation, cabling and control electronics. These are the reason why quantum computers don't scale yet to bring their expected benefits.

Figure 136: the key concepts behind gate-based quantum computing in one page. (cc) Olivier Ezratty, 2021-2022.

Linear algebra

Quantum physics and computing require some understanding of a whole bunch of concepts from linear algebra. They are associated with a mathematical formalism describing quantum phenomena. This mathematical formalism is also the cornerstone of quantum physics postulates, already covered in an earlier section, page 100. It is also essential to understand how qubit, quantum gates and quantum algorithms operate.

Linearity

Linear algebra is the branch of mathematics using vector spaces, matrices and linear transformations. In the case of quantum physics and computing, it also deals with complex numbers.

A phenomenon is linear if its effects are proportional to its causes. This translates into the verification of two simple equations pertaining to homogeneity and additivity as shown in Figure 137.

homogeneity $f(\lambda x) = \lambda f(x)\ for\ all\ x \in \mathbb{R}$

additivity $f(x+y) = f(x) + f(y)\ for\ all\ x, y \in \mathbb{R}$

Figure 137: homogeneity and additivity in linear algebra.

ℝ being a vector space, λ a real number, x being a vector of the vector space ℝ and f(x) a function applying to this vector. In a one-dimensional space, a classic example of a linear function is f(x) = ax. A polynomial function of the type $f(x) = ax^2 + b$ is obviously not linear because it evolves non-proportionally to x. Even $f(x) = ax + b$ is not linear, and for the same reason.

As already defined, an observable is a mathematical operator, a Hermitian matrix, used to measure (mathematically) a property of a physical system. It is frequently assimilated to the measured property. For a qubit, it corresponds to some measurable value by a sensor on a quantum object outputting a classical 0 or 1. The measurement causes the qubit quantum object wave function to collapse on one of the basis states. If the state of a quantum or qubit is measured twice, the measurement will yield the same result. With qubits, observables are usually based on projections on a two-level properties system, mathematically materialized by a $|0\rangle$ or $|1\rangle$, aka qubit computational basis states. But, if physics permits it, another computational basis can be used. It is the case with photons and polarization measurement where the angle can be easily changed in different parts of an experiment.

Hilbert spaces and orthonormal basis

A quantum state of a single or several quantum objects can be described by a vector in a Hilbert space. A qubit state is represented in a two-dimensional orthonormal space formed with the basis states vectors $|0\rangle$ and $|1\rangle$. It is a vector of complex numbers in a two-dimensional Hilbert space allowing length and angle measurement. A complex number is defined as a+ib where a and b are real and i^2=-1 (Figure 138).

Complex numbers are very useful in quantum physics. It relates to the wave-particle duality of all quantum objects and to the need to handle their amplitude (complex number norm, vector length or modulus) and phase (the complex number angle when using polar coordinates).

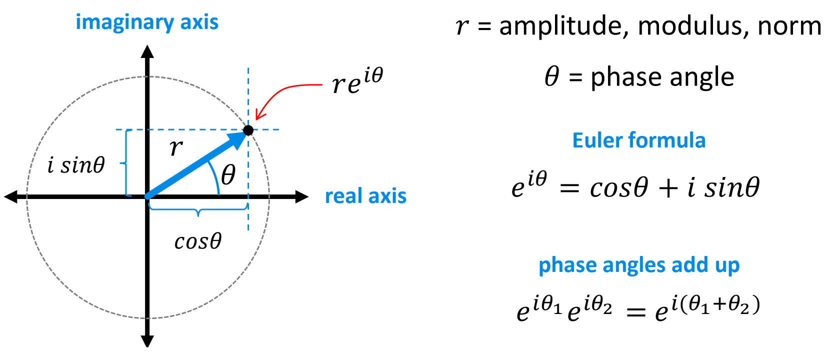

Figure 138: complex number explained by geometry and trigonometry.

With qubits, it is represented with the complex numbers α and β associated with the states |0⟩ and |1⟩ and whose sum of squares makes 1. This linear combination of the states |0⟩ and |1⟩ describes the phenomenon of superposition within a qubit.

This two-dimensional space replaces the infinite-dimensional space that characterizes a Schrödinger wave function $f(x)$, where x can take any value in space. It is thus a simplified representation of the quantum state of a qubit. By manipulating these symbols, the vectors and matrices, we forget a little the wave-like nature of the manipulated quanta, even though it is still present in the phase information embedded in the imaginary part of α and β for one qubit. It also can deal with photons which do not obey Schrödinger's equation but to Maxwell's electromagnetic equations.

An orthonormal basis of a vector space consists of base vectors which are all mathematically orthogonal with each other and whose length is 1. In the representation of a qubit state, the most common orthonormal basis is made of the states |0⟩ and |1⟩.

Other orthonormal reference basis can be used for measurement, particularly with photons, and polarization references different from the starting reference (0°/90° then 45°/135°, obtained with rotating a simple polarizer).

Another example of an orthonormal basis is the states located on the Bloch sphere on the x-axis and represented with |+⟩ and |−⟩. These are often called Schrodinger cats (Figure 139).

$$|+\rangle = \frac{|\mathbf{0}\rangle + |\mathbf{1}\rangle}{\sqrt{\mathbf{2}}} \qquad |-\rangle = \frac{|\mathbf{0}\rangle - |\mathbf{1}\rangle}{\sqrt{\mathbf{2}}}$$

Figure 139: another orthonormal basis, aka Schrodinger's cats.

Dirac Notation

In Dirac notation, in Figure 140**,** a quantum object state is represented by |Ψ⟩, the **ket** of quantum state Ψ. The **bra** of the same state vector, represented by ⟨Ψ| is the conjugate (or transconjugate, or adjoint) transpose of the "ket". It is the "horizontal" vector $[\bar{\alpha}, \bar{\beta}]$ where $\bar{\alpha}$ and $\bar{\beta}$ are the conjugates of α and β, inverting the sign of the imaginary part of the number (-i instead of +i, or the opposite).

vectors **Dirac notation**

$$|\Psi\rangle = \begin{bmatrix}\alpha \\ \beta\end{bmatrix} \qquad \bar{\alpha} = \alpha^* \quad \langle\Psi| = [\bar{\alpha}, \bar{\beta}]$$

Ψ **ket** — *Ψ* **bra**

$$(1+i)^* = 1 - i$$

complex number **conjugate**

Figure 140: introduction to Dirac vector notation.

The **scalar product** of two qubits $\langle\Psi_1|\Psi_2\rangle$ is the mathematical projection of the state vector Ψ_2 onto the vector Ψ_1. This yields a complex number. When the vectors are orthogonal, the scalar product is equal to 0. When the two vectors are identical, ⟨Ψ|Ψ⟩ is Ψ's norm and is always equal to 1. A scalar product is also named an inner product (Figure 141).

$$\langle\Psi_1|\Psi_2\rangle = [\overline{\alpha_1}, \overline{\beta_1}] \times \begin{bmatrix}\alpha_2 \\ \beta_2\end{bmatrix} = \overline{\alpha_1}\alpha_2 + \overline{\beta_1}\beta_2$$

inner **scalar** product: vector similarity

$$\langle\psi|\psi\rangle = [\bar{\alpha}, \bar{\beta}] \times \begin{bmatrix}\alpha \\ \beta\end{bmatrix} = |\alpha|^2 + |\beta|^2$$

Figure 141: inner scalar product.

An **inner product** is a generalization of a dot vector product applied to complex number vectors, according to the sigma in Figure 142.

complex vectors dot product

$$A.B = \sum_i a_i \bar{b_i}$$

Figure 142: dot product.

The **outer product** of two vectors representing a qubit, one in bra and the other in ket, gives an operator or density matrix which is a 2x2 matrix (Figure 143).

When the bra corresponds to the transconjugate of the ket, it is a density operator of a pure state. This notion of density operator will then be extended to a combination of qubits.

$$|\Psi\rangle\langle\Psi| = \begin{bmatrix}\alpha \\ \beta\end{bmatrix} \times [\bar{\alpha}, \bar{\beta}] = \begin{bmatrix}\alpha\bar{\alpha} & \alpha\bar{\beta} \\ \beta\bar{\alpha} & \beta\bar{\beta}\end{bmatrix}$$

Figure 143: outer product.

What are the use cases of this Dirac notation? It is particularly helpful for manipulating quantum states, to simplify tensor products representations and with measurement.

Eigenstuff

We also need to define the notions of **eigenvector**, **eigenvalue**, **eigenstate** and **eigenspace** which are often used in quantum mechanics and quantum computing as well as in machine learning, particularly in dimension reduction algorithms such as PCA (Principal Components Analysis). These notions allow to define the structure of certain square matrices[392].

For a square matrix A, an eigenvector x or eigenvector of A is a vector that verifies the equation Ax = λx, λ being a complex number called eigenvalue.

These eigenvectors have the particularity of not changing direction once multiplied by the matrix A. For an eigenvalue λ, the associated eigenspace, or eigenspace, is the set of vectors x that satisfy Ax = λx. These eigenvalues are evaluated by calculating the determinant of the matrix A - λI, where I is the identity matrix (1 in the diagonal boxes and 0 elsewhere). We then find the values of which solves 0 = A - λI. It is a polynomial equation having a degree less than or equal to the size of the square matrix[393].

The reference eigenvectors of a matrix A allow to reconstitute an orthonormal space linked to the matrix. For example, a projection matrix in a 3D plane will have as main eigenvectors two orthogonal vectors located in the plane and one vector orthogonal to the plane. This multiplication gives λx with λ being non-zero if the eigenvector is in the plane in question and 0 if the vector is orthogonal to the plane [394]. A matrix A can be that of a quantum gate. An eigenvector of a quantum gate is therefore a ket whose value is not modified by the quantum gate.

This is easy to imagine for the S gate, phase change, which we will see later. The $|0\rangle$ and $|1\rangle$ kets being in the rotation axis, they are not modified by it.

They are thus eigenvectors of the S gate and the corresponding eigenvalues are 1 and -1. This is always the case for quantum gate matrices since the vectors representing the quantum states, the kets, always have a length of 1. These eigenvalues are the only ones enabling this!

The search for the eigenvectors and eigenvalues of a matrix A is like diagonalizing it. For this it must be diagonalizable (“non-defective”). Hermitian and unitary matrices commonly used in quantum physics are all non-defective and diagonalizable. The diagonalization of a square matrix consists in finding the matrix which will multiply it to transform it into a matrix filled only in its diagonal. A matrix A is diagonalizable if we can find a matrix P and a diagonal matrix D such that $P^{-1}AP = D$ (P^{-1} being the inverse matrix of P, such that $P^{-1}P=PP^{-1}=I$, I being the matrix identity with 1's in the diagonal and 0's elsewhere). A square matrix of dimension n is diagonalizable if it has n mutually independent eigenvectors. The diagonalized matrix diagonal contains the eigenvalues λ_i of the origin matrix, with i=1 to N being the size of the matrix.

A diagonalized quantum state of a quantum object can look like $A = \sum_i \lambda_i |i\rangle\langle i|$. This decomposition of a pure state vector in a Hilbert space in eigenstates $|i\rangle$ and eigenvalues λ_i is also named a **spectral decomposition**. It is linked to the wave-duality aspect of all quantum objects.

[392] See a good quick review of linear algebra in Linear Algebra Review and Reference by Zico Kolter and Chuong Don 2015 (26 pages).

[393] See this nice visual explanation of eigenvectors and eigenvalues: Eigenvectors and eigenvalues | Chapter 14, Essence of linear algebra, 2016 (17 minutes).

[394] This is well explained in Gilbert Strang's lecture at MIT, 2011 (51 minutes).

A quantum object is indeed decomposed into a coherent superposition of elementary waves. In the case of photons, it is easy to grasp with several photons of different frequencies being superposed and forming a gaussian wave packet (Figure 144). It constitutes a coherent superposition of the electromagnetic field. These wave packets are commonly generated by femtosecond pulse lasers[395].

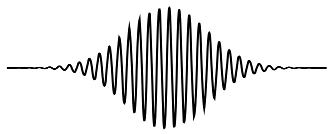

Figure 144: a photon gaussian wave packet.

And the eigenstates? This is another name given to eigenvectors, but by physicists!

Tensor products

The tensor product of two vectors of dimension m and n gives a vector of dimension m*n while the tensor product of a matrix of dimension m*n by a matrix of dimension k*l will give a matrix of dimension mk*nl. Tensor products use the sign ⊗ (Figure 145).

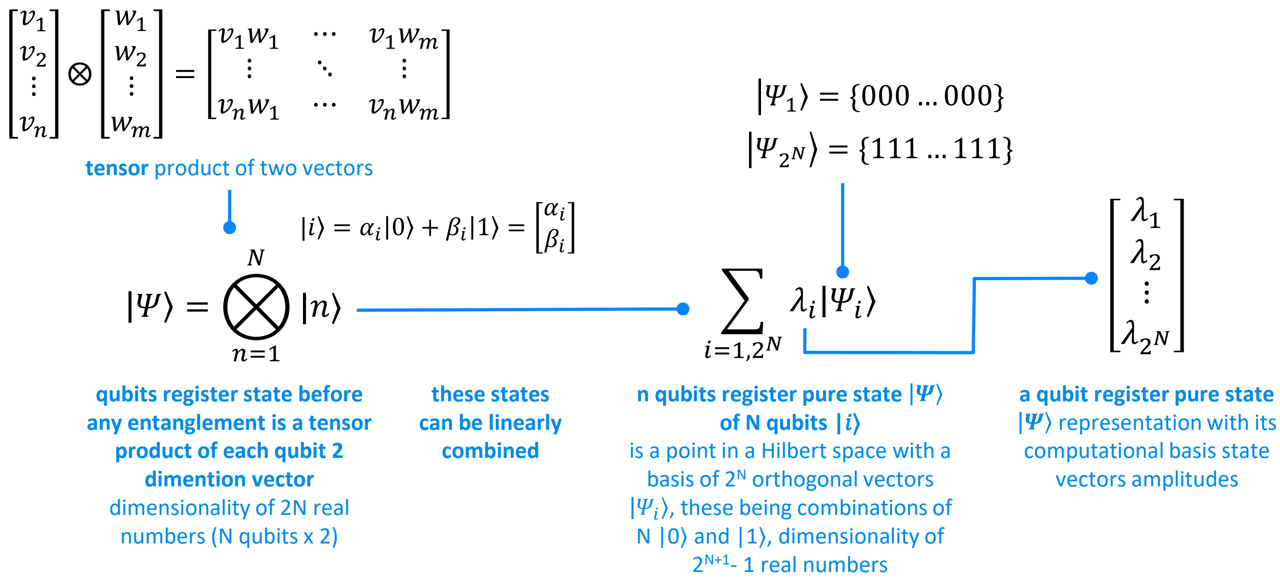

Figure 145: tensor products construction. (cc) Olivier Ezratty, 2020-2023.

Tensor products are used to compute “manually” the state of quantum registers containing several unentangled qubits. The state of a register of N non-entangled qubits is the tensor product of these N qubits represented by their vertical ket vector.

This gives a ket, a vertical vector that has 2^N different values, each representing the complex number weight of different combinations of 0s and 1s. A quantum register is a superposition of these 2^N different states complex amplitudes. The sum of these squared amplitudes gives 1 per the Born rule. By the way, the tensor product of qubits is represented by a vector, after vectorization of the tensor product matrix of 2^N dimensions.

Entanglement

Quantum states are separable when they are mathematically the result of the tensor product of each of the pure states that compose it. But these values can be assembled linearly to create another quantum state, modulo a normalization rule. This combines several vectors resulting from tensor products. These combinations can become inseparable.

[395] And when the carrier frequency is growing or decreasing through the pulse, it’s named a chirp pulse.

That's when entanglement comes into play. An entangled state of two or more qubits occurs when it cannot be factorized as the tensor product of two pure states. In other words, it cannot be the combination of independent qubits. The qubits become dependent.

This is demonstrated mathematically for the states $|00\rangle$ and $|11\rangle$ of a register of two qubits (Figure 146). In these pairs, the measurement of the value of one of the qubits determines that of the other, here identical. The creation of such entangled pairs of qubits requires preparation operations like using a combination of Hadamard and CNOT gates.

Two qubits placed side by side are not magically entangled! The pair used in the example can be generated by two quantum gates, an H gate (Hadamard) and a CNOT gate, as shown just below.

an entangled EPR pair can't be a tensor product of two qubits $|\Psi_1\rangle$ and $|\Psi_2\rangle$

$|\Psi_1\rangle = \alpha_1|0\rangle + \beta_1|1\rangle$ $\quad$ $|\Psi_2\rangle = \alpha_2|0\rangle + \beta_2|1\rangle$

$$|\Psi_1\rangle \otimes |\Psi_2\rangle = (\alpha_1|0\rangle + \beta_1|1\rangle)(\alpha_2|0\rangle + \beta_2|1\rangle)$$

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \alpha_1\alpha_2|00\rangle + \alpha_1\beta_2|01\rangle + \beta_1\alpha_2|10\rangle + \beta_1\beta_2|11\rangle)$$

$\alpha_1\beta_2 = 0$ and $\beta_1\alpha_2 = 0$

are incompatibles with $\alpha_1\alpha_2 = \frac{1}{\sqrt{2}}$ and $\beta_1\beta_2 = \frac{1}{\sqrt{2}}$

if $\alpha_1 = 0$ then $\alpha_1\alpha_2 = 0$

if $\beta_2 = 0$ then $\beta_1\beta_2 = 0$

implications: the density matrix mathematical representation of qubits registers

Figure 146: non separability of two entangled qubits.

We will define this CNOT gate after page 194. This is described as both qubits having correlated values. But these values are... random since being a perfect superposition of 0 and 1!

Only multi-qubit quantum gates generate entangled qubits in a qubit register, besides the SWAP gate which doesn't. Figure 147 shows an example of creating a Bell pair associating the states $|00\rangle$ and $|11\rangle$ with a mix of Hadamard and CNOT gates.

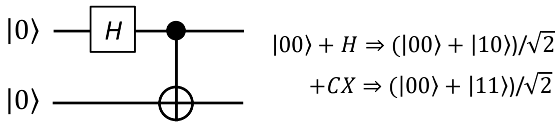

Figure 147: a Bell pair.

A so-called **GHZ** state (for Greenberger-Horne-Zeilinger, distinguishable from GHz frequencies with a capital Z) with three (or more) entangled qubits is superposing the states $|000\rangle$ and $|111\rangle$. It is a generalization of the 2-qubit Bell state $(|00\rangle + |11\rangle)/\sqrt{2}$. A GHZ is usually prepared with a Hadamard gate and two consecutive CNOTs (Figure 148).

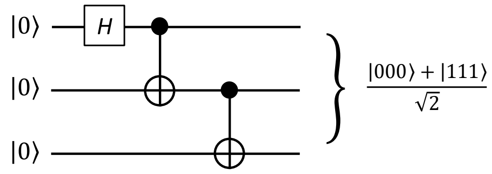

Figure 148: a GHZ state.

These pairs of Bell and GHZ states are used in error correction codes as well as in telecommunications, among other things.

Another typical entangled state is the **W state**, created in 2000 (Figure 149), that has the property of being maximally entangled and robust against particle loss. It is a generalized version of another of the four possible Bell states, $(|01\rangle + |10\rangle)/\sqrt{2}$ [396]:

$$|W\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)$$

Figure 149: a W state.

Finally, the level of entanglement of a qubit register depends on the Hamming distance between the basis states involved in the linear superposition of basis states. The far apart they are, with the greater number of non-identical 0s and 1s, the greatest the entanglement is.

[396] See Three qubits can be entangled in two inequivalent ways by Wolfgang Dür (which explains the W in W states), G. Vidal, and J. Ignacio Cirac, 2000 (12 pages) and the thesis Symmetry and Classification of Multipartite Entangled States by Adam Burchardt, September 2021 (126 pages).

Matrices

Various matrix transformations must be understood here:

- **Matrix conjugate** when all complex number see their complex part negated, or $a_{ij} = a_{ij}^*$.
- **Matrix transpose** when all matrix a_{ij} values are transformed into a_{ji}value, with i=line and j=column indices of matrix "cells".
- **Matrix transconjugate** which is a conjugate of the transpose or vice-versa, also named adjoint. It is notated as $A^\dagger$, for A « dagger ».
- **Matrix traces** are the sum of their diagonal values, usually normalized to 1, like with density matrices. It is also the sum of their eigenvalues.

We also have three important classes of matrices:

- **Hermitian matrices** are equal to their transconjugate, meaning that $a_{ij} = a_{ji}^*$.
- **Projectors** are matrix operators using a Hermitian matrix that is equal to its square. A diagonalized projector contains only zeros and ones, and a single 1 for a rank projector. A projector is a non-unitary operation. It relates to the irreversibility of quantum measurement. If $|\psi\rangle$ is a unit vector, the outer product $|\psi\rangle\langle\psi|$ is a projector that can project any vector $|\phi\rangle$ on $|\psi\rangle$.

Notation	Description
z^*	Complex conjugate of the complex number z. $(1+i)^* = 1 - i$
$\|\psi\rangle$	Vector. Also known as a *ket*.
$\langle\psi\|$	Vector dual to $\|\psi\rangle$. Also known as a *bra*.
$\langle\varphi\|\psi\rangle$	Inner product between the vectors $\|\varphi\rangle$ and $\|\psi\rangle$.
$\|\varphi\rangle \otimes \|\psi\rangle$	Tensor product of $\|\varphi\rangle$ and $\|\psi\rangle$.
$\|\varphi\rangle\|\psi\rangle$	Abbreviated notation for tensor product of $\|\varphi\rangle$ and $\|\psi\rangle$.
A^*	Complex conjugate of the A matrix.
A^T	Transpose of the A matrix.
$A^\dagger$	Hermitian conjugate or adjoint of the A matrix, $A^\dagger = (A^T)^*$. $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^\dagger = \begin{bmatrix} a^* & c^* \\ b^* & d^* \end{bmatrix}$.
$\langle\varphi\|A\|\psi\rangle$	Inner product between $\|\varphi\rangle$ and $A\|\psi\rangle$. Equivalently, inner product between $A^\dagger\|\varphi\rangle$ and $\|\psi\rangle$.

Figure 150: linear algebra key rules. Source: Quantum Computation and Quantum Information by Nielsen and Chuang, 2010 (10th edition, 704 pages).

Indeed, $(|\psi\rangle\langle\psi|)|\phi\rangle = |\psi\rangle(\langle\psi||\phi\rangle) = (\langle\psi|\phi\rangle)|\psi\rangle$, given $\langle\psi|\phi\rangle$ is a real number being the inner product of both vectors. Some of these elements are summarized in Figure 150 and Figure 152.

- **Unitary matrices** are square matrices whose inverse equals their transconjugate ($A^\dagger = A^{-1}$). A unitary matrix has several properties, one of which is to have orthogonal eigenvectors and to be diagonalizable. Unitary matrices define the reversible gates applied to qubits or sets of qubits (Figure 151).

$$A = \begin{bmatrix} 2 & i & -2i \\ -i & 1 & 3 \\ 2i & 3 & -1 \end{bmatrix}^\dagger \quad \bar{A} = \begin{bmatrix} 2 & -i & 2i \\ i & 1 & 3 \\ -2i & 3 & -1 \end{bmatrix} \quad A^\dagger = \overline{(A)}^* = \begin{bmatrix} 2 & i & -2i \\ -i & 1 & 3 \\ 2i & 3 & -1 \end{bmatrix} \quad \begin{matrix} U|x\rangle = |y\rangle \\ |x\rangle = U^\dagger|y\rangle \end{matrix}$$

transposed matrix — **hermitian matrix** (transconjugate = identity) — **unitary reversibility**

Figure 151: unitary matrices. (cc) Olivier Ezratty, 2021.

A unitary operation is the application of a unitary matrix to a computational state vector that we'll later see. Quantum computing reversibility comes from this unitary property. A unitary matrix U can also be expressed as $U = e^{iH}$, with H being a Hermitian matrix, but finding H given U is a complicated calculation problem.

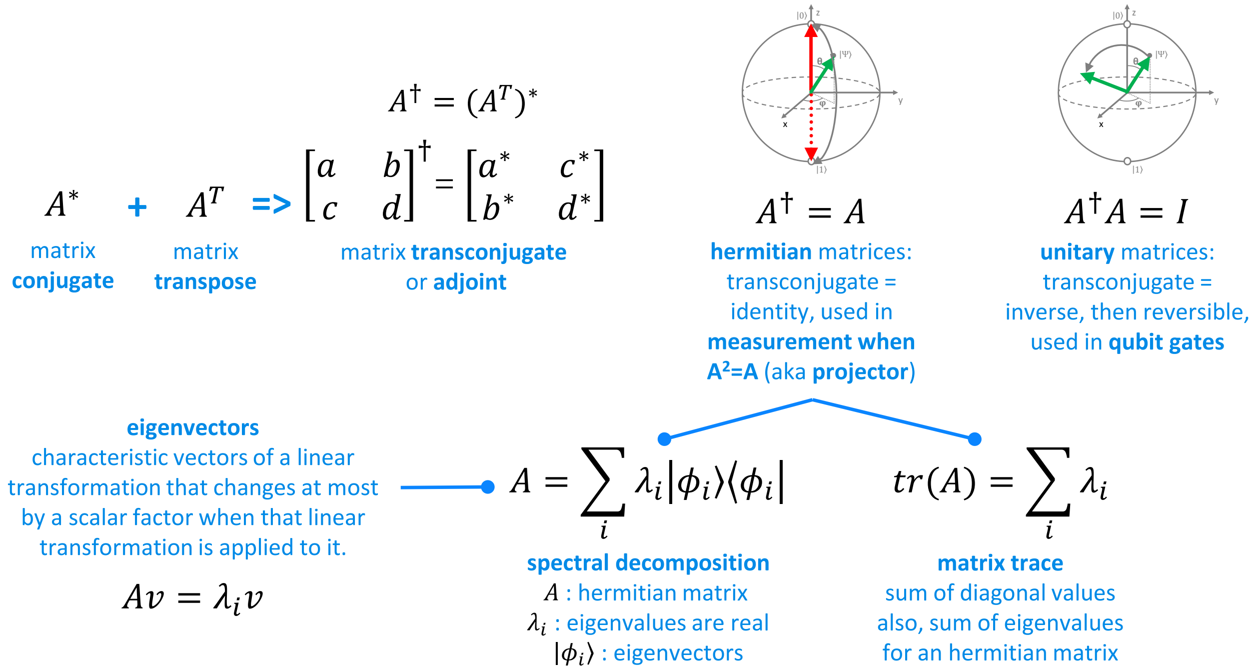

Figure 152: difference between unitary matrices and Hermitian matrices. (cc) Olivier Ezratty, 2021-2023.

Pure and mixed states

Let's now explain what the three main states of quantum objects are, basis, pure and mixed as shown in Figure 153. We'll apply it to the case of qubits, given these notions are valid with any quantum system. We are dealing with mathematical models that describe quantum objects states[397].

	basis states	pure states	mixed states
definitions	aka computational basis states, are N dimensions vectors combining 0s and 1s, with 2^N different such vectors for a N qubits register.	vectors in a Hilbert space of norm 1, specified by a single ket describing coherent superpositions of basis states with complex numbers.	or statistical mixture of pure states, are classical statistical ensemble of combination p_i of pure states Ψ_i. Ψ_i can be any combination of pure states but is usually a set of computational basis states.
randomness origin	no randomness with perfect qubits	quantum	quantum and classical
with a single qubit	$\|0\rangle$ and $\|1\rangle$	$\|\Psi\rangle = \alpha\|0\rangle + \beta\|1\rangle$ $\|\alpha\|^2 + \|\beta\|^2 = 1$	$p_1\|\psi_1\rangle, p_2\|\psi_2\rangle$ we don't add them, it's just a statistical ensemble, statistical mixture or convex sum of several systems.
with a N qubits register i = 1 to 2^N	$\|i\rangle$ $\|01101011\rangle$ for N=8 all $\|i\rangle$ form the computational basis states of the N qubits register, contains N combinations of 0 and 1, all basis states are mathematically orthogonal.	$\|\Psi\rangle = \sum_i \alpha_i \|i\rangle$ $\sum_i \|\alpha_i\|^2 = 1$ α_i = complex number a pure state is a linear superposition of computational basis states.	$\{(p_i\|\Psi_i\rangle\}$ ensemble notation $\sum_i p_i = 1$ p_i = positive real number probability to find Ψ_i in the mixed state given all p_i are 0 or a 1 in a pure state.

Figure 153: differences between basis states, pure states and mixed states. (cc) Olivier Ezratty, 2021-2023

Basis states correspond to given combinations of 0 and 1 values in a qubit register. For a single qubit, these are the states $|0\rangle$ and $|1\rangle$. For a register of N qubits, it is one of the 2^N different basis states combinations of 0s and 1s, or a tensor product of N single qubit basis states. It constitutes the computational basis in a complex numbers Hilbert space of dimension 2^N.

[397] See The Many Inconsistencies of the Purity-Mixture Distinction in Standard Quantum Mechanics by Christian de Ronde and César Massri, August 2022 (19 pages) that provides an interesting historical perspective on the pure and mixed states nuances and shortcomings.

The vectors of this basis are all mathematically orthogonal. A basis state is also named a computational basis state. When measuring individual qubits in these states, you get a deterministic result, at least with theoretically perfect qubits.

Pure states describe the state of an isolated quantum system of one or several objects as a linear superposition of the states from its computational basis. It is a vector in a Hilbert space. That's when superposition and entanglement come in. With massive particles, basis and pure states are solutions to Schrödinger's equation. It is applicable to one or several quantum objects or qubits. During computation, a qubit register is theoretically in a pure state, but quantum decoherence will gradually turn it into a mixed state. A pure state is also presented as a quantum state where we have exact information about the quantum system. This information corresponds to the famous ψ vector in the Hilbert space. When preparing a quantum state, we indeed know the parameters of the vector ψ even though actual property measurements will generate random results if the quantum state is not measured along with one of its eigenstates. The information we have about measurement potential results is their probabilistic distribution.

Mixed states are weird beasts. Literally, these are "*statistical ensembles of classical probabilistic combinations of pure states*", these being usually computational basis states, but they can also be expressed as real number linear combinations of any pure states. Basis states and pure states describe the information available for a single quantum object or qubit, or a group of such objects. A mixed state describes a large number of such systems, prepared in a similar manner, and the states they could be in when repeating an experiment followed by some measurement. However, a pure state measurement generating random results most of the time, we still also experimentally prepare and measure it on a repeated basis to have an idea of its state probability distribution. In the end, both pure states and mixed states describe the information we can extract from a system after doing repeated experiments and measurements. Their difference lies with the origin of measurement randomness. Its origin is entirely quantum for pure states and both quantum and classical (or "non-quantum") for mixed states. Got it? If not, we have a couple practical examples below to figure out what it looks like in the real world!

Typically, mixed states provide the available information describing two sorts of systems:

Random quantum objects like photons coming from an unpolarized photons source, or, when photons with different polarizations are merged like in Figure 154. The photon polarization at this point is a statistical mixture of horizontal and vertical polarization photons. Let's say this is the case where quantum objects are prepared differently and are then mixed together. The two sources are not "coherently" prepared. In the example in the left, a 45° polarizing beam splitter applied to horizontalized prepared photons produces superposed H and V photons in a pure state. On the right, the polarizing beam splitter creates 50% vertically and 50% horizontally polarized photons that can be merged by a 45° non-polarizing beam splitter. They are statistically merged, but not superposed, thus creating a mixed state.

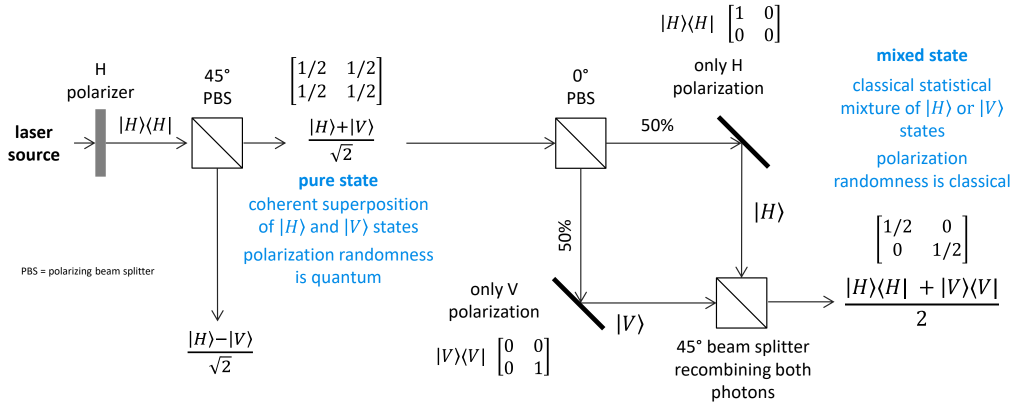

Figure 154: how to generate mixed states with photons. (cc) Olivier Ezratty, 2021-2023.

In the other example in Figure 155, two lasers are preparing coherent light that is polarized respectively horizontally and vertically and then merged by a beam combiner. The resulting photons represent a totally mixed state with uncorrelated and incoherent photons. Their statistical distribution is entirely classical with a density matrix void of any off-diagonal values.

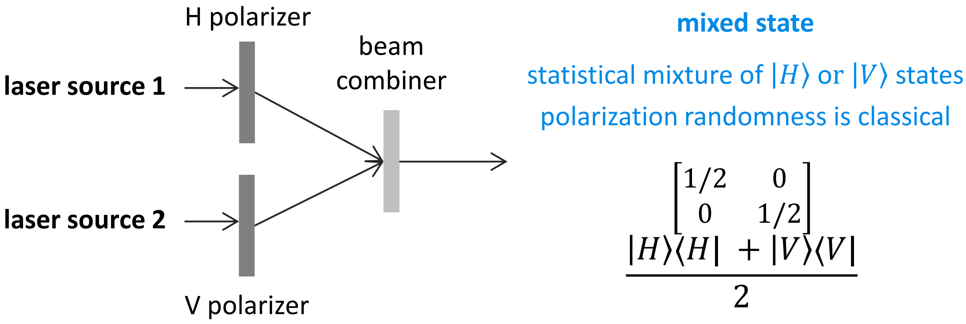

Figure 155: another method to generate a mixed state with photons. (cc) Olivier Ezratty, 2021-2023.

Subsystems of an inseparable entangled system of several quantum objects. It helps understand what we are measuring at the end of computing when the resulting qubits are still entangled. One case is a set of qubits affected by decoherence coming from interactions with the environment. It helps understand the effect of decoherence on the state of a qubits register during computing and how error correction codes are mitigating it. Decoherence comes from the entanglement between a system and its environment, thus, the observed system is not yet isolated and becomes a subsystem of a larger entangled system. Thus, it becomes a mixed state (Figure 156). Want to grasp it clearly? You need to toy with density matrices representations of these pure and mixed states.

Note that these concepts are applicable to both a single qubit and a register of N qubits.

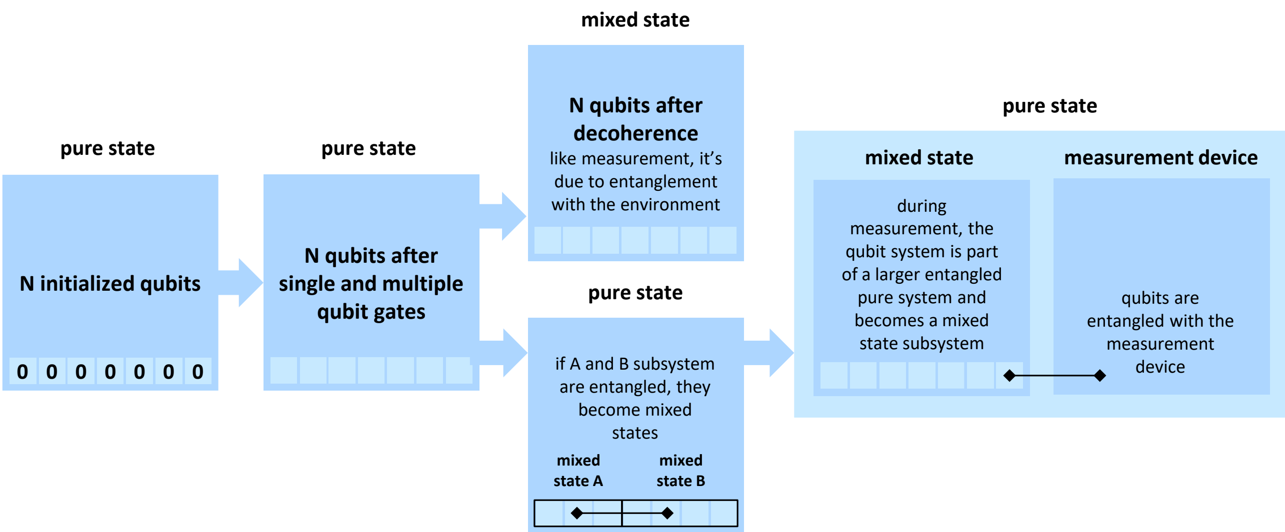

Figure 156: mixed states and pure states when using qubits. (cc) Olivier Ezratty, 2021.

Density matrices

Density matrices, also named density operators, were introduced in 1927 by **John von Neumann** and **Lev Landau** and later expanded by **Felix Bloch**. Von Neumann created this formalism to develop his theory of quantum measurements.

A density matrix is a mathematical tool used to describe quantum systems in pure or mixed states. Compared to the state vector that we saw earlier, a density matrix is the only way to mathematically describe a mixed state.

It consolidates all the physically significant information that could be retrieved from a set of quantum objects given what we know about them. Quantum and classical probabilities are boiled in the density matrix[398].

Usually represented by the sign ρ (rho), a density matrix is a square matrix of complex numbers used to describe a quantum system, like a register of several qubits. Its size is $2^N x 2^N$ where N is the number of qubits in the register.

The density matrix of a quantum register in **pure state** is the outer product of its computational basis state vector $|\Psi\rangle\langle\Psi|$ as described in Figure 157, with an example using a Bell pair of two qubits. There is no more information in the density matrix than in the basis state vector at this stage.

A density matrix for a **mixed state** adds several pure states matrices with real probability coefficients p_i. The $|\Psi_i\rangle$ pure states that are combined to form a mixed state can be themselves states from the computational basis (combination of 0s and 1s) but not necessarily. They can be any vector in the 2^N Hilbert space and made of (normalized) linear superpositions of these basis states. Mathematically speaking, a pure state density matrix is a special case of mixed state density matrix where only one p_i is not zero.

We'll repeat here what was said with pure and mixed states: a mixed state density matrix consolidates both **quantum uncertainties** (that persists even when the system state if well known) and **classical uncertainties** (due to a lack of knowledge of individual quantum sources and preparation conditions) when a pure state density matrix contains only information pertaining to **quantum uncertainties**.

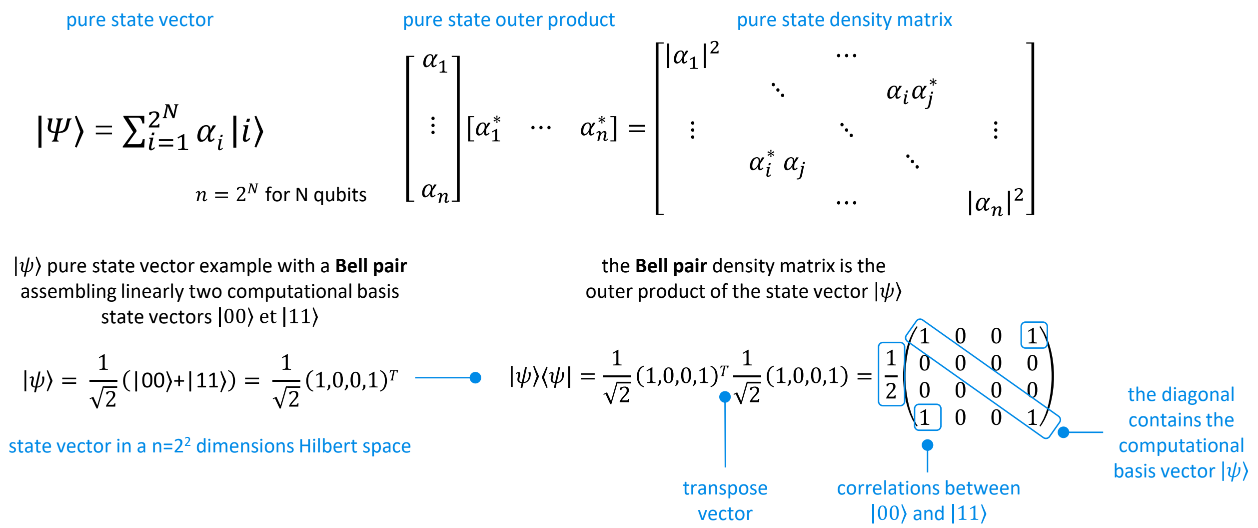

Figure 157: how a pure state matrix is constructed. (cc) Olivier Ezratty, 2021-2023.

A density matrix has several mathematical properties as described in Figure 158 and detailed afterwards with some differences between pure and mixed states density matrices.

Hermicity. A density matrix is Hermitian, meaning that it is equal to its transconjugate matrix. Consequently, the density matrix can be diagonalized in a different basis, with positive real number eigenvalues. Hermicity comes from the density matrix construction: it is a real number linear sum of Hermitian matrices resulting from the Hermitian inner product of pure states vectors. One consequence is that it removes any global phase from the quantum system it describes. You can easily understand it by evaluating on your own a density matrix of a given qubit and its global phase.

[398] See The Quantum Density Matrix and its many uses: From quantum structure to quantum chaos and noisy simulators by Apoorva D. Patel, March-August 2023 (28 pages).

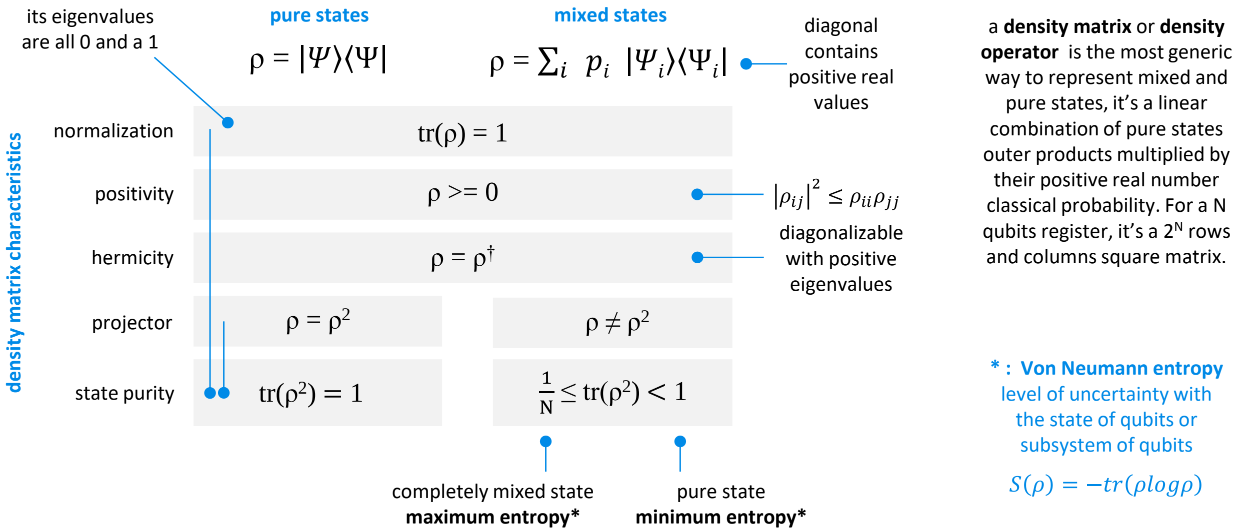

Figure 158: the various mathematical properties of pure and mixed states density matrices. 2021-2023.

Positivity. A density matrix M is positive semi-definite, meaning that $\langle x|M|x\rangle \geq 0$ for all x vectors. It is also defined as a symmetric matrix with non-negative eigenvalues (meaning... positive or zero). These eigenvalues being the values in the diagonal after matrix diagonalization. But even before diagonalization, all density matrices diagonal values are positive due to hermicity and the way they are constructed as positive probabilities combinations of outer products of pure states whose diagonal are always containing positive values.

Normalization. A density matrix trace equals 1 for both pure and mixed states. A density operator is said to be "*normalized to unit trace*". That's the sum of its diagonal values which are all positive real numbers. It comes from two rules: Born's rule applied to a pure state ($\sum_i \alpha_i^2 = 1$) and classical probabilities rules applied to the mixed state ($\sum_i p_i = 1$). As a result, a density matrix diagonal value at position j = $\sum_i p_i \alpha_{ij}^2$, α_{ij} being the weight α_j from the pure state i composing the mixed state. The diagonal is also referred to as a statistical mixture or as a population.

There are some differences between pure and mixed states density matrices.

Projector. A pure state density matrix is a projector, i.e. equal to its square and the trace of its square density matrix ρ^2 is equal to 1. Being a projector means that its eigenvalues are all zeros except a single one that is 1, for the case of a two-level quantum system like a qubit. The eigenvector associated with the eigenvalue one is the state vector of the system. Being a projector means the density matrix can be used as the way to measure a quantum state using this vector as a basis reference. In a single qubit system and the Bloch sphere, it would be any vector in the sphere and the related measurement observable, a geometrical projection of the evaluated qubit on this vector. In the case of a mixed state, the density matrix trace is inferior to 1 and its minimum is 1/N, when the state is maximally mixed with equal probabilities for all basis values. The average value obtained with applying an observable A to a pure state quantum system state vector ψ is evaluated with the formula $\langle\psi|A|\psi\rangle$, also named an expectation value. In other words, it is the dot vector product of ψ and the vector obtained by applying matrix A to vector ψ. The expectation value of a mixed state represented by a density matrix ρ is $tr(\rho A)$, a trace of the density matrix multiplied by the observable A matrix.

Off-diagonal elements can have a time-dependent phase that will describe the evolution of coherent superpositions. These elements are also named "coherences". As decoherence starts due to interactions with the environment, any pure state will progressively turn into a mixed state and the off-diagonal values will be affected. This evolution follows the Liouville–von Neumann equation.

$$\frac{|0\rangle\langle 0| + |1\rangle\langle 1|}{2} = \frac{|+\rangle\langle +| + |-\rangle\langle -|}{2} = \begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} = \frac{1}{2}\mathbb{I}$$

Mixedness defines how much "mixed" is a quantum state defined by its density matrix. It is computed with $tr(\rho^2)$ and is equal to 1 for a pure state and 1/N for a completely mixed state with N quantum objects. As a result, any time-dependent unitary transformation U applied to this quantum state won't affect the mixedness. Indeed, the density matrix over time is $\rho(t) = U(t, t_0)\rho(t_0)U^\dagger(t, t_0)$. Its mixeness is $tr(\rho^2(t)) = tr(U(t, t_0)\rho(t_0)U^\dagger(t, t_0)U(t, t_0)\rho(t_0)U^\dagger(t, t_0))$ which equals $tr(\rho^2(t_0))$.

Combinations. A mixed state can be the result of an infinite number of combinations of pure states, the most common example being, for two qubits, the half-identity mixed state being an equally mixed state of both $|0\rangle$ and $|1\rangle$ or $|+\rangle$ and $|-\rangle$. Given a density matrix, you can't compute the pure states that were combined to create it. Said otherwise, quantum states with the same density matrix can't be distinguished operationally (i.e., by a set of measurements). Also, when a unitary operation U (defined later, sorry) is applied to a mixed state defined by its density matrix ρ, the resulting state density matrix is $U\rho U^\dagger$. For the fun of a better understanding, Figure 159 a graphical segmentation of all the various matrix types we've been mentioning in the previous pages and how they are related with each other.

We forgot to define a **non-defective matrix**, which is a diagonalizable matrix. And a normal matrix A verifies $AA^\dagger = A^\dagger A$. A **trivial** matrix is both Hermitian and unitary and have orthonormal eigenvectors with eigenvalues being +1 or -1.

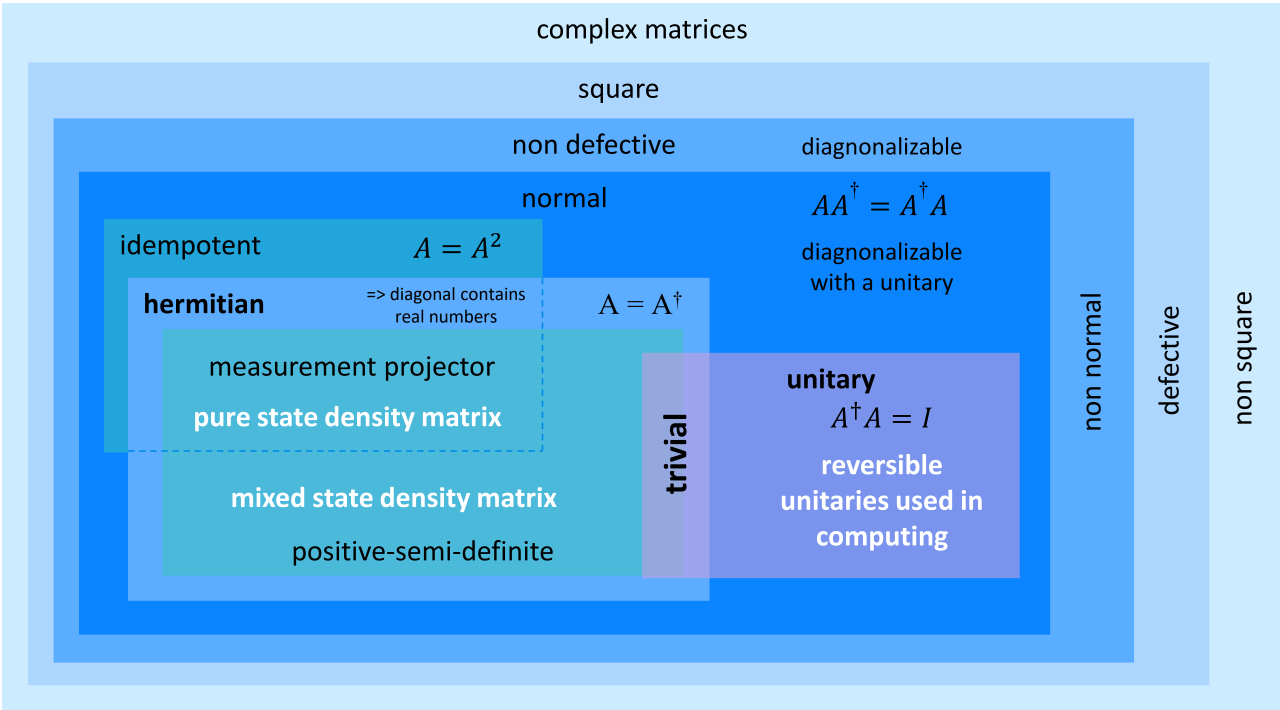

Figure 159: a Russian dolls map of matrices. (cc) Olivier Ezratty, 2021.

Single qubit mixed states can be represented by a point inside the Bloch sphere as shown in Figure 160 using a "Death Star" representation, with a statistical mixture of two pure qubit states. The mixed state is a convex sum of pure states inner products, 'convex' meaning it is a sum using positive real coefficients that sum up to 1. The geometric representation is a good way to figure out why a given mixed state can result from an infinite number of combinations of two pure states. We can combine more than two pure states to create a mixed state. By the way, the Bloch sphere becomes a Bloch ball.

Density matrix dimensionality. Although it contains 2^{2N} complex values, due to normalization, the dimensionality of a density matrix is 2^{2N}-1 real numbers. The explanation is reconstructed below. For a starter, we have 2^{2N} complex values which is the square or 2^N, the number of lines and columns in the density matrix.

We separate the matrix diagonal from the off-diagonal values. The diagonal values are real numbers because they are the positive probability sums of the diagonal values of pure states density matrices, themselves being positive as $|\alpha_i|^2$.

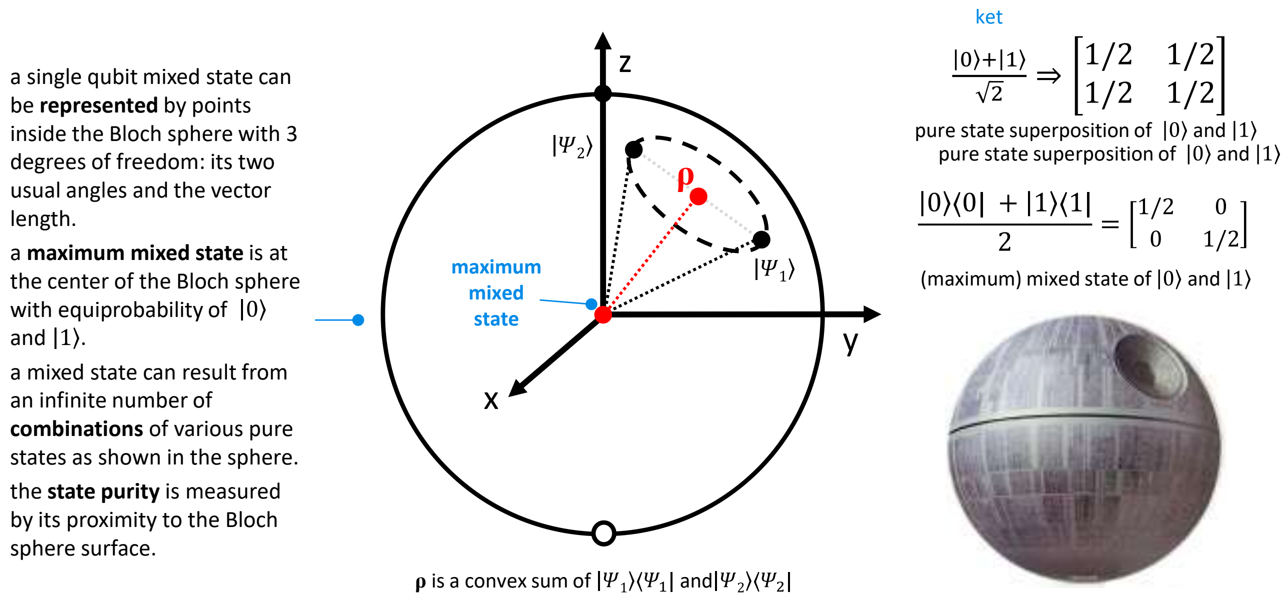

Figure 160: representation of a single qubit mixed state in the Bloch sphere. (cc) Olivier Ezratty, 2021.

The matrix trace equals 1, removing another useful dimension. The off-diagonal values are redundant since the matrix is equal to its transadjoint. So, we divide by two their dimensionality. Since these are complex numbers, we multiply it by two to get a quantity of real numbers. When summing this up, we find 2^{2N} -1 different real numbers. This dimensionality is usually presented as 2^{2N-1} complex numbers or 2^{2N} real numbers, avoiding the minus 1 which is quickly negligible as N grows (Figure 161).

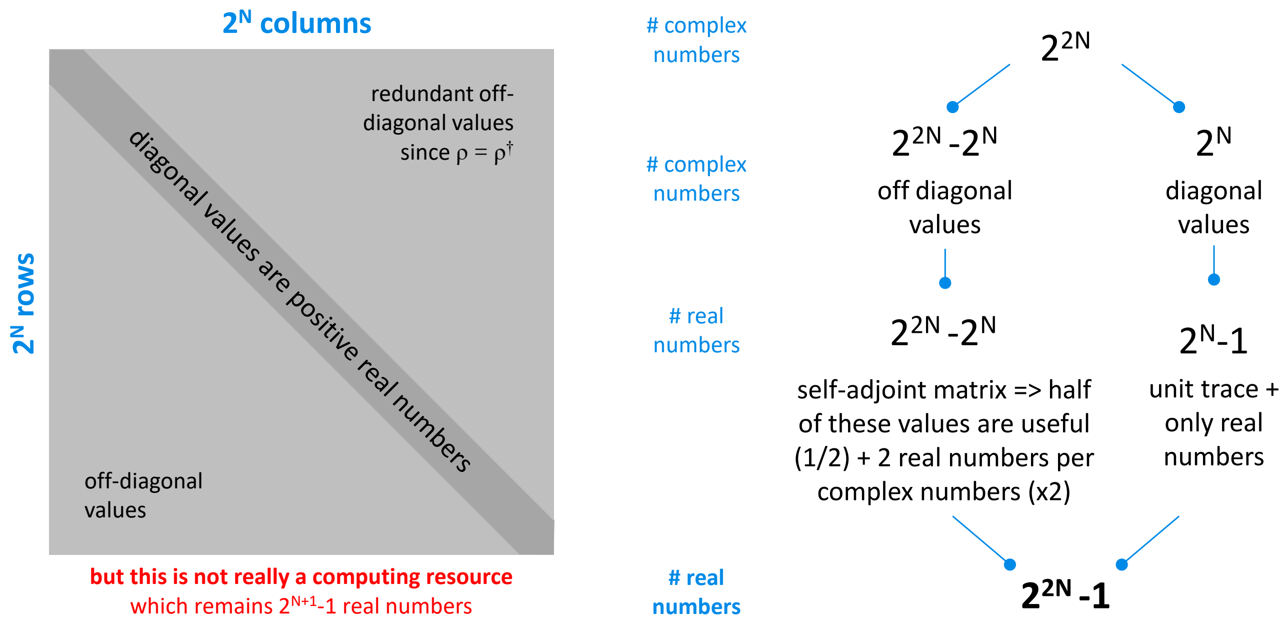

Figure 161: computing the dimensionality of a density matrix. (cc) Olivier Ezratty, 2021.

However, this dimensionality does not correspond to some useful computing resource in standard gate-based programming models although some work has been done to exploit it, but with no additional computing acceleration[399].

A theoretical perfect gate-based quantum computer is using qubits registers that are in a pure state until measurement, representing thus a dimensionality of 2^{N+1}-1 real numbers, the -1 standing for the normalization constraint of the computational basis vector[400]. So why do we care about these density matrices for mixed states? These are mostly used to understand the effects of decoherence and measurement and with qubits registers tomography which helps determine their fidelities.

The sequence of quantum gates in a quantum circuit can also be represented by a large unitary matrix of dimension $2^N*2^N=2^{2N}$ complex numbers. So, with a dimensionality close to a density matrix. But this is not an actual computing resource. It deals more with the extensive computing resources required to emulate in-memory an entire unitary algorithm in a classical computer instead of just executing gates one by one on the computational state vector (Figure 162).

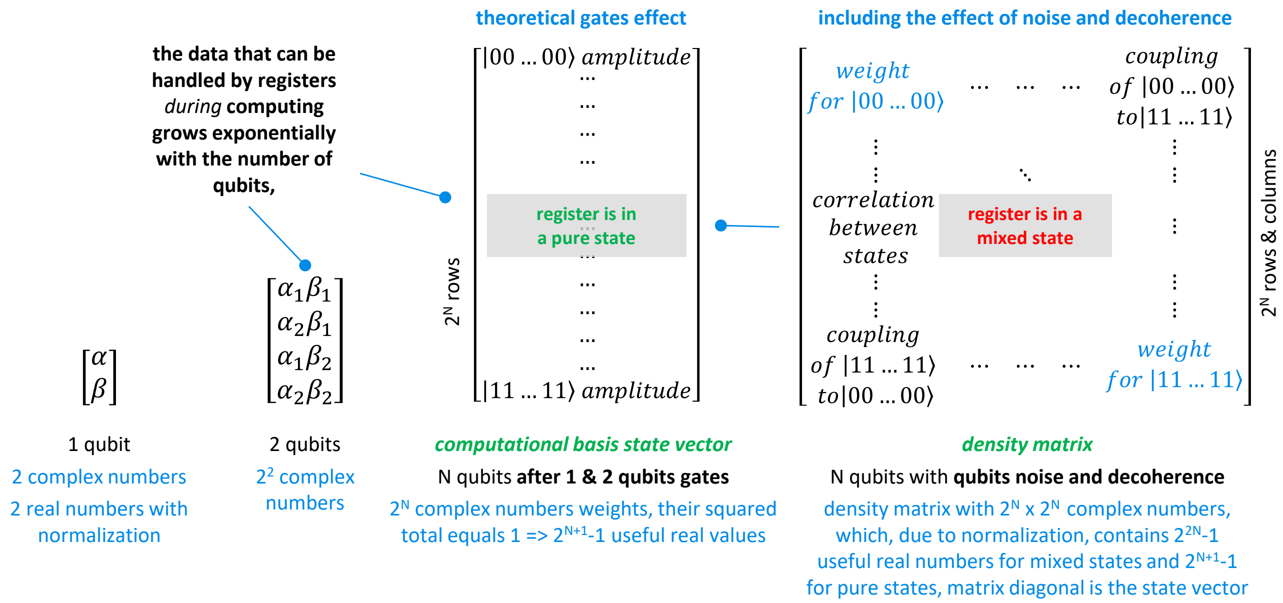

Figure 162: dimensionality of a qubit register. (cc) Olivier Ezratty, 2022.

There are many other subtleties with density matrices that we can't detail in the book. For example:

Diagonalization is possible for any mixed state density matrix. It will decompose the state into classical probabilistic combination of pure states eigenvectors forming an orthonormal basis.

Reduced density matrices are the density matrices of subsystems of composite systems. The reduced density matrix for an entangled pure state is a mixed state or mixed ensemble.

Mixed state purification consists, inversely, in integrating a mixed state in a larger system to create or reconstruct a pure state. It is used in some error-correcting codes.

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[399] See Quantum Circuits with Mixed States by Dorit Aharonov, Alexis Kitaev and Noam Nissam, 1998 (20 pages). It describes a model using not only unitary matrix operator-based quantum gates. It enables the usage of subroutines in programming. But this programming model doesn't seem adopted so far except for quantum error correction codes which implement measurement during computing. Mixed states based programming is implemented in the qGCL extension of the language pGCL as described in Quantum programming with mixed states by Paolo Zuliani, 2005 (14 pages).

[400] Thus, wrong is the statement that "*A calculation using n number of qubits on a quantum computer would need 2^n classical bits on a standard computer*" as seen in Simulating subatomic physics on a quantum computer by Sarah Charley, October 2020. Why? Because one of the 2^N quantum amplitudes in a N qubit register cannot be stored or emulated on a single bit!

Bipartite pure states are tensor products of two systems that are not entangled. A pure state system is entangled if and only if some of its reduced states are mixed rather than pure. If all were pure, it would mean that the pure state density matrix ρ would be separable into several pure states, one for each qubit in the case of a qubits register.

Schmidt decompositions are used to decompose bipartite systems and evaluate their level of entanglement. This level of entanglement can be determined with the Schmidt coefficients coming from the Schmidt decomposition.

Matrix rank. A matrix rank is the number of non-zero values in its diagonalized version. The rank of a density matrix gives an indication of the purity of the state it represents. A pure state density matrix has a rank 1, since it can be diagonalized into a matrix where only one value in the diagonal is non-zero. A maximally mixed state has a rank of 2^N, i.e., the number or lines and columns in the density matrix representing N qubits.

Schmidt rank is an indication of the level of entanglement in a density matrix. Not to be confused with the matrix rank which deals with its purity level.

Quantum Channels are transformations of a quantum state resulting from any kind of interaction with a quantum environment. They are modelized with an operator, called a superoperator, transforming a density matrix into another density matrix. Technically speaking, a superoperator is a completely positive (we've defined that already) and trace-preserving operator (self-explainable), or CPTP. Its form is a linear map from one Hilbert space to another Hilbert space. Its dimension is a square matrix with 2^{2N} columns and as many rows, so with 2^{4N} (or 16^N) complex numbers, before normalization, N being the number of qubits. It is useful to modelize quantum subsystems (which are in mixed state), decoherence, quantum error correction and qubits noise[401]. It is even possible to build a tomography with a superoperator, aka a quantum process tomography (QPT). One for example can build a QPT of a quantum gate to detect its imperfections. A QPT can also been done for a more complex operation, or unitary applied to a set of qubits, like a Quantum Fourier Transform[402].

Grad, curls and divs

In the equations of Maxwell, Schrödinger, Dirac, and others that we have seen are used notations good to remember here around the symbol nabla: $\boldsymbol{\nabla}$, sometimes used with an arrow $\vec{\boldsymbol{\nabla}}$ (Figure 163).

Nabla generally designates the gradient of a scalar or vector function, i.e. its first derivative. A scalar function applies to a vector, often of three dimensions x, y and z of a Euclidean space. It returns a number. A vector function returns a vector! This leads to the notions of **gradient** and **Laplacian** which apply to a scalar function and correspond to first and second derivatives in space, and to divergence and rotational (or curl) which apply to a vector function. A Laplacian can also be applied to a vector function. We won't go far in this book with respect to these functions.

scalar field => vector field

$$\boldsymbol{\nabla} = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

del or **nabla**: first space derivative of a vector

scalar field => vector field

$$\boldsymbol{\nabla} f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$$

gradient: scalar field vector of a scalar function space variations

vector field => scalar field

$$\boldsymbol{\nabla}.\vec{G} = \left(\frac{\partial G_x}{\partial x}, \frac{\partial G_y}{\partial y}, \frac{\partial G_z}{\partial z}\right)$$

divergence: vector function, showing its local evolution

vector field => vector field

$$\boldsymbol{\nabla}\times\vec{G} = \left(\frac{\partial G_z}{\partial y} - \frac{\partial G_y}{\partial z}, \frac{\partial G_x}{\partial z} - \frac{\partial G_z}{\partial x}, \frac{\partial G_y}{\partial x} - \frac{\partial G_x}{\partial y}\right)$$

rotational or **curl** of a vector function G transforming a vector field in a vector field describing the field variation in space

[401] See Quantum Channels by Stéphane Attal (65 pages).

[402] See Quantum Process Tomography of the Quantum Fourier Transform by Yaakov S. Weinstein, Seth Lloyd et al, 2004 (45 pages).

scalar field => scalar field

$$\nabla^2 f = \nabla.\nabla f = \left(\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y^2}, \frac{\partial^2 f}{\partial z^2}\right)$$

scalar function **laplacian**

vector field => vector field

$$\nabla^2 \vec{G} = \left(\frac{\partial^2 G_x}{\partial x^2} + \frac{\partial^2 G_x}{\partial y^2} + \frac{\partial^2 G_x}{\partial z^2}, \frac{\partial^2 G_y}{\partial x^2} + \frac{\partial^2 G_y}{\partial y^2} + \frac{\partial^2 G_y}{\partial z^2}, \frac{\partial^2 G_z}{\partial x^2} + \frac{\partial^2 G_z}{\partial y^2} + \frac{\partial^2 G_z}{\partial z^2}\right)$$

vector function **laplacian**

Figure 163: del, nabla, gradient, divergence, rotational, curl, Laplacian. You won't need them in the rest of this book, sort of. This is just informative. (cc) Olivier Ezratty, 2023.

Permanent and determinant

This inventory would not be complete without describing an even stranger mathematical object: the **permanent** of a square **matrix** n*n, invented by Louis Cauchy in 1812. The formula in Figure 164 describes its content.

$$\text{per}(A) = \sum_{\sigma \in \mathfrak{S}} \prod_{i=1}^{n} a_{i,\sigma(i)}$$

Figure 164: a permanent.

The Π denotes a multiplication of values from the index matrix i and $\sigma(i)$. σ is a permutation function of integers between 1 and n, the dimension of the matrix (number of columns and rows). The sigma relates to the set of σ functions of the permutation group S_n (also called symmetrical group) which has a size of n! (factorial of n). The values $a_{i,\sigma(i)}$ are the cells of the coordinate matrix i and $\sigma(i)$.

Figure 165 shows what it gives with $n = 2$ and $n = 3$ knowing that beyond that, it becomes less readable.

$$\text{perm}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad + bc \qquad \text{perm}\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = aei + bfg + cdh + ceg + bdi + afh$$

Figure 165: computing the permanent of 2x2 and 3x3 matrices.

The permanent is therefore a real number resulting from n! (factorial of n) additions of multiplications of n values of the matrix. The permanents are notably used to evaluate matrices that represent graphs.

They are also used in the classical numerical simulation of boson sampling that we will describe in the section dedicated to photon qubits, page 538[403]. Contrary to a determinant calculation, in Figure 166, which can be simplified, a permanent calculation remains a classical intractable problem.

The **determinant of a matrix** is a variant of its permanent. $sgn(\sigma)$ is the sign of permutations, which is +1 if the number of permutations needed to create the permutation is even and -1 if it is odd. Olé!

$$\det(A) = \sum_{\sigma \in Sn} \left(\text{sgn}(\sigma) \prod_{i=1}^{n} a_{i,\sigma(i)}\right)$$

Figure 166: a determinant.

Figure 167 shows what it gives for n=3. Note that the group of permutations includes the permutation that does not change the order of the elements.

$$\det\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = aei + bfg + cdh - ceg - bdi - afh$$

Figure 167: computing the determinant of a 3x3 matrix.

Determinants have particular properties such as det(AB)=det(A).det(B)=det(B).det(A)=det(BA) which can facilitate the calculation of the determinant of a matrix if it can be factorized into several matrices. Also, the determinant of a matrix is the product of its eigenvalues.

[403] The calculation time of a permanent increases faster than an exponential of a fixed value (Mn) as soon as n becomes very large compared to M. So, for example, with M=2, 2n is much smaller than n! as soon as n is greater than 4. As the numerical simulation of the boson requires a determinant that depends on the size of the simulation, it is even more cumbersome to compute than an exponential problem.

So much for the definition of the basics of the linear algebra of quantum computing. I've skipped a lot of other definitions and rules of computation. It was a question of clarifying certain notions that are frequently used in the scientific literature on quantum computing and in many of the reference works cited in this book. What we have just seen may be useful for you to compare some of the scientific literature on quantum computing.

If you like mathematics, linear algebra and complexity, you can have some fun exploring type III factors algebra that describes the observables in relativistic quantum fields theory[404]! Classical quantum physics and computing is based on simplistic type I factors algebra. Simpler, but still complicated.

Fourier transforms

Since quantum physics deals a lot with wave-particle duality and particularly with waves, waves signals decomposition is a key mathematical tool. That's the role of a Fourier transform that we mentioned already when dealing with Heisenberg's indeterminacy principle. It is about mathematics but not linear algebra.

The Fourier Transform implements a mathematical decomposition of a function $f(x)$ into a function $\hat{f}(\xi)$ returning a complex number containing an amplitude and phase for single frequencies ξ. It is a more generic version of Fourier series which works with periodic signals. Fourier transform are Fourier series where the signal period can approach infinite.

It can be used for example to decompose a wave packet pulse signal that is concentrated in time. A Fourier transform usually operates in the time domain with x being a time in second and ξ a frequency in Hertz (Figure 168).

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi} dx$$

Figure 168: a Fourier transform in the time domain.

It can be decomposed in Figure 169 using Euler's formula with its real and complex parts separating the amplitude and phase of the Fourier transformed signal:

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) \cos(2\pi x \xi)\, dx - i \int_{-\infty}^{\infty} f(x)\, \sin(2\pi x \xi)\, dx$$

Figure 169: Fourier transform decomposed in real and complex part.

As shown in Figure 170, the inverse Fourier transforms that frequency decomposition function $\hat{f}(\xi)$ back into its original compound time domain signal $f(x)$.

$$f(x) = \int_{-\infty}^{\infty} \hat{f}(\xi) e^{2\pi i x \xi} d\xi$$

Figure 170: inverse Fourier transform.

All of this is easier to understand with examples like in Figure 171 decomposing a time domain signal into five frequencies constituents with their respective magnitude and (equal) phases.

Computing Fourier series and transforms is done in many ways:

Discrete-time Fourier Transform (DTFT) is a form of Fourier analysis that is applicable to a sequence of values. It is often used to analyze samples of a continuous function. The term discrete-time refers to the fact that the transform operates on discrete data, often samples whose interval has some units of time.

Discrete Fourier Transform (DFT) converts a finite sequence of equally spaced samples of the function into a same-length sequence of equally spaced samples of the Discrete-Time Fourier transform (DTFT). The samples are complex numbers coming from a DTFT.

404 See The Role of Type III Factors in Quantum Field Theory by Jakob Yngvason, 2004 (15 pages).

Fast Fourier Transform (FFT) computes the discrete Fourier transform (DFT) of a sequence, or its inverse (IDFT). It is an efficient variation of the DFT.

Quantum Fourier Transform (QFT) is a linear transformation applied on qubits. It is the quantum analogue of the DFT and reverse DFT. A QFT is a Discrete Fourier Transform applied to the data stored in the 2^n computational basis states of a n qubits register. The Quantum Fourier Transform, implements a DFT on the complex amplitudes of a quantum state. We cover it later page 875.

Fourier series were created by **Joseph Fourier** (1768-1830, French) as part of his work in the book “The Analytical Theory of Heat” published in 1822. Beforehand, he accompanied Napoleon Bonaparte in his 1798-1801 Egyptian expedition as a scientific advisor. He then became a Prefect for the Isère department, based in Grenoble. Afterwards, he also drove the young Jean-François Champollion to get interested in deciphering the Rosetta Stone.

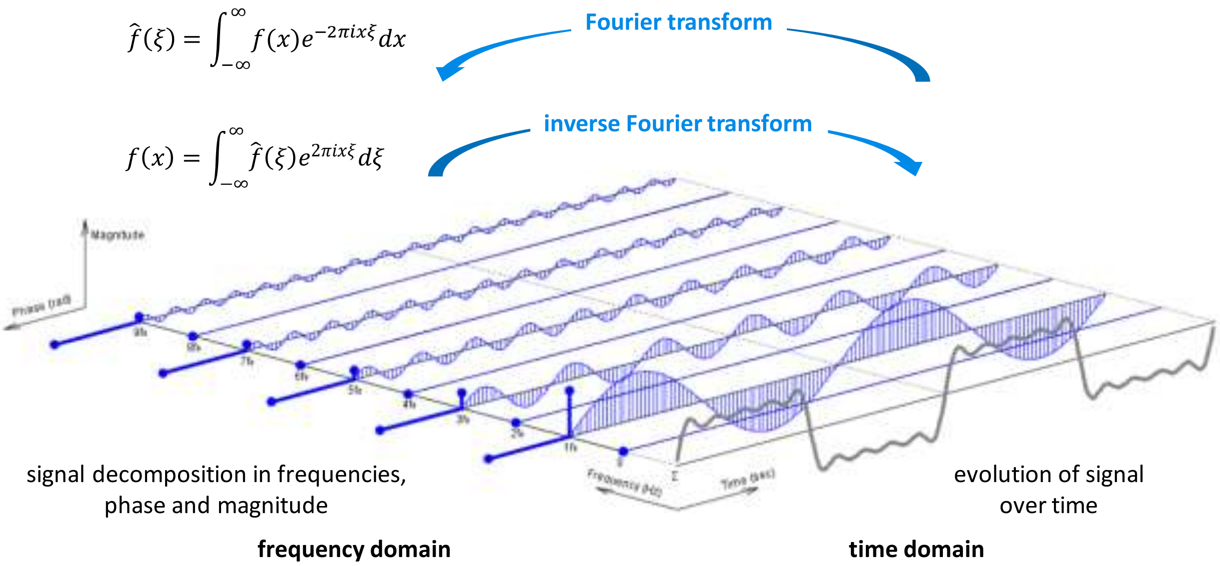

Figure 171: Fourier transform and inverse Fourier transform applied to signal decomposition and reconstruction. Source: https://www.tomasboril.cz/files/myprograms/screenshots/fourierseries3d.png, comments (cc) by Olivier Ezratty, 2021.

Lie groups

Lie groups are mathematical objects that are frequently encountered in quantum physics and quantum computing[405]. These combine the notions of a group and a smooth manifold. It was created by Sophus Lie in 1873. A typical Lie group is the SU(2) space which contains all unitary transformations applicable to a single qubit. These groups formalism is also used in quantum error correction codes. We won’t delve much into these concepts.

Group: set of elements with a binary operation (denoted as a multiplication) that combines any two elements to produce another element within the same set, with four properties: closure (the result of the operation is in the group), associativity (the order of operations doesn't matter), identity (there's an identity element that doesn't change other elements), and inverses (each element has a unique inverse that, when combined, generates the identity element).

Smooth Manifold: a manifold is a space that, when viewed locally, resembles Euclidean space (like flat space) but may have more complicated global topological structure. A smooth manifold is a type of manifold equipped with a smooth structure, meaning it is locally similar to Euclidean space and allows for smooth functions to be defined on it.

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405 See Berkeley Lectures on Lie Groups and Quantum Groups by Richard Borcherds, Mark Haiman, Theo Johnson-Freyd, Nicolai Reshetikhin, and Vera Serganova, December 2022 (384 pages).

Nonlinearities

We often hear about nonlinearities with quantum physics, particularly with the difficulty to implement it with qubits. It is also used in neural networks activation functions in classical computing. But their meaning is not the same in these different scenarios.

Superconducting qubits exploit the Josephson effect and an anharmonic oscillator to prevent the energy states of the superconducting loop oscillating current from being separated by the same energy level. This is a nonlinear effect linked to the way harmonic oscillators work when dampened in a certain way. It enables microwaves controls for changing qubits state between $|0\rangle$ and $|1\rangle$ with a larger frequency than the one that would allow a switch from the $|1\rangle$ state to the $|2\rangle$ state, which is what we are trying to avoid.

Nonlinearities are also sought after in photonics, especially to create quality two-photon quantum gates. Nonlinearities occur when solid media modify the characteristics of photons such as their polarization P and in a nonlinear way with respect to the electric field applied to the solid. The dominant chi $\chi^{(i)}$ of a nonlinear medium defines its order. A $\chi^{(3)}$ is a third order nonlinear medium.

$$P = \epsilon_0(\chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3 + \cdots) \text{ with } \epsilon_0 \text{ being the vacuum permittivity.}$$

This phenomenon happens in the Kerr effect which sees some materials refractive index changing in a nonlinear (quadratic, second order $\chi^{(2)}$ medium) manner as a function of the electric field applied to them. Conversely, the Pockels effect used in optical modulators sees the refraction changed in a linear manner as a function of the electric field applied. This nonlinearity in optics also occurs in many devices such as power lasers.

Finally, nonlinearities are classically used in neural networks activation functions. These are, for example, sigmoid based on exponential fractions.

So how can such activation functions be performed in quantum computation that relies only on linear algebra? One of the first imagined solutions consists in using a nonlinear, non-reversible and dissipative quantum gate called D [406]. Others consists in handling the nonlinearity part of algorithms in their classical parts before feeding a quantum algorithm. That's what can be done in algorithms solving Navier-Stokes fluid mechanics equations.

Qubits

Qubits are the basic elements of data manipulation in quantum computers. They are the quantum equivalents of classical computing bits. With them, we move from a deterministic to a probabilistic world but with the capability to handle more information during computing.

In conventional computing, bits used in processing units like microprocessors correspond to circulating electrical charges that reflect the passage or absence of an electrical current. A classical bit has a value of 1 if the current is flowing or 0 if the current is not flowing. The logic is transistors based. A bit readout gives 1 or 0 and deterministically, i.e., if the read operation is repeated several times, or the read operation is repeated after a re-edition of the calculation, it will yield the same result. This is true for data storage of information, for its transport and processing. This is valid, modulo the errors that can occur during this journey. These most often occur in storage and memory and are corrected via error correction systems using some data redundancy, usually with some parity bits for each stored byte, so with a rather low data overhead. In data storage, complicated redundancy systems are used like RAID disks organization mixing and matching several disks and parity error codes to consider the physical errors coming from storage.

[406] Method proposed by Sanjay Gupta in Quantum Neural Networks, 2001 (30 pages) and Quantum Algorithms for Deep Convolutional Neural Network, by Iordanis Kerenidis et al, 2020 (36 pages).

In a qubit, everything is different! While qubits are usually initialized at |0⟩, operations on them called quantum gates create a mathematical linear superposition between states |0⟩ and |1⟩. These two states correspond to two different discrete possible values of a physical property of a quantum object like an electron spin (up or down, in a given direction), a photon polarization or an atom energy level. Qubits are represented mathematically by a vector in a two-dimensional Hilbert space which describes its amplitude and phase, reminding us of the "wave" nature of quantum objects.

We'll see later how we use the Bloch sphere geometrical representation to understand how amplitude and phase are visualized. And it gets more complicated when we conditionally connect qubits together using multi-qubits quantum gates implementing quantum entanglement.

At the end of computing, we read the value of a qubit. Like all quantum object measurements, it results in a wave packet collapse onto one of the two qubit basis states. So, we get a |0⟩ or a |1⟩ and the result is probabistic, not deterministic. The wealth of information handled by a qubit during computing is lost at the end of calculation.

		bits: 0 or 1		qubits: 0 and 1
states	mathematical bit	2 possible exclusive states, 0 or 1	mathematical qubit	linear combination of |0⟩ and |1⟩
initialization		0 or 1		|0⟩ basis state
dimensionality		1 binary digit => two possible values		2 real numbers => one point on Bloch sphere
modifications		logic decision tables, irreversible		reversible unitary transformations
readout		0 ou 1, deterministic		0 or 1, probabilistic
errors		no error, perfect mathematical object		no error, perfect mathematical object
physical implementation	physical bit	current/no current in a logic electronic circuit and two states memory device bit	physical qubit	quantum object with two exclusive states for one property
computing vs memory		separate devices and parts of processors		all done in qubits and with quantum gates
computing operations		transistors based logic		amplitude and phase change + entanglement
errors sources		cosmic rays, transistors leakage		decoherence, thermal, electromagnetic, radioactivity…
error levels		2.5×10^{-11} bit per hour error rate		usually ≫ 0,1% with qubit gates and readout
information redundancy	logical bit	parity bits	logical qubit	large number of physical per logical qubits, 10^2 to 10^6
error correction		ECC (memory), MCA (CPU), RAID (storage)		quantum error correction codes
error level after correction		negligible		under an acceptable threshold for fault tolerance

(cc) Olivier Ezratty, September 2021

Figure 172: detailed comparison between classical bits and qubits with separating the mathematical logic, the physical implementation and error correction techniques. (cc) Olivier Ezratty, 2021.

The role of a quantum algorithm is to leverage this wealth of information during computing so that a simple result is generated at the end. We turn this probabilistic outcome into a deterministic one with executing the algorithm a great number of times, up to thousand times, and averaging the obtained results. It is also dependent on the structure of quantum algorithms which are designed to generate a result with qubits being as close as possible to their so-called "computational basis states", namely, |0⟩ and |1⟩.

To sort things out, it is still useful to differentiate three levels of 'qubit objects' used in computing as described in Figure 172:

Mathematically. Bits and qubits are idealized mathematical objects that implement a pure mathematical formalism with no errors. What is named a "qubit" is above all a mathematical object. Its dimensionality is different than with a bit. It is represented by two complex numbers, the amplitudes α and β from the qubit quantum state description α|0⟩+β|1⟩. Due to normalization ($\alpha^2+\beta^2=1$) and getting rid of the qubit global phase, its dimensionality becomes two real numbers, usually represented by two angles in the Bloch sphere. Bits and qubit measurement are both mathematical and physical operations. With qubits, it is mathematically based on a projective measurement on the computational basis comprised of $|0\rangle$ and $|1\rangle$, using a Hermitian matrix. Physically, it is using a measurement apparatus operating on the qubit quantum object.

Physically. Bits and qubits are implemented with different sorts of physical devices. With classical bits, we use to say they correspond to currents circulating or not circulating in transistor-based devices. While this is true with processing, it is different with memory and storage[407]. Qubits are implemented with quantum systems comprised of a single quantum object (atom, electron, photon) or several quantum objects (particularly with superconducting qubits and topological matter qubits like Majorana fermions). The $|0\rangle$ or a $|1\rangle$ states correspond to two exclusive states for one given property of a quantum object or system, that is clearly separable at measurement, like a photon polarization that is detected with a polarizer and a photon detector or an electron spin that can be detected with some magnetic sensor and a technique called electron spin resonance (ESR). These are also called two-level systems (TLS).

Physical qubits processing is using physical operations: **amplitude and phase changes** implemented by single-qubit gates and provoking **superposition,** and multiple-qubit gates generating **entanglement** which connects qubits together, **interferences** resulting from the previous operations and are at the core of most quantum algorithms, and **quantum measurement** yielding $|0\rangle$ or $|1\rangle$ for each qubit when computing has ended or when executing quantum error correction codes. Both bits and qubit physical objects are prone to physical errors. While error rates are very small with classical bits, it is currently quite high with qubits.

One simple operation like a two qubits quantum gate can generate between 0.3% and 4% error rates, depending on the qubit type and vendor, which is unacceptable for most algorithms.

Qubits errors come from the various interactions between the qubit quantum objects and their environment like thermal noise, electro-magnetic noise, cosmic rays, and gravity[408]. These errors require quantum error correction codes, which, as we'll later see, require a significant overhead of physical qubits. In NISQ quantum computers, these errors are also addressed with the quantum error mitigation technique that we'll describe later.

Logically. Error correction is thus required to create usable computing devices. In classical computing and telecommunications, "bits" are corrected with different techniques including using parity bits[409]. Bits are processed, stored, and transmitted with a very low level of errors.

Qubits must be assembled in groups called logical qubits, which are physical assemblies of a much great number of physical qubits, up to 10,000's[410]. Redundancy overhead becomes much bigger than with parity bits used in classical computing. In logical qubits, physical qubits are processed with quantum error correcting codes. The number of physical qubits assembled into logical qubits depends on their physical error rate and on the logical qubit error rate that is expected to enable practical quantum computing. For example, the famous integer factoring Shor algorithm is very demanding since using very precise small angles phase rotation gates.

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[407] These rely on electronic systems storing information like some magnetic encoding in hard disks drives or with two states transistor-based objects in SRAM (used in processors), DRAM (used around processors) or Flash memory (used in SSD and your usual USB memory key).

[408] It explains why many qubit types requires some sort of isolation: vacuum and low temperature to avoid thermal and electro-magnetic noise and multi-layered shielding to avoid other sources of electromagnetic noise. But we'll see later that for superconducting and electron spin qubits, the required low temperature is also linked to the microwaves used to control qubits.

[409] ECC (error correcting codes) are used in memories. Some systems are used in processors like the Intel MCA (Machine Check Architecture) which detects and reports errors in microprocessor. Other systems correct errors in storage like RAID redundancy for hard-disk drives and SSDs. We also have error correction codes used in classical telecoms.

[410] As of 2021, there are no commercial computers using real logical qubits. The reason is simple: the number of available physical qubits in gate-based processing units, topping at 127 with IBM's last generation of superconducting qubits, is still *under* the number of physical qubits required to build just one logical qubit!

While qubits are everywhere in quantum computing, these are not the only quantum objects available to manage quantum information.

Quantum computers can also theoretically be built with **qutrits** (with three possible quantum states), **ququarts** (with four possible states) and more generically, with **qudits** (d being the number of possible quantum states of the qubit underlying quantum system) [411]. It can deliver some computing power with a smaller number of quantum objects than with qubits (Figure 173). These are still mostly research labs tools. For example, researchers at Berkeley and at Rigetti are investigating superconducting qudits with more than two levels[412]. Complex trade-offs must be examined to look at the cost/benefits of qudits, between the increased data space and the effects of noise[413].

The most common qudits are implemented with photons by managing several of their properties.

Using qudits would have an impact on quantum algorithms design and programming. Most quantum algorithms are designed for quantum computers using qubit-based gates. However, compilers could probably automatically transform classical quantum gates into qudits-based gates.

The record so far is about creating quvigints, qudits with 20 different exclusive values for photons, that are efficiently measured with state tomography[414].

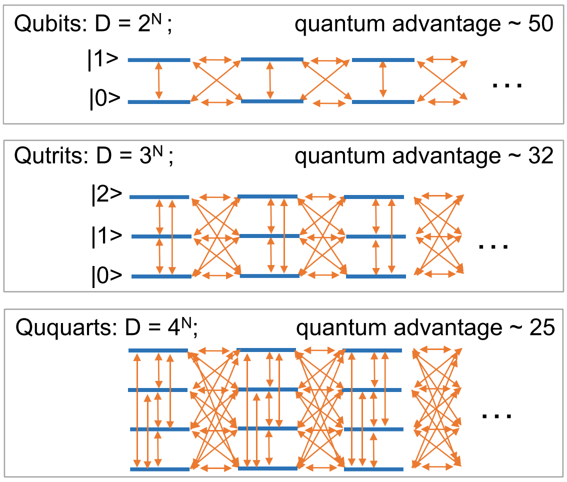

Figure 173: qubits, qutrits and ququarts. Source: Quantum Simulations with Superconducting Qubits by Irfan Siddiqi, 2019 (66 slides).

Bloch sphere

Let's first dig into the mathematical models of qubit representation. These models do not depend on the qubits underlying quantum object types. Physical qubit types have an impact on their error level and types as well as on the low-level quantum gates operations available to control qubits.

In a classical probabilistic model, a probabilistic pbit would have a probability p of having the value 0 and 1-p of having the value 1[415] (Figure 174). It would be a linear probabilistic model. We cover the niche market of probabilistic computers in a dedicated section, page 693.

Well, with qubits, these probabilistic laws are quite different!

[411] See for example Ultracold polar molecules as qudits by JM Hutson et al, 2020 which deals with qudits using fluorine-calcium and rubidium-cesium diatomic molecules allowing four quantum levels per molecule. This reduces the number of necessary qubits of $\log_2(d)$, d being the number of state levels of the qubits.

[412] See Quantum Simulations with Superconducting Qubits by Irfan Siddiqi, 2019 (66 slides).

[413] See Noisy Qudit vs Multiple Qubits : Conditions on Gate Efficiency by Denis Janković et al, February-November 2023 (10 pages).

[414] See Finding quvigints in a quantum treasure map by University of Queensland, March 2021 and Robust and Efficient High-Dimensional Quantum State Tomography by Markus Rambach et al, March 2021 (6 pages).

[415] Linear probabilistic models are used in the probabilistic processors discussed in a small dedicated chapter of this book.

A qubit vector state is defined by two complex numbers α and β according to the formula describing the qubit quantum object state |Ψ⟩ as α|0⟩+β|1⟩. Quantumly speaking, |Ψ⟩ is a linear superposition of basis states |0⟩ and |1⟩ with coefficients α and β, aka amplitudes. α is a complex number whose square module describes the probability of having the state |0⟩ and β is a complex number whose square module describes the probability of having the state |1⟩.

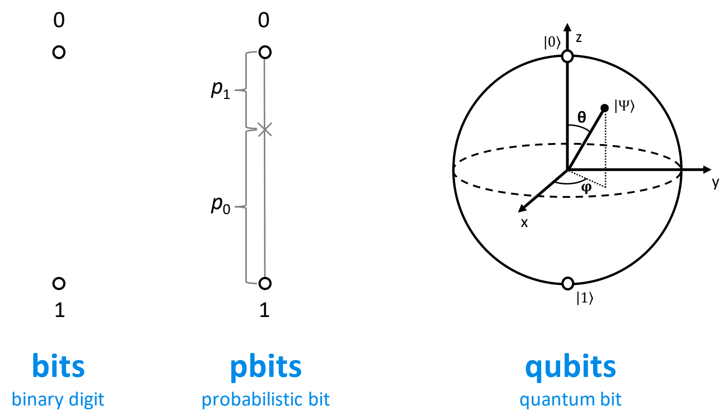

Figure 174: bits, probabilistic bits and qubits. 2020.

The sum of the probabilities of the two basis states must give 1. It is indeed not α+β but $|\alpha|^2+|\beta|^2$ that gives 1, the bars around α and β corresponding to their norm. If α=a+ib, then its norm is $\sqrt{a^2+b^2}$.

It comes from the generic probabilistic model developed by **Max Born** in 1926 and from one of the postulates of quantum physics. It gives to the square of the modulus of the wave function of a quantum the meaning of a probability density of the presence of an elementary particle in space (mostly, for electrons).

The mathematical representation model of the state of a qubit is based on complex numbers and on the geometrical metaphor of the famous **Bloch sphere**. This model is linked to the representation of the state of a qubit or any two-state quantum by a two-dimensional vector whose length, called "norm", is always 1.

Angles. The qubit state |0⟩ is a length 1 vector going from the center of the sphere to the North pole of the sphere and the state |1⟩ is a vector going from the center of the sphere to its South pole. An arbitrary qubit state $|\Psi\rangle$ is represented by a vector with an angle θ (0 to π, latitude) with respect to the vertical z-axis and an angle φ (0 to 2π, longitude) with respect to the x-axis located from the center of the sphere to its equator and around the z-axis. θ corresponds to the qubit amplitude and φ to its phase.

Orthogonality. The basis states |0⟩ and |1⟩ are opposite in the Bloch sphere and are mathematically orthogonal. This is highly counter-intuitive and linked to the angle θ that is divided by two in the formulae. When θ equals π, corresponding to a half turn in the sphere, moving from |0⟩ to |1⟩, $\cos(\theta/2) = \cos(90°) = 0$, illustrating that |0⟩ and |1⟩ are mathematically orthogonal states. This is true for any opposing states within the sphere as with the $|\Psi\rangle$ and $|\Psi'\rangle$ examples as shown in Figure 175.

These opposite states are antiparallel or antipodal, meaning parallel but in opposite directions. It explains why angle θ is halved in the equations describing a quantum state in Bloch sphere in the sine and cosine calculations of the formulas giving α and β [416]!

So, we divide θ by 2 to link the geometric representation in the sphere with the mathematical representation of the qubit state, and to allow a spreading of all the states of a qubit over the whole sphere. The whole sphere occupation of qubits representations makes it easier to describe how single qubit gates work as we'll show later in a graphical way.

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[416] This is deciphered in [Ian Glendinning's The Bloch Sphere](), 2005 (33 slides) which explains this by the mathematical orthogonality of the two states |0⟩ and |1⟩ which are nevertheless opposed in the Bloch sphere. It is even better explained in [Why is theta/2 used for a Bloch sphere instead of theta?]() which definitely clears up this mystery.

By the way, sin(θ) is a marker of the qubit coherence or level of superposition. It is easy to grasp since the sinus will be equal to zero when the qubit is in the |0⟩ and |1⟩ states. It will be maximal, at 1, when the qubit vector will sit on the equator in the Bloch sphere with an even superposition of |0⟩ and |1⟩.

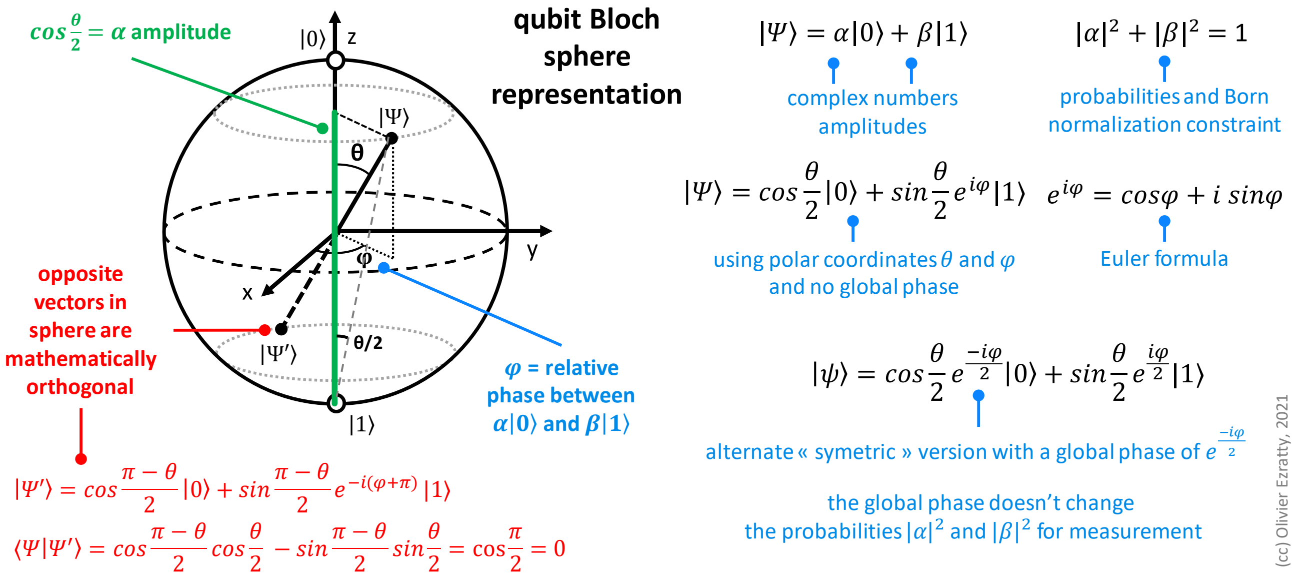

Figure 175: a thorough explanation of the Bloch sphere representation of qubits. (cc) Olivier Ezratty, 2021.

Global phase. A qubit representation is usually independent of its global phase. It can be removed from the equation to turn α into a real number. Still, a qubit is sometimes represented with a global phase of $\frac{-i\varphi}{2}$ as shown in Figure 175. When removing the global phase from α, the complex part of β integrates the phase difference between the amplitudes α and β. In that case, β is a complex number when the qubit is not in the plane crossing the x-axis (θ = 0) and the z-axis (φ = 0) of the Bloch sphere, meaning it has a non-zero phase. This complex number associates a real part for the direction z and a complex part for the dimensions x and y which are orthogonal to z. Applying a rotation around the z-axis will generally reintroduce a complex number in the α of the transformed qubit, which we do not necessarily factorize to remove the global phase of the qubit when doing hand calculations.

Information. The paradox to be understood is the following: since there is an infinite number of positions in Bloch's sphere, a single qubit could theoretically store a large amount of information, at least much more than a bit. Let's say it could be two floating point numbers, like the two angles θ and φ in the Bloch sphere.

Unfortunately, we can only obtain a classical 0 or 1 after measurement because of the Holevo theorem[417]! We could theoretically retrieve some floating-point numbers by averaging the results of a large number of runs of the quantum circuit. Their precision will depend on several factors: the number of runs or "shots", the qubit error rates and the efficiency of quantum error correction codes. Given the overhead of all of this, forget about using qubits as a high-precision floating-point number storage device!

[417] To learn more and with a better scientific accuracy, you can consult the Wikipedia sheet of the wave function and amplitude probability. Other explanations can be found in the example of the electron orbit levels in the hydrogen atom in Quantum Mechanics and the hydrogen atom (19 slides). The physical interpretation of Max Born's statistical rule remains in any case open, as explained in Arkady Bolotin's June 2018 paper, Quantum probabilities and the Born rule in the intuitionistic interpretation of quantum mechanics (14 pages).

When the qubit state vector is horizontal in the Bloch sphere, i.e., it sits in its equator, and we have an even superposition of $|0\rangle$ and $|1\rangle$, but with a variable relative phase between the $|0\rangle$ and $|1\rangle$ amplitudes which is related to the horizontal angle of the vector φ with respect to the z axis as in the diagram on the right. Two usually superposed states are $|+\rangle$ and $|-\rangle$. These are orthogonal states. These equatorial states share the same α component of $1/\sqrt{2}$ but opposite β values. This qubit-rich information is then modified by phase rotation quantum gates. If all qubits in the equator share the same 50%/50% amplitude probabilities, they have a different phase (Figure 176).

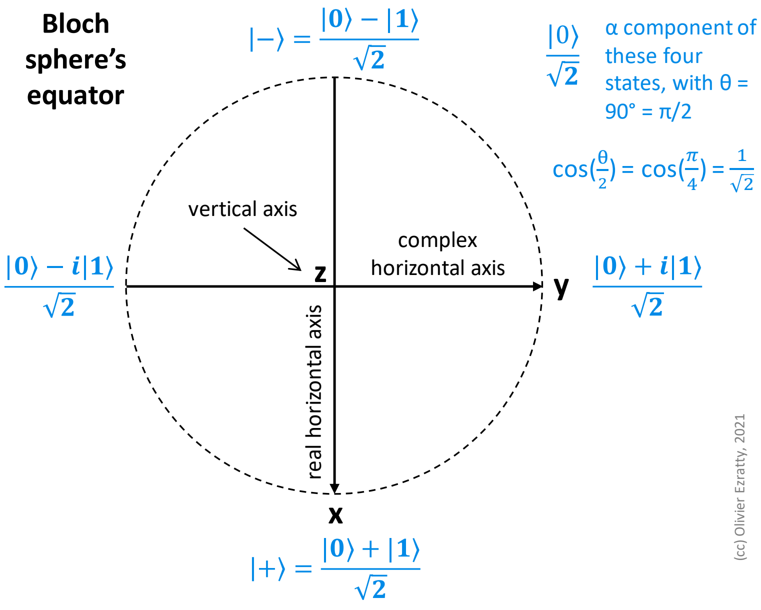

Figure 176: Bloch sphere equator and superposed states (cc) Olivier Ezratty, 2021.

A significant part of the quantum computing power comes with playing with the qubit phase that generates interferences between qubits. We'll see that later with algorithms such as phase amplitude and phase kickback.

As a general rule, most quantum gates do not generate all vector positions in the Bloch sphere. They are often half or quarter turns. The points of the sphere most often used are the cardinal points: the $|0\rangle$, $|1\rangle$, then the four points corresponding to the superposition of $|0\rangle$ and $|1\rangle$ on the equator.

To obtain all the quantum computing power, we need to make smaller turns than quarter turns, with the variable-phase R gates, usually composed with T gates, which we will see later and is outside the so-called Clifford gates group. Only these gates are supposed to enable some exponential speedup with gate-based quantum computing. Another way to look at this is that quantum advantage comes from using the full power of "analog" qubits.

Origins. We owe this Bloch sphere to three scientists: **Erwin Schrödinger** for his wave function of 1926, **Max Born** for his associated probabilistic model, created the same year, and to **Felix Bloch** (1903-1983, Switzerland) who represented the state of a two-level quantum on the sphere in 1946.

Bloch's sphere is frequently assimilated to **Poincaré's sphere**, named after **Henri Poincaré** (1854-1912, France) and created in 1892[418]. It is used to describe the polarization of light (Figure 177, *left*). The sphere polar coordinates represent the various types of light polarization: linear polarization (on the sphere equator), left elliptical polarization (upper hemisphere), right elliptical polarization (lower hemisphere) then left and right circular polarization (North and South poles). The vertical axis (circular polarization) and one of the horizontal axes (linear polarization) represent two observables for a photon. All other states can be described as linear superpositions of these couples of basis states. And contrarily to massive particle-based quantum objects whose quantum probabilities are described by Schrödinger's equation, light equations used here are just Maxwell's electro-magnetic waves equations.

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[418] Here are some sources of information associated with this section: Lectures on Quantum Computing by Dan C. Marinescu and Gabriela M. Marinescu, 2003 (274 pages), The Bloch Sphere by Ian Glendinning, 2005 (33 slides), The statistical interpretation of quantum mechanics, Max Born's 1954 Nobel Prize acceptance speech in physics (12 pages) and the excellent book The mathematics of quantum mechanics by Martin Laforest, 2015 (111 pages), which describes the mathematical basics of quantum computing with complex numbers, vectors, matrices and everything.

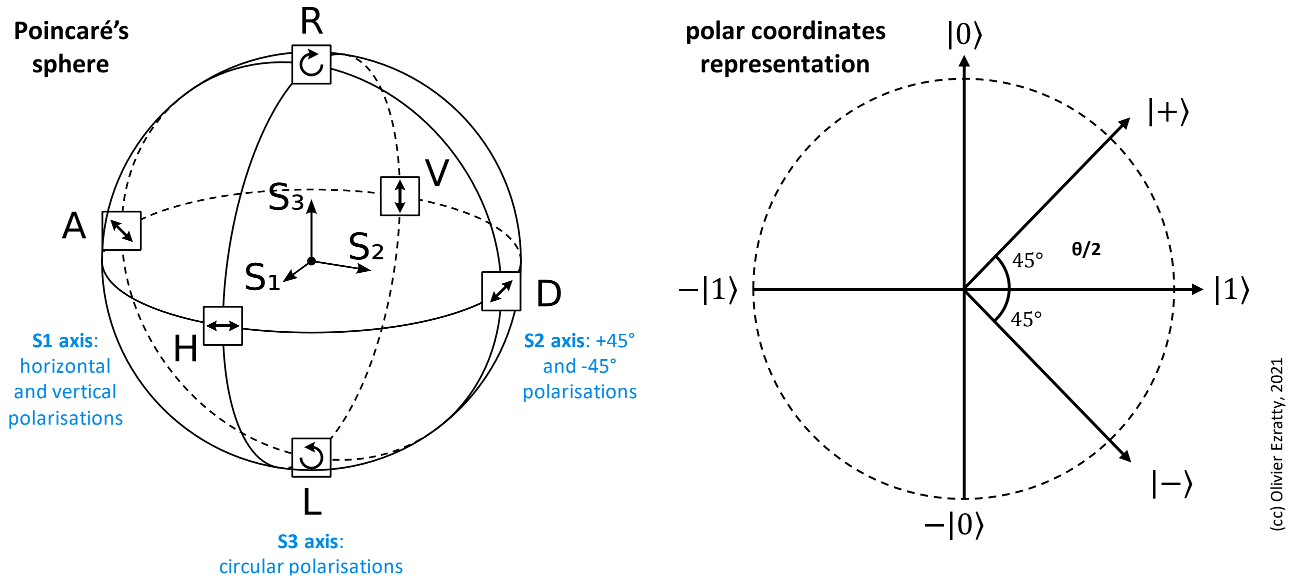

Figure 177: the Poincaré photon sphere which inspired the Bloch sphere creation and another, Euclidian, representation of a qubit.

The Bloch sphere representation is also used for representing an electron spin measured along three orthogonal axes (X, Y, Z), showing how superposition works with spins. Interestingly, the Bloch sphere corresponds to some 3D physics reality for spins but not, for example, for superconducting qubits. In that case, it is a mathematical representation of the qubit phase and amplitude that has no physical 3D meaning.

Sometimes, a system of polar coordinates is used on one circle, positioning the computational basis states of $|0\rangle$ and $|1\rangle$ as geometrically orthogonal vectors. It somewhat duplicates values since of $-|0\rangle$ and $-|1\rangle$ are similar to $|0\rangle$ and $|1\rangle$, with just a different global phase. Only the right half of the circle is useful (Figure 177, *right*).

Many other fancy qubits representations have been created with projection of the Bloch sphere onto a plane, representations of several qubit states with a single or several Bloch spheres, even some representation of quantum entanglement with three Bloch spheres for two qubits[419] or with tetrahedrons[420], torus[421] and other representations[422]. None of these have been standardized and have a practical value for most quantum developers.

Registers

In a quantum computer, qubits are organized in registers: a bit like the 32- or 64-bit registers of today's classical processors. One key difference is for now, a quantum computer has only one register and not many as with current classical microprocessors. But most other characteristics of a quantum and classical registers are different as shown in Figure 178.

The main difference between an n-qubit register and a traditional n-bit register is the amount of information that can be manipulated simultaneously. In conventional computers, 32- or 64-bit registers store integers or floating-point numbers on which elementary mathematical operations are performed.

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419 See Two-Qubit Bloch Sphere by Chu-Ryang Wie, 2020 (14 pages).

420 See Geometry of Qubits - A picture book by Yosi Avron and Oded Kenneth, 2018 (20 slides).

421 See Geometric Visualizations of Single and Entangled Qubits by Li-Heng Henry Chang et al, December 2022 (23 pages). One more try. Using a torus.

422 See Visualizing Entanglement in multi-Qubit Systems by Jonas Bley, May-October 2023 (22 pages).

A register of n qubits holds a vector in a 2^n dimensional space of complex numbers. Its dimensionality is exponentially larger than a n-bits register. Let's take for instance a register of 3 bits and 3 qubits. The first one will store one value at a time as 101 (5 in base 2) while the register of three qubits will contain complex numbers attached to each of the possible values of this register, 2 to the power of 3, i.e. 8, *aka* computational state basis. These complex numbers are the amplitude of each computational state. The total of their squares equals 1 since these are probabilities.

n bits register		n qubits register
2^n possible states once at a time	n=3 example	2^n possible states linearly superposed
evaluable		partially evaluable
independent copies		no copy
individually erasable		non individually erasable
non destructive readout		value changed after readout
deterministic		probabilistic

101

000
001
010
011
100
101
110
111

aka register pure states

Figure 178: key differences between a classical bit register and a qubit register. (cc) Olivier Ezratty, 2021.

However, these 2^n states amplitudes do not really constitute some information storage capacity. Quantum algorithm's main goal is to amplify the computational basis state amplitude that is the sought result, while reducing all the other amplitudes to near zero (Figure 180).

The output information is a set of n classical bits. The 2^n amplitudes handled during computation are not some useful information that we exploit outside the register. We'll always end with one computational state and its related classical bits. So, in the end, you don't really process "big data" with quantum computing or at least, you don't output any big data. You may still use some sort of big data to prepare the state of the register before or during calculation[423].

But it is not to the advantage of quantum computing since feeding a quantum register with classical data is quite slow[424].

The graphic representation in Figure 179 was built using the Quirk open source simulator. It is a sample of a quantum Fourier transform algorithm run on 4 qubits. The column numbers vector shows the computational base probabilities. In the beginning we have a 100% $|0000\rangle$.

After applying an X gate on the first qubit, we get a 100% amplitude for a $|1000\rangle$. After applying Hadamard gates to all qubits, we get even amplitudes of 6.3% for all computational basis states. Then the QFT finds out the result, $|1001\rangle$ which shows up on the last column[425].

423 However, exceptions are beginning to appear with hybrid methods for accelerating database access combining traditional computer-based and quantum algorithms. See Quantum computers tackle big data with machine learning by Sarah Olson, Purdue University, October 2018.

424 It's well explained in the excellent overview Quantum Computing: Progress and Prospects from the US Academy of Sciences, 2019 (272 pages) : "*Large data inputs cannot be loaded into a QC efficiently. While a quantum computer can use a small number of qubits to represent an exponentially larger amount of data, there is not currently a method to rapidly convert a large amount of classical data to a quantum state (this does not apply if the data can be generated algorithmically). For problems that require large inputs, the amount of time needed to create the input quantum state would typically dominate the computation time, and greatly reduce the quantum advantage.*".

425 In A quantum computer only needs one universe by Andrew Steane, 2003 (10 pages), the latter insists on the key role of entanglement. He considers that entanglement does not so much explain the gain in quantum computing power.

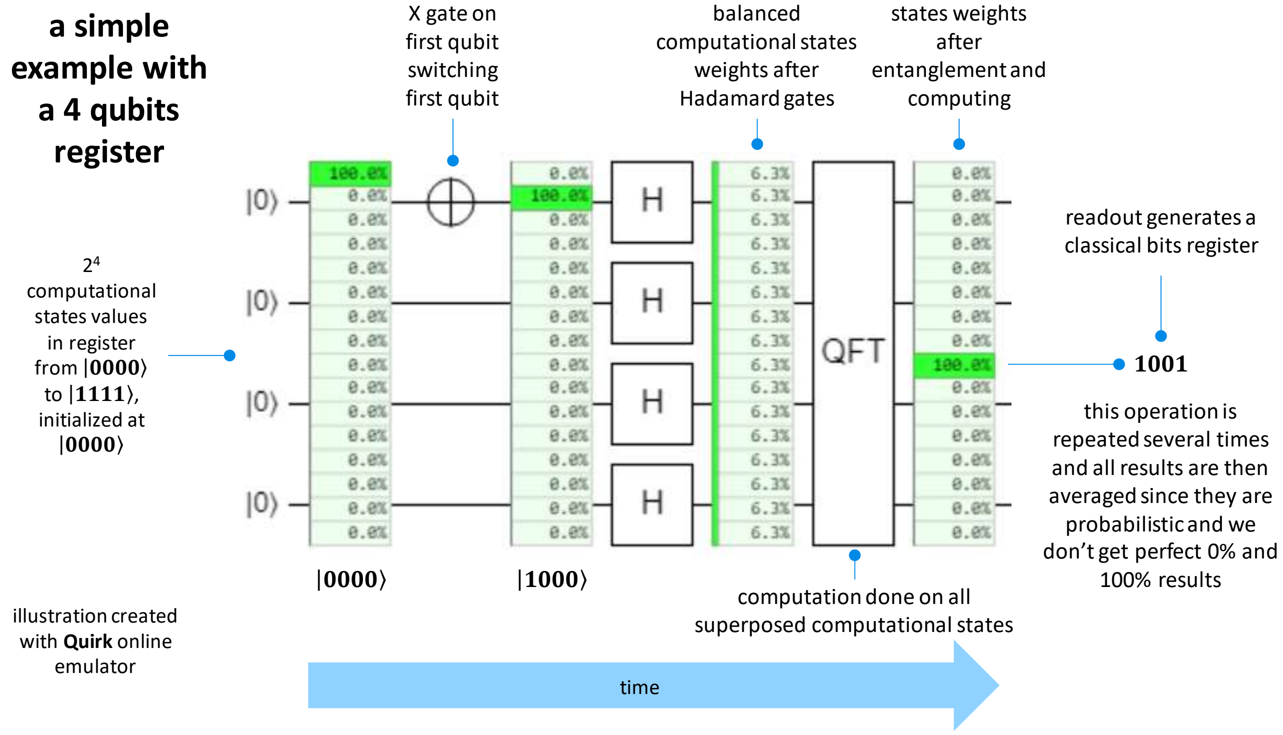

Figure 179: manipulating a 4-qubit register vector state with Quirk. (cc) Olivier Ezratty, 2021.

Another way of presenting things is a little simpler and more graphical: all the register states are on the left, the calculation generates interference between these states to make one of the states on the right come out which is the answer to the problem posed[426]. The example is based on the use of only two qubits that give four different "binary" states of the qubits. So, we do not recover 2^n values in practice, but n bits. The operation can be repeated several times to obtain an average in the form of floating numbers. But it depends on the algorithms. For most of them, a binary output is sufficient, as for Peter Shor's integer factorization algorithm.

We are anyway constrained by **Holevo's theorem** of 1973 which proves that with n qubits, we cannot recover more than n bits of information after a quantum calculation!

At the current stage of qubit development, single and two-qubit gates error rate sit between 0.1% and 1% when ideally it should be much smaller, either in NISQ regime or for implementing a fault-tolerant system. This error rate can be evaluated for each isolated qubit.

By the way, don't believe the nonsense that is the comparison of the exponential size of the qubit registers computational basis state with the number of particles in the Universe. These are not equivalent dimensions. A number of objects combination is not homothetic with a number of objects! With a given number of objects, the number of combinations of these objects will always represent a number that is much bigger than the number of objects taken as a reference. And… exponentially!

On the other hand, besides this exponential combination sizing, qubits have a lot of drawbacks in total opposition with classical bits. One can neither copy classically nor erase the value of qubits individually. Their measurement modifies their values. These are probabilistic objects that are difficult to manipulate.

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[426] See Introduction to Quantum Computing by William Oliver from MIT, December 2019 (21 slides).

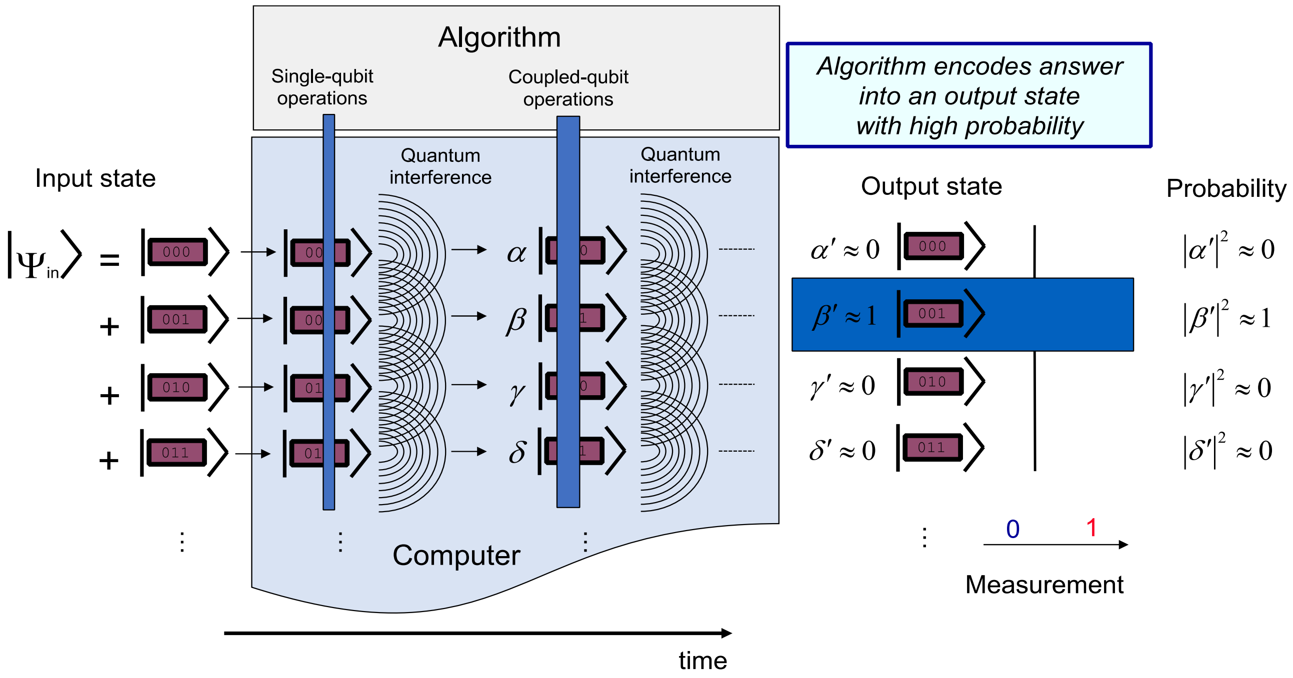

Figure 180: representing qubits manipulations with interferences.
Source: [Introduction to Quantum Computing](#) by William Oliver from MIT, December 2019 (21 slides).

Ancilla qubits. Universal gate quantum computing uses ancilla qubits or control qubits that can be combined with the computing qubits. The value of these qubits is not read at the end of the processing. It is a kind of trash can of qubits used during computations. They are used in various algorithms as well as to implement the error correction codes (QEC) explained later. We still always use a single qubit register. It can be just logically partitioned between computation qubits and ancilla qubits, these last playing the role of classical registers in a microprocessor. Their content may be scrapped at the end of some parts of computing. It is sometimes done using the "uncompute trick" which reverses part of the processing affecting these ancilla without erasing the other qubits containing the intermediate computing result.

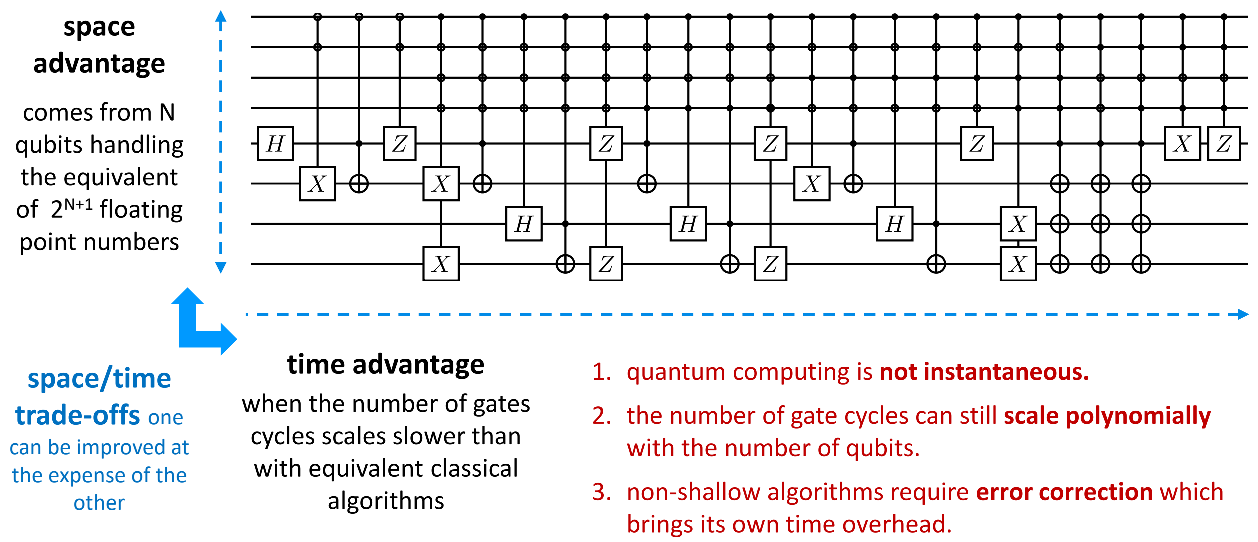

Figure 181: there's a significant difference between the space and time advantages of quantum computing. The data space of a qubit register does not guarantee a speed up in your algorithm when compared to classical algorithms. (cc) Olivier Ezratty, 2023.

Speed vs space advantage. Quantum computers potential speedup is frequently explained by the data space of registers with their 2^N amplitudes, with saying "*you can modify all these value in a single operation instead of 2^N operations in classical computing*". That is not true. Being able with a single gate to modify these amplitudes is independent from the number of such steps in your algorithm. As illustrated in Figure 181, there is indeed a difference between the space and time quantum advantages. The space exponential advantage does not warrant a time exponential speedup in your algorithm. It depends on the algorithm depth and the number of quantum gates to execute. Also, contrarily to common wisdom, quantum computing is not instantaneous. Execution time depends on many factors beyond the number of gates like the number of shots, particularly in the NISQ regime, the gate time which can be very long like with trapped ions and the cost of error mitigation and correction.

Parallelism in quantum computing is usually explained with the effects of superposition and entanglement which enables the efficient exploration of a large data space. It also comes from the fact that a single quantum gate can modify up to 2^N quantum amplitudes. But an interesting exponential quantum speedup is highly dependent on the number of these gates, which depends on the algorithm and gate types. The labyrinth metaphor is frequently proposed, explaining that quantum algorithms explore all paths in the labyrinth simultaneously to find a solution. But it is not adequate since it does not describe well the effects of superposition, entanglement, interference, and amplification during computing, nor the weird probabilistic effects of measurements at the end of an algorithm.

Gates

In classical computing, logic gates execute Boolean algebra using bit-dependent decision tables as an input. Several types of logic gates with one or two inputs are used, including the NAND gate which is interesting because it is universal and uses only two transistors. The other one- and two-bit Boolean gates can theoretically be created with NAND gates. In general, however, logic gates are mixed in the circuits (Figure 182, *left*).

An Intel Core i5/7 processor with over 10 billion transistors contains several billion logic gates. A processor is obviously very complex, with gates managing access to a cache memory and registers, and instruction pipeline executing the code defining the gates to be used in calculations. These operations are generated at the processor's clock frequency, most often expressed in GHz.

The classic two-bit logic gates (NAND, NOR, XOR, AND, OR) are irreversible because they destroy information during their execution and this information destruction generates heat!

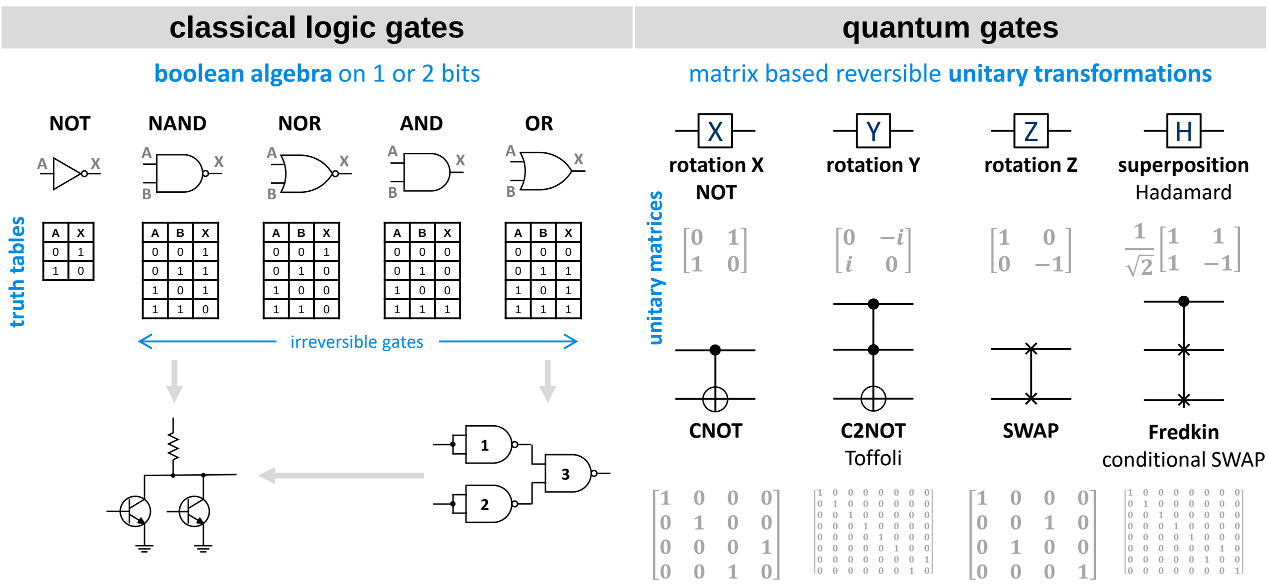

Figure 182: comparison between classical logic gates and qubit gates. (cc) Olivier Ezratty, 2021-2023.

Qubits undergo operations via quantum gates that can be applied to one or more qubits.

Single-qubit gates apply a 2x2 unitary matrices of complex numbers to the qubit state vector containing the famous α and β complex amplitudes. These always generate some rotation of the qubit vector in the Bloch sphere. The norm of the vector remains stable at 1 at least, before any decoherence happens. And quantum gates modify qubits information without reading it. A single qubit gate on a register of N qubits is a unitary operator, a large square matrix of 2^N lines and columns which results from the tensor product of the gate matrix applied to a qubit and the identity operator acting on all the other qubits, in the qubits order.

Two qubit gates apply 4x4 unitary matrices to the computational basis state vector containing 4 entries (2^2).

Three qubit gates apply an 8x8 matrix to a state vector containing $8=2^3$ entries.

We'll now look at the various quantum gates made available to quantum developers[427]. The variations come from the rotation axis in the Bloch sphere (usually, X, Y or Z) and the angle of the rotation (1/2 turn, 1/4 turn, 1/8 turn or arbitrary rotation angle)[428].

- **X** gate (or NOT) performs an inversion or bit flip. A $|0\rangle$ becomes $|1\rangle$ and vice-versa. Mathematically, it inverts the α and the β of the two-component vector that represents qubit state. It generates a 180° rotation in the Bloch sphere around the X axis.

  This gate is often used to initialize to $|1\rangle$ the state of a qubit at the beginning of a process which is by default initialized at $|0\rangle$. It has a derivative, the SX gate used in IBM QPUs, that corresponds to half an X gate. It is a sort of Hadamard rotation, but around the X axis instead of the Y axis.

  $$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

- **Y** gate performs a 180° rotation around the Y-axis in the Bloch sphere. It also turns a $|0\rangle$ into $|1\rangle$.

  $$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

- **Z** gate applies a sign change to the β component of the qubit vector (phase flip), i.e. a phase inversion and a 180° rotation with respect to the Z axis. The X, Y and Z gates complemented by the identity I are the **Pauli gates**.

  $$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

  They have several characteristics like $ZX = iY$ and $X^2 = Y^2 = Z^2 = I$. Their unitary matrices are noted σ_x, σ_y and σ_z. Any single qubit unitary transformation can be written as a linear combination of Pauli gates with real number coefficients, plus the identity I.

- **S** gate generates a phase change, or a quarter turn rotation around the Z-axis (vertical). This is the equivalent of a half Z-gate. It is also called a "phase gate".

  $$S = \begin{bmatrix} 1 & 0 \\ 0 & i \end{bmatrix}$$

- **T** gate equivalent to a half S, which generates a phase change of one eighth of a turn. With two of these gates, an S gate is generated. This gate that is not part of Clifford's group (defined ... later) has the particularity of allowing by approximation the creation of any rotation in Bloch's sphere.

  $$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{i\pi}{4}} \end{bmatrix}$$

  It is the key to universal gate-based quantum computing. It is indispensable to run a quantum Fourier transform and all derived algorithms like Shor integer factoring, HHL (linear algebra) and most quantum machine learning algorithms.

---

[427] Single qubit gates can be classified in XY and Z gates. XY gates are rotations around an axis in Bloch's sphere equator and can be viewed as amplitude change gates while Z gates are rotations around the Z axis and can be described as phase change gates.

[428] The formalism and classification of quantum gates is more sophisticated, as very well explained in the excellent lecture notes Gates, States, and Circuits - Notes on the circuit model of quantum computation by Gavin E. Crooks, January 2022 (79 pages).

- **R** phase shift gates are variations of Pauli gates, with an arbitrary rotation angle in the Bloch sphere. The R_z gate rotates around the z axis, R_x around the x axis and R_y around the y axis[429]. A R_z(angle) gate is also called a P_{angle} gate (P for phase).

$$R_m = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{2i\pi}{2^m}} \end{bmatrix}$$

  When the x, y and z axes are not specified, it is z, the vertical axis of the Bloch sphere, as in the *above* matrix. When x, y and z are specified without an angle or m, it is 90° or $\pi/2$. The rotation is carried out on a complete round divided by m. The R_z gates modify the phase of a qubit and not its amplitude. Thus, the measurement of its state $|0\rangle$ or $|1\rangle$ is not affected by this gate. It will return both $|0\rangle$ or $|1\rangle$ with the same proportions, before and after the use of an R_z gate. Only two points of a sphere do not move during a rotation around an axis connecting them.

- **H** gate, aka Hadamard-Walsh, turns a qubit at $|0\rangle$ or $|1\rangle$ in a superposed state "$|0\rangle$ and $|1\rangle$". It is fundamental to generate this superposition in the registers in most quantum algorithms. It is a rotation around the Y axis.

$$H = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

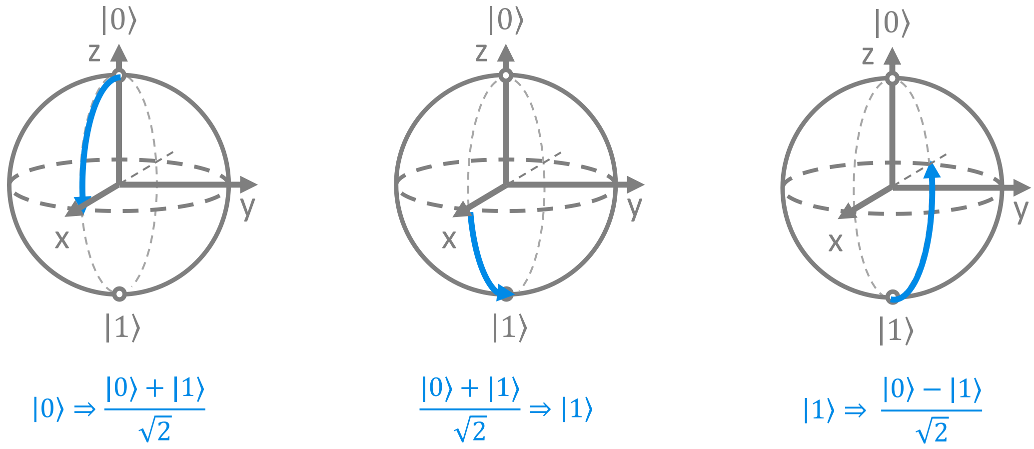

Figure 183: example of application of an Hadamard gate on |0⟩ or |1⟩ qubits (cc) Olivier Ezratty, 2023.

  It is often used to initialize a quantum register before executing an oracle-based algorithm like Grover or Simon algorithms. Figure 183 shows a representation of the effect of this gate on a qubit initialized at $|0\rangle$ or $|1\rangle$ and in-between. If we apply four Hadamard gates to a qubit, we return to the starting point. In other words: $HHHH = I$, with I being the identity operator[430].

- **I** gate is the identity gate. It may be used as a pause. In the real physical world, a real I gate is not an exact identity due to decoherence! If you "run" 20 identity gates on a $|1\rangle$ qubit, you'll end up having some amplitude flipping error transforming progressively the qubit into a $|0\rangle$.

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

- $|\mathbf{0}\rangle$ reset gate is sometimes indicated at the beginning of an algorithm to indicate that we start with initialized qubits. It is obviously irreversible.

$$|\mathbf{0}\rangle = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

The mathematical formalism applied to a single qubit simply illustrates this. But this works only in theory, only if the gate error rate is zero. Since it is not zero, you don't ever a perfect $|0\rangle$ or $|1\rangle$.

A qubit reset operation may also be used to clean up ancilla qubits after their usage, when we are not using the uncompute trick, which is a way to cleanly reset ancilla qubits and remove potential entanglements with other qubits.

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[429] This is well explained in The Prelude, Microsoft, 2017.

[430] This is also valid with X, Y and Z gates. In the usual notation, an H gate applied to $|0\rangle$ gives a state $|+\rangle$ and an H gate applied to $|1\rangle$ gives a state $|-\rangle$.

Figure 184 shows representations of the effect of these single qubit gates, also labelled unary gates, on qubits initialized in $|0\rangle$ for the gates H, X, Y, R_x and R_y and with $|+\rangle$ for the phase change gates S, T, Z and R_z. Indeed, phase shift gates have no effect on $|0\rangle$ as well as on $|1\rangle$. For $|1\rangle$, it may just change the qubit global phase, and not its relative phase between the qubit amplitudes α and β, with no material impact on most algorithms. In the examples, the R gates use an angle of 90° or $\pi/2$.

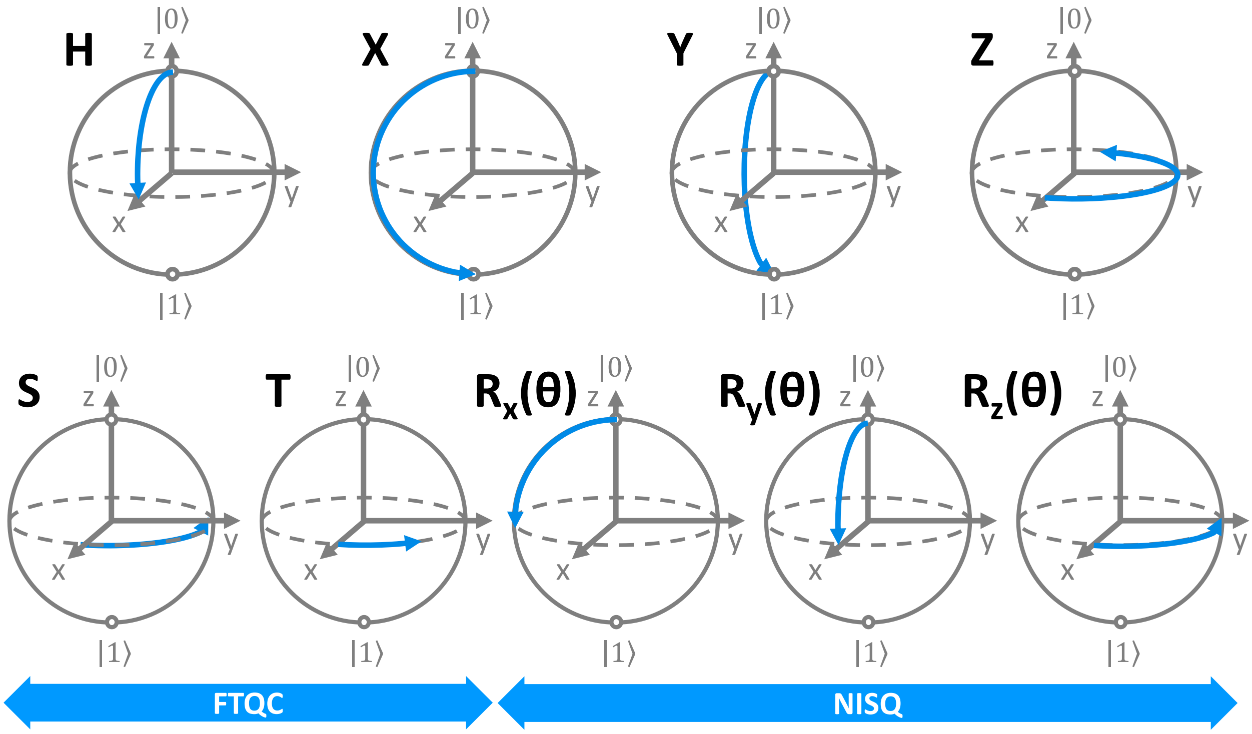

Figure 184: Bloch sphere representation of various single-qubit gates. T gates are used in FTQC models for the generation of arbitrary rotation gates while these arbitrary rotation gates and quantum error correction are used in NISQ without quantum error correction. (cc) Olivier Ezratty, 2021-2023.

We now cover two and three qubit gates. Apart from the SWAP gate, all these gates are conditional gates that apply a transformation of the state of one or two target qubits according to the state of one control qubit. These conditional gates create entanglement between the qubits that are in play. The entanglement between the involved qubits is persistent after executing these gates.

- **CNOT gate** is an inversion of the value of a qubit conditioned by the $|1\rangle$ value of another qubit. It is a quantum equivalent of the XOR gate in classical computing. Formerly called Feynman gate (C).
- **C2NOT** or **Toffoli gate** is an inversion of the value of a qubit conditioned by the $|1\rangle$ value of two other qubits.
- **CZ gate**, or Control-Z, is a conditional phase change Z gate.
- **CS gate**, or Control-S, allows a phase change of a qubit controlled by the state of a qubit.
- **SWAP gate** inverts the quantum values of two qubits (Figure 185). The SWAP gate is the only two-qubit gate that is not creating a new entanglement between the two qubits. If they were separable before the gate, they will still be separable afterwards. It can be generated from the chaining of three consecutive CNOT gates (Figure 194).

$$\text{SWAP} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Figure 185: the two-qubit SWAP gate unitary matrix.

The key role of SWAP gates is to connect qubits that are physically distant in the register physical layout. A SWAP gate may also displace some entanglement. For example, if qubits A and B are entangled, but C is not entangled with A and B, a SWAP between B and C will displace entanglement to A and C and leave B unentangled with A and C (Figure 186). SWAP is usually a costly gate. It is not used a lot when the qubit topology enables all to all qubits direct connections like with some trapped ions qubits. Therefore, most SWAP gates are created by compilers.

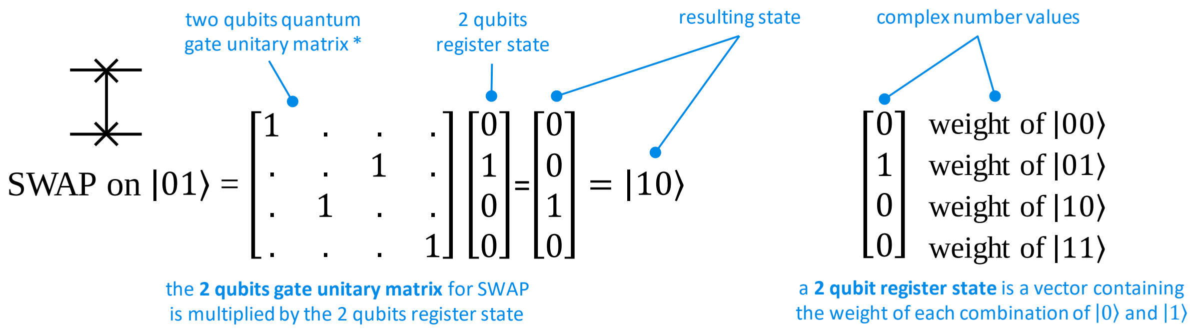

Figure 186: example of SWAP gate operation. (cc) Olivier Ezratty, 2021.

- **Fredkin gate** is a SWAP gate between two qubits that is conditioned by the state of a third qubit. So, it has three inputs.
- **Generic Control-U gate** is a two qubits gate applying a generic one qubit unitary to a qubit based on the state of a control qubit (Figure 187).

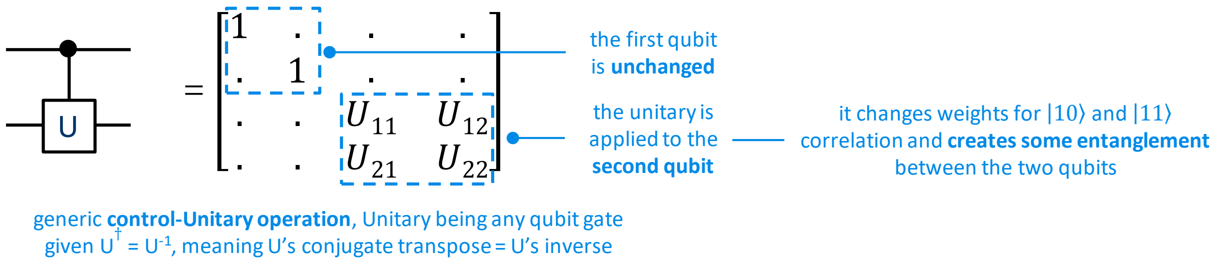

Figure 187: control-U two-qubit gate unitary matrix. (cc) Olivier Ezratty, 2021.

- **Phase-controlled R** gates are the equivalent of single-qubit phase-change R gates, conditioned by the state of a control qubit. If the algorithm, like a quantum Fourier transform, requires m to be large, it is not easy to ensure the reliability of the gate because the required precision becomes very large compared to the phase errors generated by the quantum system. However, phase errors are difficult to correct!

  A precision record of such a gate seems to have been reached by Honeywell with its trapped ions qubits presented in 2020 which have a rotation precision of 1/500 turn. This reminds us that during operations, quantum computing is analog. It is digital only at the level of commands and measured results, which become classical bits again[431].

[431] Here are a few sources of information on the subject of quantum gates: Gates, States, and Circuits by Gavin E. Crooks, July 2021 (82 pages), Universality of Quantum Gates by Markus Schmassmann, 2007 (22 slides), An introduction to Quantum Algorithms by Emma Strubell, 2011 (35 pages), Equivalent Quantum Circuits by Juan Carlos Garcia-Escartin and Pedro Chamorro-Posada, 2011 (12 pages), The Future of Computing Depends on Making It Reversible by Michael P. Frank, 2017.

There are some reasons to get confused with S, T and R phase gates angles. For example, a S gate is sometimes branded as a $\pi/2$ and sometimes as a $\pi/4$ (Figure 188). The same is applied to a T gate that is sometimes a $\pi/4$ and sometimes a $\pi/8$. The explanation is in the chart below and is related to the way a global phase is applied to the gate unitary operator. We can split hairs using a "rotation" for the large one and a "round" for the small one.

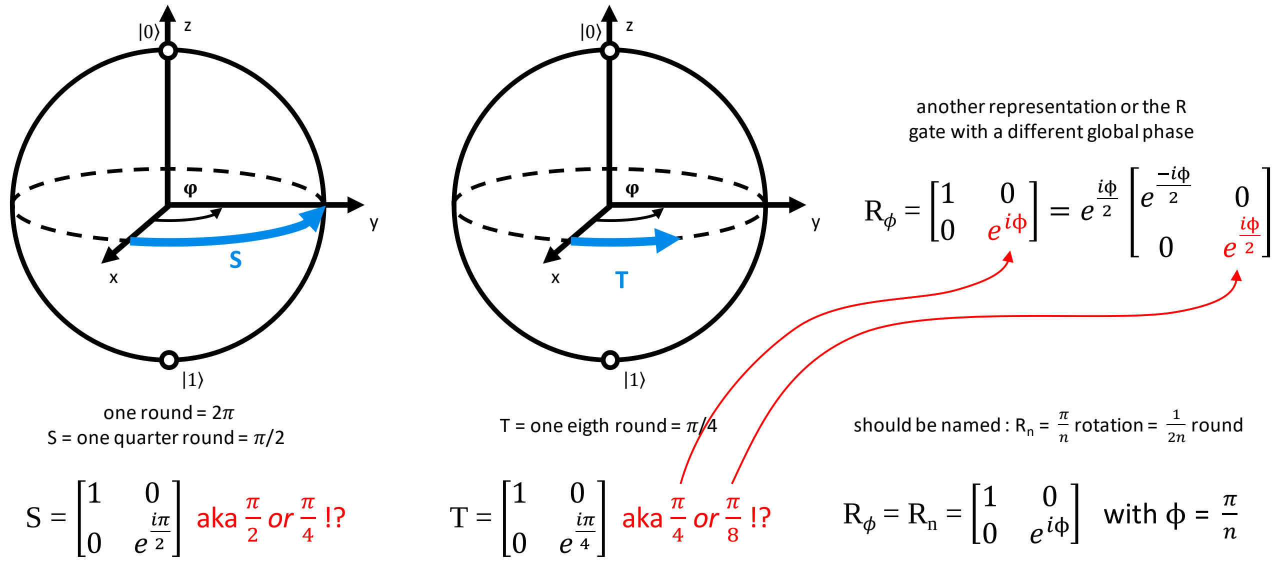

Figure 188: solving the ambiguity of phase gates labelling. (cc) Olivier Ezratty, 2021.

The effect of two-qubit gates is mostly always presented with using $|0\rangle$s and $|1\rangle$s as starting points in the control qubit, like with "*a CNOT inverts the state of a target qubit when the control qubit is* $|1\rangle$ ». But the CNOT will always have an effect on the target qubit when the control qubit is not exactly in the $|0\rangle$ state.

You just need to have a non-null β complex amplitude component in the first qubit. So, the only case a CNOT will do nothing on the target qubit is when the control qubit is exactly a $|0\rangle$.

To fully understand the effect of these gates on any qubit state and computational basis vectors for several qubits, you must look at the unitary matrices implementing these gates and their linear effects on the qubits and/or register computational basis vectors.

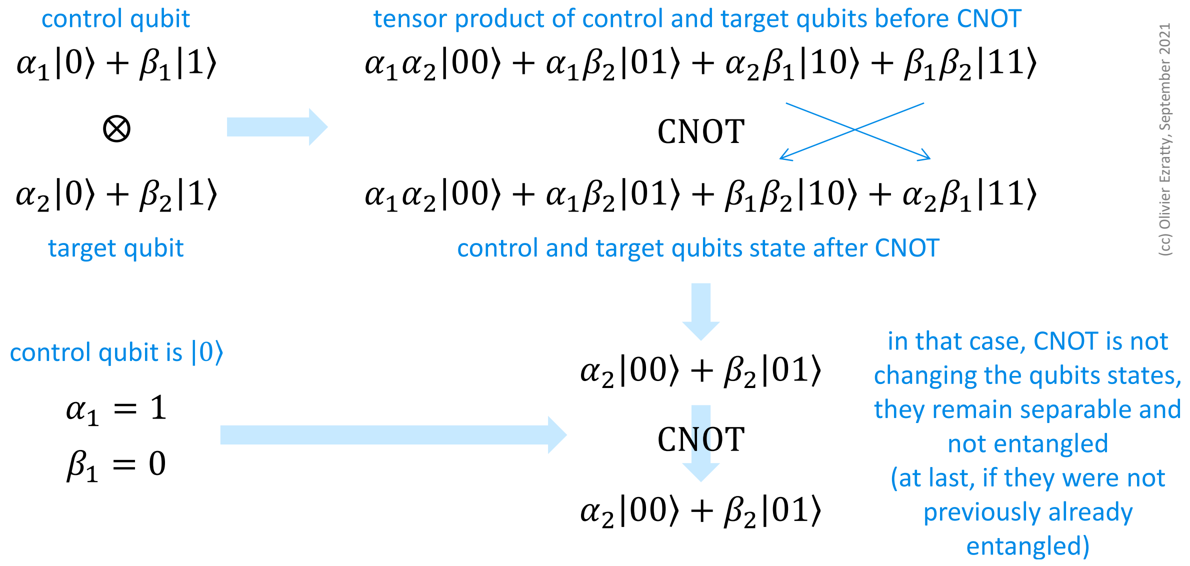

Figure 189: visualization of a CNOT two-qubit gate effect, generically and with a control qubit at $|0\rangle$, the only case when it won't generate any qubit entanglement. (cc) Olivier Ezratty, 2021.

In other words, and as demonstrated in Figure 189, unless the control qubit is $|0\rangle$, a CNOT gate will create some new entanglement between the control and target qubit. One could argue about two things: first, after a couple of operations, we never have a perfect $|0\rangle$ and are rapidly off-bounds, creating tiny entanglement with CNOT gates in that case, and second, most CNOT gates are run after a Hadamard gate was applied on the control qubit, getting off the $|0\rangle$ state!

Other two-qubit gates play a particular role. These are physical gates implemented at the lowest control level depending on the qubit type (Figure 192). They are not necessarily directly useful for developers but are the basis of some specific universal gates sets with some qubit types.

- **$\sqrt{\text{SWAP}}$ gate**, or square root SWAP, stops halfway through a SWAP (Figure 190). It is a physical level gate used to entangle electron spin qubits.
- **iSWAP gate** is a two-qubit gate that is implemented in superconducting qubits like those from IBM.

$$\sqrt{\text{SWAP}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{1}{2}(1+i) & \frac{1}{2}(1-i) & 0 \\ 0 & \frac{1}{2}(1-i) & \frac{1}{2}(1+i) & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Figure 190: a $\sqrt{SWAP}$ unitary matrix.

- **XY gate** is a generic two-qubit gate implementing a rotation by some angles β and θ between the states $|01\rangle$ and $|10\rangle$ and $\text{iSWAP} = \text{XY}(0,\pi)$ (Figure 191). This gate proposed by Rigetti can be implemented on superconducting qubits to reduce the number of two-qubits gates required to run many algorithms[432].

$$XY(\beta,\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\frac{\theta}{2}) & i\sin(\frac{\theta}{2})e^{i\beta} & 0 \\ 0 & i\sin(\frac{\theta}{2})e^{-i\beta} & \cos(\frac{\theta}{2}) & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Figure 191: an $XY(\theta,\pi)$ two-qubit gate unitary matrix.

- **ZZ gate** that is implemented with qubits coupling is a technique that can be used with qubit couplers to connect two superconducting qubits and implemented as a CZ gate[433] [434].
- **R_{xx}**, **R_{yy}** and **R_{zz}** are two-qubit gates that are implemented natively in some trapped ion quantum computers as a Mølmer-Sørensen gate. They are called Ising coupling gates. The gate R_{xx} is implemented natively in IonQ systems. These gates were also created for the first NMR quantum computing systems.
- **Mølmer-Sørensen gate**, **Cirac-Zoller gate** (C-NOT), **AC Stark shift gate** and **Bermudez gate** are various two-qubit gates implemented at the physical level with trapped ions qubits. The Mølmer-Sørensen gate is a "mixed-species" entangling gate that can couple different breeds of ions. It is also less sensitive to motion temperature. It is the main entangling gate for IonQ trapped ion computers.

	atoms		electrons & spins				photons
qubit type	trapped ions	cold atoms	super-conducting	silicon	NV centers	Majorana fermions	photons
single qubit gates	rotations	$U_{xyz}(\theta,\psi,\mu)$	$R_x(\pm\pi/2)$, $R_z(\lambda)$ (IBM, Rigetti)	R_x, R_y	R_x, R_y	T, H	X, Z, H X, Z, R, CZ (Xanadu)
two qubit gates	XX Mölmer-Sorensen	C-Z $C\text{-}U_{xy}(\theta,\psi)$	CNOT, iSWAP (IBM), ECR (IBM), C-Z (Rigetti)	$\sqrt{\text{SWAP}}$	CNOT	CNOT	CNOT

432 See Implementation of the XY interaction family with calibration of a single pulse by Deanna M. Abrams et al, 2019 (13 pages). IBM has an equivalent gate, the XXPlusYYGate.

433 See Implementation of Conditional Phase Gates Based on Tunable ZZ Interactions by Michele C. Collodo, Andreas Wallraff et al, PRL, May 2020 (10 pages).

434 Static ZZ interactions can also generate frequency shifts in fixed frequency transmon qubits. See ZZ-Interaction-Free Single-Qubit-Gate Optimization in Superconducting Qubits by Shu Watanabe, Yasunobu Nakamura et al, RIKEN, September 2023 (10 pages).

Figure 192: examples of physical qubit gates implement by specific qubit types. Consolidation (cc) Olivier Ezratty, 2021-2023.

Logical reversibility. Quantum gates have the particularity of being logically reversible. It can easily be visualized for a single qubit gate, which is a simple rotation in the Bloch's sphere and therefore, reversible with the inverse rotation. A multi-qubit gate is a rotation in a wider dimensional space, with 2^N dimensions, N being the number of qubits. Likewise, it is logically reversible with an inverse rotation, but harder to visualize.

We can rewind some parts of algorithms by applying in reverse order the quantum gates that have just been applied to a set of qubits[435]. One benefit of this process is the so-called uncompute trick used in some oracle-based algorithms. It enables resetting the ancilla qubits used in computation without doing any reading. It avoids damaging the useful qubits that we need to use for the rest of the algorithm.

That being said, qubits can undergo other operations. They could be stored, meaning transferred, in or from quantum memory. They can also be used to encode two bits instead of one, in what is called "superdense coding", which is mainly used in quantum telecommunications[436].

Gates classes. The science of quantum gates has led to the creation of many concepts and theorems about groups of quantum gates. They are associated with the notion of **universal gate sets**, capable of generating all other quantum gates.

Figure 193 contains a custom diagram summarizing these classes of quantum gates. In short, $SU(2^n)$ is the space of unitary transformations applicable on n qubits. It covers all the quantum computations that can be performed on n qubits. SU(2) includes all the unitary transformations that can be performed on one qubit (with n=1!). Clifford's group includes gates with one and discrete qubits quarter-turn rotation plus conditional gates. T (eighth turn) and R as Control-R gates with different angles from π and π/2 are not in Clifford's group. They are needed to cover SU(2) and $SU(2^n)$ well. In practice, the addition of the T gate is enough to create a universal gate set with using approximations.

The classification of the gates begins with the **Pauli gates** that apply half-turn rotations around the X, Y and Z axes of the Bloch sphere of representation of the qubits.

Pauli group includes the gates resulting from the combination of these three Pauli gates and the sign inversion operations on the α or the β of the qubits (±1 and ±i). On one qubit, the Pauli group includes the gates ±I, ±iI, ±X, ±iX, ±Y, ±iY, ±Z, and ±iZ (where I is the identity).

Clifford group includes single and multiple qubit gates that standardize the Pauli group applicable to n qubits, i.e., the U gates of this group combined with the Pauli group gates σ with UσU* generate Pauli group gates. A Clifford gate is a quantum gate that can be decomposed into Clifford group gates. These include Pauli gates (X, Y, Z) and H, S (90° rotation) and CNOT (also called CX for *control-X*) gates. The Clifford group is very large as soon as n>1. Its size is respectively 24, 11,520 and 92,897.280 elements for n=1, 2 and 3[437]. It is usually said that Clifford group gates are digital quantum gates while non-Clifford gates are analog.

Gottesman-Knill's theorem demonstrates that algorithms using gates in the Clifford group can be simulated in polynomial time on classical computers.

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[435] See Synthesis and Optimization of Reversible Circuits - A Survey by Mehdi Saeedi and Igor Markov, 2011 (34 pages), which reviews the algorithmic impact of reversibility in both classical and quantum computing.

[436] See From Classical to Quantum Shannon Theory, 2019 (774 pages) which describes the application of Shannon's information theory to quantum computing. As well as On superdense coding, August 2018, by Fred Bellaiche, an Econocom engineer who publishes very interesting and popularized scientific articles on quantum.

[437] See Clifford group by Maris Ozols, 2008 (4 pages). Clifford is the name of an English mathematician, William Kingdon Clifford (1845-1879) who is not related to the group that bears his name.

It means that they are insufficient to provide an exponential speedup compared to classical computing[438]. Another variant of this theorem from Leslie G. Valiant defines conditions for a quantum algorithm to be classically simulable in polynomial time on a classical computer[439].

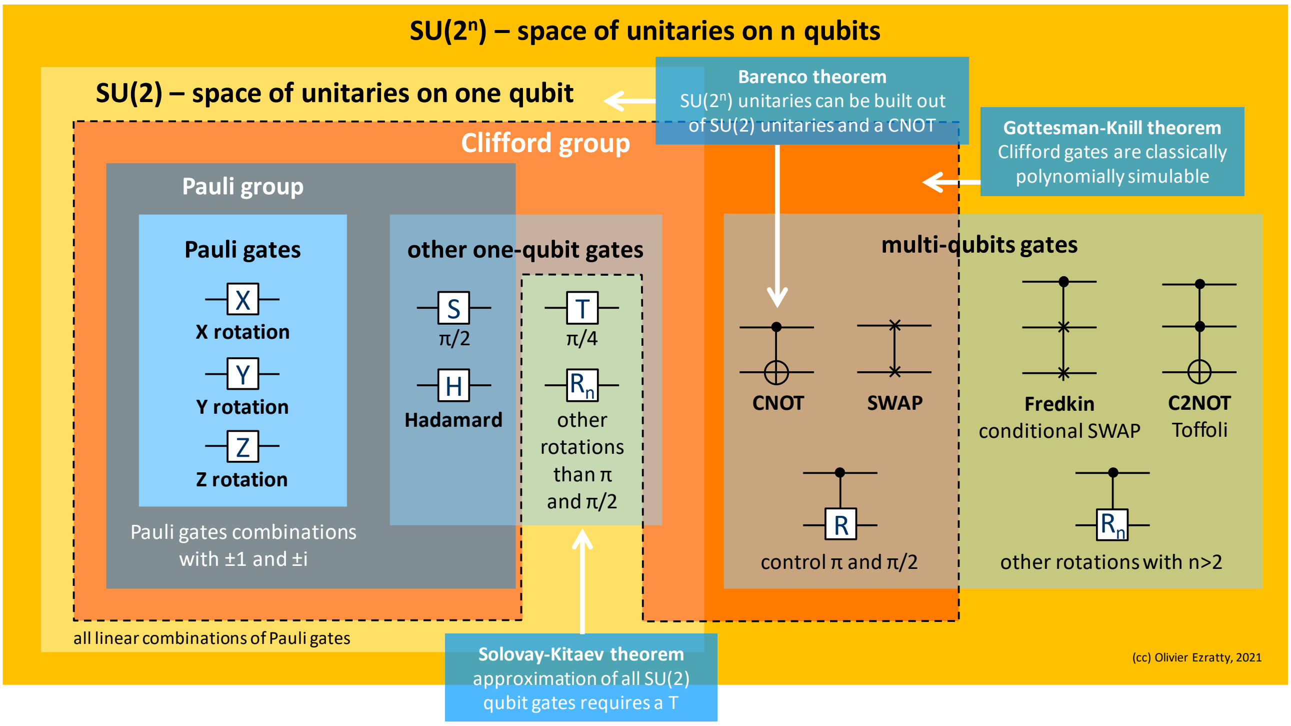

Figure 193: a visual taxonomy of qubit gates explaining the Pauli gates, the Pauli group, the Clifford group and the role of T and R gates to create a universal gate set. (cc) Olivier Ezratty, 2021.

So, how can we obtain exponential acceleration? It is necessary to use gates with more than two qubits implementing entanglement to obtain this acceleration like the Toffoli gate[440]. This can also be achieved with using phase-controlled R gates that are not part of Clifford's group, which can be approximated with adding a T gate. These non-Clifford gates have a particularity: they are difficult to correct with quantum error correction codes and to be implemented in a fault-tolerant manner. We'll see that on page 266. On top of that, a maximally entangled state is required among the used qubits which makes sense since separable subsets of the qubit register wouldn't provide a large Hilbert space for computation. To create a universal gate set, you need to use two gates that don't commute or anticommute. T and H gates don't commute whereas all Pauli gates anticommute ($XY = -YX$, $ZX = -XZ$, $YZ = -ZY$). Geometrically, commuting and anticommuting happens when the related gates rotation axis is respectively parallel (like S and T) and orthogonal in the Bloch sphere (X and Y, or X and Z). With T and H, they are neither parallel nor orthogonal but separated by a 45° rotation[441].

Continuous gates make it possible to generate rotations of any angle in the Bloch sphere. These gates enable the generation of all the phase-controlled R gates we have just seen, and which are indispensable for QFT (Quantum Fourier Transform) based algorithms.

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[438] See Positive Wigner Functions Render Classical Simulation of Quantum Computation Efficient by A. Mari and J. Eisert, December 2021 (7 pages) that generalizes the Gottesman-Knill theorem to quantum systems that preserve the positivity of the Wigner function (aka, do not use non-Gaussian photon states). It creates additional constraints on how to obtain exponential speedups with photon based quantum computers. It is also discussed in Quantum computational advantage implies contextuality by Farid Shahandeh, December 2021 (6 pages).

[439] See Quantum Computers that can be Simulated Classically in Polynomial Time by Leslie G. Valiant, 2002 (10 pages).

[440] See On the role of entanglement in quantum computational speed-up by Richard Jozsa et Noah Linden, 2002 (22 pages).

[441] See Quantum computing 40 years later by John Preskill, June 2021 (49 pages).

They can be natively implemented, particularly in NISQ platforms to create the variational algorithms ansatzes that we will cover later in the algorithms part of this book. In fault tolerant quantum computers, they can't be natively implemented and must be constructed with T or Toffoli gates which can be fault tolerant error corrected, although at a very high overhead cost.

Discrete gates are sets of (Hadamard, Z, S, CNOT) that make at best only half and quarter turns in the Bloch sphere.

Universal gate set is a group of gates that has the property of allowing the creation of all unitary operations on a set of qubits. From a practical point of view, also it allows to create all known quantum gates for one, two and three qubits. Such a gate-set must be able to create superpositions, entanglement and it must have at least one gate with no-real parameters (i.e. complex numbers instead of real numbers).

Here are some known sets of universal gates:

- CNOT + all single qubit unitaries can enable the creation of any unitary transformation on any number of qubits. This is demonstrated in the **Barenco theorem** according to which $\mathrm{SU}(2^n)$ unitaries can be built out of $\mathrm{SU}(2)$ unitaries and a CNOT two qubit gate[442]. It also demonstrates that any unitary transformation $\mathrm{SU}(2^n)$ on n qubits can be built with a maximum of 4^n elementary quantum gates.
- CNOT + T (eighth of turn) + Hadamard, using approximations, linked to the **Solovay-Kitaev's theorem**[443]. It proves that a dense and finite set of quantum gates in SU(2) space allows can be used to reconstruct any gate in this space with a maximum error rate ε.

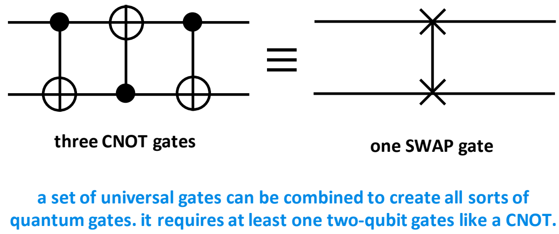

Figure 194: how to create a SWAP gate with three CNOT gates.

  The number of gates to be chained is a polynomial order of magnitude of $\log(1/\epsilon)$. The $\mathrm{SU}(2)$ space is the Special Unitary group of dimension 2 (and is a Lie group). There are various ways to optimize the number of gates to use to create arbitrary rotations[444].

  It includes unit matrices (from determinant 1) with complex coefficients and dimension 2, i.e., all single qubit gates.

$$\mathrm{SU}(2) = \left\{ \begin{pmatrix} \alpha & -\bar{\beta} \\ \beta & \bar{\alpha} \end{pmatrix} : \alpha, \beta \in \mathbb{C}, |\alpha|^2 + |\beta|^2 = 1 \right\}$$

This search for a set of discrete quantum gates allowing by approximation to generate a set of continuous gates of arbitrary rotations is important for some algorithms that we will see later, notably the discrete Fourier transform that is exploited in Shor's algorithm. You can see Figure 195 the effect of the sequence of T and H gates which, according to the combinations, allow to cover the different positions of Bloch's sphere, validating **Solovay-Kitaev**'s theorem[445].

[442] See Elementary gates for quantum computation by Adriano Barenco, Charles Bennett, David DiVincenzo, Peter Shor and al, 1995 (31 pages).

[443] This Solovay-Kitaev theorem was introduced by Robert M. Solovay in 1995 and proven by Alexei Kitaev in 1997. See Quantum computations: algorithms and error correction by Alexei Kitaev, 1997 (60 pages).

[444] See T-count and T-depth of any multi-qubit unitary by Vlad Gheorghiu, Michele Mosca and Priyanka Mukhopadhyay, Nature Quantum Information, October 2021-February 2023 (28 pages).

[445] See Shorter quantum circuits by Vadym Kliuchnikov et al, Microsoft, Facebook and the Universities of Birmingham, Oxford, Bristol and Brussels, March 2022 (83 pages) which proposes an efficient method to generate any unitary with fewer gates, and T-count and T-depth of any multi-qubit unitary by Vlad Gheorghiu, Michele Mosca, Priyanka Mukhopadhyay, October 2021-October 2022 (28 pages) which defines lower bounds for these T gate usage.

Transpilers are the parts of quantum code compilers that convert any quantum gate in the underlying universal gate set implemented by the quantum processor and handle related optimizations.

Figure 195: a visual description of Solovay-Kitaev's theorem. Source: TBD.

Gate teleportation is the application of a quantum gate on an unknown state while it is being teleported[446 447]. The practical application of this technique is to enable the creation of two-qubit gates from different systems that are connected through some entanglement. It is an enabling solution for distributed quantum computing. This feedforward operation is particularly suitable for photonic qubits but has also been tested with trapped ions[448] as well as with superconducting qubits, as shown in Figure 196. What is important here is to implement such gates in a deterministic way. Since gate teleportation is a costly resource, its use must be minimized by optimizing the partitioning of quantum code across multiple interconnected QPUs[449]. Gate teleportation is also used in the measurement based quantum computing (MBQC) technique that we will describe later in the photon qubit section, starting page 544.

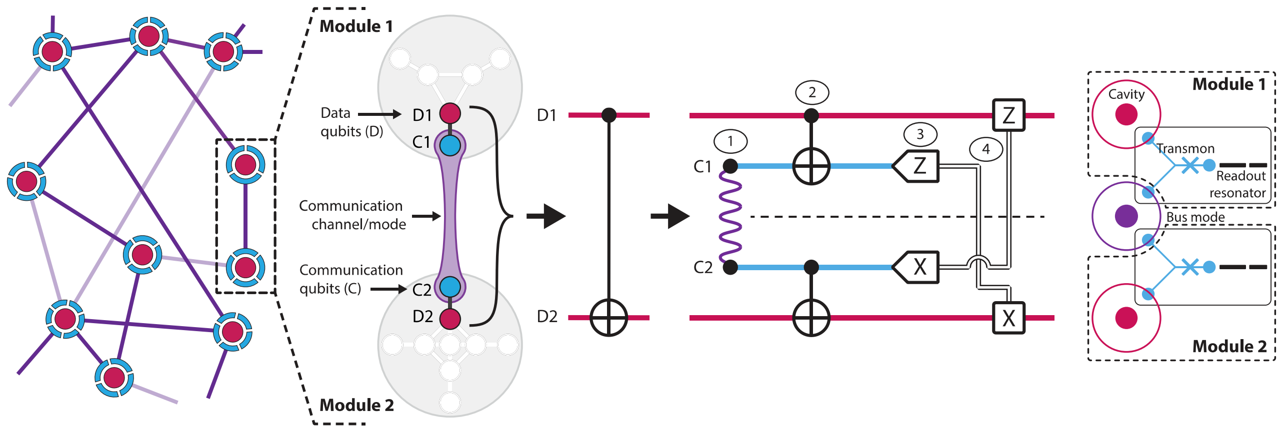

Figure 196: gate teleportation with two distant superconducting qubits (D1 and D2), using two communications qubits (C1 and C2) which are entangled, and using two classical communication lines (3 and 4). Source: Deterministic teleportation of a quantum gate between two logical qubits by Kevin S. Chou, Michel H. Devoret, Liang Jiang, Robert J. Schoelkopf et al, 2018 (33 pages). Added in 2023.

Dynamic circuits correspond to some dynamic quantum programming where the code is modified based on the execution of code and intermediate measurements, aka mid-circuit measurement. With the help from gate teleportation, this can enable long range entanglement implementation, using either sacrificial qubits in the qubit chip layout[450] or with long range microwave links in the case of solid state qubits.

[446] See Quantum Teleportation is a Universal Computational Primitive by Daniel Gottesman and Isaac L. Chuang, 1999 (6 pages).

[447] See Single-qubit gate teleportation provides a quantum advantage by Libor Caha, Xavier Coiteux-Roy and Robert Koenig, September 2022-May 2023 (31 pages).

[448] See Quantum gate teleportation between separated qubits in a trapped-ion processor by Yong Wan et al, Science, February-August 2019 (42 pages).

[449] See Applying an Evolutionary Algorithm to Minimize Teleportation Costs in Distributed Quantum Computing by Leo Sünkel et al, November 2023 (10 pages).

[450] See Efficient Long-Range Entanglement using Dynamic Circuits by Elisa Bäumer, Zlatko K. Minev et al, August 2023 (18 pages).

Inputs and outputs

Traditional microprocessors are composed of fixed logic gates, etched into the silicon, and 'moving' bits, which are electrical pulses that propagate through the circuit through the various gates. All this at a certain frequency, often in GHz, set by a quartz clock.

In a quantum computer, the first stage of processing consists of resetting the quantum register into an initial state. This is called "preparing the system".

The various registers are first physically configured in the $|0\rangle$ state. The following initialization consists in using different operators such as the Hadamard transformation to create $|0\rangle + |1\rangle$ superposition or the X gate to change this value $|0\rangle$ to $|1\rangle$. Sometimes, more preparation is required to prepare a denser register state, like with quantum machine learning algorithms where some input data must be encoded in the qubit register state vector. Once this initialization is done, computing gates operations are sequentially applied to the qubits according to the algorithm to be executed.

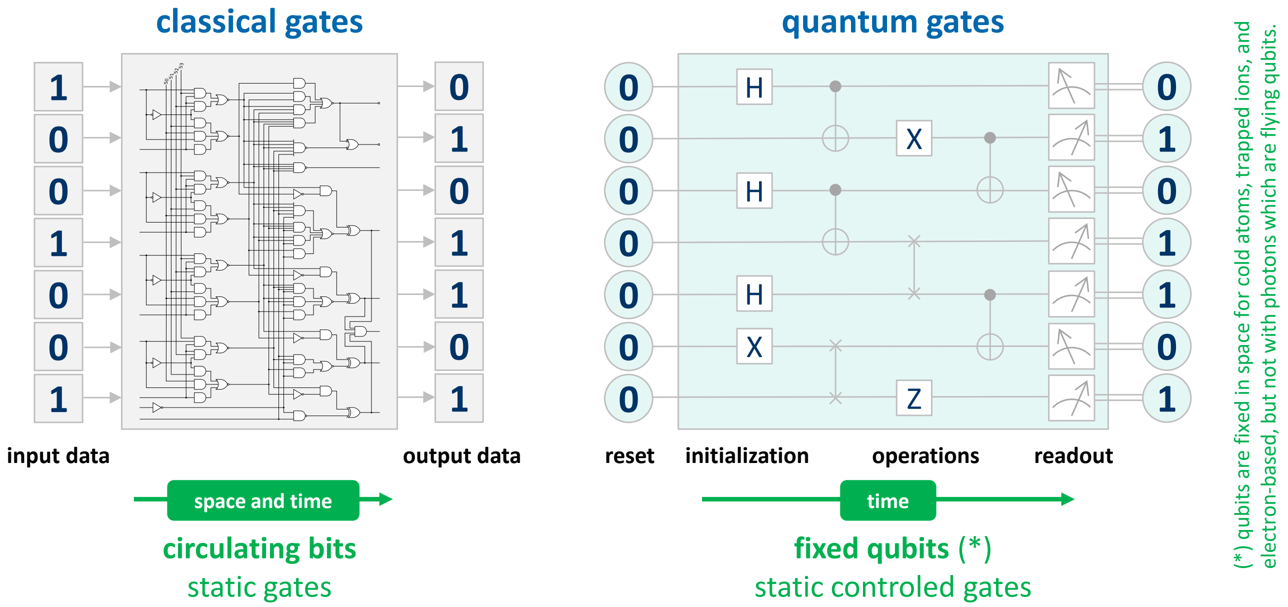

Figure 197: time and space differences with classical logic and quantum gates. (cc) Olivier Ezratty, 2021.

They always include some multi-qubits gates implementing entanglement between qubits. Finally, qubits are measured at the end of the algorithm execution, which has the effect of modifying their quantum state, even when doing what is called a non-demolition measurement, which doesn't destroy the qubit but still collapses its wave function to one of the computational basis states.

Quantum algorithms diagrams for universal gates computers, in Figure 197 on the right, are most often time diagrams, whereas for classical logic gates it is also a physical diagram. In the right part describing a quantum algorithm, there are no physical wires connecting the qubits between an input and an output, the gates being in their path. It is a time-based schema!

Unitary decomposition. A quantum algorithm is the description of a quantum circuit made of a series of sequenced timely quantum gates operating on 1, 2 and sometimes 3 qubits. It is the way to create a large unitary transformation on the initialized qubits. An arbitrary unitary transformation on N qubits can be implemented with an upper bound of $O(4^N)$ single-qubit and CNOT gates[451]. This troublesome exponential scale can hopefully be optimized in many ways, avoiding the creation of yet another exponential curse that would annihilate any quantum speedup.

[451] See Quantum Circuits for General Multiqubit Gates by Mikko Mottonen et al, 2004 (4 pages).

What is hard to understand is that a two-qubit gate creates a persistent entanglement between the related qubits, even with no persistent physical connection. This is referred to as being part of many quantum counterfactual models (see its definition in the Glossary). You understand it by toying with the qubit register state vector and computing the mathematics of gate executions "manually".

Now, let's toy a little bit with qubits and gates with Quirk, particularly to identify pure and mixed states with single or two qubits in Figure 199. It also shows the role of off-diagonal values in density matrices in Figure 200.

Figure 198: Grover algorithm with 5 qubits running on Quirk on a smartphone!

Quirk is an open source JavaScript based code emulator developed by Craig Gidney that can support up to 16 qubits on your laptop and smartphone (see Figure 198) and represent density matrices with up to 8 qubits. It is very easy to use and can help us understand how the structure of a vector state changes by applying quantum gates to a set of qubits.

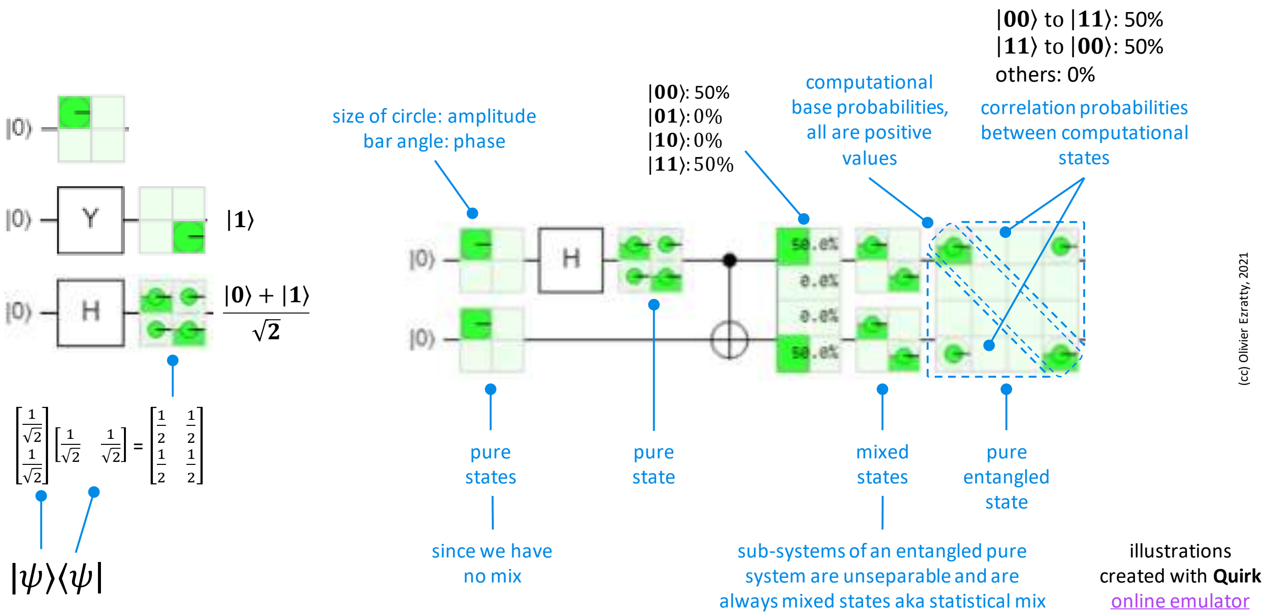

Figure 199: on examples of toying with Quirk to see how pure and mixed states look with two qubits. (cc) Olivier Ezratty, 2021.

Here, we describe a mixed state generated on two qubits after one of them is entangled with a third qubit.

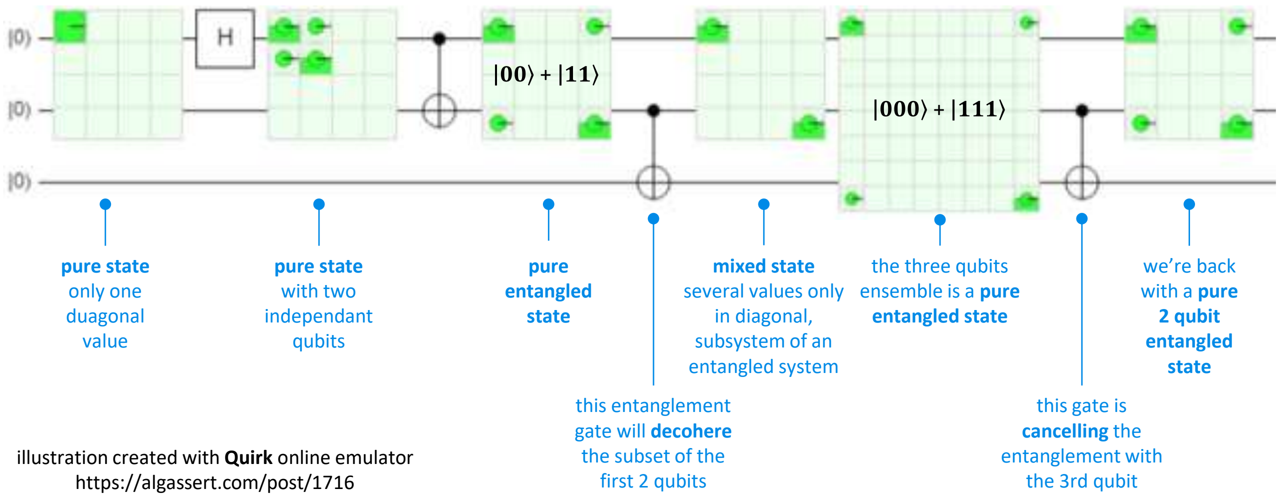

Figure 200: another example of toying with Quirk to see how pure and mixed states look with three qubits. (cc) Olivier Ezratty, 2021.

Qubit lifecycle

One way to understand how a universal gates quantum computer works is to track the life of a qubit during processing:

Initialization. A qubit is always initialized at $|0\rangle$, corresponding to the base state, usually at rest, of the qubit. This initialization consumes some energy with all known types of qubits and is not necessarily perfect[452].

Preparation. It is then programmatically prepared with quantum gates to adjust its values that are vectors in the Bloch sphere. The Hadamard gate is one of the most common one and creates a superposed state of $|0\rangle$ and $|1\rangle$ Single qubit gates apply a rotation of the qubit vector in the Bloch sphere. These rotations are based on unitaries, 2x2 complex number matrix operations applied to the qubit vector $[\alpha, \beta]$. These unitaries have a trace of 1, maintaining the vector length of 1. For most quantum algorithms, qubit preparation is usually simple with a set of X gates to set them and H gates to create superposed states. In some cases, like with quantum machine learning, qubit states preparation can be more complex, requiring a lot of gates.

Multiple-qubit gates then conditionally link qubits together. Without these quantum gates, little could be done with qubits.

Data manipulation. The qubits information that is manipulated during computing is "rich" with a dimension of two real numbers, the angles θ and ϕ, or the vector $[\alpha, \beta]$ for each qubit. But a set of N qubits holds 2^N complex number values, representing the proportion of each of the computational basis states made of the various combinations of N 0s and 1s. It creates a dimensionality of 2^{N+1}-1 real numbers, to take into account the normalization constraint for the computational basis states amplitudes. As these gates are operated on the qubits, quantum computing works in an analog way[453].

Measurement. When we measure the value of a qubit, we obtain a classical binary 0 or 1 with a probabilistic return depending on the qubit state. So, for each qubit, we have a 0 as input, a 0 or a 1 as output, and an infinite number of states in between during calculations.

All this to say that the mathematical richness of qubit-based quantum computing happens only during processing. This is the life cycle of the qubit illustrated in Figure 201.

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452 See Optimizing resetting of superconducting qubits by Ciro Micheletti Diniz et al, PRA, November 2023 (7 pages).

453 This is the position stated in Harnessing the Power of the Second Quantum Revolution by Ivan H. Deutsch, November 2020 (13 pages). Or more precisely, the author states that gate-based quantum computers are both digital and analog.

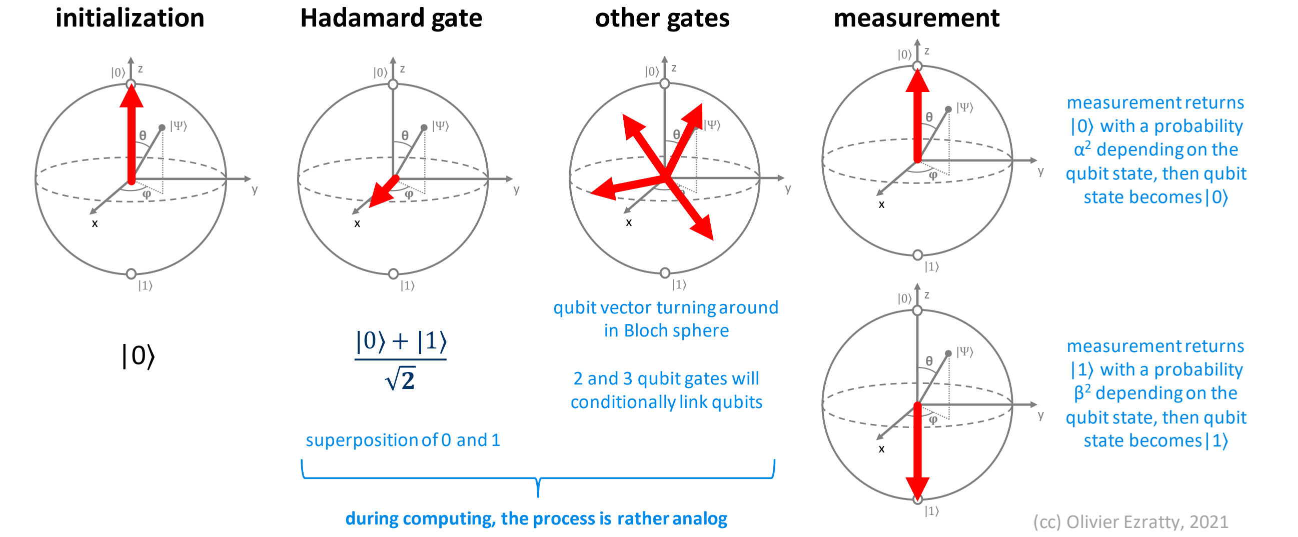

Figure 201: the effect of measurement on a single qubit. (cc) Olivier Ezratty, 2021.

Another schematic view of how classical and quantum computing are intertwined and the format of data that is handled in provided in Figure 202. What is specific to quantum computing is that the same instructions handle data and computing, i.e., quantum gates. The wealth of data in registers exists only during computing but not at the end, after measurement, where it is back in classical mode, turning the computational basis state vector of dimension 2^{N+1}-1 real numbers to a meager N classical bit. Also, two deterministic worlds are working next-door with a "probabilistic" gate between the quantum world and classical world. This is, however, not entirely true. If an algorithm like a Grover search or a Shor integer factoring is designed to output a register state with a non-superposed state and a set of |0⟩ to |1⟩ belonging to the computational basis, we will end up with having a nearly deterministic result. Practically, even with error correction, we will still probably have to run the circuit several times and average the results.

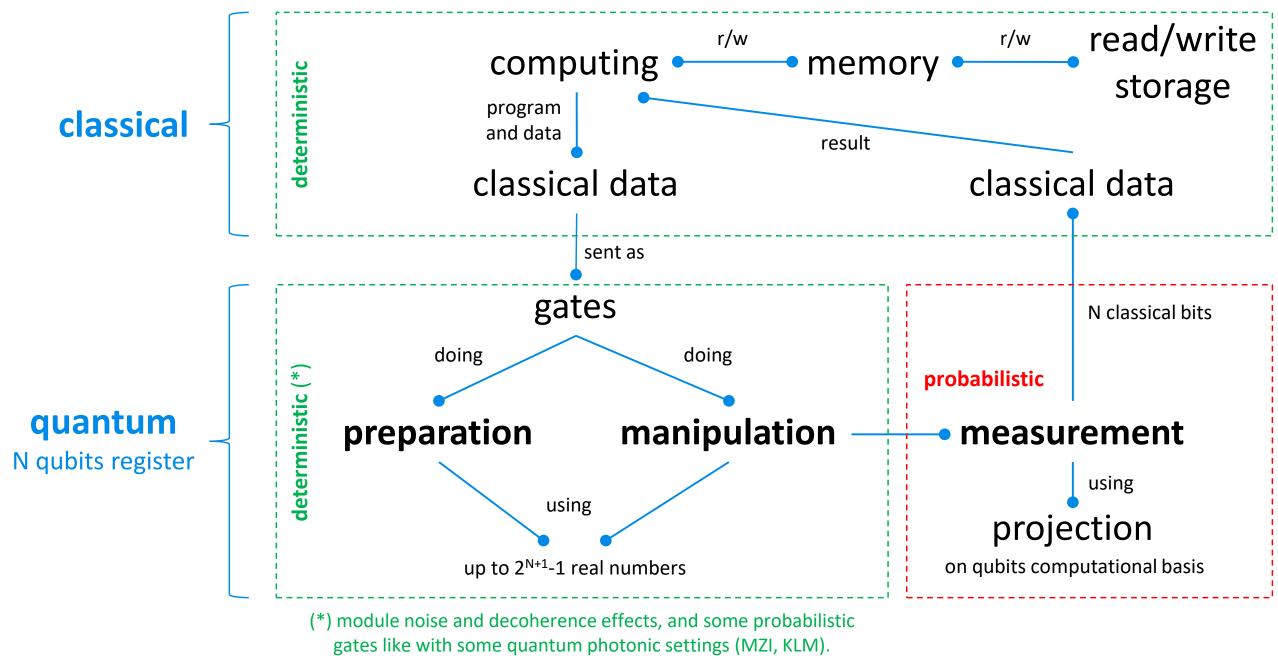

Figure 202: classical and quantum data flow in gate-based quantum computing. During quantum computing, the process is nearly deterministic, modulo the detrimental effects of noise and decoherence. It becomes probabilistic at the interface with the classical world during qubit measurement. (cc) Olivier Ezratty, 2021-2023.

Quantum switch is a curious artefact worth mentioning here. It consists in creating a series of qubit transformations that can be implemented simultaneously in different orders. Like say, A then B and

B then A, on a given register state. It defies logic and understanding of time flow, creating an indefinite causal order[454].

It can even be a useful resource for improving reliability of quantum communications[455].

Measurement

We'll now look into quantum measurement, a much broader topic than you may think. We have already explained that quantum measurement is assimilated to a wave function collapse onto basis states, in the case of qubits, $|0\rangle$ or $|1\rangle$. We've also seen that quantum computing is highly probabilistic, requiring executing several times your calculation and making an average of the obtained results.

But quantum measurement is way more subtle than that. We'll see here what can be measured in qubits and when, what is a projective measurement, what is a POVM, a CPTP map, what are gentle and weak measurements, non-selective and selective measurement, state tomography and the likes. Some of these techniques are related to quantum computing, including error corrections and some hardware benchmarking tasks and others with quantum telecommunications.

Projective measurement

A projective measurement is the most generic form of measurement used in quantum computing. We'll first describe it geometrically and then with some mathematical formalism. Projective measurement is also named a von Neumann measurement since **John Von Neumann** elaborated its formalism in 1932.

It is easy to intuitively understand what it looks like with using the Bloch sphere for a qubit. A projective measurement consists in doing a geometrical vector projection of your qubit pure state on any axis in the Bloch sphere (Figure 203 and Figure 206).

The simplest case of all is a projection on the z axis containing the $|0\rangle$ and $|1\rangle$ orthogonal vectors. It is about doing a measurement in the qubit computational basis. It could also be, theoretically, a projection on any other axis, like the $|+\rangle$ and $|-\rangle$ states that sit on the Bloch sphere equator along the x axis. We'll see later how to achieve this feat.

While quantum gates are reversible operations based on unitary operators, reading the state of the qubits is an irreversible operation. It is not a rotation in Bloch's sphere but a projection on an axis, which will yield a binary result with a probability depending on the qubit state. The projection uses a self-adjoint matrix operator, meaning that if executed several times, you'll always get the same result. Of course, the measurement of the qubit modifies its state unless it is already a perfect $|0\rangle$ or $|1\rangle$ in the computational basis.

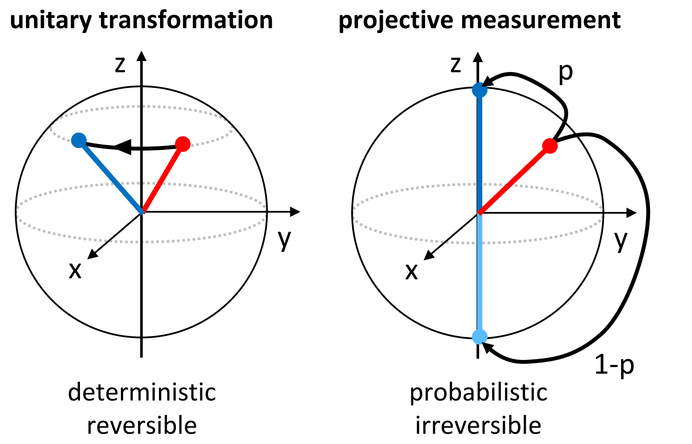

Figure 203: visual difference between a unitary transformation (single qubit gate) and a projective measurement. (cc) Olivier Ezratty, 2023.

[454] See Comparing the quantum switch and its simulations with energetically-constrained operations by Marco Fellous-Asiani, Raphaël Mothe, Léa Bresque, Hippolyte Dourdent, Patrice A. Camati, Alastair Abbott, Alexia Auffèves and Cyril Branciard, August 2022 (20 pages).

[455] See Improvement in quantum communication using quantum switch by Arindam Mitra, Himanshu Badhani and Sibasish Ghosh, September 2022 (14 pages).

After a projective measurement on the Z axis, the qubit will irreversibly collapse in the states $|0\rangle$ or $|1\rangle$. Qubits measurement is reversible only in the case when they are already perfectly in the computational basis states $|0\rangle$ or $|1\rangle$. In that case, the measurement along the Z axis is not changing the qubit value and is therefore reversible since it is an identity operation.

Mathematically, a projective measurement is using Projection-Valued Measures (PVMs) on a closed system. On a given qubit, it uses two orthogonal measurement operators, in the form of 2x2 self-adjoined (Hermitian) matrices.

When measuring a qubit along the Z axis, also named the observable Z with eigenvalues +1 and −1 and eigenvectors $|0\rangle$ and $|1\rangle$ (the observable Z is the matrix representation of a Z single qubit quantum gate!), these PVMs operators are respectively:

$$M_0 = |0\rangle\langle 0| = \begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1 & 0\end{bmatrix} = \begin{bmatrix}1 & 0\\0 & 0\end{bmatrix} \quad \text{and} \quad M_1 = |1\rangle\langle 1| = \begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0 & 1\end{bmatrix} = \begin{bmatrix}0 & 0\\0 & 1\end{bmatrix}$$

Given the Z observable operator is Z=$M_0 - M_1$, which returns $+1$ for $|0\rangle$ and -1 for $|1\rangle$.

On a general basis, with a quantum object with several distinct states, a measurement operator is a matrix M_m and the probability to get the outcome m (with m=0 and 1 in the case of a qubit, or m=0 to N-1 in the case of a N states quantum object) is $p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle$ with the completeness constraint $\sum_m M_m^\dagger M_m = I$ (I being the identity matrix).

For m=0, it reads as $p(0) = \begin{bmatrix}\alpha & \beta\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}1 & 0\\0 & 0\end{bmatrix}\begin{bmatrix}\alpha\\\beta\end{bmatrix} = \begin{bmatrix}\alpha & \beta\end{bmatrix}\begin{bmatrix}\alpha\\0\end{bmatrix} = \alpha^2$! Since $\beta^2 = 1 - \alpha^2$ due to the Born normalization rule, only one measurement is required to get both α^2 and β^2, these being not individual measurement results but their respective probabilities.

Any global phase added to $|\psi\rangle$ will disappear during measurement. If we define $|\psi'\rangle = e^{i\theta}\,|\psi\rangle$ and apply a measurement operator M_m on $|\psi'\rangle$:

$$p'(m) = \langle\psi'|M_m^\dagger M_m|\psi'\rangle = \langle\psi'|e^{-i\theta}M_m^\dagger M_m e^{i\theta}|\psi'\rangle = \langle\psi'|M_m^\dagger M_m|\psi'\rangle = p(m)$$

After the measurement with the operator M_m, the system state $|\psi\rangle$ becomes the projection of $|\psi\rangle$ on M_m divided by the probability of getting state m:

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}} \quad \text{also often written} \quad \frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m|\psi\rangle}}$$

since $M_m^\dagger = M_m$ (self-adjoint matrix) and $M_m M_m = M_m$ (projector matrix)

All these measurement equations are part of the measurement postulate (usually the third) from quantum mechanics postulates.

In Figure 204, let's make a pause to understand the $\langle A|B|C\rangle$ Dirac notation. You usually read it from the right. The ket on the right is a vertical vector that is multiplied by the middle object that is a square matrix. It creates a similar vertical vector. Then, you multiply it with the bra on the left which is a horizontal vector. It is a dot product of an inner scalar product. The result is a complex number and it is a real number when Ψ = ϕ.

Now, let's be a bit practical.

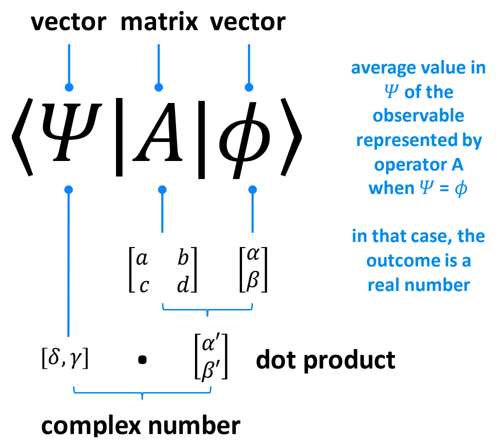

Figure 204: understanding the $\langle A|B|C\rangle$ Dirac notation.

How can we change the measurement basis with qubits, for implementing a measurement along another axis than Z? At least two options are available:

- It may be possible to *physically* implement a measurement on a different basis than the computational basis. This is, for example, the case with polarization-based photon qubits where the polarizer angle can be dynamically and programmatically modified with some electrically controlled optical settings. It looks more difficult to implement for other types of qubits.

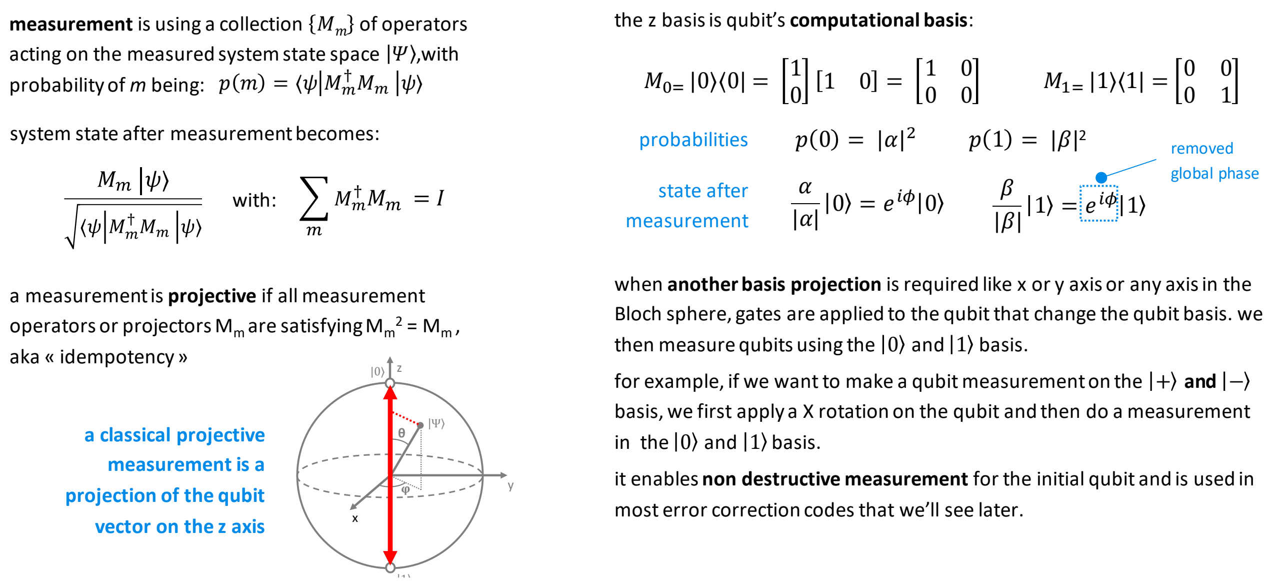

Figure 205: another explanation of projective measurement on a different basis and its usage in non-destructive measurement techniques like with error correction codes. (cc) Olivier Ezratty, 2021.

- When the *only* supported measurement is a projective measurement in the computational basis $|0\rangle$ and $|1\rangle$, any another projective measurement can be implemented with first applying a unitary transformation to the qubit that creates a rotation in the Bloch sphere equivalent to moving the measurement axis to the Z axis ($|0\rangle$ and $|1\rangle$) (Figure 205). When we say we do an "X" or "Y measurement", it means that we first apply a H or $HS^\dagger$ single gate rotation (H = Hadamard gate and S = half a Z gate or quarter phase turn) to handle this axis rotation and then, apply a (computational basis) Z-axis measurement. This is what is regularly done with quantum error correction codes as well as with MBQC (measurement-based quantum computing).

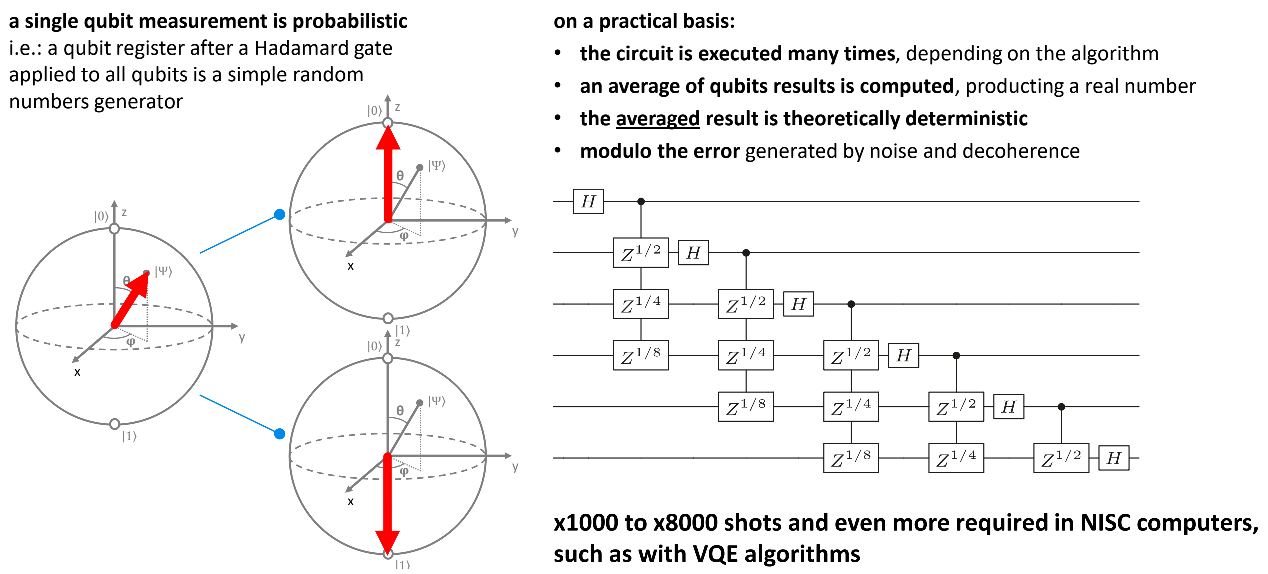

Figure 206: a qubit probabilistic measurement and the notion of computing shots. (cc) Olivier Ezratty, 2021-2023.

With QECs (quantum error correction codes), this sort of projective measurement is applied to ancilla qubits, these additional qubits that detect errors in entangled computing qubits. So, when physicists say they are doing a measurement on a basis of two orthogonal vectors, they mean they are applying first a unitary transformation and then a measurement on the computational basis.

Qubits register measurement

So far, we've just elaborated on measurement mathematical underlying tools and dealt with only one qubit. How about measuring a whole qubit register?

A N qubit register has 2^N possible computational basis states, from $|00\ldots00\rangle$ to $|11\cdots 11\rangle$. When measuring once a qubit register, you get one of these states, being a combination of N 0s and 1s.

You could stop there and think, that's my result, fine, I'm done! Well, no! Since the measurement outcome is probabilistic and prone with errors, you need to run your algorithm a certain number of times and count the number of times you will get each computational basis state (Figure 205). This will depend on what data is generated by your circuit: a value in the computational basis (series of 0s and 1s) or a superposed stated vector and on whether you are using a NISQ or FTQC QPU. Quantum algorithms like Shor and Grover generate a simple computational basis state and not a combination of several states and their respective probabilities (see Figure 743, page 871). NISQ algorithms like VQE can require up to one million shots to compute just one circuit out of many just to get a sufficient output accuracy. Otherwise, if you repeat the circuit and measurement a large number of times, you will end up recovering a probability distribution for each computational basis state and reconstruct a full state vector. But it requires an exponential number of circuit shots with regards to the number of qubits, losing any quantum computing speedup gain.

You can then run several times your algorithm and compute the average values of each qubit, giving a % of 0/1 for each then round up to the nearest 0 and 1. And there you are. What is "several"? It depends. IBM proposes to run your algorithm a couple thousand times on its cloud Q Experience platform with 5 to 133 qubits and states that this number will grow with the number of qubits, we hope linearly. How do you define the number of runs, or "shots"? It depends on the algorithm and whether you are using a NISQ or a FTQC quantum computer. All in all, one run of an algorithm is **probabilistic** and with many runs, you'll converge progressively to a **deterministic** solution being the average of all runs results[456]. Otherwise, quantum programs can also be designed to be nondeterministic if it contains mid-circuit measurements yielding some intermediate random results and conditioning the rest of the code execution[457].

From computational vector state to full state tomography

What are we measuring? A single computational state, a statistical weight of 0 and 1 or a full vector state? It depends on the algorithm and on the actual technical need of the undertaken measurement. For most algorithms, a series of runs and qubit measurement and their average will output after roundup the found computational basis state where qubits are disentangled, otherwise we will need many runs to reconstitute the register amplitudes (Figure 208).

For algorithms debugging with a reasonable number of qubits and for characterizing the quality of a small group of qubits, it may be useful to compute either a histogram of the whole computational state vector or even, a so-called quantum state tomography which will reconstitute the density matrix of the quantum register (Figure 207).

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[456] If an algorithm generates a single computational basis state (i.e., a combination of 0s and 1s without any superposition), then, the output will be deterministic or nearly deterministic considering the effect of noise, given this usually works well only with FTQC systems

[457] See [MIRAGE: Quantum Circuit Decomposition and Routing Collaborative Design using Mirror Gates](#) by Evan McKinney et al, August 2023 (13 pages).

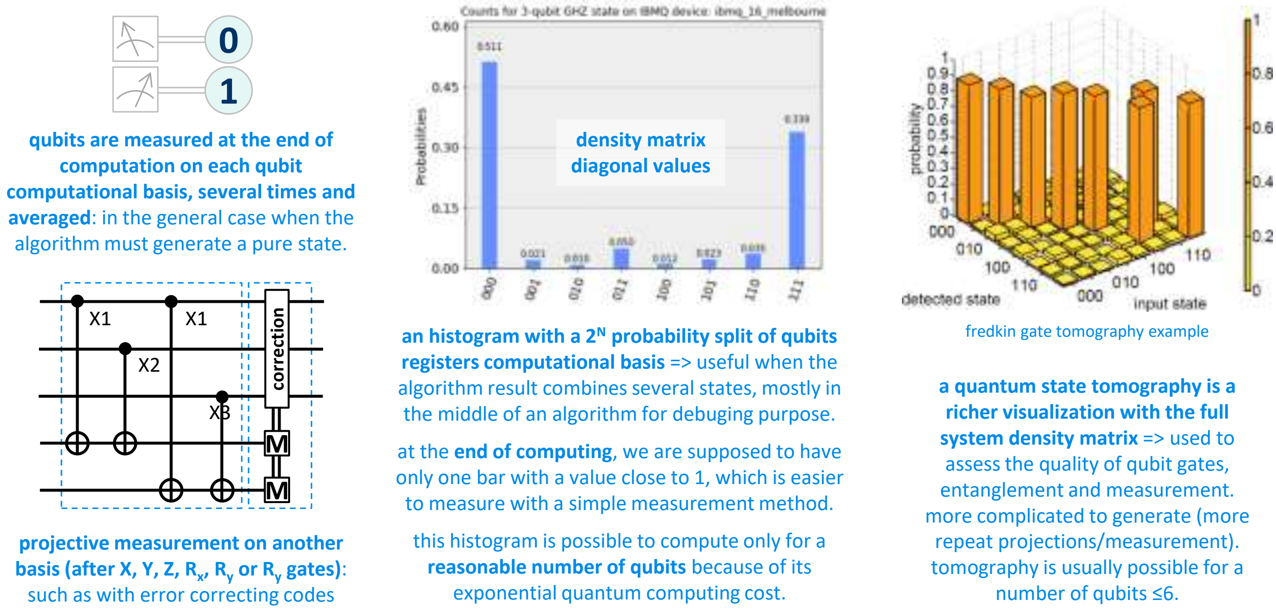

Figure 207: from a vector state to a full density matrix, the various ways to measure the state of a qubit register. Compilation (cc) Olivier Ezratty, 2021-2023.

The computational state vector is assembled with a lot of repeat runs and measurements with a number growing exponentially with the number of qubits. It will eventually provide the statistical distribution of each computational basis states. Since the number of runs grows exponentially, you understand quickly why it won't make sense to use this technique when we use a large number of qubits.

Development tools like IBM Quantum Experience dumps the vector state of your qubits only for helping you learn about how their system work and understand the impact of noise and decoherence.

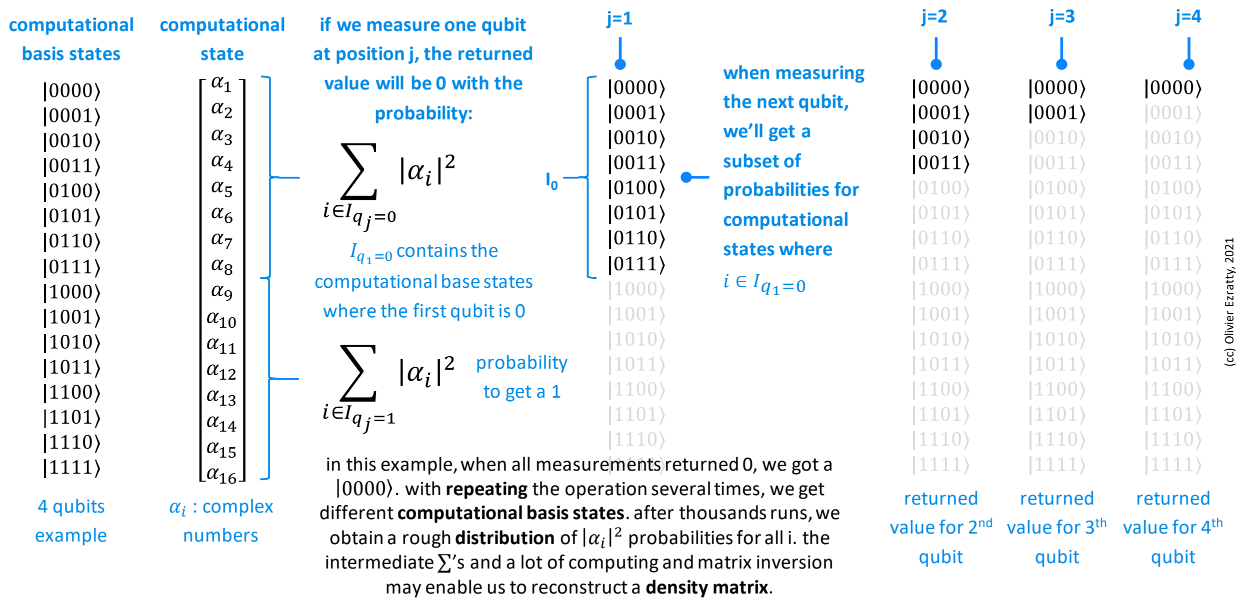

Figure 208: what happens to your qubits when you progressively measure them. (cc) Olivier Ezratty, 2021.

Reconstituting the whole system density matrix is a more tedious process. In the most basic technique used, we are keeping track of all intermediate measurements leading to getting the computational state vector and some matrix inversion is required to create it in the end. The process requires even more quantum and classical computation than for reconstituting the computational state vector. And it scales with 2^{3N}, N being the number of qubits!

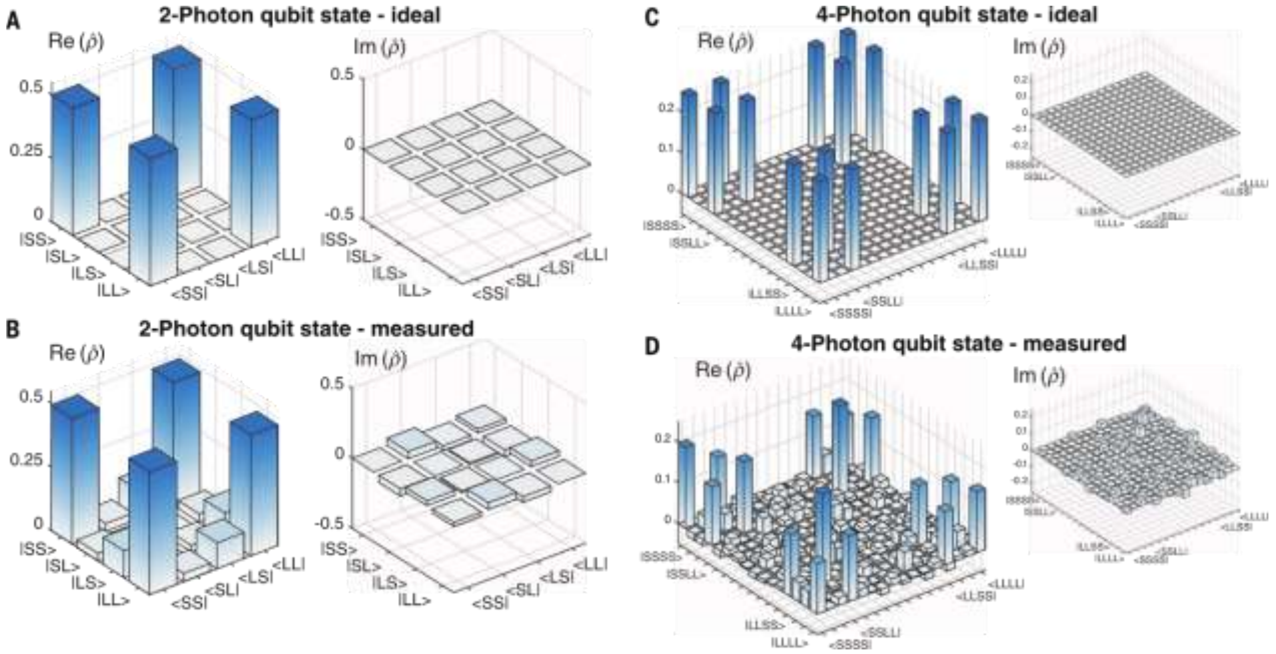

Figure 209: the difference between an ideal 2 and 4-photon density matrices and as measured in experiments. Source: Generation of multiphoton entangled quantum states by means of integrated frequency combs by Christian Reimer et al, Science, 2016 (7 pages).

This is usually applied with up to 6 qubits (shown in Figure 210[458]), and particularly with 2 qubits to characterize the quality of two qubit gates. A record state tomography of 8 qubits was achieved in 2005 with trapped ions by Rainer Blatt's group in Innsbruck[459]. Many researchers are trying to optimize this process with reducing the number of shots and measurements[460].

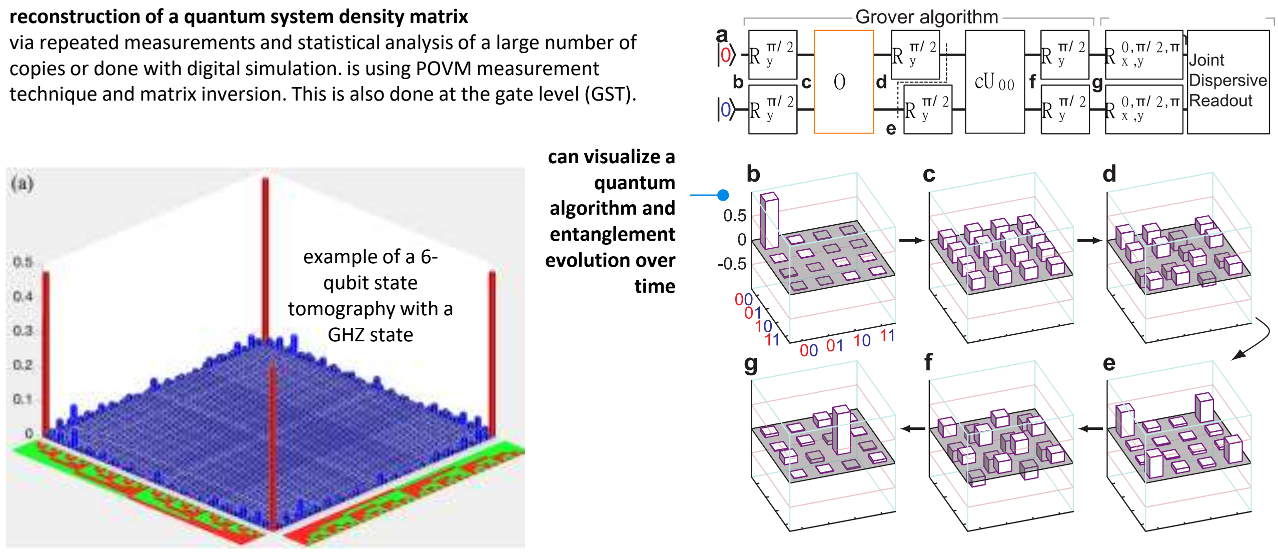

Figure 210: how do you reconstruct a quantum system density matrix. An example of a 6-qubit density matrix and how two qubit evolve over time during a Grover algorithm. Sources: Efficient Quantum Mixed-State Tomography with Unsupervised Tensor Network Machine Learning by Wen-jun Li et al, China, August 2023 (7 pages) and Demonstration of Two-Qubit Algorithms with a Superconducting Quantum Processor by L. DiCarlo et al, March-May 2009 (6 pages). Updated in 2023.

[458] See Efficient Quantum Mixed-State Tomography with Unsupervised Tensor Network Machine Learning by Wen-jun Li et al, China, August 2023 (7 pages).

[459] See Scalable multi-particle entanglement of trapped ions by H. Haffner, Rainer Blatt et al, 2006 (17 pages).

[460] See Multi-qubit State Tomography with Few Pauli Measurements by Xudan Chai et al, May 2023 (8 pages).

The graphical representation of these density matrices is often used to evaluate the fidelity of 2 or 3-qubit gates in research publications. The example in Figure 209 illustrates this with comparing the theoretical state of a density matrix for 2 and 4 qubits and measurement results. It also helps qualify the quality of qubits entanglement. Various techniques are proposed to speed-up quantum state tomographies and achieve it with better precision.

However, this is a tool for researchers and hardware designers, not for quantum software developers[461]. The next step is a Quantum Process Tomography which qualifies the quantum channel of a given process, like a series of gates, one gate, or quantum noise and decoherence. It creates an even richer matrix with 2^{2N} columns and rows, representing a linear operator on the system density matrix, *aka* a superoperator.

Non-selective and selective measurements

A non-selective measurement is a measurement that is physically done but not yet read. For any reason, its outcome is not available either because it wasn't yet used or because it is inaccessible when measurement is done by the environment. How is it different from a real measurement? It deals with the information available about the quantum states we are evaluating. This is explained in the example in Figure 211 using photons polarization and relates with pure states and mixed states.

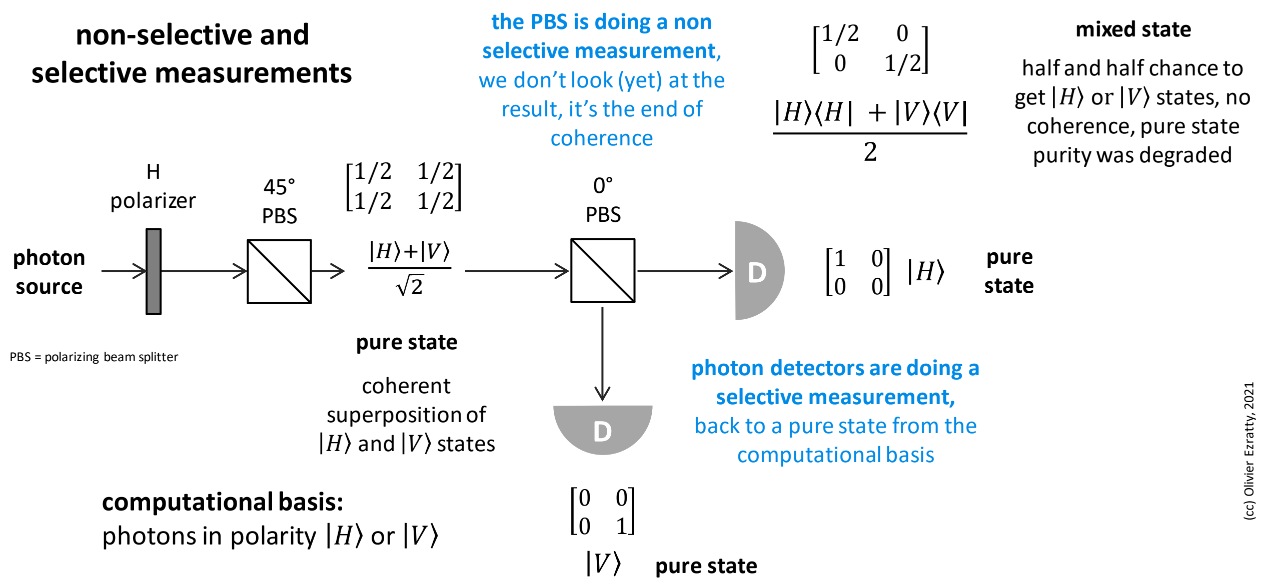

Figure 211: non-selective and selective measurements. (cc) Olivier Ezratty, 2021-2023.

A single photons source generates photons that traverse first a horizontal polarizing filter and then a 45° polarizing beam splitter (PBS). The PBS create a pure state coherent superposition of $|H\rangle$ and $|V\rangle$ states (horizontally and vertically polarized photons). Then, this coherent superposition traverses a 0° PBS. The outcome can be measured in the two PBS exits with single photon detectors. Before being measured, this output is a mixed state of $|H\rangle$ and $|V\rangle$.

There is no more coherent superposition (exit the pure state) and we don't know yet what both detectors will read. But we know that there's a 50% chance that the detector on the PBS horizontally polarized exit will detect a photon and 50% for the other detector. After detection, we'll end up with finding a single photon on one of the detectors, giving a related pure state. And nothing for the other.

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[461] See for example Quantum process tomography via completely positive and trace-preserving projection by George C. Knee et al, UK, 2020 (13 pages). But it requires some background knowledge!

This means that after measurement of a qubit in a given basis, the coherences in its density matrix in the measurement basis are erased. There's no more coherence and superposition. This happens before looking at any measurement outcomes. In other words, a non-selective measurement of a pure state degrades its purity by turning it into a totally mixed state.

This could be used in a new updated Schrodinger' cat thought experiment, replacing the disintegrating radium atom by a simple qubit in a superposed state (after a H gate). A measurement at time T would trigger the poison release if the result is $|1\rangle$. All this in a closed box. Keeping the box closed at time T+whatever would be an equivalent of a non-selective measurement, then opening the box at time T+after whatever, would become a classical measurement of an already totally mixed state.

Positive Operator-Valued Measurement (POVM)

A Positive Operator-Valued Measure (POVM) is a quantum measure generalizing Projection-Valued Measures (PVMs) which is useful when the measurement basis is not made of orthogonal states in their Hilbert space. It is of particularly interest when measuring a photon qubit in a telecommunication link with two non-orthogonal polarization basis (0° and 45° like in the BB84 protocol). Like in PVMs, the measurement operators of a POVM add up to identity matrix. POVMs are also interesting when measuring a subsystem of an open system.

POVMs that are not PVMs are called non-projective measurements. They have many use cases like enhancing quantum states tomography, help detect entanglement and allow unambiguous state discrimination of non-orthogonal states, with applications in quantum cryptography and randomness generation[462].

Other measurements concepts

I'll cover here other measurement-related tools and concepts I have encountered in various courses and scientific papers. You probably don't need to understand this if you are just a quantum software developer. It may be interesting, however, if you are involved in designing quantum systems, error correction systems, measurement systems, quantum firmware and the likes.

Gentle or Weak Measurement. It is one type of quantum measurement that retrieves little information of the measured system in average with the benefit of only slightly disturbing it. In a weak measurement, the correlations in the off-diagonal values of the system density matrix are only slightly altered. The system purity and entanglement remain mostly unaltered.

Postselected Measurement. It is a measurement where the result is chosen by the user, usually after a weak measurement. Surprising! As all measurements, it also turns a pure state into a mixed state. It refers to the process of conditioning on the outcome of a measurement on some other qubit values. The process consists in throwing away any outcome which does not allow you to do what you want to do. If the outcome you are trying to select has probability $0<p<1$, you will have to try an expected number $1/p$ times before you manage to obtain the outcome you are trying to select. If $p=1/2n$ for some large integer n, you may be waiting a very long time.

This weird technique is noticeably used to better understand quantum physics and phenomenon like measurement non-commutativity[463].

[462] See Understanding the basics of measurements in Quantum Computation by Nimish Mishra, 2019. But what is $\delta_{mm'}$ in these formulas? It is the Kronecker Delta function which is equal to 0 when m≠m' and equal to 1 when m=m'. Meaning that inner product of all measurement operators is equal to 0 when they are different. This is the definition of orthonormality between a set of operators.

[463] See for example Quantum advantage in postselected metrology by David R. M. Arvidsson-Shukur, Seth Lloyd et al, Nature Communications, 2020 (9 pages).

CPTP map. A Completely Positive and Trace Preserving map also referenced as a quantum channel is used to describe non-selective measurements, conditional expectations and quantum filters, as well as feedback networks in quantum control theory. It corresponds to the most generic operation that can be applied to a quantum system. The state of the target system is associated to a trace-one, positive semidefinite density operator and, under the assumption that no initial correlations are present with the environment, its evolution over some specified time interval is described by a completely positive, trace-preserving (CPTP) linear map.

For open quantum systems, however, the interaction between the system and environment leads to non-unitary evolution of the system (e.g., dissipation), which requires CPTP maps for full characterization[464].

In other words, a CPTP map is the mathematical operation that transforms the density matrix ρ of a quantum system during a measurement on the basis $\langle m_k|$ into the density matrix ρ' as described in Figure 212. A CPTP map is a superoperator of dimension 2^{4N} complex numbers.

$$\rho' = \sum_k M_k \, \rho M_k = \sum_k p_k \, M_k$$

$$\text{with } p_k = \langle m_k|\rho|m_k\rangle$$

Figure 212: defining a CPTP map.

Quantum Non-Demolition measurement. It is a type of measurement in which the uncertainty of the measured observable does not increase from its measured value during the subsequent normal evolution of the system. For a qubit measurement, it means that after its measurement, its value won't change anymore in subsequent measurements. QND measurements are the least disturbing type of measurement in quantum mechanics. QND measurements are extremely difficult to implement. Note that the term "non-demolition" does not imply that the wave function fails to collapse[465]. It can be implemented with photons, particularly to measure a photon number (number of photons in a superposed state of similar photons, or a single-mode Fock state), using a secondary probe field interfering with the signal field[466]. It has also been experimented to measure an electron spin with an additional ancilla quantum dot next to an operational quantum dot[467]. It also currently works well with superconducting qubits. What would be a "demolition measurement"? It would be one that, after retrieving the result, would create so significant a back-action on the measured quantum that it would either destroy it (like a classical photon counting device that absorbs the counted photons) or turn it into a state outside the computational basis (such as a different energy level than ground/excited levels for a qubit).

Quantum Steering is a quantum measurement phenomenon when one subsystem can influence the wave function of another subsystem by performing specific measurements. It is a variation of non-local correlations intermediate between Bell nonlocality and quantum entanglement[468].

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[464] Source: Quantum and classical resources for unitary design of open-system evolutions by Francesco Ticozzi and Lorenza Viola, 2017 (27 pages).

[465] QND was initially introduced in 1975 by VB Braginsky and YI Vorontsov in USSR. Source: Quantum Nondemolition Measurement,Wikipedia. See also Quantum Non-Demolition Measurement of Photons by Keyu Xia, March 2018. It was demonstrated with the detection of a single photon as described in Seeing a single photon without destroying it by G. Nogues et al, 1999 (4 pages).

[466] See Detecting an Itinerant Optical Photon Twice without Destroying It by Emanuele Distante et al, Max Planck Institute, June 2021 (6 pages) which deals with detecting twice a photon with some non demolition quantum measurement. The detectors use a single atom coupled to an optical cavity. Other methods consist in using the cross-Kerr effect where a measured photon traverses an optical medium and changes its refraction index. It provokes a phase shift for a probe photon traversing the same media, its phase being measured with a Mach-Zehnder interferometer. See a description of this old technique in Quantum non-demolition measurements in optics by Philippe Grangier, Juan Ariel Levenson and Jean-Philippe Poizat, 1998 (7 pages).

[467] See Quantum non-demolition readout of an electron spin in silicon by J. Yoneda et al, Nature, 2020 (7 pages).

[468] See Quantum Steering by Roope Uola et al, 2020 (43 pages) and Quantum steering on IBM quantum processors by Lennart Maximilian Seifert et al, PRA, April 2022 (11 pages) which shows poor entanglement with 15 qubits.

Quantum Measurement Thermodynamics. We have already mentioned the theoretical reversible aspect of gates-based quantum computing which relates to the unitary transformations applied with quantum gates. But most of the time, particularly with solid qubits, there is always some energy exchange between qubits and their control as well as measurement devices. Fundamental research is undertaken to better understand the evolution of the thermodynamic equilibrium of qubit operations particularly during entanglement and measurement and error correction. Since measurement is done on a repeated basis due to the implementation of quantum error correction codes, it makes sense to wonder whether this could be optimized. Depending on the qubit state (ground level or excited level, and in intermediate states), measurement can absorb or release some energy that is quantum and microscopic in nature and it is also powered by entanglement[469]. This research field could lead to a better understanding of the whereabouts of the energetic footprints of quantum measurement and entanglement and how it can impact the energy cost of quantum computing, particularly as it scales up[470].

Quantum Reservoir Engineering is a set of qubits management techniques using a quantum bath to reduce its energetic footprint, its measurement readout times and enable quantum non-demolition measurement[471]. It is about tightly controlling the qubit coupling with its environment. It is connected to quantum error correction techniques. The approach was initially imagined for NMR qubits, leveraging the Nuclear Overhauser effect. Then it was tested with trapped ions, using some coupling between the qubit harmonic oscillator and a reservoir of oscillator with laser radiations[472]. The technique is also branded "quantum bath", "engineered dissipation", "autonomous feedback" and "coherent feedback". It has since been tested with superconducting qubits and is the basis of the cat-qubits from Inria, Alice&Bob and Amazon[473].

Algorithmic Cooling is a related technique also named heat-bath algorithmic cooling, which consists in balancing the entropy transfers between qubits and with ancilla qubits as part of error correction codes[474]. It is used to improve the purity of a target subset of qubits quantum states in a qubit register.

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469 See also Probing nonclassical light fields with energetic witnesses in waveguide quantum electrodynamics by Maria Maffei, Patrice Camati and Alexia Auffèves, September 2021 (6 pages) which studies the thermodynamics of a qubit coupled to a waveguide, which relates well to superconducting qubit gates and readout operations but also other qubit operations (photons, cold atoms). They demonstrate that the work performed by a coherent pulse on the qubit is always larger than the work that can later be extracted from the qubit, *aka* its ergotropy. But this classical ergotropy bound is violated if the input field is a resonant single-photon pulse. This opens the door to some energy recovery at the end of computing.

470 The thermodynamics of quantum measurement is involving a few groups worldwide including the team of Alexia Auffèves from Institut Néel in Grenoble, France, IQOQI and the University of Innsbruck in Austria and Andrew Jordan's team at the University of Rochester, USA. See A two-qubit engine powered by entanglement and local measurements by Ingrid Fadelli, April 2021 which refers to Two-Qubit Engine Fueled by Entanglement and Local Measurements by Léa Bresque, Andrew Jordan, Alexia Auffèves et al, March 2021, PRL (5 pages), Alternative experimental ways to access entropy production by Zheng Tan, Alexia Auffèves, Igor Dotsenko et al, May 2021 (15 pages) and the colloquium A short story of quantum and information thermodynamics by Alexia Auffèves, March 2021 (14 pages). See also Stochastic Thermodynamic Cycles of a Mesoscopic Thermoelectric Engine by R David Mayrhofer, Cyril Elouard, Janine Splettstoesser and Andrew Jordan, October 2020 (18 pages) and Thermodynamics of quantum measurements by Noam Erez, 2018 (3 pages).

471 Quantum Reservoir Engineering must not be confused with Quantum Reservoir Computing which is an entirely different beast. Introduced by Keisuke Fujii and Kohei Nakajima in 2017, it is the quantum equivalent of a similar technique used in classical deep learning where a low-dimensional data input is projected onto a higher-dimensional dynamical system, the reservoir, generating transient dynamics that facilitates the separation of input states. It is particularly useful to analyze time series of complex data structures. See Quantum reservoir computing: a reservoir approach toward quantum machine learning on near-term quantum devices by Keisuke Fujii and Kohei Nakajima, November 2020 (13 pages).

472 See Quantum Reservoir Engineering by J.F. Poyatos, J.I. Cirac and Peter Zoller, 1996 (14 pages) and the associated presentation Quantum Reservoir Engineering by Peter Zoller, 2013 (86 slides).

473 See Measurement, Dissipation, and Quantum Control with Superconducting Circuits by Patrick Michael Harrington, 2020 (154 pages), Reservoir engineering using quantum optimal control for qubit reset by Daniel Basilewitsch et al, 2019 (13 pages), Reservoir (dissipation) engineering and autonomous stabilization of quantum systems, Quantic team, Inria, 2018 and Quantum reservoir engineering and single qubit cooling by Mazyar Mirrahimi, Zaki Leghtas and Uri Vool, 2013 (6 pages).

474 See Novel Technique for Robust Optimal Algorithmic Cooling by Sadegh Raeisi, Mária Kieferová and Michele Mosca, June 2019 (10 pages).

Gate-based quantum computing key takeaways

- Gate-based quantum computing is the main quantum computing paradigm. It relies on qubits and finite series of quantum gates acting on individual qubits or two and three qubits. Algorithms are implemented with series of quantum gates called "cirtuits". The main other paradigms belong to analog quantum computing and include quantum simulators and quantum annealers.
- To understand the effect of qubits and quantum gates, you need to learn a bit of linear algebra. It deals with Hilbert vector spaces made of vectors in highly multidimensional spaces, complex numbers, vectors and matrices. The Dirac Bra-Ket notation helps manipulate vectors and matrices in that formalism.
- A qubit is usually represented in a Bloch sphere, reminding us of the wave nature of quantum objects during computation. This wave nature is exploited with qubits phase control and entanglement which provokes interferences between qubits. Qubits entanglement is created with using conditional multi-qubit gates like the CNOT gate. These relationships are persistent in time during the execution of an algorithm.
- A qubit register of N qubits can store a linear superposition of 2^N basis states corresponding to the qubit computational basis, each associated with a complex number. But surprisingly, this exponential growth in size is not enough to create a potential polynomial or exponential speedup with quantum computing. You need a lot of entanglement and some non-obvious quantum gates like the T gate and so-called maximally entangled states to obtain interesting speedup. The nonlocalityof quantum entanglement can also explain part of the speedup of quantum computers.
- While the computational space grows exponentially with the number of qubits, a qubit register measurement at the end of quantum algorithms yields only N classical bits. You have to deal with it when designing quantum algorithms.
- Computation must usually be done a great number of times (at least in the NISQ regime) and its results averaged due to the probabilistic nature of qubits measurement. The number of "shots" however depends on the algorithm results, programming paradigm and type of error correction or error mitigation.
- Qubits measurement can be done in various ways, the main one being a classical projective measurement, if possible, a non-demolition one (QND) that will maintain the qubit in its collapsed state after measurement and not destroy it. Other techniques are used that are useful for qubits quality characterization and for quantum error corrections like a quantum state or quantum gate tomography.

Quantum computing engineering

After reviewing the basic principles of quantum physics and the logical dimension of gate-based quantum computing, let's look at the operational and physical operations of a quantum computer[475].

Quantum computer architectures depend closely on the characteristics of their qubits. In this section, we will rely on the most common universal quantum gate computer architecture, that of superconducting qubits based on the Josephson effect. It is notably used by IBM, Google, Intel, Rigetti and IQM. However, many of the architectural principles mentioned here are applicable to quantum computers using other types of qubits.

First and as a reminder, here are the main components of a classical computer that you also find in various shapes and forms in smartphones, tablets, personal computers, game consoles and servers. Its key component is its microprocessor. It retrieves data and programs from a storage system and copies them to memory (RAM) entirely or on the fly as needed. The microprocessor then reads the program's instructions from memory in its cache to execute it one after the other and use conditional branching.

Data and programs can be retrieved remotely over a network and from remote servers on the Internet. The whole system is controlled by physical interfaces at input (keyboard, mouse, touchpad, joystick, webcam, microphones, scanners) and generates output (displays, audio, printers, other peripherals). The processor can be complemented by a graphics processor (GPU). It is either external to the microprocessor, for demanding requirements such as in CAD and video games or integrated into the microprocessor as is the case for all most laptops and most desktops processors.

Depending on the configuration, the processor is surrounded by a variable number of external components that are soldered in the motherboard (Figure 213).

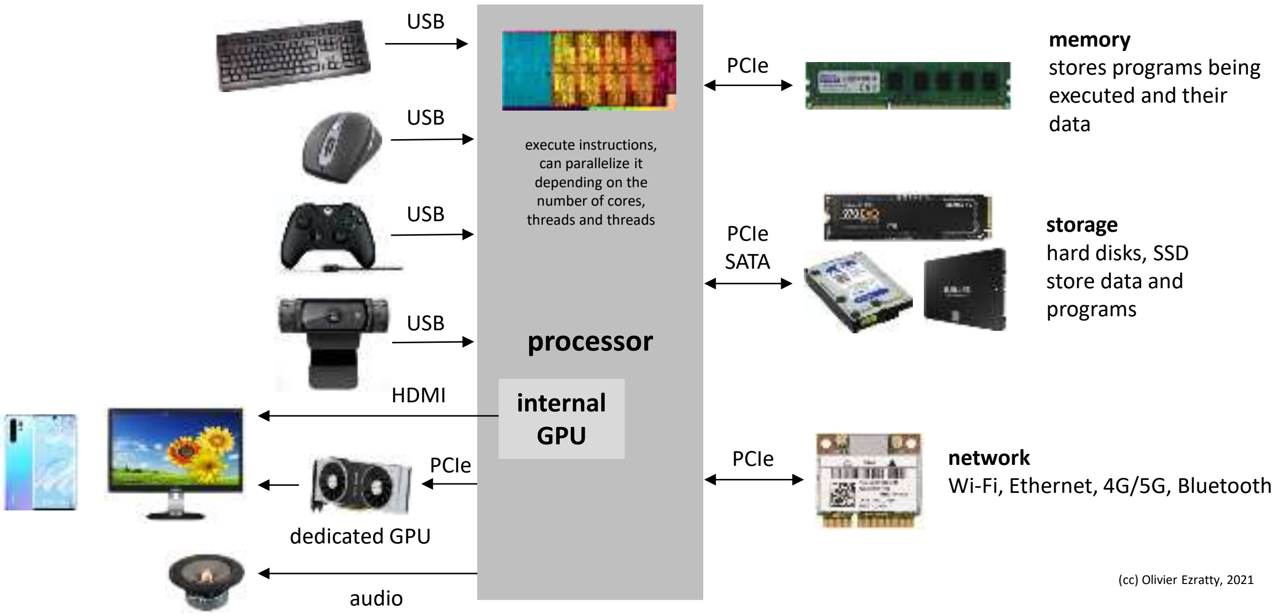

Figure 213: a classical personal computer hardware architecture. (cc) Olivier Ezratty, 2021.

475 I consulted a very large number of information sources to carry out this part, both on the research side and on the supplier side, such as IBM or D-Wave. Note [Quantum Computing Gentle Introduction]() from MIT, published in 2011 (386 pages) which describes precisely some mechanisms of quantum computers such as qubit state reading methods. It also describes quite well the mathematical foundations used in quantum computers. You can also enjoy an [8-minute video]() from Dominic Walliman, who explains the basics of the quantum computer!

This is the case of the Intel chipsets like the Z390, which complements the core processors and manages a large part of the computer's inputs/outputs. Wi-Fi and cellular modems are associated with antennas. Of course, an internal and external power supply and a battery for mobile devices must be added.

On the energy side, it is the processor and GPU that heat up the most and require passive or active cooling depending on their power drain. In embedded systems such as smartphones, this is done with heat conducts and air. In PCs, it is supplemented by one or more fans. In the most extreme cases, liquid cooling uses a water circuit to improve heat dissipation. One of the reasons why heat is generated by classical processing is the non-reversibility of classical computing.

Key parameters

Let's look at the definition of the key performance indicators of gate-based quantum computers. The best-known set of indicators was created by **David DiVincenzo** in 2000 when he was an IBM researcher (Figure 214). He is now a research professor at the University of Aachen in Germany[476].

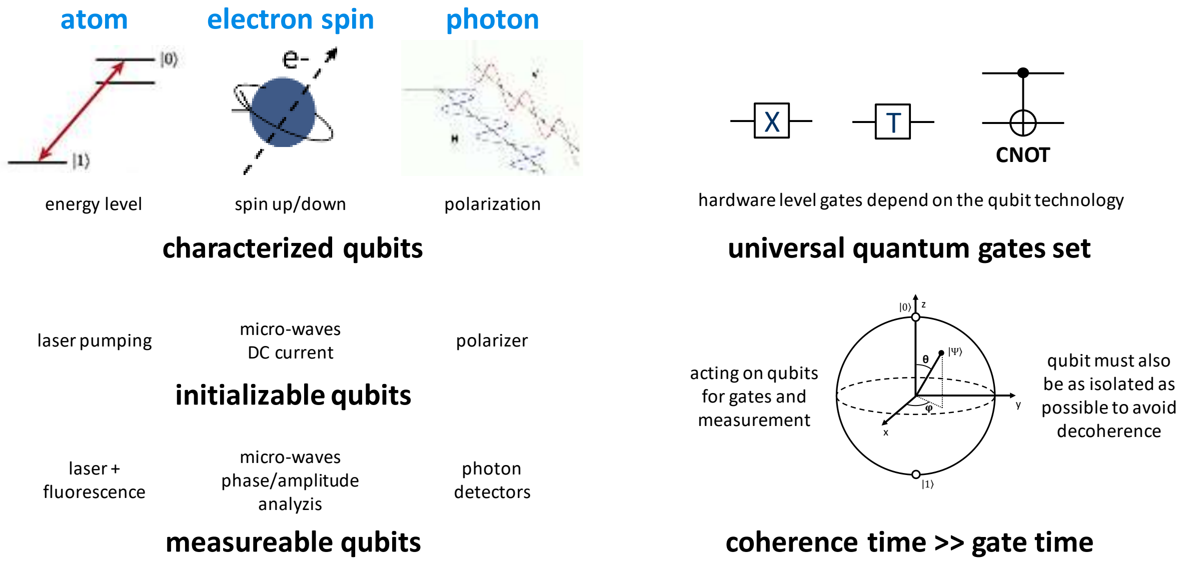

Figure 214: DiVincenzo gate-based quantum computing criteria. (cc) Olivier Ezratty, 2021, inspired by Pascale Senellart.

While individual qubits barely existed, he defined the basic technical characteristics of a universal gate-based quantum computer as follows:

Well-characterized qubits. Quantum computers use qubits that exploit quantum objects that can have two distinct and measurable states. Their physical characteristics are well known. The architecture is scalable in the sense that it can exploit many physical qubits and then, logical qubits relying on these physical qubits and quantum error correction codes.

Initializable qubits. In general, to the value |0⟩ often called "ground state" for the associated quantum objects, corresponding, for example, to the lowest energy level of an elementary particle or an artificial atom as for superconducting qubits.

Coherence times. It must be greater than quantum gates activation times. The time during which the qubits are in a coherent state must be greater than the quantum gates activation time in order to be able to execute an algorithm containing a sufficiently long sequence of quantum gates. Error correction codes using a large number of physical qubits have the benefit of extending this usable computing time.

[476] See The Physical Implementation of Quantum Computation by David DiVincenzo, 2000 (9 pages).

Universal quantum gates set. The quantum hardware must allow the creation of a universal gate set. It depends on the qubit technology. It requires a minimum set of single-qubit gates allowing the creation of any rotation in the Bloch sphere, completed by a CNOT two-qubits gate.

Measurement. With the ability to measure qubits state at the end of computing, which seems obvious. This measurement should not influence the state of other qubits in the system. Ideally, the measurement error rate should be well below 0.1%.

David DiVincenzo added two other optional criteria that are used instead for quantum communications:

Flying qubits conversion. The ability to convert static qubits into flying qubits, who are usually photons, and sometimes electrons.

Transport these moving qubits. from one point to another reliably and remotely. This will allow to manage quantum telecommunications, distributed architectures of quantum computers and to set up *blind computing* architectures allowing to distribute treatments while protecting their confidentiality. The technology will quickly become essential to enable the distribution of quantum computations over several quantum processors, a bit like we do with multi-core chips or with processing distribution architectures over several CPUs and several servers. Some vendors like IonQ have announced that they will rely on this architecture. This will be useful for qubit architectures that will be limited in the number of qubits, which may only be able to consolidate a few hundred at most. It will thus be necessary to be able to link remote processor qubits and keep them entangled. Different quantum interconnection techniques are possible. The most generic is optical and is not much constrained by distance. At rather short distances, microwave links are possible, particularly to couple superconducting qubits, as well as shuttling electrons[477].

DiVincenzo's criteria are quite basic. From a practical and operational point of view, quantum computers can also be characterized by another set of parameters as follows:

Number of qubits. It will condition the available computing power. As this power theoretically increases exponentially with the number of qubits, it is a key parameter. As of late 2021, the commercial record was 127 qubits with the largest IBM Quantum System available in the cloud. The number of qubits should be evaluated in its capacity to scale. Some technologies are easier to miniaturize and scale than others. It is necessary to integrate in this miniaturization both the quantum qubit chips and the elements that control them. On top of that, we must ensure that decoherence and noise does not increase as the number of qubits is growing. Today, trapped ions qubits have an excellent fidelity but don't scale well. Superconducting qubits seem to scale-up better but their fidelity is not stable as the number of qubits grows with existing industry vendors hardware although it could change in the future. Cold atom qubits scale a little better but with some practical limits in the number of controllable atoms. Electron spins qubits could scale best in theory.

Qubits connectivity. It will condition the quantum algorithms execution speed. The greater this physical connectivity, the faster the code execution will be. With a low connectivity, the compiler of the quantum code will have to add a lot more operations to link the qubits together, particularly relying on SWAP gates, that also add their own noise in the process. This connectivity varies greatly from one technology to another. In 2D technologies, as with superconducting and silicon qubits, it is limited to neighboring qubits. It seems better with some types of trapped ion qubits.

Qubit parallel operation. How qubit gates can be parallelized over different qubit zones without disruption will also condition the speed of execution of quantum algorithms.

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[477] Princeton University and Konstanz University in Germany are working on optical interconnection between CMOS quantum processors. This is documented in Quantum Computing Advances With Demo of Spin-Photon Interface in Silicon, 2018. The magic consists in transferring the quantum state of an electron spin to a photon at its phase level.

Qubits fidelities. When executing quantum gates and reading their state, qubit fidelity conditions the ability to execute long algorithms. It has a direct impact on the supported algorithm depth. It also impacts the capacity to run quantum error correction codes and create logical qubits with an arbitrary fidelity level. Fidelities are characterized for qubit initialization and reset, single and two qubit gates as well as for qubit readout.

Execution time. For both quantum gates and qubit state measurement. The first is obviously important to make the algorithms run as fast as possible. But the second is equally important because it is involved in error correction codes and therefore conditions the execution time of all algorithms.

Operating temperature. For the processor and their equipment which is very dependent on the type of qubits. The Holy Grail is of course to operate at room temperature. The currently operational quantum computers based on superconductors operate at a very low temperature of 15 mK (1 mK = 1 milli-kelvin, 0 kelvin = -273.15°C), but some types of qubits still in the research stage are supposed to operate at room temperature, such as those based on photons and NV centers (cavities in nitrogen-doped diamond structures like with Quantum Brilliance). However, this is not necessarily the case for associated equipment such as photon sources and detectors for photon qubits. Operating at very low temperature is a way to preserve the coherence of the qubits. But the lower the temperature, the smaller the energy that can be radiated by the qubits and their control electronics. Operating qubits at 100 mK or 1K, like with electron spin qubits, creates a much larger available cooling budget to control the qubits than operation at 15 mK. Neutral atoms and trapped ions are frequently said to operate at room temperature. As a matter of fact, they are always cooled at very low temperature, mostly using laser beams and their enclosing chamber is also frequently cooled at 4K.

Total energy consumption. We will investigate this and study it in a global manner with incorporating all quantum computer components: the processor itself, all its control electronics as well as the involved cryogenic systems, starting page 296. As of late 2023, quantum computers had a power drain sitting between 2 kW and over 150 kW depending on the qubit type and number of qubits.

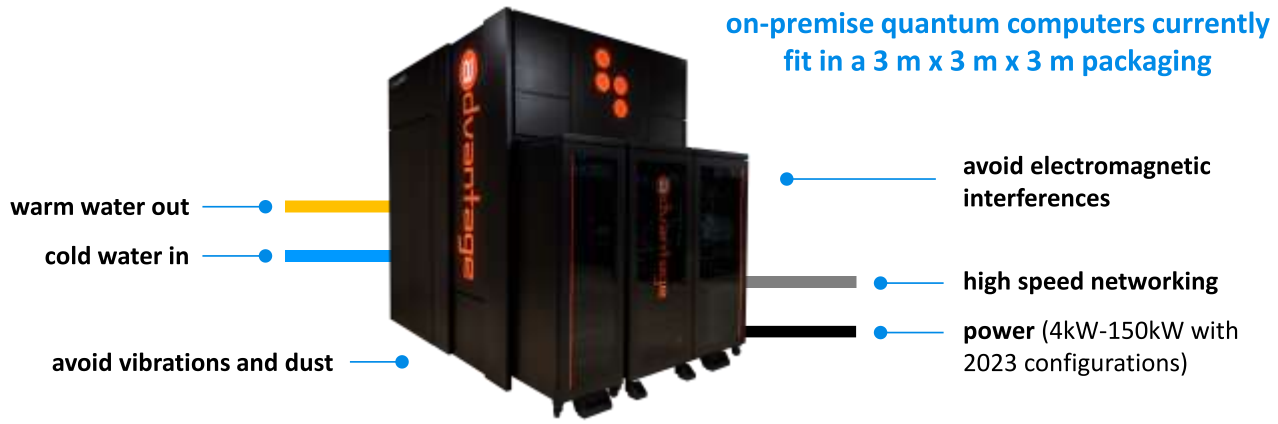

Figure 215: datacenters integration topics quantum for quantum computers. (cc) Olivier Ezratty, 2021-2023.

System rackability. How will quantum computers be deployed in data centers? Does it fit in standard rack systems? It is notably planned by the startup Pasqal, as well as for Quandela's photon generators and LightOn's optical processors, as well as micro-wave external electronics from companies like Zurich Instruments and Qblox. Alpine Quantum Technologies from Austria also announced in 2021 was fitting its trapped ion computing in two standard 19-inch racks. It is associated with issues of weight, space, cooling and power supply. What kind of fluids must be used for cooling, usually cold water, connected to the first stage compressor of cryostats, whatever their size? Quantum computers must also withstand the usual data centers conditions like vibrations, dust, and electromagnetic environment, or be separated in special isolated facilities (Figure 215). They could site in the modular building blocks used in the most recent data centers.

These last three operational parameters play a role when deploying computers or quantum accelerators in data centers. It plays a critical role since, for most applications, quantum computers will be offered through cloud services.

All these considerations to gauge the capabilities of a quantum computer involve the discipline of quantum computers benchmarking! As Kristel Michielsen points out, benchmarks can be used when the number of qubits is below 50 when comparing the rendering of algorithms between quantum computers and their emulation on supercomputers[478]. Beyond that, it will be more difficult.

Benchmarked quantum computers will generally have dissimilar characteristics: different universal quantum gates requiring compilers to assemble different quantum gates to execute the same algorithm, and different error correction codes, adapted to the qubit fidelities, circuit size and the primary quantum gate set of the compared computers. The dissimilarities will be much greater than between two Intel and AMD processors or two smartphone chips!

Quantum computers segmentation

There is not just one category of quantum computers, but many. We must at least distinguish gate-based quantum computers and analog computers, including quantum annealing computers such as the ones from D-Wave.

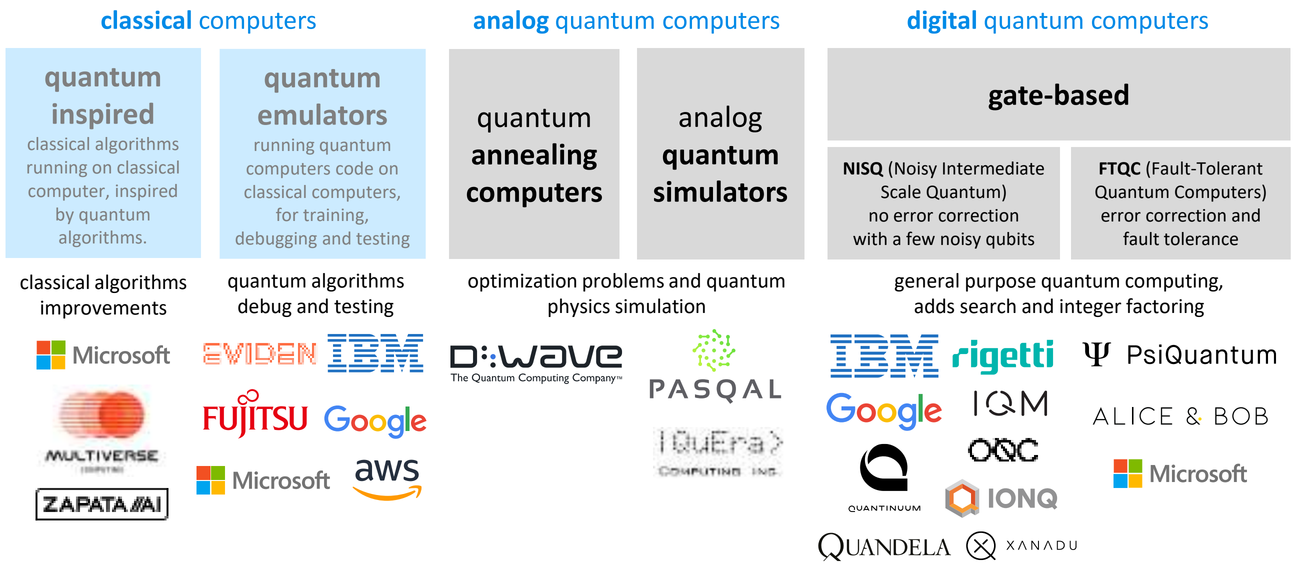

Figure 216: the different computing paradigms with quantum systems, hybrid systems and classical systems. (cc) Olivier Ezratty, 2022-2023.

But there are at least six categories of quantum computing paradigms as shown in Figure 216:

Quantum emulators (detailed page 963) are used to execute quantum algorithms on traditional computers ranging from simple laptops to supercomputers, depending on the number of qubits and algorithm depth and precision to be emulated. It is based on large vectors and matrices computing. Code emulation is used to test quantum algorithms without quantum computers. Quantum emulators are sometimes called quantum simulators, but this name should be avoided to prevent confusion with... analog quantum simulators. These are analog quantum computers simulating quantum physics phenomena, for example magnetism or the tridimensional structure of molecules. Quantum emulators may however also simulate qubit noise model like Atos/Eviden QLM emulator[479]. They can also

[478] In Benchmarking gate-based quantum computers, 2017 (33 pages).

[479] We can make a distinction between an exact digital simulation and approximate digital simulation, emulating a digital error rate that is equal or below NISQ hardware. This can help simulate a greater number of qubits.

reproduce the (quantum) physical characteristics of various qubits and in that case, they also implement some form of digital quantum simulation. To date, supercomputers can fully emulate up to the equivalent of 40 to 50 qubits. Records have however been broken with more than 100 qubits, with a low number of quantum gates and using various techniques like tensor compression. Emulating quantum computers requires a lot of power both on the memory side, to store 2^N quantum register states for N qubits, and for the associated processing that relies on floating-point matrix multiplications.

Quantum inspired computing is about using classical algorithms running on classical hardware that are inspired by quantum algorithms and bring some new efficiencies. They are not about emulating quantum code on a classical computer. Typical quantum inspired algorithms use tensor network libraries and techniques like MPS and DMRG.

Quantum annealing computers use the adiabatic theorem which consists in using a slow and controlled evolution of a set of qubits linked together according to a particular topology ("Pegasus" or "Zephyr" in the case of D-Wave). The process is first initialized in the ground state of the Hamiltonian and the adiabatic theorem guarantees the convergence of the system towards a low energy state, ideally the ground state. This technique is used to search for an energy minimum to solve various problems (simulations, optimizations, machine learning). The coefficients of the Hamiltonian are the couplings (weights of the interactions between qubits) and the self-couplings (weights of the qubits) and the variables of an instance are the spins of each qubit. Many problems can be translated into quantum annealing problems using QUBO or Ising problem formulations. D-Wave seems to bring interesting gains in computation time for some use cases, but this is strongly disputed by some specialists.

Quantum simulators work in an analog and not digital way, with continuous parameters linking the qubits together. The most common technique is based on neutral atoms, like with Pasqal and QuEra. It is also labelled “Rydberg quantum annealing”[480]. Trapped ions[481] [482] [483], superconducting qubits, spin qubits[484], photon qubits[485] and other various quantum systems can also be used for running quantum simulations[486], but no commercial vendor seems to promote this paradigm when they can also implement gate-based quantum computing which is supposed to be more generic[487]. Some vendors like ParityQC, IQM and Kipu Quantum are still proposing architectures mixing digital and analog quantum computing platforms.

Digital quantum computers *aka* universal quantum computers use qubits with quantum gates capable of executing all quantum algorithms. They are also labelled general purpose quantum computers. It is a concept associated with universal gate-based quantum computing, in opposition to analog computing which is said to have a more limited set of applications. It is also linked to an economic rationale. Indeed, computing markets usually develop naturally with generic platforms, not too narrowly specific ones.

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480 See Solving optimization problems with local light shift encoding on Rydberg quantum annealers by Kapil Goswami et al, August 2023 (18 pages).

481 See Non-equilibrium critical scaling and universality in a quantum simulator by A. De et al, September 2023 (28 pages).

482 See Interaction graph engineering in trapped-ion quantum simulators with global drives by Antonis Kyprianidis et al, Indiana University, October 2023 (27 pages).

483 See Realization of programmable Ising models in a trapped-ion quantum simulator by Yao Lu et al, November 2023 (11 pages).

484 See Analog Quantum Simulation of the Dynamics of Open Quantum Systems with Quantum Dots and Microelectronic Circuits by Chang Woo Kim, John M. Nichol, Andrew N. Jordan and Ignacio Franco, March-October 2022 (20 pages).

485 See Realizing tight-binding Hamiltonians using site-controlled coupled cavity arrays by Abhi Saxena, Arnab Manna, Rahul Trivedi and Arka Majumdar, Nature Communications, August 2023 (7 pages).

486 See Realizing tight-binding Hamiltonians using site-controlled coupled cavity arrays by Abhi Saxena et al, Nature Communications, August 2023 (7 pages).

487 Amazon is investigating it, in A scalable superconducting quantum simulator with long-range connectivity based on a photonic bandgap metamaterial by Xueyue Zhang, Oskar Painter et al, August 2022 (34 pages).

Gate-based quantum computers are currently limited to 433 qubits (IBM Osprey). Qubit quantum noise is detrimental to computing and requires the usage of logical qubits made of many physical qubits and quantum error correction codes (QEC). While waiting for these fault-tolerant quantum computers to ramp up with logical qubits, we are using non corrected qubits in the so-called NISQ for "Noisy Intermediate-Scale Quantum", an expression from John Preskill[488]. It describes existing and future general-purpose quantum computers supporting 50 to a few hundred physical qubits. These can run algorithms with a limited circuit depth due to the qubit error rates like variational quantum circuits. Their performance is improved by using quantum error suppression and quantum error mitigation techniques that we describe starting page 270. They are supposed at some point to exceed supercomputers computing capacities for solving specific problems. Then, much later, we will have fault tolerant (FTQC) quantum computers, with a very large number of physical qubits and over 100 logical qubits, which will really open the realm of useful quantum computing.

There are now several other variations of universal quantum computers that deserve some description:

Continuous variables quantum computers, or analog quantum computers with universal gates use qubits that store variable quantities between 0 and 1 and can be manipulated with quantum gates, also named 'qunats'[489]. This category of quantum computing was proposed in 1999 by Seth Lloyd and Samuel L. Braunstein[490]. They are usually based on continuous variable photons but other qubit types like trapped are used (Figure 217).

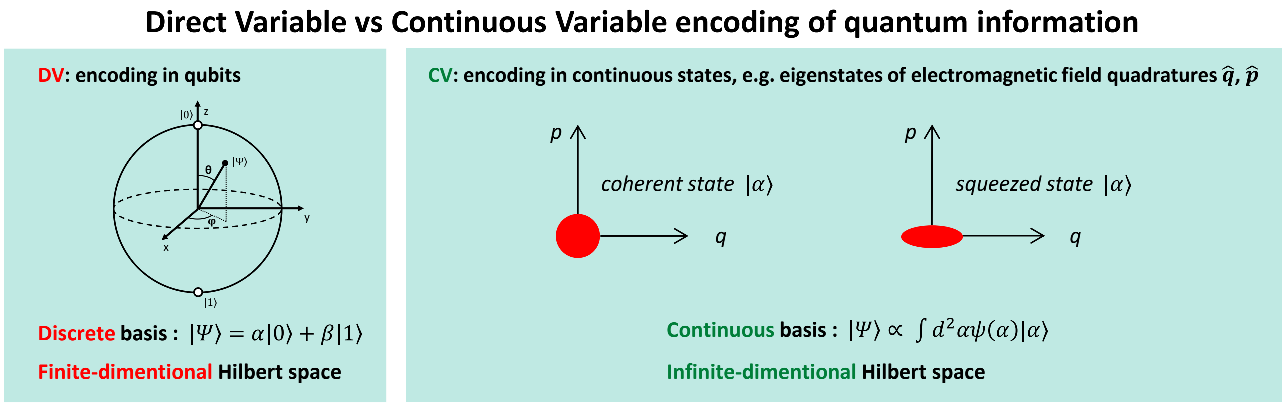

Figure 217: direct variable and continuous variable encoding of quantum information. inspired from Sub-Universal Models of Quantum Computation in Continuous Variables by Giulia Ferrini, Chalmers University of Technology, Genova, June 2018. (35 slides).

MBQC, or Measurement Based Quantum Computing, is an architecture adapted to flying qubits and particularly to photon qubits which can't easily be entangled to create two qubit gates. The process consists in creating a set of entangled qubits at the beginning of computing, aka cluster states. It is followed by qubits readouts in an ordered way, enabling the implementation of traditional gates. MBQC also implements some massive parallelism, adapted to the limited and finite processing depth of flying qubits. PsiQuantum plans to use a variant of this technique named FBQC. Quandela has also plans to implement MBQC with using its own proprietary cluster states generation technique.

Topological quantum computing is based on specific anyon qubits that are self-corrected. The low-level programming model of these qubits is much different from universal quantum computers. This is the path chosen by Microsoft. Its development seems to be quite sluggish. This technology still requires error correction codes and fault-tolerance mechanisms although with a lower overhead.

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[488] In Quantum Computing in the NISQ era and beyond in 2018.

[489] See Universal Quantum Computing with Arbitrary Continuous-Variable Encoding by Hoi-Kwan Lau and Martin B. Plenio, 2016 (5 pages) as well as Continuous-variable quantum computing in the quantum optical frequency comb by Olivier Pfister, 2019 (16 pages).

[490] See Quantum Computation over Continuous Variables by Seth Lloyd and Samuel L. Braunstein, February 1999 (9 pages).

Figure 218 shows another segmentation of these models with two dimensions: discrete or continuous data encoding and discrete of continuous variables computing given the vendor position is rough, some being positioned in various slots (Pasqal also wants to do gate-based computing)[491]:

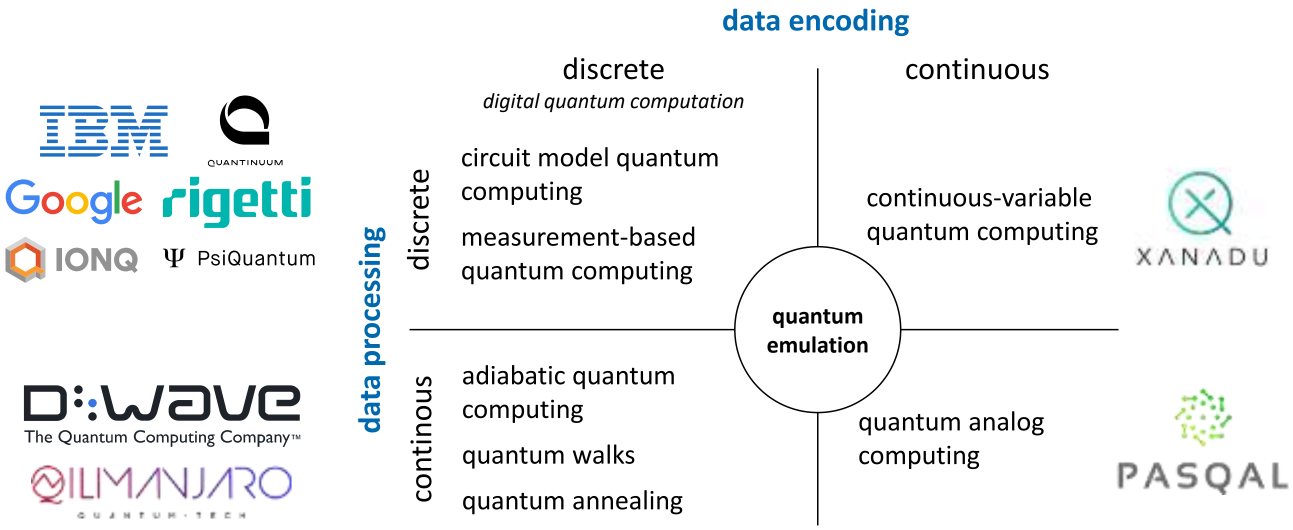

Figure 218: discrete vs continuous data encoding vs data processing.
Source: [Quantum computing using continuous-time evolution] by Viv Kendon, 2020 (19 pages).

Quantum Accelerator. It is a quantum computer used as a complement to a supercomputer or HPC, usually to run variational algorithms like VQE (Variational Quantum Eigensolvers) combining a classical part that prepares the data structure that feeds a quantum accelerator[492]. The QPU serves as an accelerator for the HPC which can be a node or the whole HPC, using CPU and/or GPUs/TPUs. GPUs/TPUs are themselves also accelerators for the CPUs. There are some design issues requiring tight integration between the HPC and the QPU, particularly with regards to batch loading and to the way the quantum algorithm is executed multiple times (Figure 219). A QPU contains itself a classical computer. It converts digital signals (gates) into analog signals (the microwaves or lasers controlling the qubits and handling their readout). This QPU computer will need to be as close as possible to the HPC computing capacities to improve the turnaround. It may lead to create custom designs integrating an HPC and one or several quantum accelerators[493].

Other quantum accelerator designs contain more or less generic higher-level software layers with connectors driving various quantum and classical architectures (annealers, gate-based, emulators)[494].

This inventory is only an appetizer. We will have the opportunity to detail these various architectures.

And we are always in for many surprise and new programming paradigms that nearly nobody in the ecosystem is evaluating like the "dark path holonomic qudit computation" coming from Sweden[495]. Go figure!

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[491] See Quantum computing using continuous-time evolution by Viv Kendon, 2020 (19 pages).

[492] See Quantum Accelerators for High-performance Computing Systems by Keith A. Britt et al, 2017 (7 pages).

[493] See Quantum Accelerator Stack: A Research Roadmap by K. Bertels et al, 2021 (39 pages) which proposes a detailed architecture for a quantum accelerator and See QPU-System Co-Design for Quantum HPC Accelerators by Karen Wintersperger, Hila Safi and Wolfgang Mauerer, Siemens AG and Technical University of Applied Sciences Regensburg, September 2022 (15 pages).

[494] See for example the proposals in Quantum Computer Architecture: Towards Full-Stack Quantum Accelerators by Koen Bertels et al, 2019 (20 pages).

[495] See Dark path holonomic qudit computation by Tomas André and Erik Sjoqvist, August 2022 (6 pages).

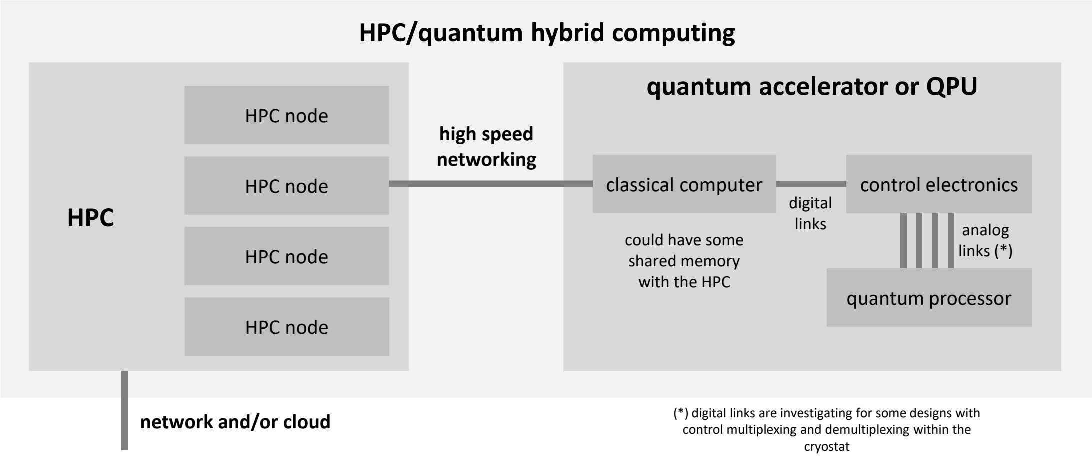

Figure 219: basics of a hybrid classical/quantum computing hardware architecture. (cc) Olivier Ezratty, 2021-2023.

Qubit types

Quantum computers physical qubits are devices that handle particles or quasiparticles with one physical property or observable that can have two possible mutually exclusive states, that can be initialized, modified with quantum gates and then measured.

They are sometimes individual quantum objects, as with atoms (trapped ions and cold atoms), electrons (quantum dots silicon qubits) or photons! And only one at a time! In the case of superconducting qubits or Majorana fermions, the quantum state is based on a large number of electrons arranged in Cooper pairs that share the same quantum state, the pairs of electrons that are created at superconducting temperature. With NV centers and some exotic qubits, qubits are constructed with ensembles of quantum objects or with heterogeneous quantum objects like mixing electron spins and atom nuclear spins.

Qubits can also be classified in two meta-breeds: stationary or moving (flying) as in Figure 220. Those based on trapped ions, cold atoms, electrons spin, NV centers and superconducting loops are stationary. Flying qubits are based on photons that physically circulate from quantum gate to quantum gate as well as on flying electrons. They move around from a source, through physical devices implementing quantum gates and land on detectors. In all cases, the quantum gates are dynamically activated by electronic circuits or lasers and operate on the qubits where they are (stationary qubits) or in the path of their transit (flying qubits).

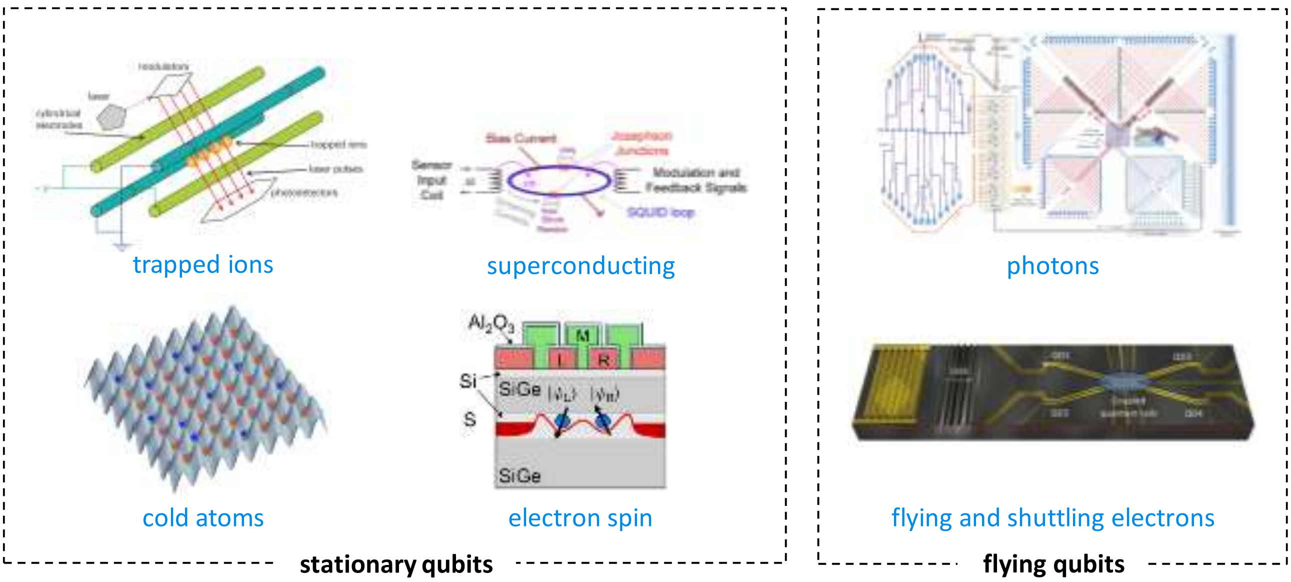

Figure 220: separating stationary and flying qubits. (cc) Olivier Ezratty, 2021.

Figure 221 and the following text describe the main types of qubits that are currently being studied, tested and sometimes commercialized[496]. Looking at these technologies reminds me of the Wacky Races movie and cartoons vehicles as well as the Tatooine podracers in Star Wars I, with an amazing technology diversity and true believers in their fate[497]. The only difference is we may end up with no single winner but several winners if not some forms of technology hybridization. Can we compare it to the Manhattan project from 1940-1945? It had only two main uranium and plutonium combustible options and some variations with the explosives, with a future project to create fusion bomb, which was implemented later in 1952. Here, with quantum computing, researchers and industry vendors are investigating many more options.

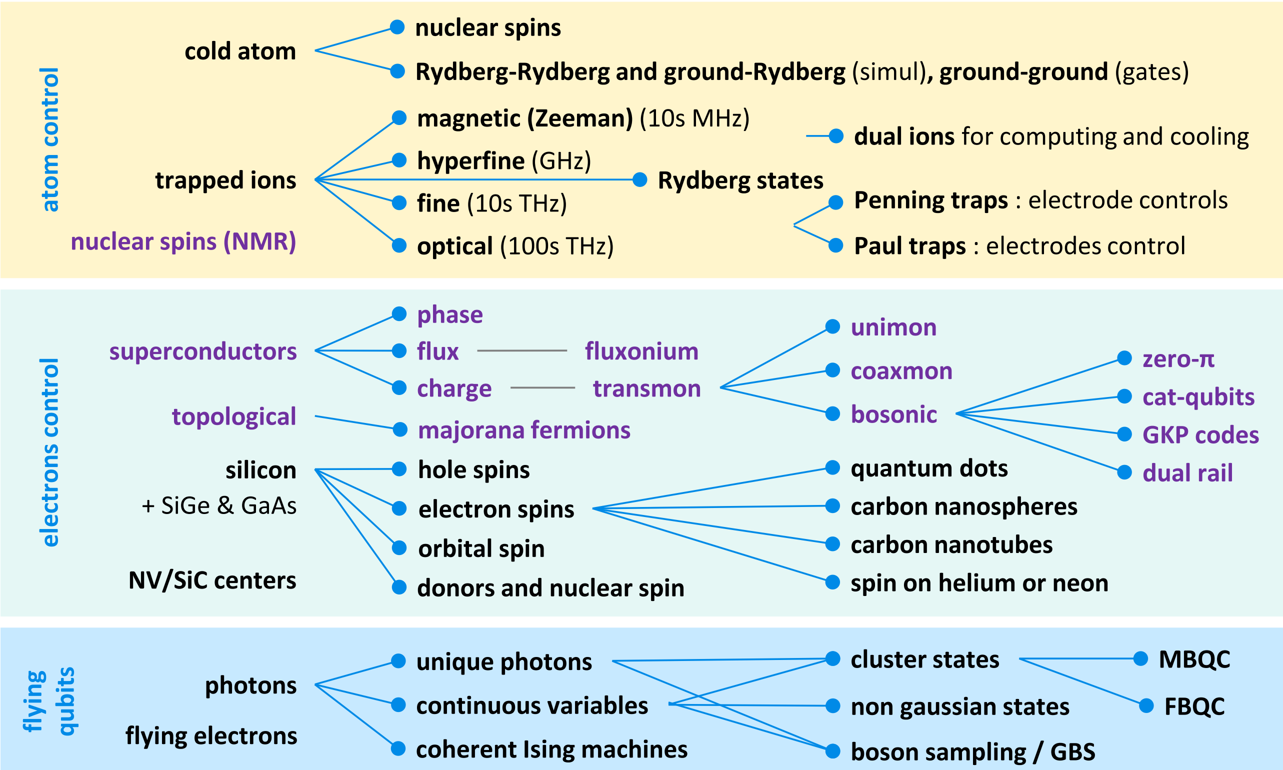

Figure 221: rough zoology of qubits classes and sub-classes. In purple, collective quantum objects, and black, individual quantum objects. (cc) Olivier Ezratty, 2021-2023.

Atoms

This is one of the oldest types of qubits. It consists in controlling atoms in vacuum with lasers, one qubit per atom. Cold atoms are neutral while trapped ions are ionized atoms. One key difference is how these atoms are controlled in space. Ions can be positioned with electrodes and magnetic fields while non-ionized atoms are only controlled by lasers. They both share a similar measurement technique using laser excitation, fluorescence and visual readout with some CCD or CMOS imaging sensor.

Trapped ions are atom ions that are kept in a vacuum and suspended by electrostatic suspension. They are electromagnetically trapped (Figure 222) and their initialization is done with laser optical pumping.

[496] See Roadmap on quantum nanotechnologies by Arne Laucht et al, 2021 (49 pages) which reviews some of these qubit types.

[497] See Noisy intermediate-scale quantum computers by Bin Cheng et al, March 2023 (50 pages) which review the main qubit types characteristics and their challenges and solutions to scale up.

Lasers are used to cool and stabilize the ions, exploiting the Doppler effect, with different energy transitions than those used to modify the state of the qubits. The most frequently used ions are calcium and strontium. Single-qubit quantum gates are activated by microwaves, lasers or magnetic dipoles. Lasers or electrodes are used for two-qubit quantum gates. While trapped ions are best-in-class for qubits fidelity and connectivity, it seems currently difficult to scale it beyond a couple dozen ions and they are very slow.

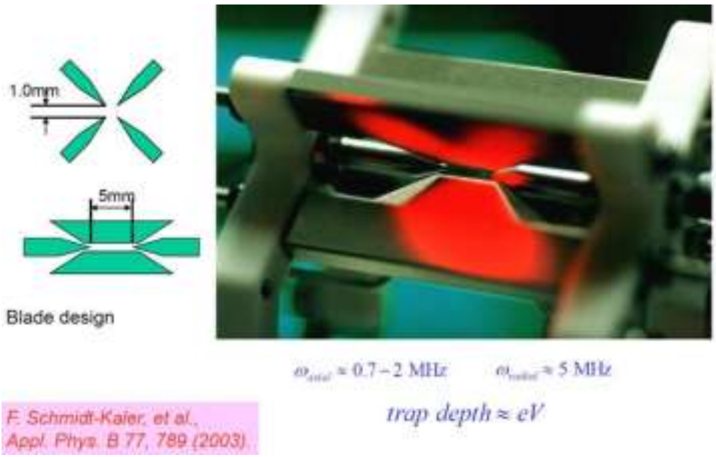

Figure 222: a typical Paul trap for trapped ions, created in 2003.

Neutral atoms, aka cold atoms, are cooled at very low temperatures, also using the Doppler effect and other laser-based techniques. The used elements can be rubidium, an alkaline metal, as well as strontium. The quantum state of these cold atoms is their energy level, which can use their Rydberg high-excitation states on some occasions, in analog quantum simulation mode, and lower-energy levels for gate-based implementations.

Nuclear magnetic resonance (NMR) qubits were tested over 20 years ago and are nearly completely abandoned besides one startup in China (SpinQ). Most of the time, they are based on using ensemble of atoms or molecules. They do not scale at all. It is a good demonstration that qubits research must remain open and cannot be settled too early around one or two technologies. Even now, it is too early to tell which qubit type will really scale to create useful quantum computers.

Electrons

This other category of qubits is about electrons that are controlled most of the time in solid-state circuits instead of vacuum like with cold atoms and trapped ions.

Superconducting qubits are based on the state of a superconducting current that crosses a very thin barrier in a loop, usually a metal oxide such as aluminum, using the Josephson effect[498] . There are several types of superconducting qubits: flux, phase and charge. The most common one is the transmon, a variation of charge superconducting qubits. In all cases, qubit observables are two very distinct states of a high frequency oscillating current flowing through the Josephson junction.

The oscillation is made possible by the fact that the loop integrates the equivalent of an inductance and a capacitance. The current oscillation is activated by microwaves pulses using frequencies between 4 and 8 GHz and transmitted by coaxial wires. In transmon qubits, the qubit observable is measured with a resonator integrated in the circuit which receives a microwave and sends it back. The readout electronic system splits out the amplitude and phase of the readout microwave to detect the qubit value. In some transmons, individual qubits activation frequency is tuned by a direct current flux bias line.

Superconducting qubits are relatively easy to manufacture because they are based on semiconductor circuit creation techniques even if some of the materials are different, such as niobium, tantalum and aluminum[499]. They are built on a dielectric substrate, usually with silicon or sapphire. These qubits are operating at 15 mK, requiring a dilution refrigerator.

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[498] See Digital readout and control of a superconducting qubit by Caleb Jordan Howington, 2019 (127 pages).

[499] See Practical realization of Quantum Computation Superconducting Qubits (36 slides).

This temperature is required for various reasons: qubits are driven by microwaves in the 4-8 GHz range and the current thermal noise is constrained by order of magnitude below the temperature corresponding to these microwaves' energy. The 4-8GHz corresponds to off-the-shelf microwave generation equipment and to the size of the capacitor and resonator used in the vicinity of the qubit Josephson junctions.

Superconducting qubits have many challenges dealing with scalability and materials quality. The microwave RF generators are usually located outside the cryogenic enclosure of the quantum processor, which requires a lot of cabling with, currently, about 3 to 4 cables per qubit. Qubits control frequencies can be different and tuned for adjacent qubits to limit the so-called crosstalk effect. Their fidelity is not best-in-class, and it seems challenging to maintain it the qubit number is growing.

Quantum dots electron spin qubits are developed with scalability in sight. Most of them use two electrons trapped in a quantum well, one containing the qubit and the other one used to measure it. These qubits are usually manufactured using silicon-based CMOS circuits. Silicon is often supplemented with various dopants. They benefit from the reuse of CMOS manufacturing processes that are already well mastered. These qubits are easy to miniaturize down to below 100 nm. They work at temperatures between 100 mK and 1K, higher than superconducting qubits, allowing the use of more electronics around the chip, to generate the microwaves and other electric signals required to create qubit gates and handle qubit readout. This promising technology is however less mature than superconducting qubits. No lab or company has really exceeded 15 functional qubits as of 2022.

NV centers (Nitrogen Vacancy) are artificial diamond structures in which a carbon atom has been replaced by a nitrogen atom near a carbon atom gap. Qubit states and control rely on a combination of electron, nitrogen and carbon ^{13}C nucleus spins. Qubit gates are implemented with microwaves, a magnetic field and an electric field. Entanglement is handled with photons, magnetic coupling or with controlling the core spin of neighboring ^{13}C carbon atoms via the use of microwaves to create a CNOT gate. Qubit readout is using a laser and fluorescence detection. There are many variants with other types of vacancies like with silicon carbide (SiC).

Majorana fermions are anyons or quasiparticles which are particular states of Cooper's pairs in condensed matter at very low temperature (Figure 223).

These qubits use braiding, a special topology that makes it possible to implement error correction at the qubit level. The promise is to enable the creation of scalable fault-tolerant quantum computers. These must also be cooled to a temperature close to absolute zero, around 10mK. This is the path chosen by Microsoft. The existence of the fermions of Majorana is not yet proven. It is one of the most hazardous paths to quantum computing. Majorana fermions are often discussed but they belong to a broader category named "topological matter" and "many-body systems".

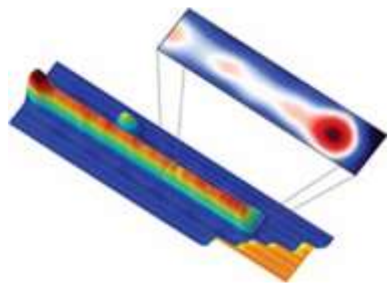

Figure 223: researchers may have seen Majorana fermions, but that's not really sure.

The main problem is… we are not sure these anyons and Majorana fermions really exist. It is still a work in progress with ups and downs.

Flying qubits

Flying qubits are special because they travel from the place where they originated, traverse physical devices acting on them and terminate their journey on a sensor measuring one observable. They have a limited time available to run any computing, including a finite and small number of quantum gates.

Photons are the most common flying qubit and there are many varieties of implementations. One type is based on a horizontal/vertical polarization observable. Others use continuous variables qubits. Boson sampling systems use multi-modes photons. It is quite difficult to implement two qubit gates with these photon qubits, thus the alternative of the MBQC architecture that is an interesting workaround. Also, photon generation follows a probabilistic pattern which makes things complicated when the number of qubits grows. Most of these qubits operate at room temperature, but the photon sources and their detectors must however usually be cooled to temperatures between 4K and 10K, which is much less demanding than the 15 mK of superconducting qubits or the 1K of silicon qubits.

Flying electrons are at a pure research stage qubit technology using traveling electrons[500]. It is based on using single-electron transport circulating on wave guide nanostructures built on semiconductors circuits, mostly GaAs, leveraging Coulomb coupling, quantum charge Hall effect and surface acoustic waves. Single- and two-qubit quantum gates can be realized on such circuits. Electrons can fly at a distance of 6 to 250 microns.

Electrons are created by producing THz photons which are converted into electrons. One qubit uses two-electron paths for states |0⟩ or |1⟩[501]. At the end of processing, these flying electrons are detected by a quantum dot (Figure 224).

This technique could also be used to create shuttling electrons qubits connecting static quantum dots-based qubits together. A few labs in the world are working on this including NPL in the UK, Ruhr-Universität Bochum, NTT, ERATO-JST and AIST in Japan, Weissman Institute in Israel, CEA-Leti and Institut Néel in Grenoble, France.

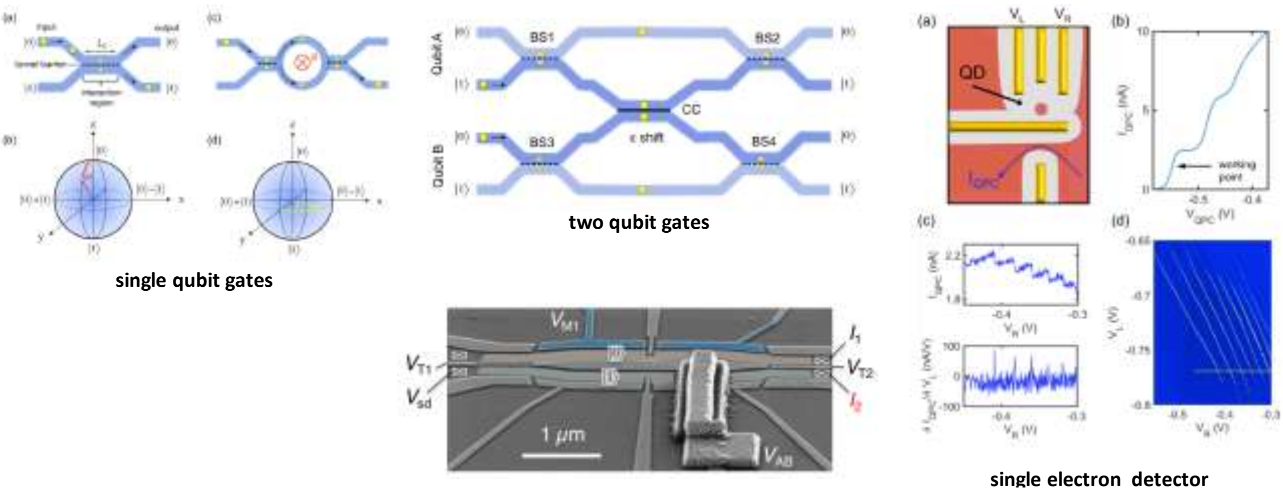

Figure 224: flying electrons in their waveguides. Their circuit architecture has some commonalities with photon circuits. Source: Coherent control of single electrons: a review of current progress by Christopher Bäuerle, Xavier Waintal et al, 2018 (35 pages).

Exotic qubits

Many research labs are working on using exotic qubits of various kinds. Most of the time, these qubits are at the fundamental research stage and far away from industrialization or even, sometimes, are not yet materialized with a real single functional qubit.

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500 See the review paper Semiconductor-based electron flying qubits: Review on recent progress accelerated by numerical modelling by Hermann Edlbauer, Xavier Waintal, et al, July 2022 (44 pages).

501 See Electrical control of a solid-state flying qubit by Michihisa Yamamoto, Christopher Bäuerle et al, 2017 (17 pages), Coherent control of single electrons: a review of current progress by Christopher Bäuerle, Xavier Waintal et al, 2018 (35 pages) and Macroscopic Electron Quantum Coherence in a Solid-State Circuit by H. Duprez et al, 2019 (10 pages).

In the **atom realm**, we can count with rare-earth ions in an insulating solid-state matrix[502], molecular ions[503], cold atom ensembles[504], 2D organic molecule networks[505], LCD base nematic qubits[506] and chemical compounds that have photon-controlled state transitions[507].

Molecular magnets are being explored[508], one variant being made with terbium and with four possible spin related quantum levels, creating qudits, with d=4. The small name of these magnets is SMM for Single-Molecule Magnets. The molecule used is TbPc2 also called "bis (phthalocyaninato) terbium(III)" (Figure 225). Their state is measured with a phase-measuring interferometer. Their advantage is their stability. But they are relatively difficult to control[509].

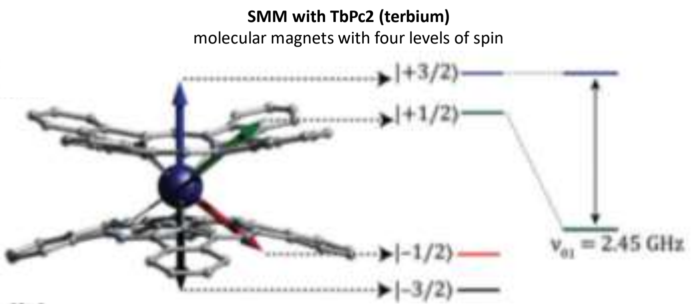

Figure 225: TbPc2 is a molecular magnet molecule used in prototype quantum processors. Source: Molecular spin qudits for quantum algorithms by Eufemio Moreno-Pineda, Clément Godfrin, Franck Balestro, Wolfgang Wernsdorfer and Mario Ruben, 2017 (13 pages).

In the **electrons realm**, you find various topological materials[510], various forms of graphene based qubits[511], carbon nanotubes-based mechanical oscillators[512], electron spin in magnetic materials in Van der Waals crystals made of chromium[513] and electrons on solid neon[514].

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502 See Universal Quantum Computing Using Electronuclear Wavefunctions of Rare-Earth Ions by Manuel Grimm et al, 2021 (19 pages).

503 See the review paper Molecular-ion quantum technologies by Mudit Sinhal and Stefan Willitsch, University of Basel, April 2022 (15 pages). Difficult to cool molecules with lasers. Destructive measurement.

504 See Quantum supremacy with spin squeezed atomic ensembles by Yueheng Shi et al, April 2022 (12 pages)

505 See Blueprint of optically addressable molecular network for quantum circuit architecture by Jiawei Chang et al, September 2022 (11 pages).

506 See Nematic bits and universal logic gates by Ziga Kos and Jörn Dunkel, August 2022 (10 pages).

507 See Functionalizing aromatic compounds with optical cycling centres by Guo-Zhu Zhu et al, UCLA, Nature Chemistry, July 2022 (6 pages).

508 See Blueprint of a Molecular Spin Quantum Processor by A. Chiesa et al, May 2023 (16 pages).

509 See Molecular spin qudits for quantum algorithms by Eufemio Moreno-Pineda, Clément Godfrin, Franck Balestro, Wolfgang Wernsdorfer and Mario Ruben, 2017 (13 pages). This work was carried out in partnership with the Karlsruhe Institute of Technology in Germany. And also the thesis Quantum information processing using a molecular magnet single nuclear spin qudit by Clement Godfrin, 2017 (191 pages).

510 See Anomalous normal fluid response in a chiral superconductor UTe_2 by Seokjin Bae et al, July 2021 (5 pages) and Multicomponent superconducting order parameter in UTe2 by I. M. Hayes, July 2021.

511 See Visualization and Manipulation of Bilayer Graphene Quantum Dots with Broken Rotational Symmetry and Non trivial Topology by Zhehao Ge et al, 2021 (19 pages).

512 See Proposal for a nanomechanical qubit by F. Pistolesi, Andrew Cleland, A. Bachtold, August 2021 (19 pages).

513 See Unique quantum material could enable ultra-powerful, compact computers by Ellen Neff, Columbia University Quantum Initiative, May 2022, referring to Coupling between magnetic order and charge transport in a two-dimensional magnetic semiconductor by Evan J. Telford et al, Nature Materials, May 2022 (15 pages). The initial title is of course quite overselling. One simple indication in the scientific paper: the words qubits, gates and entanglement are not even mentioned. So, a powerful quantum computer is very far in this roadmap even though it could operate at 132 K which is considered to be "hot" in quantum computing (ambient temperature is 300K)!

514 See Building a better quantum bit: New qubit breakthrough could transform quantum computing by Bill Wellock, Florida State University, May 2022, referring to Single electrons on solid neon as a solid-state qubit platform by Xianjing Zhou, David I. Schuster et al, Nature, May 2022 (16 pages). The team created its qubit by freezing neon gas into a solid at very low temperatures, spraying electrons from a light bulb onto the solid and trapping a single electron there.

You also have quantum neural networks using variations of superconducting qubits[515], quantum memristors[516], toponomic quantum computing which is a variant of topological computing[517] and various qubit hybridization techniques to couple fast operating qubits and long coherence time qubits for implementing some sort of quantum memories, like with associating superconducting qubits with NV centers, or superconducting qubits with yttrium iron garnet magnons[518].

Figuring out the TRL of these proposals is usually easy: it is very low (Figure 226)! Particularly when you don't have any published one and two qubit fidelities data.

The joke here is to talk about "negative TRLs". These are most of the time interesting physics experiments but no functional qubit, a fortiori, entangled qubits and related fidelities. Sometimes, there are real use cases but not in quantum computing and more in quantum sensing. It won't of course present research laboratories communications departments to fuel the hype with their stack of overpromises.

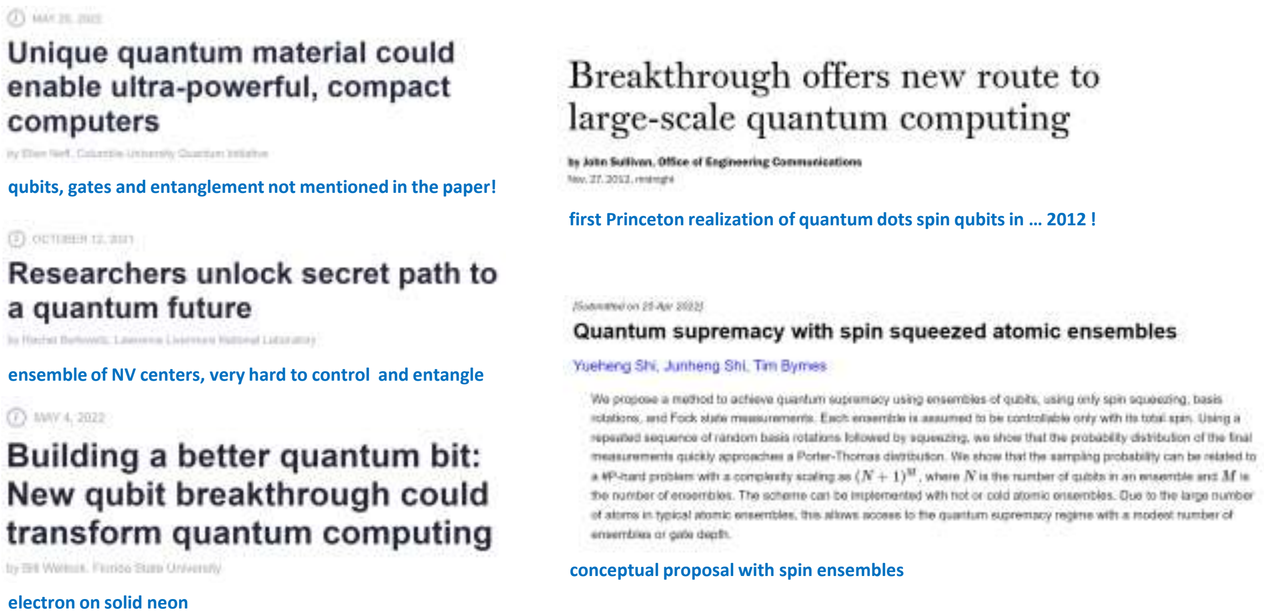

Figure 226: examples of research laboratories communication on new exotic qubits with very low TRL! 2022.

Figures of merits

Let's now inventory the various figures of merit of these qubit architectures:

Qubits stability which is evaluated with their coherence time (the T_1 we'll describe later when discussing error correction page 240). Associated with quantum gate times and error rate, it conditions the number of quantum gates that can be chained in an algorithm. The most stable qubits so far are trapped ions based but as far as you don't have too many of them.

Qubits fidelity is related to the errors level that is evaluated with single and two qubit gates as well as with initialization and readout. Again, the best-in-class are trapped ions. We cover that starting 246.

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[515] See Coherently coupled quantum oscillators for quantum reservoir computing by Julien Dudas, Julie Grollier and Danijela Marković, April 2022 (4 pages), a quantum reservoir neural network implementation on a Josephson parametric converter.

[516] See Quantum Memristors with Quantum Computers by Y.-M. Guo, F. Albarrán-Arriagada, H. Alaeian, E. Solano, G. Alvarado Barrios, PRA, December 2021 -August 2022(7 pages) and Entangled quantum memristors by Shubham Kumar, Enrique Solano et al, arXiv and PRA, July & December 2021 (9 pages).

[517] See Toponomic Quantum Computation by C. Chryssomalakos et al, February 2022 (5 pages).

[518] See Analog quantum control of magnonic cat states on-a-chip by a superconducting qubit by Marios Kounalakis et al, PRL, TU Delft, Tohoku University and CAS in China, July 2022 (14 pages).

Qubits connectivity is the way they are linked together, which conditions many aspects such as algorithms execution speed, the depth of the algorithms that can be exploited and even the types of error correction codes that can be used. Best-in-class qubits for this respect are again trapped ions in 1D structures, although they do not scale well.

Large scale entanglement, without being limited to the immediately neighboring qubits. So far, nobody does it really well.

Operating temperature and for the accompanying electronics. The best are NV centers which are supposed to work at ambient temperature, and the worst are superconducting qubits, requiring 15 mK.

Qubits density and their control electronics which impacts scalability. This rather favors quantum dots electron spin qubits.

Manufacturing process which depends on many parameters. In the case of neutral atoms, for example, it is not necessary to create specialized circuits to control the qubits, whereas it is necessary for most other qubit technologies.

Scalability potential which depends on many systems parameters, both at the fundamental level with the qubit stability and fidelities at large scale but also with the various enabling technologies. Unfortunately for your forecasting, scalability potentials do not align with qubits technologies present maturities!

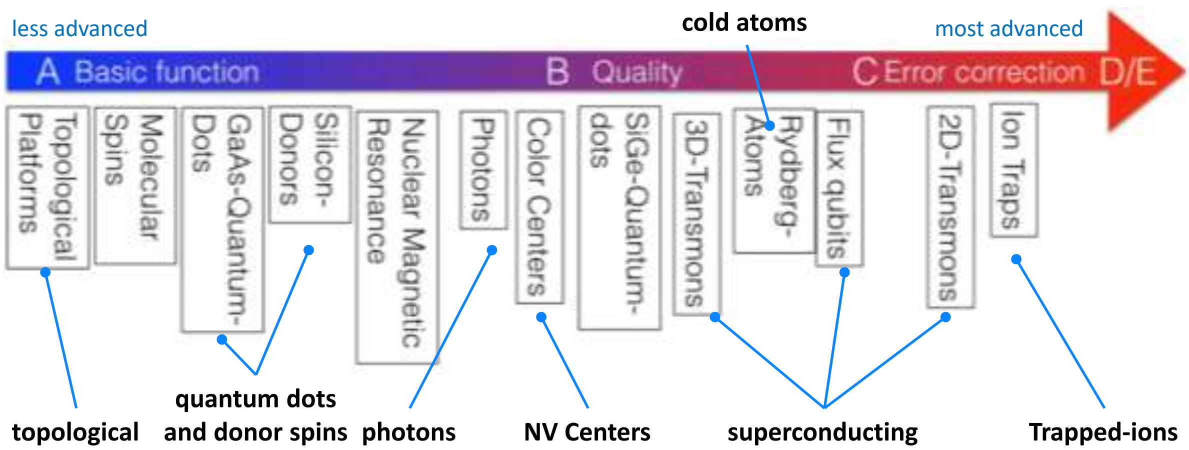

Figure 227: degree of maturity of various qubit technologies. Entwicklungsstand Quantencomputer (State of the art of quantum computing, in English, June 2020 (266 pages). It is not an indication of how these various qubit technologies can scale.

The level of qubits is evolving rapidly. It is described in this excellent document from the German cybersecurity agency, who produced Figure 227[519]. It mentions other technologies not listed in this inventory.

Figure 228 shows another way to put it[520]. It segments the types of qubits according to three dimensions: the clock frequency of the quantum gates (roughly, the gates number that can be executed per second), the number of operations before errors occur, and the quantum gates fidelity (separating the one- and two-qubit gates). These last two axis are roughly homothetic because the number of operations before errors are generated depends on the error rate.

[519] See Entwicklungsstand *Quantencomputer* (*State of the art of quantum computing,* in English, June 2020 (266 pages).

[520] See Introduction to Quantum Computing by William Oliver from MIT, December 2019 (21 slides) and Engineering Quantum Computers by William D. Oliver, December 2018 (15 slides).

Trapped ions have better gates than superconducting qubits but are quite slow.

Silicon qubits are for the moment quite fast, at least, as fast as superconducting qubits.

Neutral atoms are slower in gate based mode.

A last axis is missing: the number of qubits as of today and technology scalability. The chart was made in 2019 and may be outdated for some qubit types due to ongoing progress in qubit designs.

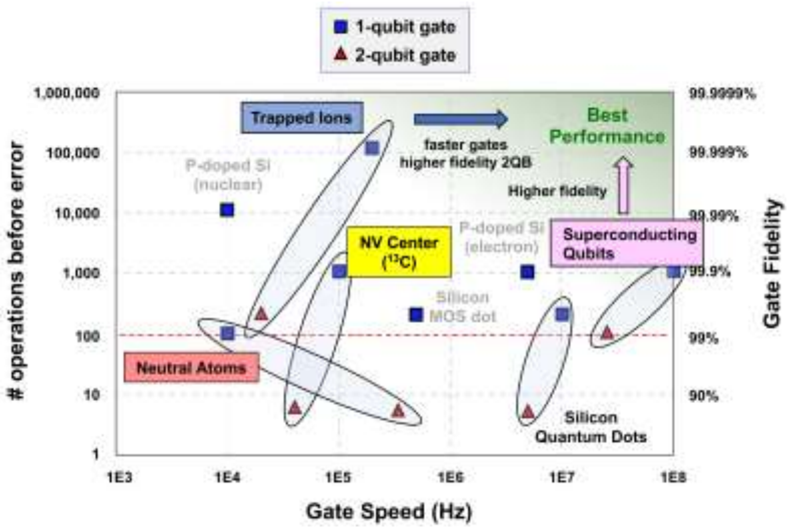

Figure 228: comparison of qubit computing depth and gate speed. Source: [Engineering Quantum Computers](#) by William D. Oliver, December 2018 (15 slides).

Architecture overview

We will provide here an overview of the general architecture of a quantum computer, using the example of a superconducting qubit accelerator (see Figure 229 and Figure 230).

First, a bit like some external GPUs, quantum computers are implemented as co-processors or accelerators of classical computers that power and control them. A quantum computer is always driven by a classical computer, as can be a GPU for video games or for training neural networks in deep learning. These conventional computers are used to run the programs that drive the quantum processor with physical operations to be performed on the qubits and are interpreting qubits readout results.

The classical computer closely controls the operation of the quantum computer by triggering at a precise rate the operations on the qubits that are performed by various electronic devices creating various electronics and photonic signals controlling quantum gates and quantum readout. It takes into account quantum gates execution time and the known qubits coherence time, i.e., the time during which the qubits remain in a state of superposition and entanglement.

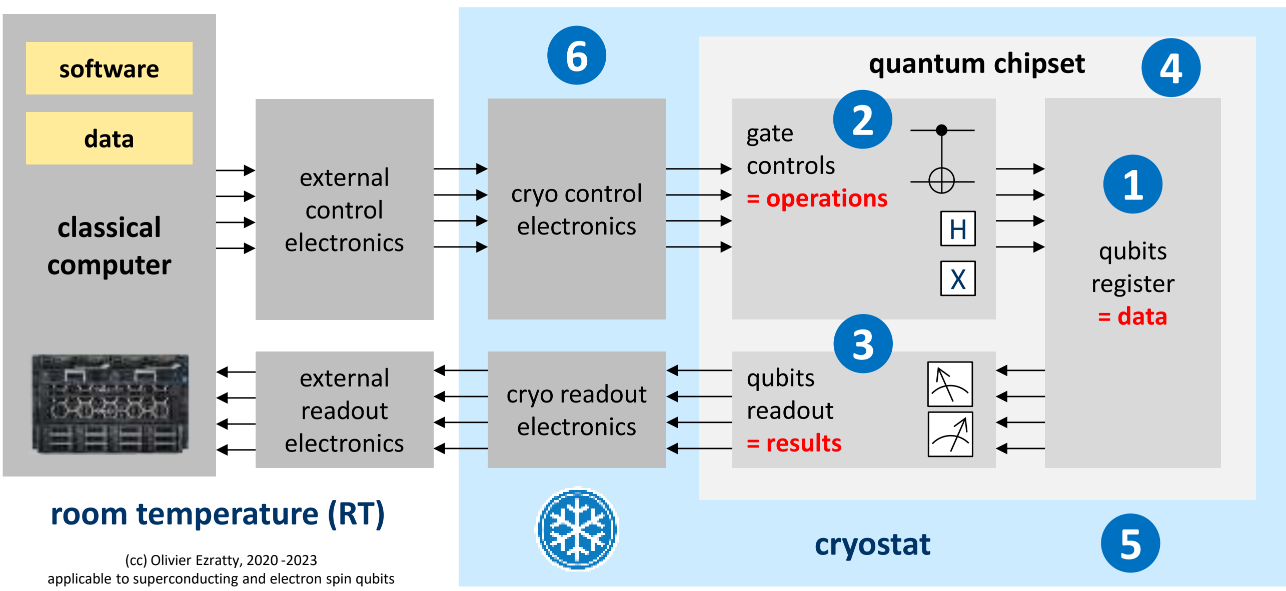

Figure 229: typical high-level architecture of a gate-based quantum computer. (cc) Olivier Ezratty, 2020-2023.

In addition to its classical control computer, our quantum computer includes at least the components labeled from 1 to 6 that we will describe one by one, first with an overview below, then later, with a more detailed view. The other types of quantum computers have similarities and differences that we will mention whenever relevant.

❶ **Quantum registers** are collections of qubits. The benchmarked record is with 433 superconducting qubits from the IBM Osprey QPU that went live in May 2023. Quantum registers store the information manipulated in the computer and exploit the principle of superposition and entanglement. To make a parallel with classical computing, quantum computing is implementing in-memory processing.

❷ **Quantum gate** controllers are physical devices that act on the quantum register qubits, both to initialize them and to perform quantum gates on them. These gates are applied iteratively, according to the algorithms to be executed. They can also be used to manage error correction codes. Quantum gates feed registers with both data and instruction. These are not separate operations like with classical microprocessors. In a way, quantum computing can be considered as a sort of in-memory processing computing architecture.

❸ **Readout** of qubit states is used to obtain the result at the end of the sequential execution of an algorithm's quantum gates and to evaluate error syndromes during quantum error correction. This cycle of initialization, calculation and measurement is usually applied several times to evaluate an algorithm result. The result is then averaged to a value between 0 and 1 for each qubit in the quantum computer's registers. The signals coming from qubit readout are then converted into digital values and transmitted to the conventional computer which controls the whole and implements results interpretation. In common cases, such as with D-Wave and IBM, computing is repeated at least a couple thousand times. The reading devices are connected to their control electronics via superconducting wires in the case of superconducting computer qubits.

❹ **Quantum chip** usually includes quantum registers, quantum gates controls and measuring devices when it comes to superconducting or electron spin qubits. These are fed by microwaves coming from outside the chip. Devices are more heterogeneous for other types of qubits, such as those that use lasers for initialization, quantum gates and qubit measurement like with trapped ions and cold atoms. Current chips are not very large. They have the size of a full-frame or dual-format photo sensor for the largest of them. Each qubit is relatively large, their size being measured in microns for superconducting qubits or down to 100 nm for electron spin qubits whereas modern CMOS processor transistors now have transistor sizes around 5 nanometers. The chip for superconducting and electron spin qubits is a chip of a few square centimeters. It is usually integrated in an OFHC (Oxygen-Free High thermal Conductivity) copper packaging which is purified and freed from oxygen, limiting thermal conductivity. This package is fitted with coaxial connectors so that it can be fed by the microwaves controlling qubit gates. In the latest superconducting processors from IBM and Google with 53 qubits, more than 160 of these connectors are required. The chip package is integrated in two small concentric aluminum and Cryoperm (from **MµShield**) magnetically insulated enclosures.

❺ **Cryogeny** usually keeps the qubit chip and its surrounding control electronics at a temperature close to absolute zero. It contains part of the control electronics and the quantum chip(s) to avoid generating disturbances that prevent the qubits from working. The Holy Grail would be to operate qubits at room temperature but the corresponding architectures such as in NV centers are not yet operational and there are still practical performance reasons to operate it at low-temperatures. The cryostat uses a mix of helium 3 and 4 to cool the components inside the chandelier while its compressor is itself cooled with cold water coming from another compressor, similar to the compressors used in classical air conditioning. Other types of qubits use cryostats in different places: with cooling photon sources or detectors in photon qubits systems, or for cooling ultra-vacuum pumps with cold atoms.

❻ **Control electronics** in the cryostat enclosure. The qubit control electronics drive the physical devices used to initialize, modify, and read the qubit status. With superconducting qubits, quantum gates are activated with microwave pulses using frequencies between 4 and 8 GHz and generally located outside the cryostat. These microwaves are transmitted on coaxial electrical wires between their source and the quantum processor, with superconducting cables below 4K. Their generators still take up a lot of space. They are not very miniaturized at this stage. Interesting work aims at integrating these microwave generators and readers inside the cryostat enclosure, if only to limit the wiring. These are frequently based on cryo-CMOS technology, CMOS components that are tailored to work at low temperature, 4K for many and as low as 20 mK for some. Figure 230 provides a rough representation of an entire superconducting qubits based quantum computer.

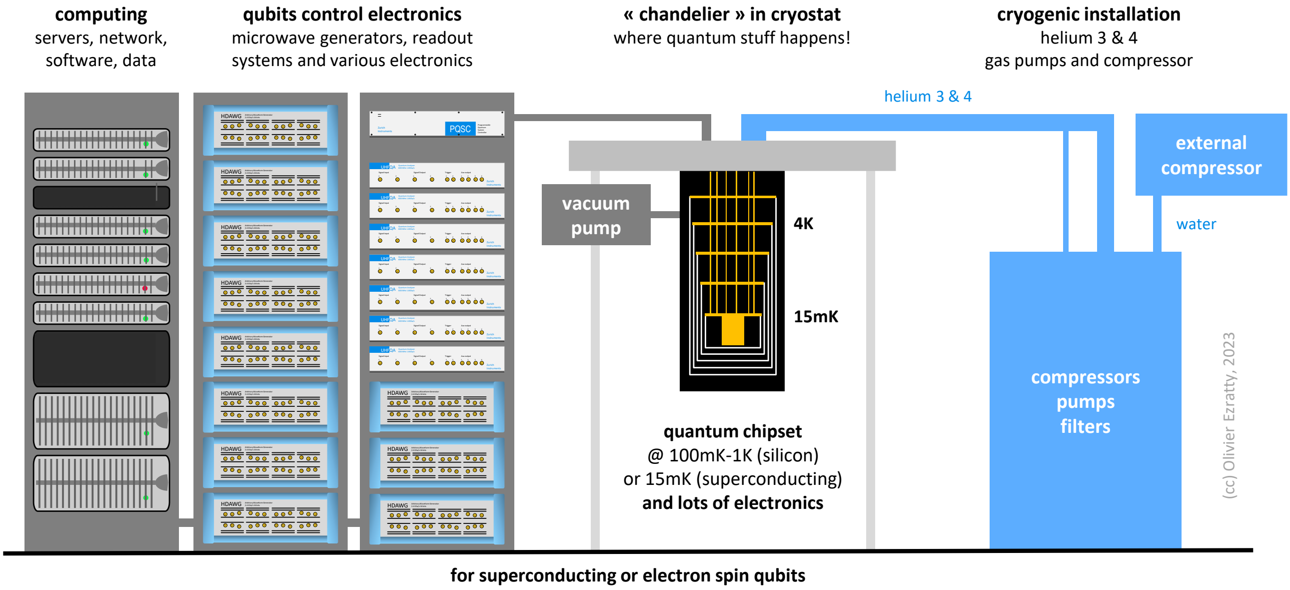

Figure 230: typical physical components of a superconducting qubit quantum computer. It contains a classical computer that drives the whole system. (cc) Olivier Ezratty, 2020-2023.

Processor layout

To better understand the previous explanation, here is a chip layout with 8 superconducting qubits, from ETH Zurich shown in Figure 231. Although it is already a few years old, the underlying concepts are generic.

- **Qubits** are located in the white rectangles. These are tiny Josephson effect superconducting circuit loop.
- **Coupling circuits** link them together. It is used to control entanglement between pairs of qubits.
- **Single-qubit gates** use the blue and purple contacts. It sends microwaves to the qubits. These pins are powered via cables by very high frequency current sources, sending microwaves photons, between 4 and 8 GHz. These frequencies must be different between adjacent qubits of the same circuit to avoid crosstalk. It is the combination of these frequencies that will trigger different types of quantum gates and entanglements between adjacent qubits.
- **Measurement** takes place with other circuits, also fixed in the component. In superconducting qubits, these are magnetometers which are then connected to the outside of the vacuum chamber and cooled by superconducting cables. These are driven by microwaves.

Qubits must interact with each other but as little as possible with their environment until measurement. This is one of the reasons why they are usually cooled to a temperature close to absolute zero and magnetically isolated from the outside. The choice of materials for the chips also plays a role in minimizing the noise that could affect the qubits and bring them out of their coherent state.

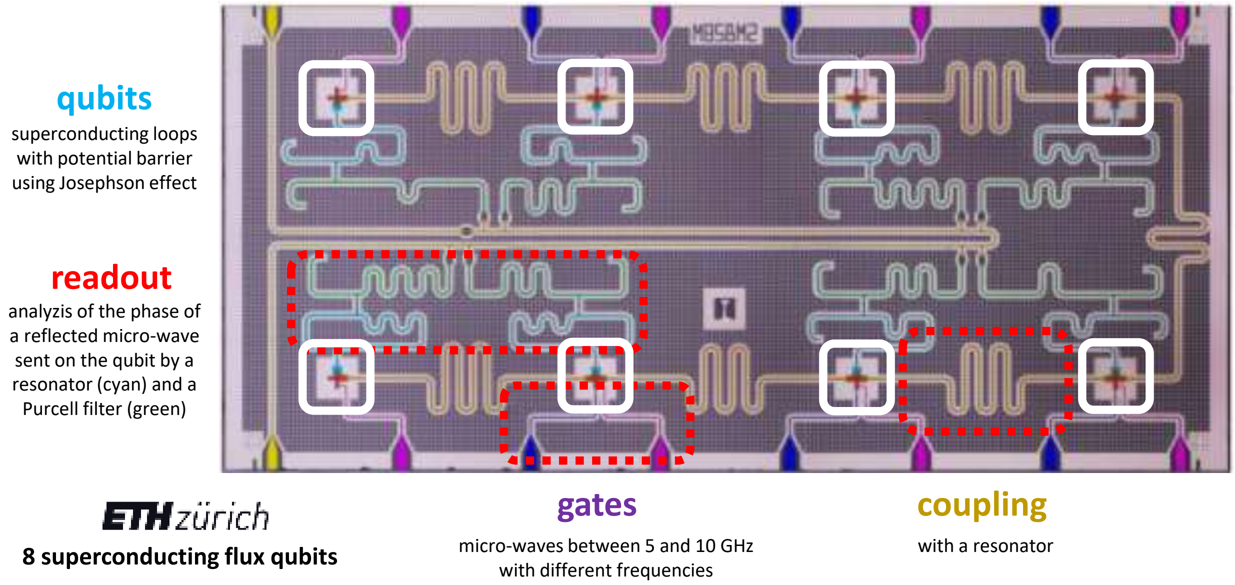

Figure 231: a small 8-qubit superconducting processor from ETH Zurich showing its various components controlling the qubits. source: The European Quantum Technologies Roadmap, 2017 (30 pages) and the thesis Digital quantum computation with superconducting qubits by Johannes Heinsoo, ETH Zurich, 2019 (271 pages).

In the diagram in Figure 232 is the relationship between qubit gates time and coherence time during which the qubits remain stable. The number of two-qubit gate cycles is limited by the qubit coherence time. In the current generations of superconducting qubit quantum computers, two-qubit gates last about 300 ns. It yields a potential of 500 gate cycles. But the current error rates in the tune of 1% damage a quantum circuit way before we reach this number of cycles. So, two-qubit error rates are more important than coherence times (T_1 and T_2). These coherence times are longer for quantum computers using trapped ions, but the gate times are also longer. In CMOS qubits, coherence times are longer and gate times are low. Quantum gates error rate will create more constraints on the computational depth, e.g. the number of quantum gates that can be chained together without the error rate of the gates mitigating the results. Algorithms must therefore optimize the number of gate cycles to be executed, which is furthermore constrained by the physical connectivity between qubits.

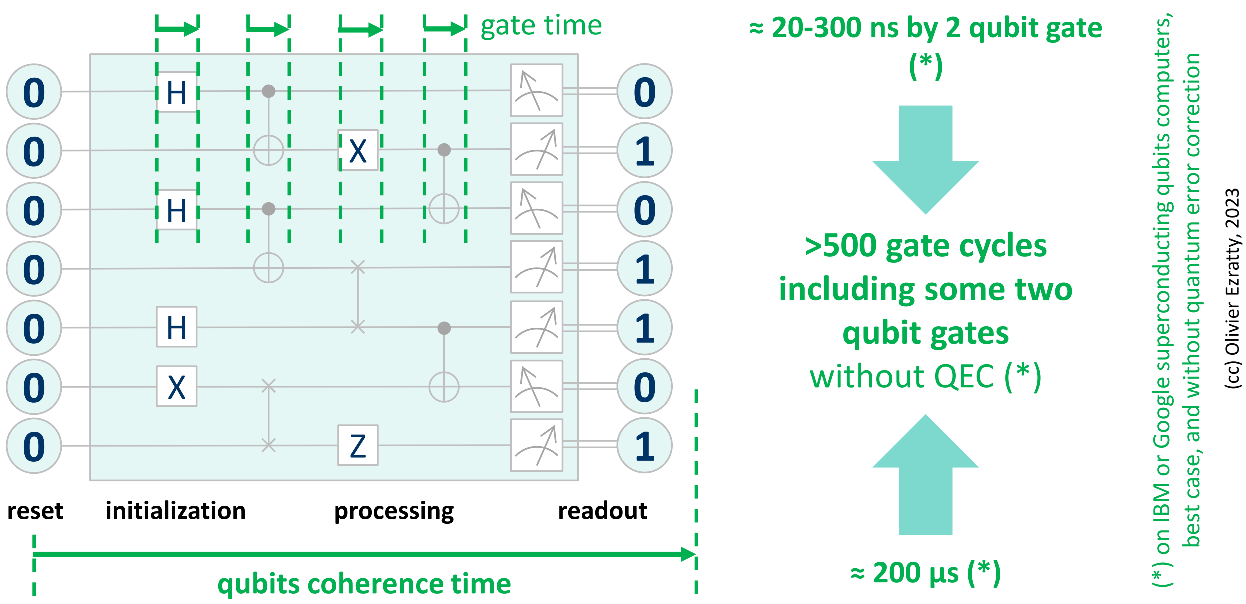

Figure 232: a timeline showing the relation between useful computing time and gate coherence times. In most cases, qubit error rates have a significant toll before we reach the T_1 and T_2 of a QPU. (cc) Olivier Ezratty, 2020-2023.

In diagrams describing quantum algorithms, such as the one in Figure 232, the double bar after measuring the state of a qubit conventionally indicates that a normal bit has been recovered, at 0 or 1. By the way, all this reminds us that there are as many output qubits as input qubits in a quantum computation since they are physically the same!

Error correction

One of the pitfalls of existing physical qubits is their significant error rates that are generated throughout the whole computing cycle, including qubit preparation, quantum gates and qubit readout and coming mostly from the fateful quantum decoherence as well as from defects from control electronics signals. Decoherence is mostly generated by the interactions between the qubits and their environment. It progressively destroys the quantum information sitting in the qubits, meaning superposition and entanglement. It leads to an inevitable failure in computation after a short time. Error rates for each operation and readouts are commonly between 0.1% and several %, depending on the qubit type and quality. But even 0.1% is an intolerable level for most calculations.

In conventional computing, errors are way less frequent but must still be corrected. While some errors may be detected and fixed during computing in microprocessors, most errors are happening in memory, storage and telecommunications. These errors are discrete, corresponding to some unwanted classical bit flips. With quantum technologies, errors happen first and foremost during computing and within qubits and they are continuous and analog by nature, involving phase errors on top of the flip error that is akin to classical bit flip errors.

Errors are corrected through various mechanisms that we'll cover here, without necessarily going into their details. This field is quite broad and is one of the most technical and cryptic you will ever find in quantum computing. It has its own weird lingua with syndromes, magic states, distillation, stabilizers and the likes. We describe here the various Quantum Error Correction (QEC) techniques that will be related to FTQC (Fault-Tolerant Quantum Computers) and Quantum Error Mitigation techniques that are adapted to NISQ (Noisy Intermediate Scale Quantum) systems.

Error types and sources

We can semantically organize quantum computing errors with gate-based systems in three dimensions as presented in Figure 233. It corresponds to a computing view, with "when" are errors happening during computation, what is their effect on the data, and then, to a physical understanding, describing the where, or their various sources at the classical and quantum physical levels.

When corresponds to the events during which these errors are happening in the quantum computing cycle, starting with qubit reset, qubit gates, idle state and qubit readout. These correspond to typical error metrics like reset[521], single and two qubit gates and readout error rates or percentages, or fidelities which are simply =1-error rate. In benchmarks that are used particularly with trapped ion qubits, SPAM (state preparation and measurement) errors measure the cumulative effect of qubit preparation and measurement. It is used and advertised by Quantinuum and IonQ. It however does not provide any indication on multiple qubit gates and qubits entanglement quality and scaling[522]. All these error types are not qubit dependent and can happen with every qubit type, even with photonic qubits.

What is the effect of these errors on the data in a qubit register. The most common effects are flip and phase errors happening on individual qubits, which, with a linear combination, can describe any type of single qubit coherent error. Here, depolarizing is frequently confused with phase errors.

[521] See Optimal Qubit Reuse for Near-Term Quantum Computers by Sebastian Brandhofer et al, July 2023 (11 pages) which shows how good qubit reset conditions their reuse, with tests on IBM QPUs.

[522] See 99.9904% SPAM Fidelity with barium-137 sets the standard and creates a further step towards solving some of the world's most intractable problems by Kortny Rolston-Duce, Quantinuum, March 2022.

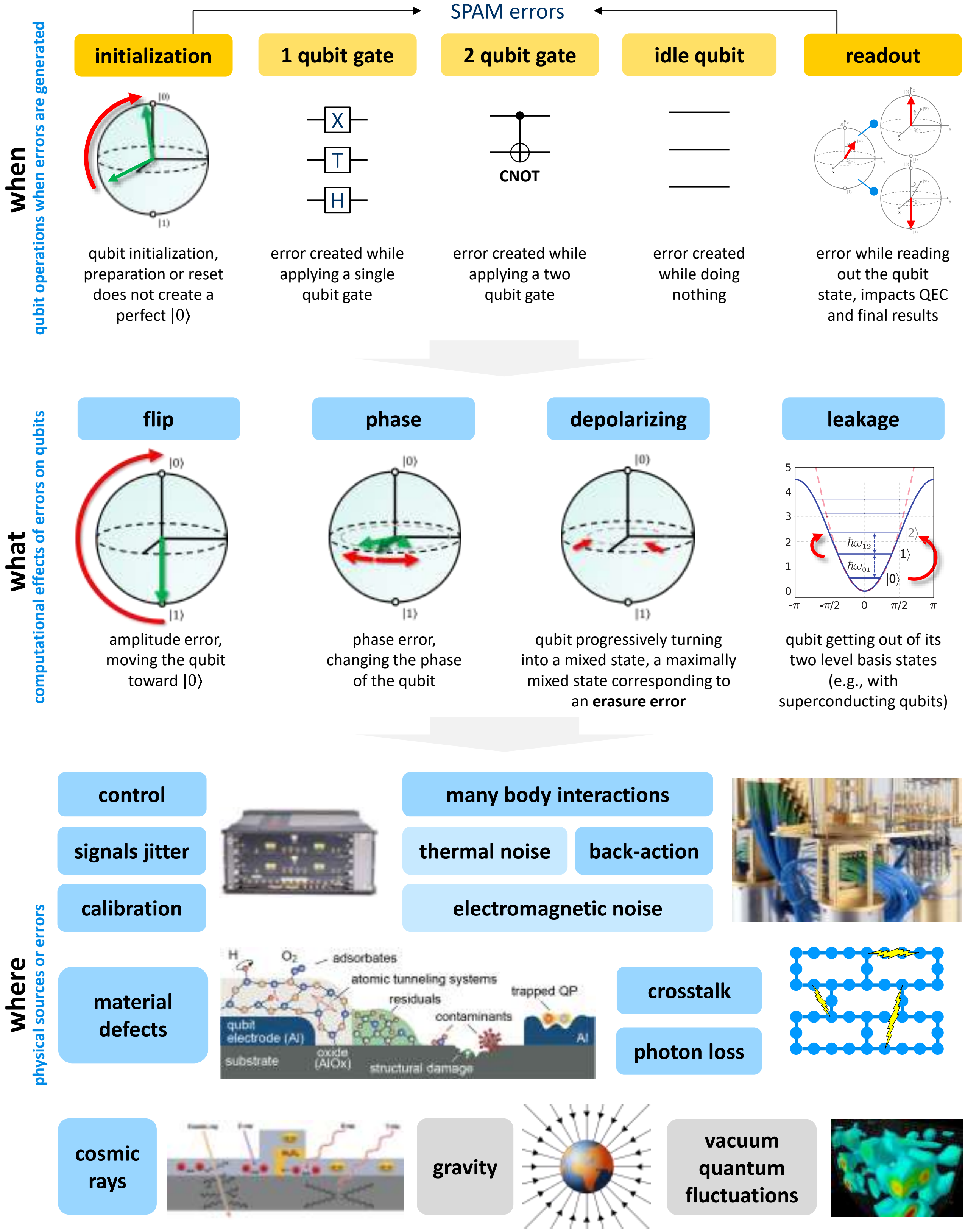

Figure 233: the when, what and where of errors affecting qubits clearly separating the logical operations when errors happen, their computational effects on the qubits and their various physical origins. Qubit fidelities correspond to measurements done in the "when" realm, T_1/T_2 to the "what" realm, and specific benchmarks are done to identify the relative weights of the various physical error sources. The impact of physical sources of errors is variable and qubit type dependent. For example, cosmic rays which impact solid state qubits are rare but have a catastrophic impact on computation while gravity and quantum vacuum fluctuations are permanent but have a minimal effect. Then, another variation to be considered is ability to dampen the impact of these errors or minimize them. For example, it seems possible to improve the quality of control signals and chips materials to avoid unwanted effects while it may be more difficult to avoid various many-body interactions. (cc) Olivier Ezratty, 2023.

Phase errors affect only the phase of a qubit which represents a rotation around the Z axis in Bloch's sphere representations of the qubit while a depolarizing error represents an evolution of the qubit toward a mixed state, which is represented by the qubit vector moving within the Bloch sphere.

We discussed it when covering mixed states and density matrices representations of qubits. These effects are collectively contributing to decoherence, the set of physical phenomena that destroy the quantum coherence of a many-body system. Decoherence progressively kills qubit superposition and entanglement. Then, leakage can happen only with specific qubit types like superconducting qubits and move a qubit state outside of its |0⟩ and |1⟩ basis states. Qubits coherence time is an indication of how long register qubits remain coherent, with stable superposition. Qubits amplitude stability is evaluated with a T_1 time while phase stability is measured with a T_2 and a T_2^*, these being sometimes confused with each other in the literature[523].

Real single qubit errors can be decomposed as linear combinations of these flip and phase errors[524]. Quantum error corrections codes are designed to separately correct flip and phase errors, the integration of which corrects any linear combination of both error types.

T_1 : flip error (relaxation, dampening)

- qubit energy loss the the environment.
- spontaneous emission, quasiparticle tunneling, flux coupling, dielectric losses and control electronics imprecision.
- more important at qubit readout.
- time to decay from |1⟩ to |0⟩.
- decay of qubit density matrix diagonal.

T_2 - T_2^* : phase error (dephasing, decoherence time)

- environment creates loss of phase memory.
- control electronics imprecision.
- important during computation.
- tied to number of consecutive gates.
- decay of qubit density matrix off-diagonal values.

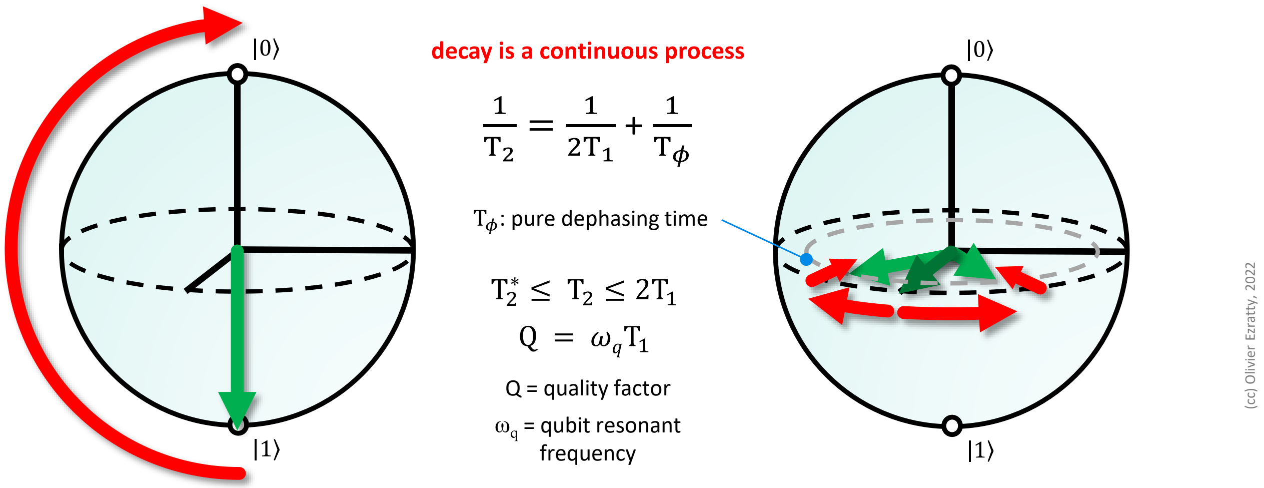

Figure 234: flip error and phase errors and their effect in the qubit Bloch sphere. (cc) Olivier Ezratty, 2022.

- **Flip errors** as shown in the Bloch sphere in Figure 234, are amplitude errors that tend to push the amplitude back to |0⟩. These errors correspond mathematically to a decay of the diagonal part of the qubit density matrix in its eigenstate basis. It is related to the T_1, which is linked to a loss of amplitude ("energy relaxation"). It is also called "longitudinal coherence time", "spontaneous emission time", "population lifetime" or "amplitude damping" and corresponds to a loss of energy

523 I found a good explanations in Dancing with qubits by Robert Suttor, pages 415 to 421, 2019 (516 pages).

524 More precisely, single qubits errors can be decomposed in quantum channels: depolarizing channel (with a bit flip error, a phase flip error or a combination of both, in which case, the qubit remains in a pure state, and the qubit moves with some rotation in the Bloch sphere), a dephasing, depolarizing or phase damping channel (vanishing off-diagonal values in the qubit density matrix, in which case the qubit moves in a mixed state and inside the Bloch sphere) and an amplitude-damping channel. Source: Lecture Notes for Ph219/CS219: Quantum Information Chapter 3 by John Preskill, Caltech, October 2018 (65 pages).

in most qubits. It is usually a dissipative process that releases some energy[525]. The flip error is measured with a simple experiment using an X gate and measuring the result n times at different t times. T_1 corresponds to the time when the probability of obtaining a $|1\rangle$ reaches 1/e. Such an error can damage the entanglement of the qubits related to the one affected by this error (Figure 235).

- **Phase errors** are rotations around the z-axis in the Bloch sphere (coherent noise, mostly due to control electronics imprecision). These errors are not dissipative, meaning, they are thermodynamically neutral. Coherent phase errors are measured in two manners, with T_2^* and T_2 and are also called the "transverse coherence time", "transverse relaxation", "phase coherence time" or "phase damping". T_2^* is evaluated with a Ramsey experiment, applying one Hadamard gate, wait time t, then applying another Hadamard gate, and readout. Without phase errors, the probability should look like a sinusoidal curve. With it, the curve slowly converges around a probability 0.5. T_2^* is obtained when the probability reaches 1/e. T_2 is obtained with a Hahn echo experiment where an X gate is added at t/2, which removes some error sources not due to qubit defects (Figure 235).

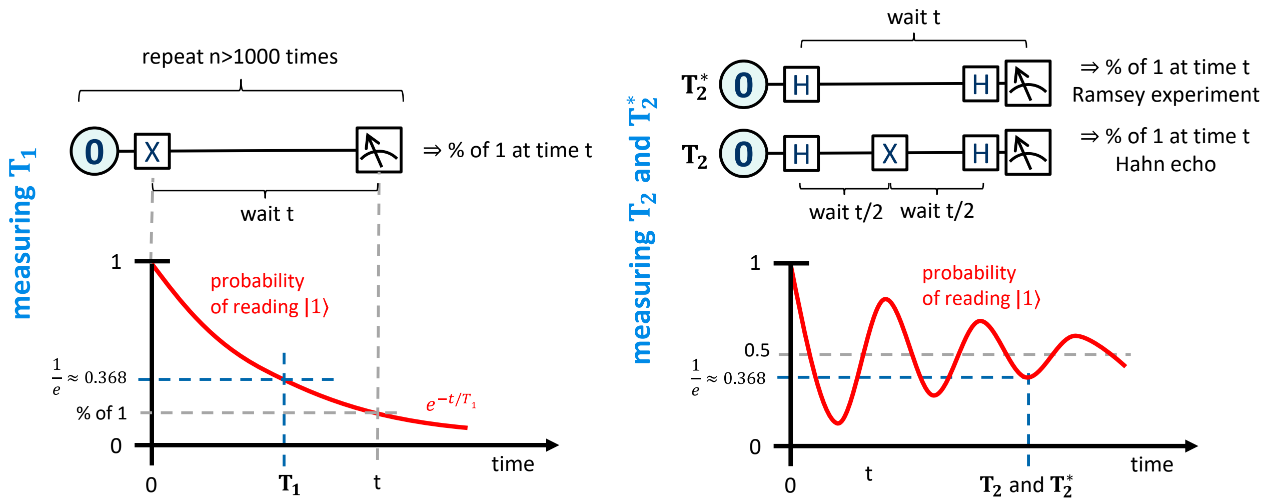

Figure 235: how are measured T1, T_2 and T_2^*. (cc) Olivier Ezratty, 2022-2023.

- **Depolarizing** or pure dephasing and decoherence noise, is related to the decay of the non-diagonal part of the qubit density matrix, creating a mixed state, shown up visually with the qubit state movin inside the Bloch sphere. An error generating a maximally mixed state is called an erasure error, with subtleties differentiating biased vs standard erasures.

- **Leakage errors** is another error type that sees a qubit drifting and stabilizing in another energy state than the basic $|0\rangle$ or $|1\rangle$. This can occur in the $|2\rangle$ level of a superconducting qubit, which we are trying to avoid, or with variations in the hyperfine energy levels of trapped ion qubits. This type of error can be benchmarked[526] and corrected with specific reset protocols[527] [528]. You don't observe such leakage errors with photon qubits using polarization as information encoding.

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[525] In superconducting qubit circuits, T_1 is proportional to the circuit quality factor $Q = \omega_q T_1$, which itself is proportional to the ratio between the energy stored in the qubit resonator and its rate of energy loss. T_1 comes from different phenomena: spontaneous emission, quasiparticle tunneling, flux coupling and dielectric losses in the Josephson junction. The Purcell effect is a spontaneous emission through the readout cavity. The Purcell decay rate is related to the speed of this phenomenon. It is reduced with using a Purcell filter which suppresses signal propagation at the qubit transition frequency. The filter is a pass-through with the readout cavity frequency that protects the qubit from decoherence channels while enabling its readout.

[526] See Leakage Benchmarking for Universal Gate Sets by Bujiao Wu et al, Alibaba, April 2023 (27 pages).

[527] See Removing leakage-induced correlated errors in superconducting quantum error correction by M. McEwen et al, Google AI, February 2021 (12 pages).

[528] See Overcoming leakage in scalable quantum error correction by Kevin C. Miao et al, Google AI, November 2022 (17 pages).

One characteristic of qubit errors and their related T_1, T_2 and T_2^* is their variability over time (it can change over time during computations or between computations, requiring frequent calibration) and space (it is qubit dependent)[529]. In the quantum errors lingua, a **Pauli noise channel** corresponds to a linera combination of flip, phase and depolarizing errors.

The goal of having long qubit energy relaxation times is in competition with that of achieving high-fidelity qubit control and measurement. One key concern is to be able to apply error corrections as fast as possible after they are detected and before qubits decoherence takes effect or gets amplified. This is the reason why readout gates and phase detection electronics (for superconducting/quantum dots spin qubits) must be as fast as possible. But fast gates and readouts can drive leakage errors! They also require high-bandwidth microwave pulses, which reduce the capacity to frequency multiplex it in readout microwave circuits.

Where are these errors coming from at the physical level. It can be various electromagnetic and thermal perturbations but various materials defects in the qubit chip circuits. It depends on the qubit type[530]. At a very low scale, some qubit types are also damaged by cosmic rays, quantum vacuum fluctuations and gravity. These error sources are multiple[531], leading notably to the progressive decoherence of qubits which affects qubits superposition and entanglement. They are linked to the various interactions between qubits and their immediate environment[532].

These include:

- **Control electronics** imperfections like clock, phase and amplitude jitter. It can be triggered by calibration errors of quantum gates that occur in particular in the calibration of superconducting qubits. They can notably trigger leakage errors. These small errors can create imprecisions with qubit operations. They are mitigated with improving the precision of control electronics. With superconducting and quantum dots spin qubits, it is mainly located at the local oscillators, mixers, AWG (arbitrary wave generation) and DAC levels (digital to analog conversion). These are coherent errors while other errors drive qubits decoherence.
- **Crosstalk** describes situations where a given qubit may affect other qubits, usually through various electromagnetic fields transmissions. One typical crosstalk effect comes from using the same microwave frequency to drive distant qubits in a chip due to the bandwidth limitations of the microwaves that can be used for this drive.
- **Thermal noise** from components around the qubits. This is the reason for the existence of attenuators around superconducting qubits. It comes among other things from shocks between atoms.
- **Material defects** which are commonplace with solid state qubits (superconducting, quantum dots spins, NV centers). One common defect comes from dielectric losses in the Josephson junction of superconducting qubits[533] [534].
- **Electrical and magnetic noise**, which can have many origins depending on the qubits. It explains why quantum chips are integrated in some multi-layer metal enclosures to limit the impact of

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[529] See Dynamics of superconducting qubit relaxation times by M. Carroll et al, npj Quantum Information, November 2022 (7 pages) that describes the variability of T_1 across time and qubits for superconducting qubits.

[530] See Fast Estimation of Physical Error Contributions of Quantum Gates by Miha Papič, Adrian Auer and Inés de Vega, IQM, May-July 2023 (28 pages).

[531] Here is a small inventory of noise sources for superconducting qubits: Sources of decoherence, ETH Zurich, 2005 (23 slides).

[532] Any operation will generate an error. An error can be generated at the time of correction, at the time of detection or at the time of application of a gate. Doing nothing on a qubit can also generate errors because of its finite coherence time.

[533] See Material Defects in Superconducting Quantum Bits: Origins and Remedies by Jürgen Lisenfeld, Les Houches, 2023 (35 slides).

[534] See Developing a Chemical and Structural Understanding of the Surface Oxide in a Niobium Superconducting Qubit by Akshay A. Murthy et al, ACS Nano, September 2022 (23 pages).

terrestrial magnetism on its qubits. Most solid state quantum processors are packaged in tight metal shielding.

- **Radioactivity**, particularly coming from cosmic rays. Radiations can be X-rays, gamma rays (whose electromagnetic nature was discovered in 1914), beta particles and their ionizing effects. The phenomenon is now well characterized[535]. It creates phonon quasiparticles in the chip substrate that can propagate to many surrounding qubits, endangering the efficiency of quantum error correction codes (Figure 236) [536] [537] [538]. Radioactivity is one of the many sources of long range correlated noise. Envisioned solutions are to shield the qubit processing unit with lead[539] or copper backside metallization[540], to trap phonons using high impedance resonators made of granular aluminum[541], adapting surface codes[542] and to implement distributed error corrections schemes[543].

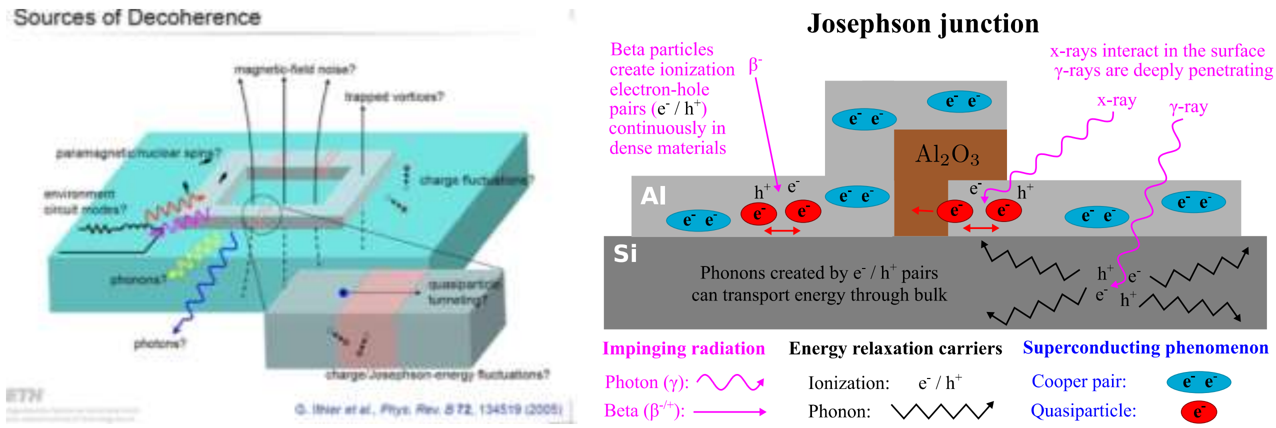

Figure 236: sources of decoherence and cosmic radiations affecting superconducting qubits. Sources: Sources of decoherence, ETH Zurich, 2005 (23 slides) and Impact of ionizing radiation on superconducting qubit coherence by Antti P. Vepsäläinen, William D Oliver et al, August 2020 (24 pages).

- **Vacuum quantum fluctuations** originating errors is an endogamous source of errors within qubits while all others are exogamous[544]. Their effect is however minimal.

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[535] See Disentangling the sources of ionizing radiation in superconducting qubits by L. Cardani et al, November 2022 (13 pages).

[536] See Correlated charge noise and relaxation errors in superconducting qubits by C.D. Wilne, Roger McDermott et al, Nature, December 2020-June 2021 (19 pages) which describes the correlated errors appearing in superconducting qubits and how it could impact the architecture of quantum error correction codes.

[537] See Resolving catastrophic error bursts from cosmic rays in large arrays of superconducting qubits by Matt McEwen, Rami Barends et al, Google AI, Nature Physics, December 2021 (13 pages) who developed a test protocol to assess the impact of radiations on 26 qubits in its Sycamore processor.

[538] See TLS Dynamics in a Superconducting Qubit Due to Background Ionizing Radiation by Ted Thorbeck et al, IBM Research, October 2022 (14 pages) which identifies the impact of ionizing radiations on qubit lifetime.

[539] See Impact of ionizing radiation on superconducting qubit coherence by Antti P. Vepsäläinen, William D Oliver et al, August 2020 (24 pages).

[540] See Phonon downconversion to suppress correlated errors in superconducting qubits by V. Iaia, Robert McDemott et a, Wisconsin and Syracuse Universities, March 2022 (21 pages).

[541] See Phonon traps reduce the quasiparticle density in superconducting circuits by Fabio Henriques et al, Applied Physics Letters, 2019 (14 pages).

[542] See Fight or Flight: Cosmic Ray-Induced Phonons and the Quantum Surface Code by Bernard Ousmane Sane et al, Keio University, July 2023 (11 pages).

[543] See Distributed quantum error correction for chip-level catastrophic errors by Qian Xu, Lian Jiang et al, March 2022 (11 pages).

[544] See Observation of quantum many-body effects due to zero point fluctuations in superconducting circuits by Sébastien Léger, Nicolas Roch et al, Institut Néel, 2019 (8 pages) which describes the phenomenon on superconducting qubits.

- **Gravity**, given this type of error and vacuum fluctuation ones appear to be minor compared to the previous ones[545].

Generally speaking, errors are generated by various interactions, electromagnetic or mechanical, between qubits and their immediate environment and are associated with the phenomenon of quantum decoherence. Some of these effects are avoided by cooling the qubits to a temperature close to absolute zero, but this is not enough. Researchers are therefore working hard to ensure that the noise affecting the qubits is as low as possible so that qubits coherence time can be as long as possible.

We have to manage this contradiction: qubits remain coherent, in a state of superposition and entanglement, if we do not disturb them, but we spend our time disturbing them with quantum gates operations! There are several ways to address these issues: improving qubit operations fidelity (preparation, gates, readout), improving the ratio between gate times and qubit coherence times, implementing quantum error mitigation techniques and quantum correction codes and at last, reduce the number of gates needed to run algorithms with various optimization techniques.

Qubits fidelities

In a gate-based quantum computer, three types of errors (or related fidelities) are usually evaluated: errors in single-qubit quantum gates, errors in two-qubit gates, and errors with qubits readout. These error rates are currently sitting between 0.1% and several 1%, which is much higher than the current error rates of traditional computing, which are negligible[546]. Qubits "fidelity" for any of these three dimensions is 100% minus the related error rate. In typical quantum parlance, when a "three-nines" 2 qubit-fidelity is mentioned, it means that it is better than 99.9%.

The chart below in Figure 237 consolidates a comparison of some fidelity levels of superconducting, trapped ion and cold atom quantum computer qubits, this information being provided by their vendors[547]. Qubit fidelities encompasses these three fidelities/errors dimensions (1Q, 2Q, readout).

Two-qubit gates and readout error rates are generally higher than one-qubit gates error rates[548]. We must therefore always pay attention to two-qubit gates, particularly given these gates are the source of much of the quantum computing power.

But these fidelities are not always measured in the same conditions. Some are measured with only a couple interacting qubits in research settings while others are done with all the register's qubits being active and with tens of qubits. It can create significant differences favoring the first kind of measurements, particularly due to the significant crosstalk between qubits. As a result, fidelities are usually much better in research lab experiments than with commercial QPUs. Thus, the choice to use only commercial settings in Figure 237.

Some best-in-class fidelities achieved as of 2022 were 99.989% for Quantinuum's trapped ions single-qubit gate and 99.68% for IBM's Egret (33 qubit) two-qubit gates.

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[545] See about gravity: A model of quantum collapse induced by gravity by Franck Laloë, 2020 (14 pages) and Gravitational Decoherence, 2017 (78 pages).

[546] In classical calculation, errors are very rare. We talk about single particle perturbations (PPI) and single event upset (SEU) which trigger "soft errors" or logical errors. The SER (Soft Error Rate) combines the SDC (Silent Data Corruption, not detected) and the DUE (Detected and Unrecoverable Error, detected but not correctable). The unit of error measurement is the FIT (Failure in Time), which corresponds to one error per billion hours of use. The MTBF of electronic equipment (Mean Time Between Failure) is generally measured in years or decades. Errors are generally caused by isolated particles (ions, electrons, photons), particularly from cosmic rays like high-energy gammy rays. This affects in particular the electronics used in aerospace, which must be hardened to withstand them, as well as those used on Earth but at altitude. Memory is often more affected than processors. Hence error correction systems that use for example a parity bit and cyclic redundancy check used in telecommunications.

[547] Source for qubit reliability data mainly comes from Qubit Quality on Quantum Computing Report website, 2020. Plus some additional data coming from vendor sites.

[548] See An introduction to quantum error correction by Mazyar Mirrahimi, 2018 (31 slides) as well as Introduction to quantum computing by Anthony Leverrier and Mazyar Mirrahimi, March 2020 (69 slides) which completes it well.

Google's Sycamore single qubit gates fidelity is 99.84% with 53 qubits. The two-qubit CZ gates fidelity of China's Zuchongzhi 2.1 66 qubits superconducting processor is 99.4%[549].

Another observation relates to IBM's most recent fidelities with their best-in-class 27, 65, 127 and 433 qubit systems as of September 2023. They showcase a zig-zag pattern with 2-gates fidelities that are decreasing from one generation to the next but then improves within a given generation as IBM. Still, the average sits around 1% error rates which is not sufficient to implement quantum error correction codes. IBM's plan is to get away from this trend and reach 99.9% fidelities.

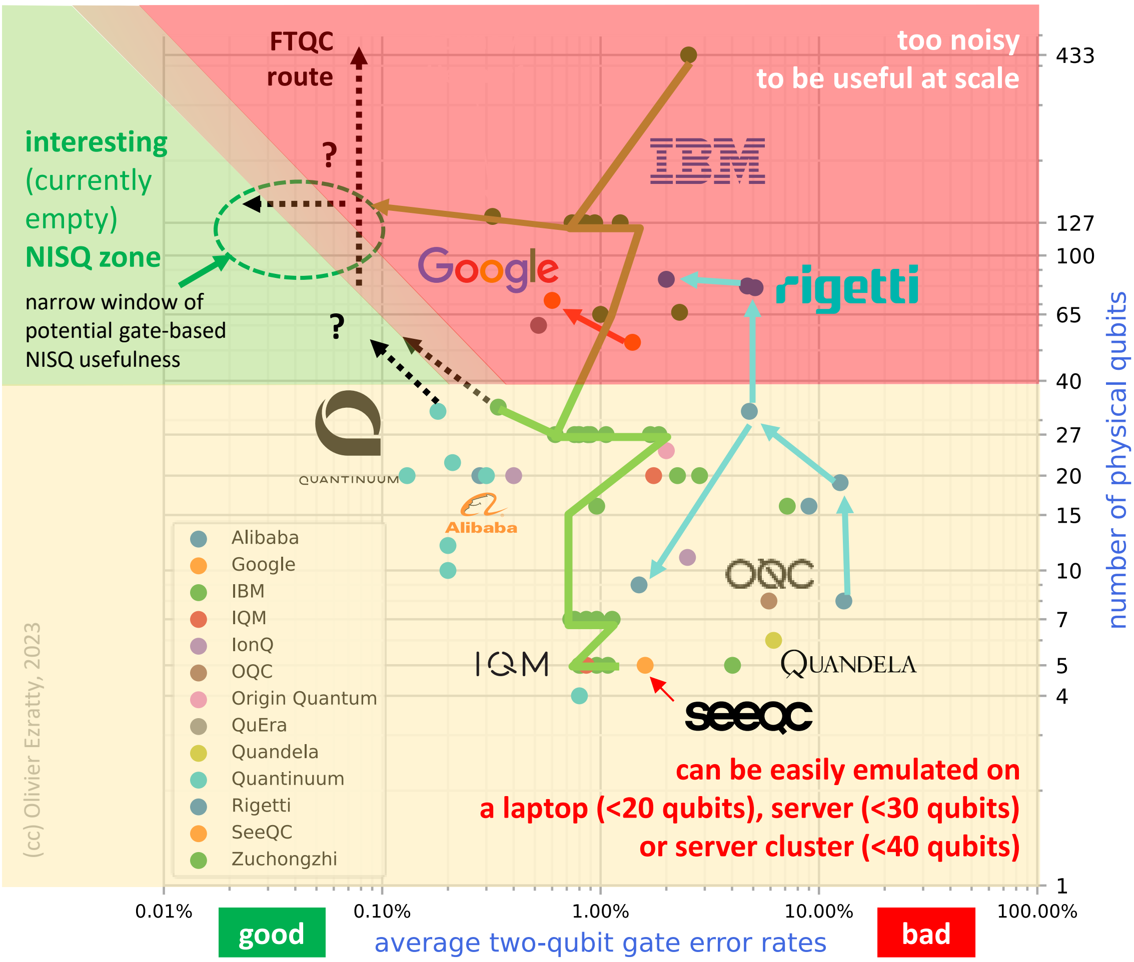

Figure 237: scatter plot with two-qubit gate fidelity and qubit number for currently available commercial gate-based quantum computing systems. Viable NISQ QPUs require figures of merit that are in the empty slanted upper-left green zone. It is slanted since, as the qubit number grows, qubit fidelities must be better to accommodate a larger quantum volume. IBM's zigzag corresponds to continuous fidelities improvement within each QPU generation having several iterations. The yellow zone corresponds to a quantum computing regime that can be easily emulated with the demanding "state vector" mode on classical computers. It is faster and cheaper under 20 qubits with a simple laptop, faster with an SV1 AWS server instance under 29 qubits and possible with a cluster server like the Eviden (Atos) QLM under 40 qubits with a 1 TB memory, and up to 44 qubits with AWS cloud servers. Two other figures of merit are missing here like qubit connectivity, which impacts algorithms depth and quantum volume which describes the useful breadth (number of qubits) and depth (actual algorithm gate cycles) on these systems. Source: Olivier Ezratty, and vendors two-qubit gate fidelities data obtained with randomized benchmarking and plotted over QPUs number of qubits in log scales, as of January 4th, 2024. Qubit fidelities data correspond to average fidelities, regardless of their standard deviation. With large standard deviations, actual NISQ algorithms are significantly damaged by gate errors. (cc) Olivier Ezratty, January 2024.

For all vendors, these error rates are currently prohibitive when executing many quantum gates in a typical algorithm. With each operation, error rates add up and the reliability rate decreases. Imagine chaining a few dozen two-qubit gates! At a rate of 1% error per operation, the error rate can very quickly exceed 50% at the end of a rather simple algorithm circuit and, generally, well before the fateful qubit coherence time limit.

[549] Data source: Superconducting Quantum Computing by Xiaobo Zhu, June 2019 (53 slides).

Hence the fact that the power of a quantum computer is always evaluated not simply by the number of available qubits but by the number of operations that can be executed with a reasonable error rate at the end of the circuit run. To avoid this quantitative constraint, we should have qubits with quantum gate error rates of 10^{-10} or even 10^{-15}. Figure 238 and Figure 241 illustrate this discrepancy between today's physical qubits and the need to perform reliable calculations (without error correction). Exceeding 10^{-16} fidelities would allow the execution of even larger and longer circuits but may have the side effect of corresponding to unpractical scales for circuit execution times.

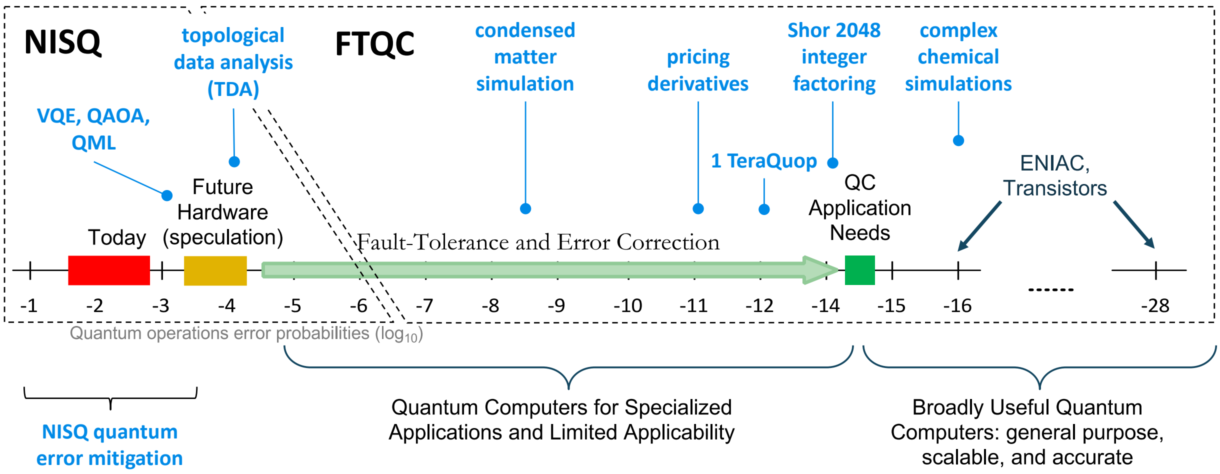

Figure 238: comparison of error levels between existing quantum hardware and what is required, with error correction codes. Source: How about quantum computing? by Bert de Jong, DoE Berkeley Labs, June 2019 (47 slides) and additions by Olivier Ezratty, 2023.

A formula is used to evaluate the dependency between quantum gates error rates (e), the number of qubits (n) and the number of usable gates (d), called "circuit depth": nd < 1/e. As the error rate decreases, the usable circuit depth increases, and the range of usable algorithms expands (Figure 239).

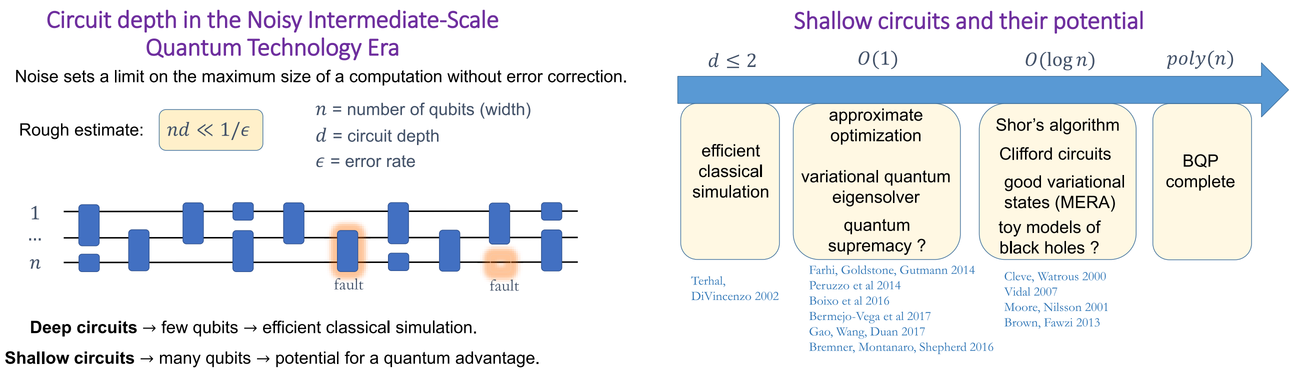

Figure 239: relationship between circuit depth and their use case.
Source: Quantum advantage with shallow circuits by Robert König, 2018 (97 slides).

For a quantum computer to be useful and scalable, you need a lot of qubits, a low error rate for quantum gates and qubits readout, and a long qubit coherence time to be able to execute algorithms without much time constraints although quantum error correction codes are indispensable workarounds for this last constraint.

How are quantum gates and measurement error rates evaluated (Figure 240)? We've seen previously how individual qubits flip and phase error rates are usually measured. Other methods are required to have an idea of the fidelities of registers with entangled qubits.

One method is the **Randomized Benchmarking** (RBM) process which consists in chaining a random sequence of quantum gates whose result is known in advance and with comparing the results obtained with the right responses. Usually, a random sequence of Clifford gates is launched and then executed backwards. The error rate increases with the number of chained quantum gates and depends on their type. We can evaluate the error rate of a given gate with the **Interleaved RBM** which injects the gate periodically into the random gate set used. We then measure the difference in error rate between the sequence with and without these added quantum gates[550].

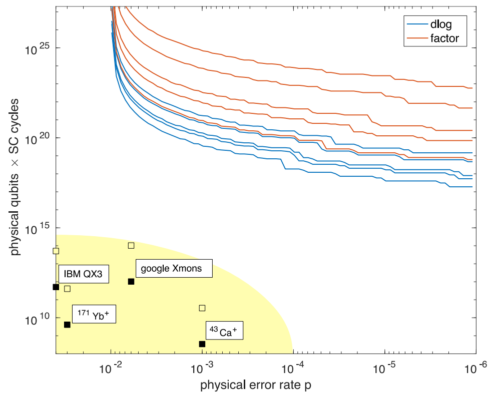

Figure 240: number of physical qubits and their physical error rates required to perform Shor dlog or integer factorization for common key sizes, with qubit performances as of 2018. Entwicklungsstand Quantencomputer. 2018.

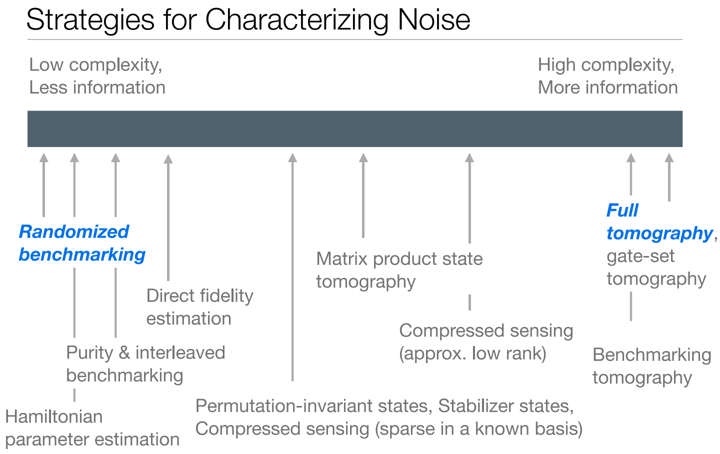

Figure 241: comparing the various strategies to characterize qubit noise. Source: Characterization of quantum devices by Steve Flammia, University of Sydney, 2017 (118 slides).

You'll have to look elsewhere to find out more data[551]. The RBM method has some drawbacks for clean noise quantification. It is apparently not suitable for the detection of any noise patterns[552].

Several other methods exist, such as **quantum state tomography** (QST) that we already covered in the section dedicated to measurement, page 212, which is based on a repeated measurement of qubit states that allows the reconstruction of a quantum system density matrix and the associated errors, for one or two qubits after some computation.

Yet another method exists that is based on some mathematical tools identifying a match between the noise rate of one and two-qubit gates of an algorithm and the total noise rate of the complete algorithm. In short, it links macro noise (algorithm) to micro noise (quantum gates).

Error correction codes zoo

Quantum error correction can't work the same as classical error correction. Qubits cannot be independently replicated with some measurement that would be performed on one replicated qubit, per the famous no-cloning theorem. On top of that, we are correcting analog errors in multiple dimensions, not just a 0/1 error flip that could be labelled as a simple "digital error"[553].

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[550] See Efficient measurement of quantum gate error by interleaved randomized benchmarking by Easwar Magesan, Jay Gambetta et al, March 2012 (5 pages). And Quantum Computing: Progress and Prospects, 2018 (206 pages), page 2-20. The process of benchmarking quantum gates is detailed in Randomized benchmarking for individual quantum gates by Emilio Onorati et al, 2018 (16 pages). The origin of the method is Scalable noise estimation with random unitary operators by Joseph Emerson et al, 2005 (8 pages).

[551] As in the aforementioned German report Entwicklungsstand Quantencomputer (*State of the art of quantum computing*), which dates from 2018 and highlights the huge gap between the performance of qubits, particularly at IBM and Google, and the need for integer factorization to break common RSA keys. See also Efficient learning of quantum noise by Robin Harper et al, Nature Communications, 2019 (15 pages) and Characterization, certification and validation of quantum systems by Martin Kliesch, April 2020 (87 pages).

[552] See Characterization of quantum devices by Steve Flammia, University of Sydney, 2017 (118 slides) which provides an excellent overview of the various qubits benchmarking techniques.

[553] The stakes of QEC are very well explained in Approaches to Quantum Error Correction by Julia Kempe, 2005 (29 pages). See also the review paper Quantum Error Correction for Quantum Memories by Barbara Terhal, April 2015 (47 pages) and the excellent Introduction to Quantum Error Correction and Fault Tolerance by Steven M. Girvin, August 2022 (99 pages) which is a transcript from a lecture at Les Houches Summer School in 2019 and (brilliantly) covers both classical and quantum error correction techniques.

The techniques explored for more than two decades consists in implementing quantum error correction codes called **QEC** for Quantum Error Correction or rather **QECC** for QEC Codes [554]. Most of these QEC schemes correct errors that are small and independent, meaning, not correlated between several close and distant qubits.

QED (Quantum Error Detection) is another error correction mechanism that is proposed for early FTQC regimes where error syndrome measurements are executed at predetermined times and the computation is discarded when an error is detected, generating an obvious computing time overhead[555].

Error correction codes apply to both universal gate quantum computing and quantum telecommunications. In the first case, they are integrated into the broader concept of fault-tolerance quantum computing (**FTQC**). Error correction's effect is to slow down qubits decoherence and to extend the available computation time. The chart in Figure 242 makes an inventory of the main quantum error correction codes with their origin and date of creation[556]. This error correction zoo is very dense and continuously expanding[557].

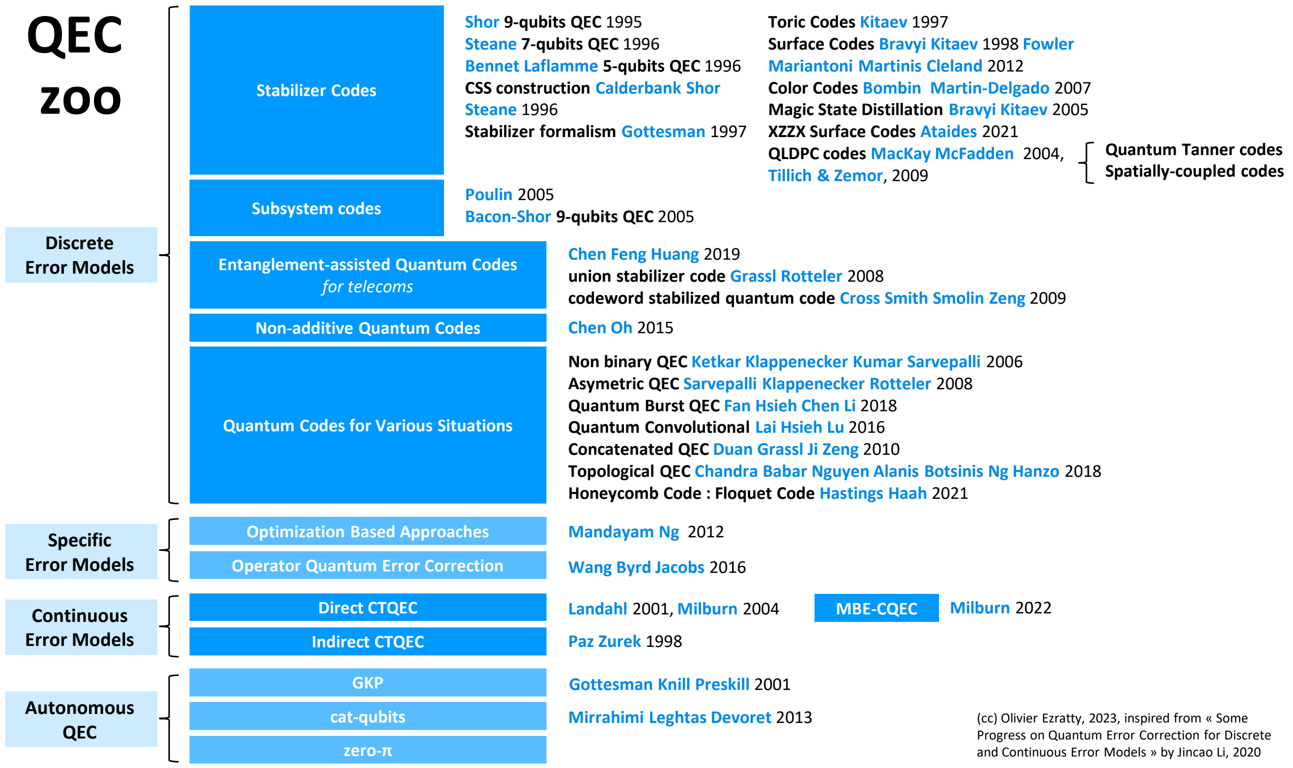

Figure 242: inventory of key quantum error correction codes. (cc) Olivier Ezratty, 2023, inspired from Some Progress on Quantum Error Correction for Discrete and Continuous Error Models by Jincao Li, 2020 (16 pages).

It includes several families of error correction codes. Their generic design requirements are to minimize the physical qubit error requirements (error threshold level), the number of physical qubits per logical qubit, their connectivity needs, and also the time to decode error syndromes.

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[554] This theme has, like many quantum specialties, its own conference. See International Conference on Quantum Error Correction and the videos with all the presentations of the 2019 edition.

[555] See Protecting Expressive Circuits with a Quantum Error Detection Code by Chris N. Self, Marcello Benedetti and David Amaro, Quantinuum, November 2022 (15 pages).

[556] Illustration inspired by a scheme discovered in Some Progress on Quantum Error Correction for Discrete and Continuous Error Models by Jincao Li, 2020 (15 pages).

[557] See the Error Correction zoo and its section on quantum error correction codes. And Quantum Error Correction for Beginners by Simon J. Devitt, William J. Munro, and Kae Nemoto, 2013 (41 pages).

Stabilizer codes correct flip and/or phase errors with five (Laflamme), seven (Steane) or nine qubits (Shor). These codes couple qubits several times using entanglement. Then, some quantum code using ancilla qubits are used to detect error syndromes without affecting the corrected qubits. It checks whether pairs of qubits have the same value on the initial basis or with using another basis after a rotation, to detect phase instead of flip errors, using so-called error syndrome measurements. All this is done without reading the value of the initial qubits which would make the whole system collapse. We then use single-qubit gates to correct the qubits for which an error was detected. It goes through some classical processing that must be as fast as possible that creates a correspondence between the error syndromes and the qubits to correct. In large surface codes, this process can be quite expensive.

Stabilizers codes have many variants including:

Quantum LDPC codes that are inspired by classical LDPC (low-density parity check) codes used in telecommunications (Wi-Fi, 5G, DVB) and seem to scale better than surface codes[558 559 560 561 562]. They require some long distance connectivity between qubits that is not possible on typical solid state 2D layouts and 3D circuitry to create long distance connections between qubits. This can be based on using multilayer connectivity chips for superconducting qubits, as is being designed by IBM[563] and MIT. It can also potentially be implemented with cold atoms qubits[564].

Quantum Tanner codes are a scalable variation of LDPC which uses a graph, with interesting scaling properties[565 566 567]. Spatially coupled codes are another breed of LDPC codes that work well with low-latency decoders[568]. LDPC codes can also correct efficiently Clifford group qubit gate errors[569] and correct (future) quantum memories[570].

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558 See Fault-Tolerant Quantum Computation with Constant Overhead by Daniel Gottesman, October-July 2014 (35 pages).

559 See the perspective Quantum Low-Density Parity-Check Codes by Nikolas P. Breuckmann and Jens Niklas Eberhardt, March 2021-October 2021 (19 pages).

560 See Qubits Can Be as Safe as Bits, Researchers Show by Mordechai Rorvig, January 2022 referring to See Asymptotically Good Quantum and Locally Testable Classical LDPC Codes by Pavel Panteleev and Gleb Kalachev, 2022 (51 pages).

561 See the review paper Opportunities and Challenges in Fault-Tolerant Quantum Computation by Daniel Gottesman, University of Maryland, October 2022 (24 pages) which covers the challenges of LDPC based FTQC.

562 See Constant-Overhead Quantum Error Correction with Thin Planar Connectivity by Maxime A. Tremblay, Nicolas Delfosse, and Michael E. Beverland, Microsoft, PRL, July 2022 (7 pages) proposed a qubit layout based on 4 3D stacked layers of long range gates, inducing a 15x reduction of the number of physical qubits required for quantum chemistry applications with logical error rate close to 10^{-15}. Providing the physical qubit error rate is 10^{-4}, presumably for two-qubit CNOT gate errors, not single qubit gate errors.

563 IBM designed surface codes for their heavy hex lattice qubit layout like in Topological and Subsystem Codes on Low-Degree Graphs with Flag Qubits by Christopher Chamberland, Guanyu Zhu, Theodore J. Yoder, Jared B. Hertzberg, and Andrew W. Cross, PRX, January 2020 (19 pages) and more recently in Empirical overhead of the adapted surface code on defective qubit arrays by Sophia Fuhui Lin et al, IBM Research, April 2023 (13 pages). But they then switched to working with LDPC when they realized they could implement long range connectivity with their qubits with only two layers of gates as proposed in High-threshold and low-overhead fault-tolerant quantum memory by Sergey Bravyi, Andrew W. Cross, Jay M. Gambetta, Dmitri Maslov, Patrick Rall and Theodore J. Yoder, IBM, August 2023 (38 pages).

564 See Constant-Overhead Fault-Tolerant Quantum Computation with Reconfigurable Atom Arrays by Qian Xu, Mikhail D. Lukin, Liang Jiang, Hengyun Zhou et al, Harvard, Caltech, University of Chicago and QuEra, August 2023 (25 pages).

565 See Quantum Tanner codes by Anthony Leverrier and Gilles Zemor, February-September 2022 (35 pages).

566 See Efficient decoding up to a constant fraction of the code length for asymptotically good quantum codes by Anthony Leverrier and Gilles Zemor, June-October 2022 (43 pages).

567 See Single-shot decoding of good quantum LDPC codes by Shouzhen Gu et al, Caltech, MIT, June 2023 (35 pages).

568 See Quantum Spatially-Coupled Codes by Siyi Yang and Robert Calderbank, Duke University, April 2023 (24 pages).

569 See Spacetime codes of Clifford circuits by Nicolas Delfosse and Adam Paetznick, Microsoft, April 2023 (33 pages). To correct Clifford gates, as part of a LDPC scheme.

570 See Hierarchical memories: Simulating quantum LDPC codes with local gates by Christopher A. Pattison, Anirudh Krishna and John Preskill, Caltech, Stanford University and AWS, March 2023 (70 pages) and a simple explanation in LDPC codes using local gates by Anirudh Krishna, March 2023.

Topological codes are LDPC variations including **surface codes** and **color codes**[571] themselves derived from **toric codes**[572], and many other specimens such as the **DFS** (Decoherence Free Subspaces) protocol encode quantum information in a subspace that is unaffected by physical errors or so-called holographic codes[573] and also **Fractal Surface Codes**[574].

Color codes have much less overhead than surface codes and render possible the implementation of FTQC (Fault-Tolerant Quantum Computing). It is due to one key feature of these correction codes: they can be implemented with the transversal gates described in the section on FTQC[575] [576]. It could be used with superconducting and quantum dots spin quits. However, what is "colored" in these colored codes and how does it work? I have no clear idea[577] [578].

Floquet Codes *aka* Planar Honeycomb Codes, were introduced by Matthew B. Hastings and Jeongwan Haah from Microsoft in 2021 to simplify toric codes with fewer qubits and stabilizers[579]. It is adapted to qubits architectures which do not have a native CNOT gate in their physical gate set and instead use joint pair-wise qubit measurements on two qubits (XX, YY, ZZ) like with Majorana fermions (Microsoft) and PsiQuantum measured based computation. Using surface codes with these qubits shows lower performance because the syndrome extraction circuit is more complicated[580] [581]. The technique was improved by Google researchers[582], by Christophe Vuillot from Inria Nancy[583] and with the XYZ^2 variant in 2022[584] and by the MIT and Microsoft with so-called dynamic automorphism codes[585].

Magic state distillation. A Shor and Steane code can correct any Pauli error, including Y gate, which is equal to iZX. It can correct any linear combination of I, X, Y and Z gates with complex numbers.

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571 This is however not the only solution to the magic state distillation physical qubits cost. See Fault-tolerant magic state preparation with flag qubits by Christopher Chamberland and Andrew Cross, IBM, May 2019 (26 pages) which describes an alternative using more ancilla qubits ("flag qubits").

572 See Fault-tolerant quantum computation by anyons by Alexei Kitaev, 1997 and 2008 (27 pages).

573 Color codes are variations of stabilizing codes. See some explanations in The Steep Road Towards Robust and Universal Quantum Computation by Earl T. Campbell, Barbara Terhal and Christophe Vuillot, 2016 (10 pages).

574 See Topological Order, Quantum Codes, and Quantum Computation on Fractal Geometries by Guanyu Zhu, Tomas Jochym-O'Connor, and Arpit Dua, IBM, PRX, Quantum, September 2022 (55 pages).

575 See The domain wall color code by Konstantin Tiurev et al, HQS, June 2023 (17 pages).

576 See Facilitating Practical Fault-tolerant Quantum Computing Based on Color Codes by Jiaxuan Zhang et al, September 2023 (9 pages).

577 See for example The ABCs of the color code by Aleksander Marek Kubica, 2018 (205 pages), a rich thesis done under the supervision of John Preskill at Caltech with the help from Jason Alicea, Fernando Brandão and Alexei Kitaev.

578 See The cost of universality: A comparative study of the overhead of state distillation and code switching with color codes, by Michael E. Beverland, Aleksander Kubica and Krysta M. Svore, 2021 (69 pages).

579 See Dynamically Generated Logical Qubits by Matthew B. Hastings and Jeongwan Haah, Microsoft, Quantum Journal, October 2021 (18 pages).

580 See Optimization of the surface code design for Majorana-based qubits by Rui Chao, Michael E. Beverland, Nicolas Delfosse, and Jeongwan Haah, Microsoft, Quantum Journal, 2020 (19 pages).

581 See Performance of Planar Floquet Codes with Majorana-Based Qubits by Adam Paetznick, Christina Knapp, Nicolas Delfosse, Bela Bauer, Jeongwan Haah, Matthew B. Hastings, and Marcus P. da Silva, PRX Quantum, January 2023 (15 pages).

582 See Benchmarking the Planar Honeycomb Code by Craig Gidney and Michael Newman, Google, February-September 2022 (16 pages).

583 See Planar Floquet Codes by Christophe Vuillot, Inria, December 2021 (16 pages) and A Pair Measurement Surface Code on Pentagons by Craig Gidney, June 2022 (16 pages).

584 See The XYZ^2 hexagonal stabilizer code by Basudha Srivastava, Anton Frisk Kockum, and Mats Granath, Chalmers and University of Gothenburg, Quantum Journal, 2022 (15 pages).

585 See Quantum computation from dynamic automorphism codes by Margarita Davydova et al, MIT, Microsoft and UCSB, July 2023 (93 pages).

This comes from the fact that any unit operation on a qubit can be expressed as a combination of IXYZ with complex factors: U = aI + bX + cY + dZ. This means, indirectly, that these QECs should be able to correct analog and continuous errors such as slight variations in amplitude or phase, i.e., rotations of a few degrees in the Bloch sphere. To correct these errors corresponding to gates outside the Clifford group such as a T gate (eighth of rotation in the Bloch sphere, these gates that provide an exponential speedup for gate-based computing), however, magic states are also used which feed circuits made with gates from the Clifford group.

These states are prepared by a process called magic state distillation which uses concatenated error correction codes[586 587 588]. It has an enormous overhead with the number of required physical qubits to create a single logical qubit, of about two orders of magnitude (x100) compared to correcting Clifford group quantum gates. This explains why the resource estimations of FTQC algorithms are made with counting the number of T gates ("T count"). Magic state distillation is usually implemented in surface codes. This overhead could be avoided or reduced using 3D correction codes, which are difficult to implement with actual qubits at this time due to their high long-range connectivity requirements, which can be implemented with some qubit types like with superconducting qubits[589] and with trapped ion qubits, although with limited scaling capabilities.

There are however some alternatives to using magic states distillation to support non-Clifford gates in a fault-tolerant manner[590]. One workaround to the high cost of FTQC with T gates and the related magic states distillation is proposed by Fujitsu (Figure 243). It consists in implementing rotation gates natively in analog fashion with precision pulse controls instead of assembling them with H and T gates. The Clifford group gates are still implemented with QEC and FTQC. The end result is an architecture that scales better at small size regimes. With that, 10,000 physical qubits could enable 60 logical qubits[591]. The caveat is that it doesn't scale well beyond that level.

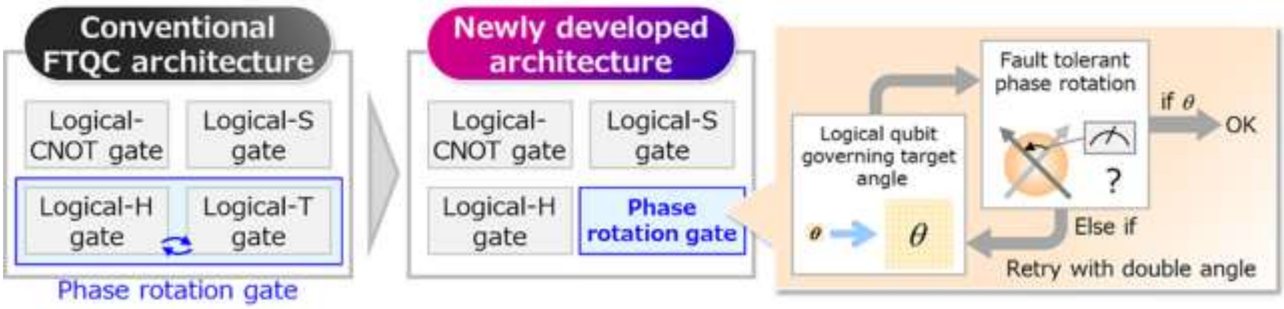

Figure 243: Fujitsu's FTQC proposal reducing the overhead of T gate FTQC correction with using arbitrary angle rotation gates. Source: Partially Fault-tolerant Quantum Computing Architecture with Error-corrected Clifford Gates and Space-time Efficient Analog Rotations by Yutaro Akahoshi et al, Fujitsu, Osaka University and RIKEN, March 2023 (20 pages).

ZXXZ surface code that would reduce the number of required physical qubits to create a logical qubit thanks to a lower error threshold. In April 2021, University of Sydney science undergraduate

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586 See Universal quantum computation with ideal Clifford gates and noisy ancillas by Sergey Bravyi and Alexei Kitaev, 2004 (15 pages).

587 See A fault-tolerant non-Clifford gate for the surface code in two dimensions by Benjamin J. Brown, May 2020, which applies to surface codes

588 See Near-Perfect Logical Magic State Preparation on a Superconducting Quantum Processor by Yangsen Ye, Jian Wei-Pan et al, May 2023 (12 pages).

589 See A lower bound on the overhead of quantum error correction in low dimensions by Nouédyn Baspin, Omar Fawzi and Ala Shayeghi, February 2023 (21 pages).

590 See Fault Tolerant Non-Clifford State Preparation for Arbitrary Rotations by Hyeongrak Choi et al, MIT, Yale, Chicago University, March 2023 (14 pages).

591 See Partially Fault-tolerant Quantum Computing Architecture with Error-corrected Clifford Gates and Space-time Efficient Analog Rotations by Yutaro Akahoshi et al, Fujitsu, Osaka University and RIKEN, March 2023 (20 pages).

Pablo Bonilla Ataides ZXXZ paper published in Nature Communications brought the attention of Amazon researchers[592]. This surface code could be used by Amazon who made a choice to use a relatively low number of photons per cat qubit (8 to 10, compared to about 30 for Alice&Bob), still requiring some first level bit-flip error correction on top of phase-flip correction. That's where a ZXXZ surface code QEC could come into play. ZXXZ QEC codes are indeed mentioned as an option QEC technique in Amazon's technical paper from December 2020. A team from Amazon, Caltech and the University of Chicago improved these codes in 2022 to work with biased noise (which has more phase than flip noise)[593].

Quantum erasure operations manipulate and modify quantum states in a controlled manner to extract useful information or facilitate error correction processes. They are used in LDPC codes[594].

Other error corrections code variations are:

Continuous error correction. In opposition to discrete quantum error correction codes (DQEC) that we've covered, there are alternative QEC using continuous measurement and correction. Juan Pablo Paz and Wojciech Zurek proposed in 1998 a continuously operating error correction code, the CTQEC, for "continuous-time QEC", or CQEC, based on differential equations and acting at reduced time intervals. There are two methods for acting directly on the information (direct CTQEC) or via auxiliary qubits (indirect CTQEC). CQEC avoids using ancilla qubits to measure the stabilizer operators by weakly measuring the physical qubits. It also enables faster measurements and error detection, reducing undetected errors. These methods were later improved by various contributors including Andrew J. Landahl and Gerard J. Milburn[595]. The latter recently proposed to use some real-time measurement-based estimator (MBE) of the real logical qubit to be protected to accurately track the actual errors occurring within the real qubits in real-time. This leads to the **MBE-CQEC** scheme that protects the logical qubit to a high degree and allows the error correction to be applied either immediately or at a later time.

Autonomous QEC implements error correction at the hardware level. The most commonplace aQEC are **bosonic codes**, including GKP error codes[596], binomial codes, 0-π[597], cat-codes or cat-qubits[598] and spherical codes which extend cat-codes[599].

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[592] See Student's physics homework picked up by Amazon quantum researchers by Marcus Strom, University of Sydney, April 2021, Sydney student helps solve quantum computing problem with simple modification by James Carmody April 2021 and The XZZX surface code by J. Pablo Bonilla Ataides et al, April 2021, Nature Communications (12 pages).

[593] See Tailored XZZX codes for biased noise by Qian Xu, Lian Jiang et al, March 2022 (16 pages). Biased noises is well explained in the thesis Quantum Error Correction with Biased Noise by Peter Brooks, Caltech, 2013 (198 pages).

[594] See Fast erasure decoder for a class of quantum LDPC codes by Nicholas Connolly, Vivien Londe, Anthony Leverrier, Nicolas Delfosse, August 2022 (5 pages).

[595] See Continuous quantum error correction via quantum feedback control by Charlene Ahn, Andrew C. Doherty and Andrew J. Landahl, PRA, March 2002 (12 pages), Practical scheme for error control using feedback by Mohan Sarovar, Charlene Ahn, Kurt Jacobs and Gerard Milburn, PRA, May 2004 (12 pages) and Measurement based estimator scheme for continuous quantum error correction by Sangkha Borah, Gerard Milburn et al, March 2022 (9 pages).

[596] See Encoding a qubit in an oscillator by Daniel Gottesman, Alexei Kitaev and John Preskill, PRA, 2001 (22 pages) and the perspective Quantum error correction with the Gottesman-Kitaev-Preskill code by Arne Grimsmo and Shruti Puri, PRX Quantum, June 2021 (20 pages).

[597] See Hardware implementation of quantum stabilizers in superconducting circuits by K. Dodge, Roger McDermott et al, Google AI, University of Wisconsin, March 2023 (31 pages).

[598] Cat-codes are used by the startup Alice&Bob. Knowing that their creation goes back to the work of Mazyar Mirrahimi and Zaki Leghtas in 2013, with whom the founders of Alice&Bob worked. Error correction codes are constantly being updated. Thus, a proposal recently emerged from QEC that goes further than cat-code and does not depend on hardware architecture. See Novel error-correction scheme developed for quantum computers, March 2020 which refers to Quantum computing with rotation-symmetric bosonic codes by Arne L. Grimsmo, Joshua Combes and Ben Q. Baragiola, September 2019.

[599] See Quantum spherical codes by Shubham P. Jain et al, NIST, February 2023 (11 pages).

The latter implements in a cavity two superposed Schrödinger cats that allows to manage a projection space used for error correction. It usually corrects flip errors autonomously with T_1 reaching 10 seconds[600]. New variations can correct both flip and phase errors[601]. It must also be engineered to correct entanglement errors[602].

Qutrits can also be built out of bosonic qubits, extending the computational space[603 604]. Bosonic qubits can also rely on using two transmons to actively correct single-photon loss and low-frequency dephasing[605].

> *"Whatever comes out of these gates, we have a better chance to survive if we work together. You understand? We stay together, we survive."*
> General Maximus Decimus Meridius (Russell Crowe) in Gladiator, 2000.

Quantum Error Mitigation. At last, one other solution being considered deals with using NISQ, for Noisy Intermediate Scale Quantum computers, those current quantum computers that use noisy and uncorrected qubits. This is done with algorithms, often hybrid classical/quantum algorithms, which are supposed to be errors resilient and with using some Quantum Error Mitigation techniques (QEM). We will detail it later in this section. Hardware specific QEC and QEM codes also exist like for spin qubits which have significant phase errors[606], photonic qubit which rely on measurement based quantum computing and entirely different QEC schemes[607 608], topological qubits which are well protected against errors but not entirely and still require some QEC[609], neutral atoms[610], trapped ions[611 612], Majorana fermions[613], and also superconducting qubits[614 615 616].

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600 See Quantum control of a cat-qubit with bit-flip times exceeding ten seconds by Ulysse Réglade, Pierre Rouchon, Alain Sarlette, Mazyar Mirrahimi, Philippe Campagne-Ibarcq, Raphaël Lescanne, Sébastien Jezouin, Zaki Leghtas et al, July 2023 (17 pages).

601 See Quantum error correction using squeezed Schrödinger cat states by David S. Schlegel et al, January 2022 (20 pages) which provides protection for both flip and phase errors.

602 See Protecting quantum entanglement between error-corrected logical qubits by Weizhou Cai et al, China, February 2023 (26 pages) which is using three 3D coaxial cavities, with tantalum transmon qubits.

603 See The 2T-qutrit, a two-mode bosonic qutrit by Aurélie Denys and Anthony Leverrier, October 2022 (20 pages).

604 See Multimode bosonic cat codes with an easily implementable universal gate set by Aurélie Denys, and Anthony Leverrier, June-September 2023 (8 pages).

605 See Autonomous error correction of a single logical qubit using two transmons by Ziqian Li, David I. Schuster et al, University of Chicago, February 2023 (20 pages).

606 See Tailoring quantum error correction to spin qubits by Bence Hetényi and James R. Wootton, IBM Research, June 2023 (14 pages).

607 See Increasing error tolerance in quantum computers with dynamic bias arrangement by Hector Bombín et al, PsiQuantum, March 2023 (11 pages).

608 See Modular decoding: parallelizable real-time decoding for quantum computers by Héctor Bombín et al, PsiQuantum, March 2023 (23 pages).

609 See Why and what is the future of the topological qubit? by Chetan Nayak, Microsoft, 2022 (21 mn).

610 See High threshold codes for neutral atom qubits with biased erasure errors by Kaavya Sahay, Shruti Puri et al, February 2023 (17 pages).

611 See Quantum error correction with metastable states of trapped ions using erasure conversion by Mingyu Kang et al, June 2023 (20 pages).

612 See Error mitigation, optimization, and extrapolation on a trapped ion testbed by Oliver G. Maupin et al, Sandia Labs, Brookaven National Laboratory, University of New Mexico, July 2023 (16 pages).

613 See Exponential suppression of Pauli errors in Majorana qubits via quasiparticle detection by Abhijeet Alase et al, UNSW, UC Berkeley and University of Calgary, July 2023 (23 pages).

614 See Relaxing Hardware Requirements for Surface Code Circuits using Time-dynamics by Matt McEwen, Dave Bacon, and Craig Gidney, Google AI, February 2023 (52 pages).

615 See Cleaner magic states with hook injection by Craig Gidney, February 2023 (12 pages).

616 See Inplace Access to the Surface Code Y Basis by Craig Gidney, February 2023 (21 pages).

Error correction principles

The general principle of a typical quantum error correction code is illustrated in Figure 244 with an eight-step correction, using the example of a simple flip-error correction code[617 618 619 620]:

1. **Encoding**: the qubit $|\psi\rangle$ to be corrected will first be coupled a certain number of times via CNOT gates with several auxiliary qubits (here 2). The resulting qubits are entangled. In the example, we get the state $\alpha|000\rangle+\beta|111\rangle$ for an input state $|\psi\rangle = \alpha|0\rangle+\beta|1\rangle$. This is called a repetition code. The three entangled qubit become a logical qubit $|\psi_L\rangle$.
2. **Channel**: single computing or transmission operation that potentially generates an error coming from various sources of noise. The error source is applied to the three entangled qubits.
3. **Detection**: one or more error syndromes are detected via quantum gates that associate qubits with other ancilla qubits and with using parity checking. The example in Figure 244 detects flip errors. The detection is usually based on stabilizer codes which check the parity of two or more qubits using CNOT gates. A change of basis with an H gate before this operation enables syndrome detection in another basis, for a phase error detection.
4. **Measurement**: the state of these parity checking ancilla qubits is measured to generate classical bits. This is also called a syndrome extraction process. This must be some non-demolition measurement for the corrected entangled qubits that does not destroy it. These measurements are labelled “mid-circuit measurements” since it occurs before the end of your circuit execution, only on a subset of the register qubits and without changing the quantum state of other qubits in the register. The syndrome measurement part is using here two “ZZ measurements” since it jointly measures along the Bloch sphere Z axis the parity of qubit pairs. If we were to change the basis with using an H gate to detect a phase error, we’d then implement an “XX measurement”.
5. **Decoding**: using the classical result of mid-circuit measurement, the syndrome detection takes place classically to determine the error to correct. In the flip error case, it generates the index of the qubit to be corrected in the replicated entangled qubits using a simple correspondence table. It cannot detect an error affecting simultaneously two or three of the entangled qubits. With more complicated error correction codes like surface codes, this decoding process can be costly and has a complexity scaling exponentially with the number of ancilla qubits.
6. **Correction**: the address obtained with syndrome measurement is used to correct the faulty qubits using a single X gate. This error correction code cannot correct an error that would affect more than one of the three entangled qubits.
7. **Recycling**: the corrected qubits in $|\psi_L\rangle$ are disentangled using CNOT gates to recreate an isolated corrected qubit $|\psi_C\rangle$. The two other ancilla qubits are reset to $|0\rangle$.
8. **Reuse**: the corrected qubit and all the ancilla qubits can be reused for subsequent operations that will also be corrected using the same process.

Error correction codes charts such as Shor's 9-qubit code on Wikipedia are usually not complete. They usually lack the measure and correction steps. It also does not specify where error correction codes are happening in a quantum algorithm. It is required at each stage of some quantum computation. Error correction codes will be repeated several times, proportionally to the quantum algorithm circuit depth. Indeed, these error correction codes can be executed in parallel with every quantum gate happening simultaneously in a quantum circuit cycle.

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[617] Based on A Tutorial on Quantum Error Correction by Andrew M. Steane, 2006 (24 pages).

[618] See also An introduction to quantum error correction by Mazyar Mirrahimi, 2018 (31 slides).

[619] See Quantum Error Correction For Dummies by Avimita Chatterjee et al, PennState University, April 2023 (12 pages).

[620] See Quantum Circuits for Stabilizer Error Correcting Codes: A Tutorial by Arijit Mondal et al, September 2023 (15 pages).

The compiler's role will be to add all QEC codes to the circuit. The QEC codes will increase computation time by at least one order of magnitude depending on the ratio of physical qubits per logical qubits, the qubit life-extension obtained with the code and the time it takes to classically decode error syndromes. Error correction must indeed be accounted for when evaluating the quantum computing speedup gain brought by a given algorithm with another one or with a classical equivalent.

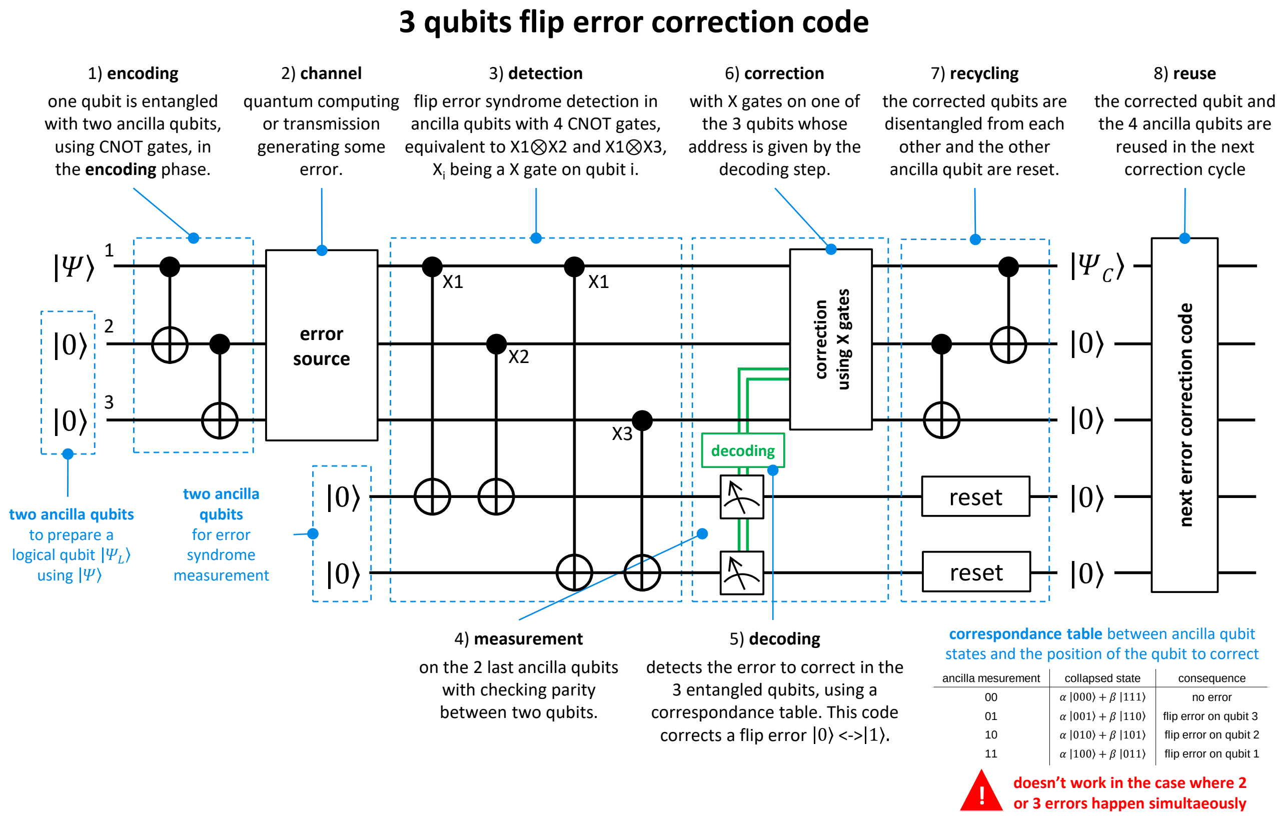

Figure 244: a simple flip-error correction code.
Adapted from A Tutorial on Quantum Error Correction by Andrew M. Steane, 2006 (24 pages). (cc) Olivier Ezratty, 2023.

Is the simple flip-error correction code in Figure 244 a three or five qubits code? It is usually described as a 3-qubit code when dealing with the size of the logical qubit $|\psi_L\rangle$ or a 5-qubit code when including all ancilla and measurement qubits used in syndrome detection (3+2=5).

A variation of this code adds some H gates to correct phase errors. The 1995 **Shor's 9-qubit error correction code** is a simple code consolidating these two methods, with the corrected qubit being replicated 8 times, as shown in Figure 245. This code corrects both flip and phase errors[621]. It uses a total of 15 qubits. In the first phase the corrected qubit is replicated two times, and each resulting qubit is again replicated two times with CNOT gates. The first three blocks of 3 qubits implement a flip error correction. It outputs 3 qubits which then implement a phase error correction.

Raymond Laflamme (1960, Canada) demonstrated in 1996 that at least five physical qubits are needed to create a "logical qubit" integrating flip and phase error correction. With **Emanuel Knill**, he also demonstrated that any single qubit error was a linear combination of flip and phase errors, leading to factoring error correction to flip and phase errors corrections[622].

[621] The details of the process are well documented in the Wikipedia sheet of quantum error correction.

[622] This is demonstrated in A Theory of Quantum Error-Correcting Codes by Emanuel Knill and Raymond Laflamme, 1996 (34 pages). But also independently in Mixed State Entanglement and Quantum Error Correction by Charles Bennett, David DiVincenzo, John A. Smolin and William K. Wootters, 1996 (82 pages). See also Magic States by Nathan Babcock (28 slides).

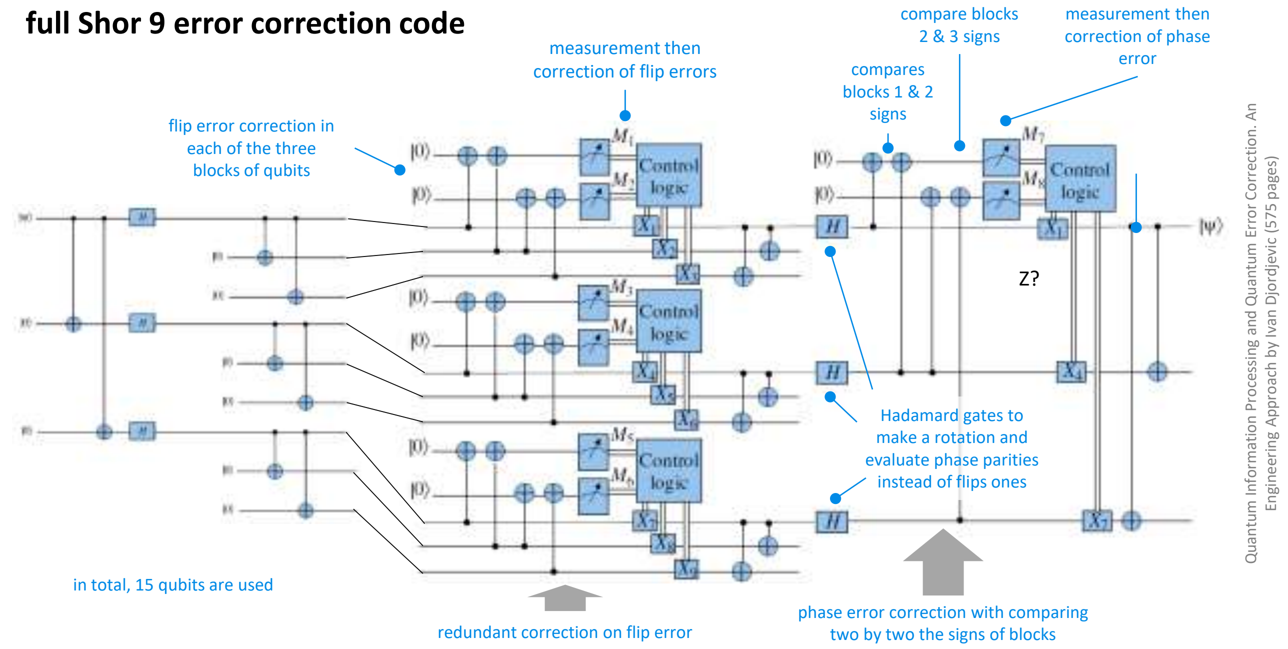

Figure 245: a full Shor 9 error correction code correcting both flip and phase errors.
Source: Quantum Information Processing and Quantum Error Correction. An Engineering Approach by Ivan Djordjevic (575 pages).

In practice, the 7-qubit **Steane** code is the most referenced because it is not redundant like the Shor code. These 3-, 5-, 7- and 9-qubit codes are part of a generic group called **stabilizer codes** formalized by **Daniel Gottesman** in 1997. We are now going to dig a little deeper into how they operate.

We will better understand how an error correction works without reading the state of the qubit to be corrected. Let's take the case of a simple flip error correction code with three qubits.

These three entangled qubits can have an error X_1, X_2 or X_3 or no error (I=identity). X is an amplitude inversion Pauli gate. It creates an amplitude inversion of the corresponding entangled qubit as shown in the equations in Figure 246. These new states correspond to three errors and the absence of errors.

$$|\psi_L\rangle = \alpha|000\rangle + \beta|111\rangle \xrightarrow{I} \alpha|000\rangle + \beta|111\rangle,$$
$$|\psi_L\rangle = \alpha|000\rangle + \beta|111\rangle \xrightarrow{X_1} \alpha|100\rangle + \beta|011\rangle,$$
$$|\psi_L\rangle = \alpha|000\rangle + \beta|111\rangle \xrightarrow{X_2} \alpha|010\rangle + \beta|101\rangle,$$
$$|\psi_L\rangle = \alpha|000\rangle + \beta|111\rangle \xrightarrow{X_3} \alpha|001\rangle + \beta|110\rangle.$$

Figure 246: amplitude inversions.

These four states have the interest of being mathematically orthogonal for all the values of the α and β defining the state of the qubit to be corrected. The trick is to perform a measurement of these values in the vector space corresponding to these four values and not in the original qubit computational base. This will not deteriorate the superposition of the original qubit. The syndrome extraction is called a "Stabilizer code" or “stabilization code”, which will feed the ancilla qubits. The process is the same to evaluate and correct a phase error but with Z gates instead of X gates.

The disadvantage of the solution is that it cannot detect errors that would occur simultaneously on two or three of the entangled qubits. No error-correcting code can correct all errors!

The stabilizer codes formalism generically describes the error correction codes we have just studied with three parameters: **[[n, k, d]]** with:

- **k** = number of logical qubits, usually 1 which is the qubit that needs to be corrected.
- **n** = number of physical qubits used in the code, with n>k. The n-k qubits store redundant information thanks to entanglement.
- **d** = smallest number of simultaneous qubit errors that can transform one valid codeword into another, *aka* the **code distance**. The complete definition is actually more complicated.

More precisely, an error correction code with a code distance d can correct errors for up to (d − 1)/2 (replicated) qubits, or say differently d ≥ 2m+1, m being the number of redundant qubits that can be corrected. You need d to be at least 3 to correct both flip and phase errors.

In this notation, Shor's 9 qubit code is a [[9, 1, 3]], Steane's is a [[7, 1, 3]] and Laflamme's is a [[5, 1, 3]]. They all have a code distance of 3. A simple 3-qubit flip or phase correction code is a [[3, 1, 1]] stabilizer code with a code distance of 1. There are larger cases like with the [[512, 174, 8]] CSS code[623].

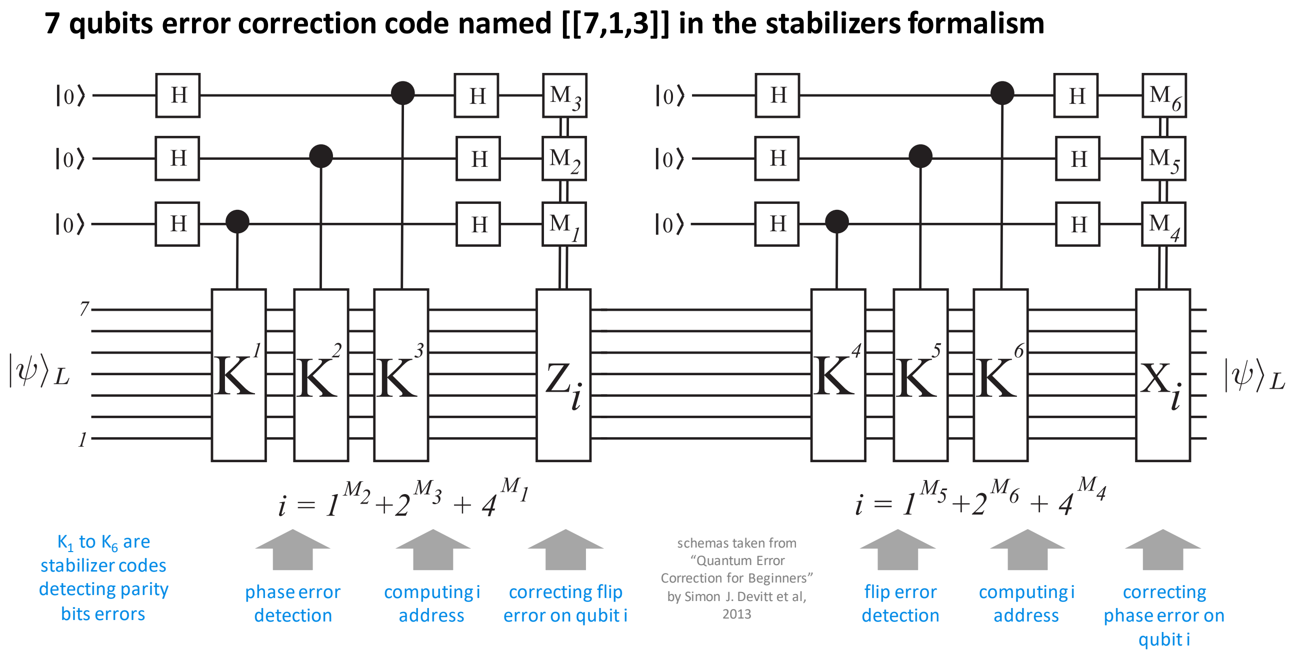

Figure 247: 7 qubits correction code with a code distance 3.
Source: [Quantum error corrections for beginners](#) by Simon J. Devitt et al, 2013.

The stabilizer codes use a syndrome table that provides a match between the errors on each qubit and the detected syndrome. The number of ancilla qubits used to create this table must therefore be sufficient to identify the qubits to be corrected in the logical qubit. In the example in Figure 247 with a logical qubit with 7 physical qubits, the 3 ancilla qubits allow the identification of eight scenarios, sufficient to determine which of the 7 physical qubits must be corrected. The 8th scenario is the absence of error, therefore needing no correction.

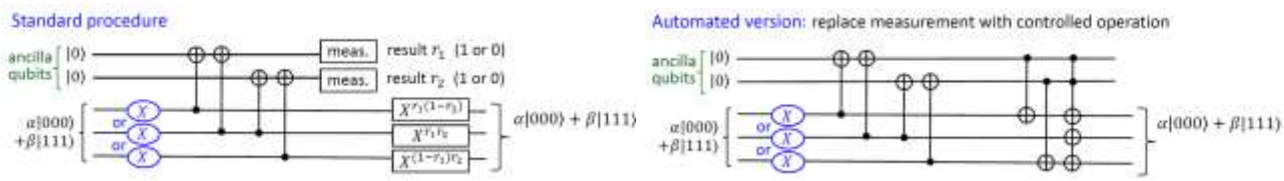

Figure 248: error correction replacing measurement with a controlled operation.
Source: [Quantum error correction (QEC)](#) by Alexander Korotkov, 2017 (39 slides).

Some quantum error correction codes detect syndrome and correct errors without using measurement and classical syndrome detection steps, parity checks and correction being directly implemented with quantum gates (see Figure 248 on the right)[624]. It is not commonplace for a reason I don't know yet.

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[623] See Classical product code constructions for quantum Calderbank-Shor-Steane codes by Dimiter Ostrev et al, 2022 (19 pages).

[624] See Quantum Error Detection Without Using Ancilla Qubits by Nicolas J. Guerrero and David E. Weeks, US Air Force Institute of Technology, April 2022 (8 pages). This is labelled a "no ancilla error detection" code (NAED).

Then, you have the autonomous methods also branded **aQEC** (autonomous quantum error correction) that is implemented at the qubit level. Some energy dissipation must be handled, using a technology called reservoir engineering, which is implemented in cat-qubits and other bosonic codes[625]. Otherwise, whatever, the ancilla qubits used in typical QEC must be reset to |0⟩ and this reset is a dissipative process. The thermal bath is just elsewhere!

Logical Qubits

With quantum computers available online such as those from IBM having up to a few dozen qubits, it is the role of software to implement dynamic error correction codes and more precisely, compilers that will transform the developer's code into executable machine code at the physical level of the qubits and integrating QEC code. Given that we have at this point just enough qubits to test small QEC like Steane's 7-qubits codes or small sized surface codes.

Conceptually, a logical qubit sits between a physical qubit (with a small lifetime and prone to significant error rates) and a mathematically perfect qubit (with infinite computing time and zero error). It lasts longer than a physical qubit and should have an error rate in the range 10^{-8} to 10^{-15} that is compatible with the constraints of your algorithm. This error rate is more or less the inverse of the number of quantum gates in your circuit.

The number of physical qubits to be assembled to create a logical qubit first depends on the error rate of the qubits. The higher the qubit error rate, the more qubits must be assembled. This number can reach several thousand qubit physical qubits[626]. As a result, QEC settings will be algorithm dependent and seem to be bound to be implemented mainly with software and on generic QPU architectures. Beyond the target error rate, the number of physical qubits in a logical qubit will also depends on many other factors such as the quantum error correction code used, the underlying physical qubits fidelities and their connectivity as shown in Figure 252.

Current estimates are around 1,000 physical qubits to create a logical qubit, but with a logical qubit precision dependent on the above mentioned factors. This corresponds to the plans published by **IBM**, **Google** and **PsiQuantum** with 100 logical qubits created out of one million physical qubits. On the physical architecture side, **topological qubits** are an analog version of surface codes that should allow to reduce this ratio of logical/physical qubits, just like cat-qubits, which are forecasted to require fewer than 100 physical qubits to create one logical qubit.

Trapped ions can use **lattice surgery** to connect and entangle these topologically corrected physical qubits[627]. **IonQ** was planning in 2020 to create logical qubits corrected with a **Bacon-Shor** code, a variation of Shor's code with 13 qubits which was the first example of a subsystem code that was proposed by David Poulin[628] [629].

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[625] See Protecting a Bosonic Qubit with Autonomous Quantum Error Correction by Jeffrey M. Gertler et al, University of Massachusetts-Amherst and Northwestern University, October 2020 (23 pages). This study investigates autonomous QEC on bosonic codes qubits using reservoir engineering. See also Autonomous quantum error correction and quantum computation by Jose Lebreuilly et al, Yale University, Amazon and University of Chicago, March 2021 (18 pages) and Autonomous quantum error correction with superconducting qubits by Joachim Cohen, ENS Paris, 2017 (164 pages).

[626] See What determines the ultimate precision of a quantum computer? by Xavier Waintal, 2019 (6 pages) which describes the limits of error correction codes. Other useful contents on error correction include: Error mitigation in quantum simulation, Xiao Yuan, IBM Research, 2017 (42 minutes), Code Used To Reduce Quantum Error In Logic Gates For First Time, 2019, Scientists find a way to enhance the performance of quantum computers by the University of Southern California, 2018 and Cramming More Power Into a Quantum Device by Jay Gambetta and Sarah Sheldon, March 2019 about the error level of the IBM Q System One announced in January 2019.

[627] See Error protected quantum bits entangled, University of Innsbruck, January 2021 referring to Entangling logical qubits with lattice surgery by Alexander Erhard et al, Nature, 2020 (15 pages).

[628] See Stabilizer Formalism for Operator Quantum Error Correction by David Poulin, PRL, 2005 (5 pages).

[629] Bacon-Shor code is documented in Operator Quantum Error Correcting Subsystems for Self-Correcting Quantum Memories by Dave Bacon, 2006 (17 pages).

It was enabled by their good physical qubit fidelities[630].

For qubits that can be physically well connected with their immediate neighbors, the most often considered error correction is the **surface code**, created between 1998 and 2001.

As shown in the diagram in Figure 249, it uses matrices of processing qubits (in white) connected to measuring qubits (in black) via **Pauli X** (amplitude flip) and **Pauli Z** (phase flip) gates operating on these data qubits as shown in yellow and green. This gives two ancilla qubits for two physical qubits organized to detect and correct flip and phase errors over 4 replicated qubits. This constitutes a stabilizer code of type [[5, 1, 2]] using four blocks with four cycles.

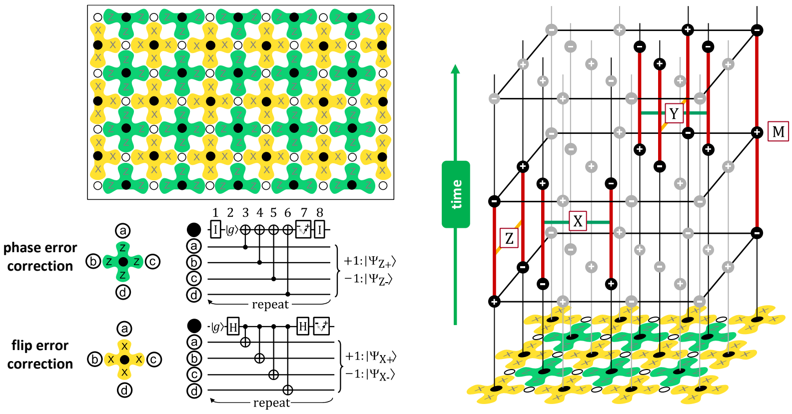

Figure 249: surface code physical layout and process.
Source: [Surface codes: Towards practical large-scale quantum computation](#) by Austin G. Fowler, Matteo Mariantoni, John M. Martinis and Andrew Cleland, 2012 (54 pages).

A surface code with distance d requires d² replicated qubits and $d^2 - 1$ measurement qubits, so a total of $2d^2 - 1$ physical qubits to correct a single qubit. d being usual odd, you then have an even number of measurement qubits divided in two equal parts with flip (Z) measurement) and phase (X) measurement qubits as shown below with two example of distance 3 and 5 surface codes.

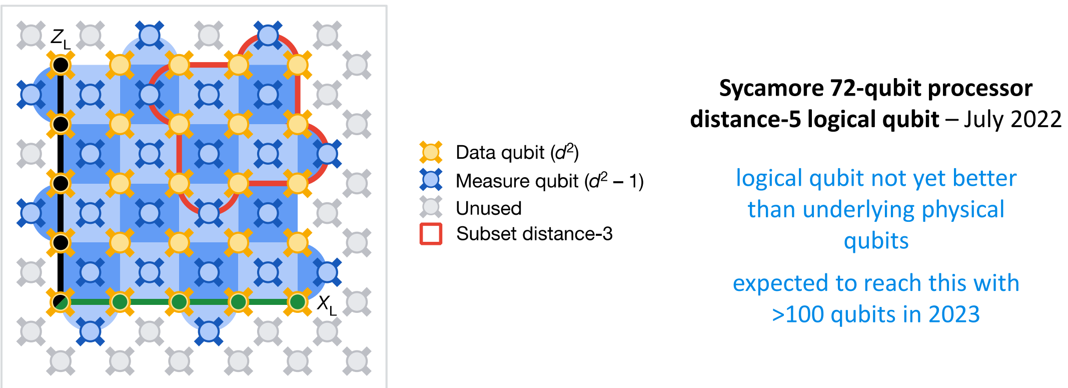

Figure 250: the last logical qubit created by Google AI assembles a distance-5 surface code with 49 qubits but its fidelity is not as good as the underlying physical qubits. See [Suppressing quantum errors by scaling a surface code logical qubit](#) by Rajeev Acharya et al, Google AI, Nature, July 2022-February 2023 (44 pages). Google projects to create a logical qubit with better fidelities than their physical qubits with a distance-7 surface code and 98 qubits.

A surface code logical qubit error rate is $P_L \approx 0.03(p/p_{th})^{d_e}$ with p being the physical qubit error rate, p_{th} being the threshold physical error rate below which logical errors falls with d, and d_e being linked to the surface code distance d with $d_e = d/2$ when d is even, and $= (d+1)/2$ when d is odd. See Figure 251 and an example in Figure 250 of a surface code with a distance-5 logical qubit created by Google in 2022[631].

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[630] And Fault-Tolerant Operation of a Quantum Error-Correction Code by Laird Egan, Christopher Monroe et al, 2020 (23 pages).

[631] Although this logical qubit was not better than its physical qubits, these Sycamore qubits were still improved compared with Sycamore's original 53-qubit version in 2019. It increased qubit relaxation and dephasing lifetimes through an improved fabrication process and environmental noise reduction near the quantum processor, reduced cross-talk between all physical qubits during parallel operation, reduced drift and improved qubit control fidelity through upgraded custom electronics, implemented faster and higher-fidelity readout and reset operations, reduced calibration errors, improved control electronics calibration, and enhanced dynamical decoupling protocols to protect physical qubits from noise and cross-talk during idling operations.

How are logical qubits delimited and logical gates implemented is another (complicated) story. Some more efficient surface codes using long range connectivity are also proposed for trapped ions and cold atoms[632] [633].

A quantum error correcting code has a certain **threshold** level that defines the higher bound of physical qubit error rates when a logical qubit will have an error rate inferior to that of the physical qubits. It depends on the code itself and on the qubit type. Surface codes have a higher threshold in the 1% range and are thus tolerant to higher qubit error rates. But they require a larger number of physical qubits per logical qubits. Physical qubits must be connected to their immediate neighbors in a 2D structure or with honeycomb variations [634]. For example, if the physical error rate is 10 times below the threshold, d should be greater than 17 to achieve a logical error rate below 10^{-10} [635].

Lattice surgery is used with surface codes to encode and manipulate logical qubits in 2D qubit lattices. It relies on local measurements, magic states distillation and their fusion that implement logical gates in a fault-tolerant manner and corrections to perform logical operations in a fault-tolerant manner. Due to the significant overhead of lattice surgery, many techniques are proposed to optimize it[636].

MWPM (minimum weight perfect matching) is an algorithm used in surface codes that interprets the syndromes and determines the most likely error configuration that was detected[637]. It involves finding the minimum-weight matching of the syndrome graph, which is a bipartite graph representing the connections between the syndrome qubits and the corresponding data qubits. The graph's edges are assigned weights based on the syndromes obtained from the syndrome measurement. The weights indicate the likelihood of an error occurring between the connected qubits. The algorithm finds a matching that minimizes the total weight of the edges, which corresponds to the most probable error configuration. Once obtained, the identified errors can be corrected using recovery single Pauli gate operations. MWPM complexity scales as $O(d^3)$ to $O(d^5)$ with a surface code of distance d, and, of course, researchers are trying to minimize this significant classical computing overhead[638] [639].

Union-find decode is another syndrome decoding technique that that doesn't find the perfect matching of error configuration but is more efficient than MWPM since it scales as $O(d^2)$ [640].

Classical cost. The classical part of quantum error correction is significant. It is handled by a classical computer that is as close as possible to the electronics controlling the qubit, whatever the qubit type. The turn-around correction cycle must be as fast as possible, under the constraint of physical qubit decoherence time.

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632 See Long-range-enhanced surface codes by Yifan Hong et al, University of Colorado and NIST, September 2023 (16 pages).

633 See An Architecture for Improved Surface Code Connectivity in Neutral Atoms by Joshua Viszlai et al, September 2023 (13 pages).

634 Surface codes are well documented in Surface codes: Towards practical large-scale quantum computation, by Austin G. Fowler, Matteo Mariantoni, John M. Martinis and Andrew Cleland, 2012 (54 pages) but their source of inspiration is older and comes from Quantum codes on a lattice with boundary by Sergey Bravyi and Alexei Kitaev, 1998 (6 pages). In practice, the structure of surface codes is quite complex and involves activated and deactivated substructures in the qubit matrix.

635 See Scalable Quantum Error Correction for Surface Codes using FPGA by Namitha Liyanage et al, Yale University, January 2023 (12 pages).

636 See New magic state distillation factories optimized by temporally encoded lattice surgery by Prithviraj Prabhu et al, AWS, USC and IQIM, October 2022 (35 pages).

637 See How to choose a decoder for a fault-tolerant quantum computer? The speed vs accuracy trade-off by Nicolas Delfosse, Andres Paz, Alexander Vaschillo and Krysta M. Svore, Microsoft, October 2023 (19 pages) which describes the challenges of choosing the right decoder and their performance limits and their trade-offs between speed and accuracy.

638 See Hardness results for decoding the surface code with Pauli noise by Alex Fischer et al, September 2023 (36 pages).

639 See Splitting decoders for correcting hypergraph faults by Nicolas Delfosse, Adam Paetznick, Jeongwan Haah and Matthew B. Hastings, Microsoft, September 2023 (12 pages).

640 See An interpretation of Union-Find Decoder on Weighted Graphs by Yue Wu, Namitha Liyanage, and Lin Zhong, Yale, November 2022 (20 pages).

One key bottleneck here is the syndrome decoding that is theoretically an NP-hard problem[641 642]. QEC dynamic code compilation is also a challenge[643]. Surface code compiling with many logical qubits seems possible in a reasonable time[644]. Surface codes syndrome decoding is a NP-hard problem[645], thus the need to find solutions to simplify it, like using deep learning based techniques[646].

Instruction bandwidth bottleneck is yet another engineering challenge for FTQC and error correction. Thousands of physical qubits must be driven by software-based quantum error correction. It creates a digital workload from the classical control computer down to the physical qubits and their many ancilla qubits, in a range exceeding several tens of TB/s just for factoring a 1024 bits integer with Shor's algorithm! Specific architectures can be designed to handle QEC as close as possible to the physical qubits, ideally in cryo-electronics components and with some microcode sitting at the lowest possible stage in the cryostat (for solid-state qubits), starting at 4K[647].

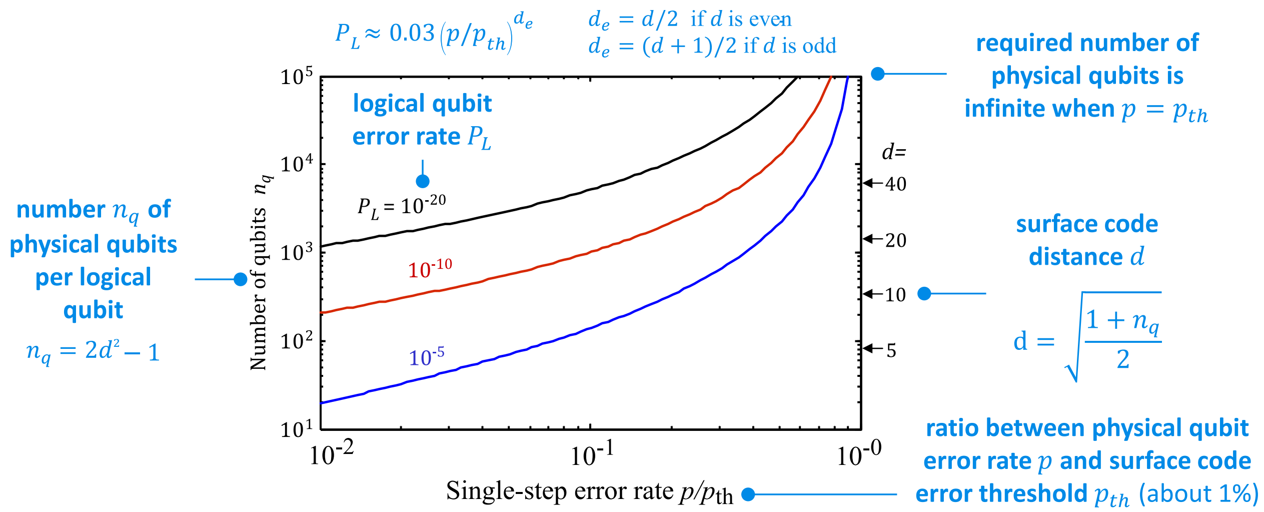

Figure 251: relationship between physical and logical qubit error rate with the number of physical qubits in a logical qubit and the surface code distance. Source: the excellent review paper An introduction to the surface code by Andrew Cleland, University of Chicago, 2022 (68 pages). Added in 2023.

Other proposals consist in improving syndrome decoding and corrections parallelism[648]. The current record for a QEC cycle is about 300 ns to one millisecond which starts to fit the speed of superconducting qubits, which are at this point the most challenging of all[649].

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641 See Sparse Blossom: correcting a million errors per core second with minimum-weight matching by Oscar Higgott and Craig Gidney, Google AI, March 2023 (33 pages) with a fast implementation of the minimum-weight perfect matching (MWPM) decoder, that works as fast as the rate at which syndrome data is generated by superconducting quantum computers.

642 See Decoding quantum color codes with MaxSAT by Lucas Berent et al, March 2023 (17 pages).

643 See Enabling Full-Stack Quantum Computing with Changeable Error-Corrected Qubits by Anbang Wu et al, UCSB, May 2023 (13 pages).

644 See A High Performance Compiler for Very Large Scale Surface Code Computations by George Watkins et al, Aalto University, Simon Fraser University and TUM Munich, February 2023 (10 pages).

645 See On the Hardnesses of Several Quantum Decoding Problems by Kao-Yueh Kuo and Chung-Chin Lu, National Tsing Hua University, Quantum Information Processing, 2013 (6 pages).

646 See Deep Quantum Error Correction by Yoni Choukroun and Lior Wolf, Tel Aviv University, January 2023 (11 pages).

647 See the QuEST architecture proposal in Taming the Instruction Bandwidth of Quantum Computers via Hardware Managed Error Correction by Swamit Tannu et al, GeorgiaTech, Stanford and Microsoft, 2017 (13 pages slides).

648 See Parallel window decoding enables scalable fault tolerant quantum computation by Luka Skoric et al, Riverlane, UCL and University of Sheffield, September 2022 (12 pages) which describes a solution to optimize the ingestion of this data thanks to parallelization using a sliding window mechanism (which I don't understand well). One million physical qubits represent 3.3TB of data per second!

649 See the perspective Real-Time Decoding for Fault-Tolerant Quantum Computing: Progress, Challenges and Outlook by Francesco Battistel et al, Qblox, AWS, QuTech, Riverlane et al, Nano Futures, February-August 2023 (15 pages).

How about real implementations of logical qubits? They are now plentiful but are not yet creating logical qubits with higher fidelities than their underlying physical qubits.

- A team from **Maryland University** led by Christopher Monroe implemented in January 2021 a logical qubit using a Bacon-Shor 13 code with a chain of 15 trapped ytterbium ions that was correcting single qubit errors[650]. They then used two such logical qubits in a configuration of 32 qubits to implement fault-tolerant 2-qubit gates.
- A **China** research team implemented a distance 3 surface code using 17 physical qubits ($=3^2+(3^2-1)$) on the 66 qubits Zuchongzhi 2.1 superconducting qubits QPU. It implements repeated error corrections and post-processing error corrections[651].
- **Quantinuum** created a single Steane color-code also using 10 trapped ion qubits. They created a break-even logical qubit with better fidelities than their underlying physical qubits in August 2022[652]. They extended it in 2023 to three logical qubits used to simulate an H_2 molecule using a Bayesian quantum phase estimation[653] and an optimized version for a 1-qubit adder[654].
- A team led by **Andreas Wallraff** from ETH Zurich did a similar experiment with 17 superconducting qubits in 2021[655].
- Another team in **Austria** and **Germany** developed a proof of concept of two logical qubits made with trapped ions and using a T gate and magic state distillation[656].
- A joint **QuTech-Fujitsu-Element Six** team demonstrated in 2022 a fault-tolerant operation of a NV centers based QPU with logical qubits made of 5 physical spin qubits and two additional measurement qubits in a 29-qubit QPU running at 10K[657].
- **Google** announced in July 2021 the creation of their first logical qubits with 5 and 21 physical qubits, showing a x100 improvement in the error rate between the 5 and 21 version[658]. In 2022, they created a distance 5 surface code logical qubit with 49 qubits ($=5^2+(5^2-1)$) that improves logical qubit error rates as physical qubits per logical qubits grows. But they have not yet reached the QEC efficiency threshold where logical qubit errors would be lower than physical qubit

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[650] See Fault-tolerant control of an error-corrected qubit by Laird Egan, Christopher Monroe et al, January 2021 (9 pages). The 15 used qubits contain the 9 for the Bacon-Shor correction code, 4 for the stabilizers ancilla and two unused ions at the edges of the 1D set of ions.

[651] See Realization of an Error-Correcting Surface Code with Superconducting Qubits by Youwei Zhao et al, PRL, December 2021 (10 pages). "*Future work will concentrate on realizing larger-scale surface codes, to achieve the important goal of suppressing the logical error rate as the code distance increases. This necessitates further improvements to the quantum computing system's performance, such as the number and quality of qubits, the fidelity of quantum gate operations, and rapid feedback of digital electronics*".

[652] See Realization of Real-Time Fault-Tolerant Quantum Error Correction by C. Ryan-Anderson et al, PRX, December 2021 (29 pages), a follow-up from the previous paper. It uses a 10 qubit trapped ion quantum computer to encode a single logical qubit using the Steane [[7, 1, 3]] color code. They later implemented a [[7, 1, 3]] color code with 20 qubits with a transversal CNOT gate on two logical qubits in Implementing Fault-tolerant Entangling Gates on the Five-qubit Code and the Color Code by C. Ryan-Anderson et al, August 2022 (17 pages) and could obtain a logical qubit with 99.94% fidelity, compared to 99.68% for the underlying physical qubits.

[653] See Demonstrating Bayesian Quantum Phase Estimation with Quantum Error Detection by Kentaro Yamamoto, Samuel Duffield, Yuta Kikuchi, David Muñoz Ramo, Quantinuum and RIKEN, June 2023 (16 pages).

[654] See Fault-Tolerant One-Bit Addition with the Smallest Interesting Colour Code by Yang Wang et al, September 2023 (11 pages).

[655] See Realizing Repeated Quantum Error Correction in a Distance-Three Surface Code by Sebastian Krinner, Alexandre Blais, Andreas Wallraff et al, Nature, December 2021-May 2022 (28 pages).

[656] See Demonstration of fault-tolerant universal quantum gate operations by Lukas Postler, Rainer Blatt et al, Nature, December 2021 (14 pages).

[657] See QuTech and Fujitsu realise the fault-tolerant operation of a qubit by Qutech, May 2022.

[658] For Google's logical qubit, see Exponential suppression of bit or phase errors with cyclic error correction by Zijun Chen et al, February 2021 in arXiv and in Nature in July 2021 (6 pages) and supplemental materials (30 pages).

errors[659]. Google researchers indicate that their logical qubits would be better than their underlying physical qubit starting with a distance 7 surface code, requiring about 100 physical qubits, which they are not far from obtaining.

- A proposal coming from Italy consists in **encoding a logical qubit into a qudit**, a quantum object with many distinct states. On paper, it reduces the number of individual physical objects needed to create logical qubits[660]. But this theoretical proposal moves the goalpost elsewhere, like making sure that quantum gates have good fidelities and not too much overhead in such settings.

What are the figures of merit of a quantum error correction code and a logical qubit architecture? As shown in Figure 252, a key one is the logical qubit fidelities. You can't claim the creation of a logical qubit without adding its target fidelity. The end goal is to reach between 10^{-9} and 10^{-15} error rates. These rates differ according to the target algorithms. 10^{-9} may be enough for condensed matter and Hubbard models simulations, 10^{-15} is required for Shor's integer factoring for 2048-bit RSA keys while even better error rates are required for complex chemical simulations. The inverse of these error rates corresponds approximatively to the number of T gates to execute in these algorithms, which are the costlier to correct in surface codes. A milestone in this quest is the **TeraQuop** quantum computer, that could execute a trillion quantum operations before a single logical error occurs (10^{12}).

Then comes the overhead with the number of physical qubits per logical qubit but also code time cost, meaning, how will it slow down quantum computing. A logical qubit must also be able to implement non-Clifford quantum gates in a fault-tolerant manner, an aspect we'll describe in the next part. There are some documented lower bounds for physical per logical qubit number[661].

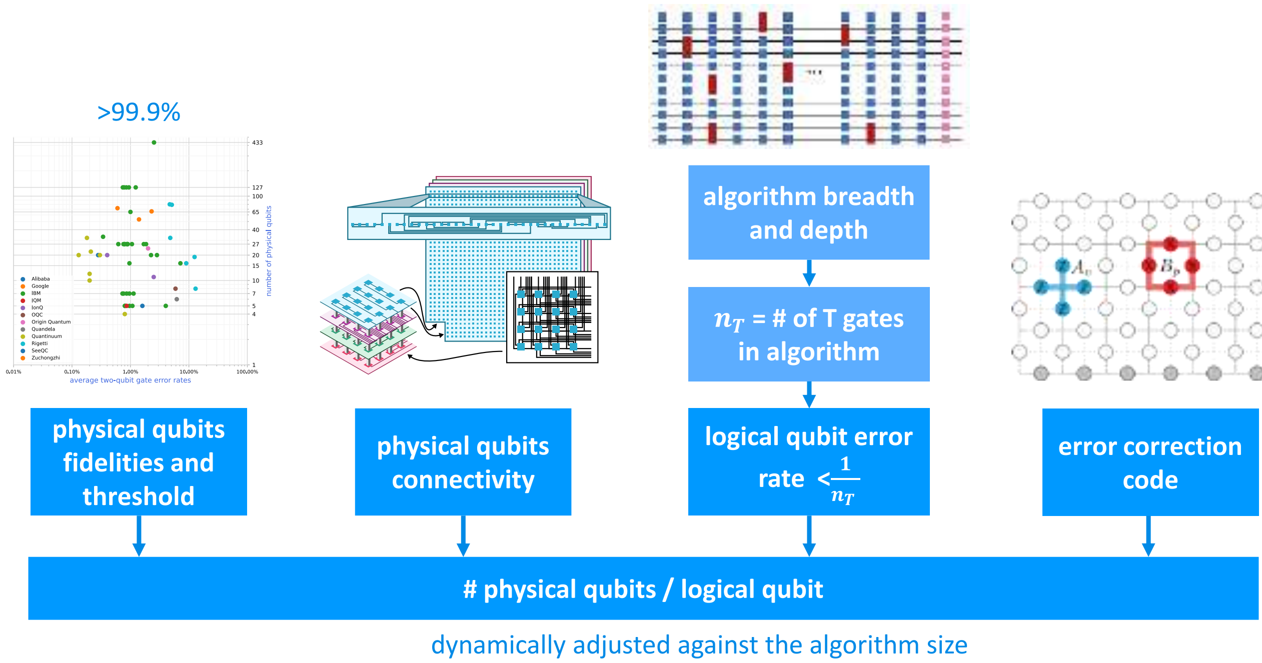

Figure 252: the various parameters influencing the number of physical qubits to build a logical qubit. (cc) Olivier Ezratty, 2023-2024.

Will logical qubit entirely fit in a monolithic chip? It is far from being obvious when looking at the various vendor roadmaps. IBM plans to scale-out their chips starting with 133 qubits (Heron) first with microwave interconnect but then at around 1,500 qubits, with photonic based interconnect.

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[659] See Suppressing quantum errors by scaling a surface code logical qubit by Rajeev Acharya et al, Google AI, July 2022 (44 pages).

[660] See Fault-Tolerant Computing with Single Qudit Encoding by Matteo Mezzadri et al, July 2023 (7 pages).

[661] See A Converse for Fault-tolerant Quantum Computation by Uthirakalyani G et al, November 2022 (10 pages) which quotes A lower bound on the space overhead of fault-tolerant quantum computation by Omar Fawzi et al, January 2022 (23 pages).

Trapped ions vendors like IonQ also have various scale-out approaches also using photonic interconnect. Silicon spin qubits could scale to higher levels thanks to being smaller, but their large scale entanglement must be demonstrated like with each and every other platform.

Fault-tolerant quantum computing

FTQC (fault-tolerance quantum computing) was defined by Peter Shor in 1997[662] and is based on a few general principles related to implementing a practically useful QEC scheme with logical qubits: error-tolerant state preparation, error-tolerant quantum gates, error-tolerant measurement and error-tolerant error correction. Error correction codes can themselves introduce errors since they use quantum gates and state measurements which themselves generate errors. Moreover, error correction codes do not correct all possible errors. They just increase the apparent fidelity rate of the corrected qubits.

Also, QEC codes used repeatedly during long calculations must not introduce more errors than are corrected and should not spread errors in an uncontrollable way to various qubits in the computing register. As Peter Shor recounts: "*To be able to build a quantum computer, it's not enough to be able to correct errors with noiseless gates; you need to be able to correct errors using noisy gates. This means you have to correct the errors faster than you introduce new ones*"[663]. This is where you understand why qubit gates, qubit readout time and the classical processing of readout data have all to be as fast as possible.

FTQC theoretically allows the execution of algorithms of arbitrary length, whereas without it, we are limited to a few series of gates. The challenge is to ensure that the calculation and QEC prevents errors from cascading. We must avoid linking one qubit with too many qubits with multi qubit gates in QECs. For this respect, a 7-qubits Steane code is appropriate.

And let's not forget that a CNOT gate propagates flip errors from the control qubit to the target qubit and phase errors from the target to the control. From an operational standpoint, FTQCs creation involves minimizing the number of ancilla qubits and optimizing the choice of QECs according to the type of errors generated by each type of qubit and quantum gates[664].

Transversal gates are implemented with FTQC to avoid propagating errors beyond the corrected qubits. It is an arrangement of links between logical qubits linked together by two-qubit gates. The diagram in Figure 253 illustrates these links between two logic qubits using a 7 qubit Steane code via CNOT gates. Each of the physical qubits of the logical qubits is connected one by one between the two logical qubits. This is still very theoretical, besides trapped ions, no qubit topology enables this kind of connectivity.

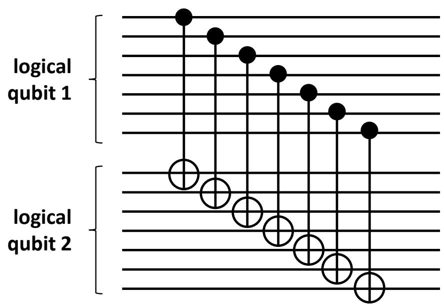

Figure 253: transversality connecting two logical qubits.

However, transversal gates can only be implemented within the Clifford group. According to the **Eastin-Knill** no-go theorem, no QEC code can transversally implement a universal gate set. That's why we usually need a costly QEC named **magic state distillation** to implement FTQC with T and Toffoli gates which lie outside the Clifford group. It has a huge cost of two orders of magnitude for physical per logical gates, explaining why it is often estimated said that logical qubits require overs 10K physical qubits (on top of the effect of code concatenations or surface code)[665].

[662] See Fault-tolerant quantum computation by Peter Shor, March 1997 (11 pages).

[663] In The Early Days of Quantum Computation by Peter Shor, August 2022 (10 pages).

[664] See A Comparative Code Study for Quantum Fault Tolerance by David DiVincenzo, Barbara Terhal and Andrew Cross, 2009 (34 pages).

[665] See Roads towards fault-tolerant universal quantum computation by Earl T. Campbell et al, 2018 (9 pages).

One of the problems is that error correction generates an overhead that grows faster than the exponential gain of the quantum computer (2^{4n} vs 2^n according to Quantum Benchmark).

We can get some comfort from the **threshold theorem** demonstrated by Dorit Aharonov and Michael Ben-Or in 1999 according to which it is possible to perform error correction up to an arbitrary desired apparent error rate if the error rate of the single-qubit gates is below a given threshold which is dependent on the error correction code used and the characteristics of the qubits[666].

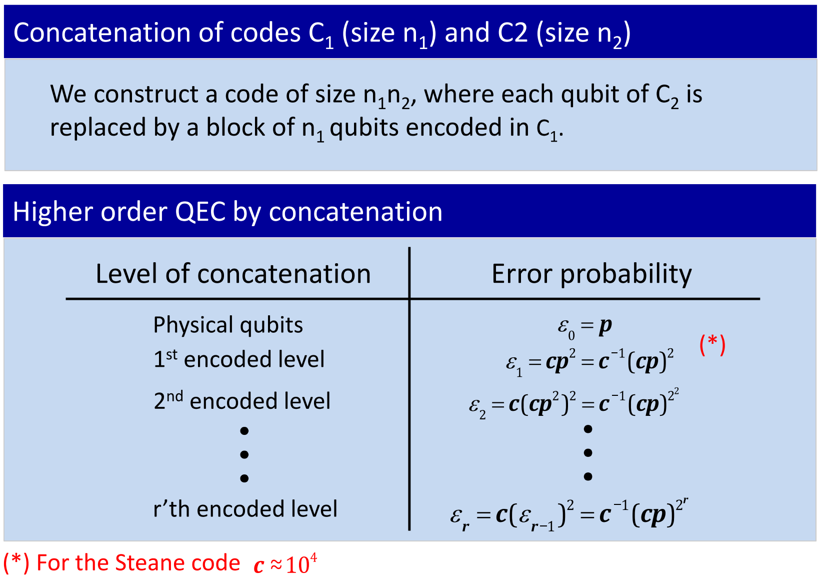

Figure 254: how concatenated codes are reducing the error rate. Source: Introduction to quantum computing by Anthony Leverrier and Mazyar Mirrahimi, March 2020 (69 slides).

This rate would be between 0.1% and 1% but is subject to change. The consequence of this theorem is to allow the application of error correction codes recursively until reaching the desirable error rate to execute a given algorithm (Figure 254). This is however based on the assumption that qubit fidelities are stable as their number is growing, a feat that is not yet achievable!

It also doesn't correct various sources of errors like isotropic errors affecting simultaneously several qubits[667]. On the other hand, a variation of the threshold theorem was recently demonstrated that takes into account a stable percentage of defects in planar arrays of qubits and includes a QEC protocol for large arrays of defective qubits[668].

The standard specification in vendor roadmaps for FTQC QPUs is one million physical qubits with 99.9% two-qubit gate fidelity running a surface code QEC. It might enable 100 to 1,000 logical qubits, taking into account the significant overhead of T-gates or Toffoli gates synthesis and correction.

How about the number of logical qubits of a FTQC QPU? It should provide a space-related quantum advantage vs classical computing so we need at least 50 to 55 data qubits. Most algorithms requiring an equivalent of about 50 additional qubits (ancilla, transit, …), we end up with needing about 100 logical qubits and a number of physical qubits that depends on the architecture, ranging from 30 to 10,000 physical qubits per logical qubits (Figure 255). The sheer number of qubits required to build a FTQC awarded it another nickname: Fairy-Tale Quantum Computing!

Most quantum processors won't have more than a couple hundred qubits implemented in a "monolithic" way. Scaling these systems to implement FTQC will require the assembly and interconnection of several QPUs though entangled resources links and the use of the gate teleportation technique. It will drive the creation of specific architectures, particularly for implementing gate teleportation between these qubits using photon entangled resources in the microwave or optical regime[669].

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[666] See Fault-Tolerant Quantum Computation With Constant Error Rate by Dorit Aharonov and Michael Ben-Or, 1999 (63 pages).

[667] See Quantum codes do not increase fidelity against isotropic errors by J. Lacalle et al, January 2022 (18 pages).

[668] See Quantum computing is scalable on a planar array of qubits with fabrication defects by Armands Strikis, Simon C. Benjamin and Benjamin J. Brown, November 2021 (16 pages).

[669] See Microarchitectures for Heterogeneous Superconducting Quantum Computers by Samuel Stein, Andrew A. Houck et al, May 2023 (16 pages).

QEC concatenation is exploiting this recursivity of error correction codes. A QEC generates logical qubits which can then be used as virtual physical qubits for a new QEC, and so on. With each recursion, the apparent error rate decreases. We stop concatenating QEC codes when we reach an error rate compatible with the expected usage of the qubits. Concatenation can be optimized by using different types of QEC at each level of recursivity[670]. This theorem was demonstrated only for a 7-qubit Steane error correction code and for error rates that are not growing with the number of physical qubits. This is unfortunately not what is currently observed with most qubit types! Surface codes and their various derivatives are not concatenated but rather expanded in 2D with a growing number of qubits. But their relative noise and number of qubit scaling are different.

With concatenated codes, the noise is reduced by the exponential law ϵ^{2^k} with a number of qubits in X^k, ϵ being the error rate, k the number of concatenations and X the number of qubits within a single concatenation, which can reach about 91 depending on the implementation and on the way ancilla qubits are managed and recycled, using Steane's method (not to be confused with Steane's code)[671].

But concatenated codes threshold is quite low, in the range of 10^{-6}, which is currently inaccessible for all breeds of qubits. With surface codes, the noise is reduced according to $\epsilon^{d/2}$ and the number of required physical qubits grows by d^2, d being the distance of the surface code, more or less corresponding to the edge of the surface code squares as shown in Figure 250 for distances 3 and 5.

All in all, concatenated error correction codes have a better impact on noise, but at the expense of a large number of physical qubits while surface codes scale slower in error reduction and physical qubits requirements. It seems that surface codes are more appropriate for more noisy physical qubits while concatenated codes will be better, for less noisy qubits.

Leakage errors. Most quantum error correction codes are focused on Pauli channel noise errors correction (flip, phase and depolarizing errors). Leakage errors require their own correction methods[672].

Qubits lifetime extension. A nagging question may arise: if we need to accumulate error correction codes, don't we risk running into the wall of qubit decoherence, particularly with superconducting qubits? Well, no. As said before, error correction codes have the direct effect of artificially extending the coherence time of the qubit registers by several orders of magnitude[673]. Each correction is equivalent to a reset of the qubits decoherence times T_1 (flip) and T_2 (phase). This explains how Google could publish an optimized version of the Shor integer factoring algorithm with 20 million qubits and requiring 8 hours of run-time, which is many orders of magnitude longer than their qubits coherence time that sits way under a tiny 100 µs.

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[670] See Dynamic Concatenation of Quantum Error Correction in Integrated Quantum Computing Architecture by Ilkwon Sohn et al, 2019 (7 pages).

[671] 91 is based on using a Steane 7-qubit [[7; 1; 3]] code, including the ancilla factory and 4x7=28 qubits ancilla factory times 3 because the preparation, verification and measurement of ancillas is three times longer than the data qubit operations (9 versus 3 time-steps). Hence, while ancillas are in used for a given gate, ancillas must be being prepared for the next two gates. Thus each level of error correction replaces one qubit by 91 qubits (7 data qubits and 3x28 ancilla qubits). Source: Optimizing resource efficiencies for scalable full-stack quantum computers by Marco Fellous-Asiani, Jing Hao Chai, Yvain Thonnart, Hui Khoon Ng, Robert S. Whitney and Alexia Auffèves, arXiv and PRX Quantum, September 2022-October 2023 (39 pages). Flag qubits could reduce this significant overhead and reduce X. But this doesn't take into account T gates magic state distillation that adds a minimum of 15 qubits! See Fault-tolerant quantum error correction on near-term quantum processors using flag and bridge qubits by Lao Lingling et al, 2020 (12 pages) and Fault-tolerant quantum error correction for Steane's seven-qubit color code with few or no extra qubits by Ben W. Reichardt, April 2018 (11 pages). See Overhead analysis of universal concatenated quantum codes by Christopher Chamberland, Raymond Laflamme et al, 2017 (25 pages) which describes a fault-tolerant QEC of 105 qubits.

[672] See ERASER: Towards Adaptive Leakage Suppression for Fault-Tolerant Quantum Computing by Suhas Vittal et al, Georgiatech and University of Texas Austin, September 2023 (17 pages).

[673] See Extending the lifetime of a quantum bit with error correction in superconducting circuits by Nissim Ofek, Zaki Leghtas, Mazyar Mirrahimi, Michel Devoret et al, 2016 (5 pages) which shows that thanks to a cat-code-based QEC, the lifetime of superconducting qubits can be extended by a factor of 20!

A record qubit lifetime doubling extension to 1.8 ms was achieved by a Yale and Sherbrooke University team led by Michel H. Devoret in 2022 with superconducting qubits[674]. It was a GKP bosonic mode coupled to a tantalum transmon and using reinforcement learning to implement an autonomous error correction. It was not really a logical qubit made of several identical physical qubits.

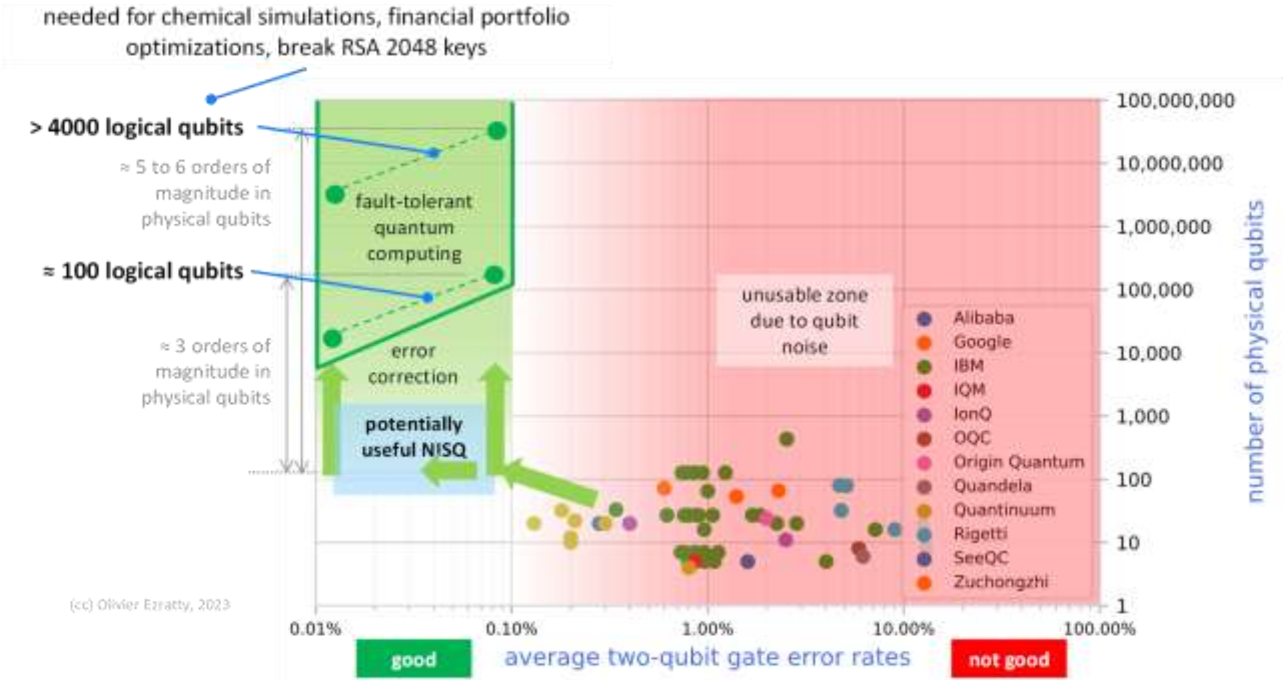

Figure 255: qubit number and fidelities requirements for FTQC. (cc) Olivier Ezratty, 2023.

EFTQC stands for Early FTQC. It corresponds to the early stages of FTQC with small logical qubits improving logical qubit fidelities in an intermediate range[675]. Researchers are already trying to design FTQC algorithms that would work in such situations, like the randomized Fourier estimation (RFE) which can be used in several quantum machine learning algorithms[676].

Another proposal is about using 3D color codes to solve a sparse IQP (Instantaneous Quantum Polynomial-time) circuits polynomial problem showing a superpolynomial quantum advantage[677].

Approximate QEC

One QEC group named Approximate QEC or Quasi-Exact fault-tolerant Quantum (QEQ) computation sits in-between NISQ and FTQC[678]. It is an intermediate solution implementing some error correction, but not to the point of creating perfect logical qubits[679]. It still uses some variations of stabilizers and surface codes.

[674] See Real-time quantum error correction beyond break-even by V. V. Sivak, Steve M. Girvin, Robert J. Schoelkopf, Michel H. Devoret et al, Nature, March 2023 (8 pages).

[675] See Early Fault-Tolerant Quantum Computing by Amara Katabarwa et al, Zapata AI and ICFO, November 2023 (27 pages).

[676] See On proving the robustness of algorithms for early fault-tolerant quantum computers by Rutuja Kshirsagar et al, Zapata Computing, September 2022 (27 pages).

[677] See Robust sparse IQP sampling in constant depth by Louis Paletta, Anthony Leverrier, Alain Sarlette, Mazyar Mirrahimi and Christophe Vuillot, July 2023 (15 pages).

[678] See Theory of quasi-exact fault-tolerant quantum computing and valence-bond-solid codes by Dong-Sheng Wang, Raymond Laflamme et al, May 2021 (22 pages).

[679] See Complexity and order in approximate quantum error-correcting codes by Jinmin Yi, Weicheng Ye, Daniel Gottesman and Zi-Wen Liu, Perimeter Institute, October 2023 (27 pages) which defines the field of application of AQEC.

One of these methods named NISQ+ combines aQEC and SFQ driving circuits[680]. It can help boost the "simple" quantum volume of NISQ QPUs (Figure 256).

The simple quantum volume is computed with multiplying the number of useful qubits and a number of doable quantum gates under a certain error threshold. The related paper raises an interesting point: slow QEC decoders make applications take exponential time to complete, which is kind of problematic! It is explained by the ratio between QEC data generation and QEC data processing is around 2 for syndrome data processing ratio using classical controls. With superconducting SFQ circuits, the ratio is of 0.125 thanks to a very low latency. The proposed SFQ circuit uses a circuit map similar to the qubit circuits topology. It implements an "Approximate SFQ decoder" stabilizer-based algorithm using a union-find algorithm, resets (stopping signal propagation once pairs are found), boundaries (match signals to boundaries) and tie-breaking (chooses single paths among an equal set).

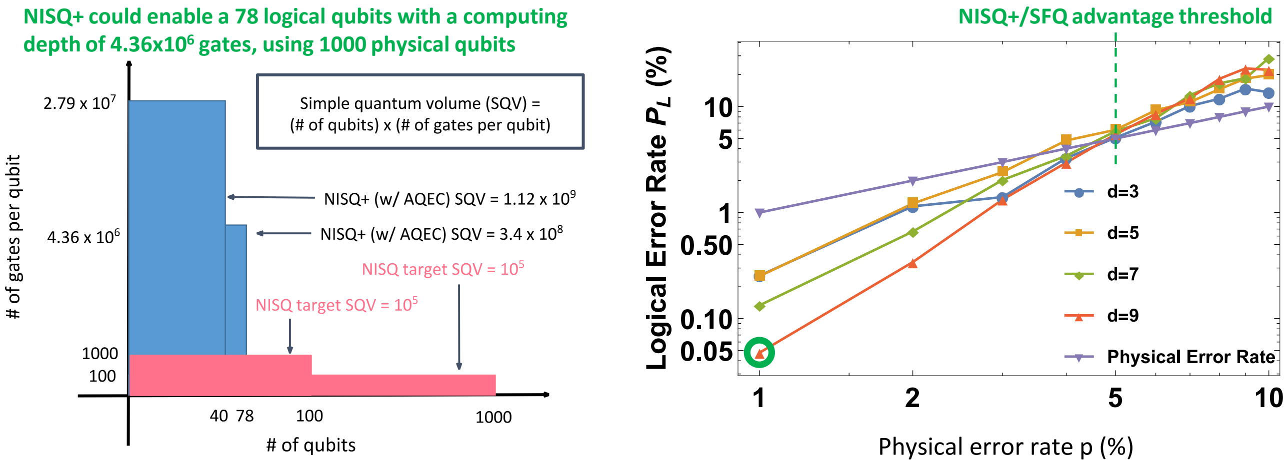

Figure 256: NISQ+ could potentially enable the creation of 78 logical qubits and a good computing depth. They use the notion of Simple Quantum Volume (SQV) which is the qubit number times their available gate depth. Still, satisfying logical qubit error rates require a surface code of distance 7 to 9. Source: NISQ+: Boosting quantum computing power by approximating quantum error correction by Adam Holmes et al, Intel, University of Chicago and USC, April 2020 (13 pages).

The accuracy threshold of this SFQ circuit is at 5% of physical error rate and significantly interesting at 1%, yielding then a logical qubit error rate of 0.05% with a code distance d=9. It is a much lower required code distance compared to other correction techniques like those using neural networks.

The SFQ circuit power consumption is 13 µW for a full circuit with a logical depth of 6, has a real-estate of 1.3 mm^2 and a latency of 20 ns for QEC. It seems to run at 4K. It could enable the creation of a 78 logical qubits system using 1,000 physical qubits and a computing depth of $4.36x10^6$ gates.

In another work, a team led by Microsoft created a concept architecture to implement a FTQC with a scalable decoder running the QEC, but without details on the required hardware (room temperature or cryoelectronics, CMOS or SFQ)[681].

Quantum error mitigation

Quantum error mitigation (QEM) is about reducing quantum algorithms errors with combining classical post-processing techniques with some potential circuits modifications on top of running the algorithm several times and averaging its results (*aka* the "*expectation values of an observable*", i.e., the combination of 0s and 1s).

[680] See NISQ+: Boosting quantum computing power by approximating quantum error correction by Adam Holmes et al, Intel, University of Chicago and USC, April 2020 (13 pages) and explained in this video (21 mn).

[681] See A Scalable Decoder Micro-architecture for Fault-Tolerant Quantum Computing by Poulami Das, Krysta Svore, Nicolas Delfosse et al, Microsoft, GeorgiaTech and Caltech, January 2020 (19 pages).

QEM has a much lower overhead in qubits and running time vs QEC. It is a NISQ-era solution aiming at creating a quantum computing advantage before FTQC shows up in the longer term. QEM reduces the influence of quantum errors using multiple runs and subsequent measurements coupled to some classical processing as opposed to QEC-based active qubits measurement and fast feedback-based corrections impacting the results of individual runs.

QEM proposals started to pop-up around 2016[682]. Most of them consist in learning the effects of noise on qubit evolutions and creating predictive (if not linear) noise models that can be applied to tune the results of quantum computations. It is adapted to rather shallow circuits[683] and we're not really sure yet it brings a quantum advantage on useful problems. Most QEM methods do not increase the required qubits count for a given algorithm. You will notice that many contributions in the QEM space come from IBM Research[684] and also Google AI.

Here are some identified QEM techniques:

Zero noise extrapolation (ZNE) builds error models based on solving linear equations. It supposes the noise is stable. It cancels noise perturbations by an application of Richardson's deferred approach to the limit and works on short-depth (or shallow) circuits[685] [686]. A similar protocol was designed for (hybrid) variational quantum simulators used to simulate the dynamics of quantum systems[687]. Some ZNE methods are splitting decoherence error mitigation from depolarizing error mitigation[688] [689]. ZNE can even be adapted to quantum annealers[690].

Probabilistic error cancellation is about detecting circuit bias with finding noise quantum channels, represented as density matrices for quantum gates, using quasi-probability decomposition[691] [692] [693]. There's a sampling overhead in the process. It then inverts a well-characterized noise channel to produce noise-free estimates of the algorithm observables (the 0s and 1s they're supposed to generate)[694].

[682] See the review papers Hybrid Quantum-Classical Algorithms and Quantum Error Mitigation by Suguru Endo, Zhenyu Cai, Simon C. Benjamin and Xiao Yuan, 2020 (39 pages), Quantum Error Mitigation by Zhenyu Cai, Ryan Babbush, Simon C. Benjamin et al, October 2022 (40 pages) and Testing platform-independent quantum error mitigation on noisy quantum computers by Vincent Russo, Andrea Mari, Nathan Shammah, Ryan LaRose and William J. Zeng, October 2022 (17 pages).

[683] See Fundamental limits of quantum error mitigation by Ryuji Takagi, Suguru Endo, Shintaro Minagawa and Mile Gu,, npj, September 2022 (11 pages).

[684] See What's the difference between error suppression, error mitigation, and error correction? by Matthias Steffen, IBM Research, October 2022.

[685] See the beginning of Error mitigation for short-depth quantum circuits by Kristan Temme, Sergey Bravyi and Jay M. Gambetta, 2016 (15 pages).

[686] See Scalable error mitigation for noisy quantum circuits produces competitive expectation values by Youngseok Kim, Jay M. Gambetta, Kristan Temme et al, August 2021 (7 pages).

[687] See Efficient Variational Quantum Simulator Incorporating Active Error Minimization by Ying Li and Simon C. Benjamin, PRX, 2017 (14 pages).

[688] See Folding-Free ZNE: A Comprehensive Quantum Zero-Noise Extrapolation Approach for Mitigating Depolarizing and Decoherence Noise by Hrushikesh Patil et al, NC State University, May 2023 (12 pages).

[689] See Development of Zero-Noise Extrapolated Projection Based Quantum Algorithm for Accurate Evaluation of Molecular Energetics in Noisy Quantum Devices by Chinmay Shrikhande et al, Indian Institute of Technology Bombay, June 2023 (12 pages).

[690] See Quantum error mitigation in quantum annealing by Mohammad H. Amin et al, D-Wave, Simon Fraser University, University of British Columbia, November 2023 (10 pages).

[691] See Probabilistic error cancellation with sparse Pauli-Lindblad models on noisy quantum processors by Ewout van den Berg, Zlatko K. Minev, Abhinav Kandala and Kristan Temme, IBM Research, January 2022 (30 pages).

[692] See Unfolding Quantum Computer Readout Noise by Benjamin Nachman et al, October 2019-May 2020 (13 pages).

[693] See Compressed quantum error mitigation by Maurits S. J. Tepaske and David J. Luitz, February 2023 (11 pages).

[694] See the second part of Error mitigation for short-depth quantum circuits by Kristan Temme, Sergey Bravyi and Jay M. Gambetta, 2016 (15 pages).

It can be implemented in mid-circuit measurement operations[695]. It is also called Bayesian error mitigation and Bayesian read-out error mitigation (BREM)[696].

Learning Based Methods QEM are based on machine learning techniques using training data to learn the effect of quantum noise in various situations[697]. It is proposed by companies like QuantrolOx (UK), by the University of Erlangen in Germany[698] and Quantum Machines (Israel). One of these is Clifford circuit learning or Clifford Data Regression is a variation of the previous technique that learns the effect of noise from Clifford gates using data comparing quantum emulation on classical hardware and runs on quantum processors. It then uses rather simple linear regression techniques to correct errors in post-processing[699]. It can also be applicable to fault-tolerant T gates[700]. You have also QuantumNAS, a noise adaptative search method[701] as well as some specific deep reinforcement learning (RL) techniques to improve qubit control precision[702] [703], including a novel technique based on LSTM neural networks[704].

Error suppression by derangement (ESD) which provides an exponential error suppression with increasing the qubit count by n≥2 but is still adapted to NISQ architecture and shallow circuits[705].

Dynamic Decoupling involves decoupling idle qubits from other qubits under certain conditions. The technique is proposed by IBM[706] and also proposed by AWS[707]. It seems that under certain circumstances, it can generate a good quantum speedup for oracle-based algorithms[708]. It is also the base of the Space Curve Quantum Control (SCQC) codes designed at Virginia Tech. They are suppressing errors caused by quasi-static noises on qubits and work particularly with spin qubits[709] [710] [711].

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[695] See Probabilistic error cancellation for measurement-based quantum circuits by Riddhi S. Gupta, Ewout van den Berg, Maika Takita, Kristan Temme and Abhinav Kandala, IBM, October 2023 (16 pages).

[696] See Scalable Measurement Error Mitigation via Iterative Bayesian Unfolding by Siddarth Srinivasan et al, October 2022 (13 pages).

[697] See Machine Learning for Practical Quantum Error Mitigation by Haoran Liao, Zlatko K. Minev et al, IBM, September 2023 (15 pages).

[698] See Neural networks enable learning of error correction strategies for quantum computers, October 2018 and Reinforcement Learning with Neural Networks for Quantum Feedback, Thomas Fösel et al, 2018 (7 pages).

[699] See Error mitigation with Clifford quantum-circuit data by Piotr Czarnik et al, May 2020 (16 pages) and Improving the efficiency of learning-based error mitigation by Piotr Czarnik, Michael McKerns, Andrew T. Sornborger and Lukasz Cincio, April 2022 (13 pages).

[700] See Error mitigation for universal gates on encoded qubits by Christophe Piveteau, David Sutter, Sergey Bravyi, Jay M. Gambetta and Kristan Temme, IBM Research, March 2021 (11 pages).

[701] See QuantumNAS: Noise-Adaptive Search for Robust Quantum Circuits by Hanrui Wang et al, January 2022 (19 pages).

[702] See Deep Reinforcement Learning for Quantum State Preparation with Weak Nonlinear Measurements by Riccardo Porotti, Antoine Essig, Benjamin Huard and Florian Marquardt, June 2021 (15 pages).

[703] See Reinforcement learning pulses for transmon qubit entangling gates by Ho Nam Nguyen et al, November 2023 (26 pages).

[704] See Quantum Circuit Fidelity Improvement with Long Short-Term Memory Networks by Yikai Mao, Shaswot Shresthamali and Masaaki Kondo, Keio University and RIKEN, March-May 2023 (17 pages).

[705] See Exponential error suppression for near-term quantum devices by Balint Koczor, PRX, 2021 (34 pages).

[706] See Pulse-level Noise Mitigation on Quantum Applications by Siyuan Niu and Aida Todri-Sanial, LIRMM Montpellier France, April 2022 (11 pages) and Analyzing Strategies for Dynamical Decoupling Insertion on IBM Quantum Computer by Siyuan Niu and Aida Todri-Sainal, LIRMM France, April 2022 (11 pages).

[707] See Suppressing errors with dynamical decoupling using pulse control on Amazon Braket by Palash Goiporia, Pranav Gokhale, Michael A. Perlin, Yunong Shi, and Martin Suchara, AWS, December 2022.

[708] See Demonstration of algorithmic quantum speedup by Bibek Pokharel and Daniel A. Lidar, July 2022 (12 pages).

[709] See Designing dynamically corrected gates robust to multiple noise sources using geometric space curves by Hunter T. Nelson et al, Virginia Tech, November 2022 (12 pages).

[710] See Dynamically corrected gates from geometric space curves by Edwin Barnes et al, Virginia Tech, March 2021 (25 pages).

[711] See the thesis Study and Application of the Space Curve Quantum Control Formalism by Fei Zhuang, Virginia Tech, May 2023 (205 pages) based on dynamic decoupling.

Measurement Error Mitigation techniques are focused on correcting errors happening at the end of circuit with qubit measurement[712] [713], using machine learning methods[714], some being tailored for variational quantum algorithms[715] and optimization algorithms[716].

Other methods include symmetry constraints verification, distillation using randomized benchmarking[717] [718], randomized compiling[719], applying gates simulating the reverse effect of errors[720], depolarizing noise[721], quantum verification and post-selection[722], doubling quantum resources in time with echo verification or in space with virtual distillation[723], virtual distillation with derangement operators[724], focusing on Pauli gates error mitigation[725], using matrix product operators[726], and also mixing various QEM and QEC techniques[727] [728] [729].

Segmenting, detailing and comparing all these various methods remains an open challenge[730] [731]. See Figure 257 that provides a high-level overview of the various quantum error mitigation and suppression techniques.

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[712] See Error estimation in IBM Quantum Computers by Unai Aseguinolaza et al, various Universities in Spain and aQuantum, February 2023 (9 pages).

[713] See Development and Demonstration of an Efficient Readout Error Mitigation Technique for use in NISQ Algorithms by Andrew Arrasmith et al, Rigetti, Mar 2023 (19 pages).

[714] See Enhancing Qubit Readout with Autoencoders by Piero Luchi et al, November 2022 (16 pages).

[715] See Application-tailored Measurement Error Mitigation for Variational Quantum Algorithms by Siddharth Dangwal et al, June 2023 (17 pages).

[716] See Error Propagation in NISQ Devices for Solving Classical Optimization Problems by Guillermo González-García, Rahul Trivedi, and J. Ignacio Cirac, MPI and MCQST, PRX Quantum, December 2022 (17 pages) and Quantum entanglement can be a double-edged sword, December 2022.

[717] See Shadow Distillation: Quantum Error Mitigation with Classical Shadows for Near-Term Quantum Processors by Alireza Seif, Liang Jiang, March 2022 (16 pages).

[718] See Virtual Distillation for Quantum Error Mitigation by William J. Huggins et al, Google AI, PRX, 2021 (25 pages).

[719] See Crucial leap in error mitigation for quantum computers by Monica Hernandez and William Schulz, Lawrence Berkeley National Laboratory, December 2021, referring to Randomized Compiling for Scalable Quantum Computing on a Noisy Superconducting Quantum Processor by Akel Hashim, Irfan Siddiqi et al, 2021 (12 pages).

[720] See Quantum Error Mitigation via Quantum-Noise-Effect Circuit Groups by Yusuke Hama et al, May 2022 (22 pages).

[721] See Mitigating Depolarizing Noise on Quantum Computers with Noise-Estimation Circuits by Miroslav Urbanek, Benjamin Nachman, Vincent R. Pascuzzi, Andre He, Christian W. Bauer, and Wibe A. de Jong, PRA, December 2021 (7 pages).

[722] See Mitigating errors by quantum verification and post-selection by Rawad Mezher, James Mills and Elham Kashefi, September 2021 and May 2022 (15 pages).

[723] See Purification-based quantum error mitigation of pair-correlated electron simulations by T. E. O'Brien et al, Google AI, October 2022 (23 pages). It identifies 175 authors working at Google AI!

[724] See Virtual Distillation for Quantum Error Mitigation by William J. Huggins, Ryan Babbush et al, August 2021 (26 pages).

[725] See Single-shot error mitigation by coherent Pauli checks by Ewout van den Berg, Sergey Bravyi, Jay M. Gambetta, Petar Jurcevic, Dmitri Maslov and Kristan Temme, December 2022 (30 pages).

[726] See Quantum error mitigation via matrix product operators by Yuchen Guo et al, January-October 2022 (13 pages) which accounts for correlated errors between different gates.

[727] Like in Quantum Error Mitigation as a Universal Error Reduction Technique: Applications from the NISQ to the Fault-Tolerant Quantum Computing Eras by Yasunari Suzuki et al, NTT, PRX Quantum, March 2022 (33 pages).

[728] See Error Suppression for Arbitrary-Size Black Box Quantum Operations by Gideon Lee, Steve M. Girvin et al, October 2022 (20 pages) which is proposing the technique of Error Filtration (EF), an intermediate between NISQ and FTQC error processing.

[729] See Zero noise extrapolation on logical qubits by scaling the error correction code distance by Misty A. Wahl et al, Unitary Fund, April 2023 (8 pages).

[730] See Benchmarking Noisy Intermediate Scale Quantum Error Mitigation Strategies for Ground State Preparation of the HCl Molecule by Tim Weaving et al, March 2023 (18 pages).

[731] See Hypothesis Testing for Error Mitigation: How to Evaluate Error Mitigation by Abdullah Ash Saki et al, Zapata Computing and UCL, Jan 2023 (19 pages).

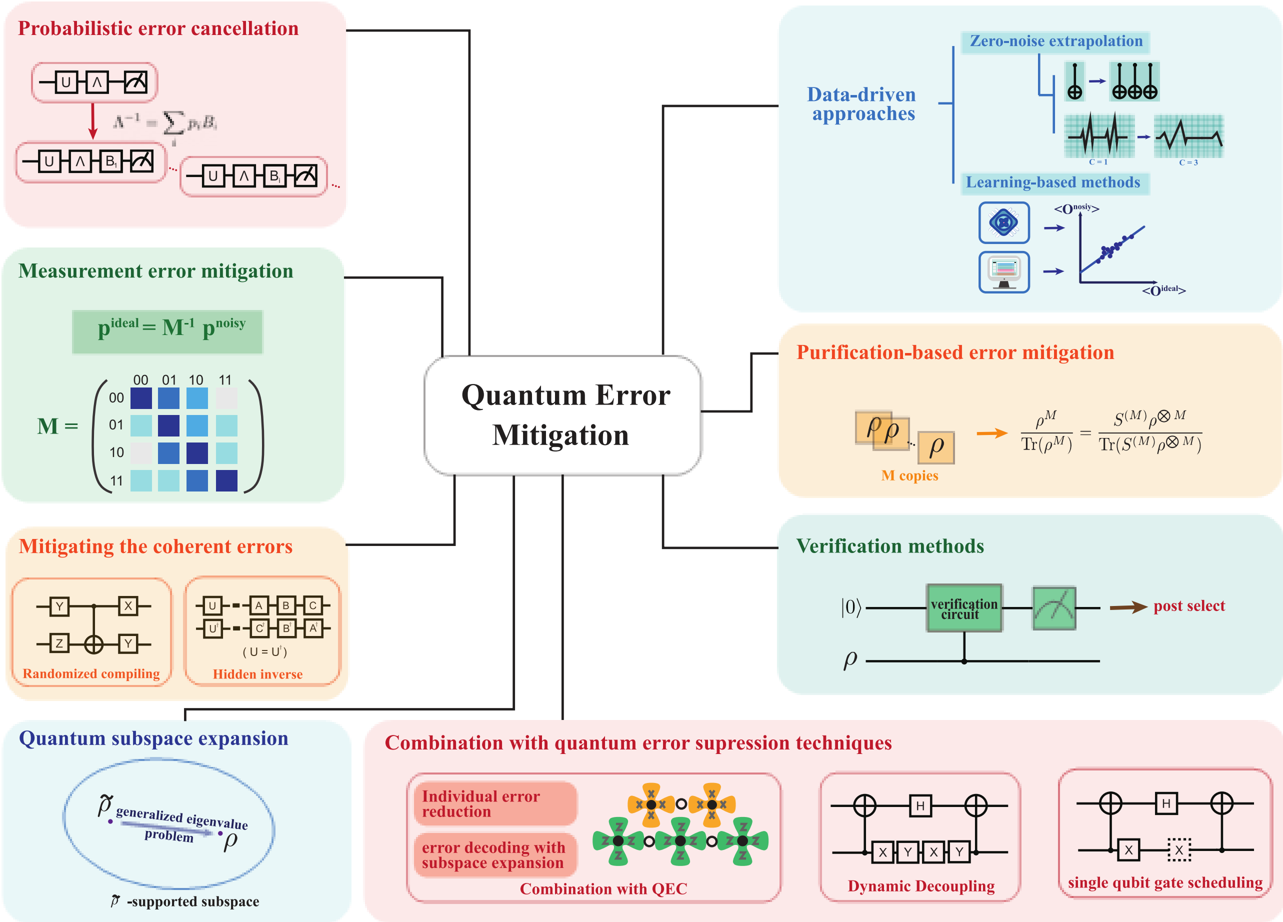

Figure 257: a segmentation of various quantum error mitigation and suppression techniques. Source: Near-Term Quantum Computing Techniques: Variational Quantum Algorithms, Error Mitigation, Circuit Compilation, Benchmarking and Classical Simulation by He-Liang Huang et al, November-December 2022 (46 pages). Added in 2023.

In the commercial world, QEM can lead some vendors to display some outlandish claims like when Q-CTRL announces that its error correction scheme boosts algorithms performance by up to 9,000x thanks to some autonomous error correction[732]. Why not, but 9,000x of what? Looking at the details, this is achieved with back-end and front-end optimization compilation and some error mitigation techniques. When you read their chart, shown in Figure 258, you find that the x9,000 ratio pertains to the success rate of running a Bernstein-Vazirani algorithm on a superconducting qubits processor, in the case of 16 qubits. But the related success factor is below 20% and is an extreme case.

You must remind yourself that 16 qubits can be easily emulated on your own laptop and is way below any quantum computing advantage. If you were to extend their chart beyond 30 qubits, you'd be hundreds of thousands better than their competitors but with a very small success probability.

QEM scalability is still questioned since its cost can scale exponentially with the number of qubits[733] [734] [735] [736]. Also, QEM doesn't work well for quantum simulations of continuous dynamics[737].

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[732] See Firing up quantum algorithms - boosting performance up to 9,000x with autonomous error suppression by Michael J. Biercuk, March 2022.

[733] See Making quantum error mitigation practical by Will Zeng and Nathan Shammah, Unitary Fund, May 2023 which lays out the various challenges to implement QEM.

[734] See Error statistics and scalability of quantum error mitigation formulas by Dayue Qin et al, December 2021-April 2023 (25 pages).

[735] See Exponentially tighter bounds on limitations of quantum error mitigation by Yihui Quek et al, October 2022 (41 pages).

[736] See Error Mitigation Thresholds in Noisy Quantum Circuits by Pradeep Niroula et al, NIST, February 2023 (10 pages).

[737] See Limitations of quantum error mitigation for open dynamics beyond sampling overhead by Yue Ma et al, August 2023 (12 pages).

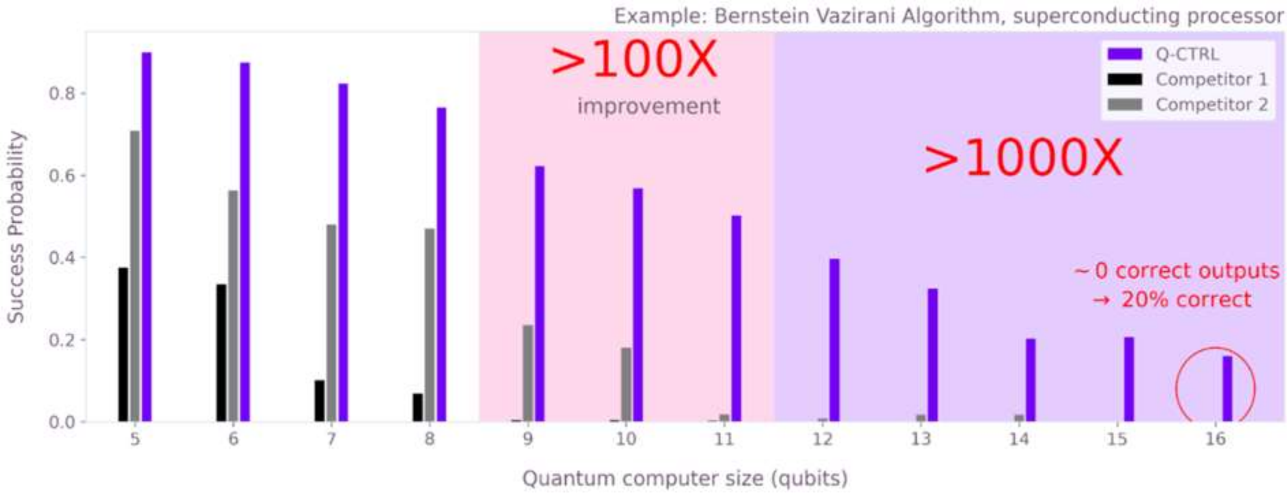


Figure 258: charting the Q-CTRL improvements.
Sources: [Firing up quantum algorithms - boosting performance up to 9,000x with autonomous error suppression]() by Michael J. Biercuk, March 2022 and [Experimental benchmarking of an automated deterministic error suppression workflow for quantum algorithms]() by Pranav S. Mundada, Michael J. Biercuk, Yuval Baum et al, September 2022 (16 pages).

Gate-based computing classes

I summarized these various gate-based computing classes in the below schema in Figure 259.

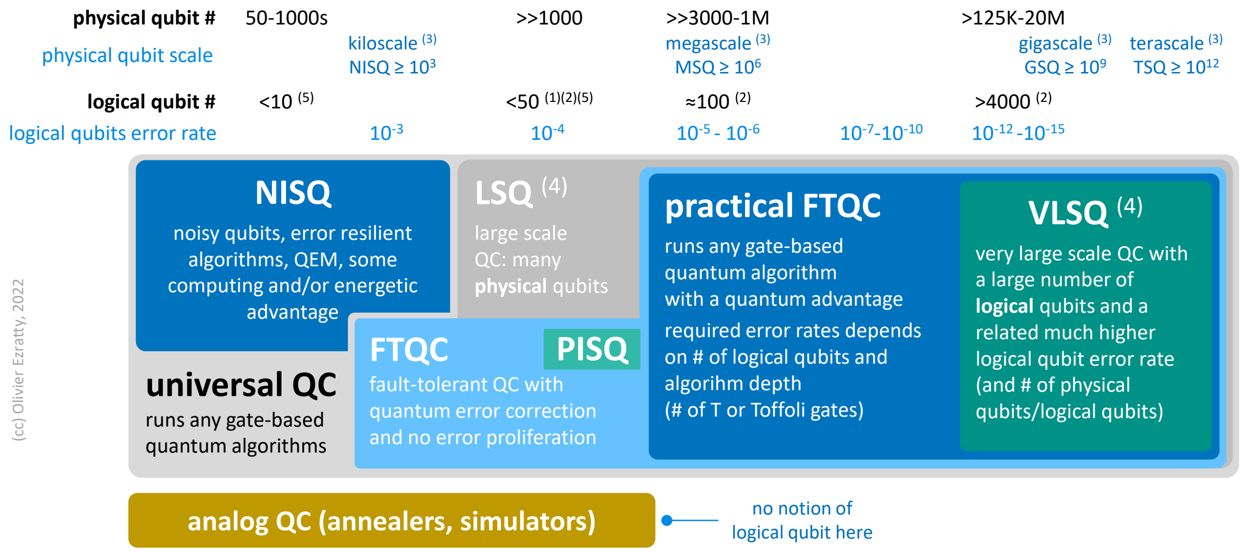
It requires a lot of comments and annotations and is still work in progress:

- **Universal quantum computing** is the quantum computing paradigm in which all quantum algorithms can be implemented from a mathematical standpoint. It must support non-Clifford quantum gates. This feature is implemented at a narrow and noisy scale with NISQ systems and with FTQC.
- **NISQ** definition is not really agreed upon. Is it starting today, or will we need more physical qubits and generate some proven generic quantum computing advantage? I added in blue a scale proposal presented by Dave Bacon at Google in March 2022[738], which deals with a simple scale of number of physical qubits with NISQ being in the thousand, MSQ in the million, GSQ in the billion and TSQ in the tera number of qubits. NISQ is powered by quantum error mitigation in the near term and with approximated QEC in the mid-term. It will extend the usability of quantum computers with a larger number of qubits and circuit depth.
- **LSQ** stands for large scale quantum computer and is about having a QPU with a large number of qubits. But are these physical or logical qubits and how does it relate to FTQC? The dust has not yet settled for its definition. What we know is a large scale quantum system without error correction would not be very usable. On average, the depth of gate-based computing is limited by qubit error rates and many quantum algorithms have a breadth (number of qubits) that is in line with their depth (number of gate cycles). So, we have a sort of gap between the upper stages of NISQ and early stages of practical FTQC with logical qubits.
- **FTQC** can start with a few logical qubits of average error correction with target error rates of about 10^{-3} to 10^{-4}. We'll maybe have a continuum in the FTQC progress with error rates growing progressively until it reaches 10^{-15} in the long term as the number of logical qubits will grow. These error rates will have to shrink at a faster rate than the increase of logical qubit numbers.
- **Practical FTQC** is about FTQC providing a generic quantum advantage. It would require at least 100 logical qubits given half of them are used for data with a space exceeding the memory capabilities of equivalent classical systems, and the other half providing the ancilla qubits required for many algorithms like the QFT and its derivatives. The number of physical qubits corresponding

[738] See [QIP 2022 | Software of QIP, by QIP, and for QIP]() by Dave Bacon from Google, March 2022 (1 hour).

to the logical qubit thresholds depend on autonomous error correction and the number of physical qubits required per logical qubits, the best case being with cat-qubits[739]. The target error rate of 10^{-5} to 10^{-6} is a rough estimate, below the inverse square of the number of logical qubits.

- **PISQ** for Perfect Intermediate Scale Quantum is a proposal from Qutech scientists that corresponds to the arrival of 50+ FTQC "perfect" qubits QPUs. They advocate to get ready for it in parallel with all the efforts related to NISQ systems[740]. Basically, it is about creating quantum algorithms with using classical emulation solutions in a higher regime than mostly used nowadays, above 40 qubits.
- **VLSQ**: this is large scale FTQC with several orders of magnitude larger number of logical qubits used to run chemical simulations, Shor integer factoring and large scale optimization and industry scale algorithms. So, in the below chart in Figure 259, I position these various definitions for universal quantum computing, FTQC, LSQ and VLSQ, with one scale for physical qubits and one for logical qubits as well as with logical qubit error rates.
- **FTDQC** is a new term, meaning « fault tolerant distributed quantum computation », which could potentially be implement with long distance quantum communication, even with satellites[741].



(1) 100 logical qubits # is based on a mathematical advantage starting with about 55 qubits and for a similar number of ancilla qubits for most algorithms (QFT, HHL, QML).
(2) the # of logical qubits depends on autonomous error correction and the number of physical qubits required per logical qubits, the best case being with cat-qubits.
(3) this scale was presented in March 2022 by Dave Bacon from Google. It positions NISQ above the 1000 physical qubits threshold.
(4) VLSQ is an equivalent to VLSI in classical semiconductors computing.
(5) a few <50 logical qubits are not sufficient to run a quantum gate-based algorithm with some quantum computing advantage. Thus, the practical FTQC threshold at 100 logical qubits.

Figure 259: positioning all the concepts: NISQ, PISQ, LSQ, FTQC, Universal quantum computing and the related error correction codes. (cc) Olivier Ezratty, 2022-2023.

QEC impact on computing time

FTQC algorithms papers sometimes evaluate how long it would take to execute specific quantum algorithms in an "end-to-end" fashion. We know that, theoretically, with a FTQC with 22 million physical qubits, an RSA 2048-bit key could be factorized in 8 hours with using superconducting qubits. Gate time is quite variable from 12 ns for superconducting qubits to over 100 µs for trapped ions qubits.

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739 I saved you the EFTQC variation, for early FTQC that is used in On proving the robustness of algorithms for early fault-tolerant quantum computers by Rutuja Kshirsagar et al, September 2022 (27 pages) which deals with an interesting question: what is the error rate of logical qubits in the FTQC realm that would be required for some key algorithms?

740 See Quantum Computing -- from NISQ to PISQ by Koen Bertels et al, April 2022 (11 pages).

741 See Upper Bounds for the Clock Speeds of Fault-Tolerant Distributed Quantum Computation using Satellites to Supply Entangled Photon Pairs by Hudson Leone et al, University of Technology Sidney, August 2022 (9 pages).

You can get an idea of the timing overhead coming from three mechanisms (see Figure 260):

- **Non-Clifford gates** creation overhead like R/Control-R gates with arbitrary phases, based on the Solovay-Kitaev theorem. It creates a x127 to x235 gates overhead!
- **Quantum error correction** (QEC) overhead in the case of FTQC. It creates a x10 to x20 gates overhead, minimum! It may be much bigger for large surface codes and concatenated codes. With surface code QEC, this runtime overhead scales with the code distance.
- **Number of runs** or shots required to average probabilistic results. This number is variable and depending on the algorithm. IBM advises using 4,000 runs but this number may grow with the number of used qubits and it is for NISQ algorithms which require more runs than FTQC algorithms. For example, with a FTQC QPE (quantum phase estimate) algorithm, the number of runs may be much smaller than its NISQ VQE equivalent (see details on these in the algorithm part starting page 855). We trade here the high ansatz and shots overhead of NISQ vs the overhead of error correction and magic state distillation to fault-correct T gates.

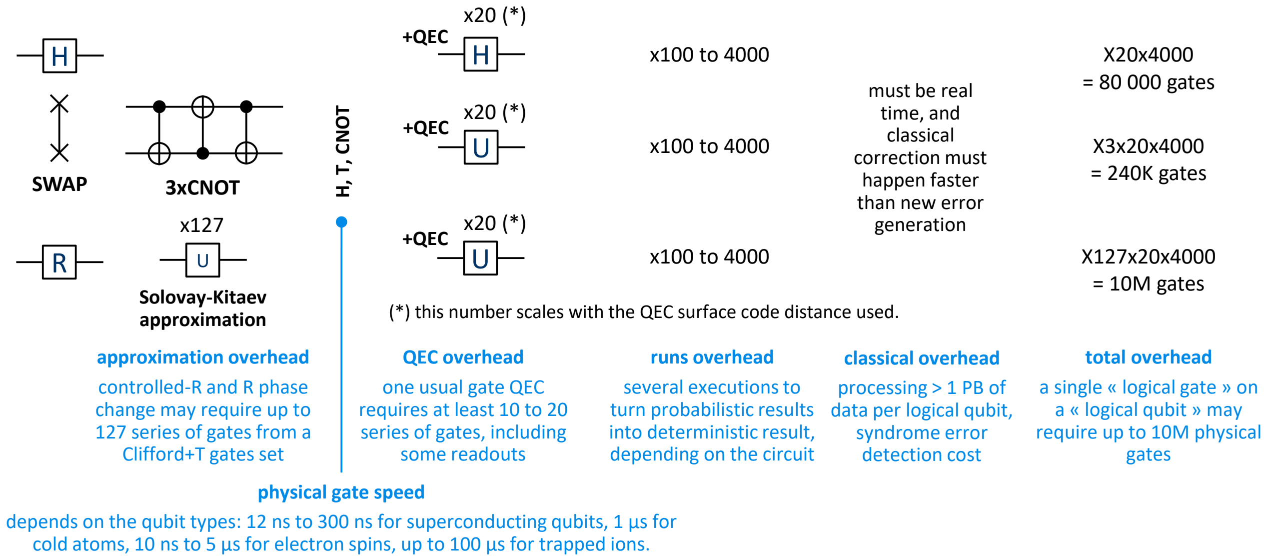

Figure 260: assessing the overhead of quantum error correction on a practical basis. (cc) Olivier Ezratty, 2021-2023.

I tentatively added these mechanisms for three scenarios: an H gate, a SWAP gate assembled with three CNOT gates and an arbitrary rotation R gate created with a Clifford gate set plus a T gate using the Solovay-Kitaev approximation theorem. Adding all these timing overheads, you obtain between 80K and 10M gates to run to execute a single physical gate. The runs overhead of x4000 may still be overestimated here since it is more relevant for a NISQ paradigm than a FTQC/QEC one. Still, a FTQC QPE (quantum phase estimation) circuit has to be run $1/\epsilon$ time, ϵ being the sought precision.

Interestingly, the longer the gates, like with trapped ions qubits, the better fidelity they have, creating a balancing effect between the QEC overhead and the gates times. All this should be taken into account when dealing with so-called quantum algorithms speedups, particularly with non-exponential speedups.

But these estimates are very raw and deserve scrutiny. It depends on the QEC technique that is being used, on the qubit type, on their fidelities, and so on.

At last, as a sort of conclusion for this part on error corrections, Figure 261 contains a summary of the key points differentiating quantum error correction and quantum error mitigation techniques, given that some quantum error mitigation techniques may be used in the FTQC regime along with quantum error correction codes.

criteria	QEM quantum error mitigation	QEC quantum error correction
goals	reduce circuit errors with classical **post-processing** techniques	create longer lifetime **logical qubits** with apparent lower error rates
techniques	**probabilistic approach**, circuits modifications, several runs, regressions and machine learning	**surface codes**, color codes, qLDPC codes, Floquet codes, Steane codes, CSS codes, etc.
figures of merit	circuit shots overhead, classical overhead	logical qubit target error rate, physical/logical qubit ratios, classical overhead
overhead	**more circuit shots** classical post-processing	**more physical qubits** longer circuit execution time
timing	**short term** NISQ realm	**long-term** FTQC realm

Figure 261: multi-criteria comparison between quantum error correction and quantum error mitigation. (cc) Olivier Ezratty, 2023-2024.

Quantum memory

We would guess that quantum memory is some memory capable of storing the quantum state of qubits and then using them to feed quantum computer registers[742]. It should be able to store superposed and entangled qubits and deliver it to whatever computing is needed.

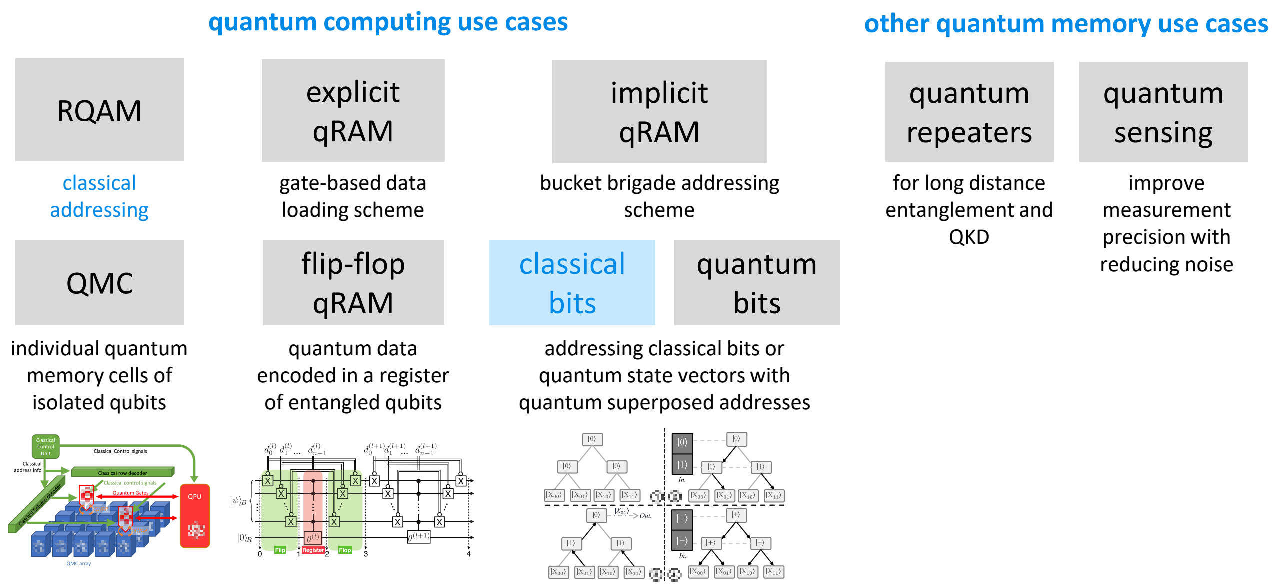

Figure 262: various classes of quantum memories and use cases. (cc) Olivier Ezratty, 2021-2023. Image sources: Quantum Memory: A Missing Piece in Quantum Computing Units by Chenxu Liu et al, September-November 2023 (43 pages), Circuit-based quantum random access memory for classical data with continuous amplitudes by Daniel K. Park et al, 2019 (9 pages) and Quantum Random Access Memory For Dummies by Koustubh Phalak et al, May 2023 (12 pages).

742 See Architectures for a quantum random access memory, by the Vittorio Giovannetti and Lorenzo Maccone and Seth Lloyd, 2008 (12 pages).

But it is part of a broader category defined as "quantum RAM" or qRAM, which can store either classical or quantum data, the data being queried with superposed quantum addresses[743]. Quantum memory is also required in quantum key distribution repeaters[744] and can be useful in various situations like with quantum sensing and for creating deterministic sources of photons[745]. However, we focus here on the first category of quantum memory, aimed at quantum computing. It is a very diverse one with different logical and physical architectures (Figure 262)[746] [747]. We'll look at quantum memories for repeaters in the section dedicated to quantum telecommunications hardware.

Quantum algorithms requirements

One anticipated usage of quantum memory is to temporarily store the state of a qubit register during a data preparation process, a usual lengthy process, before transferring it to a faster quantum processing unit. With N qubits, this memory would be able to store in theory 2^N different computational vector states amplitude values.

According to the no-cloning theorem, the content of this memory cannot be the copy of the state of other quantum registers. In computing, quantum memory is used to store data into some quantum memory to be later used in quantum processing. Data preparation and encoding depends on the algorithm. It is necessary for certain types of quantum algorithms such as Grover's search and quantum machine learning algorithms that we will describe later on[748]. The most demanding encoding is when you encode a vector of 2^N values (well, minus 1 for normalization constraints) in the whole computational state vector[749].

$$\sum_{i=1,\, 2^N} x_i^2 = 1$$

normalization constraint

$$\sum_{i=1,\, 2^N} x_i |i\rangle = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_{2^N-1} \end{bmatrix}$$

encoded state vector

This creates a superposition with all or some of the basis states from the computational basis. Namely, we encode a vector $\boldsymbol{x}$ containing 2^N real (or even complex) number values from $\boldsymbol{x_0}$ to $\boldsymbol{x_{2^N-1}}$ with the normalization constraint that the square of these values is equal to 1. It ends up creating the state vector on the right with 2^N amplitudes $\boldsymbol{x_i}$ associated with the vectors $|i\rangle$ from the computational basis. This is called amplitude encoding.

Since this data encoding grows exponentially with the number of qubits, it may erase any computing speedup we would gain later. So, this is efficient only if we find a way to make this fast. One solution is to encode only sparse vectors where only a few values are nonzero.

743 See the excellent review paper Quantum Memory: A Missing Piece in Quantum Computing Units by Chenxu Liu et al, September-November 2023 (43 pages).

744 Here's one example with One-hour coherent optical storage in an atomic frequency comb memory by Yu Ma et al, April 2021 (6 pages) and another one with Space-borne quantum memories for global quantum communication by Mustafa Gündoğan et al, 2020 (11 pages).

745 See Quantum memories - A review based on the European integrated project "Qubit Applications (QAP)" by C. Simon et al, 2010 (22 pages).

746 See Systems Architecture for Quantum Random Access Memory by Shifan Xu et al, June 2023 (14 pages).

747 See Approximate Quantum Random Access Memory Architectures by Koustubh Phalak et al, October 2022 (5 pages).

748 See Quantum Machine Learning and qRAM by Behnam Kia, 2018 (59 slides) as well as Quantum Algorithms for Linear Algebra and Machine Learning by Anupam Prakash, 2014 (91 pages).

749 See Quantum 101: Do I need a quantum RAM? by Olivia Di Matteo, May 2020 (58 slides).

Quantum memory types

There are several types of quantum memories used in quantum computing (Figure 263):

- **Explicit qRAM** encodes data in physical qubits and then, use quantum circuits to extract the encoded data[750]. There is no specific addressing system to selectively access parts of this memory. This is the scenario depicted above. Also named QAQM for Quantum Access Quantum Memory and Quantum Access Memory[751]. There, data is quantum and data access is quantum.
- **Flip-flop qRAM** is a variant of explicit qRAM based on qubits circuit algorithms used to efficiently load classical data in a qubit register[752].
- **Implicit qRAM** was proposed by Seth Lloyd et al in 2008 with the bucket brigade addressing system, based on a qutrits tree (three states quantum objects) containing wait/left/right flags[753], sort of decision trees to reach the right memory cell. It is also named QACM for Quantum Access Classical Memory[754].

  This quantum addressing system can be used for accessing both *classical* bits and *coherent states* in qubits. The first case may be useful when building some oracles for algorithms like a Grover search[755].

  $$\sum_j \alpha_j |j\rangle|0\rangle$$

  α_j weighted superposition of adresses corresponding to computational basis states $|j\rangle$

  In the full quantum case, the coherent superposition of these addresses enables a readout of a superposition of many state amplitudes in the computational basis. Namely, we can query a given amplitude α_i of the computational basis vector at the i address, encoded in binary with N classical bits or several of these, encoded in superposition[756].

  $$\sum_j \alpha_j \; |j\rangle|b_j\rangle$$

  result of query, weights are applied to $|b_j\rangle$ j-th memory location

  This is the best case to obtain some exponential computing time separation for some machine learning algorithms between using or not using some quantum memory[757].

  In classical RAM, the memory array of N bits (2^n) is usually organized in a 2-dimensional lattice which requires $O(\sqrt{N})$ switches, precisely, usually a fixed number of address data to address lines and columns in memory chips.

[750] See Optimal QRAM and improved unitary synthesis by quantum circuits with any number of ancillary qubits by Pei Yuan and Shengyu Zhang, Tencent Quantum Laboratory, February 2022 (19 pages) proposes an optimized method for feeding QRAM with amplitude QSP (quantum state preparation).

[751] See Quantum Associative Memory by Dan Ventura and Tony Martinez, 1998 (31 pages) and this implementation proposal that sees quite optimistic despite the support of prestigious folks like John Preskill, in Quantum Data Center: Theories and Applications by Junyu Liu et al, University of Chicago, Caltech and AWS, July 2022 (24 pages).

[752] See Circuit-based quantum random access memory for classical data with continuous amplitudes by Tiago M. L. de Veras et al, 2020 (11 pages) referring to Circuit-based quantum random access memory for classical data with continuous amplitudes by Daniel K. Park et al, 2019 (9 pages).

[753] See Quantum random access memory by Vittorio Giovannetti, Seth Lloyd et al, 2008 (4 pages) and Architectures for a quantum random access memory by Vittorio Giovannetti, Seth Lloyd and Lorenzo Maccone, 2008 (12 pages).

[754] See QRAM: A Survey and Critique by Samuel Jaques and Arthur G. Rattew, University of Orford, May 2023 (38 pages).

[755] See Quantum algorithm for finding minimum values in a Quantum Random Access Memory by Anton S. Albino et al, Zapata Computing, January 2023 (14 pages).

[756] See Circuit-Based Quantum Random Access Memory for Classical Data by Daniel K. Park et al, Nature, 2019 (8 pages) which proposes an optimized implementation.

[757] See Exponential separations between learning with and without quantum memory by Sitan Chen et al, UC Berkeley, Harvard University, Caltech and Microsoft, November 2021 (77 pages).

In bucket brigade qRAM, this can decrease to $O(logN)$ to adress a particular computational basis vector amplitude. But this has to take into account the burden of any quantum error correction[758] and other physics limitations[759]. Various implementations of the bucket brigade solution have been proposed so far, including one using quantum walks, with the benefit of being more robust to decoherence and easier to parallelize[760].

Before any qRAM data transfer to computation qubits can be done, an uncompute processing must be implemented that remove the selected computational basis vectors addresses from the related data.

- **RQAM (Random Quantum Access Memory)** or Random Access Quantum Memory which can encode arbitrary data in single of multiple qubits[761].
- **Probabilistic Quantum Memory** (PQM) stores and simultaneously analyzes r patterns while using only n qubits. A quantum computer therefore would need $O(n)$ qubits as opposed to $O(rn)$ bits of associative memory on a classical computer[762].
- **Quantum Read Only Memories** (QROM) are also proposed to allow only the retrieval of stored quantum information which cannot be updated[763].

TABLE OF COMPARISON BETWEEN DIFFERENT QRAM TECHNOLOGIES.

Feature	**Bucket-Brigade QRAM [36]**	**Fanout QRAM [37]**	**Flip-Flop QRAM [44]**	**Qudits-based memory [48]**	**Approximate PQC-based [49] & EQGAN QRAM [46]**
Structure	Bifurcation graph	Bifurcation graph	Quantum circuit	Higher states	Parametric Quantum Circuit
Circuit width (n = #address lines)	O(2^n)	O(2^n)	O(n)	Dependent on d (# qudit states)	O(n)
Circuit depth (n = #address lines)	O(2^n)	O(2^n)	O(2^n)	Dependent on d (# qudit state)	O(1)
Unique qualities	Qubits are routed in a sequential fashion	Qubits controlling exponential quantum switches	Quantum circuit-based	Reduces requirements of ancilla qubits to 0	Trainable like a machine learning model
Implementation technology	Photons, trapped atoms	Photons, microwave cavities	Superconducting qubits, trapped ion qubits	Superconducting qudits, trapped ion qudits, OAM photonic qudits	Superconducting qubits, trapped ion qubits
Drawbacks	Exponential circuit width and depth	Exponential circuit width and depth, susceptible to decoherence	Exponential circuit depth	Unstable higher states	Performance degradation under noise (approx. QRAM), store only simple dataset (EQGAN)

Figure 263: another segmentation of various forms of qRAM. Source: Quantum Random Access Memory For Dummies by Koustubh Phalak et al, PennState University, May 2023 (12 pages). Added in 2023. I must be really dummy since I still can't figure out how classical to quantum data connection works with qRAM accessing classical data.

In the end, when quantum data is transferred from quantum memory to computing qubits, it is achieved with teleporting the memory qubits to the computing one by one, usually with using

[758] The QEC burden may be significant. also On the Robustness of Bucket Brigade Quantum RAM by Srinivasan Arunachalam et al, 2015 (19 pages) which shows that the timing advantage of qRAM bucket brigade addressing may be quickly lost due to QEC overhead. See also Quantum Random Access Memory by Aaron Green and Emily Kaplitz, 2019 (12 pages) and Methods for parallel quantum circuit synthesis, fault-tolerant quantum RAM, and quantum state tomography by Olivia Di Matteo, 2019 (111 pages) and Fault tolerant resource estimation of quantum random-access memories by Olivia Di Matteo et al, 2020 (14 pages).

[759] See Fundamental causal bounds of quantum random access memories by Yunfei Wang et al, July 2023 (32 pages).

[760] See Quantum random access memory via quantum walk by Ryo Asaka et al, 2021 (13 pages).

[761] See Realization of a programmable multi-purpose photonic quantum memory with over-thousand qubit manipulations by Sheng Zhang et al, November 2023 (23 pages).

[762] See Probabilistic Quantum Memories by Carlo A. Trugenberger, PRL, 2000 (4 pages) and a recent implementation improvement proposal in EP-PQM: Efficient Parametric Probabilistic Quantum Memory with Fewer Qubits and Gates by Mushahid Khan et al, University of Toronto, January 2022 (27 pages).

[763] See Optimization of Quantum Read-Only Memory Circuits by Koustubh Phalak et al, PennState and IBM, April 2022 (6 pages). It uses amplitude encoding with qubits for address and qubits for memory.

entangled photons and, in many cases, some conversion from solid qubits to photon qubits (spin or charge to photons and the other way around). This teleportation is supposed to preserve the superposition and entanglement between the memory qubits during this transfer. Given there must be some errors generated during the transfer, which will require their own error correction codes.

Quantum memory physical implementations

None of the different quantum memory architectures studied over the last two decades is working yet. However, research is making progress, with targeted use cases that are more related to secure telecommunications and for quantum optical repeaters. At this stage, the advent of qRAM for quantum computing is more difficult to predict than scalable quantum computing!

The most promising quantum memory technologies are coupling cold atoms and photon polarization[764]:

- **Optical memories** are tested with ytterbium[765], a rare earth that can be controlled at high frequency. The process is like the previous one and consists in preserving the polarization of a single photon in a magnetic trap, rather for optical repeater applications in long-distance secure communication lines[766].
- **NV centers**[767] and other crystal defects are also tested.
- **Cold atoms and light polarization**. Chinese scientists used in 2019 the storage of the circular polarization state of a single photon trapped in a laser-cooled rubidium structure in a magneto-optical trap and thus made transparent[768] (Figure 264). Rubidium atoms are cooled with lasers to 200 µK. The same year, another team in China created a 105 qubits memory using 210 memory cells and dual-rail representation of a photon-based qubits with fidelities of 90% but these qubits seem not entangled and thus, not able to store a full state vector with 2^N values, but only a N or 2N values using basis encoding in each individual qubit[769]. Other techniques are based on cesium with fidelities reaching 99%[770].

  This is also the technique developed by Julien Laurat at ENS LKB in Paris and implemented by Welinq for both quantum repeaters and quantum computers interconnect. A related work in Canada is dynamically controlling rubidium's transparency to trap single photons[771].

[764] As in Highly-efficient quantum memory for polarization qubits in a spatially-multiplexed cold atomic ensemble, 2017 (13 pages), a paper to which Julien Laurat from CNRS contributed.

[765] See Nuclear spin-wave quantum register for a solid-state qubit by Andrei Ruskuc et al, Caltech, Nature, February 2022 (32 pages). It uses ytterbium nuclear spin in yttrium orthovanadate crystal (YVO_4, V for vanadium) arranged in nanophotonic cavity. It stores polarization information in spin ensembles. Bell states are created with ytterbium and vanadium. Control is made with 675 and 991 MHz microwaves and optical readout at 984 nm. It operated at 460 mK.

[766] See Simultaneous coherence enhancement of optical and microwave transitions in solid-state electronic spins, December 2017 (10 pages). This is a joint work between the University of Geneva, notably Nicolas Gisin, and the CNRS in France.

[767] See Storing quantum information in spins and high-sensitivity ESR, by two researchers including Patrice Bertet of the Quantronics group at CEA/CNRS, September 2017 (13 pages). See also A Ten-Qubit Solid-State Spin Register with Quantum Memory up to One Minute by C. E. Bradley et al, QuTech and TU Delft, 2019 (12 pages) and Multiplexed control of spin quantum memories in a photonic circuit by D. Andrew Golter et al, MITRE Corporation, Sandia Labs, University of Arizona, September 2022 (18 pages).

[768] As reported in HKUST Physicist Contributes To New Record Of Quantum Memory Efficiency, 2019, which refers to Efficient quantum memory for single-photon polarization qubits by Yunfei Wang et al, 2019 (8 pages).

[769] See Experimental realization of 105-qubit random access quantum memory by N. Jiang et al, 2019 (6 pages).

[770] See Highly-efficient quantum memory for polarization qubits in a spatially-multiplexed cold atomic ensemble by Pierre Vernaz-Gris, Julien Laurat et al, Nature Communications, January 2018 (6 pages) and Efficient reversible entanglement transfer between light and quantum memories by M. Cao, Julien Laurat et al, LKB France, April 2021 (6 pages).

[771] See Physicists create new, simpler-than-ever quantum 'hard drive for light', by Kate Willis, University of Alberta, 2018, which refers to Coherent storage and manipulation of broadband photons via dynamically controlled Autler-Townes splitting, October 2017 (17 pages).

In practice, photons are stored for a thousandth of a second, but this would be sufficient for optical telecommunication repeaters. Another work in France from the Pasqal team used cold atoms to store quantum information[772].

- **Trapped ions** as experimented in 2022 in China with 218 ions in a 1D trap with over 300 ms stability [773].

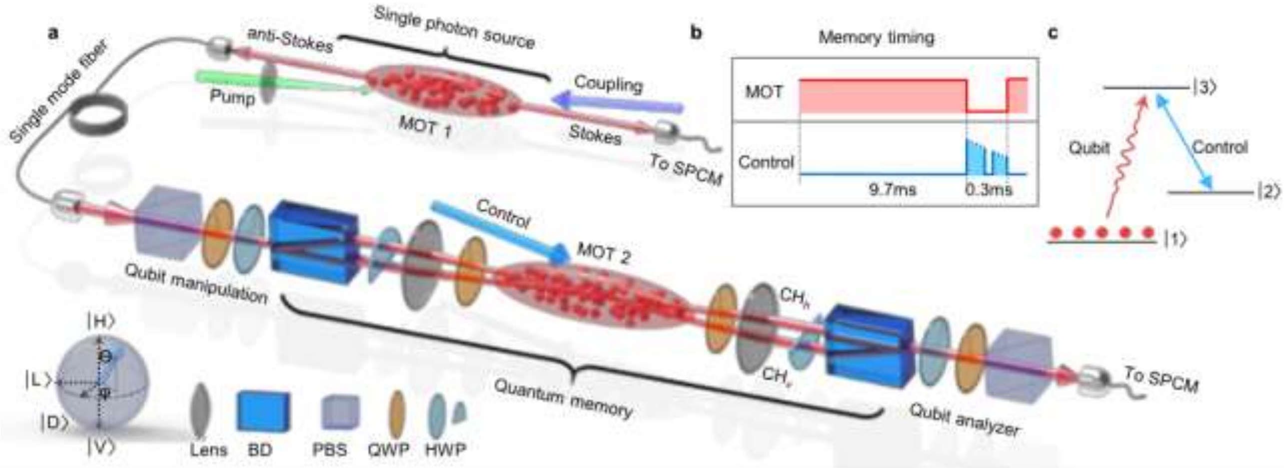

FIG. 1: **Experimental setup and energy level scheme of the single-photon quantum memory**. **a.** Schematics of experimental optical setup. The cold atoms in the first magneto-optical trap (MOT_1) serve as a nonlinear optical medium for producing time-frequency entangled photon pairs, while the cold atoms in the second magneto-optical trap (MOT_2) are the medium for the quantum memory. The anti-Stokes photon is coded with an arbitrary polarization state through the qubit manipulation unit (QMU) consisting of a quart-wave plate (QWP) and half-wave plate (HWP). After the QMU, the two orthogonal linear polarizations are separated into two beams by a polarization beam displacer (BD) which are coupled into the two balanced spatial channels CH_h and CH_v of the quantum memory. The memory read outs are recombined at the second BD and the polarization state is measured by the qubit analyzer. **b.** The memory operation timing shows the MOT sequence and the optimized control laser intensity time-varying profile in each experimental cycle. **c.** The atomic energy level scheme of the quantum memory based on electromagnetically induced transparency (EIT).

Figure 264: a cold atom base single qubit memory.
Source: Efficient quantum memory for single-photon polarization qubits by Yunfei Wang et al, 2019 (8 pages).

- **Electron spins**[774] and **donors spins**[775] are other options for the storage of quantum states.
- **Nuclear spins** qubits storage[776] like with ionized $^{31}P^{+}$ donors in isotopically purified ^{28}Si [777].
- **Superconducting cavities**, as explored at Yale[778].
- **Photons** trapped in cavities[779].

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[772] See Storage and Release of Subradiant Excitations in a Dense Atomic Cloud by Giovanni Ferioli, Antoine Glicenstein, Loic Henriet, Igor Ferrier-Barbut and Antoine Browaeys, PRX, May 2021 (12 pages).

[773] See Experimental realization of a 218-ion multi-qubit quantum memory by R. Yao et al, September 2022 (6 pages).

[774] See Researchers achieve on-demand storage in integrated solid-state quantum memory by Liu Jia, Chinese Academy of Sciences, January 2021.

[775] See Random-access quantum memory using chirped pulse phase encoding by James O'Sullivan, March 2021-June 2022 (27 pages) which deals with using ensembles of bismuth donors spin in natural silicon, coupled to a planar superconducting niobium resonator, all operating at 100 mK with a resonant frequency of 7.093 GHz. Pulses are made of 1,200 photons. It seems to be used with individual qubits memory, not entangled qubits and amplitude encoding.

[776] See Nuclear Spin Quantum Memory in Silicon Carbide by Benedikt Tissot et al, April-August 2022 (12 pages). It uses an all-optical O-band (in the 1,260 nm-1,360 nm range, adapted to long distance communication) to control vanadium defect spins in SiC

[777] See Room temperature quantum bit storage exceeding 39 minutes using ionized donors in 28-silicon by Kamyar Saeedi et al, Oxford University, UCL, Simon Fraser University, PTB, March 2023 (5 pages).

[778] See QRAM architectures using superconducting cavities by D. K. Weiss, Shruti Puri, Steve M. Girvin, October 2023 (19 pages).

[779] See Toward a Quantum Memory in a Fiber Cavity Controlled by Intracavity Frequency Translation by Philip J. Bustard et al, March 2022 (7 pages). Here, the memory traps photons in a low-loss cavity.

* **Cat-qubits** can implement passively corrected quantum memory[780].
* Memory with **error correction** using the honeycomb technique or Floquet codes[781].

Many new quantum memory proposals pop up from time to time. An interesting one from the University of Cambridge stores some quantum bit information in an electron spin hidden in haystack of 100,000 atom nuclei. The electron spin and the whole haystack are controlled by a laser. But the nuclei surrounding the electron make it difficult to entangle several qubits. End of story[782]!

Energetics

The main motivation for creating quantum computers is their computing capacity, which theoretically could increase exponentially with their number of high fidelity logical qubits. This should make it possible to perform calculations that are inaccessible to conventional supercomputers. In some other cases, like with some NISQ architecture using quantum error mitigation, it will only be "just" faster or sometimes, provide better results, like with quantum machine learning or some many-body quantum physics simulations.

How does this computing capacity translate in terms of energy consumption is a key open question. At first glance, it looked like the energetic cost of quantum computing was several orders of magnitude lower than classical computers. That was a naïve interpretation of Google Sycamore's 2019 quantum supremacy demonstration which did show a ratio of about one to one million in energy consumption compared to the IBM Summit supercomputer that was used as a comparison, and even when using the optimized algorithm and configuration proposed afterwards by IBM that ran in 2.5 days instead of the 10,000 years advertised by Google.

But the benchmark was comparing apples and oranges with a randomized benchmark with no input data nor any useful output data. It was later shown by Xavier Waintal et al that, with accounting for its high error rate and noise and using tensor networks, Sycamore's performance could be emulated on a rather simpler classical server cluster[783] [784]. In 2023, Google itself did show that Sycamore 53-qubit randomized benchmarking could run in 6 seconds on the DoE Frontier Aurora. It turned that this turned into giving Google's solution a 1 to 30 ratio advantage vs classical simulation with regards to energy consumption[785].

On the other hand, another commonplace view is that the sheer power of about 15 kW that is required for cooling current small scale superconducting qubits processors is a showstopper. It gives the impression that quantum computers will be high-power consuming devices. This *may* not be true and forgets that a rack of Nvidia GPGPUs used for machine learning tasks has a power consumption above 30 kW. But how to compare these various beasts remains a new investigation field.

Real comparisons should be made in the future, with large-scale quantum computers that will bring a quantum computing advantage to classical supercomputers. These will require a large number of physical qubits to implement error correction in a fault tolerant manner (FTQC).

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[780] See Candidate for a self-correcting quantum memory in two dimensions by Simon Lieu et al, May 2022 (11 pages).

[781] See A Fault-Tolerant Honeycomb Memory by Craig Gidney et al, August 2021 (17 pages).

[782] See Light used to detect quantum information stored in 100,000 nuclear quantum bits by University of Cambridge, February 2021 and A different type of cloud computing: Quantum breakthrough uses lasers to find data in a giant cloud of atomic nuclei by Daphne Leprince-Ringuet, February 2021. And Quantum sensing of a coherent single spin excitation in a nuclear ensemble by D. M. Jackson et al, Nature Physics, 2021 (21 pages).

[783] See What limits the simulation of quantum computers? by Yiqing Zhou, Edwin Miles Stoudenmire and Xavier Waintal, PRX, November 2020 (14 pages).

[784] See A density-matrix renormalization group algorithm for simulating quantum circuits with a finite fidelity by Thomas Ayral, Thibaud Louvet, Yiqing Zhou, Cyprien Lambert, E. Miles Stoudenmire and Xavier Waintal, August 2022 (25 pages).

[785] See Phase transition in Random Circuit Sampling by A. Morvan et al, Google AI, April 2023 (39 pages).

Controlling these qubits uses energy-consuming conventional electronics. In that case, will quantum computers provide some energy advantage on top of a computing advantage, or do they risk turning into energy hogs[786]? The same questions should be asked for other quantum technologies that could potentially be deployed at a large scale like quantum telecommunications and cryptography as well as quantum sensors.

Digital energy footprint

The energy footprint of all our digital tools is already significant with between 4-11% of global electricity consumption in 2020[787] [788] [789]. Some forecasts estimate that this bill could grow to 20% by 2030, mostly due to the increase of power consumption in data centers[790].

Since quantum computers will mostly sit and data centers and add another source of power drain to existing systems, it is interesting to look at classical comparables. Quantum computers are usually compared in performance and energy footprint with supercomputers. The world's largest supercomputers have powers in the tens of MW (megawatts) range like the Aurora Frontier from the DoE Oak Ridge Laboratory in Tennessee and its 21 MW for 1.1 exaflops and 700 petabytes of storage, 9,400 AMD CPUs and 37,000 AMD GPUs. It followed the IBM Summit in 2019 and its 13 MW of peak power for 200 petaflops, including 3.9 MW just for cooling. These MW came from the thousands of Power9s CPU chips and general purpose Nvidia GPUs requiring a complex water-cooling system that uses two tons of water per minute. IBM Summit occupied 500 m^2 and weighed 349 tons, compared to about 2 tons for a superconducting qubits QPU fitting into a room of about 20 m^2, the device being a square cube of about 2.75m, which also gives a "mass advantage" and a "surface advantage" in its current state, provided we also obtain a computing advantage, which has yet to be proven.

New supercomputers are deployed every year and we are entering the “exaflops” era. These supercomputers won’t however be replaced by quantum computers. Many of the scientific applications they are used for are not suitable for quantum computing, like any digital simulation requiring large sets of data such as in weather forecasts or using the finite elements method and other methods to solve differential equations. We will always need them. On the other hand, when quantum computers scale up, they will be able to perform computations inaccessible to conventional supercomputers, like molecular simulations and, we hope, with a smaller energy footprint.

The energy efficiency of classical systems is the ratio of their performance to their energy consumption. Classical server efficiency can be expressed in FLOPS/W, where FLOPS is the number of floating-point operations per second. Since the birth of computing, this efficiency has doubled about every 18 months. This is Koomey's law, with a current record for supercomputers of 52 GFLOPS/W for the DoE’s full size Frontier Aurora launched in 2022[791]. This figure of merit is easy to use but is unfortunately not applicable to quantum computing. QPUs performance cannot be evaluated in FLOPS! We can only assess the capability and resource to solve problems of given sizes.

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[786] Like in this evaluation of Shor’s energetic cost seen in Energy Cost of Quantum Circuit Optimisation: Predicting That Optimising Shor’s Algorithm Circuit Uses 1 GWh by Alexandru Paler et al, ACM Transactions on Quantum Computing, March 2022.

[787] See Energy consumption of ICT by Aimee Ross and Lorna Christie, UK Parliament POSTnote, September 2022 (7 pages).

[788] See Spintronic devices for energy-efficient data storage and energy harvesting by Jorge Puebla et al, Communication Materials, 2020 (9 pages).

[789] See Usage impact on data center electricity needs: A system dynamic forecasting model by Martijn Koot and Fons Wijnhoven, June 2021 (13 pages).

[790] See The nexus between data centres, efficiency and renewables: a role model for the energy transition by Sean Ratka and Francisco Boshell, EnergyPost.eu, June 2020.

[791] See https://www.top500.org/ and the June 2022 Top 500 charts. See also Compute and energy consumption trends in deep learning inference by R. Desislavov, F. Martinez-Plumed, and J. Hernandez-Orallo, 2021 (26 pages). The Frontier TDS has an energetic performance of 62 GFLOPS/W but was only a small scale testbed for the full Frontier.

The supercomputing energetic efficiency gains have not prevented an explosion in global energy consumption to power digital technologies. First, supercomputers account for a small share of global digital energetic footprint. It is increasing as usage grows. These phenomena are simply a new manifestation of the rebound effect, formalized by William Stanley Jevons in 1865.

Efficiency gains automatically lead to a decrease in the cost of resources. Without regulation of markets and uses, they lead to an increase in global consumption (see Figure 265). However, this does not mean that improving the energy efficiency of computers is inherently wrong. On the contrary, it is the only solution to maintain performance with limited energy and material resources.

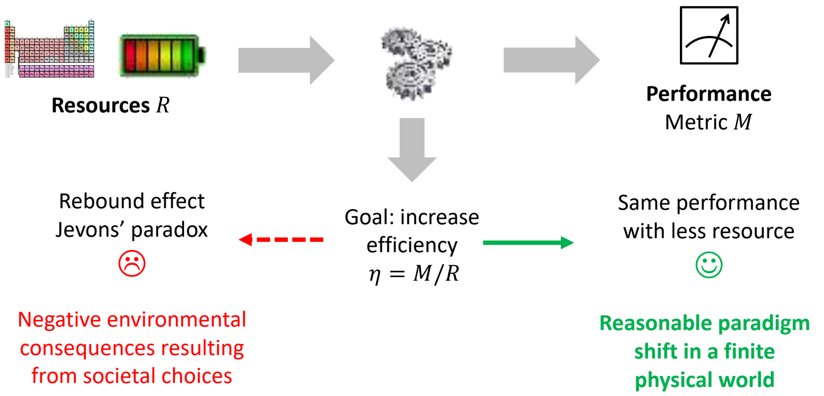

Figure 265 Energy efficiency and the rebound effect. A machine consumes material and energy resources to perform a task with a performance M. Its efficiency is defined by the ratio $\eta = M/R$. Source: Alexia Auffèves, France Quantum June 2022 presentation.

One key under tapped avenue to reduce classical computing energetic footprint lies with software optimizations and even the choice of programming languages[792].

The debate is also raging about the potentially large energetic footprint of cryptocurrencies, with various questionable comparison methodologies[793]. And let's forget about the metaverse which may turn into yet another digital energetic hog if it gets used on a large scale.

Quantum Energy Initiative

Scaling quantum computing is one of the most challenging scientific and technology endeavors ever launched by mankind on top of space exploration, nuclear fusion, DNA sequencing and genome based therapies creations. It should be undertaken with behaving responsibly as early as possible.

One way is to embed in research and systems design an approach integrating the environmental footprint of quantum technologies. This footprint is of course energetic but also encompasses raw materials, manufacturing processes and product lifecycle estimations and optimizations. Addressing these questions are both scientific, technological, and societal challenges.

We can learn a couple lessons from what happened with artificial intelligence and deep learning. It became trendy starting in 2012 with a peak around 2020 when deep learning use cases became mainstream and embedded from smartphones to cloud datacenters. It was then discovered that AI had a significant energetic cost, both for training large deep learning models and to run them whether on end-user devices or on servers[794].

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[792] See Ranking programming languages by energy efficiency by Rui Pereira, Science of Computer Programming, May 2021 (63 pages).

[793] On Bitcoin's Energy consumption: a quantitative approach on a subjective question by Rachek Rybarczyk, Galaxy Digital Mining, May 2021 (13 pages) and Fact sheet: Climate and Energy Implications of Crypto-Assets in the United States, White House, September 2022. In the Blockchain realm, Ethereum switched in September 2022 from proof-of-work to proof-of-stake for mining, with a significant energy saving of several orders of magnitude. See Ethereum energy consumption, Ethereum, September 2022 which provides a lot of energy consumption related data for various Internet services.

[794] See Compute and Energy Consumption Trends in Deep Learning Inference by Radosvet Desislavov, 2021 (26 pages) which describes how GLOPS/W have recently evolved depending on the type of AI problem (CNN for convolutional networks, NLP for natural language processing).

The “frugal AI” topic then emerged. Solutions were proposed to reduce the energetic footprint on AI mainly with less data-hungry machine learning models[795], so-called data “quantization” (using 8-bit and even 1-bit numbers instead of 16-32-64 floating-point numbers) and with optimizing the power consumption of dedicated hardware like GPGPUs and embedded systems chips (in smartphones, connected objects and also cars). What if the environmental footprint or AI had been taken care of earlier?

The same question deserves to be asked for quantum technologies. Why not take care right now of their environmental footprint? One could argue that the first challenge is scientific before being environmental. Some are advocating to create high-fidelity qubits and useful fault-tolerant quantum computers first, and later address their related environmental aspects. Looking at how research labs and industry vendors were working until now on addressing the scalability challenges of quantum computers demonstrate that despite its relative technology immaturity and high scientific uncertainty, it is time to take environmental concerns into account right now. In a world of doubts on the role of science and technology, it is also a way to demonstrate that in emerging technologies, it is possible to implement responsible innovation practices from the start and not as afterthoughts and under pressure.

#QEI
the quantum energy initiative

This is the reasoning behind the creation of the **Quantum Energy Initiative** (QEI) in 2022[796]. The idea came out from a research team led by Alexia Auffèves (CNRS MajuLab Director, Singapore) with Robert Whitney (CNRS LPMMC in Grenoble), along with Janine Splettstoesser (Chalmers University) and Olivier Ezratty (this book’s author)[797].

The QEI is there first to try to answer to several key questions related to quantum computing:

- Is there a **quantum energy advantage** as the processors scale up and how different is it from the quantum computational advantage?
- What is the fundamental **minimal energetic cost** of quantum computing?
- How to **avoid energetic dead-ends** on the road to large scale quantum computing? Can we create optimization tools and models for qubit technology, enabling technologies and software engineering?

The seed of Quantum Energy Initiative is described in a June 2022 PRX Quantum perspective[798]. It lays the ground for a transversal initiative, connecting quantum thermodynamics, quantum information science, quantum physics and engineering. It makes the connection between classical and quantum thermodynamics, qubit architectures, qubit noise models, room temperature control electronics and cryo-electronics, quantum error correction codes, algorithms and compiler designs. It proposes a methodology to assess the energetic performance of quantum technologies, dubbed MNR [799].

After having first modeled the energy consumption of an idealized scalable fault-tolerant superconducting qubits quantum computer and learned some lessons on the conditions of a related energetic advantage, the QEI aims to apply this methodology to all types of qubits developed by research laboratories and industry vendors. All quantum computing paradigms will also need to be evaluated, namely gate-based, quantum annealing and quantum simulation. This will allow the energy dimension to be exploited for comparison and scaling. These efforts also involve the entire quantum computing software chain, in particular error correction codes, algorithms and compilers.

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[795] See Frugal Machine Learning by Mikhail Evchenko, Joaquin Vanschoren, Holger H. Hoos, Marc Schoenauer and Michèle Sebag, November 2021 (31 pages).

[796] See the QEI website: https://quantum-energy-initiative.org/.

[797] I participated to the launch of the Quantum Energy Initiative from its inception in 2022 until August 2023.

[798] See Quantum technologies need a Quantum Energy Initiative by Alexia Auffèves, PRX Quantum, June 2022 (11 pages).

[799] See also the thesis The resource cost of large scale quantum computing by Marco Fellous-Asiani, November 2021 (215 pages).

As of September 2023, the QEI had gathered the support of a community of over 350 participants from 46 countries and 29 partners in research, with industry vendors, hybrid HPC-quantum services and others. It has a scientific board covering 5 continents, launched its own video seminar series and is organizing its foundational workshop in Singapore in November 2023. It garnered visibility in various quantum related events and even in Nature[800]. It also led to the creation of the IEEE Working Group for creating a Standard for Quantum Computing Energy Efficiency (P3329).

The QEI is not limited to quantum computing and is intended to expand to all quantum technologies, namely quantum telecommunications[801] and quantum sensing.

Modeling a quantum computing energetic advantage

Thanks to quantum coherence, superposition and entanglement, quantum computers could showcase an exponential computing speedup compared to their classical counterparts, depending on the size and nature of the problems to be solved and on the used quantum algorithm. This computational advantage is usually predicted for ideal, error-free processors. In reality, quantum processors are noisy, with error rates currently exceeding 0.1% per operation, a prohibitive level for most algorithms and many quantum error correction codes.

In the short term, variational algorithms are developed for noisy processors in the quantum computing paradigm called NISQ (Noisy Intermediate Scale Quantum) and with using quantum error mitigation techniques. These have not yet demonstrated any clear quantum utility or advantage.

In the longer term, we will rely on quantum error correction using many physical qubits assembled as logical qubits. Their number varies from a couple dozens to millions depending on the qubit technology and the target logical qubits error rates. Thanks to a very low logical error rate, it will enable longer calculations with deeper algorithms.

In both cases, demonstrating a computational advantage for real quantum processors is an open field covering fundamental and applied research as well as technological developments. In some cases, quantum computers could be less energy-intensive than conventional computers to solve the same problem.

With larger scale FTQC could emerge a sort of quantum energy supremacy when a quantum computer solves a problem that no classical computer could process with a "reasonable" energy footprint like the power of a large supercomputer (20-30 MW) or, to be extreme, a nuclear plant reactor (1 GW)[802].

Modeling and optimizing the energetic efficiency of quantum computers must consider all classical resources used for control and error correction. As a ratio of a performance to a resource, this energy efficiency is a hybrid quantity:

- Computational performance emerges at the fundamental quantum level, and results from the ability to control the noisy quantum processor to perform an algorithm with a certain accuracy. Understanding and optimizing these mechanisms is a matter of quantum control, quantum thermodynamics, quantum error correction, algorithms and compilers.

[800] See Are quantum computers really energy efficient? by Sophia Chen, Nature Computational Science, June 2023 (4 pages).

[801] Work has already started there. See for example Reducing energy consumption of fiber networks via quantum communication technology by Janis Notzel and Matteo Rosati, February 2022 (25 pages). With some proposal of a quantum receiver that would reduce the power consumption of classical fiber optic lines amplifiers.

[802] Interestingly, one paper shows how Shor algorithm is near this threshold, in Energy Cost of Quantum Circuit Optimisation: Predicting That Optimising Shor's Algorithm Circuit Uses 1 GWh by Alexandru Paler and Robert Basmadjian, ACM Transactions on Quantum Computing, March 2022 (no free access). Since this GWh is to be consumed in 8 hours, we're off with a power of 125 MW, which is in the low end spectrum of small modular reactors (SMRs).

- Establishing satisfactory control at the quantum level requires the provision of resources at the macroscopic level, which determines the energy consumption necessary to carry out the calculation. This is the domain of enabling technologies including cryogenics, control electronics, cabling, lasers, amplifiers, detectors, classical computing resources, whose mix depends on the qubit type.

It is essential to set up a full-stack quantum computer model coupling these different levels, as well as common language and concepts[803]. On this basis, the methodology proposed in the QEI is simple. It sets a target performance at the microscopic level defining an implicit relationship between the different parameters of the model and a macroscopic energy consumption that is then minimized under this constraint.

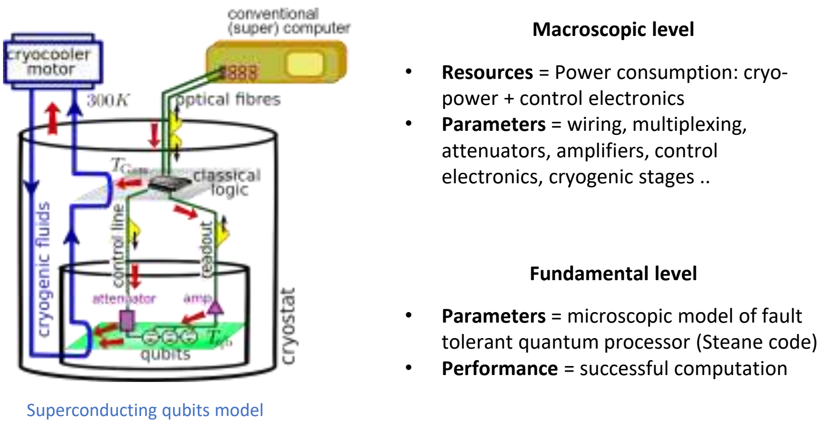

Figure 266 Full-stack model of a superconducting quantum computer coupling a quantum level and a macroscopic level of description. Source: Alexia Auffèves and Robert Whitney.

Marco Fellous-Asiani et al applied this methodology in 2022 to some idealized superconducting qubits to see if and how some quantum energetic advantage could be envisioned[804]. They considered typical algorithms used for optimization, physical simulations, quantum machine learning, and cryptanalysis for integer factorization. Their full-stack modeling integrated the sources of quantum noise affecting qubits, the conventional qubit control resources such as electronics that generate microwave pulses and voltages, filters and attenuators, cryogenics, cables and amplifiers used for reading the state of the qubits, then the sources of heat dissipation involved in the whole material chain and in particular in the cryostat (see Figure 266).

It also accounts for the size of the error correction code, initially a concatenated Steane code and then a surface code. The model established a relationship between microscopic processor parameters such as qubit fidelity, with macroscopic qubit control parameters. It was an interesting basis to optimize the energy consumption of the whole system, under the constraint of reaching a targeted computational performance.

Naturally, the results depend strongly on the qubits fidelity. A gain of a factor of 10 could lead to an energy gain of a factor of 100. The model can help find out the optimal temperature for control electronics. For CMOS type control electronics technologies and even with a highly optimistic assumed power consumption of 2 mW per qubit[805], room temperature seems preferable to run the electronics. It would be similar with higher power drains electronics. The technological constraint then lies in the wiring, which must be simplified, essentially by using advanced (and future) multiplexing techniques.

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[803] See Energy use in quantum data centers: Scaling the impact of computer architecture, qubit performance, size, and thermal parameters by Michael James Martin et al, NREL, IEEE Explore, March 2021- December 2022 (18 pages) that proposes a modelling of QPU energy consumption but not in a full-stack manner. It does not take into account algorithm specifics and is very generic with regards to all enabling technologies where many technology choices can impact the total system power consumption like efficient electronics.

[804] All of this modeling comes out of Optimizing resource efficiencies for scalable full-stack quantum computers by Marco Fellous-Asiani, Jing Hao Chai, Yvain Thonnart, Hui Khoon Ng, Robert S. Whitney and Alexia Auffèves, arXiv and PRX Quantum, September 2022-October 2023 (39 pages). See also the thesis The resource cost of large scale quantum computing by Marco Fellous-Asiani, November 2021 (215 pages).

[805] A Scalable Cryo-CMOS 2-to-20GHz Digitally Intensive Controller for 4×32 Frequency Multiplexed Spin Qubits/Transmons in 22nm FinFET Technology for Quantum Computers by Bishnu Patra et al, 2020 (4 pages). This consumption model should still be full stack, up to analyzing readout microwaves after traversing parametric amplifiers, HEMTs and ADCs. It is not sure 2 mW are enough to do all of this. One key question to ask is what is the theorical lower bound of microwave packets generation energetic costs?

Another option would be to use superconducting electronics running at the processor level or at the 4K stage, like the ones developed by SEEQC that we describe later in the enabling technologies part of this book.

With this model, the possibility of an energy-based quantum advantage was investigated. It computes the minimum energy consumption of a fault-tolerant quantum computer to factor an N-bit integer and compare it to the classical record[806]. A classical record was obtained in 2021 by an Inria team on a Germany-based supercomputer, for factoring a 829-bit key[807] with a consumption of 965 GJ, or 1.3MW of power over 8.6 days.

The model shows that a quantum computer operating with qubits 2,000 times more faithful than Google Sycamore, combined with Steane's code would require 2.7GJ = 2.9MW for 16 minutes, which is the amount of energy contained in about 75 liters of fuel oil. That would be 350 times less energy than used on the equivalent supercomputer. Breaking a 2048-bit RSA key is beyond the reach of a conventional supercomputer. On a quantum computer of the same type as above, the energy consumption would be 38 GJ =7 MW for 1.5 hours. Using surface codes error correction would lighten the constraint of qubit fidelities. Estimations were made for different key sizes in the classical and quantum cases, giving access to some energy efficiency in each case. An energetic quantum advantage would start to show up with N=848. It would be different in nature from the computational advantage, which considers only the computation time[808]. Both advantages would thus be achieved for different key sizes. Let us recall that the proposed corrector code is resource-intensive and that the result would be much lower with, for example, a surface code.

So, are we sure to get this sort of energetic advantage before any computational advantage? Not yet. Their model was highly theoretical and with many optimistic technological assumptions and it could not account for the classical computing cost of error correction, which is significant although not well documented. Still, the interest of the model is to highlight technology interdependencies, which can help make sound choices in quantum computer design. It deserves to be applied to various quantum computing architectures, qubit types and paradigms.

After the QEI was launched, several preprints were published in 2023 with future quantum computer energetic consumption estimations. Most of the time, their underlying methodology missed several sources of power drain and they were drawing optimistic conclusions way too rapidly.

In one example related to the QPU energetic cost for Bitcoin mining, the energetic consumption was based on Landauer’s (tiny) lower bound applied to the cost per qubit gate, and is not considering any qubit control, cryogenics and classical error correction energetic costs[809]. The advertised result touting some quantum computing energetic advantage for Bitcoin mining is then entirely misleading.

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[806] The method is different from the one proposed in Is quantum computing green? An estimate for an energy-efficiency quantum advantage by Daniel Jaschke and Simone Montangero, November 2022 (13 pages) which compares NISQ systems and their classical emulation equivalent, but not best in-class classical algorithms equivalents. This makes the energetic reasoning incomplete. They also remind us that a quantum advantage comes from maximally entangled states, the overarching question of quantum computing scalability.

[807] See The State of the Art in Integer Factoring and Breaking Public-Key Cryptography by Fabrice Boudot, Pierrick Gaudry, Aurore Guillevic, Nadia Heninger, Emmanuel Thomé and Paul Zimmermann, June 2022 (9 pages).

[808] This is the topic of The impact of hardware specifications on reaching quantum advantage in the fault tolerant regime by Mark Webber et al, September 2021 (16 pages) which shows that the number of qubits to achieve a given task that is inaccessible to a classical computer depends on the target precision and computing time. 13 to 317 million qubits would be necessary to break Bitcoin signatures and about the same order of magnitude to compute the ground state of the FeMo cofactor (FeMoCo). See also Nitrogen, Bitcoin, and Qubits The Shape of Transmons to Come by The Observer, September 2021, and From FeMoco to Bitcoin: Universal Quantum answers two major quantum advantage questions by Universal Quantum, January 2022, which advertises the benefits of trapped ions qubits, and points to Blueprint for a microwave trapped ion quantum computer by Bjoern Lekitsch et al, 2017 (12 pages).

[809] See Quantum Blockchain Miners Provide Massive Energy Savings by Joseph Kearney and Carlos A Perez-Delgado, June 2023 (6 pages) and Quantum Miners: Revolutionizing Energy Efficiency in Blockchain by Uvin Vindula, June 2023

In another example, Florian Meier and Hayata Yamasaki demonstrate that "*quantum computation achieves an exponential energy-consumption advantage over classical computation for Simon's problem*". The estimate is based on a single (Simon) oracle based algorithm without specifying how the oracle is implemented. It also determines lower and upper energy consumption bounds formula. It shows an asymptotic exponential gain in energy consumption but do not account for the classical cost of quantum error correction[810].

The Rand Corporation produced an estimation of the energetic cost of a QPU able to break RSA-2048 codes in 2023 by consolidating various resource estimations. It estimated a rough, power cost of 6.25 W per physical qubits, but reminded readers of the many technology uncertainties about scaling[811]. In all these cases, one difficulty is to avoid projecting current energetic footprint to larger scales without accounting for some potential progress in electronics design, cryo-electronics, error correction code improvements and the likes.

At last, a 2023 preprint from Junyu Liu *et al* created some economical laws showing the energy economic advantage of quantum computers but seems off the mark when estimating the energetic costs of these systems[812]. For example, the authors write that "*it has been reported that Google's quantum devices require approximately 15kW for the complete experiment. Since this energy demand is primarily tied to cooling, it does not significantly fluctuate with the number of qubits*" which contains three factual errors. First, 15 kW is power, not energy consumption. Energy consumption is in Wh and depends on the total classical and quantum computing time. Second, Sycamore's experiment had a power drain of 26 kW as mentioned page 51 in the Google paper supplemental materials[813]. Third, energetic costs scale with the number of physical qubits! Then, they make some assumptions on the energetic cost of gate-based Rydberg atoms quantum computing using kJ per two-qubit gates. But they don't take into account the number of shots, error correction costs or atoms preparation costs.

Other preprints are looking at the details of some detail aspects like an energy efficient way to implement LDPC quantum error correction codes with using analog classical control circuits[814], with studying the lower bounds for the classical control of the qubits and the trade-offs between its energetics and speed[815] and, on how to optimize cryogenics[816]. At last, another paper explores the interesting field of quantum code classical emulation, showing that emulating 43 qubits for 1.64 hours costs 568.77kg of CO^2 [817].

As a rule, estimating energetic resources for quantum computing requires a full-stack approach encompassing quantum physics, control electronics, cryogenics, error correction, compilers and algorithms, including the number of circuit or Hamiltonian shots and the total classical computing cost of the solution (see Figure 267). Not many organizations can do that in an integrated way.

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[810] See Energy-consumption advantage of quantum computation by Florian Meier and Hayata Yamasaki, May-September 2023 (46 pages).

[811] See Estimating the Energy Requirements to Operate a Cryptanalytically Relevant Quantum Computer by Edward Parker and Michael J. D. Vermeer, April 2023 (17 pages).

[812] See Potential Energy Advantage of Quantum Economy by Junyu Liu, Hansheng Jiang and Zuo-Jun Max Shen, August 2023 (23 pages).

[813] See Supplementary information for "Quantum supremacy using a programmable superconducting processor" by Frank Arute, John Martinis et al, October 2019 (58 pages).

[814] See Gradient Flow Decoding for LDPC Codes by Tadashi Wadayama et al, Nagoya Institute of Technology, March 2023 (6 pages).

[815] See Fidelity and energetics of driven quantum systems for quantum computing by Sagar Silva Pratapsi, Lorenzo Buffoni and Stefano Gherardini, May 2023 (11 pages).

[816] See Designing Energy-Efficient Quantum Computers Through Prediction and Reduction of Cooling Requirements for Cryogenic Electronics by Michael Martin, Caroline Hughes, Gilbert Moreno, Eric Jones, David Sickinger, Sreekant Narumanchi and Ray Grout, NREL, 2021 (22 slides).

[817] See Carbon Emissions of Quantum Circuit Simulation: More than You Would Think by Jinyang Li et al, July 2023 (3 pages).

We can suspect that relatively large quantum computer vendors will be best positioned to advance this field thanks to them having large interdisciplinary teams covering all these aspects.

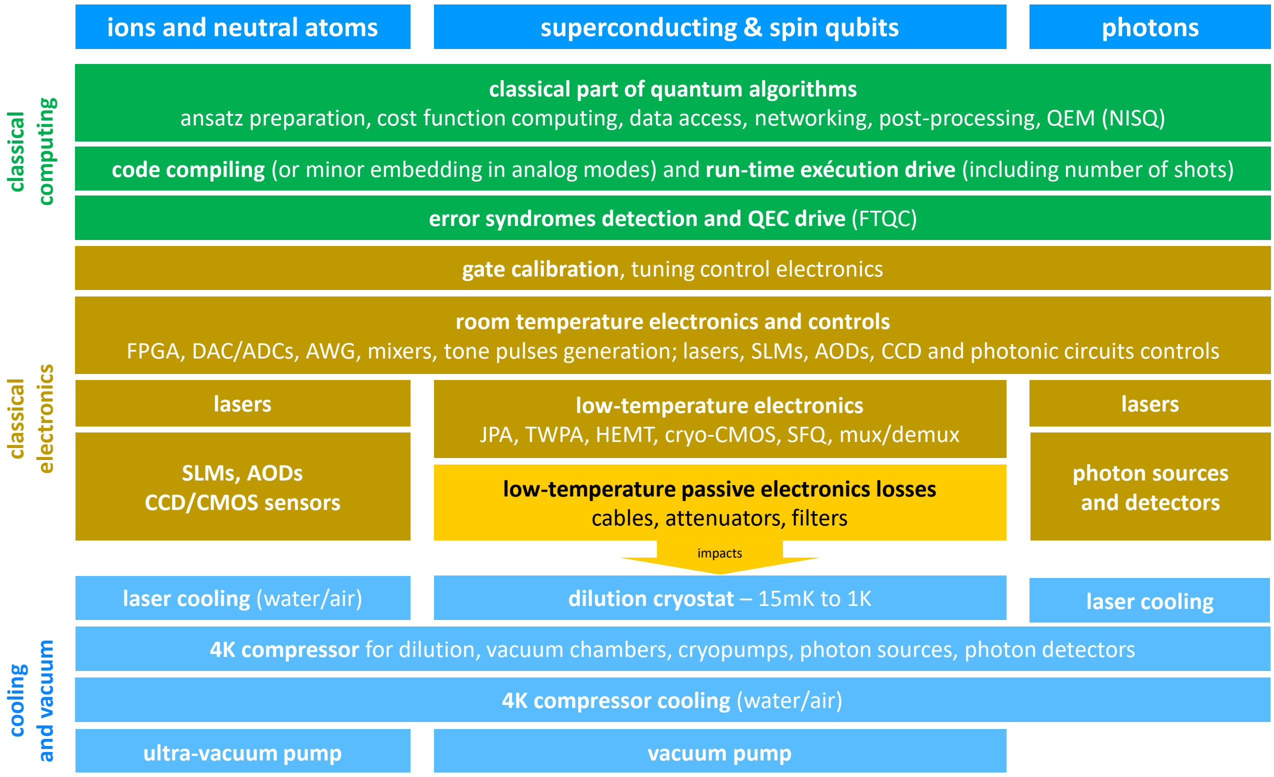

Figure 267: a laundry list of items to account for when estimating the power and energetic cost of quantum computing to solve a given problem. And don't confuse power (kW) and energy (kWh)! (cc) Olivier Ezratty, 2023.

Microscopic energetics of quantum technologies

The fundamental quantum level mentioned before is already a rich field of research[818].

The energy and entropy at stake when dealing with quantum systems are the kingdom of the broad field of quantum thermodynamics[819] [820], *aka* quantum energetics[821].

As Kater Murch & al wrote in a 2022 review paper "*Quantum information processing relies on precise control of non-classical states in the presence of many uncontrolled environmental degrees of freedom—requiring careful orchestration of how the relevant degrees of freedom interact with that environment. These interactions are often viewed as detrimental, as they dissipate energy and decohere quantum states. Nonetheless, when controlled, dissipation is an essential tool for manipulating quantum information: Dissipation engineering enables quantum measurement, quantum state preparation, and quantum state stabilization*" [822].

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[818] See the colloquium A short story of quantum and information thermodynamics by Alexia Auffèves, March 2021 (14 pages).

[819] See for example Third law of thermodynamics and the scaling of quantum computers by Lorenzo Buffoni et al, March-October 2022 (9 pages) which looks a fundamental issue related to the limits of the preparation of a qubit ground state.

[820] See the review paper Nonequilibrium boundary-driven quantum systems: Models, methods, and properties by Gabriel T. Landi, Dario Poletti, and Gernot Schaller, Review of Modern Physics, December 2022 (59 pages).

[821] At the quantum level, quantum energetics is a more appropriate term since many exchanges of energy in quantum systems are not necessarily linked to the field of thermodynamics.

[822] See the review papers Engineered Dissipation for Quantum Information Science by Patrick M. Harrington, Erich Mueller and Kater Murch, February 2022 (28 pages) and Energy dynamics, heat production and heat-work conversion with qubits: towards the development of quantum machines by Liliana Arrachea, May 2022 (63 pages).

Likewise, Matteo Carlesso and Mauro Paternostro recommend studying energetics at the fundamental quantum level to design more energy-efficient quantum devices. They promote the use of machine learning to optimize the energy exchange mechanisms in quantum computing[823].

There are many quantum thermodynamics and energetics concepts in play in qubits inner working and with other quantum technologies, some of them still relating to the famous Maxwell's demon[824].

Each qubit technology comes with its own assets and challenges regarding their energy consumption.

Superconducting qubits are a well investigated area at the quantum level. Qubits microwaves spontaneous emission is a dissipative process engendering error and decoherence. Energy exchanges happen between the qubit and its controlling microwave during a qubit gate operation[825]. Even qubit dephasing drives energy dissipation[826]. Error mitigation can make use (among various other techniques) of dissipation engineering with energy baths whether it is handled with bosonic qubits like cat-qubits or with programmed error correction. Engineering dissipation can also help efficiently prepare (entangled) Bell states with two qubits. Measurement energetics is also studied with the Zeno effect with measurement backactions, how to optimize measurement operations with various types of microwave light (single photon, coherent light, thermal light)[827], the connection between quantum measurement and error correction[828] and ways to purify the state of a single qubit with a quantum thermodynamic method[829]. There are also specific cooling mechanisms for superconducting qubits and even some connections between qubit thermodynamics and the way to optimize computing at the compiler level. At last, in the internal debates between the types of superconducting qubits, let's note that fluxonium qubits have a lower energy consumption since being driven by lower frequency microwaves and need less cooling, but at the price of various connectivity and other constraints.

Silicon spin energetics are also studied. Their operating parameter and controls are a bit different than with superconducting qubits, with a richer mix of microwave pulses and direct current and operations at a potentially higher temperature in the 1K range. Some quantum energetic advantage can even be found at the scale of one-qubit full adder implemented with three quantum dots silicon spin qubits with a gain of three orders of magnitude[830] and with entanglement generation between electron spin and photons in devices mixing static and flying qubits like quantum memories, repeaters and interconnections between quantum computing units[831].

Trapped-ions qubits operations can also be optimized with regards to the energetics of their gates or even for implementing a half-adder[832].

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[823] See From basic science to technological development: the case for two avenues by Matteo Carlesso and Mauro Paternostro, Queens University Belfast, May 2023 (22 pages).

[824] See Illusory cracks in the second law of thermodynamics in quantum nanoelectronics by Robert S. Whitney, April 2023 (75 pages).

[825] See Energetics of a Single Qubit Gate by J. Stevens, Andrew Jordan, Audrey Bienfait, Alexia Auffèves, Benjamin Huard et al, PRL, September 2021-September 2022 (19 pages).

[826] See Calorimetry of a phase slip in a Josephson junction by E. Gümüş, J. P. Pekola, H. Courtois, W. Belzig, C. B. Winkelmann et al, Nature Physics, January 2023 (6 pages).

[827] See Energetic cost of measurements using quantum, coherent, and thermal light by Xiayu Linpeng, Léa Bresque, Maria Maffei, Andrew N. Jordan, Alexia Auffèves and Kater W. Murch, PRL, June 2022 (13 pages).

[828] There are also debates about what are heat and work in quantum thermodynamics and qubits.

[829] See Quantum thermodynamic method to purify a qubit on a quantum processing unit by Andrea Solfanelli, Alessandro Santini and Michele Campisi, March 2022 (5 pages).

[830] See Quantum dynamics for energetic advantage in a charge-based classical full-adder by João P. Moutinho, Silvano De Franceschi et al, July 2022 (18 pages).

[831] See Energy-efficient entanglement generation and readout in a spin-photon interface by Maria Maffei, Andrew Jordan, Alexia Auffèves et al, May 2022 (6 pages).

[832] See Classical Half-Adder using Trapped-ion Quantum Bits: Towards Energy-efficient Computation by Sagar Silva Pratapsi et al, October 2022.

Photon qubits are different beasts with regards to the thermodynamics of the whole food chain between single photon generations, entanglement preparation, computing (usually, using MBQC), spin-photon interfaces and single photon readout[833]. A first seed of optical computing energetics was launched in Pascale Senellart's C2N team[834]. Also, the H2020 OPTOlogic project aimed at creating light-induced and controlled topology for energy-efficient logic operations in quantum photonic computing systems.

And in general, there is a direct link between quantum thermodynamics and physics with the speed of the quantum gates a quantum computer could execute, with fundamental **Quantum Speed Limits** governed by energetic levels and operating time trade-offs (QSL)[835 836 837 838 839].

Quantum sensing is also an interesting field of research with regards to quantum thermodynamics and energetics, particularly to find theoretical lower bounds of energy consumption in quantum sensors[840]. Quantum thermodynamics can also help optimize quantum sensors precision.

Classical and quantum computing reversibility

Here we study the impact of theoretical reversibility of gate-based quantum computing on its energetic cost. We first need to define the notion of logical reversibility of computation and its thermodynamic impact.

Logical reversibility of a calculation is linked to the ability to reverse it after one or more operations and recover input data from output data. This can be done at the scale of a classical logic gate or an elementary quantum gate and then up to a complete calculation. If logical reversibility is possible at the level of any gate used, then it becomes ipso-facto doable for a complete calculation[841]. Today's classical computers are logically irreversible. They rely on two-bit logic gates that destroy information since they generate one bit with two bits, and we don't keep the information from the two initial qubits. One bit is thrown away every time. You can't reverse a simple NAND, OR or AND logic operation.

We could use reversible logic gates that do not destroy information and generate as many output bits as input bits. This would lead to a logically reversible calculation. All of this was theorized by Charles Bennett in 1973 and Tommaso Toffoli in 1980. Classical computing is a big energy spender because logic gates are not logically reversible. The lower bound of energy consumption of current classical computing comes from Landauer's famous limit of kT ln(2) energy dissipated per irreversible bit operation, which can be the erasure of a bit or the merging of two computation paths.

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[833] See Energy-efficient quantum non-demolition measurement with a spin-photon interface by Maria Maffei, Bruno O. Goes, Stephen C. Wein, Andrew N. Jordan, Loïc Lanco and Alexia Auffèves, Quantum Journal, May 2022-August 2023 (18 pages).

[834] See Coherence-Powered Charge and Discharge of a Quantum Battery by Ilse Maillette de Buy Wenniger, M. Maffei, N. Somaschi, A. Auffèves, P. Senellart et al, February 2022 (19 pages).

[835] See From quantum speed limits to energy-efficient quantum gates by Maxwell Aifer and Sebastian Deffner, February 2022 (19 pages). It mentions that Amazon Web Services (AWS) classical computing is charged with about $4x10^{-13}$ cents per classical floating point operation when a single quantum circuit evaluation currently costs 1 cent on an AWS-owned Rigetti QPU (pricing source).

[836] See Fundamental speed limits on entanglement dynamics of bipartite quantum systems by Vivek Pandey er al, July 2023 (13 pages).

[837] See Exact Quantum Speed Limits by Arun K. Pati et al, May 2023 (7 pages).

[838] See A Unifying Quantum Speed Limit For Time-Independent Hamiltonian Evolution by H. F. Chau, and Wenxin Zeng, University of Hong Kong, Octobre 2023 (20 pages).

[839] See Quantum speed limit for complex dynamics by Mao Zhang, Huai-Ming Yu and Jing Liu, Nature, October 2023 (7 pages).

[840] See Thermodynamic principle for quantum metrology by Yaoming Chu and Jianming Cai, Huazhong University of Science and Technology, March 2022 (19 pages) and Notes on Thermodynamic Principle for Quantum Metrology by Yaoming Chu and Jianming Cai, Huazhong University of Science and Technology, August 2022 (6 pages).

[841] See these detailed explanations on the reversibility of classical calculus: Synthesis of Reversible Logic Circuits by Vivek Shende et al, 2002 (30 pages).

Even though we are far off this limit with current classical computing technologies, this lower bound could be avoided with logical reversible computing.

The implementation of this logical reversibility by rewinding calculations would reduce the energetic cost of classical computing, the energy spent in the forward calculation being potentially recovered in the reverse calculation. It was not a chosen path for various reasons. First was the steadiness of Moore's empirical law for many decades. Second is a reversible classical architecture has significant overhead in the number of transistors used.

Gate-based quantum computing is logically reversible (Figure 268). All gates implement a unitary transformation that is on paper reversible, modulo the effects of qubit errors. We'll see later the role of the uncompute trick which uses it to reverse computation on some qubits.

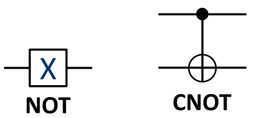
Thermodynamic reversibility is another matter and can be obtained when the system is continuously balanced with its thermal bath. It requires handling operations in a quasi-static way, namely, slowly and with logical gates requiring a minimum energy spending. This is the field of adiabatic computing.

Gate-based quantum computing is logically reversible because it uses unitary operations which are all mathematically reversible. Qubits readout is the only logically irreversible operation when it collapses qubit states to a basis state[842].

Qubits readout is reversible only when the qubit states are perfectly aligned with the basis qubit states $|0\rangle$ and $|1\rangle$, i.e., when the readout doesn't change the qubit quantum state.

However, quantum computing is not really thermodynamically reversible. It would be reversible in the absence of noise and if measurements were not changing qubit's internal states. Achieving physical irreversibility would also mandate that all non-quantum qubit control electronics rely on physically and thermodynamically reversible processes or at least be energy-saving operations.

all quantum gates are mathematically reversible,
this is a property of the matrix linear transformations



$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

we could theorically run an algorithm and rewind it entirely to return to the initial state, which could help recover part of the energy spent in the system

can be useful for some sub-parts of algorithms run before the end of computing and measurement, used in the "uncompute trick" at the end of some algorithms like for solving linear equations with HHL. it keeps the result x with resetting all other qubits without any measurement.

on a practical basis:

- **the gates are not physically and thermodynamically reversible** due to some irreversible processes like microwave generations and DACs (digital analog converters) and because gates are analog and noisy.
- **part of the digital processes taking place before microwaves generation and after their readout conversion back to digital** could be implemented in classical adiabatic / thermodynamically reversible fashion.
- **being investigated** at Sandia Labs, Wisconsin University and with SeeQC, with their RSFQ superconducting based logic, microwaves DACs and ADCs.

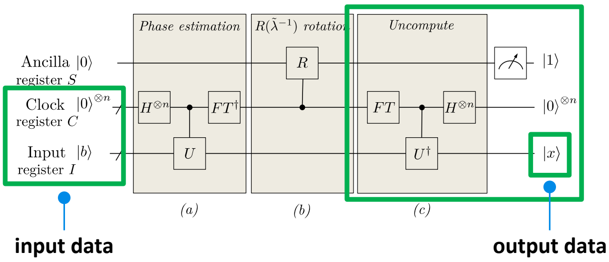


Figure 268: reversibility in quantum computing. Source: Olivier Ezratty, 2021-2022.

One way to achieve this would be to use adiabatic and reversible electronic components working from within the cryostat, but it is not really possible, particularly at the DAC/ADC levels, given these analog/digital pulse signals conversions are seemingly not reversible processes.

[842] Measurement Based Quantum Computing, which relies mainly on measurement during the entire calculation, is irreversible by construction. This is why it is also called 1WQC for one way quantum computing.

Another explored avenue is ABQC for **Asynchronous Ballistic Quantum Computing**, promoted by Michael P. Frank's team at the DoE Sandia Labs in the USA.

They plan to implement it with Josephson junction circuits[843].

Quantum reversible computing can also be used in quantum memory and with the uncompute trick of results that are no longer necessary, such as those sitting in ancilla qubits[844]. However, quantum computing reversibility is not the key to reducing the energy consumption of quantum computing.

Macroscopic energetics of quantum computing

We look here with more details at the "classical" and "macroscopic" power consumption of a quantum computer, taking first the example of a superconducting qubits QPU.

To date, the energy consumption of a quantum computer is relatively reasonable as shown in Figure 269. A current quantum computer with superconducting qubits consumes about 25 kW, of which 16 kW comes from cryogenics. Cold atoms or photons quantum computers consume even less energy, in particular because they do not require cryogenic cooling to 15 mK.

Photon qubits only require photon sources and detectors cooled at around 4K which is less energy hungry at face-value provided it scales well with the number of used qubits.

When thousands of qubits will fit in these machines, their power consumption will increase due to the energetic cost of qubit control for initialization, quantum gates, error correction and qubit readout[845]. Most of qubits energetic costs come from the signals used for gate activations and readout.

These signals are microwave pulses, direct current pulses and laser beams. The related spent power seems to increase linearly with the number of qubits. But error correction requires a large number of physical qubits per logical qubits, adding another power consumption multiplying factor.

It will depend on the fidelity of the physical qubits and the ratio of physical qubits per logical qubits. The higher the fidelity, the lower this ratio will be. On top of that, the cryogenic cost of the qubits grows very fast as the temperature is lower in proportion to the mass to cool.

Let's breakdown the power consumption of a typical quantum computer:

Control electronics power consumption varies greatly from one technology to another and depends on the number of physical qubits managed, which will be counted in millions with large scale quantum computers (FTQC).

Superconducting qubits are driven with microwaves produced outside the cryostat with electronics coming from Zurich Instruments, Qblox, Quantum Machines, Keysight and the likes. Microwave readout is costly in bandwidth, requiring Gbits/s of data streams per qubit.

It is mind blowing to see the complex systems used to just get a 0 or 1 out of a qubit. Pulsed microwaves can be generated by cryo-CMOS chips sitting in the cryostat at the 4K stage. It can help reduce the wiring clutter entering the cryostat but is not necessarily reducing the energetic footprint since it increases the cooling budget requirements. It seems to fit the needs of intermediate systems with only a couple hundred or thousand superconducting or silicon spin qubits. Another option is to use superconducting electronics (SFQ) like the ones developed by SEEQC but it still requires some work to avoid side effects like back-action on the qubits.

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843 See Pathfinding Thermodynamically Reversible Quantum Computation by Karpur Shukla and Michael P. Frank, January 2020 (28 slides) and Asynchronous Ballistic Reversible Computing using Superconducting Elements by Michael P. Frank et al, April 2020 (27 slides).

844 See Putting Qubits to Work - Quantum Memory Management by Yongshan Ding and Fred Chong, July 2020.

845 This is the thesis of Joni Ikonen, Juha Salmilehto and Mikko Mottonen in Energy-Efficient Quantum Computing 2016 (12 pages).

D-Wave quantum annealers chips embed their own superconducting electronics controls but with only DC outputs and inputs and not the more costly microwave pulses used in gate-based models.

Trapped ions qubits control is performed with lasers and conventionally generated electromagnetic pulses. Its cost scales with the number of qubits.

Cold atoms qubits control exploits a couple lasers and a mix of SLM and AOD systems that potentially support a thousand qubits with modest power consumption. But laser power can grow significantly as the qubit number exceeds a couple hundred.

Photon qubits power drain seems more important for photon detection (about 7.5W per qubit) than for photon generation (about 1mW per qubit, source: Quandela). Power consumption also comes from the control electronics driving the nanophotonic circuits (phase controls, …). Superconducting based photon detectors are more demanding with cooling.

Vacuum is generated with superconducting and silicon spin qubits while cold atoms and trapped ions qubits use ultra-high vacuum. Photons do not need it. With superconducting and silicon qubits, vacuum comes from pumps and dilution refrigeration cooling. Cold atoms require only 100W for the ultra-vacuum pump plus about 300W for its cooling at 4K. Trapped ions systems use heating strips covering the vacuum chamber with a process that can take weeks. This is a fixed cost because when vacuum is in place, heating is stopped, and vacuum remains stable during computations. As with cold atoms, ion traps chambers may be cooled at 4K to avoid thermal photons entering the system.

Cryogenics consumes up to 16 kW[846] for existing superconducting and silicon qubits and a little less for other types of qubits due to higher temperatures, such as the 4K of photon sources and detectors used with photon qubits. Cryogenics will be required for cold atoms at the ultra-vacuum pump level but will not significantly scale with the number of injected atoms. These are cooled with laser beams and tweezers and under ultra-high vacuum. The cryogenics consumption is usually continuous, without variations between thermalization and production. Thermalization lasts about 24 hours for dilution refrigerator systems used with superconducting and electron spins qubits. The cryogenic cost has different scale properties depending on the qubit types.

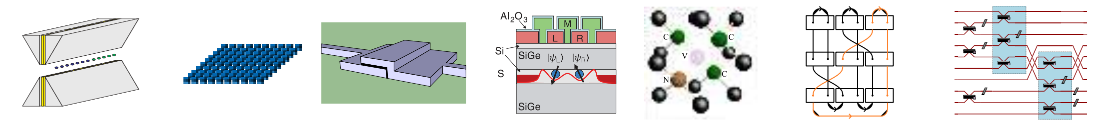
With superconducting qubits, it scales with the number of physical qubits but also depends on the way they are controlled, the nature of control signals (microwave pulses, DC pulses) and how control signals are multiplexed and demultiplexed. One must not forget to account for the cost of water cooling of the cryostat compressors. More powerful cryostats can be created with more pulse tubes and dilutions with a gain of an order of magnitude for the available cooling power. Other optimizations can be implemented to increase the available cooling power at very low temperature with getting closer to the theoretical Carnot efficiency. It seems possible with large cryostats using more stages. There are still significant scale constraints for FTQC QPUs which will require millions of physical qubits.

Computer control is used with all types of qubits. They all require one to three control servers that drive the qubit gates and readout devices by exploiting compiled quantum software, that transforms qubit gates into low-level instructions for qubits initialization, control and readout. These servers are networked, either on premises or in the cloud and via conventional network switches. They represent a limited fixed cost with an estimated consumption of between 300W and 1 kW. Part of the control computing could be moved into the cryostat for superconducting and electron spin qubits, in order to implement autonomous error correction codes. The control computer would then only drive logical qubits and not the physical qubits of the configuration.

[846] This cooling power usually doesn't take into account the cost of cooling the water circulating in the cryostat compressor. It is an estimate for a 50ish superconducting qubit QPU. Larger QPUs like the latest from IBM with hundreds of qubits will have cryostats with a power of over 100 kW.

Error correction conditions the power consumption of a fault-tolerant quantum computer. It is related to the classical electronics and computing resources used for qubit control (AWGs, mixers, DACs), qubit readout (mixers, ADCs, FPGAs), and error syndrome detection and correction. One key parameter is the ratio between the number of physical qubits and logical qubits, which depends on physical qubit fidelities. The higher this one is, the lower the ratio of physical/logical qubits. It also depends on the algorithm size and its target performance. In the FTQC regime, the number of qubits to control will be multiplied and generate high energy consumption. However, error correction codes may run up against another wall: the scale dependence of qubit noise. Namely, qubit gates and readout fidelities may decrease with their number. This may have the consequence of reversing the effect of increasing the number of qubits used in error correction. The error rate of logical qubits gates then increases, instead of decreasing[847]. The same problem arises with surface codes although their overhead seems lower than with concatenated codes. For error correction codes to be effective, the error rate of qubits should be at least ten times lower than their current level. In their work modelling a full-stack energetic cost of a superconducting qubits computer, Fellous-Asiani et al found out that, from the energetic footprint standpoint, it is way more efficient to use room temperature electronics than cryo-CMOS due to the overhead cost of their cooling. This result moves the scalability burden cost on the wiring and its multiplexing. On top of that, control electronics have an energetic bill that is much bigger than the cryogeny used for the electronic components sitting in the cryostat (cables, filters, attenuators, qubit chips, circulators, amplifiers).

Many of these quantum computer components have a variable energy cost depending on the number of qubits, including the cryogenic side. Indeed, the electronics embedded in cryostats release heat in approximate proportion to the number of physical qubits used. This heat must be evacuated within the cryostat. The consumption of control electronics also generally depends on the number of qubits. It seems that, up to a thousand qubits, this control electronics is a fixed cost for cold atoms. Only vacuum creation and the control computer seem to be fixed costs.



	atoms		electron superconducting loops & controlled spin				photons
qubit type	**trapped ions**	**cold atoms**	**supercond.**	**silicon**	**NV centers**	**Majorana**	**photons**
cryogeny	300 W-6 kW	7-10 kW (2)	16-105 kW	12 kW	< 1 kW	16 kW	3 kW
vacuum pumps[1]	ultra-vacuum	ultra-vacuum	vacuum	vacuum	vacuum	vacuum	vacuum
qubits gate controls	<1.4 kW ions heating, lasers, micro-aves generation, CMOS readout electronics	1.8 kW atoms heater, lasers, control (SLM, AOD), readout sensor + electronics	from 20 mW to 100 W / qubit depending on architectures with micro-wave generation outside or inside the cryostat		N/A	<25 mW / qubit	300 W for photons sources and detectors, qubit gates controls
computing	300 W	300 W	<1 kW	<1 kW	<1 kW	<1 kW	700 W
# qubits used	24	100/256 (1) - 300-1000 (2)	53-433	12	<10	N/A	20
total	**2 KW** (5)	**3** (1)-**20 KW** (2)	**25-140 KW** (3)	**21 KW**	**N/A**	**N/A**	**4 KW** (4)

[1] : fixed energetic cost, for preping stage

typical configurations for Pasqal and QuEra (1), neutral atoms with 4K pump/chamber cooling (2), Google Sycamore with 53 qubits, and guestimate for IBM System 2 with its KIDE cryostat(3), Quandela/QuiX (4), AQT (5) rough estimates for others

Figure 269: rough estimations of current quantum computers total power and decomposition. It is too early to extrapolate these numbers to useful QPUs in the FTQC regime where several orders of magnitude more physical qubits will be necessary and their related control electronics and, sometimes, cryogenic resources. (cc) Olivier Ezratty, November 2023.

[847] This is what comes out of [Limitations in quantum computing from resource constraints](#) by Marco Fellous-Asiani, Jing Hao Chai, Robert S. Whitney, Alexia Auffèves and Hui Khoon Ng, PRX Quantum, November 2021 (8 pages).

In the industry vendors scene, it is interesting to observe that the total power consumption of a quantum computer recently starting to become a selling point, although not yet being perceived as being an important one.

For example, AQT (trapped ions) explain that their 20-qubit system can be attached to a regular 220V/110W plug with their <2kW power drain, similar to a kitchen oven. Pasqal is using a similar selling point for its Fresnel quantum simulator although their future generation of quantum simulator with over 300 qubits will require a 4K cryostat to cool the ultra-vacuum pump and vacuum chamber hosting the neutral atoms with a total power footprint of about 12 to 20 kW.

Creating a scalable quantum computer is clearly an optimization problem taking into account many energetic constraints. Qubits systems that operate at cryogenic temperature are constrained by the cryostat cooling power and by the heat released within the cryostat. Superconducting and silicon spins qubits are the most challenging for that respect. Heat is generated by the inbound cable microwave attenuation filters and in the qubit readout related microwave amplifiers.

In addition, the part of microwave generation and readout systems that is integrated in the cryostat have their own thermal footprint. All this must fit into the current thermal budget of the cryostats that is currently limited to 1W at the 4K stage and to about 25 µW at the lower 15 mK stage.

The other way to be less constrained is to run the qubits at higher temperatures. This is what is possible with silicon spin qubits, which only require a temperature between 100mK and 1K instead of 15 mK for superconductors. This increases the thermal budget for the control electronics at the qubit stage.

Some significant engineering is required to optimize a multi-parameter system, at least with superconducting and silicon spin qubits:

1. Physical scalability requires putting as much as possible **qubits control electronics inside the cryostat**… but it is not efficiency energy wise unless control electronics are of the superconducting breed (SFQ). Another option is to find ways to multiplex inbound control signals to remove the cable clutter and, potentially, reduce their thermal dissipation.
2. These electronics thermal dissipation is **constrained** by the available cryostat cooling power.
3. Two paths must be investigated simultaneously: increase the available **cryostat cooling power** and reduce these **electronics thermal footprint** as low as possible.
4. Find an efficient way to **handle digital communication** between the inside and outside of the cryostat. Fiber optics, wireless, multiplexing, up/down signal conversions, whatever!
5. Look at various ways to **reduce qubits power drain**, with optimizing their own quantum thermodynamics, particularly when implementing error correction codes. It can also come from algorithm and compiler designs. Reducing the number of physical qubits per logical qubit is an option pursued for example with bosonic codes who are self-correcting flip errors.
6. With **scale-out solutions** involving connecting several quantum computing processing units with some microwaves and photonic links, look at the energetic footprint of this connectivity, on top of its probable impact on qubit links fidelity.

Another critical aspect to account for is the algorithm computing duration. Current time estimates for useful algorithms either in the NISQ or the FTQC regimes can be very large, in the days-months and even years and decades. Energy consumption is the system power multiplied by the computing time. With large times come large energy consumptions. And if algorithms have prohibitive times, various parallelization solutions will have to be found, which may increase systems power requirements. These solutions can be envisioned with NISQ and variational algorithms where several circuit shots could be run in parallel with similar systems. In the FTQC regime, parallelization seems way less obvious.

Given the history of supercomputers, we can suspect that the acceptable level of power consumption for FTQC quantum computers will be on par or below their classical counterpart, particularly given these will be often deployed in the same datacenters. So, avoid design systems with tens of MW!

Energetic cost of distributed architectures

The temptation is great to create ever larger quantum computers, with giant cryostats in the case of superconducting qubits, like we'll see with IBM and Google's roadmaps. Another approach would be to create distributed architectures of quantum computers linked together by quantum connection based on entangled photons, a choice made by IonQ with their trapped ions qubits, noticeably because it is difficult to scale these qubits beyond a couple dozens.

In theory, this would make it possible to create computing clusters that, seen from the outside, would create a single computer, a bit like a large classical server cluster.

This will be conditioned by the capability of converting qubits states to photons qubits states and by the resulting qubit connectivity between the various quantum processors units of this quantum cluster. But this is not just about "connecting" qubits. Interconnect architectures are about creating fragile entangled states between qubits using the intermediary of photons. These may create some statistical overhead on their own, which must be boiled in for both assessing the real obtained quantum computing scalability and the related energetic footprint.

Use cases energetic assessment

Another longer-term question deserves to be asked: does the potential energetic advantage of quantum computing depend on algorithms and applications? What will happen if and when quantum computing becomes widespread? Are we finally going to create a new source of energy consumption that will be added to existing sources, which are already growing fast in the digital world? What will be its impact? How can it be limited?

At this stage, it is too early to have a clear idea. Answers will largely come from the emergence or not of quantum solutions for volume applications, such as autonomous vehicle routing or personalized health solutions.

Without volume-oriented applications, quantum computers will be dedicated to niche applications equivalent to those of current supercomputers, which are mainly used in fundamental and applied research or for public services like weather forecasts.

On their end, volume applications will only be achievable once the quantum computing scalability will work, and millions of low-noise qubits can be operated. This scalability will probably come from fixing some of energetic consumption issues of quantum computing. And we'll close the loop!

Then, we'll have to look at the externalities of these applications and potential Jevon's effects. Namely, some new solutions will have a given quantum computing energetic cost but may help reduce the environmental footprint in other domains like in transportation. If it is well balanced, that's fine. If, on the other hand, the externalities are not positive, like, say in finance portfolio optimization tasks, you will have to think about it.

Economics

Given we are at the very early stage of the quantum computing era, it is still difficult to assess the economics of this industry, on both the cost and business benefit sides. It is too small to generate economies of scale giving some indications on the cost and price of a regular quantum computer. Still, we can make some projections based on a couple assumptions.

The only "priced" quantum computers on the market today are coming from D-Wave. Their units are priced at about $14M. They have sold only a few of these. Most D-Wave customers are using D-Wave computers sitting on the cloud either with D-Wave itself or with Amazon. Some customers pay in excess of $200K per year to benefit from premium access to these machines.

As far as we know, the other "volume" manufacturer of quantum computers is IBM. They first installed a couple ones in Germany, South Korea, Japan and Canada in their own facilities, to serve these markets through various local research, university and industry partners, plus about 24 QPUs installed in their Poughkeepsie datacenter in New York State. They started to sell and install actual QPUs in third party sites in 2023, like in Cleveland Clinic, Ohio. The price tag is not public but probably sits in the tens of millions of dollars. IonQ also sold QPUs to QuantumBasel and India in 2023 at undisclosed prices. Various QPUs have also been ordered by various public hybrid classical/quantum data centers in Europe and other places.

We can make an economic distinction between **cost** (of R&D, goods and manufacturing), **price** (how much is it sold or rented) and **value** (what value is it bringing to customers, particularly, compared with existing classical computing solutions). Right now, the equation is simple: costs are quite high, prices are high as well when computers are sold on premise (particularly superconducting qubits ones) and value is low at this point and is positioned in the educational and proof-of-concept realms.

A quantum computer cost and price depend on several parameters including its underlying R&D, bill of materials of off-the-shelf and custom-designed components, manufacturing and integration costs, economies of scale, marketing and sales costs, the cost of maintenance and consumables if any, and finally, the manufacturer's profit. The higher the sales volume, the greater the economies of scale. Volumes are currently very low given most quantum computers are just prototypes that are not yet useful for production-grade applications.

At some point, when and if we reach some quantum advantage threshold, useful applications will emerge. It will first target niche b2b and government markets[848]. Then, when applications and innovation ramp-up, we may have a larger number of corporate users. It will justify scaling manufacturing capacities. R&D fixed costs will then be easier to amortize with volume. Cost of goods may also decrease, particularly if technology progress can help get rid of the complicated wirings and electronics that we have today in some of these devices.

Cost could also be better shared thanks to the emerging practice of open hardware in quantum technologies[849].

Let's look one by one at the major hardware components of a quantum computer looking at how it will benefit from economies of scale:

- **Control computer(s)**: these are standard rack-mounted servers as well as the associated networking connection. These are the most generic parts of a quantum computer.
- **Chip**: quantum registers chips are the cornerstone of electron-based quantum computers, such as with superconducting and electron spin qubits. Even if they are manufactured in CMOS or similar technologies, their manufacturing volume is very low. Economies of scale are therefore almost non-existent. You don't need such components with cold atoms and trapped ions qubits. It is replaced by specialized optical components to direct the laser beams controlling the qubit atoms. With NV centers, chips can be cheap to manufacture if done in volume.

[848] Some economists think that quantum computers may offer an economic advantage compared to classical computing even without reaching a computing advantage, thanks to asymmetries in cost structures. This still is conjecture based since these economists didn't really analyze the real possibility of pre-quantum-advantage NISQ computers to bring any usefulness. The proposed model is only based on economies of scale and the effects of competition. See Quantum Economic Advantage by Francesco Bova, Avi Goldfarb and Roger G. Melko, National Bureau of Economic Research, February 2022 (28 pages).

[849] See Open Hardware in Quantum Technology by Nathan Shammah, Irfan Siddiqi, William J. Zeng en al, Septemner 2023 (22 pages).

- **Electronic components**: these are used to create, process, transmit and send quantum gate signals to the qubits. Their technology depends on the type of qubit. These signals are microwaves for superconducting and electron spins qubits, laser-based photons for cold atoms and trapped ions, and some other varieties of electro-magnetic signals otherwise. Standard and expensive laboratory equipment is used for microwave generations such as those from Rohde & Schwarz. More integrated equipment is sold by companies like Zurich Instruments, Qblox, Quantum Machines and Keysight. It is using customized FPGA and rather standard electronic components. When these tools are miniaturized as ASICs running at room temperature or cryo-CMOS running at temperatures below 4K, their small economies of scale make it rather expensive.
- **Cabling**: niobium-titanium superconducting cabling used to feed superconducting and electron spin qubits with microwaves are very expensive, costing about $3K each. And we need about three such cables for each and every qubit. This drives high-costs for manufacturing superconducting and electron-spin based qubits systems. The main companies providing these cables are Coax&Co and Delft Circuits.
- **Cryogenics**: these are standard systems but marketed in low volumes. They can cost up to $1M for superconducting and silicon qubits. Their cost is one to two orders of magnitude lower for the cryogenics of components such as photon qubits. Large cryostats use an enclosed cooled system with many cylindrical layers of protection, a GHS (gas handling system), a compressor (such as those coming from CryoMech and Sumitomo) and another compressor used to cool the water feeding the primary compressor.
- **Consumables**: in quantum computers operating at very low temperatures, there is at least some liquid nitrogen and gaseous helium 3 and 4. The latter two are not consumables and operate in a closed circuit in dry dilution systems. It is still expensive.
- **Casing**: this is about steel, glass and design with some specifics linked to vibrations isolation.

As quantum technologies mature, some cost structures will increase, and others will decrease. Economies of scale will do the rest. Comparisons will also be key. We can also expect that quantum computers will have a price constrained by the most expensive supercomputer prices given the available budgets of their users, mostly public research and large corporation organizations. In practice, many quantum computers will be usable as resources in the cloud and at a relatively moderate cost, although currently much higher than classical HPC cloud resources. This is what IBM, Rigetti, D-Wave, Microsoft and AWS (with third-party machines for the latter two) are already offering. Microsoft and AWS quantum cloud pricing is already quite high. Then, one can wonder about its added value.

Some early estimates of the cost of fault-tolerant quantum computers supporting thousands of logical qubits have been done by companies like Google. If you just scale existing costs with the required physical qubits, you easily land in the billion dollars scale, which is definitively not acceptable and will require various optimizations to say the least.

Some economists are trying to quantify the economic advantage of quantum computers even without them bringing some quantum computing advantage. Their assumptions are often quite sketchy without a good understanding of the underlying use cases[850].

Other economists from the University of Cambridge and Bandung Institute of Technology in Indonesia tried to evaluate the productivity gains or losses generated by quantum computers adoption[851].

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[850] See Quantum Economic Advantage by Francesco Bova, Avi Goldfarb and Roger G. Melko, Management Science, December 2022 (11 pages).

[851] See How to introduce quantum computers without slowing economic growth by Chander Velu and Fathiro H. R. Putra, Nature, August 2023.

Their conclusion is rather negative, similar to what supposedly happened with the advent of classical computing in the 1970s and 1980s with a productivity paradox coming from the initial need to invest in new equipment and know-how. These costs could be higher with quantum computing for various reasons, one being the difficulty to find relevant use cases and the other being the threats generated by quantum computers on cybersecurity.

The experts then propose to demonstrate the value of quantum computers for societal challenges. Why not, but these are not necessarily aligned with the corporation goals whatever they say in their CSR (Corporate Sustainability Responsibility) reports. Then, they advise the creation of a unified and standardized language to identify the right problems for quantum computers and prepare data in a quantum-ready format. At last, they recommend deploying either PQC or QKD cryptography. But they have no idea of the cost structure of quantum computing and how the industry would consolidate around standards and how its size would generate economies of scale.

Quantum uncertainty

Estimating if and when scalable and useful quantum computers will be available is a difficult art and science. The opinion spread between optimists and pessimists is quite large. Some entrepreneurs expect to achieve miracles in less than one decade while some scientists, on the other hand, think that this will never happen. In between, other scientists are moderately optimists and expect the wait to last at least a couple decades. Let's look at these various opinions.

Optimism

Google said it achieved quantum supremacy in October 2019. It was not a true supremacy since their Sycamore setting was doing no programmable computing solving a specific problem. It was found later that, due to the qubit noise in their system, it was relatively easy to emulate it on a classical server cluster. So much for any quantum supremacy or advantage! It was the same with the so-called boson sampling experiments quantum advantages coming from China in 2019 and 2020. These were unprogrammable random photon mixers. Later boson sampling experiments in 2021 and 2022, like withy Xanadu, were programmable, but did not show a practical computing advantage.

As published in their 2020 roadmaps, Google, IBM and Amazon expect to achieve true quantum supremacy relatively quickly and create a quantum computer with 100 logical qubits in less than a decade.

Kenneth Regan thought in 2017 that an industry vendor - probably Google - would claim to have reached quantum supremacy in 2018 and that it would quickly be contradicted by the scientific community[852]. This happened in 2019 and the contradictions came fast. That was quite a good prediction! For the specialists who can dissect their scientific publications, the view is obviously more nuanced, especially concerning the reliability of the qubits they generate. They communicate a lot about their efforts to reduce the noise of qubits to make them more reliable[853].

Alain Aspect does not see any strong scientific obstacle preventing the creation of reliable quantum computers. He believes that uncertainty is mostly a technological and engineering one, but that it will take a few decades to create one reliable advantage-grade quantum computer. However, there is nothing to prevent this process from being accelerated, if it is fueled by good talent and public/private investments. **John Preskill** has the same opinion: it will work, but it will take several decades. **Nicolas Gisin** estimates that the pace to quantum usability is accelerating [854].

[852] In Predictions we didn't make, January 2018.

[853] See The Era of quantum computing is here.Outlook: cloudy by Philipp Ball, in Science, April 2018.

[854] See Quantum computing at the quantum advantage threshold: a down-to-business review by A.K. Fedorov, Nicolas Gisin, Sergei Beloussov et al, March 2022 (55 pages).

Jian-Wei Pan is even more optimistic, forecasting some regular quantum advantage before 2027 [855].

Optimists also include the many hardware quantum computing startups, all with solutions that are expected to work on a large scale within the next five years. They are found in all types of qubits: superconductors (IQM, QCI), electron spins (Quantum Motion, SQC), cold atoms (Pasqal, Atom Computing, QuEra), trapped ions (IonQ, Quantinuum, Universal Quantum) and photons (PsiQuantum which predicts one million qubits in 2030, Quandela, Xanadu).

At last, you have ultra-optimists like **Michio Kaku**, a Japanese physicist turned into a futurist and best-selling author and who cocreated the string field theory seems affected by a variant of the Nobel disease. His 2023 book "Quantum Supremacy", predicts that quantum computing will soon "*solve some of humanity's biggest problems, like global warming, world hunger, and incurable disease*". He definitively lives in the overpromising wonderland.

Pessimism

Pessimism comes from a few researchers, who are not necessarily specialized in the field in which they express themselves. Above all, they are pessimistic about the ability to fix the noise that affects qubits, whatever their type[856].

The first and best-known of these pessimists is the Israeli researcher **Gil Kalai** who believes that we will never be able to create quantum computers with a low error rate[857]. According to him, we cannot create stable quantum computers because of the noise that affects the qubits. This is illustrated in the scale below in Figure 270, which sets the lowest reasonably achievable noise level well above the level required to create a scalable quantum computer.

He is working on the creation of some mathematical model that would prove the impossibility of overriding these errors, even with quantum error correction codes. In 2022, he published another paper to prove his point, with a philosophical approach related to the notion of free will[858].

Another skeptic of quantum computing is **Mikhail Dyakonov** (born in 1940 in the USSR) who works in the Charles Coulomb Laboratory (L2C) of the CNRS and the University of Montpellier in France.

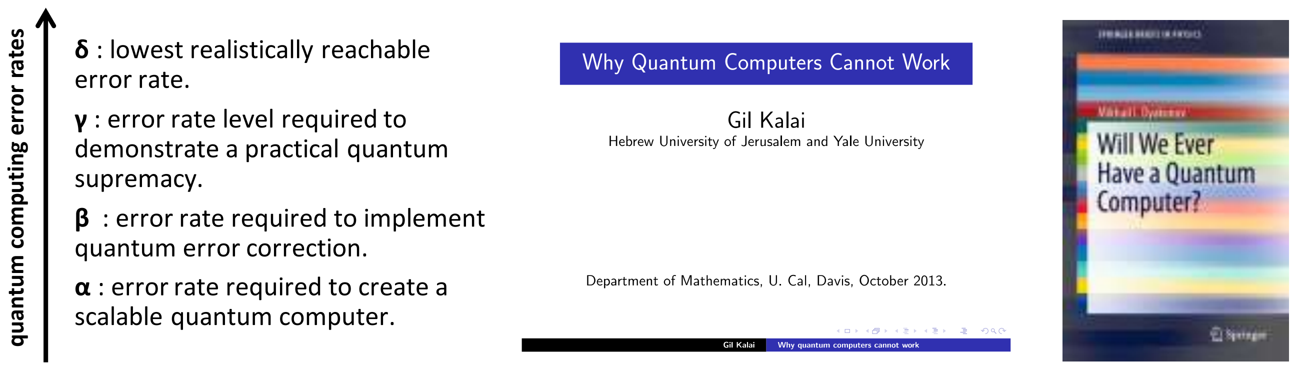

Figure 270: Gil Kalai's quantum computing errors complexity scale.

[855] See Jian-Wei Pan Sees Routine Quantum Advantage Within Five Years by Matt Swayne, The Quantum Insider, February 2022.

[856] See The different forms of quantum computing skepticism by Boaz Barak, 2017.

[857] See Why Quantum Computers Cannot Work, 2013 (60 slides) illustrating How Quantum Computers Fail: Quantum Codes, Correlations in Physical Systems, and Noise Accumulation, 2011 (16 pages) and The Argument Against Quantum Computers by Katia Moskwitch, February 2017. Gil Kalai declares: "*My expectation is that the ultimate triumph of quantum information theory will be in explaining why quantum computers cannot be built*".

[858] See Quantum Computers, Predictability, and Free Will by Gil Kalai, April 2022 (33 pages).

He detailed his views in an article at the end of 2018, which he later turned into a book[859]. His argument is more intuitive but less documented than the work of Gil Kalai[860].

Serge Haroche believes universal quantum computing is an unreachable dream, also because of that damned noise. On the other hand, he thinks that the path of quantum simulation, especially based on cold atoms, is reasonable and realistic.

Xavier Waintal (CEA-IRIG in Grenoble, France) also has serious reservations about the possibility of creating large-scale quantum computers. Here again, the culprit is noise. His reasoning is based on physical explanations different from those of Gil Kalai. Qubit operations rely on very complex n-body quantum problems and error correction codes generally deal with only two types of errors (flip, phase) but not with all sources of error. He recommends exploiting the mean-fields theory which allows to model the complex interactions between qubits and their environment[861]. He published with others various papers (mentioned elsewhere in this book) showing the serious limitations of Grover's algorithm and also (NISQ) VQE and (FTQC) QPE algorithms used in quantum chemical simulations.

These are very fundamental questions to address. His arguments are both the most documented I have seen but which, well used, may fuel interesting research to find solutions. He also exemplify the huge progress made with classical algorithms and architectures to solve the problems envisioned for quantum computers, mainly thanks to the advances with tensor networks (MPS, DMRG), covered page 936 in this book.

Cristian Calude and **Alastair Abbott** point out that the advantage of the main quantum algorithms usable in practice would generate a modest quadratic acceleration (square root of classical computing time) that could be achieved on classical computers with heuristic approaches[862].

Quantum skepticism is also evident in **Ed Sperling's** November 2017 review of the field, which included a reminder of all the obstacles to be overcome[863].

Leonid Levin (a Russian scientist who defined the NP complete complexity class in 1973) and **Oded Goldreich** (an Israeli professor in computer science from the Weizmann Institute) are other quantum computing skeptics mentioned by Scott Aaronson[864].

Another argument against scalable quantum computing deals with the computational state vector amplitudes values becoming tiny as the number of qubits grows. After just applying a set of H gates on N qubits, this amplitude becomes $1/2^N$ for each computational basis state in the qubits register. It becomes quite small as N grows beyond 50. Are these values corresponding to some physical observable that would have a value way below the physical error rate in the system? Or even below some physical Planck constant? Well, this is good food for thought. At least, the computational state vector always has a norm of 1. And the physical observables in the system remain based on the individual qubits basis states $|0\rangle$ and $|1\rangle$.

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859 See The Case Against Quantum Computing, 2018. He even made a short book about it, Will We Ever Have a Quantum Computer?, 2020 (54 pages, free download). As well as a debate on the subject launched by Scott Aaronson in Happy New Year! My response to M. I. Dyakonov. See also Skepticism of Computing by Scott Aaronson who dissects 11 objections on quantum computing capabilities. See also Noise stability, noise sensitivity and the quantum computer puzzle by Gil Kalai, 2018 (1h04mn).

860 See a response to this argument in The Case Against 'The Case Against Quantum Computing' by Ben Crige, January 2019 and a highly documented response from Scott Aaronson in Happy New Year! My response to M. I. Dyakonov, 2013.

861 See What determines the ultimate precision of a quantum computer? by Xavier Waintal, 2017 (6 pages) that we have already mentioned. Xavier Waintal has notably developed classical algorithms for the simulation of N-body problems. They are used by various teams of researchers in condensed matter physics, notably those working on topological matter and Majorana fermions. They run on laptops and supercomputers.

862 In The development of a scientific field by Alastair Abbott and Cristian Calude, June 2016.

863 In Quantum Madness by Ed Sterling, November 2017.

864 In Lecture 14: Skepticism of Quantum Computing.

Managing uncertainty

One key challenge is to make a distinction between scientific unfeasibility, scientific uncertainty and technological uncertainty. This set of uncertainties raises existential questions about how to manage such a long innovation cycle. When should we invest? When are market positions being settled? Is fundamental research decoupled from industrialization? Is quantum computing a simple engineering matter? Or, on the other hand, will it be impossible to control very large swaths of maximally entangled physical qubits?

Note that the pessimists quoted above are not Americans and most of the optimists are. Is there a cultural bias here? These variations in innovation and economic cultures have an impact on industry approaches. Major IT companies such as IBM, Google, Intel, Amazon and Microsoft can fund quantum computing R&D investments with a very long-term vision. They have the profitability, the cash and the ability to attract skills to do so. You may still note that, at this point in time, these large IT vendors have not yet engaged in a startup acquisition spree like they did in the fields of artificial intelligence and other emerging technologies.

> *"For me, the most important application of a quantum computer is disproving the people who said it's impossible. The rest is just icing on the cake."*
> Scott Aaronson
> 2019.

Well-funded startups such as D-Wave, Rigetti, IonQ, PsiQuantum, OQC and IQM can also adopt a fairly long-term view, even if it still depends on their ability to commercialize quantum computer prototypes and to attract long-term oriented investors. The corresponding amounts are not necessarily crazy.

The engineering problems to be solved deal with qubits materials and manufacturing techniques, quantum error correction, control electronics, large-scale cryogenics and of course algorithmic and software advances. The required approach is multidisciplinary with mathematics, fundamental quantum physics, thermodynamics and chemistry, and finally, code, including machine learning which is notably used for qubits calibration.

We can try to extrapolate the evolutions of the last ten years in quantum computing. When he was the co-founder of D-Wave, Geordie Rose enacted in 2003 his own equivalent of Moore's empirical law, Rose's Law, predicting a doubling every year of the number of qubits in a quantum computer, as show in Figure 271 [865].

Since 2007, D-Wave delivered well on this promise, but the progress has been sluggish for many figures of merit of gate-based quantum computers. While the number of qubits has steadily progressed, their quality has not followed on. There is unfortunately no certain equivalent of Moore's law to assess the progress in quantum computing as shown in a 2023 preprint of mine[866].

Most of the charts produced in the late 2010s have not been updated and some even include numbers corresponding to nonoperational systems like Google's 2018 72-qubit Bristlecone or IonQ's 129 qubits which never saw the day of light. You then understand why you must be cautious when interpreting these "exponential charts" with looking at a similar chart created in 2015 that positioned NMR qubits as best-in-class fort their scalability potential[867] (Figure 272). In reality, NMR qubits didn't really scale well.

Some exponential law can however be observed in the evolution of other operating parameters of quantum computers such as the stability time of qubits, their error rate and the number of consecutive

[865] See Quantum computing Rose's Law is Moore's Law on steroids by Matthew Griffin, Fanatical Futurist, August 2016.

[866] See Is there a Moore's law for quantum computing? by Olivier Ezratty, March 2023 (34 pages).

[867] See Recent advances in nuclear magnetic resonance quantum information processing by Ben Criger, Gina Passante, Daniel Park and Raymond Laflamme, The Royal Society Publishing, 2015 (16 pages).

operations performed reliably. **Rob Schoelkopf** from Yale University created his own law showcasing the progress with superconducting qubits coherence times and gates fidelities and times.

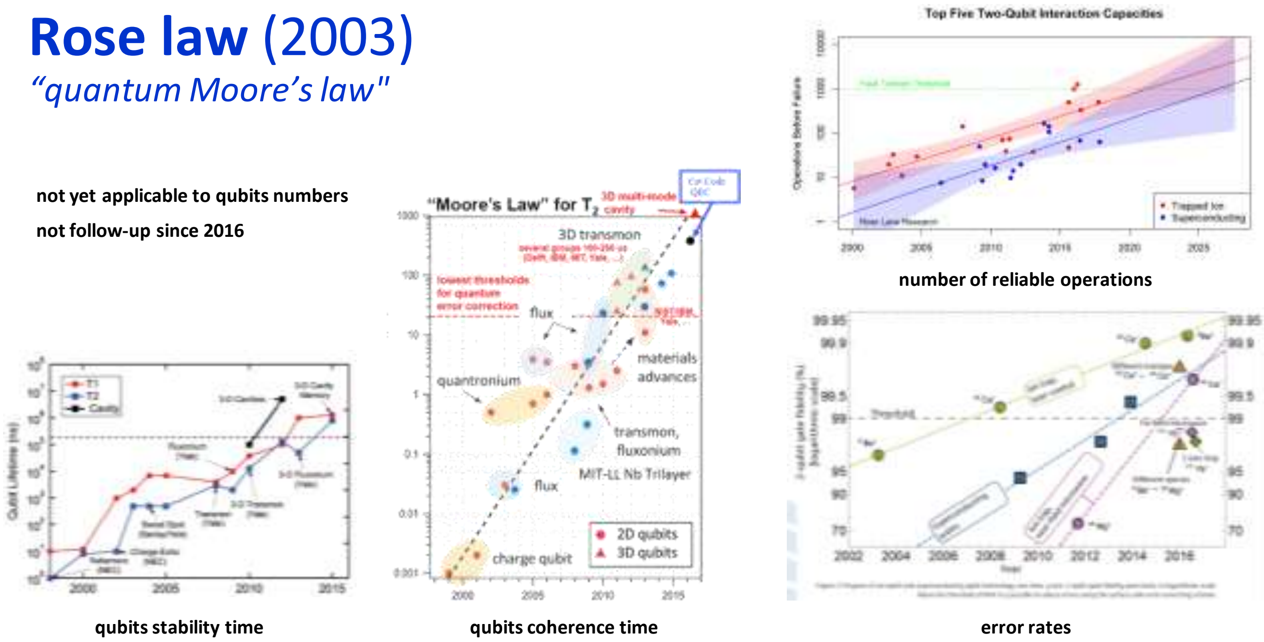

Figure 271: a compilation of the putative equivalents of Moore’s law in quantum computing. They all need updates! Otherwise, you can’t prove there’s a real ongoing acceleration of progress in quantum computing science and technology. Sources: Technical Roadmap for Fault-Tolerant Quantum Computing, a UK report published in October 2016 for the lower-right chart. The first one on the left is from Michel Devoret.

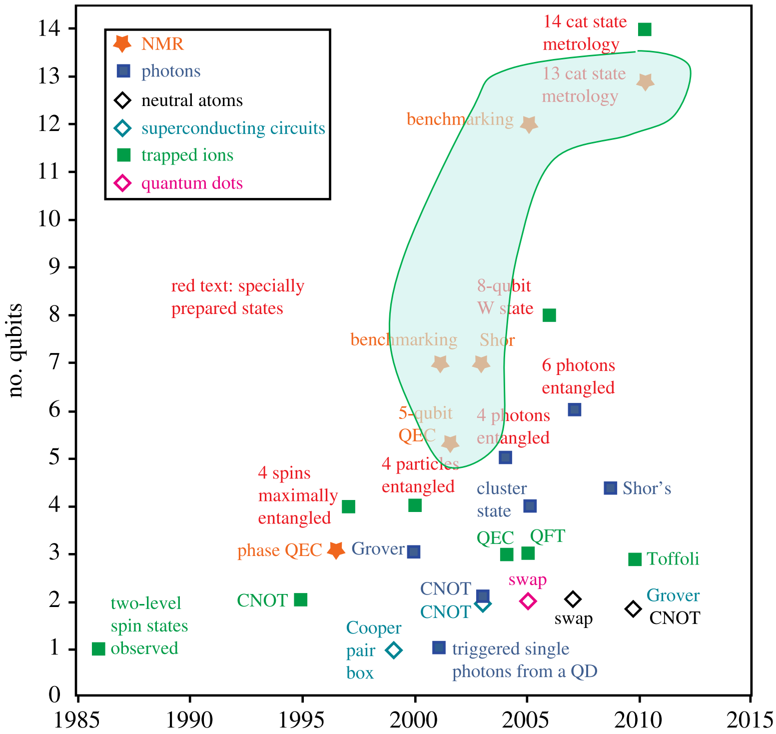

Figure 272: a chart with number of qubits per technology and year, as of 2015. It gave the impression, back then, that NMR qubits were the most scalable. They are not! Source: Recent advances in nuclear magnetic resonance quantum information processing by Ben Criger, Gina Passante, Daniel Park and Raymond Laflamme, The Royal Society Publishing, 2015 (16 pages).

I tried to understand the reasons why the predictions of creating viable quantum computers were always quite long-term, between 5 and 50 years. The first one is that the scientific uncertainty is very high in quantum computing. We have no idea about the size of controllable entangled sets of quantum objects. It is the mother of any functional large scale quantum computer.

There are tons of scientific and engineering challenges ahead, like creating quantum memories and quantum interconnect solutions.

Another reason comes from the length of cycles in the associated research and manufacturing processes. For example, creating a prototype silicon-based qubit chip takes up to nine months of manufacturing with up to 160 manufacturing steps. After this manufacturing process, component characterization and packaging stage add more time. Components characterization qualitatively tests and selects the manufactured components.

This can last up to a month and in the best case down to a week. Then, to carry out the tests, the thermalization of the computer takes about 24 hours and the change of the chip to be tested takes at least 3 to 7 hours as we'll discovered in the section on cryogenics.

The design to manufacturing whole cycle lasts about 2.5 years. Finally, the *test & learn* cycles are often very long, much longer than with software! This long cycle may be shorter with other solid-state qubits like supercomputing qubits, and also with photon qubits for which semiconducting photonic circuits are also required.

Challenges ahead

Whether you are an optimist or a pessimist with regards to the advent of scalable quantum computers, you need to adopt an educated view of the challenges ahead. Over time, as my understanding of these challenges grew, I tended to shift from "optimism" to "neutralism" or, at least, to being a "documented optimist". Some of the challenges ahead are indeed enormous.

Xavier Waintal uses the scale in Figure 273, with 5 difficulty levels, to build a quantum computing machine. It positions where we are right now and the challenges ahead. It goes beyond large scale computing given some quantum memory would be mandatory for some key algorithms like QML and HHL.

difficulty scale	technology	use cases	examples
1	quantum simulator (analog-no gates)	quantum simulations	D-Wave, Pasqal
2	gates-based analog systems, low fidelity	system validation NISQ algorithms	Google, IBM, Rigetti, …
3	gates-based analog systems, low fidelity	variational calculations in quantum chemistry	Possibly PsiQuantum
4	ideal quasi-deterministic gates-based systems (FTQC/LSQ)	factoring large numbers, exact quantum chemistry	TBD
5	4 + quantum memory	quantum machine learning, linear algebra (HHL)	TBD

Figure 273: Source: Xavier Waintal. 2021.

Figure 274 goes into some details with laying out some of these challenges, most of which being covered extensively in various parts of this document.

In this book, every tough challenge is already addressed by many researchers and usually with many competing approaches and low TRLs (technology readiness level). In such cases, I frequently make an inventory of these variations in the book, but without the ability to rank them properly.

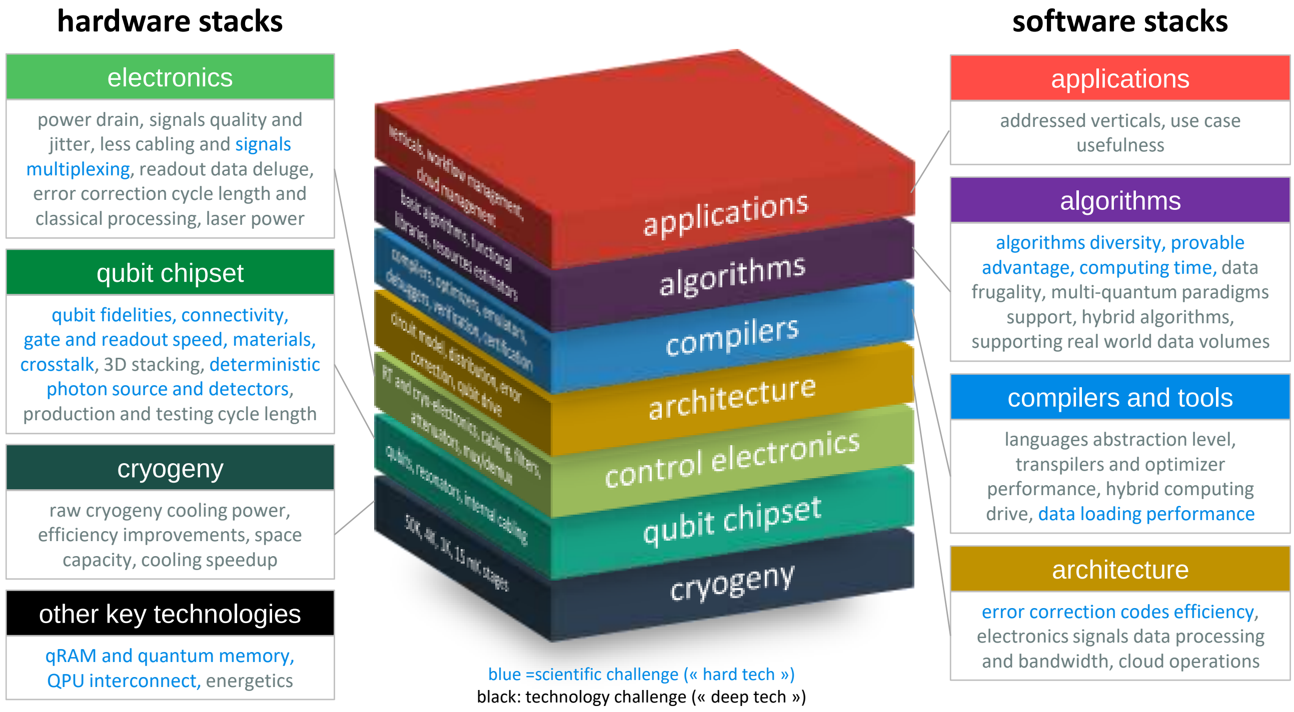

Figure 274: full stack software and hardware challenges for QPU scalability. (cc) Olivier Ezratty, 2023.

Two things come to mind: one is that quantum computers scalability is the most challenging issue to tackle with. If quantum computing capacity is known to theoretically scale exponentially with the number of qubits, you may wonder whether the scale challenge itself is also an exponential one.

One way to grasp it is to look at IBM and Google's progress with their superconducting qubits. It has been sluggish since 2019 with 72/433 qubits, given that most benchmarks show that only fewer than 20 qubits are practically usable due to a small available quantum volume[868]. There is still some hope with bosonic codes and cat-qubits which could limit the logical/physical scaling overhead. Also, scale-out options with qubits interconnect options using microwaves or photons are interesting but have their own scalability challenges. Other qubit types like electron spins and photons also look promising.

The second challenge deals with real algorithms speedups. Not all algorithms showcase an exponential theoretical speedup. Grover's algorithm speedup is only polynomial. All non-exponential speedups may end up being useless due to their implementation cost. The trick of the trade is that all speedups are theoretical but not yet practical. Another way to look at this is to envision, even with moderate algorithms speedups, an energetic advantage for quantum computing, as discussed in the related part that we just saw, starting on page **Error! Bookmark not defined.**.

These speedups are rarely documented with taking into account all the quantum computing food chain: data preparation, oracle operations, quantum memory access when it is required, quantum error correction, non-Clifford group quantum gates generation (particularly for all algorithms using a quantum Fourier transform, and there are many) and the number of shots/runs required (with or without quantum error corrections).

I wish somebody did produce such evaluations with actual and projected data on these different aspects of gate-based quantum computing, even if it brings bad news! When bad news travel fast, fixes arrive faster, if there are any! And this is valid for the dominant wave of NISQ hybrid algorithms.

[868] See one example here from Google with experiments on Sycamore stopping at 20 qubits: Efficient and Noise Resilient Measurements for Quantum Chemistry on Near-Term Quantum Computers by William J. Huggins et al, Google AI, June 2020 (17 pages).

In the end, it looks like analog quantum simulators may be one very viable short-term option, but we also still lack data to prove it. Some algorithms are being evaluated to run on these quantum simulators, like the ones from Pasqal. Quantum annealing and photonic coherent Ising machines could also bring their share of hope. The debate is still out to assess what is the quantum advantage of D-Wave with its latest annealer generation, relying on their 5,000 qubits Pegasus chip.

We can still count on two things to reach quantum computing scalability and practicality. One is the great diversity of paths chosen by scientists and entrepreneurs. This creates a sort of fault-tolerance for innovation. The second, more generally, is we can believe and bet on scientists and engineers' creativity to solve these highly complicated problems. It is still a very generic commonplace reasoning that has not much value per se.

Quantum computing engineering key takeaways

- A quantum computer is based on physical qubits of different nature, the main ones being superconducting qubits, electron spin qubits, NV centers, cold atoms, trapped ions and photons. They all have pros and cons, and no one is perfect at this stage. Future systems may combine several of these technologies.
- Many key parameters are required to create a functional quantum computer. It must rely on two-levels quantum objects (qubits). These must be initializable and manipulable with a set of universal gates enabling the implementation of any linear transformation of qubit states. Qubits must be measurable at the end of algorithms. Their coherence time must allow the execution of a sufficient number of quantum gates. Decoherence and errors must be as low as possible.
- Most quantum computers are composed of several parts: the qubit circuit (for solid-state qubits but also for trapped ions), vacuum enclosures (particularly with cold atoms and trapped ions) or waveguides (photons), usually housed in a cryogenic vacuum chamber (with the exceptions of photons and some NV centers), some electronics sending laser beams or coaxial cables guided microwaves or direct currents onto qubits and a classical computer driving these electronic components.
- Since qubits are noisy, scientists have devised quantum error correcting (during computing) and quantum error mitigation (after computing) schemes. Quantum error correction relies on creating logical "corrected" qubits composed of a lot of physical qubits, up to 10,000. The number of physical qubits per logical qubit depends on many parameters: the algorithm size and its error rates requirements, the quantum error correction code type and the qubits connectivity. This creates huge scalability challenges, many of them with classical enabling technologies like cabling, electronics and cryogeny. The science of quantum error correction, quantum error mitigation and fault-tolerant quantum computing is a realm in itself.
- Many quantum algorithms also require some form of quantum memory, either for data preparation and loading (such as with quantum machine learning) or to access efficiently classical data (such as with oracle based algorithms like a Grover search). These quantum memories don't exist yet and are at a very early research stage.
- The energetic cost of quantum computing is both a potential benefit but also an immense challenge, particularly when a large number of physical qubits are required to create large scale fault-tolerant computers. All components must be carefully designed to take into account the cryogenic cooling power, control electronics, cabling as well as the available space to house cabling and cryo-electronics. This explains the creation of the Quantum Energy Initiative in 2022, which created a community of researchers and industry vendors and organizations working collectively on this topic.
- The economics of quantum computers are still uncertain due to their immaturity and the current low manufacturing volume. Uncertainty is also strong with regards to the feasibility of scalable quantum computers. The scalability challenges ahead are enormous. One of them is to benefit from actual algorithm speedups when including all end-to-end computing operations.

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The remainder of “Understanding Quantum Technologies” is in Volume 2 and Volume 3.

Volume 2 contains the following parts: quantum computing hardware, enabling technologies, unconventional computing, quantum telecommunications and cryptography and quantum sensing

Volume 3 contains the following parts: algorithms, software tools, case studies, quantum technologies around the world, corporate adoption, quantum technologies and society, and quantum fake sciences, plus a glossary, table of figures and index for both documents using a continuous pagination numbering.

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