Q2B | 2024 Paris

the road to quantum computing scalability, from QEC to FTQC

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Q2B Paris, March 8th, 2024
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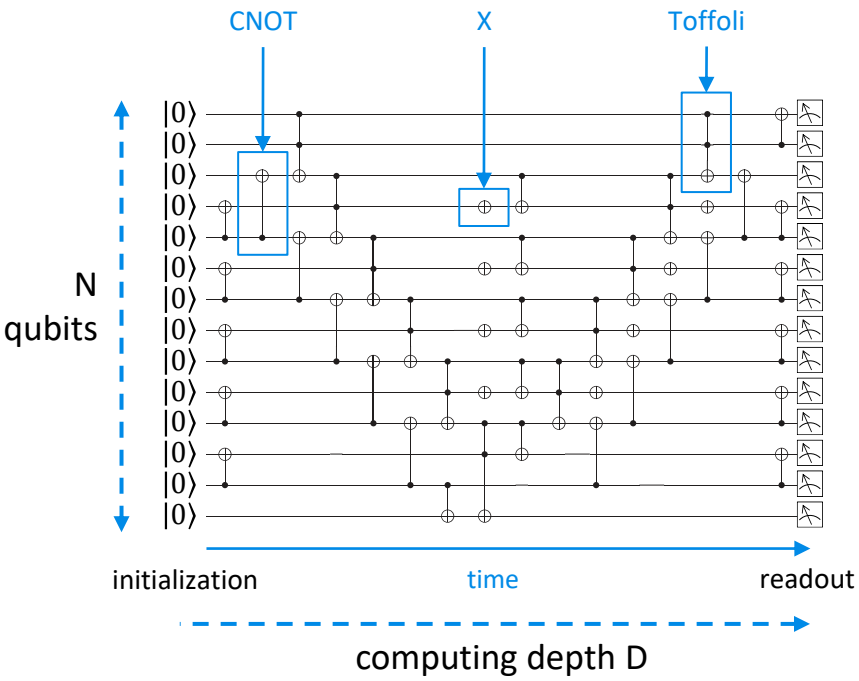
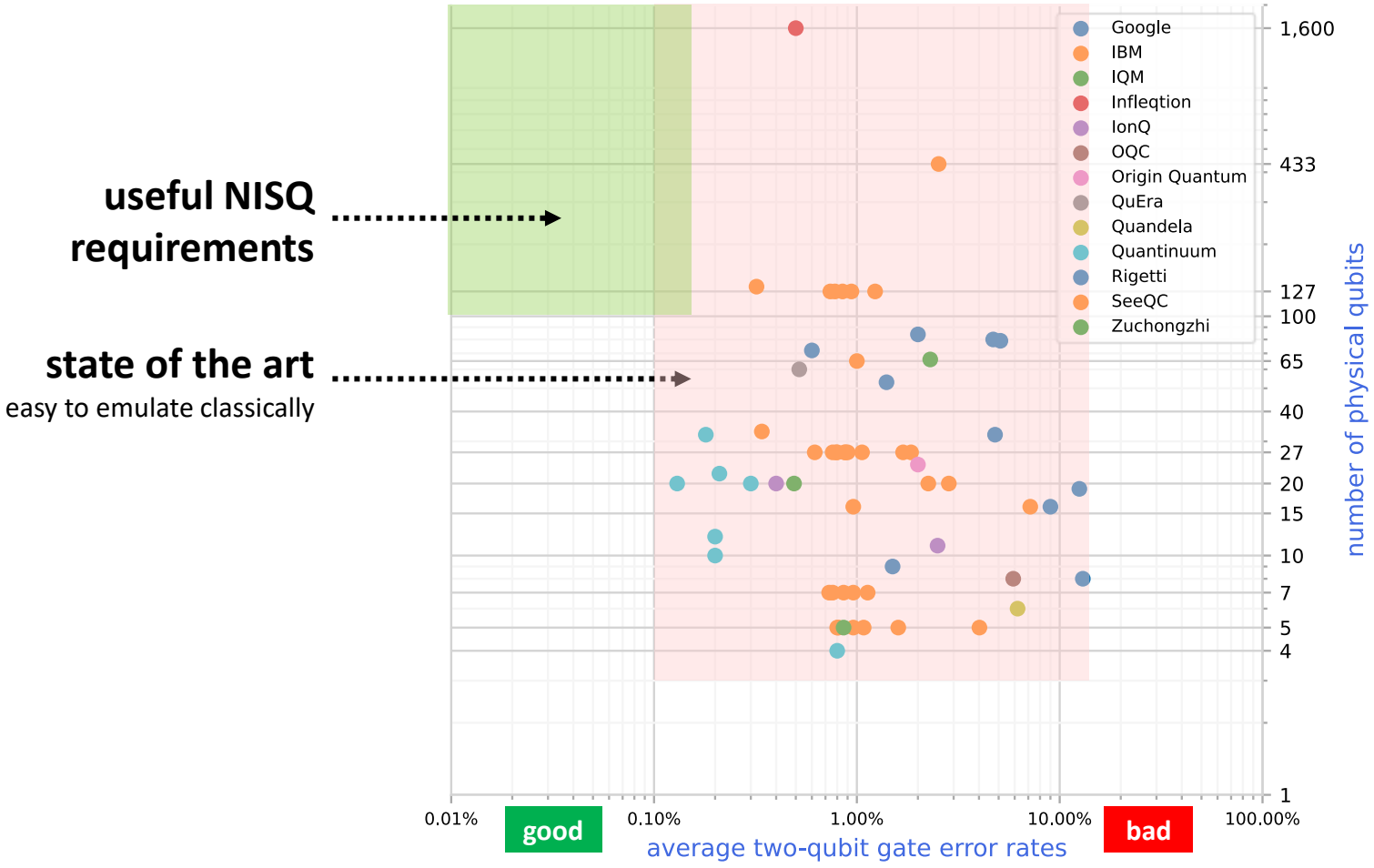
raw algorithm fidelities requirements



$$\text{desired error rate} < \frac{1}{N \times D}$$

N qubits	D depth	required error rate (%)	required fidelity (%)	available fidelity (%)
50	100	0.02000%	99.98%	**99.30%**
133	300	0.00251%	99.9975%	**99.6%**
433	1000	0.00023%	99.9998%	**98%**
1121	2000	0.00004%	99.99996%	N/A

but… QEC cost discrepancy between Clifford and non-Clifford gates


useful NISQ requirements
state of the art
easy to emulate classically
Google
IBM
IQM
Infleqtion
IonQ
OQC
Origin Quantum
QuEra
Quandela
Quantinuum
Rigetti
SeeQC
Zuchongzhi
1,600
433
127
100
65
40
27
20
15
10
7
5
4
1
number of physical qubits
0.01%
0.10%
1.00%
10.00%
100.00%
good
bad
average two-qubit gate error rates

qubit errors sources

where
physical sources or errors

control

signals jitter

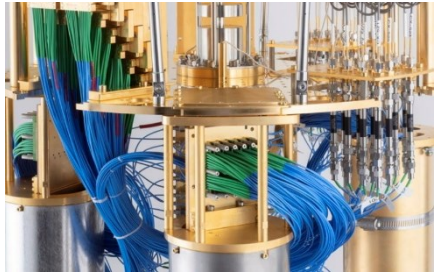
calibration

many body interactions

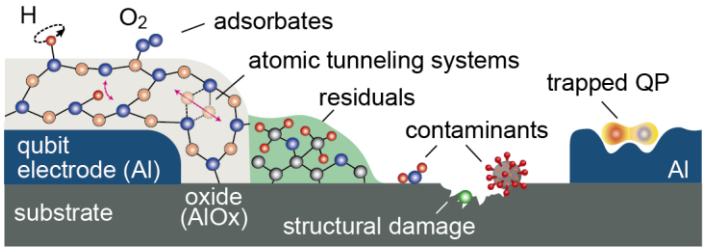
thermal noise

back-action

electromagnetic noise

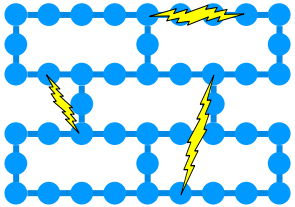


material defects

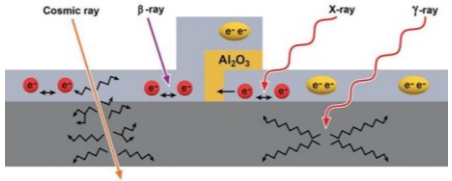


crosstalk

photon loss

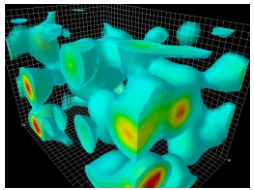


cosmic rays

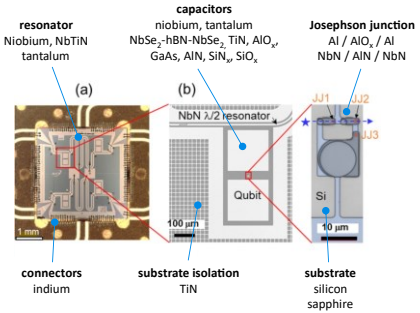


gravity

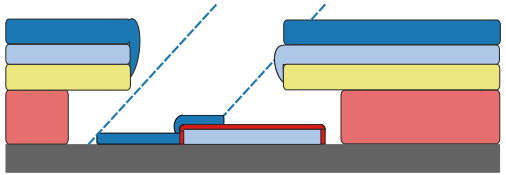
vacuum quantum fluctuations



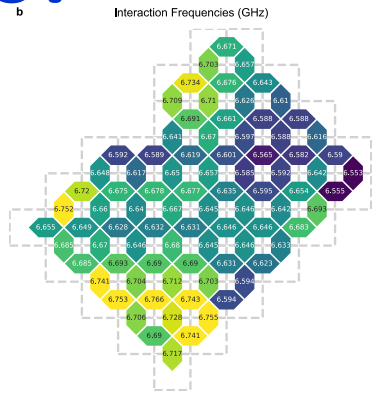
how to improve qubit fidelities? *



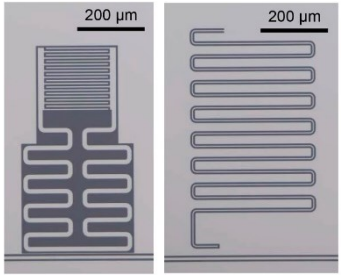
materials



manufacturing



reduce crosstalk



tune qubit parameters
bosonic qubits

Cross-Cross Resonance Gate

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[2] *Research Center for Advanced Science and Technology (RCAST), The University of Tokyo, Meguro-ku, Tokyo 153-8904, Japan*

High-fidelity three-qubit *i*Toffoli gate for fixed-frequency superconducting qubits

Yosep Kim,[1,*] Alexis Morvan,[1] Long B. Nguyen,[1] Ravi K. Naik,[1,2] Christian Jünger,[1]
Larry Chen,[2] John Mark Kreikebaum,[2,3] David I. Santiago,[1,2] and Irfan Siddiqi[1,2,3]

[1] *Computational Research Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
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[3] *Materials Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*
(Dated: December 21 2022)

use different primary gates

improve control signals quality

* using here the example of superconducting qubits

John Preskill @ Q2B SV 2023

What we have now. NISQ is valuable for scientific exploration. But there is no proposed application of NISQ computing with *commercial* value for which quantum advantage has been demonstrated *when compared to the best classical hardware running the best algorithms for solving the same problems*.

What we can reasonably foresee. Nor are there persuasive theoretical arguments indicating that commercially viable applications will be found that do *not* use quantum error-correcting codes and fault-tolerant quantum computing.

logical qubits and FTQC

physical qubit

error rates ≈0.1%

+

error correction code

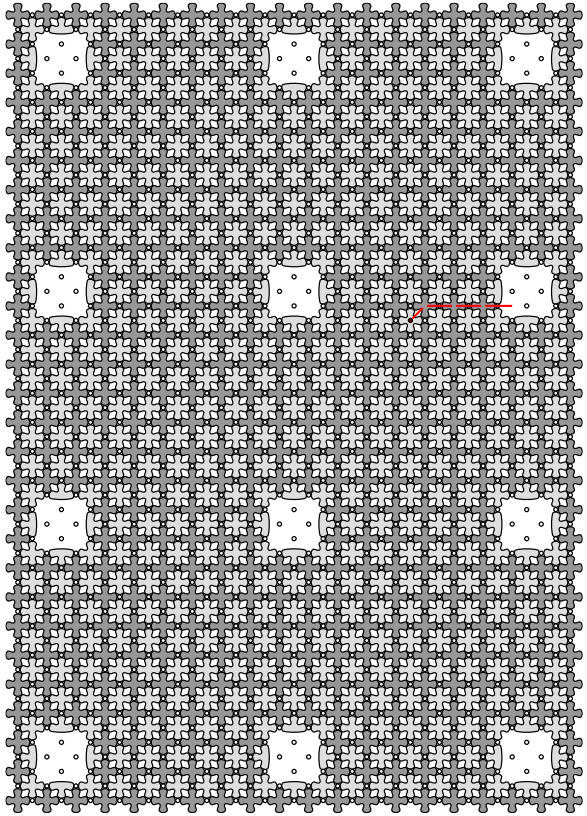
threshold, physical qubits overhead, connectivity requirements, syndrome decoding and scale

logical qubit

error rate $<10^{-8}$ to $<10^{-15}$

fault tolerance

avoid error propagation and amplification
implement a universal gate set
fault-tolerant results readout



tens to thousands qubits

https://arxiv.org/abs/1202.2639

threshold and break-even

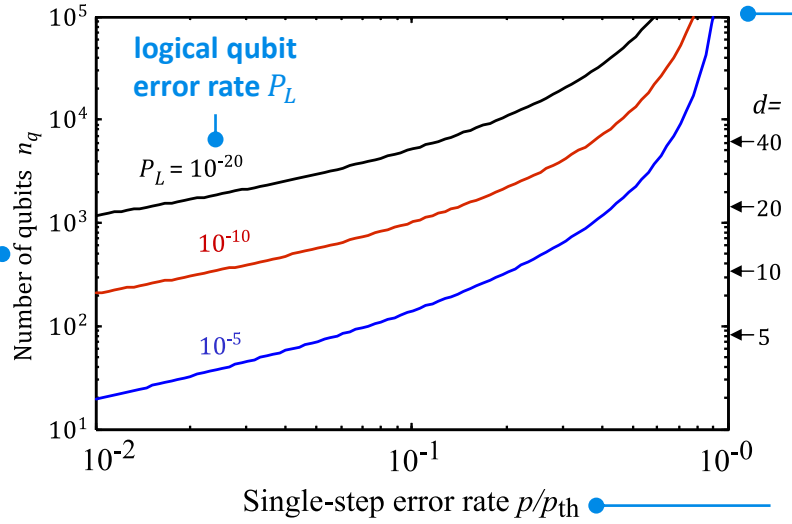
$P_L \approx 0.03\,(p/p_{th})^{d_e}$

$d_e = d/2$ if d is even
$d_e = (d+1)/2$ if d is odd

required number of physical qubits is infinite when $p = p_{th}$

number n_q of physical qubits per logical qubit

$n_q = 2d^2 - 1$



surface code distance d

$$d = \sqrt{\frac{1+n_q}{2}}$$

ratio between physical qubit error rate p and surface code error threshold p_{th} (about 1%)

break-even

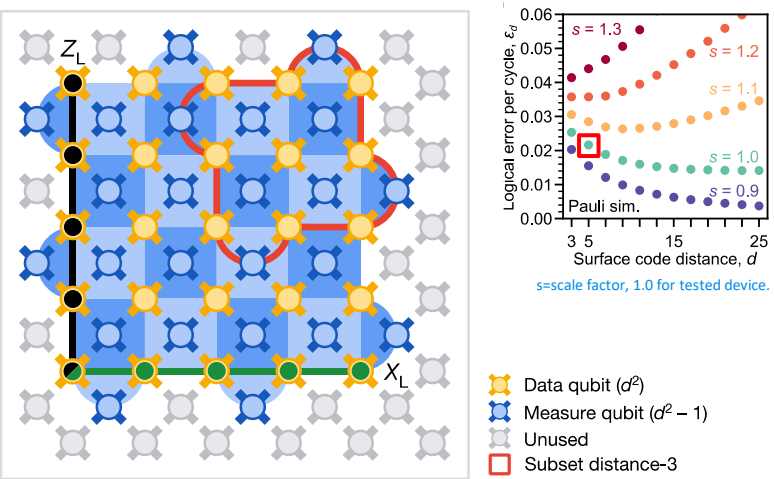
logical qubit error rate lower than physical qubit error rate. **$p_L < p$**, depends on p_{th}, p and d

threshold

physical qubit fidelity needed to create logical qubits with better fidelities. at threshold, # of required physical qubits for surface codes is infinite! practically needs **$p/p_{th} < 10\%$**

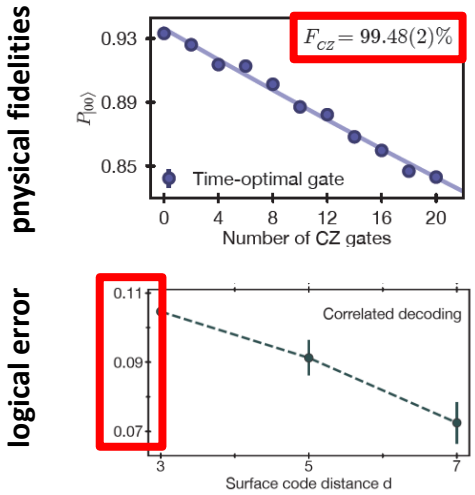
source: An introduction to the surface code by Andrew N. Cleland, 2022 (68 pages)

existing logical qubits *above* break-even



Sycamore 72-qubit processor
single distance-5 logical qubit – July 2022

it slightly outperforms a distance-3 logical qubit but with providing a higher error rate (2.1%) than the underlying physical qubits (0.7%).



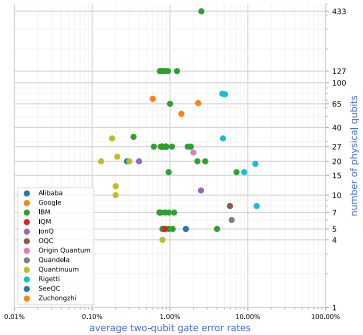
Harvard-MIT-QuEra 48 logical qubits – December 2023

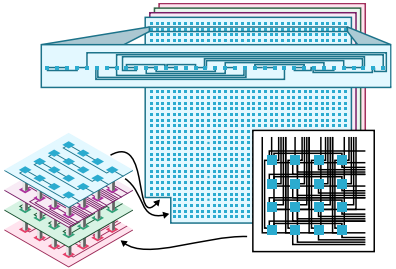
logical 2-qubit gate error rates at 7% with distance-7 surface code while physical qubit error rate is 0.5%.

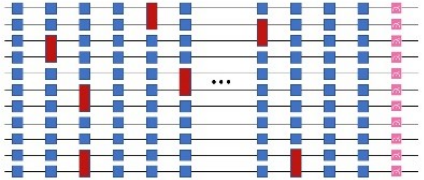
sources: Suppressing quantum errors by scaling a surface code logical qubit by Rajeev Acharya et al, Google AI, Nature, July 2022-February 2023 (44 pages) and Logical quantum processor based on reconfigurable atom arrays by Dolev Bluvstein, Mikhail D. Lukin et al, December 2023 (32 pages)..

qubits for QEC-FTQC?

>99.9%

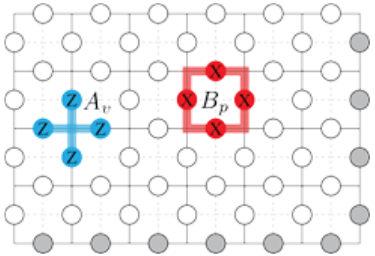






algorithm breadth and depth

n_T = # of T gates in algorithm



physical qubits fidelities and threshold

physical qubits connectivity

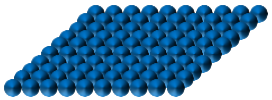
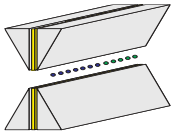
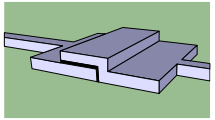
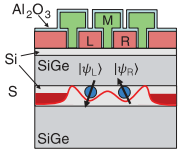
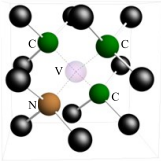
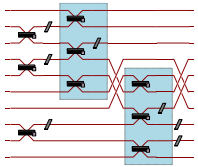
logical qubit error rate $< \frac{1}{n_T}$

error correction code

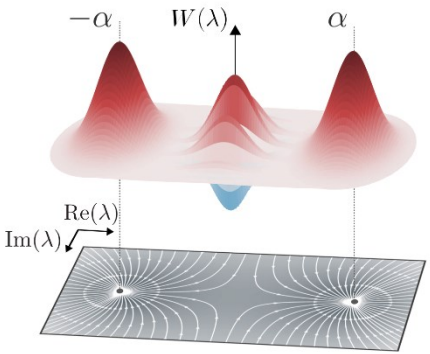
 # physical qubits / logical qubit

dynamically adjusted against the algorithm size

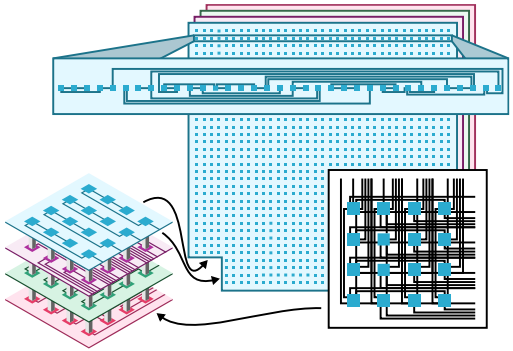
favored QEC codes per qubit types

	atoms		electrons superconducting & spins			photons
						
	cold atoms	**trapped ions**	**superconducting**	**silicon**	**NV centers**	**photons**
surface codes	Yes		Yes	Yes		Yes
color codes	Yes			Yes		
QLDPC	Yes	Yes	Yes			
Stabilizers					Yes	
Bacon-Shor		Yes				
Steane		Yes				
Bosonic codes			Yes			Yes (GKP)
MBQC based						Yes

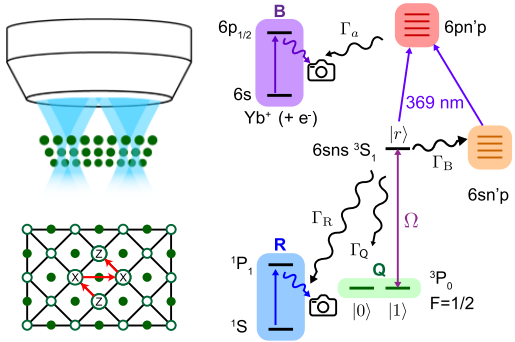
some advanced QEC enabling techniques



biased noise with bosonic qubits
qubits self-correct bit flip errors and QEC is involved to correct phase flips. Flip-error reduction is exponential when phase-error cost is linear [Alice&Bob, AWS].

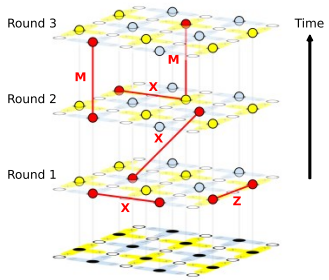


qLDPC codes
constant depth, lower overhead, but nonlocal syndrome measurements required requiring n-to-n qubit connectivity or movable qubits [IBM, Alice&Bob, QuEra, Quantinuum, …].

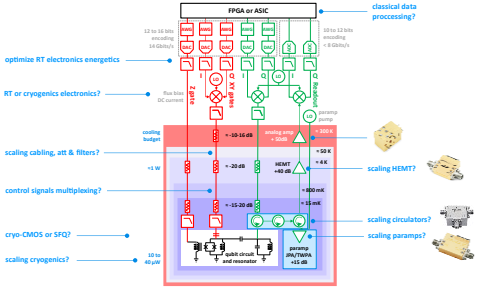


erasure conversion
dominant errors occur at known locations are easier to detect and correct. E.g. with dual-rails superconducting qubits and some cold-atoms setups.

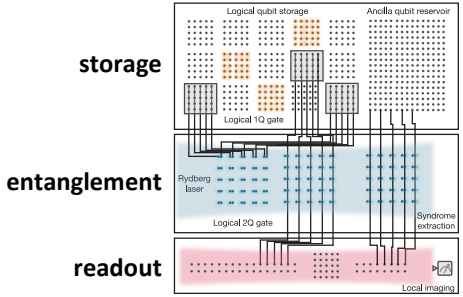
some QEC/FTQC challenges



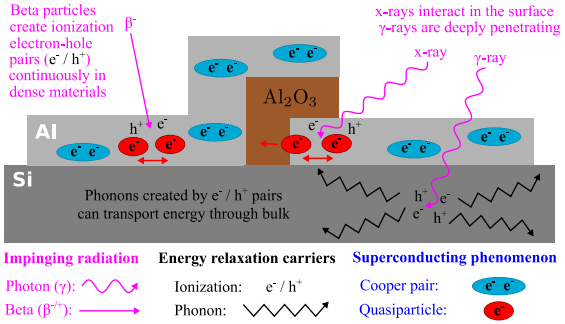
syndrome decoding
speed and scale



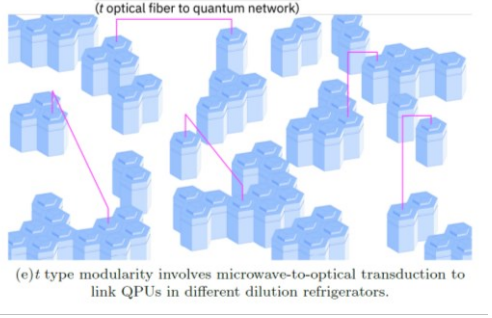
enabling technologies scalability
and energy consumption



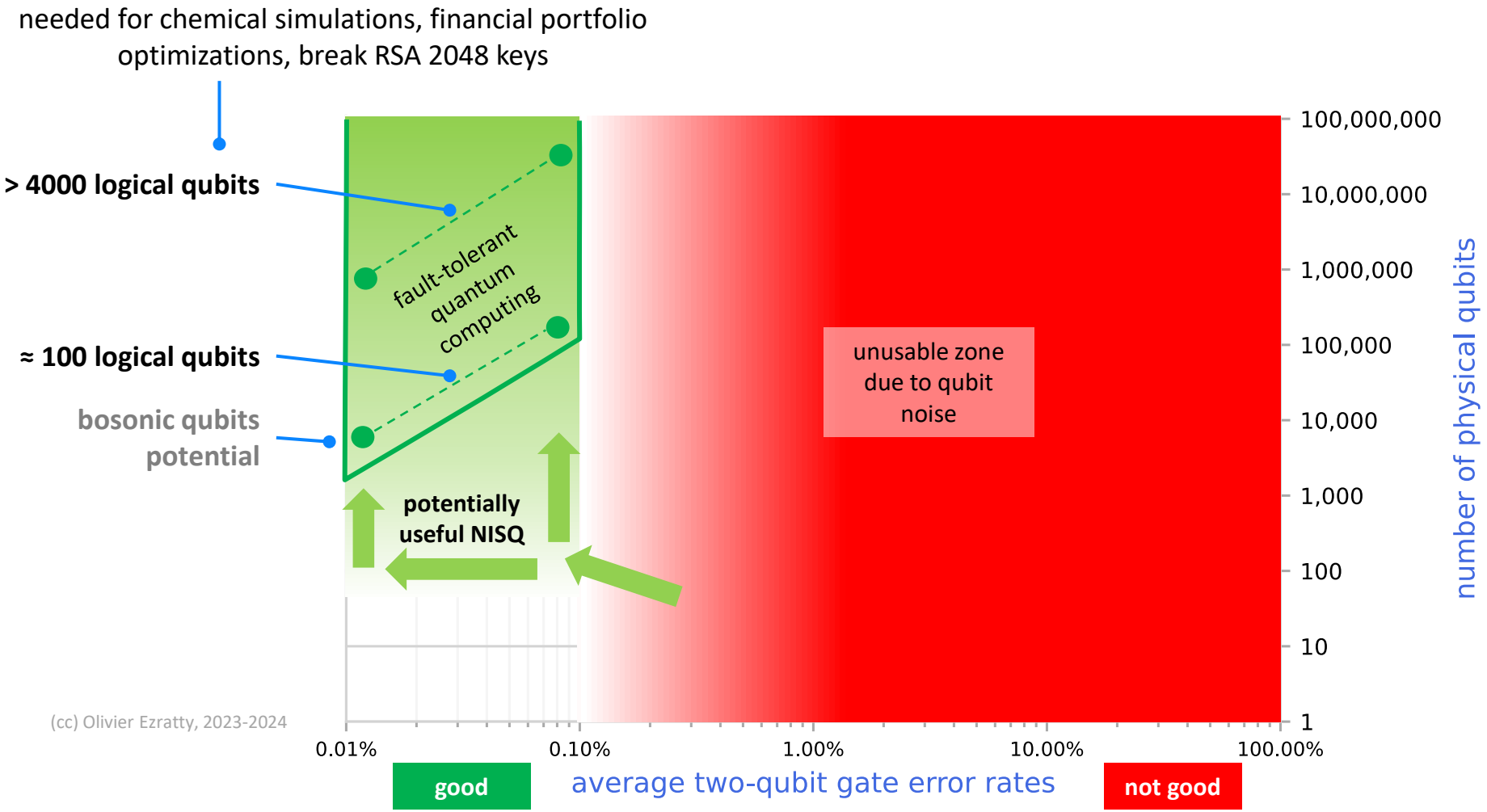
long distance coupling
speed-losses trade-offs



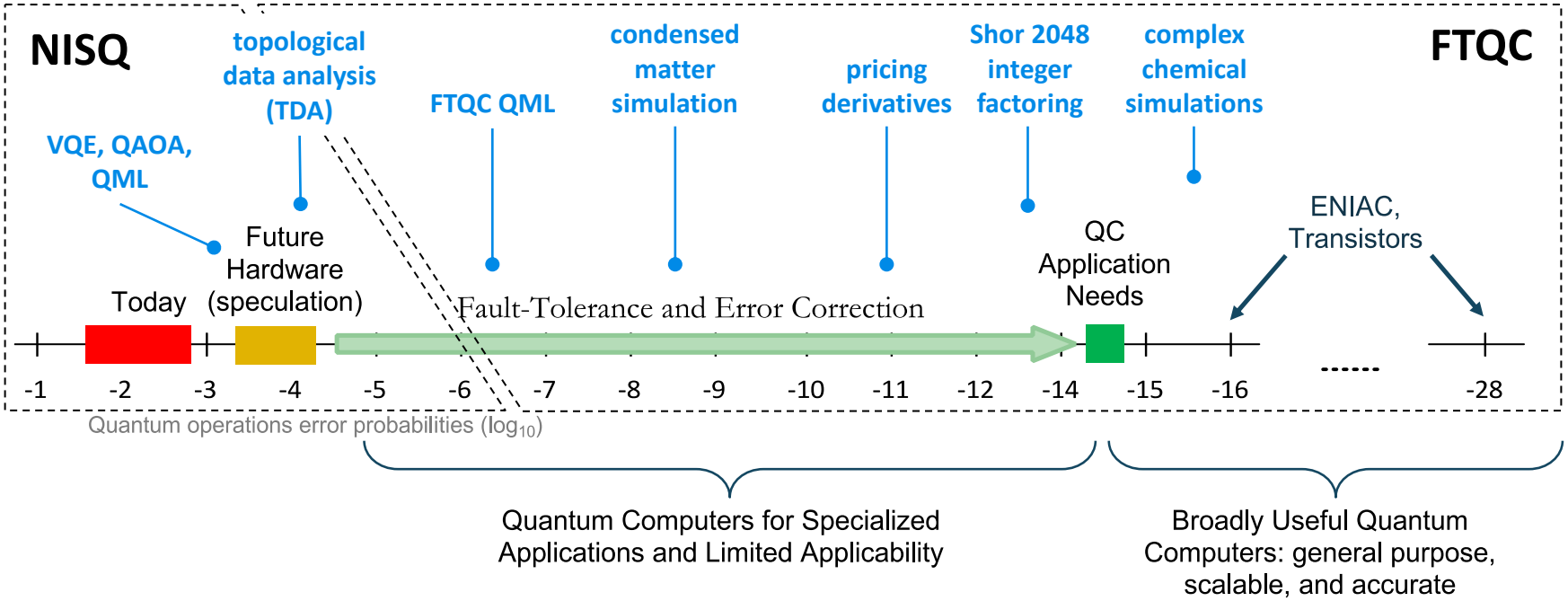
uncorrelated and crosstalk errors



scaling QEC and FTQC across QPU interconnects


needed for chemical simulations, financial portfolio optimizations, break RSA 2048 keys
> 4000 logical qubits
≈ 100 logical qubits
bosonic qubits potential
fault-tolerant quantum computing
potentially useful NISQ
unusable zone due to qubit noise
100,000,000
10,000,000
1,000,000
100,000
10,000
1,000
100
10
1
number of physical qubits
0.01%
0.10%
1.00%
10.00%
100.00%
good
average two-qubit gate error rates
not good
(cc) Olivier Ezratty, 2023-2024

logical qubits requirements



source: How about quantum computing? by Bert de Jong, DoE Berkeley Labs, June 2019 (47 slides) + Olivier Ezratty additions, 2021-2024.

compute FeMoCo… energy ground state

needs…

2,142 logical qubits

4M physical qubits

4 days computing time

“simulating the ground state of active-space models of FeMoCo.”

PRX QUANTUM **2,** 030305 (2021)

Even More Efficient Quantum Computations of Chemistry Through Tensor Hypercontraction

Joonho Lee,[1,*,§] Dominic W. Berry,[2,†,§] Craig Gidney,[3] William J. Huggins,[3] Jarrod R. McClean,[3] Nathan Wiebe,[4,5] and Ryan Babbush[3,‡]

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(Received 12 December 2020; revised 7 April 2021; accepted 24 May 2021; published 8 July 2021)

	Reiher *et al.* FeMoCo [23]		Li *et al.* FeMoCo [36]	
Algorithm	Logical qubits	Toffoli count	Logical qubits	Toffoli count
Reiher *et al.* [23] (Trotter)	111	5.0×10^{13}	—	—
Campbell and Kivlichan *et al.* [52,53] (qDRIFT) (D16), (D17)	288	5.2×10^{27}	328	1.8×10^{28}
qDRIFT with 95% confidence interval (D34)	270	1.9×10^{16}	310	1.0×10^{16}
Berry *et al.* [9] (single factorization) (B16), (B17)	3,320	9.5×10^{10}	3,628	1.2×10^{11}
Berry *et al.* [9] (sparse) (A17), (A18)	2,190	8.8×10^{10}	2,489	4.4×10^{10}
von Burg *et al.* [10] (double factorization) (C39), (C40)	3,725	1.0×10^{10}	6,404	6.4×10^{10}
This work (tensor hypercontraction) (44) (46)	2,142	5.3×10^{9}	2,196	3.2×10^{10}

source: Even More Efficient Quantum Computations of Chemistry Through Tensor Hypercontraction by Joonho Lee, Craig Gidney et al, July 2021 (62 pages). https://journals.aps.org/prxquantum/pdf/10.1103/PRXQuantum.2.030305

LNO battery simulation

Google

simulation of $LiNiO_2$ batteries

from 75K to 3M logical qubits and 91M to 6G physical qubits

computing time from one year to 2,739 years

System	LCU	k-mesh	λ	Num. Spin-Orbs.	Toffolis	Logical Qubits	Physical Qubits [M]	run time [days]
$R\bar{3}m$	Sparse	$[2,2,2]$	120382.037	116	6.16×10^{13}	166946	242.72	1.51×10^{4}
		$[3,3,3]$	718377.133	116	3.57×10^{15}	1625295	2808.82	9.82×10^{5}
	SF	$[2,2,2]$	183778.821	116	7.86×10^{13}	89162	129.77	1.93×10^{4}
		$[3,3,3]$	2966279.293	116	4.60×10^{15}	404723	699.68	1.27×10^{6}
	DF	$[2,2,2]$	10730.422	116	4.97×10^{12}	149939	180.16	1.08×10^{3}
		$[3,3,3]$	44794.803	116	7.28×10^{13}	598286	869.02	1.79×10^{4}
C2/m	Sparse	$[2,2,1]$	58422.522	116	1.03×10^{13}	83532	100.47	2.53×10^{3}
		$[4,4,2]$	893339.394	116	5.37×10^{15}	3051285	5272.93	1.48×10^{6}
	SF	$[2,2,1]$	95803.204	116	2.05×10^{13}	44657	53.90	5.05×10^{3}
		$[4,4,2]$	2899609.300	116	5.23×10^{15}	405310	700.69	1.44×10^{6}
	DF	$[2,2,1]$	4873.648	116	1.18×10^{12}	75178	90.44	2.56×10^{2}
		$[4,4,2]$	51416.281	116	9.82×10^{13}	598736	869.68	2.41×10^{4}
P2/c	Sparse	$[1,1,1]$	84977.359	464	2.06×10^{13}	99918	120.21	5.07×10^{3}
		$[2,2,2]$	1627121.892	464	1.67×10^{16}	3182362	6454.14	4.59×10^{6}
	SF	$[1,1,1]$	201894.726	464	8.74×10^{13}	92786	135.04	2.15×10^{4}
		$[2,2,2]$	5666363.179	464	2.07×10^{16}	839487	1450.95	5.68×10^{6}
	DF	$[1,1,1]$	2753.901	464	9.72×10^{11}	75834	91.23	2.11×10^{2}
		$[2,2,2]$	40788.113	464	1.40×10^{14}	1192900	1732.40	3.44×10^{4}
$P2_1/c$	Sparse	$[1,2,1]$	105584.297	232	3.39×10^{13}	182864	265.83	8.34×10^{3}
		$[2,4,2]$	1714723.913	232	1.50×10^{16}	3116825	6321.24	4.12×10^{6}
	SF	$[1,2,1]$	271178.934	232	8.92×10^{13}	96882	140.98	2.19×10^{4}
		$[2,4,2]$	7798992.981	232	2.13×10^{16}	438080	757.32	5.85×10^{6}
	DF	$[1,2,1]$	3958.111	232	1.27×10^{12}	75383	90.69	2.76×10^{2}
		$[2,4,2]$	46189.645	232	1.23×10^{14}	1192758	1732.20	3.02×10^{4}

TABLE VI. Quantum Resource estimates for all four LNO structures normalized by the number of formula units represented in each simulation cell. $R\bar{3}m$ and C2/m are both one formula unit while P2/c is four formula units and $P2_1/c$ is two formula units. The sparse threshold is selected to be 1.0×10^{-4}, the SF the auxiliary index is truncated at eight times the number of molecular orbitals, and the DF the second factorization is truncated at 1.0×10^{-4}.

source: Fault-tolerant quantum simulation of materials using Bloch orbitals
Nicholas C. Rubin et al, February 2023 (58 pages).

discussion